# Riesel problems

## Definition

For the original Riesel problem, it is finding and proving the smallest $k$ such that $k{\times}b^n$-1 is not prime for all integers $n \geq 1$ and GCD($k$-1, $b$-1)=1.

## Extended definiton

Finding and proving the smallest $k$ such that ($k{\times}b^n$-1)/GCD($k$-1, $b$-1) is not prime for all integers $n \geq 1$.

## Notes

All $n$ must be >= 1.

$k$-values that make a full covering set with all or partial algebraic factors are excluded from the conjectures.

$k$-values that are a multiple of base ($b$) and where ($k$-1)/gcd($k$-1,$b$-1) is not prime are included in the conjectures but excluded from testing.

Such $k$-values will have the same prime as $k$ / $b$.

## Table

| Base | Conjectured smallest Riesel $k$ | Covering set | $k$'s that make a full covering set with all or partial algebraic factors | Remaining $k$ to find prime ($n$ testing limit) | Top 10 $k$'s with largest first primes: $k$ ($n$) (sorted by $n$ only) | Comments |
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| 2 | 509203 | 3, 5, 7, 13, 17, 241 | | 2293, 23669, 31859, 38473, 46663, 67117, 74699, 81041, 93839, 97139, 107347, 121889, 129007, 143047, 161669, 192971, 206039, 206231, 215443, 226153, 234343, 245561, 250027, 315929, 319511, 324011, 325123, 327671, 336839, 342847, 344759, 351134, 362609, 363343, 364903, 365159, 368411, 371893, 384539, 386801, 397027, 409753, 444637, 470173, 474491, 477583, 478214, 485557, 494743 (k = 351134 and 478214 at n=6.65M, other k at n=11.4M) | 9221 (11392194) 146561 (11280802) 273809 (8932416) 502573 (7181987) 402539 (7173024) 40597 (6808509) 304207 (6643565) 398023 (6418059) 252191 (5497878) 353159 (4331116) | |

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|---|---|---|---|---|---|---|
| **3** | 12119 | 2, 5, 7, 13, 73 | | 1613, 1831, 1937, 3131, 3589, 5755, 6787, 7477, 7627, 7939, 8713, 8777, 9811, 10651, 11597 (all at n=50K) | 8059 (47256) 11753 (36665) 6119 (28580) 7511 (26022) 313 (24761) 11251 (24314) 9179 (21404) 997 (20847) 6737 (17455) 7379 (16856) | |
| **4** | 361 | 3, 5, 7, 13 | All k = m^2 for all n; factors to: (m*2^n - 1) * (m*2^n + 1) | none - proven | 106 (4553) 74 (1276) 219 (206) 191 (113) 312 (51) 247 (42) 223 (33) 274 (22) 234 (18) 91 (17) | k = 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, 256, 289, and 324 proven composite by full algebraic factors. |
| **5** | 13 | 2, 3 | | none - proven | 2 (4) 1 (3) 11 (2) 8 (2) 12 (1) 9 (1) 7 (1) 6 (1) 4 (1) 3 (1) | |

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| 6 | 84687 | 7, 13, 31, 37, 97 | | 1597, 6236, 9491, 37031, 49771, 50686, 53941, 55061, 57926, 76761, 79801, 83411 (k = 1597 at n=5.4M, other k at n=40K) | 36772 (1723287) 43994 (569498) 77743 (560745) 51017 (528803) 57023 (483561) 78959 (458114) 59095 (171929) 48950 (143236), 29847 (141526) 9577 (121099) | |
| 7 | 457 | 2, 3, 5, 13, 19 | | none - proven (with probable primes that have not been certified: k = 197 and 367) | 197 (181761) 367 (15118) 313 (5907) 159 (4896) 429 (3815) 419 (1052) 391 (938) 299 (600) 139 (468) 79 (424) | |
| 8 | 14 | 3, 5, 13 | All k = $m^3$ for all n; factors to: $(m*2^n - 1) * (m^2*4^n + m*2^n + 1)$ | none - proven | 11 (18) 5 (4) 12 (3) 7 (3) 2 (2) 13 (1) 10 (1) 9 (1) 6 (1) 4 (1) | k = 1 and 8 proven composite by full algebraic factors. |

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| **9** | 41 | 2, 5 | All k = m^2 for all n; factors to: (m*3^n - 1) * (m*3^n + 1) | none - proven | 11 (11)<br>24 (8)<br>14 (8)<br>38 (3)<br>18 (3)<br>39 (2)<br>34 (2)<br>32 (2)<br>29 (2)<br>27 (2) | k = 1, 4, 9, 16, 25, and 36 proven composite by full algebraic factors. |
| **10** | 334 | 3, 7, 13, 37 | | none - proven | 121 (483)<br>109 (136)<br>98 (90)<br>230 (60)<br>289 (35)<br>89 (33)<br>32 (28)<br>233 (18)<br>324 (17)<br>100 (17) | |
| **11** | 5 | 2, 3 | | none - proven | 1 (17)<br>3 (2)<br>2 (2)<br>4 (1) | |

| 12 | 376 | 5, 13, 29 | (Condition 1): All k where k = m^2 and m = = 5 or 8 mod 13: for even n let k = m^2 and let n = 2*q; factors to: (m*12^q - 1) * (m*12^q + 1) odd n: factor of 13 (Condition 2): All k where k = 3*m^2 and m = = 3 or 10 mod 13: even n: factor of 13 for odd n let k = 3*m^2 and let n=2*q-1; factors to: [m*2^(2q-1)*3^q - 1] * [m*2^(2q-1)*3^q + 1] | none - proven | 298 (1676) 157 (285) 46 (194) 304 (40) 259 (40) 94 (36) 292 (30) 147 (28) 301 (27) 349 (25) | k = 25, 64, and 324 proven composite by condition 1. k = 27 and 300 proven composite by condition 2. |
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| 13 | 29 | 2, 7 | | none - proven | 25 (15) 28 (14) 20 (10) 1 (5) 22 (3) 17 (3) 16 (3) 27 (2) 21 (2) 12 (2) | |
| 14 | 4 | 3, 5 | | none - proven | 2 (4) 1 (3) 3 (1) | |

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| **15** | 622403 | 2, 17, 113, 1489 | | 47, 203, 239, 407, 437, 451, 889, 893, 1945, 2049, 2245, 2487, 2507, 2689, 2699, 2863, 2940, 3059, 3163, 3179, 3261, 3409, 3697, 3701, 3725, 4173, 4249, 4609, 4771, 4877, 5041, 5243, 5425, 5441, 5503, 5669, 5857, 5913, 5963, 6231, 6447, 6787, 6879, 6999, 7386, 7407, 7459, 7473, 7527, 7615, 7683, 7687, 7859, 8099, 8610, 8621, 8671, 8839, 8863, 9025, 9267, 9409, 9655, 9663, 9707, 9817, 9955 (for k <= 10K) (all at n=1.5K) | 2940 (13254) 8610 (5178) 2069 (1461) 3917 (1427) 1145 (1349) 1583 (1330) 7027 (1316) 8831 (1296) 5305 (1273) 4865 (1265) | |
| **16** | 100 | 3, 7, 13 | All k = m^2 for all n; factors to: (m*4^n - 1) * (m*4^n + 1) | none - proven | 74 (638) 78 (26) 48 (15) 58 (12) 31 (12) 95 (8) 46 (8) 88 (6) 44 (6) 39 (6) | k = 1, 4, 9, 16, 25, 36, 49, 64, and 81 proven composite by full algebraic factors. |

| 17 | 49 | 2, 3 | | none - proven | 44 (6488)<br>29 (4904)<br>13 (1123)<br>36 (243)<br>10 (117)<br>26 (110)<br>5 (60)<br>11 (46)<br>46 (25)<br>35 (24) | |
|----|-----|------------|--------------------------------------------------------------------------------------------------------------------------------------------------------|---------------|-------------------------------------------------------------------------------------------------------|-------------------------------------------------------------|
| 18 | 246 | 5, 13, 19 | | none - proven | 151 (418)<br>78 (172)<br>50 (110)<br>79 (63)<br>237 (44)<br>184 (44)<br>75 (44)<br>215 (36)<br>203 (32)<br>93 (32) | |
| 19 | 9 | 2, 5 | All k where k = m^2 and m = = 2 or 3 mod 5: for even n let k = m^2 and let n = 2*q; factors to: (m*19^q - 1) * (m*19^q + 1) odd n: factor of 5 | none - proven | 1 (19)<br>7 (2)<br>3 (2)<br>8 (1)<br>6 (1)<br>5 (1)<br>2 (1) | k = 4 proven composite by partial algebraic factors. |
| 20 | 8 | 3, 7 | | none - proven | 2 (10)<br>1 (3)<br>6 (2)<br>5 (2)<br>7 (1)<br>4 (1)<br>3 (1) | |

