Minimal elements for the base b representations of the primes which are > b

Introduction

A <u>string</u> x is a <u>subsequence</u> of another string y, if x can be obtained from y by deleting zero or more of the <u>characters</u> in y (this should not to be confused with <u>substring</u>, a substring is a contiguous sequence of characters within a string). For example, 514 is a subsequence of 352148. The <u>empty string</u> λ is a subsequence of every string. There are 2^n subsequences of a string with length n, e.g. the subsequences of 123456 are (totally $2^6 = 64$ subsequences)

λ, 1, 2, 3, 4, 5, 6, 12, 13, 14, 15, 16, 23, 24, 25, 26, 34, 35, 36, 45, 46, 56, 123, 124, 125, 126, 134, 135, 136, 145, 146, 156, 234, 235, 236, 245, 246, 256, 345, 346, 356, 456, 1234, 1235, 1236, 1245, 1246, 1256, 1345, 1346, 1356, 1456, 2345, 2346, 2356, 2456, 3456, 12345, 12346, 12356, 12456, 13456, 23456, 123456

(in this article, we only consider the subsequences with length ≥2, and not consider the subsequences beginning with 0 and/or ending with 0, e.g. for the string 123456, we have these subsequences: 12, 13, 14, 15, 16, 23, 24, 25, 26, 34, 35, 36, 45, 46, 56, 123, 124, 125, 126, 134, 135, 136, 145, 146, 156, 234, 235, 236, 245, 246, 256, 345, 346, 356, 456, 1234, 1235, 1236, 1245, 1246, 1256, 1345, 1346, 1356, 1456, 2345, 2345, 2346, 2356, 2456, 3456, 12345, 12346, 12356, 12456, 13456, 23456, 123456, totally 57 substrings, and for a string with length n with no character 0, there are $2^n - n - 1$ substrings)

Two strings x and y are <u>comparable</u> if either x is a subsequence of y, or y is a subsequence of x. A surprising result from formal language theory is that every set of pairwise incomparable strings is finite. This means that from any set of strings we can find its minimal elements. A string x in a set of strings S is a minimal string if whenever y (an element of S) is a subsequence of x, we have y = x.

research of this set in other bases is the target of this article. The set of the minimal strings of the latter set is {16, 25, 36, 49, 64, 81, 100, 121, 144, 289, 324, 400, 441, 484, 529, 576, 676, 729, 784, 900, 961, 1024, 1089, 2209, 2304, 2401, 2601, 2704, 3721, 3844, 4761, 5041, 5184, 6561, 6889, 7056, 7569, 7744, 7921, 21904, 22201, 28224, 29241, 29929, 31329, 35344, 38809, 46656, 47524, 55696, 62001, 63001, 69696, 79524, 80089, 80656, 82944, 88209, 88804, 91204, 91809, 97344, 97969, 98596, 99856, 138384, 139129, 173889, 182329, 199809, 300304, 301401, 304704, 305809, 332929, 339889, 345744, 374544, 393129, 473344, 505521, 515524, 558009, 559504, 567009, 589824, 595984, 657721, 660969, 665856, 683929, 695556, 702244, 719104, 743044, 777924, 779689, 842724, 850084, 876096, 877969, 896809, 898704, 929296, 935089, 1317904, 1557504, 1882384, 1898884, 2022084, 2027776, 2039184, 2070721, 2477476, 2802276, 2979076, 2999824, 3055504, 3073009, 3139984, 3323329, 3415104, 3794704, 4477456, 4545424, 4575321, 5053504, 5067001, 5071504, 5280804, 5303809, 5513104, 5527201, 5531904, 5574321, 5579044, 5707321, 5750404, 5755201, 5987809, 6517809, 6568969, 6620329, 6901129, 7006609, 7011904, 7033104, 7096896, 7177041, 7474756, 7551504, 7557001, 7573504, 7941124, 8020224, 8054244, 8282884, 8340544, 8508889, 8538084, 8620096, 8809024, 9229444, 9535744, 9809424, 9847044, 9935104, 9998244, 13118884, 13337104, 15038884, 15578809, 18939904, 19775809, 20903184, 20912329, 20994724, 23902321, 27709696, 29833444, 31102929, 31899904, 33039504, 33085504, 33315984, 33500944, 35533521, 35545444, 37797904, 38093584, 39980329, 40755456, 45535504, 47073321, 47444544, 50098084, 50566321, 50580544, 50608996, 50808384, 51151104, 53333809, 53993104, 55011889, 55517401, 55666521, 57501889, 57775201, 58247424, 58339044, 58859584, 59089969, 60575089, 60590656, 61199329, 65658609, 66650896, 66863329, 69072721, 69338929, 70006689, 70543201, 70997476, 71351809, 72233001, 73153809, 73994404, 74407876, 74632321, 75968656, 77668969, 77686596, 77757124, 77898276, 78907689, 78960996, 78978769, 79869969, 84052224, 85507009, 86992929, 88059456, 88096996. 88585744. 88868329. 89056969. 91833889. 94303521. ...}. although this set seems to be endless, but by the theorem that there are no infinite antichains for the subsequence ordering, this set must be finite, but this set is extremely difficult to found, and it is also difficult to determine the number of elements in this set, and is much more difficult than that of the first set in every base $2 \le b \le 36$. For example, it is not even known whether the non-simple family 3^m5ⁿ8^r44 contain a square or not, this situation usually not occur for primes in any base, i.e. every non-simple family which can not be ruled out as containing no primes > base usually contain a small prime > base.

Equivalently, a string x in a set of strings S is a minimal string if and only if any proper subsequence of x (subsequence of x which is unequal to x, like proper subset) is not in S.

The minimal set M(L) of a language L is interesting, this is because it allows us to compute two natural and related languages, defined as follows:

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sub(L) := \{x \in \Sigma^* : \text{there exists } y \in L \text{ such that } x \text{ is a subsequence of } y\};
 sup(L) := \{x \in \Sigma^* : \text{there exists } y \in L \text{ such that } y \text{ is a subsequence of } x\}.
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An amazing fact is that sub(L) and sup(L) are always regular. This follows from the following classical theorem:

Theorem: For every language L, there are only finitely many minimal strings.

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Indeed, we have sup(L) = sup(M(L)) and \Sigma^* - sub(L) = sup(M(\Sigma^* - sub(L))), and the superword language of a finite language is regular, since sup(\{w_1, ..., w_n\}) = \bigcup_{i=1}^n \quad \Sigma^* w_{i,1} \Sigma^* ... \quad \Sigma^* w_{i,|w_i|} \Sigma^* where w_i = w_{i,1} ... \quad w_{i,|w_i|} with w_{i,j} \in \Sigma.
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Although the set M(S) of minimal strings is necessarily finite, determining it explicitly for a given S can be a difficult computational problem. We use some <u>numbertheoretic heuristics</u> to compute $M(L_b)$, where L_b is the language of <u>base-b</u> representations of the <u>prime numbers</u> which are > b, for $2 \le b \le 16$. (Also, I left as a challenge to readers the task of computing $M(L_b)$ for $17 \le b \le 36$)

Notation

In what follows, if x is a string of symbols over the alphabet $\Sigma_b := \{0, 1, ..., b-1\}$ we let $[x]_b$ denote the evaluation of x in base b (starting with the most significant digit), and $[\lambda]_b := 0$ where λ is the empty string. This is extended to languages as follows: $[L]_b := \{ [x]_b : x \in L \}$. We use the convention that A := 10, B := 11, C := 12, ..., Z := 35, to conveniently represent strings of symbols in base b > 10. We let $(x)_b$ be the canonical representation of x in base b, that is, the representation without leading zeros. Finally, as usual, for a language L we let $L^n := LLL...LLL$ with $L^n := LLL...LL$ with $L^n := LLLL...LLL$ with $L^n := LLL...LLL$ with $L^n := LLL...LLL$ with $L^n := LLLL...LLL$ with $L^n := LLLL...LL$ with $L^n :$

This is a list for L_b for bases $2 \le b \le 36$.

b	L _b
2	11, 101, 111, 1011, 1101, 10001, 10011, 10111, 11101, 11111, 100101, 101001, 101011, 101111, 110101, 111011, 111101, 1000011, 1000111, 1001001, 1001111, 1010011, 1011001, 1100101, 1100101, 1100101, 1100101, 1100101, 1101011, 1101011, 1101011, 1001011, 1001011, 1001011, 1001011, 1001011, 1001011, 1001011, 10100111, 1010011, 10110111, 1010011, 10110111, 1100011, 1110011, 11100011, 11100011, 11100011, 11100011, 11100011, 11100011, 11100011, 10001001, 10001101, 100011011, 100011011, 100011011, 10011001
3	12, 21, 102, 111, 122, 201, 212, 1002, 1011, 1101, 1112, 1121, 1202, 1222, 2012, 2021, 2111, 2122, 2201, 2221, 10002, 10022, 10121, 10202, 10211, 10222, 11001, 11012, 11201, 11212, 12002, 12011, 12112, 12121, 12211, 20001, 20012, 20102, 20122, 20201, 21002, 21011, 21022, 21101, 21211, 22021, 22102, 22111, 22122, 22212, 22221, 100022, 100112, 100202, 100222, 101001, 101021, 101102, 101111, 101212, 102101, 102112, 102121, 102202, 110021, 110111, 110212, 110221, 111002, 1111022, 111121, 111211,

	112001, 112012, 112102, 112201, 112212, 120011, 120112, 120121, 120222, 121001, 121021, 121102, 121122, 121221, 122002, 122011, 122022, 122202, 200001, 200012, 200111, 200122, 200212, 201022, 201101, 202001, 202021, 202122,
4	11, 13, 23, 31, 101, 103, 113, 131, 133, 211, 221, 223, 233, 311, 323, 331, 1003, 1013, 1021, 1033, 1103, 1121, 1201, 1211, 1213, 1223, 1231, 1301, 1333, 2003, 2021, 2023, 2111, 2113, 2131, 2203, 2213, 2231, 2303, 2311, 2333, 3001, 3011, 3013, 3103, 3133, 3203, 3211, 3221, 3233, 3301, 3323, 10001, 10013, 10031, 10033, 10111, 10121, 10123, 10211, 10303, 10313, 10321, 10331, 11023, 11101, 11123, 11131, 11201, 11213, 11233, 11311, 11323, 11333, 12011, 12031, 12101, 12121, 12203, 12211, 12233, 12301, 12313, 12323, 13001, 13021, 13031, 13033, 13103, 13133, 13213, 13223, 13303, 13313, 13331, 20021, 20023, 20131, 20203, 20231,
5	12, 21, 23, 32, 34, 43, 104, 111, 122, 131, 133, 142, 203, 214, 221, 232, 241, 243, 304, 313, 324, 342, 401, 403, 412, 414, 423, 1002, 1011, 1022, 1024, 1044, 1101, 1112, 1123, 1132, 1143, 1204, 1211, 1231, 1233, 1242, 1244, 1321, 1343, 1402, 1404, 1413, 1424, 1431, 2001, 2012, 2023, 2034, 2041, 2102, 2111, 2113, 2133, 2212, 2221, 2223, 2232, 2311, 2322, 2342, 2344, 2403, 2414, 2432, 2443, 3004, 3013, 3024, 3042, 3101, 3114, 3134, 3141, 3211, 3213, 3224, 3233, 3244, 3312, 3321, 3323, 3332, 3404, 3422, 3431, 3444, 4003, 4014, 4041, 4043, 4131, 4142, 4212, 4223,
6	11, 15, 21, 25, 31, 35, 45, 51, 101, 105, 111, 115, 125, 135, 141, 151, 155, 201, 211, 215, 225, 241, 245, 251, 255, 301, 305, 331, 335, 345, 351, 405, 411, 421, 431, 435, 445, 455, 501, 515, 521, 525, 531, 551, 1011, 1015, 1021, 1025, 1035, 1041, 1055, 1105, 1115, 1125, 1131, 1141, 1145, 1151, 1205, 1231, 1235, 1241, 1245, 1311, 1321, 1335, 1341, 1345, 1355, 1411, 1421, 1431, 1435, 1445, 1501, 1505, 1521, 1535, 1541, 1555, 2001, 2011, 2015, 2025, 2041, 2045, 2051, 2055, 2115, 2131, 2135, 2151, 2155, 2205, 2225, 2231, 2301, 2311, 2325, 2335,
7	14, 16, 23, 25, 32, 41, 43, 52, 56, 61, 65, 104, 113, 115, 124, 131, 133, 142, 146, 155, 166, 203, 205, 212, 214, 221, 241, 245, 254, 256, 302, 304, 313, 322, 326, 335, 344, 346, 362, 364, 401, 403, 421, 436, 443, 445, 452, 461, 463, 506, 515, 524, 533, 535, 544, 551, 553, 566, 616, 623, 625, 632, 652, 661, 1004, 1006, 1013, 1022, 1033, 1042, 1051, 1055, 1064, 1105, 1112, 1123, 1136, 1141, 1154, 1156, 1165, 1202, 1211, 1222, 1226, 1231, 1235, 1253, 1264, 1301, 1312, 1316, 1325, 1343, 1345, 1402, 1411, 1424, 1433, 1442,
8	13, 15, 21, 23, 27, 35, 37, 45, 51, 53, 57, 65, 73, 75, 103, 107, 111, 117, 123, 131, 141, 145, 147, 153, 155, 161, 177, 203, 211, 213, 225, 227, 235, 243, 247, 255, 263, 265, 277, 301, 305, 307, 323, 337, 343, 345, 351, 357, 361, 373, 401, 407, 415, 417, 425, 431, 433, 445, 463, 467, 471, 475, 513, 521, 533, 535, 541, 547, 557, 565, 573, 577, 605, 615, 621, 631, 643, 645, 657, 661, 667, 673, 701, 711, 715, 717, 723, 737, 747, 753, 763, 767, 775, 1011, 1013, 1035, 1043, 1055, 1063, 1071,
9	12, 14, 18, 21, 25, 32, 34, 41, 45, 47, 52, 58, 65, 67, 74, 78, 81, 87, 102, 108, 117, 122, 124, 128, 131, 135, 151, 155, 162, 164, 175, 177, 184, 201, 205, 212, 218, 221, 232, 234, 238, 241, 254, 267, 272, 274, 278, 285, 287, 308, 315, 322, 328, 331, 337, 342, 344, 355, 371, 375, 377, 382, 407, 414, 425,

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	427, 432, 438, 447, 454, 461, 465, 472, 481, 485, 504, 515, 517, 528, 531, 537, 542, 548, 557, 562, 564, 568, 582, 601, 605, 614, 618, 625, 638, 641, 661, 667, 678, 685, 702,
10	11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107, 109, 113, 127, 131, 137, 139, 149, 151, 157, 163, 167, 173, 179, 181, 191, 193, 197, 199, 211, 223, 227, 229, 233, 239, 241, 251, 257, 263, 269, 271, 277, 281, 283, 293, 307, 311, 313, 317, 331, 337, 347, 349, 353, 359, 367, 373, 379, 383, 389, 397, 401, 409, 419, 421, 431, 433, 439, 443, 449, 457, 461, 463, 467, 479, 487, 491, 499, 503, 509, 521, 523, 541, 547, 557, 563, 569,
11	12, 16, 18, 21, 27, 29, 34, 38, 3A, 43, 49, 54, 56, 61, 65, 67, 72, 76, 81, 89, 92, 94, 98, 9A, A3, 106, 10A, 115, 117, 126, 128, 133, 139, 142, 148, 153, 155, 164, 166, 16A, 171, 182, 193, 197, 199, 1A2, 1A8, 1AA, 209, 214, 21A, 225, 227, 232, 236, 238, 247, 25A, 263, 265, 269, 281, 287, 296, 298, 2A1, 2A7, 304, 30A, 315, 319, 324, 331, 335, 342, 351, 353, 362, 364, 36A, 373, 379, 386, 38A, 391, 395, 3A6, 403, 407, 414, 418, 423, 434, 436, 452, 458, 467, 472, 478, 47A,
12	11, 15, 17, 1B, 25, 27, 31, 35, 37, 3B, 45, 4B, 51, 57, 5B, 61, 67, 6B, 75, 81, 85, 87, 8B, 91, 95, A7, AB, B5, B7, 105, 107, 111, 117, 11B, 125, 12B, 131, 13B, 141, 145, 147, 157, 167, 16B, 171, 175, 17B, 181, 18B, 195, 19B, 1A5, 1A7, 1B1, 1B5, 1B7, 205, 217, 21B, 221, 225, 237, 241, 24B, 251, 255, 25B, 267, 271, 277, 27B, 285, 291, 295, 2A1, 2AB, 2B1, 2BB, 301, 307, 30B, 315, 321, 325, 327, 32B, 33B, 347, 34B, 357, 35B, 365, 375, 377, 391, 397, 3A5, 3AB, 3B5, 3B7,
13	14, 16, 1A, 23, 25, 2B, 32, 34, 38, 41, 47, 49, 52, 56, 58, 61, 65, 6B, 76, 7A, 7C, 83, 85, 89, 9A, A1, A7, A9, B6, B8, C1, C7, CB, 104, 10A, 10C, 119, 11B, 122, 124, 133, 142, 146, 148, 14C, 155, 157, 164, 16A, 173, 179, 17B, 184, 188, 18A, 197, 1A8, 1AC, 1B1, 1B5, 1C6, 1CC, 209, 20B, 212, 218, 223, 229, 232, 236, 23C, 247, 24B, 256, 263, 265, 272, 274, 27A, 281, 287, 292, 296, 298, 29C, 2AB, 2B6, 2BA, 2C5, 2C9, 302, 311, 313, 328, 331, 33B, 344, 34A, 34C, 355,
14	13, 15, 19, 21, 23, 29, 2D, 31, 35, 3B, 43, 45, 4B, 51, 53, 59, 5D, 65, 6D, 73, 75, 79, 7B, 81, 91, 95, 9B, 9D, A9, AB, B3, B9, BD, C5, CB, CD, D9, DB, 101, 103, 111, 11D, 123, 125, 129, 131, 133, 13D, 145, 14B, 153, 155, 15B, 161, 163, 16D, 17D, 183, 185, 189, 199, 1A1, 1AB, 1AD, 1B3, 1B9, 1C3, 1C9, 1D1, 1D5, 1DB, 205, 209, 213, 21D, 221, 22B, 22D, 235, 239, 241, 249, 24D, 251, 255, 263, 26B, 271, 279, 27D, 285, 293, 295, 2A9, 2B1, 2BB, 2C3, 2C9, 2CB, 2D3,
15	12, 14, 18, 1E, 21, 27, 2B, 2D, 32, 38, 3E, 41, 47, 4B, 4D, 54, 58, 5E, 67, 6B, 6D, 72, 74, 78, 87, 8B, 92, 94, 9E, A1, A7, AD, B2, B8, BE, C1, CB, CD, D2, D4, E1, ED, 102, 104, 108, 10E, 111, 11B, 122, 128, 12E, 131, 137, 13B, 13D, 148, 157, 15B, 15D, 162, 171, 177, 182, 184, 188, 18E, 197, 19D, 1A4, 1A8, 1AE, 1B7, 1BB, 1C4, 1CE, 1D1, 1DB, 1DD, 1E4, 1E8, 1EE, 207, 20B, 20D, 212, 21E, 227, 22B, 234, 238, 23E, 24B, 24D, 261, 267, 272, 278, 27E, 281, 287,
16	11, 13, 17, 1D, 1F, 25, 29, 2B, 2F, 35, 3B, 3D, 43, 47, 49, 4F, 53, 59, 61, 65, 67, 6B, 6D, 71, 7F, 83, 89, 8B, 95, 97, 9D, A3, A7, AD, B3, B5, BF, C1, C5, C7, D3, DF, E3, E5, E9, EF, F1, FB, 101, 107, 10D, 10F, 115, 119, 11B, 125, 133,

