Minimal elements for the base *b* representations of the primes which are > *b*

Keywords

prime number, number theory, minimal element, partially ordered set, subsequence, formal language theory, positional notation, radix, algorithm, computer science, primality test, Miller–Rabin primality test, Baillie–PSW primality test, sieving, heuristic algorithm, conjecture, open problem, mathematical proof

Target of this article

Find the minimal set (the set of the minimal elements) of the set of the "prime number > *b*" digit strings in the positional numeral system with base (or radix) *b* for the subsequence ordering for 2 ≤ *b* ≤ 36 (i.e. from binary (base 2) to hexatridecimal (base 36)) (e.g. for *b* = 10 (decimal), this set has exactly 77 elements: {11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 227, 251, 257, 277, 281, 349, 409, 449, 499, 521, 557, 577, 587, 727, 757, 787, 821, 827, 857, 877, 881, 887, 991, 2087, 2221, 5051, 5081, 5501, 5581, 5801, 5851, 6469, 6949, 8501, 9001, 9049, 9221, 9551, 9649, 9851, 9949, 20021, 20201, 50207, 60649, 80051, 666649, 946669, 5200007, 22000001, 60000049, 66000049, 66600049, 80555551, 555555555551, 5000000000000000000000000000027}).

Introduction

A string *x* is a subsequence of another string *y*, if *x* can be obtained from *y* by deleting zero or more of the characters (in this article, digits) in *y*. For example, 514 is a subsequence of 352148, "*string*" is a subsequence of "*Meistersinger*". In contrast, 758 is not a subsequence of 378259, since the characters (in this article, digits) must be in the same order. The empty string λ is a subsequence of every string. There are 2^n subsequences of a string with length *n*, e.g. the subsequences of 123456 are (totally 2^6 = 64 subsequences):

λ, 1, 2, 3, 4, 5, 6, 12, 13, 14, 15, 16, 23, 24, 25, 26, 34, 35, 36, 45, 46, 56, 123, 124, 125, 126, 134, 135, 136, 145, 146, 156, 234, 235, 236, 245, 246, 256, 345, 346, 356, 456, 1234, 1235, 1236, 1245, 1246, 1256, 1345, 1346, 1356, 1456, 2345, 2346, 2356, 2456, 3456, 12345, 12346, 12356, 12456, 13456, 23456, 123456

(In this article, we only consider the subsequences with length ≥2, and not consider the subsequences beginning with 0 and/or ending with 0, e.g. for the string 123456, we have these subsequences: 12, 13, 14, 15, 16, 23, 24, 25, 26, 34, 35, 36, 45, 46, 56, 123, 124, 125, 126, 134, 135, 136, 145, 146, 156, 234, 235, 236, 245, 246, 256, 345, 346, 356, 456, 1234, 1235, 1236, 1245, 1246, 1256, 1345, 1346, 1356, 1456, 2345, 2346, 2356, 2456, 3456, 12345, 12346, 12356, 12456, 13456, 23456, 123456, totally 57 subsequences, and for a string with length *n* with no character 0, there are 2^n-n-1 subsequences)

Subsequence should not be confused with substring, a substring is a contiguous sequence of characters within a string, e.g. 397 is a subsequence of 163975, "*string*" is a substring of "*substring*". In contrast, 514 is a subsequence of 352148, but not a substring. The empty string λ is a substring of every string. There are $n*(n+1)/2+1$ substrings of a string with length *n*, e.g. the substrings of 123456 are (totally 6*(6+1)/2+1 = 22 substrings):

λ, 1, 2, 3, 4, 5, 6, 12, 23, 34, 45, 56, 123, 234, 345, 456, 1234, 2345, 3456, 12345, 23456, 123456

There are 64−22 = 42 subsequences of 123456 which are not substrings:

13, 14, 15, 16, 24, 25, 26, 35, 36, 46, 124, 125, 126, 134, 135, 136, 145, 146, 156, 235, 236, 245, 246, 256, 346, 356, 1235, 1236, 1245, 1246, 1256, 1345, 1346, 1356, 1456, 2346, 2356, 2456, 12346, 12356, 12456, 13456

Substring also called "subword", while subsequence also called "scattered subword".