| 21 | 45 | 2, 11 | | none - proven | 29 (98)<br>34 (17)<br>43 (10)<br>32 (4)<br>5 (4)<br>6 (3)<br>1 (3)<br>44 (2)<br>37 (2)<br>31 (2) | |
|---|---|---|---|---|---|---|
| 22 | 2738 | 5, 23, 97 | | 208, 211, 898, 976, 1036, 1885, 1933, 2050, 2161, 2278, 2347, 2434 (all at n=13K) | 1013 (26067)<br>185 (11433)<br>1335 (11155)<br>2719 (9671)<br>2083 (8046)<br>883 (5339)<br>2529 (3700)<br>2116 (3371)<br>2230 (3236)<br>1119 (2849) | |
| 23 | 5 | 2, 3 | | none - proven | 3 (6)<br>2 (6)<br>4 (5)<br>1 (5) | |

| 24 | 32336 | 5, 7, 13, 73, 577 | (Condition 1): All k where k = m^2 and m = = 2 or 3 mod 5: for even n let k = m^2 and let n = 2*q; factors to: (m*24^q - 1) * (m*24^q + 1) odd n: factor of 5 (Condition 2): All k where k = 6*m^2 and m = = 1 or 4 mod 5: even n: factor of 5 for odd n let k = 6*m^2 and let n=2*q-1; factors to: [m*2^(3q-1)*3^q - 1] * [m*2^(3q-1)*3^q + 1] | 389, 461, 1581, 1711, 2094, 2606, 3006, 3754, 4239, 5356, 5784, 5791, 6116, 6579, 6781, 6831, 7321, 7809, 10219, 10399, 10666, 11101, 11516, 12326, 12429, 12674, 13269, 13691, 15019, 15151, 15614, 15641, 16124, 16234, 16616, 17019, 17436, 18054, 18454, 18964, 19116, 20026, 20576, 20611, 20879, 21004, 21464, 21524, 21639, 21809, 23549, 24404, 25046, 25136, 25349, 25389, 25419, 25646, 25731, 26176, 26229, 26661, 27049, 27154, 28001, 28384, 28849, 28859, 29211, 29531, 29569, 29581, 31071, 31466, 31734, 31854, 31994, 31996, 32099 (k = 1 mod 23 at n=12.4K, other k at n=260K) | 10171 (259815) 11906 (252629) 23059 (252514) 21411 (252303) 28554 (239686) 20804 (233296) 8894 (210624) 2844 (203856) 25379 (175842) 22604 (169372) | k = 2^2, 3^2, 7^2, 8^2, 12^2, 13^2, 17^2, 18^2 (etc. pattern repeating every 5m) proven composite by condition 1. k = 6*1^2, 6*4^2, 6*6^2, 6*9^2, 6*11^2, 6*14^2, 6*16^2, 6*19^2 (etc. pattern repeating every 5m) proven composite by condition 2. |

| 25 | 105 | 2, 13 | All k = m^2 for all n; factors to: (m*5^n - 1) * (m*5^n + 1) | none - proven | 86 (1029) 58 (26) 72 (24) 67 (24) 79 (21) 37 (17) 38 (14) 92 (13) 57 (10) 98 (9) | k = 1, 4, 9, 16, 25, 36, 49, 64, 81, and 100 proven composite by full algebraic factors. |
|----|-----|-------|------|------|------|------|
| 26 | 149 | 3, 7, 31, 37 | | none - proven | 115 (520277) 32 (9812) 121 (1509) 73 (537) 80 (382) 128 (300) 124 (249) 37 (233) 25 (133) 65 (100) | |
| 27 | 13 | 2, 7 | All k = m^3 for all n; factors to: (m*3^n - 1) * (m^2*9^n + m*3^n + 1) | none - proven | 9 (23) 11 (10) 12 (2) 7 (2) 6 (2) 3 (2) 10 (1) 5 (1) 4 (1) 2 (1) | k = 1 and 8 proven composite by full algebraic factors. |

| 28 | 3769 | 5, 29, 157 | (Condition 1): All k where k = m^2 and m = = 12 or 17 mod 29: for even n let k = m^2 and let n = 2*q; factors to: (m*28^q - 1) * (m*28^q + 1) odd n: factor of 29 (Condition 2): All k where k = 7*m^2 and m = = 5 or 24 mod 29: even n: factor of 29 for odd n let k = 7*m^2 and let n=2*q-1; factors to: [m*2^(2q-1)*7^q - 1] * [m*2^(2q-1)*7^q + 1] | 233, 376, 943, 1132, 1422, 2437 (k = 233 and 1422 at n=1M, other k at n=20.3K) | 2319 (65184) 3232 (9147) 3019 (7073) 460 (5400) 1688 (4760) 2406 (4634) 2464 (4324) 849 (3129) 1507 (2938) 472 (2414) | k = 144, 289, 1681, and 2116 proven composite by condition 1. k = 175 proven composite by condition 2. |
| 29 | 4 | 3, 5 | | none - proven | 2 (136) 1 (5) 3 (1) | |

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|---|---|---|---|---|---|---|
| **30** | 4928 | 13, 19, 31, 67 | k = 1369: for even n let n=2*q; factors to: (37*30^q - 1) * (37*30^q + 1) odd n: covering set 7, 13, 19 | 659, 1024, 1580, 1936, 2293, 2916, 3719, 4372, 4897 (all at n=500K) | 1642 (346592) 239 (337990) 2538 (262614) 249 (199355) 3256 (160619) 225 (158755) 774 (148344) 1873 (50427) 3253 (43291) 1654 (38869) | |
| **31** | 145 | 2, 3, 7, 19 | | 5, 19, 51, 73, 97 (all at n=6K) | 123 (1872) 124 (1116) 113 (643) 49 (637) 115 (464) 21 (275) 39 (250) 70 (149) 142 (140) 33 (107) | |
| **32** | 10 | 3, 11 | All k = m^5 for all n; factors to: (m*2^n - 1) * (m^4*16^n + m^3*8^n + m^2*4^n + m*2^n + 1) | none - proven | 3 (11) 2 (6) 9 (3) 8 (2) 5 (2) 7 (1) 6 (1) 4 (1) | k = 1 proven composite by full algebraic factors. |

| 33 | 545 | 2, 17 | (Condition 1): All k where k = m^2 and m = = 4 or 13 mod 17: for even n let k = m^2 and let n = 2*q; factors to: (m*33^q - 1) * (m*33^q + 1) odd n: factor of 17 (Condition 2): All k where k = 33*m^2 and m = = 4 or 13 mod 17: [Reverse condition 1] (Condition 3): All k where k = m^2 and m = = 15 or 17 mod 32: for even n let k = m^2 and let n = 2*q; factors to: (m*33^q - 1) * (m*33^q + 1) odd n: factor of 2 | 257, 339 (both at n=12K) | 186 (16770) 254 (3112) 142 (2568) 370 (1628) 272 (1418) 222 (919) 108 (360) 213 (233) 387 (191) 277 (187) | k = 16, 169, and 441 proven composite by condition 1. k = 528 proven composite by condition 2. k = 225 and 289 proven composite by condition 3. |

| 34 | 6 | 5, 7 | All k where k = m^2 and m = = 2 or 3 mod 5: for even n let k = m^2 and let n = 2*q; factors to: (m*34^q - 1) * (m*34^q + 1) odd n: factor of 5 | none - proven | 1 (13) 5 (2) 3 (1) 2 (1) | k = 4 proven composite by partial algebraic factors. |
| --- | --- | --- | --- | --- | --- | --- |
| 35 | 5 | 2, 3 | | none - proven | 1 (313) 3 (6) 2 (6) 4 (1) | |