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	137, 139, 13D, 14B, 151, 15B, 15D, 161, 167, 16F, 175, 17B, 17F, 185, 18D, 191, 199, 1A3, 1A5, 1AF, 1B1, 1B7, 1BB, 1C1, 1C9, 1CD, 1CF, 1D3, 1DF, 1E7, 1EB, 1F3, 1F7, 1FD, 209, 20B, 21D, 223, 22D, 233, 239, 23B, 241,
17	12, 16, 1C, 1E, 23, 27, 29, 2D, 32, 38, 3A, 3G, 43, 45, 4B, 4F, 54, 5C, 5G, 61, 65, 67, 6B, 78, 7C, 81, 83, 8D, 8F, 94, 9A, 9E, A3, A9, AB, B4, B6, BA, BC, C7, D2, D6, D8, DC, E1, E3, ED, F2, F8, FE, FG, G5, G9, GB, 104, 111, 115, 117, 11B, 128, 12E, 137, 139, 13D, 142, 14A, 14G, 155, 159, 15F, 166, 16A, 171, 17B, 17D, 186, 188, 18E, 191, 197, 19F, 1A2, 1A4, 1A8, 1B3, 1BB, 1BF, 1C6, 1CA, 1CG, 1DB, 1DD, 1EE, 1F3, 1FD, 1G2, 1G8, 1GA, 1GG, 209,
18	11, 15, 1B, 1D, 21, 25, 27, 2B, 2H, 35, 37, 3D, 3H, 41, 47, 4B, 4H, 57, 5B, 5D, 5H, 61, 65, 71, 75, 7B, 7D, 85, 87, 8D, 91, 95, 9B, 9H, A1, AB, AD, AH, B1, BD, C7, CB, CD, CH, D5, D7, DH, E5, EB, EH, F1, F7, FB, FD, G5, H1, H5, H7, HB, 107, 10D, 115, 117, 11B, 11H, 127, 12D, 131, 135, 13B, 141, 145, 14D, 155, 157, 15H, 161, 167, 16B, 16H, 177, 17B, 17D, 17H, 18B, 191, 195, 19D, 19H, 1A5, 1AH, 1B1, 1C1, 1C7, 1CH, 1D5, 1DB, 1DD, 1E1, 1EB,
19	14, 1A, 1C, 1I, 23, 25, 29, 2F, 32, 34, 3A, 3E, 3G, 43, 47, 4D, 52, 56, 58, 5C, 5E, 5I, 6D, 6H, 74, 76, 7G, 7I, 85, 8B, 8F, 92, 98, 9A, A1, A3, A7, A9, B2, BE, BI, C1, C5, CB, CD, D4, DA, DG, E3, E5, EB, EF, EH, F8, G3, G7, G9, GD, H8, HE, I5, I7, IB, IH, 106, 10C, 10I, 113, 119, 11H, 122, 12A, 131, 133, 13D, 13F, 142, 146, 14C, 151, 155, 157, 15B, 164, 16C, 16G, 175, 179, 17F, 188, 18A, 199, 19F, 1A6, 1AC, 1AI, 1B1, 1B7, 1BH, 1C4,
20	13, 19, 1B, 1H, 21, 23, 27, 2D, 2J, 31, 37, 3B, 3D, 3J, 43, 49, 4H, 51, 53, 57, 59, 5D, 67, 6B, 6H, 6J, 79, 7B, 7H, 83, 87, 8D, 8J, 91, 9B, 9D, 9H, 9J, AB, B3, B7, B9, BD, BJ, C1, CB, CH, D3, D9, DB, DH, E1, E3, ED, F7, FB, FD, FH, GB, GH, H7, H9, HD, HJ, I7, ID, IJ, J3, J9, JH, 101, 109, 10J, 111, 11B, 11D, 11J, 123, 129, 12H, 131, 133, 137, 13J, 147, 14B, 14J, 153, 159, 161, 163, 171, 177, 17H, 183, 189, 18B, 18H, 197, 19D,
21	12, 18, 1A, 1G, 1K, 21, 25, 2B, 2H, 2J, 34, 38, 3A, 3G, 3K, 45, 4D, 4H, 4J, 52, 54, 58, 61, 65, 6B, 6D, 72, 74, 7A, 7G, 7K, 85, 8B, 8D, 92, 94, 98, 9A, A1, AD, AH, AJ, B2, B8, BA, BK, C5, CB, CH, CJ, D4, D8, DA, DK, ED, EH, EJ, F2, FG, G1, GB, GD, GH, H2, HA, HG, I1, I5, IB, IJ, J2, JA, JK, K1, KB, KD, KJ, 102, 108, 10G, 10K, 111, 115, 11H, 124, 128, 12G, 12K, 135, 13H, 13J, 14G, 151, 15B, 15H, 162, 164, 16A, 16K, 175,
22	11, 17, 19, 1F, 1J, 1L, 23, 29, 2F, 2H, 31, 35, 37, 3D, 3H, 41, 49, 4D, 4F, 4J, 4L, 53, 5H, 5L, 65, 67, 6H, 6J, 73, 79, 7D, 7J, 83, 85, 8F, 8H, 8L, 91, 9D, A3, A7, A9, AD, AJ, AL, B9, BF, BL, C5, C7, CD, CH, CJ, D7, DL, E3, E5, E9, F1, F7, FH, FJ, G1, G7, GF, GL, H5, H9, HF, I1, I5, ID, J1, J3, JD, JF, JL, K3, K9, KH, KL, L1, L5, LH, 103, 107, 10F, 10J, 113, 11F, 11H, 12D, 12J, 137, 13D, 13J, 13L, 145, 14F, 14L,
23	16, 18, 1E, 1I, 1K, 21, 27, 2D, 2F, 2L, 32, 34, 3A, 3E, 3K, 45, 49, 4B, 4F, 4H, 4L, 5C, 5G, 5M, 61, 6B, 6D, 6J, 72, 76, 7C, 7I, 7K, 87, 89, 8D, 8F, 94, 9G, 9K, 9M, A3, A9, AB, AL, B4, BA, BG, BI, C1, C5, C7, CH, D8, DC, DE, DI, E9, EF, F2, F4, F8, FE, FM, G5, GB, GF, GL, H6, HA, HI, I5, I7, IH, IJ, J2, J6, JC, JK, K1, K3, K7, KJ, L4, L8, LG, LK, M3, MF, MH, 10C, 10I, 115, 11B, 11H, 11J, 122, 12C, 12I, 131,
24	15, 17, 1D, 1H, 1J, 1N, 25, 2B, 2D, 2J, 2N, 31, 37, 3B, 3H, 41, 45, 47, 4B, 4D, 4H, 57, 5B, 5H, 5J, 65, 67, 6D, 6J, 6N, 75, 7B, 7D, 7N, 81, 85, 87, 8J, 97, 9B,

	9D, 9H, 9N, A1, AB, AH, AN, B5, B7, BD, BH, BJ, C5, CJ, CN, D1, D5, DJ, E1, EB, ED, EH, EN, F7, FD, FJ, FN, G5, GD, GH, H1, HB, HD, HN, I1, I7, IB, IH, J1, J5, J7, JB, JN, K7, KB, KJ, KN, L5, LH, LJ, MD, MJ, N5, NB, NH, NJ, 101, 10B, 10H, 10N,
25	14, 16, 1C, 1G, 1I, 1M, 23, 29, 2B, 2H, 2L, 2N, 34, 38, 3E, 3M, 41, 43, 47, 49, 4D, 52, 56, 5C, 5E, 5O, 61, 67, 6D, 6H, 6N, 74, 76, 7G, 7I, 7M, 7O, 8B, 8N, 92, 94, 98, 9E, 9G, A1, A7, AD, AJ, AL, B2, B6, B8, BI, C7, CB, CD, CH, D6, DC, DM, DO, E3, E9, EH, EN, F4, F8, FE, FM, G1, G9, GJ, GL, H6, H8, HE, HI, HO, I7, IB, ID, IH, J4, JC, JG, JO, K3, K9, KL, KN, LG, LM, M7, MD, MJ, ML, N2, NC, NI, NO,
26	13, 15, 1B, 1F, 1H, 1L, 21, 27, 29, 2F, 2J, 2L, 31, 35, 3B, 3J, 3N, 3P, 43, 45, 49, 4N, 51, 57, 59, 5J, 5L, 61, 67, 6B, 6H, 6N, 6P, 79, 7B, 7F, 7H, 83, 8F, 8J, 8L, 8P, 95, 97, 9H, 9N, A3, A9, AB, AH, AL, AN, B7, BL, BP, C1, C5, CJ, CP, D9, DB, DF, DL, E3, E9, EF, EJ, EP, F7, FB, FJ, G3, G5, GF, GH, GN, H1, H7, HF, HJ, HL, HP, IB, IJ, IN, J5, J9, JF, K1, K3, KL, L1, LB, LH, LN, LP, M5, MF, ML, N1,
27	12, 14, 1A, 1E, 1G, 1K, 1Q, 25, 27, 2D, 2H, 2J, 2P, 32, 38, 3G, 3K, 3M, 3Q, 41, 45, 4J, 4N, 52, 54, 5E, 5G, 5M, 61, 65, 6B, 6H, 6J, 72, 74, 78, 7A, 7M, 87, 8B, 8D, 8H, 8N, 8P, 98, 9E, 9K, 9Q, A1, A7, AB, AD, AN, BA, BE, BG, BK, C7, CD, CN, CP, D2, D8, DG, DM, E1, E5, EB, EJ, EN, F4, FE, FG, FQ, G1, G7, GB, GH, GP, H2, H4, H8, HK, I1, I5, ID, IH, IN, J8, JA, K1, K7, KH, KN, L2, L4, LA, LK, LQ, M5,
28	11, 13, 19, 1D, 1F, 1J, 1P, 23, 25, 2B, 2F, 2H, 2N, 2R, 35, 3D, 3H, 3J, 3N, 3P, 41, 4F, 4J, 4P, 4R, 59, 5B, 5H, 5N, 5R, 65, 6B, 6D, 6N, 6P, 71, 73, 7F, 7R, 83, 85, 89, 8F, 8H, 8R, 95, 9B, 9H, 9J, 9P, A1, A3, AD, AR, B3, B5, B9, BN, C1, CB, CD, CH, CN, D3, D9, DF, DJ, DP, E5, E9, EH, ER, F1, FB, FD, FJ, FN, G1, G9, GD, GF, GJ, H3, HB, HF, HN, HR, I5, IH, IJ, J9, JF, JP, K3, K9, KB, KH, KR, L5, LB,
29	12, 18, 1C, 1E, 1I, 1O, 21, 23, 29, 2D, 2F, 2L, 2P, 32, 3A, 3E, 3G, 3K, 3M, 3Q, 4B, 4F, 4L, 4N, 54, 56, 5C, 5I, 5M, 5S, 65, 67, 6H, 6J, 6N, 6P, 78, 7K, 7O, 7Q, 81, 87, 89, 8J, 8P, 92, 98, 9A, 9G, 9K, 9M, A3, AH, AL, AN, AR, BC, BI, BS, C1, C5, CB, CJ, CP, D2, D6, DC, DK, DO, E3, ED, EF, EP, ER, F4, F8, FE, FM, FQ, FS, G3, GF, GN, GR, H6, HA, HG, HS, I1, IJ, IP, J6, JC, JI, JK, JQ, K7, KD, KJ, KL,
30	11, 17, 1B, 1D, 1H, 1N, 1T, 21, 27, 2B, 2D, 2J, 2N, 2T, 37, 3B, 3D, 3H, 3J, 3N, 47, 4B, 4H, 4J, 4T, 51, 57, 5D, 5H, 5N, 5T, 61, 6B, 6D, 6H, 6J, 71, 7D, 7H, 7J, 7N, 7T, 81, 8B, 8H, 8N, 8T, 91, 97, 9B, 9D, 9N, A7, AB, AD, AH, B1, B7, BH, BJ, BN, BT, C7, CD, CJ, CN, CT, D7, DB, DJ, DT, E1, EB, ED, EJ, EN, ET, F7, FB, FD, FH, FT, G7, GB, GJ, GN, GT, HB, HD, I1, I7, IH, IN, IT, J1, J7, JH, JN, JT, K1,
31	16, 1A, 1C, 1G, 1M, 1S, 1U, 25, 29, 2B, 2H, 2L, 2R, 34, 38, 3A, 3E, 3G, 3K, 43, 47, 4D, 4F, 4P, 4R, 52, 58, 5C, 5I, 5O, 5Q, 65, 67, 6B, 6D, 6P, 76, 7A, 7C, 7G, 7M, 7O, 83, 89, 8F, 8L, 8N, 8T, 92, 94, 9E, 9S, A1, A3, A7, AL, AR, B6, B8, BC, BI, BQ, C1, C7, CB, CH, CP, CT, D6, DG, DI, DS, DU, E5, E9, EF, EN, ER, ET, F2, FE, FM, FQ, G3, G7, GD, GP, GR, HE, HK, HU, I5, IB, ID, IJ, IT, J4, JA, JC, JI,
32	15, 19, 1B, 1F, 1L, 1R, 1T, 23, 27, 29, 2F, 2J, 2P, 31, 35, 37, 3B, 3D, 3H, 3V,