(For the references of the difference between "subsequence" and "substring", see this post and this post, and see the list below)

Subsequence	**Substring**
A071062	A033274
A130448	A238334
A039995	A039997
A039994	A039996
A094535	A093301
A350508	A038103
https://primes.utm.edu/glossary/xpage/MinimalPrime.html	https://www.mersenneforum.org/showthread.php?p=235098#post235098
longest common subsequence problem	longest common substring problem

The longest common subsequence problem and the longest common substring problem are two hard problems on strings, the former is NP-hard and NP-complete, while the latter is not.

divisibility ordering	subset ordering	subsequence ordering	substring ordering
greatest common divisor of natural	intersection of sets	longest common subsequence of	longest common substring of strings

numbers		strings	

Note: The comment by Charles R Greathouse IV in https://oeis.org/A062115 is wrong, it should be A033274 instead of A071062, however, A062115 is a 10-automatic sequence is really true, currently there is no analog of A062115 with subsequence instead of substring in OEIS (searching of this sequence in OEIS), the first difference of such sequence and A062115 is that such sequence does not have the term 169 (since the prime number 19 is a subsequence but not a substring, of 169), but A062115 has.

(In this article, we only research subsequence and not research substring, the reason is the minimal set of subsequence ordering must be finite even if the set is infinite (by the theorem that there are no infinite antichains for the subsequence ordering), and hence we may find this set, but the minimal set of substring ordering may be infinite, and it is highly possible that we cannot find this set, e.g. the minimal set of subsequence ordering of the set of prime number digit strings with length ≥2 in decimal (proofs for that this set is infinite) is known to be finite and contain exactly 77 elements, and the largest element is $50^{28}27$, where 0^{28} means the string with 28 0's, but the minimal set of substring ordering of the set of prime number digit strings with length ≥2 in decimal is very likely to be infinite, since all primes of the form 1{0}3 (10^n+3, A159352) or 3{0}1 ($3*10^n+1$, A259866) are minimal elements of substring ordering of the set of prime number digit strings with length ≥2 in decimal, and there is likely infinitely many primes of the form 1{0}3 and infinitely many primes of the form 3{0}1 (see the "Proof" section of this article, also see this reference), thus the minimal set of substring ordering is not discussed in this article) (Another reason: the problem of the minimal set of substring ordering cannot cover the Sierpinski problems and the Riesel problems and the problem 197 in World! Of Numbers, while the problem of the minimal set of subsequence ordering can, since (for example) 1223 is not a substring of 12223, and 12223 is not a substring of 122223, and hence cannot contain a large number 1222…2223, thus the problem of the minimal set of substring ordering is less-number-theory-related then the problem of the minimal set of subsequence ordering)

The set of all strings ordered by subsequence (i.e. under the binary relation "is a subsequence of") is a partially ordered set (i.e. the binary relation "is a subsequence of" is a partial order relation, since this binary relation is reflexive, antisymmetric, and transitive), hence, any given (finite or infinite) set (e.g. the set of the "prime numbers > *b*" strings in base *b*, for 2 ≤ *b* ≤ 36), which is the target of this article) of strings ordered by subsequence is also a partially ordered set, and thus we can draw its Hasse diagram and find its greatest element, least element, maximal elements, and minimal elements, however, the greatest element and least element may not exist, and for an infinite set, the maximal elements also may not exist, thus we are only interested on finding the minimal elements of such sets, and we define *minimal set* of a set as the set of the minimal elements of this set, under a given partially ordered binary relation (this binary relation is "is a subsequence of" in this article), and we use *M*(*S*) to denote the minimal set of the set *S*.