| 36 | 33791 | 13, 31, 43, 97 | All k = m^2 for all n; factors to: (m*6^n - 1) * (m*6^n + 1) | 1148, 1555, 2110, 2133, 3699, 4551, 4737, 6236, 6883, 7253, 7362, 7399, 7991, 8250, 8361, 8363, 8472, 9491, 9582, 11014, 12320, 12653, 13641, 14358, 14540, 14836, 14973, 14974, 15228, 15687, 15756, 15909, 16168, 17354, 17502, 17946, 18203, 19035, 19646, 20092, 20186, 20630, 21880, 22164, 22312, 23213, 23901, 23906, 24236, 24382, 24645, 24731, 24887, 25011, 25159, 25161, 25204, 25679, 25788, 26160, 26355, 27161, 29453, 29847, 30970, 31005, 31634, 32302, 33047, 33627 (all at n=10K) | 13800 (9790) 20485 (9140) 19389 (9119) 20684 (8627) 19907 (8439) 11216 (7524) 28416 (7315) 32380 (7190) 27296 (7115) 10695 (6672) | k = 1^2, 2^2, 3^2, 4^2, 5^2, 6^2, 7^2, 8^2, 9^2, 10^2, 11^2, 12^2, 13^2, 14^2, 15^2, 16^2, etc. proven composite by full algebraic factors. |
| --- | --- | --- | --- | --- | --- | --- |
| 37 | 29 | 2, 5, 7, 13, 67 | | none - proven | 5 (900) 19 (63) 18 (14) 1 (13) 8 (4) 25 (3) 23 (3) 14 (3) 6 (3) 4 (3) | |

| 38 | 13 | 3, 5, 17 | | none - proven | 11 (766)<br>9 (43)<br>7 (7)<br>1 (3)<br>12 (2)<br>8 (2)<br>5 (2)<br>2 (2)<br>10 (1)<br>6 (1) | |
|---|---|---|---|---|---|---|
| 39 | 9 | 2, 5 | All k where k = m^2 and m = = 2 or 3 mod 5: for even n let k = m^2 and let n = 2*q; factors to:<br>(m*39^q - 1) *<br>(m*39^q + 1)<br>odd n:<br>factor of 5 | none - proven | 1 (349)<br>7 (2)<br>3 (2)<br>2 (2)<br>8 (1)<br>6 (1)<br>5 (1) | k = 4 proven composite by partial algebraic factors. |

| 40 | 25462 | 3, 7, 41, 223 | (Condition 1): All k where k = m^2 and m = = 9 or 32 mod 41: for even n let k = m^2 and let n = 2*q; factors to: (m*40^q - 1) * (m*40^q + 1) odd n: factor of 41 (Condition 2): All k where k = 10*m^2 and m = = 18 or 23 mod 41: even n: factor of 41 for odd n let k = 10*m^2 and let n=2*q-1; factors to: [m*2^(3q-1)*5^q - 1] * [m*2^(3q-1)*5^q + 1] | 157, 490, 520, 534, 618, 709, 739, 787, 862, 955, 1067, 1114, 1174, 1242, 1352, 1544, 1559, 1762, 1795, 1805, 2254, 2290, 2830, 2887, 3033, 3034, 3156, 3342, 3361, 3418, 3650, 3750, 3830, 3859, 3922, 4006, 4132, 4183, 4219, 4297, 4582, 4673, 4724, 4771, 5218, 5233, 5308, 5431, 5629, 6107, 6192, 6220, 6436, 6463, 6582, 6618, 6682, 6684, 6709, 6946, 7089, 7094, 7126, 7258, 7282, 7381, 7504, 7602, 7678, 7702, 7795, 8032, 8035, 8173, 8461, 8572, 8899, 8959, 9121, 9226, 9347, 9424, 9472, 9511, 9716, 9748, 9874, 9964, 10003, 10060, 10285, 10615, 10744, 11030, 11470, 11479, 11560, 11847, 11971, 12178, 12193, | 23977 (982) 13072 (982) 20952 (963) 8749 (962) 18103 (957) 22759 (939) 220 (939) 23795 (935) 8113 (918) 13654 (905) | k = 81, 1024, 2500, 5329, 8281, 12996, 17424, and 24025 proven composite by condition 1. k = 3240 and 5290 proven composite by condition 2. |

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|---|---|---|---|---|---|---|
| | | | | 12226, 12250, 12256, 12299, 12301, 12422, 12505, 12544, 12547, 12568, 12709, 12742, 12750, 12873, 13005, 13022, 13039, 13165, 13191, 13212, 13624, 13666, 13777, 13894, 13939, 14146, 14262, 14272, 14362, 14494, 14513, 14636, 14766, 14802, 14980, 15046, 15154, 15271, 15374, 15376, 15388, 15417, 15496, 15579, 15661, 15730, 15907, 15967, 16108, 16235, 16579, 16705, 16728, 16891, 16897, 16932, 17014, 17137, 17275, 17287, 17344, 17536, 17653, 17707, 17801, 17860, 17896, 17923, 17998, 18114, 18292, 18397, 18697, 18787, 18818, 18853, 18949, 19117, 19310, 19510, 19606, 19722, 19751, 19756, 19761, 19780, 19825, 19927, 20158, 20212, 20253, 20428, 20458, 20479, 20491, 20583, 20632, 20747, | | |

| | | | | 20788, 20809, 21058, 21082, 21276, 21321, 21403, 21493, 21731, 21817, 21895, 21975, 22114, 22130, 22262, 22263, 22303, 22344, 22570, 22706, 22879, 23371, 23615, 23851, 24184, 24189, 24268, 24337, 24397, 24421, 24448, 24483, 24519, 24805, 24979 (all at n=1K) | | |
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| 41 | 8 | 3, 7 | | none - proven | 7 (153)<br>5 (10)<br>1 (3)<br>6 (2)<br>2 (2)<br>4 (1)<br>3 (1) | |
|----|----|----|----|----|----|----|
| 42 | 15137 | 5, 43, 353 | | 603, 1049, 1600, 2538, 4299, 4903, 5118, 5978, 6836, 6964, 6971, 7309, 8297, 8341, 9029, 9201, 9633, 9848, 11267, 11781, 11911, 11996, 12125, 12127, 12213, 12598, 13288, 13347, 14884 (k = 1600, 6971 and 14884 at n=8K, other k at n=200K) | 7051 (188034)<br>5417 (179220)<br>13898 (152983)<br>1633 (128734)<br>13757 (126934)<br>7913 (108747)<br>15024 (104613)<br>8453 (89184)<br>7658 (79316)<br>10923 (61071) | |
| 43 | 21 | 2, 11 | | 13 (50K) | 4 (279)<br>12 (203)<br>17 (79)<br>3 (24)<br>1 (5)<br>19 (4)<br>15 (4)<br>7 (4)<br>11 (2)<br>10 (2) | |
| 44 | 4 | 3, 5 | | none - proven | 1 (5)<br>2 (4)<br>3 (1) | |

| 45 | 93 | 2, 23 | | none - proven | 24 (153355)<br>53 (582)<br>70 (167)<br>29 (146)<br>76 (102)<br>85 (82)<br>91 (50)<br>77 (26)<br>1 (19)<br>33 (11) | |
|---|---|---|---|---|---|---|
| 46 | 928 | 3, 7, 103 | | 281, 436, 800<br>(k = 800 at<br>n=500K, other<br>k at n=28K) | 870 (51699)<br>86 (26325)<br>93 (24162)<br>561 (5011)<br>576 (3659)<br>100 (2955)<br>386 (2425)<br>338 (1478)<br>597 (950)<br>121 (935) | |
| 47 | 5 | 2, 3 | | none - proven | 4 (1555)<br>1 (127)<br>2 (4)<br>3 (2) | |
| 48 | 3226 | 5, 7, 461 | | 313, 384, 708,<br>909, 916, 1093,<br>1457, 1686,<br>1877, 1896,<br>1898, 2071,<br>2148, 2172,<br>2402, 2589,<br>2682, 2927,<br>2939, 3044,<br>3067 (all at<br>n=200K) | 2157<br>(169491)<br>2549<br>(169453)<br>1478<br>(167541)<br>2822<br>(129611)<br>2379<br>(116204)<br>118<br>(107422)<br>692<br>(103056)<br>1842<br>(87175)<br>953 (81493)<br>2582<br>(75696) | |

| 49 | 81 | 2, 5 | All k = m^2 for all n; factors to: (m*7^n - 1) * (m*7^n + 1) | none - proven | 79 (212)<br>44 (122)<br>69 (42)<br>30 (24)<br>59 (16)<br>53 (15)<br>70 (14)<br>24 (14)<br>31 (9)<br>74 (6) | k = 1, 4, 9, 16, 25, 36, 49, and 64 proven composite by full algebraic factors. |
| --- | --- | --- | --- | --- | --- | --- |
| 50 | 16 | 3, 17 | | none - proven | 14 (66)<br>13 (19)<br>5 (12)<br>11 (6)<br>6 (6)<br>1 (3)<br>8 (2)<br>2 (2)<br>15 (1)<br>12 (1) | |
| 51 | 25 | 2, 13 | | none - proven | 1 (4229)<br>23 (96)<br>3 (8)<br>12 (4)<br>14 (3)<br>4 (3)<br>22 (2)<br>19 (2)<br>18 (2)<br>15 (2) | |
| 52 | 25015 | 3, 7, 53, 379 | (Condition 1): All k where k = m^2 and m = = 23 or 30 mod 53: for even n let k = m^2 and let n = 2*q; factors to: | 82, 139, 233, 239, 349, 363, 372, 472, 476, 478, 547, 557, 607, 613, 654, 657, 796, 813, 902, 931, 991, 1012, 1069, 1104, 1161, 1167, 1231, 1234, 1271, | 13298 (1000)<br>19006 (994)<br>10592 (993)<br>427 (992)<br>10687 (989)<br>14621 (982)<br>20044 (980)<br>8959 (980)<br>19084 (977) | k = 529, 900, 5776, 6889, 16641, and 18496 proven composite by condition 1.<br>k = 637 |