	43, 49, 4B, 4L, 4N, 4T, 53, 57, 5D, 5J, 5L, 5V, 61, 65, 67, 6J, 6V, 73, 75, 79, 7F, 7H, 7R, 81, 87, 8D, 8F, 8L, 8P, 8R, 95, 9J, 9N, 9P, 9T, AB, AH, AR, AT, B1, B7, BF, BL, BR, BV, C5, CD, CH, CP, D3, D5, DF, DH, DN, DR, E1, E9, ED, EF, EJ, EV, F7, FB, FJ, FN, FT, G9, GB, GT, H3, HD, HJ, HP, HR, I1, IB, IH, IN, IP, IV,
33	14, 18, 1A, 1E, 1K, 1Q, 1S, 21, 25, 27, 2D, 2H, 2N, 2V, 32, 34, 38, 3A, 3E, 3S, 3W, 45, 47, 4H, 4J, 4P, 4V, 52, 58, 5E, 5G, 5Q, 5S, 5W, 61, 6D, 6P, 6T, 6V, 72, 78, 7A, 7K, 7Q, 7W, 85, 87, 8D, 8H, 8J, 8T, 9A, 9E, 9G, 9K, A1, A7, AH, AJ, AN, AT, B4, BA, BG, BK, BQ, C1, C5, CD, CN, CP, D2, D4, DA, DE, DK, DS, DW, E1, E5, EH, EP, ET, F4, F8, FE, FQ, FS, GD, GJ, GT, H2, H8, HA, HG, HQ, HW, I5, I7, ID,
34	13, 17, 19, 1D, 1J, 1P, 1R, 1X, 23, 25, 2B, 2F, 2L, 2T, 2X, 31, 35, 37, 3B, 3P, 3T, 41, 43, 4D, 4F, 4L, 4R, 4V, 53, 59, 5B, 5L, 5N, 5R, 5T, 67, 6J, 6N, 6P, 6T, 71, 73, 7D, 7J, 7P, 7V, 7X, 85, 89, 8B, 8L, 91, 95, 97, 9B, 9P, 9V, A7, A9, AD, AJ, AR, AX, B5, B9, BF, BN, BR, C1, CB, CD, CN, CP, CV, D1, D7, DF, DJ, DL, DP, E3, EB, EF, EN, ER, EX, FB, FD, FV, G3, GD, GJ, GP, GR, GX, H9, HF, HL, HN, HT,
35	12, 16, 18, 1C, 1I, 1O, 1Q, 1W, 21, 23, 29, 2D, 2J, 2R, 2V, 2X, 32, 34, 38, 3M, 3Q, 3W, 3Y, 49, 4B, 4H, 4N, 4R, 4X, 54, 56, 5G, 5I, 5M, 5O, 61, 6D, 6H, 6J, 6N, 6T, 6V, 76, 7C, 7I, 7O, 7Q, 7W, 81, 83, 8D, 8R, 8V, 8X, 92, 9G, 9M, 9W, 9Y, A3, A9, AH, AN, AT, AX, B4, BC, BG, BO, BY, C1, CB, CD, CJ, CN, CT, D2, D6, D8, DC, DO, DW, E1, E9, ED, EJ, EV, EX, FG, FM, FW, G3, G9, GB, GH, GR, GX, H4, H6, HC,
36	11, 15, 17, 1B, 1H, 1N, 1P, 1V, 1Z, 21, 27, 2B, 2H, 2P, 2T, 2V, 2Z, 31, 35, 3J, 3N, 3T, 3V, 45, 47, 4D, 4J, 4N, 4T, 4Z, 51, 5B, 5D, 5H, 5J, 5V, 67, 6B, 6D, 6H, 6N, 6P, 6Z, 75, 7B, 7H, 7J, 7P, 7T, 7V, 85, 8J, 8N, 8P, 8T, 97, 9D, 9N, 9P, 9T, 9Z, A7, AD, AJ, AN, AT, B1, B5, BD, BN, BP, BZ, C1, C7, CB, CH, CP, CT, CV, CZ, DB, DJ, DN, DV, DZ, E5, EH, EJ, F1, F7, FH, FN, FT, FV, G1, GB, GH, GN, GP, GV,

The primes in $M(L_b)$ are called **minimal prime base** b in this article, although in fact this name should be used for L_b is the language of base-b representations of the prime numbers, where primes > b is not required, this problem is an extension of the <u>original minimal prime problem</u>, to include <u>CRUS Sierpinski/Riesel</u> conjectures base b with k-values < b, i.e. the smallest prime of the form k^*b^n+1 and k^*b^n-1 for all k < b. The original minimal prime base b puzzle does not cover CRUS Sierpinski/Riesel conjectures base b with $\underline{CK} < b$, since in Riesel side, the prime is not minimal prime in original definition if either k-1 or b-1 (or both) is prime, and in Sierpinski side, the prime is not minimal prime in original definition, since it is OT^{34205} in base 30, and T (= 29 in decimal) is prime, but it is minimal prime in base 30 if only primes > b ase are counted), but this extended version of minimal prime base b problem does, this requires a restriction of prime > b, and the primes $\le b$ (including the k-1, b-1, k) are not allowed, also, this problem is better than the original minimal prime problem since this problem does not regard whether 1 is considered as prime or not.

For example, 857 is a minimal prime in decimal because there is no prime > 10 among the shorter subsequences of the digits: 8, 5, 7, 85, 87, 57. The subsequence does not have to consist of consecutive digits, so 149 is not a minimal prime in decimal (because 19 is prime

and 19 > 10). But it does have to be in the same order; so, for example, 991 is still a minimal prime in decimal even though a subset of the digits can form the shorter prime 19 > 10 by changing the order.

A summary of the results of our <u>algorithm</u> is presented in the table in the next section; I completely solved all bases up to 16 except for bases 14, 16, and the odd bases >6 (the proofs are at the end of this article), for bases 14, 16, and the odd bases >6, I only found all minimal primes up to certain limit (about 2³²) and some larger minimal primes (such as 3¹⁶1 in base 7 and 54¹¹ in base 9). I left as a challenge to readers the task of solving (finding all minimal primes and proving that these are all such primes) bases 7, 9, 11, 13, 14, 15, 16, and bases 17 through 36 (this will be a hard problem, e.g. base 23 has a minimal prime 9E⁸⁰⁰⁸⁷³, and base 30 has a minimal prime OT³⁴²⁰⁵).

Problems about the digits of prime numbers have a long history, and many of them are still <u>unsolved</u>. For example, are there infinitely many primes, all of whose base-10 digits are 1? Currently, there are only five such "<u>repunits</u>" known, corresponding to $(10^p - 1)/9$ for $p \in \{2, 19, 23, 317, 1031\}$. It seems likely that four more are given by $p \in \{49081, 86453, 109297, 270343, 5794777, 8177207\}$, but this has not yet been <u>rigorously proven</u>. This problem also exists for other bases, e.g. for base 12, there are only nine proven such numbers, corresponding to $(12^p - 1)/11$ for $p \in \{2, 3, 5, 19, 97, 109, 317, 353, 701\}$. It seems likely that five more are given by $p \in \{9739, 14951, 37573, 46889, 769543\}$, but this has not yet been rigorously proven. However, for some bases there exists no such primes, these bases are 9, 25, 32, 49, 64, 81, 121, 125, 144, ..., this is because the numbers with all digits 1 in these bases can be factored algebraically (this strategy (algebraic factorization) will also be used in our problem, to show that some families (such as 10^n1 in base 8 and 38^n in base 9) contains no primes > base).

Table

In the " $max(x, x \in L_b)$ " column, xy^nz means xyyy...yyyz with n y's (the n-value is written in decimal), not y to the nth power.

b	$ M(L_b) $	$max(x, x \in M(L_b))$	$max(x , x \in M(L_b))$	Algebraic form of $max(x, x \in M(L_b))$
2	1	11	2	3
3	3	111	3	13
4	5	221	3	41
5	22	10 ⁹³ 13	96	5 ⁹⁵ +8
6	11	40041	5	5209
7 ^a	≥71	3 ¹⁶ 1	17	$\frac{7^{17}-5}{2}$

8	75	4 ²²⁰ 7	221	$\frac{4\cdot 8}{7} \frac{221}{7} + 17$
9 ^a	≥149	30115811	1161	3·9 ¹¹⁶⁰ +10
10	77	50 ²⁸ 27	31	5·10 ³⁰ +27
11 ^a	≥914	557 ¹⁰¹¹ or 57 ⁿ with <i>n</i> >50000	1013	$\frac{607 \cdot 11^{-1011} - 7}{10}$
12	106	40 ³⁹ 77	42	4·12 ⁴¹ +91
13 ^{ab}	≥2492	80 ³²⁰¹⁷ 111 or 95 ⁿ with <i>n</i> >50000 or A3 ⁿ A with <i>n</i> >50000	32021	8·13 ³²⁰²⁰ +183
14 ^a	≥605	4D ¹⁹⁶⁹⁸	19699	5·14 ¹⁹⁶⁹⁸ -1
15 ^a	≥1171	7 ¹⁵⁵ 97	157	$\frac{15^{-157} + 59}{2}$
16 ^{ab}	≥2050	DB ³²²³⁴	32235	$\frac{206 \cdot 16^{-32234} - 13}{15}$

^a I have not proved these bases, these are the largest elements in $M(L_b)$ known to me, and they are just the <u>lower bounds</u>.

Lists

The sets of bases 14, 16, and the odd bases >6 are possibly not complete, only tested to the test limit in the discussion of these bases and found the smallest (probable) prime in some unsolved families of these bases, but there may be more unsolved families not found by me.

There are five unsolved families found by me and searched to length 50000 with no (probable) prime found, they are 57ⁿ in base 11, 95ⁿ in base 13, A3ⁿA in base 13, 3ⁿAF in base 16, and 4ⁿDD in base 16.

base 2

11

base 3

12, 21, 111

^b Data based on results of <u>probable primality tests</u>.

base 4

11, 13, 23, 31, 221

base 5

12, 21, 23, 32, 34, 43, 104, 111, 131, 133, 313, 401, 414, 3101, 10103, 14444, 30301, 33001, 33331, 44441, 300031,

base 6

11, 15, 21, 25, 31, 35, 45, 51, 4401, 4441, 40041

base 7 (not proved, only checked to the prime 5100000001)

base 8

base 9 (not proved, only checked to the prime 8333333335)

12, 14, 18, 21, 25, 32, 34, 41, 45, 47, 52, 58, 65, 67, 74, 78, 81, 87, 117, 131, 135, 151, 155, 175, 177, 238, 272, 308, 315, 331, 337, 355, 371, 375, 377, 438, 504, 515, 517, 531, 537, 557, 564, 601, 638, 661, 702, 711, 722, 735, 737, 751, 755, 757, 771, 805, 838, 1011, 1015, 1101, 1701, 2027, 2207, 3017, 3057, 3101, 3501, 3561, 3611, 3688, 3868, 5035, 5051, 5071, 5101, 5501, 5554, 5705, 5707, 7017, 7075, 7105, 7301, 8535, 8544, 8555, 8854, 20777, 22227, 22777, 30161, 33388, 50161, 50611, 53335, 55111, 55535, 55551, 57061,

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57775, 70631, 71007, 77207, 100037, 100071, 100761, 105007, 270707, 301111, 305111,
333035, 333385, 333835, 338885, 350007, 500075, 530005, 555611, 631111, 720707,
2770007, 3030335, 7776662, 30300005, 30333335, 38333335, 51116111, 70000361,
300030005, 300033305, 3511111111, 1300000007, 51611111111, 8333333335, ...,
30000000035, ..., 311111111161, ..., 54444444444, ..., 200000000007, ...,
570000000001, ..., 88888888833335, ..., 10000000000507, ..., 5111111111111161, ...,
888888888888888888335, ..., 3000000000000000000051, ...,
777777777777777777777777777777777
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base 10

11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 227, 251, 257, 277, 281, 349, 409, 449, 499, 521, 557, 577, 587, 727, 757, 787, 821, 827, 857, 877, 881, 887, 991, 2087, 2221, 5051, 5081, 5501, 5581, 5801, 5851, 6469, 6949, 8501, 9001, 9049, 9221, 9551, 9649, 9851, 9949, 20021, 20201, 50207, 60649, 80051, 666649, 946669,

base 11 (not proved, only checked to the prime 1500000001)

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base 12

base 13 (not proved, only checked to the prime 1010008001, also the numbers B0⁶⁵⁴⁰BBA and 80³²⁰¹⁷111 are only probable primes, i.e. not proven primes)

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base 14 (not proved, only checked to the prime 108000000D)

13, 15, 19, 21, 23, 29, 2D, 31, 35, 3B, 43, 45, 4B, 51, 53, 59, 5D, 65, 6D, 73, 75, 79, 7B, 81, 91, 95, 9B, 9D, A9, AB, B3, B9, BD, C5, CB, CD, D9, DB, 101, 111, 11D, 161, 17D, 1A1, 1AD, 1D1, 205, 22B, 255, 26B, 285, 2BB, 30D, 33D, 349, 389, 3D3, 40D, 41D, 44D, 469, 471, 499, 4AD, 4C1, 4D1, 50B, 525, 52B, 55B, 585, 58B, 60B, 61B, 683, 689, 6A3, 701, 71D, 741, 771, 77D, 7DD, 803, 80B, 825, 82B, 833, 839, 84D, 86B, 88D, 893, 8AD, 8BB, 8D3, 983, 9A3, A0D, A1D, A25, A41, A4D, AAD, AC1, AC3, AD1, B05, B41, B5B, B6B, B85, BA1, BB1, C49, C61, C83, C89, CC1, D01, D03, D33, D4D, D61, D71, D7D, D83, DA1, DA5, DC3, DD1, 10BB, 10DD, 128B, 18DD, 1B71, 1B8B, 1C41, 1D8D, 2BA5, 33A3, 347D, 3863, 3A7D, 40A1, 478D, 4809, 48C9, 48DD, 4C09, 4D8D, 56BB, 6049, 60C1, 6171, 61C1, 628B, 6409, 6461, 64A1, 6711, 6761, 67A1, 6A11, 6A71, 6B71, 6B8B, 708D, 748D, 7611, 780D, 7CA1, 8009, 8055, 807D, 8089, 80C9, 80DD, 837D, 8505, 88A3, 89C9, 8A05, 8A85, 8C63, 8C99, 8CC9, 9489, 94C9, 9869, 9899, A063, A071, A0A1, A0A3, A303, A603, A611, A633, A663, A83D, A883, A8A5, AA01, AD8D, B02B, B061, B08B, B10B, BC01, C0A3, C141, C171, C401, C441, CCA3, D005, D055, D08D, D18D, D1C1, D225, D80D, D885, DC11, 1062B, 11BBB, 1668B, 1B00B, 1BBBB, 1D00D, 1DD0D, 1DDDD, 2B225, 30083, 308A3, 33383, 338C3, 37A8D, 38883, 38AA3, 38DDD, 3A033, 3A8DD, 3AA83, 3AAA3, 3CA63, 40061, 400C9, 40601, 40641, 44141, 444C9, 44601, 44661, 44849, 44A01, 44AA1, 46061, 46411, 48489, 5B555, 5BA55, 5BBB5, 60A01, 60AA1, 64401, 66411, 66601, 66649, 6666B, 666B1, 66949, 66B11, 6BC11, 766C1, 7A661, 7AA11, 80649, 80669, 80699, 80885, 80949, 80AA5, 84409, 84849, 84889, 85A55, 86099, 86449, 86609, 86999, 86C09, 8700D, 884C9, 88805, 88809, 88899, 88B55, 89069, 89099, 89449, 89609, 89889, 89999, 8A5A5, 8AA55, 8AAA3, 8B555, 8BAA5, 8CAA3, 908C9, 90989, 94449, 98C09, 99089, 99409, 99949, A0085, A0A85, A7A11, A7A61, A8005, AA383, AA711, AA7A1, AA855, ADDD5, B011B, B07C1, B0C71, B11BB, B2225, B5555, B5AA5, B67C1, B76C1, B7C11, BB2B5, BB88B, BBB55, C04A1, C0A01, C0AA1, C3A03, D0ADD, D3DDD, DA8DD, DD38D, DDA63, DDD25, DDD55, DDDAD, 10006B, 11088B, 116B2B, 166B2B, 20008B, 300A33, 30A363, 3CA003, 400041, 400489, 401441, 404001, 404089, 404411, 404441, 404CC9, 406611, 40CCC9, 440001, 440409, 444041, 444611, 444641, 460011, 460041, 466401, 4A0001, 4A6AA1, 5BAAA5, 600411, 604041, 640011, 660441, 6666C1, 666A61, 6A0061, 6A0601, 6A6061, 6AAA61, 76A6A1, 8000A5, 85B5A5, 869669, 884049, 8885A5, 888669, 8886C3, 888BA5, 888C69, 889849, 896669, 898049, 900049, 900649, 908449, 940009, 969649, 988849, 990649, A08555, A33333, A3A333, A3A363, A6A6A1, A6AAA1, A88855, AAA085, AAA3A3, ADAAA3, ADD085, B0001B, B000C1, B00711, B2000B, B2AAA5, B60071, B66011, B66071, B666C1, B66C11, BA5A55, BAA5A5, BAAA55, C00A11, C00A71, C3A333, CA0333, CA3AA3, CAAA03, CAAA11, CAAAA1, D1000D, D3DA8D, DDAAA3, 100008B, 100020B, 3000A03, 3000CA3, 308CCC3, 38CCCC3, 4000011, 4000449, 4040449, 4400089, 4440009, 4440011, 4440449, 4440889, 4444441, 4664441, 4666AA1, 46AAAA1, 4A66A61, 4CCCCC9, 6000001, 6000141, 6000441, 6000A61,

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base 15 (not proved, only checked to the prime 555555557)