A partially ordered set is a totally ordered set if the elements in this set are pairwise comparable, two elements *x* and *y* are comparable with respect to a binary relation "≤" if at least one of *x* ≤ *y* or *y* ≤ *x* is true, thus, under the binary relation "is a subsequence of", two strings *x* and *y* are comparable if either *x* is a subsequence of *y*, or *y* is a subsequence of *x*.

A surprising result from formal language theory is that every set of pairwise incomparable (i.e. not comparable) strings is finite (note that this is not true for general partially ordered binary relations, e.g. the set of the positive integers, under the binary relation “is a divisor of”, the infinite set of the prime numbers (proofs for that this set is infinite) is pairwise incomparable, in fact, this set is exactly the minimal set of the set of the positive integers >1 under this binary relation). This means that from any set of strings we can find its minimal elements. A string *x* in a set of strings *S* is a *minimal string* (minimal element of a set of strings ordered by subsequence) if whenever *y* (an element of *S*) is a subsequence of *x*, we have *y* = *x*.

The set of all minimal strings of *S* is denoted *M*(*S*), *M*(*S*) is the **kernel** of the set *S*, and the set *M*(*S*) must be finite! Even if *S* is an infinite set, such as the set of prime number digit strings with length ≥2 in decimal (proofs for that this set is infinite) and the set of square number digit strings with length ≥2 in decimal, although the set of the minimal strings of the latter set is not known and extremely difficult to compute. The set of the minimal strings of the former set has exactly 77 elements, and it is {11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 227, 251, 257, 277, 281, 349, 409, 449, 499, 521, 557, 577, 587, 727, 757, 787, 821, 827, 857, 877, 881, 887, 991, 2087, 2221, 5051, 5081, 5501, 5581, 5801, 5851, 6469, 6949, 8501, 9001, 9049, 9221, 9551, 9649, 9851, 9949, 20021, 20201, 50207, 60649, 80051, 666649, 946669, 5200007, 22000001, 60000049, 66000049, 66600049, 80555551, 555555555551, 5000000000000000000000000000027}, and we will prove that this set is complete, and the research of this set in other bases is exactly the target of this article. The set of the minimal strings of the latter set is {16, 25, 36, 49, 64, 81, 100, 121, 144, 289, 324, 400, 441, 484, 529, 576, 676, 729, 784, 900, 961, 1024, 1089, 2209, 2304, 2401, 2601, 2704, 3721, 3844, 4761, 5041, 5184, 6561, 6889, 7056, 7569, 7744, 7921, 21904, 22201, 28224, 29241, 29929, 31329, 35344, 38809, 46656, 47524, 55696, 62001, 63001, 69696, 79524, 80089, 80656, 82944, 88209, 88804, 91204, 91809, 97344, 97969, 98596, 99856, 138384, 139129, 173889, 182329, 199809, 300304, 301401, 304704, 305809, 332929, 339889, 345744, 374544, 393129, 473344, 505521, 515524, 558009, 559504, 567009, 589824, 595984, 657721, 660969, 665856, 683929, 695556, 702244, 719104, 743044, 777924, 779689, 842724, 850084, 876096, 877969, 896809, 898704, 929296, 935089, 1317904, 1557504, 1882384, 1898884, 2022084, 2027776, 2039184, 2070721, 2477476, 2802276, 2979076, 2999824, 3055504, 3073009, 3139984, 3323329, 3415104, 3794704, 4477456, 4545424, 