| 53 | 13 | 2, 3 | | none - proven | 12 (71) 10 (71) 2 (44) 7 (11) 1 (11) 8 (8) 11 (6) 9 (3) 5 (2) 6 (1) | |
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| **54** | 21 | 5, 11 | (Condition 1): All k where k = m^2 and m = = 2 or 3 mod 5: for even n let k = m^2 and let n = 2*q; factors to: (m*54^q - 1) * (m*54^q + 1) odd n: factor of 5 (Condition 2): All k where k = 6*m^2 and m = = 1 or 4 mod 5: even n: factor of 5 for odd n let k = 6*m^2 and let n=2*q-1; factors to: [m*2^q*3^(3q-1) - 1] * [m*2^q*3^(3q-1) + 1] | none - proven | 20 (8) 19 (6) 10 (4) 17 (3) 1 (3) 14 (2) 7 (2) 3 (2) 18 (1) 16 (1) | k = 4 and 9 proven composite by condition 1. k = 6 proven composite by condition 2. |

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| **55** | 13 | 2, 7 | | none - proven | 3 (76)<br>1 (17)<br>11 (8)<br>9 (3)<br>7 (2)<br>6 (2)<br>12 (1)<br>10 (1)<br>8 (1)<br>5 (1) | |
| **56** | 20 | 3, 19 | | none - proven | 14 (26)<br>10 (23)<br>1 (7)<br>18 (4)<br>17 (4)<br>7 (3)<br>11 (2)<br>8 (2)<br>5 (2)<br>2 (2) | |
| **57** | 144 | 5, 13, 29 | All k where k = m^2 and m = = 3 or 5 mod 8: for even n let k = m^2 and let n = 2*q; factors to: (m*57^q - 1) * (m*57^q + 1) odd n: factor of 2 | none - proven | 87 (242)<br>54 (157)<br>100 (109)<br>59 (83)<br>115 (34)<br>124 (31)<br>88 (27)<br>63 (22)<br>139 (20)<br>38 (20) | k = 9, 25, and 121 proven composite by partial algebraic factors. |
| **58** | 547 | 3, 7, 163 | | 71, 130, 169, 178, 319, 456, 493, 499 (k = 71 and 456 at n=100K, other k at n=14K) | 382 (7188)<br>400 (5245)<br>421 (4526)<br>176 (2854)<br>473 (1641)<br>487 (1412)<br>312 (1079)<br>334 (724)<br>53 (645)<br>457 (492) | |

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| **59** | 4 | 3, 5 | | none - proven | 3 (8)<br>1 (3)<br>2 (2) | |
| **60** | 20558 | 13, 61, 277 | (Condition 1):<br>All k where k = m^2<br>and m = = 11 or 50 mod 61:<br>for even n let k = m^2<br>and let n = 2*q; factors to:<br>(m*60^q - 1) *<br>(m*60^q + 1)<br>odd n:<br>factor of 61<br>(Condition 2):<br>All k where k = 15*m^2<br>and m = = 22 or 39 mod 61:<br>even n:<br>factor of 61<br>for odd n let k = 15*m^2<br>and let n=2*q-1;<br>factors to:<br>[m*2^(2q-1)*15^q - 1] *<br>[m*2^(2q-1)*15^q + 1] | 36, 1770, 4708, 5317, 5611, 6101, 6162, 6274, 7060, 7870, 8722, 9212, 9454, 9881, 10249, 11101, 12061, 12072, 12098, 12479, 12996, 13297, 13480, 14275, 14851, 15800, 16167, 17185, 17620, 18055, 18965, 18972, 19336, 19394, 19397 (k = 16167 and 18055 at n=8K, other k at n=100K) | 1024 (90701)<br>12121 (84208)<br>15227 (80625)<br>15185 (79350)<br>8649 (79159)<br>20131 (71977)<br>19457 (68854)<br>16333 (61172)<br>18776 (60164)<br>1486 (58932) | k = 121, 2500, 5184, 14641, and 17689 proven composite by condition 1.<br>k = 7260 proven composite by condition 2. |
| **61** | 125 | 2, 31 | | 37, 53, 100 (all at n=10K) | 13 (4134)<br>77 (3080)<br>10 (1552)<br>41 (755)<br>42 (174)<br>22 (117)<br>57 (89)<br>109 (86)<br>103 (78)<br>93 (60) | |

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| **62** | 8 | 3, 7 | | none - proven | 3 (59)<br>4 (9)<br>1 (3)<br>6 (2)<br>5 (2)<br>2 (2)<br>7 (1) | |
| **63** | 857 | 2, 5, 397 | | 37, 65, 93, 129, 139, 177, 211, 231, 237, 251, 271, 281, 291, 333, 417, 457, 471, 473, 491, 493, 497, 513, 587, 599, 633, 669, 677, 679, 687, 691, 695, 717, 733, 771, 817, 819, 821, 831, 853 (all at n=1K) | 64 (1483)<br>372 (1320)<br>839 (940)<br>495 (916)<br>183 (904)<br>97 (851)<br>39 (848)<br>277 (835)<br>775 (710)<br>411 (678) | |
| **64** | 14 | 5, 13 | All k = m^2 for all n; factors to: (m*8^n - 1) * (m*8^n + 1) -or- All k = m^3 for all n; factors to: (m*4^n - 1) * (m^2*16^n + m*4^n + 1) | none - proven | 11 (9)<br>12 (6)<br>5 (2)<br>13 (1)<br>10 (1)<br>7 (1)<br>6 (1)<br>3 (1)<br>2 (1) | k = 1, 4, 8, and 9 proven composite by full algebraic factors. |
| **65** | 10 | 3, 11 | | none - proven | 1 (19)<br>8 (10)<br>4 (9)<br>2 (4)<br>5 (2)<br>9 (1)<br>7 (1)<br>6 (1)<br>3 (1) | |

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| **66** | 63717671 | 7, 67, 613, 4423 | | 681, 1056, 1205, 1575, 1669, 1944, 2182, 2916, 2949, 3014, 3083, 3148, 3221, 3526, 3684, 3911, 3946, 4423, 5329, 5361, 5897, 5898, 5959, 5972, 6096, 6189, 6263, 6451, 6768, 6796, 7168, 7237, 7357, 7572, 7614, 7927, 8156, 8173, 8348, 8432, 8510, 8825, 8866, 9017, 9111, 9406, 9409, 9781, 9801, 9906, 9998 (for k <= 10K) (all at n=1K) | 7578 (988) 1252 (956) 2746 (918) 5248 (916) 5476 (873) 5929 (795) 6699 (790) 8843 (780) 5435 (762) 2946 (748) | |
| **67** | 33 | 2, 17 | All k where k = m^2 and m = = 4 or 13 mod 17: for even n let k = m^2 and let n = 2*q; factors to: (m*67^q - 1) * (m*67^q + 1) odd n: factor of 17 | none - proven | 25 (2829) 2 (768) 23 (42) 21 (27) 1 (19) 31 (10) 19 (8) 18 (7) 13 (7) 11 (6) | k = 16 proven composite by partial algebraic factors. |

| 68 | 22 | 3, 23 | | none - proven | 7 (25395) 5 (13574) 11 (198) 8 (62) 10 (53) 3 (10) 1 (5) 14 (4) 2 (4) 9 (3) | |
|---|---|---|---|---|---|---|
| 69 | 6 | 3, 5 | All k where k = m^2 and m = = 2 or 3 mod 5: for even n let k = m^2 and let n = 2*q; factors to: (m*69^q - 1) * (m*69^q + 1) odd n: factor of 5 | none - proven | 5 (4) 1 (3) 3 (1) 2 (1) | k = 4 proven composite by partial algebraic factors. |
| 70 | 853 | 13, 29, 71 | | 811 (50K) | 729 (28625) 376 (6484) 496 (4934) 434 (3820) 489 (2096) 278 (1320) 550 (764) 31 (545) 174 (441) 778 (356) | |
| 71 | 5 | 2, 3 | | none - proven | 2 (52) 1 (3) 3 (2) 4 (1) | |