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49A8, 4E34, 5037, 507D, 5091, 509B, 5107, 5161, 5202, 53C7, 5552, 570B, 590B, 590D, 59C7, 5A5B, 5C97, 5D0D, 5DAB, 6061, 6151, 6191, 6511, 6601, 6911, 707B, 7091, 7097, 70AE, 70BB, 70CE, 70DB, 7561, 760E, 7691, 76CE, 7907, 7961, 7A0E, 7A3B, 7AEE, 7B0B, 7BAB, 7C0E, 7C77, 7CAE, 7D0B, 7D61, 7DAB, 7E5B, 7E6E, 7E7B, 7EBB, 8098, 811D, 8191, 835D, 853D, 8881, 8908, 8951, 8968, 899D, 8D3D, 8D5D, 8D6E, 8DDD, 8E98, 9011, 9037, 9097, 90D7, 9301, 93C7, 95C7, 9611, 9631, 96A8, 9811, 9851, 989D, 990B, 990D, 998D, 99AB, 99C7, 99D8, 9A08, 9A9B, 9AA8, 9ABB, 9B61, 9BC7, 9D0B, 9DAB, 9DC7, 9DD8, A052, A304, A502, A55B, A9BB, AB04, AB64, B09D, B107, B10B, B161, B1AB, B1C7, B30D, B3C7, B50B, B664, B691, B6A4, B707, B761, B90D, B961, BA5B, BABB, BBAB, BBB4, BC37, BC77, C777, C937, C997, D011, D03D, D05D, D09B, D0B1, D0BD, D101, D10B, D30D, D3AB, D507, D50D, D66E, D761, D7DE, D811, D85D, D86E, D89D, D8C8, D8E8, D9AB, D9D8, DA3B, DA9B, DABB, DB01, DB61, DBAB, DC88, DD07, DD0B, DD7E, DD8D, DDE7, DE6E, E252, 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900DB, 901C7, 90C57, 90D8D, 91007, 91061, 9199B, 95997, 96068, 96561, 99397, 99537, 9999B, 999B7, 999D7, 999DB, 999DD, 99BBB, 99DBB, 99DD7, 99DDD, 9B007, 9B00B, 9B0AB, 9BB11, 9BBBB, 9D007, 9D08D, 9D537, 9D9BB, 9D9DB, 9DD57, 9DDB7, 9DDDB, 9DDDD, A0A34, A0B5B, A0BBB, A0E34, A2E52, A330B, A8434, A8834, A8E34, A909B, AAA34, AAE52, AB0BB, AB334, ABB34, AE034, AE834, AE99B, AEA52, AEE52, B0011, B0071, B0077, B00B1, B0611, B0A64, B500D, B599D, B6101, B7771, B7911, BA064, BAAA4, BAB34, BB061, BB304, BB53D, BB601, BBB91, BBB9D, BBBBD, BDA0B, BDBBB, D0088, D00D7, D0307, D05C7, D070D, D0888, D0B07, D0BC7, D0C08, D0DC7, D0DD8, D1661, D59DD, D5D3D, D5DDD, D6611, D700D, D8D0D, D900B, D9908, D999D, D9BBB, D9D9D, D9DDB, D8007, D800D, D81B1, DB53D, DB59D, DB99D, DBBB1, DD0D8, DD33B, DD3B7, DD3BB, DD57D, DD898, DD9DD, DDB37, DDBDB, DDD08, DDD3D, DDD5D, DDD7D, DDD88, DDD9D, DDDB7, DDDC8, DDDD7, DDE98, DE037, DE998, DEB07, E0098, E00C7, E0537, E0557, E077B, E0834, E0968, E3334, E37AB, E39C7, E4034, E5307, 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DDDEEE8E, DDDEEEE8, DEE80008, E0777E97, E0E0E397, E0E77797, E0EE0397,
E7777797, E9066668, EE00E397, EE077797, EE0E0397, EEE00797, EEE07E97,
EEE0AA52, EEE55397, EEE55557, EEEAAA52, EEEEE834, EEEEEA52, 300003331,
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B000000000000000000005B. .... B700000000000000000000001. ....
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base 16 (not proved, only checked to 100000000, also the number DB³²²³⁴ is only a probable prime, i.e. not proven prime)

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AA4000005, AAA0AA8FF, AAAA0A8FF, AAAA0AA8F, B00000881, B00009801,
B00090081, B00BBBABB, B0EB0000B, B444444D, B77777777, B7E777777,
BB00000BD, BB0C0000D, BBBBBA00B, BBBBBBABB, BE0EEEE0B, BE7777777,
C00000CAF, C00006AAF, C000082CD, C00063AFF, C000820CD, C00F00023,
C0444444D, C66666AFF, CCCD99999, CF0000023, CF66666AF, D00000009,
D0000044D, D0044000D, D040E000D, D0440000D, D0DD000D9, DAAAAAA45,
E004044DD, E004444DD, E044400DD, E0C00008D, E0C08000D, E0EAAAAA1,
E2000000D, E400044DD, EAAA4AAA1, EAAAAEAA1, EAAAEA041, EBBBBC00D,
EEEE00CCB, F00000545, F02600003, F066AAAAF, F0FF5666F, F3FFF3F23,
F60AAAA0F, F7777777, FFEEEEEE7, FFFF33323, FFFF5666F, FFFFF2CC3,
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..., 700000077B, ..., 8000000AA1, ..., 800AAAAA01, ..., 8886888AAF, ..., 88888888AF, ...,
88888888F, ..., 888AAFFFFF, ..., 9000000019, ..., 9000000109, ..., 908AAAAA01, ...,
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..., CCCCCCAAF, ..., CFFFFFAAF, ..., E44444441, ..., E4444444DD, ..., EAAAAAAAA1,
..., F260000003, ..., FEEEEEEE7, ..., FFFFFF56F, ..., 2200000007, ..., 400000004B,
..., 400000000A5, ..., 52CCCCCCCD, ..., 80AAAAAAA01, ..., 87000000007, ...,
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C000CCCC6AF, ..., C0AF000000F, ..., EAAAEAAAAA1, ..., FAAAAAAAA8F, ...,
5888888887. .... 800AAAAAAAA1. .... 888888AFFFFF. .... 88AFFFFFFFF. ....
8CCCCCCFCF, ..., A0000000AA8F, ..., A4000000005, ..., A4404444441, ...,
AAAAAAA00A8F, ..., C00000000C8F, ..., CA0F0000000F, ..., CCCCCCCC6AF, ....
D1000000005, ..., E0A04AAAAAA1, ..., 1A000000000B, ..., 5BBBBBBBBBBBB, ...,
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88888F8888F. .... 88F88888888F. .... A00000000A8F. .... A0FFFFFFFF45. ....
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EEEEEEEEEEEEEEEEEEEEEEEEEEEEEE.....
C000000000000000000000000000000000000DD, ....
CCCCCCCCF, ...,
1. ....
CCCCCCCCCCCAF, ...,
BBBBBBBBBBBBBBBBBBBBB, ...,
44444444444444444D. ....
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CCCCCD,

Proof

Determining M(L) for arbitrary L is in general <u>unsolvable</u>, and can be difficult even when L is relatively simple.

The following is a "semi-algorithm" that is guaranteed to produce M(L), but it is not so easy to implement:

- (1) M = 0
- (2) while $(L \neq \emptyset)$ do

```
(3) choose x, a shortest string in L
```

- (4) $M = M \cup \{x\}$
- $(5) L = L \underline{\sup}(\{x\})$

In practice, for arbitrary L, we cannot feasibly carry out step (5). Instead, we work with L', some regular overapproximation to L, until we can show $L' = \emptyset$ (which implies $L = \emptyset$). In practice, L' is usually chosen to be a finite <u>union</u> of sets of the form $L_1L_2*L_3$, where each of L_1 , L_2 , L_3 is finite. In the case we consider in this paper, we then have to determine whether such a language contains a prime or not.

However, it is not even known if the following simpler <u>decision problem</u> is recursively solvable:

Problem: Given strings x, y, z, and a base b, does there exist a prime number whose base-b expansion is of the form xy^nz for some $n \ge 0$?

An algorithm to solve this problem, for example, would allow us to decide if there are any additional Fermat primes (of the form $2^{2^n} + 1$) other than the known ones (corresponding to n = 0, 1, 2, 3, 4). To see this, take b = 2, x = 1, y = 0, and $z = 0^{16}1$. Since if $2^n + 1$ is prime then n must be a power of two, a prime of the form $(xy^*z)_b$ must be a new Fermat prime. Besides, it would allow us to decide if there are infinitely many Mersenne primes (of the form $2^p - 1$ with prime p). To see this, take $b = 2, x = \lambda$ (the empty string), y = 1, and $z = 1^{n+1}$, where p is the exponent of the Mersenne prime which we want to know whether it is the largest Mersenne prime or not. Since if $2^n - 1$ is prime then p must be a prime, a prime of the form $(xy^*z)_b$ must be a new Mersenne prime. Also, it would allow us to decide if 21181 is a Sierpinski number (take p = 2, p = 101110001111101, p = 0, and p = 1 and if 23669 is a Riesel number (take p = 2, p = 1011110001110100, p = 1, and p = p (the empty string).

Therefore, in practice, we are forced to try to rule out prime representations based on <u>heuristics</u> such as <u>modular techniques</u> and <u>factorizations</u>.

It will be necessary for our algorithm to determine if families of the form $(xy^*z)_b$ contain a prime > b or not. We use two different heuristic strategies to show that such families contain no primes > b.

In the first strategy, we mimic the well-known technique of "covering congruences", by finding some finite set S of primes p such that every number in a given family is divisible by some element of S. In the second strategy, we attempt to find an <u>algebraic factorization</u>, such as difference-of-<u>squares</u> factorization, difference-of-<u>cubes</u> factorization, and <u>Aurifeuillian factorization</u> for x^4+4y^4 .

Examples of the first strategy: (we can show that the corresponding numbers are > all elements in S, if n makes corresponding numbers > b (i.e. $n \ge 1$ for 51^n in base 9 and 25^n in base 11 and 4^n D in base 16 and 8^n F in base 16, $n \ge 0$ for other examples), thus these factorizations are nontrivial)

^{*} In base 10, all numbers of the form 46ⁿ9 are divisible by 7

- * In base 6, all numbers of the form 40ⁿ1 are divisible by 5
- * In base 15, all numbers of the form 96ⁿ8 are divisible by 11
- * In base 9, all numbers of the form 51^n are divisible by some element of $\{2,5\}$ (note: the prime 5 is not allowed since the prime must be > base)
- * In base 11, all numbers of the form 25^n are divisible by some element of $\{2,3\}$ (note: the prime 2 is not allowed since the prime must be > base)
- * In base 14, all numbers of the form B0ⁿ1 are divisible by some element of {3,5}
- * In base 8, all numbers of the form 64ⁿ7 are divisible by some element of {3,5,13}
- * In base 13, all numbers of the form 30ⁿ95 are divisible by some element of {5,7,17}
- * In base 16, all numbers of the form 4^nD are divisible by some element of $\{3,7,13\}$ (note: the prime D is not allowed since the prime must be > base)
- * In base 16, all numbers of the form 8"F are divisible by some element of {3,7,13}

Examples of the second strategy: (we can show that both factors are > 1, if n makes corresponding numbers > b (i.e. $n \ge 2$ for 1^n in base 9, $n \ge 0$ for $10^n 1$ in base 8 and B4ⁿ1 in base 16, $n \ge 1$ for other examples), thus these factorizations are nontrivial)

- * In base 9, all numbers of the form 1^n factored as (3^n-1) * $(3^n+1)/8$
- * In base 8, all numbers of the form $10^{n}1$ factored as $(2^{n+1}+1)$ * $(4^{n+1}-2^{n+1}+1)$
- * In base 9, all numbers of the form 38^n factored as $(2*3^n-1)*(2*3^n+1)$ (note: the prime 3 is not allowed since the prime must be > base)
- * In base 16, all numbers of the form $8F^n$ factored as $(3*4^n-1)*(3*4^n+1)$
- * In base 16, all numbers of the form F^n 7 factored as $(4^{n+1}-3)$ * $(4^{n+1}+3)$ (note: the prime 7 is not allowed since the prime must be > base)
- * In base 9, all numbers of the form 31^n factored as $(5*3^n-1)*(5*3^n+1)/8$ (note: the prime 3 is not allowed since the prime must be > base)
- * In base 16, all numbers of the form $4^{n}1$ factored as $(2^{*}4^{n+1}-7)$ * $(2^{*}4^{n+1}+7)/15$
- * In base 16, all numbers of the form 15^n factored as $(2*4^n-1)*(2*4^n+1)/3$
- * In base 16, all numbers of the from C^nD factored as $(2*4^{n+1}-2*2^{n+1}+1)*(2*4^{n+1}+2*2^{n+1}+1) / 5$ (note: the prime D is not allowed since the prime must be > base)
- * In base 16, all numbers of the form B4ⁿ1 factored as $(13*4^{n+1}-7)*(13*4^{n+1}+7)/15$

Examples of combine of the two strategies: (we can show that for the part of the first strategy, the corresponding numbers are > all elements in S, and for the part of the second strategy, both factors are > 1, if n makes corresponding numbers > b (i.e. $n \ge 0$ for $B^n \ni B$ in base 12, $n \ge 1$ for other examples), thus these factorizations are nontrivial)

- * In base 14, numbers of the form $8D^n$ are divisible by 5 if n is odd and factored as $(3*14^{n/2}-1)*(3*14^{n/2}+1)$ if n is even
- * In base 12, numbers of the form Bⁿ9B are divisible by 13 if n is odd and factored as $(12^{(n+2)/2}-5)$ * $(12^{(n+2)/2}+5)$ if n is even
- * In base 14, numbers of the form $D^n 5$ are divisible by 5 if n is even and factored as $(14^{(n+1)/2}-3)$ * $(14^{(n+1)/2}+3)$ if n is odd (note: the prime 5 is not allowed since the prime must be > base)
- * In base 17, numbers of the form 19^n are divisible by 2 if n is odd and factored as $(5*17^{n/2}-3)*(5*17^{n/2}+3)/16$ if n is even
- * In base 19, numbers of the from 16^n are divisible by 5 if n is odd and factored as $(2*19^{n/2}-1)*(2*19^{n/2}+1)/3$ if n is even

As previously mentioned, in practice to compute $M(L_b)$ one works with an underapproximation M of $M(L_b)$ and an overapproximation L of $L_b - sup(M)$. One then refines such approximations until $L = \emptyset$ from which it follows that $M = M(L_b)$.

For the initial approximation, note that every minimal prime in base b with at least 4 digits is of the form xY^*z , where $x \in \{x \mid x \text{ is base-}b \text{ digit}, x \neq 0\}$, $z \in \{z \mid z \text{ is base-}b \text{ digit}, gcd(z,b) = 1\}$, and Y^* (for this (x,z) pair) = $\{y \mid xy, xz, yz, xyz \text{ are all composites}\}$. (Of course, if xz is prime, then the Y^* set for this (x,z) pair is \emptyset)

Making use of this, our algorithm sets M to be the set of base-b representations of the minimal primes with at most 3 digits (which can be found simply by brute force) and L to be $\bigcup_{x,z} (xY *z)$ as described above.

All remaining minimal primes are members of L, so to find them we explore the families in L. During this process, each family will be decomposed into possibly multiple other families. For example, a simple way of exploring the family xY^*z where $Y = \{y_1, ..., y_n\}$ is to decompose it into the families xY^*y_1z , ..., xY^*y_nz . If the smallest member (say xy_iz) of any such family happens to be prime, it can be added to M and the family xY^*y_iz removed from consideration. Furthermore, once M has been updated it may be possible to simplify some families in L. In this case, xY^*y_jz (for $j \neq i$) can be simplified to $x(Y-y_i)^*y_jz$ since no minimal prime contains xy_iz as a proper subsequence.

We call families of the form xy^*z (where $x, z \in \Sigma_b^*$ and $y \in \Sigma_b$) *simple* families. Our algorithm then proceeds as follows:

1. Let

 $M := \{ minimal primes in base b of length \leq 3 \}$

$$L := \cup_{x,z \in \Sigma h} (xY *z)$$

where $x \neq 0$ and Y is the set of digits y such that xyz has no subword in M.

- 2. While *L* contains non-simple families:
- (a) Explore each family of *L*, and update *L*.
- (b) Examine each family of *L*:
- i. Let *w* be the shortest string in the family. If *w* has a subword in *M*, then remove the family from *L*. If *w* represents a prime, then add *w* to *M* and remove the family from *L*. ii. If possible, simplify the family.
- iii. Check if the family can be proven to contain no primes > base, and if so then remove the family from L.
- (c) As much as possible and update *L*; after each split examine the new families as in (b).

At the conclusion of the algorithm described, L will consist of simple families (of the form xy^*z) which have not yet yielded a prime, but for which there is no obvious reason why there

can't be a prime of such a form. In such a case, the only way to proceed is to test the primality of larger and larger numbers of such form and hope a prime is eventually discovered (we usually conjecture that there must be a prime at some point if there is no obvious reason why there can't be a prime, since by the prime number theorem, the chance that a random n-digit number is prime is approximately 1/n. If one conjectures the numbers xy^*z behave similarly you would expect $\sum_{n=2}^{\infty} \frac{1}{n} = \infty$ primes of the form xy^*z , of course, this does not always happen, since some xy*z families can be proven to contain no primes > base, and every xy*z family has its own Nash weight (or difficulty), xy*z families which can be proven to contain no primes > base have Nash weight (or difficulty) 0, thus xy*z families are not "completely" random. They are random enough that the prime number theorem can be used to predict their primality, but divisibility by small primes is not as random and can easily be predicted: Once one candidate is found to be divisible by a prime p or to have an algebraic factorization (e.g. difference-of-squares factorization, difference-of-cubes factorization, Aurifeuillian factorization for x^4+4y^4), another predictable candidate will also be divisible by p or also have the same algebraic factorization. This decreases the probability of expected primes. Sometimes though, the candidates will never be divisible by a prime p. which increases the probability of expected primes. However, it is at least a reasonable conjecture in the absence of evidence to the contrary). For example, the base 11 family 57^n , this family have already been searched to length 50000 with no prime or PRP found. however the algebraic form of this family is $(57*11^n-7)/10$, and there is no n satisfying that 57*11ⁿ and 7 are both r-th powers for some r>1 to make this number have difference-of-r-th powers factorization (since 7 is not perfect power), nor there is n satisfying that 57*11ⁿ and -7 are (one is 4th power, another is of the form 4*m⁴) to make this number have <u>Aurifeuillian</u> factorization for x^4+4y^4 (since -7 is neither 4th power nor of the form 4*m⁴), thus, base 11 family 57^n has no algebraic factorization for any n, thus 57^n eventually should yield a prime unless it can be proven to contain no primes > base using covering congruence, and we have:

```
57^n is divisible by 2 for n == 1 \mod 2

57^n is divisible by 13 for n == 2 \mod 12

57^n is divisible by 17 for n == 4 \mod 16

57^n is divisible by 5 for n == 0 \mod 5

57^n is divisible by 23 for n == 6 \mod 22

57^n is divisible by 601 for n == 8 \mod 600

57^n is divisible by 97 for n == 12 \mod 48

57^n is divisible by 1279 for n == 16 \mod 426

...
```

and it does not appear to be any covering set of primes, so there must be a prime at some point.