4575321, 5053504, 5067001, 5071504, 5280804, 5303809, 5513104, 5527201, 5531904, 5574321, 5579044, 5707321, 5750404, 5755201, 5987809, 6517809, 6568969, 6620329, 6901129, 7006609, 7011904, 7033104, 7096896, 7177041, 7474756, 7551504, 7557001, 7573504, 7941124, 8020224, 8054244, 8282884, 8340544, 8508889, 8538084, 8620096, 8809024, 9229444, 9535744, 9809424, 9847044, 9935104, 9998244, 13118884, 13337104, 15038884, 15578809, 18939904, 19775809, 20903184, 20912329, 20994724, 23902321, 27709696, 29833444, 31102929, 31899904, 33039504, 33085504, 33315984, 33500944, 35533521, 35545444, 37797904, 38093584, 39980329, 40755456, 45535504, 47073321, 47444544, 50098084, 50566321, 50580544, 50608996, 50808384, 51151104, 53333809, 53993104, 55011889, 55517401, 55666521, 57501889, 57775201, 58247424, 58339044, 58859584, 59089969, 60575089, 60590656, 61199329, 65658609, 66650896, 66863329, 69072721, 69338929, 70006689, 70543201, 70997476, 71351809, 72233001, 73153809, 73994404, 74407876, 74632321, 75968656, 77668969, 77686596, 77757124, 77898276, 78907689, 78960996, 78978769, 79869969, 84052224, 85507009, 86992929, 88059456, 88096996, 88585744, 88868329, 89056969, 91833889, 94303521, ...}, although this set seems to be endless, but by the theorem that there are no infinite antichains for the subsequence ordering, this set must be finite, but this set is extremely difficult to found (reference), and it is also difficult to determine the number of elements in this set, and is much more difficult than that of the first set in every base 2 ≤ *b* ≤ 36 (to find these two sets in bases 2 ≤ *b* ≤ 36 (the prime or square = *b* (i.e. the prime or square "10") is also excluded when the base (*b*) is itself prime or square), we can use some theorems in number theory, e.g. a digit in base *b* can be the last digit of a prime number > *b* if and only if this digit is coprime to *b* (i.e. this digit is in the reduced residue system mod *b*, there are *eulerphi*(*b*) such digits), and a digit in base *b* can be the last digit of a square number > *b* if and only if this digit is a quadratic residue mod *b*). For example, it is not even known whether there is a square composed of digits 6, 7, 8 (except $676 = 26^2$) (reference and reference and reference), also, it is not even known whether the non-simple family $3^m5^n9^r44$ contain a square or not, this situation usually not occur for primes in any base, i.e. every non-simple family which can not be ruled out as containing no primes > base usually contain a small prime > base, thus although the problem in this article (i.e. finding the minimal set of the primes > *b* in base *b*, for 2 ≤ *b* ≤ 36) is hard, it is much easier than finding the minimal set of the squares > 10 in decimal (also finding the minimal set of the squares > *b* in base *b* for any base *b* > 4), thus the latter set is not discussed in this article. (another reason for we research the minimal strings of the prime numbers instead of the minimal strings of the square numbers is that the prime numbers behave similarly to a random sequence of numbers, while the square numbers do not, thus prime numbers are more mysterious than square numbers) (reference of primes written in other bases)