| 72 | 293 | 5, 17, 73 | | none - proven | 4 (1119849)<br>79 (28009)<br>291 (26322)<br>116 (13887)<br>118 (4599)<br>67 (4308)<br>197 (3256)<br>24 (2648)<br>11 (2445)<br>18 (1494) | |
|---|---|---|---|---|---|---|
| 73 | 112 | 5, 13, 37 | (Condition 1):<br>All k where k = m^2<br>and m = = 6 or 31 mod 37:<br>for even n let k = m^2<br>and let n = 2*q; factors to:<br>(m*73^q - 1) *<br>(m*73^q + 1)<br>odd n:<br>factor of 37<br>(Condition 2):<br>All k where k = m^2<br>and m = = 3 or 5 mod 8:<br>for even n let k = m^2<br>and let n = 2*q; factors to:<br>(m*73^q - 1) *<br>(m*73^q + 1)<br>odd n:<br>factor of 2 | none - proven (with probable primes that have not been certified: k = 79) | 79 (9339)<br>101 (2146)<br>105 (102)<br>48 (73)<br>54 (63)<br>42 (50)<br>26 (50)<br>97 (47)<br>61 (39)<br>89 (32) | k = 36 proven composite by condition 1.<br>k = 9 and 25 proven composite by condition 2. |
| 74 | 4 | 3, 5 | | none - proven | 2 (132)<br>1 (5)<br>3 (2) | |

| 75 | 37 | 2, 19 | | none - proven | 35 (1844)<br>16 (119)<br>18 (54)<br>30 (41)<br>3 (16)<br>22 (15)<br>5 (9)<br>17 (5)<br>4 (5)<br>23 (4) | |
| --- | --- | --- | --- | --- | --- | --- |
| 76 | 34 | 7, 11 | | none - proven | 1 (41)<br>27 (40)<br>20 (22)<br>25 (11)<br>15 (11)<br>30 (7)<br>21 (4)<br>19 (4)<br>13 (4)<br>10 (4) | |
| 77 | 13 | 2, 3 | | none - proven | 2 (14)<br>1 (3)<br>12 (2)<br>11 (2)<br>8 (2)<br>5 (2)<br>3 (2)<br>10 (1)<br>9 (1)<br>7 (1) | |

| 78 | 90059 | 5, 79, 1217 | | 274, 302, 631, 1816, 2292, 2381, 3872, 3949, 4344, 4383, 4489, 4937, 5057, 5766, 5782, 6077, 6436, 7032, 7800, 8469, 8499, 8649, 8758, 10263, 10924, 10928, 10942, 11044, 11936, 12167, 12187, 12244, 12286, 12332, 12622, 13212, 13287, 13668, 13824, 14059, 14456, 14526, 14932, 15722, 15799, 16451, 16688, 17029, 17039, 17221, 17271, 17732, 17886, 18013, 18663, 19614, 19846, 19909, 19986, 20027, 20182, 20462, 20879, 21197, 21631, 21961, 23052, 23079, 23801, 23899, 24214, 24949, 25061, 25532, 25901, 26377, 26385, 26804, 27021, 27096, 27175, 27256, 27399, 27439, 27842, 29073, 29389, 29668, 29863, 30444, 31046, 31053, 31742, 31836, 31917, 31994, 32705, 33298, 33412, 33671, 33888, 33892, 34728, 35179, 35568, 36233, 36344, 36609, 37024, 38354, 38438, | 3633 (94500) 68571 (91386) 51476 (88677) 78053 (84433) 58412 (83824) 45661 (73022) 11412 (72798) 72638 (70230) 23462 (69162) 23543 (62677) | |

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| **79** | 9 | 2, 5 | All k where k = m^2 and m = = 2 or 3 mod 5: for even n let k = m^2 and let n = 2*q; factors to: (m*79^q - 1) * (m*79^q + 1) odd n: factor of 5 | none - proven | 1 (5)<br>7 (4)<br>3 (4)<br>6 (3)<br>8 (1)<br>5 (1)<br>2 (1) | k = 4 proven composite by partial algebraic factors. |
| **80** | 253 | 3, 37, 173 | | 10, 31, 214 (all at n=400K) | 170 (148256)<br>106 (16237)<br>154 (9753)<br>46 (5337)<br>232 (2997)<br>157 (2613)<br>169 (1959)<br>45 (1156)<br>218 (776)<br>244 (653) | |
| **81** | 74 | 7, 13, 73 | All k = m^2 for all n; factors to: (m*9^n - 1) * (m*9^n + 1) | none - proven | 53 (268)<br>42 (99)<br>23 (68)<br>18 (15)<br>35 (14)<br>30 (12)<br>71 (4)<br>60 (4)<br>40 (4)<br>24 (4) | k = 1, 4, 9, 16, 25, 36, 49, and 64 proven composite by full algebraic factors. |

| 82 | 22326 | 5, 83, 269 | | 118, 133, 290, 331, 334, 439, 625, 649, 667, 748, 757, 763, 829, 878, 883, 898, 997, 1163, 1252, 1279, 1327, 1348, 1351, 1531, 1741, 1827, 1936, 1991, 2050, 2157, 2263, 2278, 2419, 2431, 2539, 2543, 2588, 2635, 2668, 2797, 2836, 2896, 2929, 2971, 2974, 3079, 3121, 3156, 3293, 3319, 3436, 3653, 3796, 3817, 4068, 4078, 4083, 4118, 4372, 4399, 4447, 4481, 4483, 4780, 4801, 4867, 4898, 4972, 5053, 5182, 5230, 5311, 5329, 5401, 5560, 5562, 5713, 5893, 5899, 5975, 6028, 6122, 6124, 6143, 6178, 6186, 6226, 6296, 6343, 6418, 6427, 6571, 6631, 6925, 6994, 7054, 7056, 7303, 7386, 7388, 7396, 7474, 7615, 7723, 7801, 7813, 7822, 7884, 7892, 7969, 8065, 8314, 8368, 8384, 8499, 8629, | 15978 (99999) 21429 (96772) 18989 (96049) 17592 (83837) 22233 (75716) 12912 (74869) 5811 (72615) 16091 (65850) 18576 (64927) 4482 (63245) | |

| 83 | 5 | 2, 3 | | none - proven | 2 (8)<br>1 (5)<br>3 (2)<br>4 (1) | |
|----|---|------|---|---------------|-----------------|---|
| 84 | 16 | 5, 17 | All k where k = m^2 and m = = 2 or 3 mod 5: for even n let k = m^2 and let n = 2*q; factors to:<br>(m*84^q - 1) *<br>(m*84^q + 1)<br>odd n:<br>factor of 5 | none - proven | 1 (17)<br>14 (8)<br>11 (7)<br>8 (4)<br>12 (3)<br>15 (1)<br>13 (1)<br>10 (1)<br>7 (1)<br>6 (1) | k = 4 and 9 proven composite by partial algebraic factors. |
| 85 | 173 | 2, 43 | | 61 (8K) | 169 (6939)<br>64 (1253)<br>105 (403)<br>112 (394)<br>97 (287)<br>109 (230)<br>16 (171)<br>27 (160)<br>93 (90)<br>145 (77) | |
| 86 | 28 | 3, 29 | | none - proven | 23 (112)<br>14 (38)<br>18 (26)<br>27 (14)<br>1 (11)<br>2 (10)<br>25 (9)<br>11 (8)<br>22 (5)<br>19 (5) | |

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| **87** | 21 | 2, 11 | | none - proven | 19 (372)<br>9 (91)<br>16 (17)<br>18 (15)<br>5 (15)<br>13 (11)<br>11 (10)<br>1 (7)<br>7 (6)<br>12 (5) | |
| **88** | 571 | 3, 7, 13, 19 | k = 400:<br>for even n let n=2*q;<br>factors to:<br>(20*88^q - 1) * (20*88^q + 1)<br>odd n:<br>covering set 3, 7, 13 | 46, 94, 277, 508 (all at n=10K) | 464 (20648)<br>444 (19708)<br>544 (8904)<br>380 (8712)<br>79 (7665)<br>477 (5816)<br>212 (5511)<br>179 (4545)<br>346 (2969)<br>68 (2477) | |
| **89** | 4 | 3, 5 | | none - proven | 2 (60)<br>3 (5)<br>1 (3) | |
| **90** | 27 | 7, 13 | All k where k = m^2<br>and m = = 5 or 8 mod 13:<br>for even n let k = m^2<br>and let n = 2*q; factors to:<br>(m*90^q - 1) * (m*90^q + 1)<br>odd n:<br>factor of 13 | none - proven | 6 (20)<br>11 (10)<br>10 (10)<br>13 (6)<br>15 (5)<br>12 (4)<br>7 (4)<br>24 (3)<br>1 (3)<br>20 (2) | k = 25 proven composite by partial algebraic factors. |