The numbers in simple families are of the form $\frac{a \cdot b}{gcd(a+c,b-1)}^{n+c}$ for some fixed integers a, b, c where $a \ge 1$, $b \ge 2$ (b is the base), $c \ne 0$, gcd(a,c)=1, gcd(b,c)=1. Except in the special case $c=\pm 1$ and gcd(a+c,b-1)=1, when n is large the known primality tests for such a number are too inefficient to run. In this case one must resort to a probable primality test such as a Miller-Rabin test or a Baillie-PSW test, unless a divisor of the number can be found. Since

we are testing many numbers in an exponential sequence, it is possible to use a <u>sieving</u> <u>process</u> (such as *srsieve* software) to find divisors rather than using <u>trial division</u>.

To do this, we made use of Geoffrey Reynolds' <u>srsieve</u> software. This program uses the baby-step giant-step algorithm to find all primes p which divide $a \cdot b^n + c$ where p and n lie in a specified range. Since this program cannot handle the general case $\frac{a \cdot b^{-n} + c}{gcd(a+c,b-1)}$ when gcd(a+c,b-1) > 1 we only used it to sieve the sequence $a \cdot b^n + c$ for primes p not dividing gcd(a+c,b-1), and initialized the list of candidates to not include p for which there is some prime p dividing gcd(a+c,b-1) for which p divides $\frac{a \cdot b^{-n} + c}{gcd(a+c,b-1)}$. The program had to be modified slightly to remove a check which would prevent it from running in the case when p, and p were all p (since then 2 divides p divides p and p were all p divides p divides p and p divides p div

Once the numbers with small divisors had been removed, it remained to test the remaining numbers using a probable primality test. For this we used the software \underline{LLR} by Jean Penné. Although undocumented, it is possible to run this program on numbers of the form $\frac{a \cdot b^{-n} + c}{gcd(a+c,b-1)}$ when gcd(a+c,b-1) > 1, so this program required no modifications (also, LLR can prove the primality for numbers of the form $a \cdot b^n \pm 1$ (i.e. the special case $c = \pm 1$ and gcd(a+c,b-1) = 1) with $b^n > a$). A script was also written which allowed one to run srsieve while LLR was testing the remaining candidates, so that when a divisor was found by srsieve on a number which had not yet been tested by LLR it would be removed from the list of candidates. In the cases where the elements of $M(L_b)$ could be proven prime rigorously, we employed PRIMO by Marcel Martin, an elliptic curve primality proving implementation.

Some families xy^*z could not be ruled out as containing no primes > base, but no primes > base could be found in the family, even after searching through numbers with over 50000 digits. Many xy^*z families contain no small primes even though they do contain very large primes, for example:

- * In base 5, the smallest prime in the family $10^{n}13$ is $10^{93}13$
- * In base 8, the smallest prime in the family 4^n 7 is 4^{220} 7 (the prime 7 is not counted since the prime must be > base)
- * In base 9, the smallest prime in the family 30ⁿ11 is 30¹¹⁵⁸11
- * In base 9, the smallest prime in the family 27ⁿ07 is 27⁶⁸⁶07
- * In base 11, family 57ⁿ can not be ruled out as containing no primes > base but no primes > base found in the family after searching to length 50000 (the prime 5 is not counted since the prime must be > base)
- * In base 13, the smallest prime in the family 80^n 111 is 80^{32017} 111 (this prime is only a probable prime, i.e. not proven prime)
- * In base 13, the smallest prime in the family 2B30ⁿ1 is 2B3¹⁵¹⁹⁷1
- * In base 13, the smallest prime in the family B0ⁿBBA is B0⁶⁵⁴⁰BBA (this prime is only a probable prime, i.e. not proven prime)
- * In base 13, the smallest prime in the family 390ⁿ1 is 390⁶²⁶⁶1
- * In base 13, the smallest prime in the family 720ⁿ2 is 720²²⁹⁷2
- * In base 13, family 95ⁿ can not be ruled out as containing no primes > base but no primes > base found in the family after searching to length 50000

- * In base 13, family A3ⁿA can not be ruled out as containing no primes > base but no primes
- > base found in the family after searching to length 50000
- * In base 14, the smallest prime in the family 4Dⁿ is 4D¹⁹⁶⁹⁸
- * In base 16, family 3ⁿAF can not be ruled out as containing no primes > base but no primes
- > base found in the family after searching to length 50000
- * In base 16, family 4ⁿDD can not be ruled out as containing no primes > base but no primes
- > base found in the family after searching to length 50000
- * In base 16, the smallest prime in the family DB^n is DB^{32234} (this prime is only a probable prime, i.e. not proven prime) (the prime D is not counted since the prime must be > base)

For any given base b, we find all (x,z) digits-pair such that $x \ne 0$ and gcd(z,b) = 1, and find the corresponding sets Y^* , see below.

Bold for minimal primes in base b, i.e. elements of the set $M(L_b)$

base 2

The possible (first digit, last digit) for an element with >=3 digits in the minimal set of the strings for primes with at least two digits are:

(1,1)

* Case (1,1):

** 11 is prime, and thus the only minimal prime in this family.

base 3

The possible (first digit, last digit) for an element with >=3 digits in the minimal set of the strings for primes with at least two digits are:

```
(1,1), (1,2), (2,1), (2,2)
```

- * Case (1,1):
- ** Since 12, 21, **111** are primes, we only need to consider the family 1{0}1 (since any digits 1, 2 between them will produce smaller primes)
- *** All numbers of the form 1{0}1 are divisible by 2, thus cannot be prime.
- * Case (1,2):
- ** 12 is prime, and thus the only minimal prime in this family.
- * Case (2,1):
- ** 21 is prime, and thus the only minimal prime in this family.

- * Case (2,2):
- ** Since 21, 12 are primes, we only need to consider the family 2{0,2}2 (since any digits 1 between them will produce smaller primes)
- *** All numbers of the form 2{0,2}2 are divisible by 2, thus cannot be prime.

The possible (first digit, last digit) for an element with >=3 digits in the minimal set of the strings for primes with at least two digits are:

```
(1,1), (1,3), (2,1), (2,3), (3,1), (3,3)
```

- * Case (1,1):
- ** 11 is prime, and thus the only minimal prime in this family.
- * Case (1,3):
- ** 13 is prime, and thus the only minimal prime in this family.
- * Case (2,1):
- ** Since 23, 11, 31, **221** are primes, we only need to consider the family 2{0}1 (since any digits 1, 2, 3 between them will produce smaller primes)
- *** All numbers of the form 2{0}1 are divisible by 3, thus cannot be prime.
- * Case (2,3):
- ** 23 is prime, and thus the only minimal prime in this family.
- * Case (3,1):
- ** 31 is prime, and thus the only minimal prime in this family.
- * Case (3,3):
- ** Since 31, 13, 23 are primes, we only need to consider the family 3{0,3}3 (since any digits 1, 2 between them will produce smaller primes)
- *** All numbers of the form 3{0,3}3 are divisible by 3, thus cannot be prime.

base 5

The possible (first digit, last digit) for an element with >=3 digits in the minimal set of the strings for primes with at least two digits are:

```
(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (3,4), (4,1), (4,2), (4,3), (4,4)
* Case (1,1):
** Since 12, 21, 111, 131 are primes, we only need to consider the family 1{0,4}1 (since any digits 1,
2, 3 between them will produce smaller primes)
*** All numbers of the form 1{0,4}1 are divisible by 2, thus cannot be prime.
* Case (1,2):
** 12 is prime, and thus the only minimal prime in this family.
* Case (1,3):
** Since 12, 23, 43, 133 are primes, we only need to consider the family 1{0,1}3 (since any digits 2, 3,
4 between them will produce smaller primes)
*** Since 111 is prime, we only need to consider the families 1{0}3 and 1{0}1{0}3 (since any digit
combo 11 between (1,3) will produce smaller primes)
**** All numbers of the form 1{0}3 are divisible by 2, thus cannot be prime.
**** For the 1{0}1{0}3 family, since 10103 is prime, we only need to consider the families 1{0}13 and
11{0}3 (since any digit combo 010 between (1,3) will produce smaller primes)
***** The smallest prime of the form 1{0}13 is
000000000013, which can be written as 10<sub>93</sub>13 and equal the prime 5^95+8
***** All numbers of the form 11{0}3 are divisible by 3, thus cannot be prime.
* Case (1,4):
** Since 12, 34, 104 are primes, we only need to consider the families 1{1,4}4 (since any digits 0, 2, 3
between them will produce smaller primes)
*** Since 111, 414 are primes, we only need to consider the family 1{4}4 and 11{4}4 (since any digit
combo 11 or 41 between them will produce smaller primes)
**** The smallest prime of the form 1{4}4 is 14444.
**** All numbers of the form 11{4}4 are divisible by 2, thus cannot be prime.
* Case (2,1):
** 21 is prime, and thus the only minimal prime in this family.
* Case (2,2):
```

- ** Since 21, 23, 12, 32 are primes, we only need to consider the family 2{0,2,4}2 (since any digits 1, 3 between them will produce smaller primes)
- *** All numbers of the form 2{0,2,4}2 are divisible by 2, thus cannot be prime.
- * Case (2,3):
- ** 23 is prime, and thus the only minimal prime in this family.
- * Case (2,4):
- ** Since 21, 23, 34 are primes, we only need to consider the family 2{0,2,4}4 (since any digits 1, 3 between them will produce smaller primes)
- *** All numbers of the form 2{0,2,4}4 are divisible by 2, thus cannot be prime.
- * Case (3,1):
- ** Since 32, 34, 21 are primes, we only need to consider the family 3{0,1,3}1 (since any digits 2, 4 between them will produce smaller primes)
- *** Since 313, 111, 131, **3101** are primes, we only need to consider the families 3{0,3}1 and 3{0,3}11 (since any digit combo 10, 11, 13 between (3,1) will produce smaller primes)
- **** For the 3{0,3}1 family, we can separate this family to four families:
- ***** For the 30(0,3)01 family, we have the prime 30301, and the remain case is the family 30(0)01.
- ****** All numbers of the form 30{0}01 are divisible by 2, thus cannot be prime.
- ***** For the 30{0,3}31 family, note that there must be an even number of 3's between (30,31), or the result number will be divisible by 2 and cannot be prime.
- ****** Since 33331 is prime, any digit combo 33 between (30,31) will produce smaller primes.
- ******* Thus, the only possible prime is the smallest prime in the family 30{0}31, and this prime is **300031**.
- ***** For the 33{0,3}01 family, note that there must be an even number of 3's between (33,01), or the result number will be divisible by 2 and cannot be prime.
- ****** Since 33331 is prime, any digit combo 33 between (33,01) will produce smaller primes.
- ******* Thus, the only possible prime is the smallest prime in the family 33{0}01, and this prime is **33001**.
- ***** For the 33{0,3}31 family, we have the prime 33331, and the remain case is the family 33{0}31.
- ****** All numbers of the form 33{0}31 are divisible by 2, thus cannot be prime.
- * Case (3,2):

- ** 32 is prime, and thus the only minimal prime in this family.
- * Case (3,3):
- ** Since 32, 34, 23, 43, **313** are primes, we only need to consider the family 3{0,3}3 (since any digits 1, 2, 4 between them will produce smaller primes)
- *** All numbers of the form 3{0,3}3 are divisible by 3, thus cannot be prime.
- * Case (3,4):
- ** **34** is prime, and thus the only minimal prime in this family.
- * Case (4,1):
- ** Since 43, 21, **401** are primes, we only need to consider the family 4{1,4}1 (since any digits 0, 2, 3 between them will produce smaller primes)
- *** Since 414, 111 are primes, we only need to consider the family 4{4}1 and 4{4}11 (since any digit combo 14 or 11 between them will produce smaller primes)
- **** The smallest prime of the form 4{4}1 is 44441.
- **** All numbers of the form 4{4}11 are divisible by 2, thus cannot be prime.
- * Case (4,2):
- ** Since 43, 12, 32 are primes, we only need to consider the family 4{0,2,4}2 (since any digits 1, 3 between them will produce smaller primes)
- *** All numbers of the form 4{0,2,4}2 are divisible by 2, thus cannot be prime.
- * Case (4,3):
- ** 43 is prime, and thus the only minimal prime in this family.
- * Case (4,4):
- ** Since 43, 34, **414** are primes, we only need to consider the family 4{0,2,4}4 (since any digits 1, 3 between them will produce smaller primes)
- *** All numbers of the form 4{0,2,4}4 are divisible by 2, thus cannot be prime.

The possible (first digit, last digit) for an element with >=3 digits in the minimal set of the strings for primes with at least two digits are:

$$(1,1), (1,5), (2,1), (2,5), (3,1), (3,5), (4,1), (4,5), (5,1), (5,5)$$

* Case (1,1):

```
** 11 is prime, and thus the only minimal prime in this family.
* Case (1,5):
** 15 is prime, and thus the only minimal prime in this family.
* Case (2,1):
** 21 is prime, and thus the only minimal prime in this family.
* Case (2,5):
** 25 is prime, and thus the only minimal prime in this family.
* Case (3,1):
** 31 is prime, and thus the only minimal prime in this family.
* Case (3,5):
** 35 is prime, and thus the only minimal prime in this family.
* Case (4,1):
** Since 45, 11, 21, 31, 51 are primes, we only need to consider the family 4{0,4}1 (since any digits 1,
2, 3, 5 between them will produce smaller primes)
*** Since 4401 and 4441 are primes, we only need to consider the families 4{0}1 and 4{0}41 (since
any digits combo 40 and 44 between them will produce smaller primes)
**** All numbers of the form 4{0}1 are divisible by 5, thus cannot be prime.
**** The smallest prime of the form 4{0}41 is 40041
* Case (4,5):
** 45 is prime, and thus the only minimal prime in this family.
* Case (5,1):
** 51 is prime, and thus the only minimal prime in this family.
* Case (5,5):
** Since 51, 15, 25, 35, 45 are primes, we only need to consider the family 5{0,5}5 (since any digits 1,
2, 3, 4 between them will produce smaller primes)
*** All numbers of the form 5{0,5}5 are divisible by 5, thus cannot be prime.
```

The possible (first digit, last digit) for an element with >=3 digits in the minimal set of the strings for primes with at least two digits are:

```
(1,1), (1,3), (1,5), (1,7), (2,1), (2,3), (2,5), (2,7), (3,1), (3,3), (3,5), (3,7), (4,1), (4,3), (4,5), (4,7), (5,1), (5,3), (5,5), (5,7), (6,1), (6,3), (6,5), (6,7), (7,1), (7,3), (7,5), (7,7)
```

- * Case (1,1):
- ** Since 13, 15, 21, 51, **111**, **141**, **161** are primes, we only need to consider the family 1{0,7}1 (since any digits 1, 2, 3, 4, 5, 6 between them will produce smaller primes)
- *** Since 107, 177, 701 are primes, we only need to consider the number 171 and the family 1{0}1 (since any digits combo 07, 70, 77 between them will produce smaller primes)
- **** 171 is not prime.
- **** All numbers of the form $1{0}1$ factored as $10^n+1=(2^n+1)*(4^n-2^n+1)$, thus cannot be prime.
- * Case (1,3):
- ** 13 is prime, and thus the only minimal prime in this family.
- * Case (1,5):
- ** 15 is prime, and thus the only minimal prime in this family.
- * Case (1,7):
- ** Since 13, 15, 27, 37, 57, **107**, **117**, **147**, **177** are primes, we only need to consider the family 1{6}7 (since any digits 0, 1, 2, 3, 4, 5, 7 between them will produce smaller primes)
- *** The smallest prime of the form 1{6}7 is 16667 (not minimal prime, since 667 is prime)
- * Case (2,1):
- ** 21 is prime, and thus the only minimal prime in this family.
- * Case (2,3):
- ** 23 is prime, and thus the only minimal prime in this family.
- * Case (2,5):
- ** Since 21, 23, 27, 15, 35, 45, 65, 75, **225**, **255** are primes, we only need to consider the family 2{0}5 (since any digits 1, 2, 3, 4, 5, 6, 7 between them will produce smaller primes)
- *** All numbers of the form 2{0}5 are divisible by 7, thus cannot be prime.
- * Case (2,7):