	the last digit of a prime number > *b* in base *b*	the last digit of a square number > *b* in base *b*
condition	coprime to *b*	a quadratic residue mod *b*
number of such digits	A000010	A000224
irregular triangle read by rows, row *b* is such digits in base *b*	A038566	A096008
bases *b* such that all such digits are (primes or 1, squares, respectively), thus the corresponding minimal set problems are easy to solve if single-digit numbers are not excluded, there are only finitely many such bases *b*	A048597 (2, 3, 4, 6, 8, 12, 18, 24, 30)	A254328 (2, 3, 4, 5, 8, 12, 16)

In this article, we want to find the set of the minimal strings of the "prime number > *b*" digit strings in bases 2 ≤ *b* ≤ 36, since decimal (base 10) is not special in mathematics, there is no reason to only find this set in decimal (base 10), also, finding this set in decimal (base 10) is too easy to be researched in an article (only harder than bases 2, 3, 4, 6), thus it is necessary to research this set in other bases *b*.

Equivalently, a string x in a set of strings S is a minimal string if and only if any proper subsequence of x (subsequence of x which is unequal to x, like proper subset) is not in S.

The minimal set $M(L)$ of a language L is interesting, this is because it allows us to compute two natural and related languages, defined as follows:

$sub(L) := \{x \in \Sigma^* :$ there exists $y \in L$ such that x is a subsequence of $y\}$;
$sup(L) := \{x \in \Sigma^* :$ there exists $y \in L$ such that y is a subsequence of $x\}$.

An amazing fact is that $sub(L)$ and $sup(L)$ are always regular. This follows from the following classical theorem:

Theorem: For every language L, there are only finitely many minimal strings. (Equivalently, there are no infinite antichains for the subsequence ordering) (references: https://books.google.com.tw/books?id=-HrTBwAAQBAJ&pg=PA255&lpg=PA255&dq=every+set+of+pairwise+incomparable+strings+is+finite&source=bl&ots=U7D1b_pfao&sig=ACfU3U2_pcwWftogmSFA03C6D7_xR5ux-g&hl=zh-TW&sa=X&ved=2ahUKEwjP272ytqX2AhWMHKYKHfqVCOAQ6AF6BAgTEAM#v=onepage&q=every%20set%20of%20pairwise%20incomparable%20strings%20is%20finite&f=false https://www.jstor.org/stable/44161544 http://www.ams.org/mathscinet-getitem?mr=84g:05002 (article is not yet available) https://hal.archives-ouvertes.fr/hal-01888614/document https://books.google.com.tw/books?id=wkVbDAAAQBAJ&pg=PA84&lpg=PA84&dq=every+set+of+pairwise+incomparable+strings+is+finite&source=bl&ots=Mt9t8xpcS3&sig=ACfU3U1pKFNjoyS5UelN-SfNX7wKE2NLqg&hl=zh-TW&sa=X&ved=2ahUKEwjP272ytqX2AhWMHKYKHfqVCOAQ6AF6BAgREAM#v=onepage&q=every%20set%20of%20pairwise%20incomparable%20strings%20is%20finite&f=false https://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.65.3806&rep=rep1&type=pdf http://www.combinatorics.org/Volume_7/PDF/v7i1n2.pdf https://www.researchgate.net/publication/233917563_Large_infinite_antichains_of_permutations http://www.lsv.fr/~phs/course1.pdf)

Indeed, we have $sup(L) = sup(M(L))$ and $\Sigma^* - sub(L) = sup(M(\Sigma^* - sub(L)))$, and the superword language of a finite language is regular, since $sup(\{w_1, ..., w_n\}) = \cup_{i=1}^{n} \quad \Sigma^* w_{i,1} \Sigma^* \ldots \Sigma^* w_{i,|w_i|} \Sigma^*$ where $w_i = w_{i,1} \ldots w_{i,|w\_i|}$ with $w_{i,j} \in \Sigma$.

Since there are no infinite antichains for the subsequence ordering of strings whose characters belong to a fixed finite set (e.g. the "prime number > b" digit strings in positional numeral system with radix b (which is exactly the target of this article), whose characters always belong to the set of the digits in base b: $\{0, 1, ..., b - 1\}$, which is a finite set with b elements, note that the set must be finite (an easy counterexample for an infinite set S is the set of all strings with length 2 whose characters belong to the set S, which is clearly an infinite antichain for the subsequence ordering), thus, e.g. in factorial base there may exist infinitely many minimal primes, i.e. the minimal set of the prime strings of subsequence ordering may be infinite, since the set of the digits in factorial base is infinite, it includes *all* nonnegative integers, and thus this is not discussed in this article, just as the minimal set of substring ordering) (note that there can be infinite antichains for general ordering, e.g. the set of primes is an infinite antichain for the divisibility ordering (proofs for that this set is infinite), also, the set of strings {*abc*, *abbc*, *abbbc*, *abbbbc*, ...} is an infinite antichain for the substring ordering of strings whose characters are in a fixed finite set {*a*, *b*, *c*}), the set $M(S)$ of minimal strings of any set S of strings must be finite.