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| **91** | 45 | 2, 23 | | none - proven (with probable primes that have not been certified: k = 27) | 27 (5048)<br>1 (4421)<br>37 (159)<br>15 (14)<br>43 (6)<br>39 (6)<br>31 (6)<br>24 (5)<br>20 (4)<br>36 (3) | |
| **92** | 32 | 3, 31 | | none - proven | 1 (439)<br>29 (272)<br>28 (99)<br>13 (35)<br>14 (32)<br>18 (26)<br>22 (25)<br>20 (6)<br>6 (6)<br>17 (4) | |
| **93** | 189 | 2, 47 | | 33, 69, 109, 113, 125, 149, 177 (all at n=8K) | 97 (1179)<br>29 (496)<br>92 (476)<br>46 (434)<br>121 (271)<br>141 (262)<br>101 (142)<br>122 (126)<br>85 (86)<br>166 (66) | |
| **94** | 39 | 5, 19 | All k where k = m^2 and m = = 2 or 3 mod 5: for even n let k = m^2 and let n = 2*q; factors to: (m*94^q - 1) * (m*94^q + 1) odd n: factor of 5 | 29 (1M) | 16 (21951)<br>37 (254)<br>13 (163)<br>14 (154)<br>7 (95)<br>34 (54)<br>25 (41)<br>24 (12)<br>26 (9)<br>36 (7) | k = 4 and 9 proven composite by partial algebraic factors. |

| 95 | 5 | 2, 3 | | none - proven | 1 (7)<br>3 (2)<br>2 (2)<br>4 (1) | |

| 96 | 38995 | 7, 67, 97, 1303 | (Condition 1): All k where k = m^2 and m = = 22 or 75 mod 97: for even n let k = m^2 and let n = 2*q; factors to: (m*96^q - 1) * (m*96^q + 1) odd n: factor of 97 (Condition 2): All k where k = 6*m^2 and m = = 9 or 88 mod 97: even n: factor of 97 for odd n let k = 6*m^2 and let n=2*q-1; factors to: [m*2^(5q-1)*3^q - 1] * [m*2^(5q-1)*3^q + 1] | 431, 591, 701, 831, 872, 956, 1006, 1126, 1648, 1681, 1810, 2036, 2386, 2424, 2878, 3001, 3431, 3461, 3671, 3856, 3881, 3956, 3996, 4261, 4351, 4366, 4406, 4451, 4461, 5046, 5836, 5918, 6031, 6261, 6481, 6586, 6670, 6786, 7091, 7116, 7121, 7131, 7249, 7274, 7461, 7801, 8016, 8202, 8291, 8546, 8816, 9022, 9131, 9156, 9326, 9441, 9463, 9476, 9677, 9681, 9921, 10036, 10204, 10375, 10453, 10551, 10651, 10721, 11056, 11156, 11196, 11458, 11553, 11766, 11831, 12676, 12901, 13216, 13231, 13288, 13571, 14011, 14061, 14276, 14517, 14551, 14646, 15341, 15461, 15573, 15596, 16176, 16306, 16392, 16586, 16641, 16645, 17116, 17421, 17636, | 3769 (92879) 28907 (89447) 13528 (86114) 19882 (82073) 37155 (76817) 9160 (71178) 5179 (66965) 32960 (60312) 7565 (59052) 4754 (56909) | k = 484, 5625, 14161, and 29584 proven composite by condition 1. k = 486 proven composite by condition 2. |

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| | | | | 17653, 17792, 18311, 19136, 19191, 19246, 19486, 19681, 20091, 20396, 20464, 20502, 20936, 21488, 21776, 22541, 22811, 22846, 22931, 23010, 23161, 23271, 23301, 23570, 23766, 24076, 24216, 24386, 24506, 24831, 24916, 24929, 25306, 25706, 25966, 26038, 26161, 26183, 26571, 26772, 26801, 26846, 27045, 27106, 27126, 27450, 27646, 27700, 27741, 28365, 28558, 28774, 28776, 28921, 29093, 29196, 29561, 29681, 30086, 30120, 30151, 30421, 30581, 30662, 31021, 31136, 31936, 32205, 32881, 33099, 33141, 33391, 33406, 33501, 33621, 33701, 33711, 33951, 33986, 34116, 34236, 34436, 34531, 34921, 35016, 35113, 35271, 35406, 35446, 35781, 35966, 36158, 36551, 36945, 36981, 37031, 37036, 37166, | | |

| | | | | 37222, 37471, 37991, 38156, 38301, 38316, 38986 (k = 1 mod 5 and k = 1 mod 19 at n=1K, other k at n=100K) | | |
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| 97 | 43 | 3, 5, 7, 37, 139 | | 22 (35.8K) | 8 (192335)<br>16 (1627)<br>4 (621)<br>28 (184)<br>1 (17)<br>34 (16)<br>32 (9)<br>27 (8)<br>37 (5)<br>31 (5) | |
|----|----|----|----|----|----|----|
| 98 | 10 | 3, 11 | | none - proven | 1 (13)<br>5 (10)<br>7 (3)<br>4 (3)<br>8 (2)<br>2 (2)<br>9 (1)<br>6 (1)<br>3 (1) | |
| 99 | 9 | 2, 5 | All k where k = m^2 and m = = 2 or 3 mod 5: for even n let k = m^2 and let n = 2*q; factors to:<br>(m*99^q - 1) *<br>(m*99^q + 1) odd n: factor of 5 | none - proven | 5 (135)<br>3 (4)<br>1 (3)<br>7 (2)<br>8 (1)<br>6 (1)<br>2 (1) | k = 4 proven composite by partial algebraic factors. |
| 100 | 211 | 7, 13, 37 | All k = m^2 for all n; factors to:<br>(m*10^n - 1) *<br>(m*10^n + 1) | none - proven (with probable primes that have not been certified: k = 133) | 74 (44709)<br>133 (5496)<br>102 (209)<br>193 (155)<br>203 (133)<br>95 (96)<br>109 (68)<br>55 (56)<br>98 (45)<br>37 (36) | k = 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, and 196 proven composite by full algebraic factors. |

| 101 | 13 | 2, 3 | | none - proven | 5 (350)<br>8 (112)<br>2 (42)<br>11 (24)<br>12 (11)<br>4 (3)<br>1 (3)<br>6 (2)<br>10 (1)<br>9 (1) | |
|---|---|---|---|---|---|---|
| 102 | 1635 | 7, 19, 79 | | 191, 207, 1082, 1369 (all at n=500K) | 1451 (188973)<br>1208 (178632)<br>653 (117255)<br>1607 (82644)<br>254 (58908)<br>1527 (49462)<br>1037 (43460)<br>32 (43302)<br>1296 (37715)<br>142 (22025) | |
| 103 | 25 | 2, 13 | | none - proven | 19 (820)<br>22 (442)<br>23 (216)<br>14 (189)<br>16 (57)<br>11 (54)<br>24 (32)<br>15 (32)<br>1 (19)<br>20 (5) | |
| 104 | 4 | 3, 5 | | none - proven | 1 (97)<br>2 (68)<br>3 (1) | |

| 105 | 297 | 2, 37, 149 | All k where k = m^2 and m = = 3 or 5 mod 8: for even n let k = m^2 and let n = 2*q; factors to: (m*57^q - 1) * (m*57^q + 1) odd n: factor of 2 | 73, 137 (both at n=8K) | 148 (3645) 265 (1666) 162 (294) 255 (222) 154 (139) 145 (119) 80 (91) 68 (56) 66 (47) 223 (21) | k = 9, 25, 121, and 169 proven composite by partial algebraic factors. |

| 106 | 13624 | 3, 19, 199 | | 64, 81, 163, 332, 391, 400, 429, 511, 526, 582, 596, 643, 676, 841, 862, 897, 913, 1024, 1223, 1261, 1283, 1294, 1417, 1428, 1546, 1597, 1713, 1869, 2056, 2116, 2248, 2389, 2458, 2605, 2623, 2656, 2674, 2719, 2743, 2780, 2781, 2813, 2888, 2965, 3047, 3073, 3130, 3136, 3142, 3241, 3277, 3336, 3425, 3427, 3478, 3481, 3617, 3622, 3646, 3655, 3694, 3746, 3883, 4045, 4067, 4096, 4153, 4162, 4177, 4219, 4336, 4339, 4416, 4628, 4662, 4666, 4696, 4713, 4722, 4801, 5135, 5283, 5359, 5395, 5468, 5485, 5623, 5692, 5707, 5752, 5776, 5777, 5872, 5878, 5937, 5971, 5992, 5993, 6040, 6094, 6100, 6103, 6181, 6220, | 8272 (998) 508 (998) 13417 (994) 4908 (970) 5179 (969) 3700 (968) 577 (947) 3583 (943) 9814 (935) 1321 (913) | |