- ** 27 is prime, and thus the only minimal prime in this family.
- * Case (3,1):
- ** Since 35, 37, 21, 51, **301**, **361** are primes, we only need to consider the family 3{1,3,4}1 (since any digits 0, 2, 5, 6, 7 between them will produce smaller primes)
- *** Since 13, 343, 111, 131, 141, 431, **3331**, **3411** are primes, we only need to consider the families 3{3}11, 33{1,4}1, 3{3,4}4{4}1 (since any digits combo 11, 13, 14, 33, 41, 43 between them will produce smaller primes)
- **** All numbers of the form 3{3}11 are divisible by 3, thus cannot be prime.
- **** For the 33{1,4}1 family, since 111 and 141 are primes, we only need to consider the families 33{4}1 and 33{4}11 (since any digits combo 11, 14 between them will produce smaller primes)
- ***** The smallest prime of the form 33{4}1 is 3344441
- ***** All numbers of the form 33{4}11 are divisible by 301, thus cannot be prime.
- **** For the 3{3,4}4{4}1 family, since 3331 and 3344441 are primes, we only need to consider the families 3{4}1, 3{4}31, 3{4}341, 3{4}3441, 3{4}34441 (since any digits combo 33 or 34444 between (3,1) will produce smaller primes)
- ***** All numbers of the form 3{4}1 are divisible by 31, thus cannot be prime.
- ***** Since 4443 is prime, we only need to consider the numbers 3431, 34431, 34341, 343441, 343441, 3434441 (since any digit combo 444 between (3,3{4}1) will produce smaller primes)
- ****** None of 3431, 34431, 34341, 344341, 343441, 3443441, 3434441, 34434441 are primes.
- * Case (3,3):
- ** Since 35, 37, 13, 23, 53, 73, **343** are primes, we only need to consider the family 3{0,3,6}3 (since any digits 1, 2, 4, 5, 7 between them will produce smaller primes)
- *** All numbers of the form 3{0,3,6}3 are divisible by 3, thus cannot be prime.
- * Case (3,5):
- ** **35** is prime, and thus the only minimal prime in this family.
- * Case (3,7):
- ** 37 is prime, and thus the only minimal prime in this family.
- * Case (4,1):
- ** Since 45, 21, 51, **401**, **431**, **471** are primes, we only need to consider the family 4{1,4,6}1 (since any digits 0, 2, 3, 5, 7 between them will produce smaller primes)

- *** Since 111, 141, 161, 661, **4611** are primes, we only need to consider the families 4{4}11, 4{4,6}4{1,4,6}1, 4{4}6{4}1 (since any digits combo 11, 14, 16, 61, 66 between them will produce smaller primes)
- **** The smallest prime of the form 4{4}11 is 4444444444411 (not minimal prime, since 444444441 is prime)
- **** For the 4{4,6}4{1,4,6}1 family, we can separate this family to 4{4,6}41, 4{4,6}411, 4{4,6}461
- ****** For the 4{4,6}41 family, since 661 and 6441 are primes, we only need to consider the families 4{4}41 and 4{4}641 (since any digits combo 64 or 66 between (4,41) will produce smaller primes)
- ****** The smallest prime of the form 4{4}41 is 444444441
- ****** The smallest prime of the form 4{4}641 is 444641
- ***** For the 4{4,6}411 family, since 661 and 6441 are primes, we only need to consider the families 4{4}411 and 4{4}6411 (since any digits combo 64 or 66 between (4,411) will produce smaller primes)
- ****** The smallest prime of the form 4{4}411 is 44444441
- ****** The smallest prime of the form 4{4}6411 is 44444444444446411 (not minimal prime, since 444444441 and 444641 are primes)
- ***** For the 4{4,6}461 family, since 661 is prime, we only need to consider the family 4{4}461
- ****** The smallest prime of the form 4{4}461 is 4444444461 (not minimal prime, since 444444441 is prime)
- **** For the 4{4}6{4}1 family, since 6441 is prime, we only need to consider the families 4{4}61 and 4{4}641 (since any digits combo 44 between (4{4}6,1) will produce smaller primes)
- ***** The smallest prime of the form 4{4}61 is 4444444461 (not minimal prime, since 444444441 is prime)
- ***** The smallest prime of the form 4{4}641 is 444641
- * Case (4,3):
- ** Since 45, 13, 23, 53, 73, **433**, **463** are primes, we only need to consider the family 4{0,4}3 (since any digits 1, 2, 3, 5, 6, 7 between them will produce smaller primes)
- *** Since **4043** and **4443** are primes, we only need to consider the families 4{0}3 and 44{0}3 (since any digits combo 04, 44 between them will produce smaller primes)
- **** All numbers of the form 4{0}3 are divisible by 7, thus cannot be prime.
- **** All numbers of the form 44{0}3 are divisible by 3, thus cannot be prime.
- * Case (4,5):
- ** 45 is prime, and thus the only minimal prime in this family.

- * Case (4,7):
- ** Since 45, 27, 37, 57, **407**, **417**, **467** are primes, we only need to consider the family 4{4,7}7 (since any digits 0, 1, 2, 3, 5, 6 between them will produce smaller primes)
- *** Since 747 is prime, we only need to consider the families 4{4}7, 4{4}77, 4{7}7, 44{7}7 (since any digits combo 74 between (4,7) will produce smaller primes)
- **** The smallest prime of the form 4{4}7 is

- **** The smallest prime of the form 4{4}77 is 4444477
- **** The smallest prime of the form 4{7}7 is 47777
- * Case (5,1):
- ** **51** is prime, and thus the only minimal prime in this family.
- * Case (5,3):
- ** **53** is prime, and thus the only minimal prime in this family.
- * Case (5,5):
- ** Since 51, 53, 57, 15, 35, 45, 65, 75 are primes, we only need to consider the family 5{0,2,5}5 (since any digits 1, 3, 4, 6, 7 between them will produce smaller primes)
- *** Since 225, 255, **5205** are primes, we only need to consider the families 5{0,5}5 and 5{0,5}25 (since any digits combo 20, 22, 25 between them will produce smaller primes)
- **** All numbers of the form 5{0,5}5 are divisible by 5, thus cannot be prime.

**** For the 5{0,5}25 family, since 500025 and 505525 are primes, we only need to consider the number 500525 the families 5{5}25, 5{5}025, 5{5}0025, 5{5}0525, 5{5}00525, 5{5}050525 (since any digits combo 000, 055 between (5,25) will produce smaller primes) **** 500525 is not prime. ***** The smallest prime of the form 5{5}25 is **55555555555555555** ***** The smallest prime of the form 5{5}025 is 55555025 ***** The smallest prime of the form 5{5}0025 is primes) ***** The smallest prime of the form 5{5}0525 is **5550525** ***** The smallest prime of the form 5{5}00525 is 5500525 ***** The smallest prime of the form 5{5}05025 is 5555555555555555555555550025 (not minimal prime, since 5550525, 55555025, and 555555555555525 are primes) * Case (5,7): ** 57 is prime, and thus the only minimal prime in this family. * Case (6.1): ** Since 65, 21, 51, 631, 661 are primes, we only need to consider the family 6{0,1,4,7}1 (since any digits 2, 3, 5, 6 between them will produce smaller primes) *** Numbers containing 4: (note that the number cannot contain two or more 4's, or 6441 will be a subsequence) **** The form is 6{0,1,7}4{0,1,7}1 ***** Since 141, 401, 471 are primes, we only need to consider the family 6{0,7}4{1}1 ****** Since 111 is prime, we only need to consider the families 6(0,7)41 and 6(0,7)411 ******* For the 6{0,7}41 family, since 60741 is prime, we only need to consider the family 6{7}{0}41 ******** Since 6777 is prime, we only need to consider the families 6(0)41, 67(0)41, 677(0)41 ******** All numbers of the form 6{0}41 are divisible by 3, thus cannot be prime. ******* All numbers of the form 67{0}41 are divisible by 13, thus cannot be prime. ******** All numbers of the form 677{0}41 are divisible by 3, thus cannot be prime. ******* For the 6{0,7}411 family, since 60411 is prime, we only need to consider the family 6{7}411

- ******** The smallest prime of the form 6{7}411 is 67777411 (not minimal prime, since 6777 is prime)
- *** Numbers not containing 4:
- **** The form is 6{0,1,7}1
- ***** Since 111 is prime, we only need to consider the families 6{0,7}1 and 6{0,7}1{0,7}1
- ****** All numbers of the form 6{0,7}1 are divisible by 7, thus cannot be prime.
- ****** For the $6\{0,7\}1\{0,7\}1$ family, since 711 and **6101** are primes, we only need to consider the family $6\{0\}1\{7\}1$
- ******* Since 60171 is prime, we only need to consider the families 6{0}11 and 61{7}1
- ******* All numbers of the form 6{0}11 are divisible by 3, thus cannot be prime.
- ******** The smallest prime of the form 61{7}1 is 617771 (not minimal prime, since 6777 is prime)
- * Case (6,3):
- ** Since 65, 13, 23, 53, 73, **643** are primes, we only need to consider the family 6{0,3,6}3 (since any digits 1, 2, 4, 5, 7 between them will produce smaller primes)
- *** All numbers of the form 6{0,3,6}3 are divisible by 3, thus cannot be prime.
- * Case (6,5):
- ** 65 is prime, and thus the only minimal prime in this family.
- * Case (6,7):
- ** Since 65, 27, 37, 57, **667** are primes, we only need to consider the family 6{0,1,4,7}7 (since any digits 2, 3, 5, 6 between them will produce smaller primes)
- *** Since 107, 117, 147, 177, 407, 417, 717, 747, **6007**, **6477**, **6707**, **6777** are primes, we only need to consider the families 60{1,4,7}7, 6{0}17, 6{0,4}4{4}7, 6{0}77 (since any digits combo 00, 10, 11, 14, 17, 40, 41, 47, 70, 71, 74, 77 between them will produce smaller primes)
- **** All numbers of the form 6{0}17 or 6{0}77 are divisible by 3, thus cannot be prime.
- **** For the 60{1,4,7}7 family, since 117, 147, 177, 417, 6477, 717, 747, 6777 are primes, we only need to consider the numbers 6017, 6047, 6077 and the family 60{4}7 (since any digit combo 11, 14, 17, 41, 47, 71, 74, 77 between (60,7) will produce smaller primes)
- ***** None of 6017, 6047, 6077 are primes.
- **** All numbers of the form 60{4}7 are divisible by 21, thus cannot be prime.
- **** For the 6{0,4}4{4}7 family, since 6007 and 407 are primes, we only need to consider the families 6{4}7 and 60{4}7 (since any digit combo 00, 40 between (6,4{4}7) will produce smaller primes)

***** All numbers of the form 6{4}7 are divisible by 3, 5, or 15, thus cannot be prime. ***** All numbers of the form 60{4}7 are divisible by 21, thus cannot be prime. * Case (7,1): ** Since 73, 75, 21, 51, **701**, **711** are primes, we only need to consider the family 7{4,6,7}1 (since any digits 0, 1, 2, 3, 5 between them will produce smaller primes) *** Since 747, 767, 471, 661, **7461**, **7641** are primes, we only need to consider the families 7{4,7}4{4}1, 7{7}61, 7{7}7{4,6,7}1 (since any digits combo 46, 47, 64, 66, 67 between them will produce smaller primes) **** For the 7{4,7}4{4}1 family, since 747, 471 are primes, we only need to consider the family 7{7}{4}1 (since any digits combo 47 between (7,4{4}1) will produce smaller primes) ***** The smallest prime of the form 7{7}1 is 77777777771 ***** The smallest prime of the form 7{7}41 is (not minimal prime, since 77777777771 is prime) ***** The smallest prime of the form 7{7}441 is 777441 (not minimal prime, since 77777777771 is prime) ***** The smallest prime of the form 7{7}4441 is minimal prime, since 77777777771 is prime) ***** The smallest prime of the form 7{7}44441 is since 77777777771 is prime) ***** All numbers of the form 7{7}4444441 are divisible by 7, thus cannot be prime. ***** The smallest prime of the form 7{7}4444441 is **77774444441** ****** Since this prime has just 4 7's, we only need to consider the families with <=3 7's ****** The smallest prime of the form 7{4}1 is 744444441 ****** All numbers of the form 77{4}1 are divisible by 5, thus cannot be prime.

******* The smallest prime of the form 777{4}1 is 77744444444441 (not minimal prime, since

* Case (7,3):

44444441 and 74444441 are primes)

- ** 73 is prime, and thus the only minimal prime in this family.
- * Case (7,5):
- ** 75 is prime, and thus the only minimal prime in this family.
- * Case (7,7):
- ** Since 73, 75, 27, 37, 57, **717**, **747**, **767** are primes, we only need to consider the family 7{0,7}7 (since any digits 1, 2, 3, 4, 5, 6 between them will produce smaller primes)
- *** All numbers of the form 7{0,7}7 are divisible by 7, thus cannot be prime.

The possible (first digit,last digit) for an element with >=3 digits in the minimal set of the strings for primes with at least two digits are:

```
(1,1), (1,3), (1,7), (1,9), (2,1), (2,3), (2,7), (2,9), (3,1), (3,3), (3,7), (3,9), (4,1), (4,3), (4,7), (4,9), (5,1), (5,3), (5,7), (5,9), (6,1), (6,3), (6,7), (6,9), (7,1), (7,3), (7,7), (7,9), (8,1), (8,3), (8,7), (8,9), (9,1), (9,3), (9,7), (9,9)
```

- * Case (1,1):
- ** 11 is prime, and thus the only minimal prime in this family.
- * Case (1,3):
- ** 13 is prime, and thus the only minimal prime in this family.
- * Case (1,7):
- ** 17 is prime, and thus the only minimal prime in this family.
- * Case (1,9):
- ** 19 is prime, and thus the only minimal prime in this family.
- * Case (2,1):
- ** Since 23, 29, 11, 31, 41, 61, 71, **251**, **281** are primes, we only need to consider the family 2{0,2}1 (since any digits 1, 3, 4, 5, 6, 7, 8, 9 between them will produce smaller primes)
- *** Since **2221** and **20201** are primes, we only need to consider the families 2{0}1, 2{0}21, 22{0}1 (since any digits combo 22 or 020 between them will produce smaller primes)
- **** All numbers of the form 2{0}1 are divisible by 3, thus cannot be prime.
- **** The smallest prime of the form 2{0}21 is 20021

**** The smallest prime of the form 22{0}1 is 22000001 * Case (2,3): ** 23 is prime, and thus the only minimal prime in this family. * Case (2,7): ** Since 23, 29, 17, 37, 47, 67, 97 227, 257, 277 are primes, we only need to consider the family 2{0,8}7 (since any digits 1, 2, 3, 4, 5, 6, 7, 9 between them will produce smaller primes) *** Since 887 and 2087 are primes, we only need to consider the families 2(0)7 and 28(0)7 (since any digit combo 08 or 88 between them will produce smaller primes) **** All numbers of the form 2{0}7 are divisible by 3, thus cannot be prime. **** All numbers of the form 28{0}7 are divisible by 7, thus cannot be prime. * Case (2,9): ** 29 is prime, and thus the only minimal prime in this family. * Case (3,1): ** 31 is prime, and thus the only minimal prime in this family. * Case (3,3): ** Since 31, 37, 13, 23, 43, 53, 73, 83 are primes, we only need to consider the family 3{0,3,6,9}3 (since any digits 1, 2, 4, 5, 7, 8 between them will produce smaller primes) *** All numbers of the form 3{0,3,6,9}3 are divisible by 3, thus cannot be prime. * Case (3,7): ** 37 is prime, and thus the only minimal prime in this family. * Case (3,9): ** Since 31, 37, 19, 29, 59, 79, 89, **349** are primes, we only need to consider the family 3{0,3,6,9}9 (since any digits 1, 2, 4, 5, 7, 8 between them will produce smaller primes) *** All numbers of the form 3{0,3,6,9}9 are divisible by 3, thus cannot be prime. * Case (4,1): ** 41 is prime, and thus the only minimal prime in this family. * Case (4,3):

** 43 is prime, and thus the only minimal prime in this family.

- * Case (4,7):
- ** 47 is prime, and thus the only minimal prime in this family.
- * Case (4,9):
- ** Since 41, 43, 47, 19, 29, 59, 79, 89, **409**, **449**, **499** are primes, we only need to consider the family 4{6}9 (since any digits 0, 1, 2, 3, 4, 5, 7, 8, 9 between them will produce smaller primes)
- *** All numbers of the form 4{6}9 are divisible by 7, thus cannot be prime.
- * Case (5,1):
- ** Since 53, 59, 11, 31, 41, 61, 71, **521** are primes, we only need to consider the family 5{0,5,8}1 (since any digits 1, 2, 3, 4, 6, 7, 9 between them will produce smaller primes)
- *** Since 881 is prime, we only need to consider the families 5{0,5}1 and 5{0,5}8{0,5}1 (since any digit combo 88 between them will produce smaller primes)
- **** For the 5{0,5}1 family, since **5051** and **5501** are primes, we only need to consider the families 5{0}1 and 5{5}1 (since any digit combo 05 or 50 between them will produce smaller primes)
- ***** All numbers of the form 5{0}1 are divisible by 3, thus cannot be prime.
- ***** The smallest prime of the form 5{5}1 is 5555555555551
- **** For the 5{0,5}8{0,5}1 family, since **5081**, **5581**, **5801**, **5851** are primes, we only need to consider the number 581
- ***** 581 is not prime.
- * Case (5,3):
- ** 53 is prime, and thus the only minimal prime in this family.
- * Case (5,7):
- ** Since 53, 59, 17, 37, 47, 67, 97, **557**, **577**, **587** are primes, we only need to consider the family 5{0,2}7 (since any digits 1, 3, 4, 5, 6, 7, 8, 9 between them will produce smaller primes)
- *** Since 227 and **50207** are primes, we only need to consider the families 5{0}7, 5{0}27, 52{0}7 (since any digits combo 22 or 020 between them will produce smaller primes)
- **** All numbers of the form 5{0}7 are divisible by 3, thus cannot be prime.
- **** The smallest prime of the form 52{0}7 is 5200007
- * Case (5,9):

** **59** is prime, and thus the only minimal prime in this family. * Case (6,1): ** **61** is prime, and thus the only minimal prime in this family. * Case (6,3): ** Since 61, 67, 13, 23, 43, 53, 73, 83 are primes, we only need to consider the family 6(0,3,6,9)3 (since any digits 1, 2, 4, 5, 7, 8 between them will produce smaller primes) *** All numbers of the form 6{0,3,6,9}3 are divisible by 3, thus cannot be prime. * Case (6,7): ** 67 is prime, and thus the only minimal prime in this family. * Case (6.9): ** Since 61, 67, 19, 29, 59, 79, 89 are primes, we only need to consider the family 6{0,3,4,6,9}9 (since any digits 1, 2, 5, 7, 8 between them will produce smaller primes) *** Since 449 is prime, we only need to consider the families 6(0,3,6,9)9 and 6(0,3,6,9)4(0,3,6,9)9 (since any digit combo 44 between them will produce smaller primes) **** All numbers of the form 6{0,3,6,9}9 are divisible by 3, thus cannot be prime. **** For the 6{0,3,6,9}4{0,3,6,9}9 family, since 409, 43, 6469, 499 are primes, we only need to consider the family 6{0,3,6,9}49 ***** Since 349, 6949 are primes, we only need to consider the family 6(0,6)49 ****** Since 60649 is prime, we only need to consider the family 6{6}{0}49 (since any digits combo 06 between {6,49} will produce smaller primes) ****** The smallest prime of the form 6{6}49 is 666649 ******* Since this prime has just 4 6's, we only need to consider the families with <=3 6's ******* The smallest prime of the form 6{0}49 is 60000049 ******** The smallest prime of the form 66{0}49 is 66000049 ********* The smallest prime of the form 666{0}49 is 66600049 * Case (7,1): ** 71 is prime, and thus the only minimal prime in this family. * Case (7,3): ** 73 is prime, and thus the only minimal prime in this family.