Although the set $M(S)$ of minimal strings is necessarily finite, determining it explicitly for a given S can be a difficult computational problem. We use some numbertheoretic heuristics to compute $M(L_b)$, where L_b is the language of base-b representations of the prime numbers which are > b, for $2 \le b \le 16$ (the set $M(L_b)$ can be called ***b*-kernel**, since this set is the kernel of the set L_b). (Also, I left as a challenge to readers the task of computing $M(L_b)$ for $17 \le b \le 36$) (we stop at base 36 since this base is a maximum base for which it is possible to write the numbers with the symbols 0, 1, ..., 9 (the 10 Arabic numerals) and A, B, ..., Z (the 26 Latin letters) of the Latin alphabet, references: http://www.tonymarston.net/php-mysql/converter.html https://www.dcode.fr/base-36-cipher http://www.urticator.net/essay/5/567.html http://www.urticator.net/essay/6/624.html https://docs.python.org/3/library/functions.html#int https://reference.wolfram.com/language/ref/BaseForm.html https://baseconvert.com/ https://www.calculand.com/unit-converter/zahlen.php?og=Base+2-36&ug=1 https://linesegment.web.fc2.com/application/math/numbers/RadixConversion.html (in Japanese), also see https://primes.utm.edu/notes/words.html for English words which are prime numbers when viewed as a number base 36)

This problem is very hard, since determining $M(L)$ for arbitrary L is in general unsolvable and can be difficult even when L is relatively simple, the set $M(L)$ is an antichain of L for the subsequence ordering (although may not be the "maximum antichain" (an antichain that has cardinality at least as large as every other antichain), which may not exist even for the subsequence ordering, although there cannot be an infinite antichains for the subsequence ordering), the problems in this article (i.e. determining $M(L_b)$ for $2 \le b \le 36$) are very hard open problems in number theory when b is large (say > 16) and may be NP-complete or NP-hard or an undecidable problem, or an example of Gödel's incompleteness theorems (like the continuum hypothesis and the halting problem, in fact, if the halting problem can be solved, then the problem in this article can also be solved (we only need to write a computer program for this problem, since this problem is discrete), however, the halting problem is known to be undecidable, i.e. a general algorithm to solve the halting problem for all possible program-input pairs cannot exist) (even in the weaker case that probable primes are allowed in place of proven primes, i.e. not including primality proving of the probable primes in $M(L_b)$), or as hard as the unsolved problems in mathematics, such as the Riemann hypothesis and the *abc* conjecture (which are the two famous hard problems in number theory), determining $M(L_b)$ is much harder when b > 24 and/or *eulerphi*(b) is larger, since *eulerphi*(b) is the number of possible last digits of a prime number > b in base b (these digits are exactly the base b digits coprime to b, all these bases are possible and for all such digits, there are infinitely many such primes (by Dirichlet's theorem), and for digits not coprime to b (let d be the greatest common divisor (GCD) of the digit and b), all such numbers are divisible by d and $d \le b$, thus cannot be primes > b). Maybe only God knows the set $M(L_b)$ when b > 24 (and only God knows the largest element in the set $M(L_b)$ when b > 24, and only God knows the number of the elements in the set $M(L_b)$ when b > 24). We can imagine an alien force, vastly more powerful than us, landing on Earth and demanding $M(L_b)$ for b = 17 (or 18, 19, 20, 21, 22, 23, 24, 28, 30, 36) (including primality proving of all primes in this set) or they will destroy our planet. In that case, I claim, we should marshal all our computers and all our mathematicians and attempt to find the set and to prove the primality of all numbers in this set. But suppose, instead, that they ask for $M(L_b)$ for b = 25 (or 26, 27, 29, 31, 32, 33, 34, 35). In that case, I believe, we should attempt to destroy the aliens. (like Paul Erdős for the Ramsey numbers, I do not think that finding $M(L_b)$ for $b > 16$ is easier than finding the Ramsey numbers $R(m,n)$ for $m > 4$, $n > 4$)