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|---|---|---|---|---|---|---|
| | | | | 6376, 6421, 6505, 6547, 6613, 6716, 6736, 6832, 6955, 7069, 7156, 7202, 7246, 7273, 7297, 7331, 7336, 7345, 7356, 7398, 7402, 7496, 7540, 7561, 7744, 7771, 7894, 7906, 7915, 8023, 8181, 8266, 8323, 8329, 8371, 8386, 8428, 8521, 8561, 8572, 8637, 8779, 8788, 8861, 8950, 8956, 8962, 8975, 9031, 9096, 9190, 9238, 9294, 9366, 9415, 9469, 9589, 9634, 9736, 9774, 9787, 9790, 9796, 9808, 9859, 9877, 9973, 9976, 10033, 10072, 10117, 10150, 10166, 10186, 10271, 10273, 10446, 10451, 10627, 10646, 10651, 10660, 10699, 10816, 10876, 10894, 11097, 11173, 11278, 11299, 11419, 11420, 11426, 11506, 11639, 11671, 11833, 11884, | | |

| | | | | 11901, 12066, 12076, 12090, 12145, 12252, 12269, 12321, 12352, 12361, 12490, 12627, 12851, 12856, 12910, 12916, 12970, 12978, 12991, 13023, 13027, 13162, 13174, 13269, 13366, 13374, 13378, 13387, 13497, 13511, 13516, 13528, 13543, 13553, 13558, 13567 (all at n=1K) | | |
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| 107 | 5 | 2, 3 | | none - proven (with probable primes that have not been certified: k = 3) | 2 (21910) 3 (4900) 4 (251) 1 (17) | |
|-----|---|------|---|---|---|---|
| **108** | 13406 | 7, 13, 61, 109 | (Condition 1): All k where k = m^2 and m = = 33 or 76 mod 109: for even n let k = m^2 and let n = 2*q; factors to: (m*108^q - 1) * (m*108^q + 1) odd n: factor of 109 (Condition 2): All k where k = 3*m^2 and m = = 20 or 89 mod 109: even n: factor of 109 for odd n let k = 3*m^2 and let n=2*q-1; factors to: [m*2^(2q-1)*3^(3q-1) - 1] * [m*2^(2q-1)*3^(3q-1) + 1] | 137, 411, 437, 873, 1634, 1769, 1782, 1961, 2508, 2617, 2962, 2963, 3002, 3029, 3474, 3499, 3596, 3646, 4007, 4066, 4084, 4121, 4184, 4328, 4468, 4499, 4744, 4904, 5015, 5142, 5212, 5351, 5625, 5821, 5892, 5923, 5994, 6212, 6284, 6432, 6528, 6570, 6614, 6866, 7107, 7211, 7302, 7304, 7419, 7848, 8037, 8144, 8374, 8383, 8503, 8524, 8638, 8986, 9346, 9852, 10052, 10129, 10136, 10245, 10699, 10926, 11089, 11164, 11278, 11619, 11881, 11918, 12262, 12861, 12863, 13162, 13291, 13297 (k = 5351, 6528, and 13162 at | 10322 (88080) 1999 (85188) 7557 (84180) 11882 (81547) 3439 (79524) 4686 (79010) 1159 (77107) 3573 (76352) 1465 (75209) 2148 (75018) | k = 1089 and 5776 proven composite by condition 1. k = 1200 proven composite by condition 2. |

| | | | | n=2K, other k at n=100K) | | |
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| | | | | | | |

| 109 | 9 | 2, 5 | All k where k = m^2 and m = = 2 or 3 mod 5: for even n let k = m^2 and let n = 2*q; factors to: (m*109^q - 1) * (m*109^q + 1) odd n: factor of 5 | none - proven | 8 (19) 1 (17) 5 (2) 2 (2) 7 (1) 6 (1) 3 (1) | k = 4 proven composite by partial algebraic factors. |
|---|---|---|---|---|---|---|
| 110 | 38 | 3, 37 | All k where k = m^2 and m = = 6 or 31 mod 37: for even n let k = m^2 and let n = 2*q; factors to: (m*110^q - 1) * (m*110^q + 1) odd n: factor of 37 | none - proven | 23 (78120) 17 (2598) 37 (1689) 9 (77) 11 (42) 10 (17) 2 (16) 31 (9) 5 (6) 22 (5) | k = 36 proven composite by partial algebraic factors. |
| 111 | 13 | 2, 7 | | none - proven | 2 (24) 7 (6) 6 (4) 1 (3) 12 (2) 11 (2) 3 (2) 10 (1) 9 (1) 8 (1) | |

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|---|---|---|---|---|---|---|
| **112** | 1357 | 5, 13, 113 | All k where k = m^2 and m = = 15 or 98 mod 113: for even n let k = m^2 and let n = 2*q; factors to: (m*112^q - 1) * (m*112^q + 1) odd n: factor of 113 | 31, 79, 310, 340, 421, 424, 451, 529, 703, 940, 1018, 1051, 1204 (all at n=7.5K) | 948 (173968) 1268 (50536) 758 (35878) 1353 (7751) 187 (7524) 498 (6038) 9 (5717) 1024 (5681) 619 (5441) 981 (2858) | k = 225 proven composite by partial algebraic factors. |
| **113** | 20 | 3, 19 | | none - proven | 14 (308) 1 (23) 7 (15) 19 (11) 5 (8) 16 (5) 3 (5) 12 (3) 4 (3) 18 (2) | |
| **114** | 24 | 5, 23 | All k where k = m^2 and m = = 2 or 3 mod 5: for even n let k = m^2 and let n = 2*q; factors to: (m*114^q - 1) * (m*114^q + 1) odd n: factor of 5 | none - proven | 3 (63) 1 (29) 11 (27) 18 (21) 22 (20) 20 (3) 19 (2) 17 (2) 14 (2) 10 (2) | k = 4 and 9 proven composite by partial algebraic factors. |

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| **115** | 57 | 2, 29 | | 13, 43 (both at n=8K) | 45 (5227)<br>4 (4223)<br>51 (2736)<br>23 (1116)<br>53 (165)<br>21 (127)<br>35 (50)<br>15 (38)<br>39 (28)<br>32 (28) | |
| **116** | 14 | 3, 13 | | none - proven | 9 (249)<br>5 (156)<br>11 (118)<br>1 (59)<br>2 (32)<br>13 (15)<br>10 (11)<br>12 (2)<br>8 (2)<br>7 (1) | |
| **117** | 149 | 2, 5, 37 | | 5, 17, 33, 141 (all at n=8K) | 83 (442)<br>59 (352)<br>19 (336)<br>110 (232)<br>143 (222)<br>41 (209)<br>87 (177)<br>129 (165)<br>118 (136)<br>92 (129) | |
| **118** | 50 | 7, 17 | | 43 (37K) | 27 (860)<br>29 (599)<br>18 (393)<br>6 (210)<br>22 (191)<br>8 (85)<br>19 (72)<br>7 (52)<br>42 (30)<br>37 (27) | |
| **119** | 4 | 3, 5 | | none - proven | 2 (28)<br>3 (6)<br>1 (3) | |

| 120 | 166616308 | 11, 13, 1117, 14281 | | 384, 386, 419, 483, 551, 672, 824, 846, 890, 901, 991, 1024, 1077, 1095, 1132, 1134, 1255, 1309, 1385, 1394, 1693, 1797, 1921, 2036, 2133, 2177, 2258, 2354, 2386, 2410, 2452, 2650, 2696, 2716, 3004, 3025, 3123, 3178, 3189, 3214, 3290, 3343, 3347, 3400, 3407, 3433, 3596, 3786, 3994, 4003, 4082, 4320, 4399, 4423, 4460, 4500, 4577, 4676, 4685, 4819, 4830, 4839, 4936, 5105, 5125, 5255, 5378, 5630, 5686, 5730, 6112, 6241, 6332, 6357, 6425, 6581, 6676, 6678, 6755, 6821, 6852, 6951, 6982, 6997, 7008, 7413, 7470, 7523, 7545, 7549, 7789, 7803, 7820, 7910, 7985, 8100, 8205, 8464, 8647, 8810, 8812, 8869, | 8063 (997) 6434 (976) 2980 (958) 5180 (938) 164 (878) 4234 (876) 7085 (843) 4390 (833) 9354 (829) 2726 (822) | |