- * Case (7,7):
- ** Since 71, 73, 79, 17, 37, 47, 67, 97, **727**, **757**, **787** are primes, we only need to consider the family 7{0,7}7 (since any digits 1, 2, 3, 4, 5, 6, 8, 9 between them will produce smaller primes)
- *** All numbers of the form 7{0,7}7 are divisible by 7, thus cannot be prime.
- * Case (7,9):
- ** 79 is prime, and thus the only minimal prime in this family.
- * Case (8,1):
- ** Since 83, 89, 11, 31, 41, 61, 71, **821**, **881** are primes, we only need to consider the family 8{0,5}1 (since any digits 1, 2, 3, 4, 6, 7, 8, 9 between them will produce smaller primes)
- *** Since **8501** is prime, we only need to consider the family 8{0}{5}1 (since any digits combo 50 between them will produce smaller primes)
- **** Since **80051** is prime, we only need to consider the families 8{0}1, 8{5}1, 80{5}1 (since any digits combo 005 between them will produce smaller primes)
- ***** All numbers of the form 8{0}1 are divisible by 3, thus cannot be prime.
- ***** The smallest prime of the form 8{5}1 is 855555555555555555551 (not minimal prime, since 55555555555551 is prime)
- ***** The smallest prime of the form 80{5}1 is 80555551
- * Case (8,3):
- ** 83 is prime, and thus the only minimal prime in this family.
- * Case (8,7):
- ** Since 83, 89, 17, 37, 47, 67, 97, **827**, **857**, **877**, **887** are primes, we only need to consider the family 8{0}7 (since any digits 1, 2, 3, 4, 5, 6, 7, 8, 9 between them will produce smaller primes)
- *** All numbers of the form 8{0}7 are divisible by 3, thus cannot be prime.
- * Case (8,9):
- ** 89 is prime, and thus the only minimal prime in this family.
- * Case (9,1):
- ** Since 97, 11, 31, 41, 61, 71, **991** are primes, we only need to consider the family 9{0,2,5,8}1 (since any digits 1, 3, 4, 6, 7, 9 between them will produce smaller primes)

- *** Since 251, 281, 521, 821, 881, **9001**, **9221**, **9551**, **9851** are primes, we only need to consider the families 9{2,5,8}0{2,5,8}1, 9{0}2{0}1, 9{0}5{0,8}1, 9{0,5}8{0}1 (since any digits combo 00, 22, 25, 28, 52, 55, 82, 85, 88 between them will produce smaller primes)
- **** For the 9{2,5,8}0{2,5,8}1 family, since any digits combo 22, 25, 28, 52, 55, 82, 85, 88 between (9,1) will produce smaller primes, we only need to consider the numbers 901, 9021, 9051, 9081, 9201, 9501, 9801, 90581, 95081, 95801
- ***** 95801 is the only prime among 901, 9021, 9051, 9081, 9201, 9501, 9801, 90581, 95081, 95801, but it is not minimal prime since 5801 is prime.
- **** For the 9{0}2{0}1 family, since 9001 is prime, we only need to consider the numbers 921, 9201, 9021
- ***** None of 921, 9201, 9021 are primes.
- **** For the 9{0}5{0,8}1 family, since 9001 and 881 are primes, we only need to consider the numbers 951, 9051, 9501, 9581, 90581, 95081, 95801
- ***** 95801 is the only prime among 951, 9051, 9501, 9581, 90581, 95081, 95801, but it is not minimal prime since 5801 is prime.
- **** For the 9{0,5}8{0}1 family, since 9001 and 5581 are primes, we only need to consider the numbers 981, 9081, 9581, 9801, 90581, 95081, 95801
- ***** 95801 is the only prime among 981, 9081, 9581, 9801, 90581, 95081, 95801, but it is not minimal prime since 5801 is prime.
- * Case (9,3):
- ** Since 97, 13, 23, 43, 53, 73, 83 are primes, we only need to consider the family 9{0,3,6,9}3 (since any digits 1, 2, 4, 5, 7, 8 between them will produce smaller primes)
- *** All numbers of the form 9{0,3,6,9}3 are divisible by 3, thus cannot be prime.
- * Case (9,7):
- ** 97 is prime, and thus the only minimal prime in this family.
- * Case (9,9):
- ** Since 97, 19, 29, 59, 79, 89 are primes, we only need to consider the family 9{0,3,4,6,9}9 (since any digits 1, 2, 5, 7, 8 between them will produce smaller primes)
- *** Since 449 is prime, we only need to consider the families 9{0,3,6,9}9 and 9{0,3,6,9}4{0,3,6,9}9 (since any digit combo 44 between them will produce smaller primes)
- **** All numbers of the form 9{0,3,6,9}9 are divisible by 3, thus cannot be prime.
- **** For the $9\{0,3,6,9\}4\{0,3,6,9\}9$ family, since **9049**, 349, **9649**, **9949** are primes, we only need to consider the family $94\{0,3,6,9\}9$

***** Since 409, 43, 499 are primes, we only need to consider the family 94{6}9 (since any digits 0, 3, 9 between (94,9) will produce smaller primes)

****** The smallest prime of the form 94{6}9 is 946669

base 12

The possible (first digit, last digit) for an element with >=3 digits in the minimal set of the strings for primes with at least two digits are:

```
(1,1), (1,5), (1,7), (1,B), (2,1), (2,5), (2,7), (2,B), (3,1), (3,5), (3,7), (3,B), (4,1), (4,5), (4,7), (4,B), (5,1), (5,5), (5,7), (5,B), (6,1), (6,5), (6,7), (6,B), (7,1), (7,5), (7,7), (7,B), (8,1), (8,5), (8,7), (8,B), (9,1), (9,5), (9,7), (9,B), (A,1), (A,5), (A,7), (A,B), (B,1), (B,5), (B,7), (B,B)
```

- * Case (1,1):
- ** 11 is prime, and thus the only minimal prime in this family.
- * Case (1,5):
- ** 15 is prime, and thus the only minimal prime in this family.
- * Case (1,7):
- ** 17 is prime, and thus the only minimal prime in this family.
- * Case (1,B):
- ** **1B** is prime, and thus the only minimal prime in this family.
- * Case (2,1):
- ** Since 25, 27, 11, 31, 51, 61, 81, 91, **221**, **241**, **2A1**, **2B1** are primes, we only need to consider the family 2{0}1 (since any digits 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B between them will produce smaller primes)
- *** The smallest prime of the form 2{0}1 is 2001
- * Case (2,5):
- ** 25 is prime, and thus the only minimal prime in this family.
- * Case (2,7):
- ** **27** is prime, and thus the only minimal prime in this family.
- * Case (2,B):
- ** Since 25, 27, 1B, 3B, 4B, 5B, 6B, 8B, AB, **2BB** are primes, we only need to consider the family 2{0,2,9}B (since any digits 1, 3, 4, 5, 6, 7, 8, A, B between them will produce smaller primes)

- *** Since 90B, **200B**, **202B**, **222B**, **229B**, **299B** are primes, we only need to consider the numbers 20B, 22B, 29B, 209B, 220B (since any digits combo 00, 02, 22, 29, 90, 92, 99 between them will produce smaller primes)
- **** None of 20B, 22B, 29B, 209B, 220B are primes.
- * Case (3,1):
- ** 31 is prime, and thus the only minimal prime in this family.
- * Case (3,5):
- ** 35 is prime, and thus the only minimal prime in this family.
- * Case (3,7):
- ** 37 is prime, and thus the only minimal prime in this family.
- * Case (3,B):
- ** **3B** is prime, and thus the only minimal prime in this family.
- * Case (4,1):
- ** Since 45, 4B, 11, 31, 51, 61, 81, 91, **401**, **421**, **471** are primes, we only need to consider the family 4{4,A}1 (since any digit 0, 1, 2, 3, 5, 6, 7, 8, 9, B between them will produce smaller primes)
- *** Since A41 and **4441** are primes, we only need to consider the families 4{A}1 and 44{A}1 (since any digit combo 44, A4 between them will produce smaller primes)
- **** All numbers of the form 4{A}1 are divisible by 5, thus cannot be prime.
- **** The smallest prime of the form 44{A}1 is 44AAA1
- * Case (4,5):
- ** 45 is prime, and thus the only minimal prime in this family.
- * Case (4,7):
- ** Since 45, 4B, 17, 27, 37, 57, 67, 87, A7, B7, **447**, **497** are primes, we only need to consider the family 4{0,7}7 (since any digit 1, 2, 3, 4, 5, 6, 8, 9, A, B between them will produce smaller primes)
- *** Since **4707** and **4777** are primes, we only need to consider the families 4{0}7 and 4{0}77 (since any digit combo 70, 77 between them will produce smaller primes)
- **** All numbers of the form 4{0}7 are divisible by B, thus cannot be prime.
- * Case (4,B):

- ** **4B** is prime, and thus the only minimal prime in this family. * Case (5,1): ** 51 is prime, and thus the only minimal prime in this family. * Case (5,5): ** Since 51, 57, 5B, 15, 25, 35, 45, 75, 85, 95, B5, **565** are primes, we only need to consider the family 5{0,5,A}5 (since any digits 1, 2, 3, 4, 6, 7, 8, 9, B between them will produce smaller primes) *** All numbers of the form 5{0,5,A}5 are divisible by 5, thus cannot be prime. * Case (5,7): ** 57 is prime, and thus the only minimal prime in this family. * Case (5,B): ** **5B** is prime, and thus the only minimal prime in this family. * Case (6,1): ** 61 is prime, and thus the only minimal prime in this family. * Case (6,5): ** Since 61, 67, 6B, 15, 25, 35, 45, 75, 85, 95, B5, 655, 665 are primes, we only need to consider the family 6{0,A}5 (since any digits 1, 2, 3, 4, 5, 6, 7, 8, 9, B between them will produce smaller primes) *** Since 6A05 and 6AA5 are primes, we only need to consider the families 6{0}5 and 6{0}A5 (since any digit combo A0, AA between them will produce smaller primes) **** All numbers of the form 6{0}5 are divisible by B, thus cannot be prime. **** The smallest prime of the form 6{0}A5 is 600A5 * Case (6,7): ** 67 is prime, and thus the only minimal prime in this family. * Case (6,B): ** **6B** is prime, and thus the only minimal prime in this family. * Case (7,1):
- *** Since 7BB, **7441** and **7B41** are primes, we only need to consider the numbers 741, 7B1, 74B1

** Since 75, 11, 31, 51, 61, 81, 91, **701**, **721**, **771**, **7A1** are primes, we only need to consider the family 7{4,B}1 (since any digits 0, 1, 2, 3, 5, 6, 7, 8, 9, A between them will produce smaller primes)

**** None of 741, 7B1, 74B1 are primes. * Case (7,5): ** **75** is prime, and thus the only minimal prime in this family. * Case (7,7): ** Since 75, 17, 27, 37, 57, 67, 87, A7, B7, **747**, **797** are primes, we only need to consider the family 7{0,7}7 (since any digits 1, 2, 3, 4, 5, 6, 8, 9, A, B between them will produce smaller primes) *** All numbers of the form 7{0,7}7 are divisible by 7, thus cannot be prime. * Case (7,B): ** Since 75, 1B, 3B, 4B, 5B, 6B, 8B, AB, **70B**, **77B**, **7BB** are primes, we only need to consider the family 7{2,9}B (since any digits 0, 1, 3, 4, 5, 6, 7, 8, A, B between them will produce smaller primes) *** Since 222B, 729B is prime, we only need to consider the families 7{9}B, 7{9}2B, 7{9}22B (since any digits combo 222, 29 between them will produce smaller primes) **** The smallest prime of the form 7{9}B is 7999B **** The smallest prime of the form 7{9}2B is 79992B (not minimal prime, since 992B and 7999B are primes) **** The smallest prime of the form 7{9}22B is 79922B (not minimal prime, since 992B is prime) * Case (8,1): ** **81** is prime, and thus the only minimal prime in this family. * Case (8,5): ** **85** is prime, and thus the only minimal prime in this family. * Case (8,7): ** 87 is prime, and thus the only minimal prime in this family. * Case (8,B): ** **8B** is prime, and thus the only minimal prime in this family. * Case (9,1): ** 91 is prime, and thus the only minimal prime in this family. * Case (9,5):

** 95 is prime, and thus the only minimal prime in this family.

- * Case (9,7):
- ** Since 91, 95, 17, 27, 37, 57, 67, 87, A7, B7, **907** are primes, we only need to consider the family 9{4,7,9}7 (since any digit 0, 1, 2, 3, 5, 6, 8, A, B between them will produce smaller primes)
- *** Since 447, 497, 747, 797, **9777**, **9947**, **9997** are primes, we only need to consider the numbers 947, 977, 997, 9477, 9977 (since any digits combo 44, 49, 74, 77, 79, 94, 99 between them will produce smaller primes)
- **** None of 947, 977, 997, 9477, 9977 are primes.
- * Case (9,B):
- ** Since 91, 95, 1B, 3B, 4B, 5B, 6B, 8B, AB, **90B**, **9BB** are primes, we only need to consider the family 9{2,7,9}B (since any digit 0, 1, 3, 4, 5, 6, 8, A, B between them will produce smaller primes)
- *** Since 27, 77B, **929B**, **992B**, **997B** are primes, we only need to consider the families 9{2,7}2{2}B, 97{2,9}B, 9{7,9}9{9}B (since any digits combo 27, 29, 77, 92, 97 between them will produce smaller primes)
- **** For the 9{2,7}2{2}B family, since 27 and 77B are primes, we only need to consider the families 9{2}2{2}B and 97{2}2{2}B (since any digits combo 27, 77 between (9,2{2}B) will produce smaller primes)
- ***** The smallest prime of the form 9{2}2{2}B is 9222B (not minimal prime, since 222B is prime)
- ***** The smallest prime of the form 97{2}2{2}B is 97222222222B (not minimal prime, since 222B is prime)
- **** For the 97{2,9}B family, since 729B and 929B are primes, we only need to consider the family 97{9}{2}B (since any digits combo 29 between (97,B) will produce smaller primes)
- ***** Since 222B is prime, we only need to consider the families 97{9}B, 97{9}2B, 97{9}2B (since any digit combo 222 between (97,B) will produce smaller primes)
- ****** All numbers of the form 97{9}B are divisible by 11, thus cannot be prime.
- ****** The smallest prime of the form 97{9}2B is 979999992B (not minimal prime, since 9999B is prime)
- ****** All numbers of the form 97{9}22B are divisible by 11, thus cannot be prime.
- **** For the 9{7,9}9{9}B family, since 77B and 9999B are primes, we only need to consider the numbers 99B, 999B, 979B, 9799B, 9979B
- ***** None of 99B, 999B, 979B, 9799B, 9979B are primes.
- * Case (A,1):
- ** Since A7, AB, 11, 31, 51, 61, 81, 91, **A41** are primes, we only need to consider the family A{0,2,A}1 (since any digits 1, 3, 4, 5, 6, 7, 8, 9, B between them will produce smaller primes)

- *** Since 221, 2A1, **A0A1**, **A201** are primes, we only need to consider the families A{A}{0}1 and A{A}{0}21 (since any digits combo 0A, 20, 22, 2A between them will produce smaller primes)
- **** For the A{A}{0}1 family:
- ***** All numbers of the form A{0}1 are divisible by B, thus cannot be prime.
- ***** The smallest prime of the form AA{0}1 is AA000001
- ***** The smallest prime of the form AAA{0}1 is AAA0001
- ***** The smallest prime of the form AAAA{0}1 is AAAA1
- ****** Since this prime has no 0's, we do not need to consider the families {A}1, {A}01, {A}001, etc.
- **** All numbers of the form A{A}{0}21 are divisible by 5, thus cannot be prime.
- * Case (A,5):
- ** Since A7, AB, 15, 25, 35, 45, 75, 85, 95, B5 are primes, we only need to consider the family A{0,5,6,A}5 (since any digits 1, 2, 3, 4, 7, 8, 9, B between them will produce smaller primes)
- *** Since 565, 665, **A605**, **A605**, **A6A5**, **AA65** are primes, we only need to consider the families A{0,5,A}5 and A{0}65 (since any digits combo 56, 60, 65, 66, 6A, A6 between them will produce smaller primes)
- **** All numbers of the form A{0,5,A}5 are divisible by 5, thus cannot be prime.
- **** The smallest prime of the form A{0}65 is A00065
- * Case (A,7):
- ** A7 is prime, and thus the only minimal prime in this family.
- * Case (A,B):
- ** **AB** is prime, and thus the only minimal prime in this family.
- * Case (B,1):
- ** Since B5, B7, 11, 31, 51, 61, 81, 91, **B21** are primes, we only need to consider the family B{0,4,A,B}1 (since any digits 1, 2, 3, 5, 6, 7, 8, 9 between them will produce smaller primes)
- *** Since 4B, AB, 401, A41, **B001**, **B0B1**, **BB01**, **BB41** are primes, we only need to consider the families B{A}0{4,A}1, B{0,4}4{4,A}1, B{0,4,A,B}A{0,A}1, B{B}B{A,B}1 (since any digits combo 00, 0B, 40, 4B, A4, AB, B0, B4 between them will produce smaller primes)
- **** For the B{A}0{4,A}1 family, since A41 is prime, we only need consider the families B0{4}{A}1 and B{A}0{A}1
- ***** For the B0{4}{A}1 family, since **B04A1** is prime, we only need to consider the families B0{4}1 and B0{A}1

****** The smallest prime of the form B0{4}1 is B04441 (not minimal prime, since 4441 is prime) ****** The smallest prime of the form B0{A}1 is B0AAAAA1 (not minimal prime, since AAAA1 is prime) ***** For the B{A}0{A}1 family, since A0A1 is prime, we only need to consider the families B{A}01 and B0{A}1 ****** The smallest prime of the form B{A}01 is **BAA01** ****** The smallest prime of the form B0{A}1 is B0AAAAA1 (not minimal prime, since AAAA1 is prime) **** For the B{0,4}4{4,A}1 family, since 4441 is prime, we only need to consider the families B{0}4{4,A}1 and B{0,4}4{A}1 ***** For the B{0}4{4,A}1 family, since B001 is prime, we only need to consider the families B4{4,A}1 and B04{4,A}1 ****** For the B4{4,A}1 family, since A41 is prime, we only need to consider the family B4{4}{A}1 ******* Since 4441 and BAAA1 are primes, we only need to consider the numbers B41, B441, B4A1, B44A1, B4AA1, B44AA1 ******** None of B41, B441, B4A1, B44A1, B4AA1, B44AA1 are primes. ****** For the B04{4,A}1 family, since **B04A1** is prime, we only need to consider the family B04{4}1 ******* The smallest prime of the form B04{4}1 is B04441 (not minimal prime, since 4441 is prime) ***** For the B{0,4}4{A}1 family, since 401, 4441, B001 are primes, we only need to consider the families B4{A}1, B04{A}1, B44{A}1, B044{A}1 (since any digits combo 00, 40, 44 between (B,4{A}1) will produce smaller primes) ****** The smallest prime of the form B4{A}1 is B4AAA1 (not minimal prime, since BAAA1 is prime) ****** The smallest prime of the form B04{A}1 is B04A1 ****** The smallest prime of the form B44{A}1 is B44AAAAAAA1 (not minimal prime, since BAAA1 is prime) ****** The smallest prime of the form B044{A}1 is B044A1 (not minimal prime, since B04A1 is prime) **** For the B{0,4,A,B}A{0,A}1 family, since all numbers in this family with 0 between (B,1) are in the B{A}0{4,A}1 family, and all numbers in this family with 4 between (B,1) are in the B{0,4}4{4,A}1 family, we only need to consider the family B{A,B}A{A}1 ***** Since BAAA1 is prime, we only need to consider the families B{A,B}A1 and B{A,B}AA1 ****** For the B{A,B}A1 family, since AB and BAAA1 are primes, we only need to consider the families B{B}A1 and B{B}AA1 ****** All numbers of the form B{B}A1 are divisible by B, thus cannot be prime.

****** The smallest prime of the form B{B}AA1 is BBBAA1 ****** For the B{A,B}AA1 family, since **BAAA1** is prime, we only need to consider the families B{B}AA1 ****** The smallest prime of the form B{B}AA1 is BBBAA1 **** For the B{B}B{A,B}1 family, since AB and BAAA1 are primes, we only need to consider the families B{B}B1, B{B}BA1, B{B}BAA1 (since any digits combo AB or AAA between (B{B}B,1) will produce smaller primes) ***** The smallest prime of the form B{B}B1 is BBBB1 ***** All numbers of the form B{B}BA1 are divisible by B, thus cannot be prime. ***** The smallest prime of the form B{B}BAA1 is BBBAA1 * Case (B,5): ** **B5** is prime, and thus the only minimal prime in this family. * Case (B,7): ** **B7** is prime, and thus the only minimal prime in this family. * Case (B,B): ** Since B5, B7, 1B, 3B, 4B, 5B, 6B, 8B, AB, **B2B** are primes, we only need to consider the family B{0,9,B}B (since any digits 1, 2, 3, 4, 5, 6, 7, 8, A between them will produce smaller primes) *** Since 90B and 9BB are primes, we only need to consider the families B{0,B}{9}B **** Since 9999B is prime, we only need to consider the families B(0,B)B, B(0,B)9B, B(0,B)99B, B{0,B}999B ***** All numbers of the form B{0,B}B are divisible by B, thus cannot be prime. ***** For the B{0,B}9B family: ****** Since B0B9B and BB09B are primes, we only need to consider the families B{0}9B and B{B}9B (since any digits combo 0B, B0 between (B,9B) will produce smaller primes) ******* All numbers of the from B{B}9B is either divisible by 11 (if totally number of B's is even) or

factored as $10^{(2*n)-21} = (10^n-5) * (10^n+5)$ (if totally number of B's is odd number 2^n-1), thus

cannot be prime.

***** For the B{0,B}99B family:

****** Since B0B9B and BB09B are primes, we only need to consider the families B{0}99B and B{B}99B (since any digits combo 0B, B0 between (B,99B) will produce smaller primes)

****** The smallest prime of the form B{0}99B is B00099B

******* The smallest prime of the form B{B}99B is BBBBBB99B

***** For the B{0,B}999B family:

****** Since B0B9B and BB09B are primes, we only need to consider the families B{0}999B and B{B}999B (since any digits combo 0B, B0 between (B,999B) will produce smaller primes)

****** The smallest prime of the form B{0}999B is

prime, since B00099B and B00000000000000000000000009B are primes)

References

BBBBBB99B is prime)

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- [5] <u>https://github.com/xayahrainie4793/non-single-digit-primes</u> (my data for these $M(L_b)$ sets for $2 \le b \le 16$)
- [6] http://recursed.blogspot.com/2006/12/prime-game.html (Shallit's The Prime Game page)
- [7] http://www.cs.uwaterloo.ca/~shallit/Papers/minimal5.pdf (Shallit's proof of base 10 minimal primes, when the restriction of prime>base is not required)
- [8] https://scholar.colorado.edu/downloads/hh63sw661 (proofs of minimal primes in bases b≤10, when the restriction of prime>base is not required)
- [9] https://cs.uwaterloo.ca/~cbright/reports/mepn.pdf (the article for this minimal prime problem in bases *b*≤30, when the restriction of prime>base is not required)
- [10] https://cs.uwaterloo.ca/~cbright/talks/minimal-slides.pdf (the article for this minimal prime problem in bases b≤30, when the restriction of prime>base is not required)
- [11] <u>https://doi.org/10.1080/10586458.2015.1064048</u> (the article for this minimal prime problem in bases $b \le 30$, when the restriction of prime>base is not required) (need access)
- [12] <u>https://github.com/curtisbright/mepn-data</u> (data for these $M(L_b)$ sets and unsolved families for $2 \le b \le 30$, when the restriction of prime>base is not required, search limits of lengths: 1000000 for b=17, 707000 for b=19, 506000 for b=21, 292000 for b=25, 486000 for b=26, 543000 for b=28, 233000 for b=29)
- [13] <u>https://github.com/RaymondDevillers/primes</u> (data for these $M(L_b)$ sets and unsolved families for $2 \le b \le 50$, when the restriction of prime>base is not required, search limits of lengths: 10000 for all b)
- [14] http://www.bitman.name/math/article/730 (article for minimal primes, when the restriction of prime>base is not required)
- [15] <u>http://www.bitman.name/math/table/497</u> (data for minimal primes in bases $2 \le b \le 16$, when the restriction of prime>base is not required)
- [16] https://oeis.org/A071071/a071071.pdf (research of minimal sets of powers of 2, when the restriction of >base is not required)
- [17] <u>http://nntdm.net/papers/nntdm-25/NNTDM-25-1-036-047.pdf</u> (research of minimal set of totients+n for $0 \le n \le 5$, when the restriction of >base is not required)

- [18] http://www.prothsearch.com/sierp.html (the Sierpinski problem)
- [19] http://www.prothsearch.com/rieselprob.html (the Riesel problem)
- [20] http://www.primegrid.com/ (with projects for the Sierpinski problem, the Riesel problem, the Prime Sierpinski problem, the Extended Sierpinski problem, Sierpinski/Riesel base 5 problem, generalized Fermat prime search)
- [21] <u>http://www.prothsearch.com/</u> (lists for primes of the form k^*2^n+1 for odd k<1200, also factoring status of generalized Fermat numbers of the form $a^{2^n} + b^{2^n}$ for $1 \le b < a \le 12$)
- [22] http://www.15k.org/ (lists for primes of the form $k*2^n-1$ for odd k<10000)
- [23] <u>https://www.rieselprime.de/default.htm</u> (lists for primes of the form $k^*2^n\pm 1$)
- [24] http://www.noprimeleftbehind.net/crus/Sierp-conjectures.htm (generalized Sierpinski conjectures in bases $b \le 1030$, some primes found in these conjectures are minimal primes in base b, especially, all primes for k < b (if exist for a (k,b) combo) are always minimal primes in the base b) (also some examples for simple families contain no primes > b)
- [25] http://www.noprimeleftbehind.net/crus/Riesel-conjectures.htm (generalized Riesel conjectures in bases $b \le 1030$, some primes found in these conjectures are minimal primes in base b, especially, all primes for k < b (if exist for a (k,b) combo) are always minimal primes in the base b) (also some examples for simple families contain no primes > b)
- [26] http://www.noprimeleftbehind.net/crus/tab/CRUS_tab.htm (list for the status of the generalized Sierpinski conjectures and the generalized Riesel conjectures in bases *b*≤1030)
- [27] https://www.utm.edu/staff/caldwell/preprints/2to100.pdf (article for generalized Sierpinski conjectures in bases *b*≤100)
- [28] https://oeis.org/A076336/a076336c.html (the dual Sierpinski problem)
- [29] https://mersenneforum.org/showthread.php?t=10761 (list of large (probable) primes for the dual Sierpinski problem)
- [30] http://www.kurims.kyoto-u.ac.jp/EMIS/journals/INTEGERS/papers/i61/i61.pdf (article for the mixed (original+dual) Sierpinski problem)
- [31] https://mersenneforum.org/showthread.php?t=6545 (research for the mixed (original+dual) Riesel problem)
- [32] https://mersenneforum.org/showthread.php?t=26328 (research for the mixed (original+dual) Sierpinski base 5 problem)
- [33] <u>http://www.fermatquotient.com/</u> (generalized repunit primes (primes of the form $(b^n-1)/(b-1)$) in bases $b \le 160$, the smallest such prime for base b (if exists) is always minimal

prime in base b, also list of generalized half Fermat primes (primes of the form $(b^{2^n} + 1)/2$) sorted by n, the smallest such prime for base b (if exists) is always minimal prime in base b)

[34]

https://web.archive.org/web/20021111141203/http://www.users.globalnet.co.uk/~aads/prime s.html (generalized repunit primes (primes of the form $(b^n-1)/(b-1)$) in bases $b \le 1000$, the smallest such prime for base b (if exists) is always minimal prime in base b)

- [35] <u>http://jeppesn.dk/generalized-fermat.html</u> (generalized Fermat primes (primes of the form $b^{2^n} + 1$) in even bases $b \le 1000$, the smallest such prime for base b (if exists) is always minimal prime in base b)
- [36] <u>http://www.noprimeleftbehind.net/crus/GFN-primes.htm</u> (generalized Fermat primes (primes of the form $b^{2^n} + 1$) in even bases $b \le 1030$, the smallest such prime for base b (if exists) is always minimal prime in base b)
- [37] <u>https://harvey563.tripod.com/wills.txt</u> (primes of the form $(b-1)*b^n-1$ for bases $b \le 2049$, the smallest such prime for base b (if exists) is always minimal prime in base b)
- [38] <u>https://www.rieselprime.de/ziki/Williams_prime_MM_least</u> (the smallest primes of the form $(b-1)^*b^n-1$ for bases $b \le 2049$, these primes (if exists) is always minimal prime in base b)
- [39] <u>https://www.rieselprime.de/ziki/Williams_prime_MP_least</u> (the smallest primes of the form $(b-1)*b^n+1$ for bases $b \le 1024$, these primes (if exists) is always minimal prime in base b)
- [40] <u>https://www.rieselprime.de/ziki/Riesel_prime_small_bases_least_n</u> (the smallest primes of the form k^*b^n -1 for $k \le 12$ and bases $b \le 1024$, these primes (if exists) is always minimal prime in base b if b > k)
- [41] <u>https://www.rieselprime.de/ziki/Proth_prime_small_bases_least_n</u> (the smallest primes of the form k^*b^n+1 for $k \le 12$ and bases $b \le 1024$, these primes (if exists) is always minimal prime in base b if b > k)
- [42] https://docs.google.com/spreadsheets/d/e/2PACX-1vTKkSNKGVQkUINlp1B3cXe90FWPwiegdA07EE7-U7sqXntKAEQrynol1sbFvvKriieda3LfkqRwmKME/pubhtml (my list for the smallest primes in given simple family in bases *b*≤1024)
- [43] https://www.rose-hulman.edu/~rickert/Compositeseg (a problem related to this project)
- [44] http://www.worldofnumbers.com/Appending%201s%20to%20n.txt (a problem related to this project)
- [45] http://www.worldofnumbers.com/deplat.htm (list of plateau and depression primes)

- [46] http://www.worldofnumbers.com/wing.htm (list of palindromic wing primes)
- [47] https://stdkmd.net/nrr/prime/primecount.txt (near- and quasi- repdigit (probable) primes sorted by count)
- [48] https://stdkmd.net/nrr/prime/primedifficulty.txt (near- and quasi- repdigit (probable) primes sorted by difficulty)
- [49] https://stdkmd.net/nrr/coveringset.htm (covering set of near-repdigit-related sequences)
- [50] http://www.rieselprime.de/dl/CRUS pack.zip (srsieve, sr1sieve, sr2sieve, pfgw, and llr softwares)
- [51] https://www.bc-team.org/app.php/dlext/?cat=3 (srsieve, sr1sieve, sr2sieve, sr5sieve software)
- [52] https://sourceforge.net/projects/openpfgw/ (pfgw software)
- [53] http://jpenne.free.fr/index2.html (*llr* software)
- [54] http://www.ellipsa.eu/public/primo/primo.html (PRIMO software)
- [55] https://primes.utm.edu/prove/index.html (website for primality proving)
- [56] https://www.rieselprime.de/ziki/Primality_test (list of known primality tests and probable primality tests)
- [57] https://primes.utm.edu/notes/prp_prob.html (the probability that a random PRP is composite)
- [58] https://oeis.org/wiki/User:Charles R Greathouse IV/Tables of special primes (expected number of primes in first *n* terms of a given sequence)
- [59] https://primes.utm.edu/curios/page.php?number_id=22380 (the largest base 10 minimal prime in Prime Curios!)
- [60] https://oeis.org/A071062 (OEIS sequence for base 10 minimal primes, when the restriction of prime>base is not required)
- [61] https://oeis.org/A326609 (OEIS sequence for the largest base *b* minimal prime, when the restriction of prime>base is not required)
- [62] https://primes.utm.edu/primes/lists/all.txt (top proven primes)
- [63] http://www.primenumbers.net/prptop/prptop.php (top PRPs)
- [64] http://factordb.com (online factor database, including many primes which are minimal primes in a small base)