| | | | | 8922, 8964, 8966, 8997, 9010, 9019, 9057, 9070, 9395, 9564, 9626, 9712, 9889, 9921, 9954, 9993 (for k <= 10K) (all at n=1K) | | |
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| **121** | 100 | 3, 7, 37 | All k = m^2 for all n; factors to: (m*11^n - 1) * (m*11^n + 1) | none - proven | 62 (13101) 79 (4545) 43 (68) 7 (60) 30 (24) 60 (12) 87 (11) 39 (11) 57 (10) 50 (10) | k = 1, 4, 9, 16, 25, 36, 49, 64, and 81 proven composite by full algebraic factors. |
| **122** | 14 | 3, 5, 13 | | none - proven | 13 (43) 8 (26) 11 (10) 2 (6) 12 (5) 1 (5) 10 (3) 6 (2) 5 (2) 3 (2) | |
| **123** | 13 | 2, 5, 17 | | 11 (8K) | 1 (43) 3 (8) 2 (8) 12 (7) 6 (7) 9 (5) 7 (2) 10 (1) 8 (1) 5 (1) | |
| **124** | 92881 | 3, 5, 7, 5167 | (Condition 1): All k where k = m^2 | 101, 136, 146, 175, 179, 199, 204, 236, 259, | 1194 (998) 1611 (989) 659 (986) | k = 2^2, 3^2, 7^2, 8^2, 12^2, |
| **125** | 8 | 3, 7 | All k = m^3 for all n; factors to: (m*5^n - 1) * (m^2*25^n + m*5^n + 1) | none - proven | 6 (24) 7 (5) 3 (3) 5 (2) 2 (2) 4 (1) | k = 1 proven composite by full algebraic factors. |

| 126 | 480821 | 13, 19, 127, 829 | | 380, 406, 438, 729, 893, 1132, 1523, 1654, 1810, 1855, 2707, 2744, 2804, 3285, 3566, 3573, 3631, 3721, 4335, 4416, 4436, 4596, 4772, 5081, 5164, 5285, 5784, 5820, 6026, 6041, 6204, 6605, 6990, 7075, 7107, 7183, 7479, 7580, 7673, 7876, 8061, 8099, 8238, 8256, 8323, 8336, 8485, 8527, 8836, 9025, 9127, 9166, 9220, 9524, 9606, 9651, 9936, 10195, 10728, 10818, 11012, 11287, 11366, 11475, 11493, 11683, 11696, 12013, 12416, 12424, 12433, 12594, 12794, 12820, 12868, 13006, 13016, 13023, 13027, 13134, 13302, 13389, 13824, 14225, 14270, 14509, 14790, 14831, 15167, 15348, 15366, 15577, 15596, 15620, 15752, 15898, 16130, 16367, 16636, 16723, | 16604 (2475) 26728 (2429) 3428 (2428) 16844 (2365) 15239 (2348) 13759 (2324) 4698 (2302) 13672 (2239) 8177 (2224) 8682 (2162) | |

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|---|---|---|---|---|---|---|
| | | | | 16974, 17351, 17436, 17826, 17920, 18001, 18058, 18067, 18162, 18430, 18437, 18543, 18571, 18617, 18638, 18849, 19314, 19686, 19759, 19847, 19940, 19996, 20192, 20216, 20439, 20497, 20520, 20573, 20575, 20608, 20635, 20744, 20907, 20983, 20993, 21060, 21209, 21306, 21316, 21342, 21583, 21849, 22031, 22224, 22389, 22478, 22790, 22837, 22938, 23180, 23264, 23390, 23466, 23533, 23692, 23748, 23830, 23903, 24001, 24060, 24176, 24319, 24390, 24579, 24706, 24748, 24779, 24832, 24963, 25012, 25106, 25130, 25886, 26159, 26279, 26326, 26490, 26822, 27182, 27296, 27730, 27842, 27920, 28447, 28453, 28659, 28791, 28928, 29001, 29012, 29228, 29329, 29477, 29551, 29617, 29719, 29844, 29942 | | |

| | | | | (for k <= 30K) (k = 1 mod 5 at n=1K, other k at n=2.5K) | | |
|---|---|---|---|---|---|---|
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| | | | | (for k <= 30K) (k = 1 mod 5 at n=1K, other k at n=2.5K) | | |

| 127 | 2593 | 2, 5, 17, 137 | | 13, 17, 25, 27, 33, 35, 79, 83, 91, 113, 121, 139, 159, 179, 191, 231, 233, 235, 236, 237, 239, 250, 251, 264, 279, 288, 293, 333, 353, 361, 367, 379, 443, 451, 459, 471, 473, 511, 513, 517, 523, 531, 537, 551, 553, 557, 561, 597, 599, 604, 617, 631, 639, 649, 659, 679, 699, 715, 725, 731, 733, 737, 739, 747, 751, 755, 763, 773, 778, 783, 797, 809, 838, 848, 863, 871, 895, 919, 937, 939, 950, 953, 964, 982, 997, 999, 1013, 1019, 1025, 1031, 1037, 1039, 1043, 1051, 1106, 1107, 1117, 1119, 1127, 1157, 1173, 1185, 1196, 1199, 1211, 1231, 1232, 1233, 1245, 1253, 1259, 1279, 1288, 1291, 1313, 1327, 1333, 1335, 1337, 1347, 1353, 1359, 1371, 1377, 1401, 1407, 1417, 1421, | 667 (1000) 1775 (994) 2497 (989) 2199 (972) 1759 (936) 2015 (910) 343 (904) 1113 (899) 1962 (893) 1543 (872) | |

| | | | | 1429, 1432, 1439, 1473, 1481, 1491, 1513, 1525, 1539, 1549, 1551, 1573, 1577, 1579, 1589, 1593, 1595, 1597, 1599, 1611, 1612, 1618, 1631, 1639, 1641, 1661, 1677, 1693, 1699, 1709, 1711, 1731, 1732, 1737, 1751, 1771, 1792, 1793, 1803, 1837, 1839, 1903, 1911, 1921, 1928, 1933, 1936, 1939, 1943, 1951, 1957, 1959, 1999, 2013, 2017, 2032, 2039, 2045, 2072, 2073, 2079, 2092, 2097, 2099, 2129, 2155, 2168, 2179, 2191, 2197, 2215, 2231, 2247, 2253, 2273, 2279, 2303, 2313, 2339, 2367, 2377, 2389, 2411, 2427, 2431, 2433, 2479, 2501, 2543, 2548, 2559, 2565, 2573, 2583 (all at n=1K) | | |
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| **128** | 44 | 3, 43 | All k = m^7 for all n; factors to: (m*2^n - 1) * (m^6*64^n + m^5*32^n + m^4*16^n + m^3*8^n + m^2*4^n + m*2^n + 1) | none - proven | 29 (211192) 23 (2118) 26 (1442) 37 (699) 16 (459) 42 (246) 35 (98) 30 (66) 36 (59) 12 (46) | k = 1 proven composite by full algebraic factors. |
| **256** | 100 | 3, 7, 13 | All k = m^2 for all n; factors to: (m*16^n - 1) * (m*16^n + 1) | none - proven | 74 (319) 47 (228) 42 (224) 92 (143) 68 (87) 61 (54) 35 (28) 65 (24) 70 (18) 75 (17) | k = 1, 4, 9, 16, 25, 36, 49, 64, and 81 proven composite by full algebraic factors. |
| **512** | 14 | 3, 5, 13 | All k = m^3 for all n; factors to: (m*8^n - 1) * (m^2*64^n + m*8^n + 1) | none - proven | 4 (2215) 13 (2119) 9 (7) 11 (6) 6 (6) 5 (2) 3 (2) 2 (2) 12 (1) 10 (1) | k = 1 and 8 proven composite by full algebraic factors. |
| **1024** | 81 | 5, 41 | All k = m^2 for all n; factors to: (m*32^n - 1) * (m*32^n + 1) -or- All k = m^5 for all n; factors to: (m*4^n - 1) * (m^4*256^n + m^3*64^n + m^2*16^n + m*4^n + 1) | 29, 31, 56, 61 (k = 29 at n=1M, other k at n=3K) | 74 (666084) 39 (4070) 43 (2290) 13 (1167) 78 (424) 65 (93) 69 (54) 3 (47) 71 (41) 44 (36) | k = 1, 4, 9, 16, 25, 32, 36, 49, and 64 proven composite by full algebraic factors. |