There are many unsolved problems (open problems) in number theory:

* Grand Riemann hypothesis
** Extended Riemann hypothesis
*** Generalized Riemann hypothesis
**** Riemann hypothesis
* *n* conjecture
** *abc* conjecture
*** Fermat–Catalan conjecture
**** Beal conjecture
*** Lander, Parkin, and Selfridge conjecture
*** Pillai's conjecture
** Szpiro's conjecture

and unsolved problems (open problems) about the prime numbers, which are related to this article:

* Are there infinitely many Mersenne primes (the Lenstra–Pomerance–Wagstaff conjecture)? (Equivalently, are there infinitely many even perfect numbers?)
* Are there infinitely many Wagstaff primes?
* Are there infinitely many repunit primes?
* Is every Fermat number $2^{2^n}+1$ composite for $n > 4$?
* Is every generalized half Fermat number $(3^{2^n}+1)/2$ composite for $n > 6$?
* Is every double Mersenne number $2^{2^n-1}-1$ composite for $n > 7$?
* Are all Mersenne numbers of prime index square-free?
* Are all Wagstaff numbers of prime index square-free?
* Are all Fermat numbers square-free?
* For any given natural number $b \geq 2$ which is not perfect power, are there infinitely many generalized repunit primes in base b (primes of the form $(b^n-1)/(b-1)$)? (If b is perfect power, then generalized repunits in base b can be factored algebraically, and thus there is at most one generalized repunit prime in base b, reference: https://oeis.org/A084740 https://oeis.org/A128164 https://oeis.org/A096059 https://oeis.org/A126589)
* For any given natural number $b \geq 2$ which is neither perfect odd power (A070265) nor of the form $4m^4$ (A141046), are there infinitely many generalized Wagstaff primes in base b (primes of the form $(b^n+1)/(b+1)$)? (If b is either perfect odd power (A070265) or of the form $4m^4$ (A141046), then generalized Wagstaff numbers in base b can be factored algebraically, and thus there is at most one generalized Wagstaff prime in base b)

* For any given even natural number $b \ge 2$, are there only finitely many generalized Fermat primes in base b (primes of the form $b^{2^n}+1$)? (If b is odd, then all generalized Fermat numbers in base b are divisible by 2, and if b is either perfect odd power (A070265), then generalized Fermat numbers in base b can be factored algebraically, and thus there is no generalized Fermat prime in base b)
* For any given odd natural number $b \ge 3$, are there only finitely many generalized half Fermat primes in base b (primes of the form $(b^{2^n}+1)/2$)? (If b is either perfect odd power (A070265), then generalized half Fermat numbers in base b can be factored algebraically, and thus there is no generalized half Fermat prime in base b)
* For any given natural number $b \ge 2$, are there infinitely Williams primes of the first kind base b (primes of the form $(b-1)*b^n-1$)?
* For any given natural number $b \ge 2$, are there infinitely Williams primes of the second kind base b (primes of the form $(b-1)*b^n+1$)?
* For any given natural number $b \ge 2$, are there infinitely Williams primes of the third kind base b (primes of the form $(b+1)*b^n-1$)?
* For any given natural number $b \ge 2$ which is not == 1 mod 3, are there infinitely Williams primes of the fourth kind base b (primes of the form $(b+1)*b^n+1$)? (If b is == 1 mod 3, then all Williams numbers of the fourth kind in base b are divisible by 3, and thus there is no Williams primes of the fourth kind in base b)
* Is 78557 the lowest Sierpiński number (the Selfridge conjecture)?
* Is 509203 the lowest Riesel number?
* Is 125050976086 the lowest Sierpiński number to base 3?
* Is 63064644938 the lowest Riesel number to base 3?
* Is 66741 the lowest Sierpiński number to base 4?
* Is 39939 the lowest Riesel number to base 4 which is not square (for square k, $k*4^n-1$ can be factored algebraically)?
* Is 159986 the lowest Sierpiński number to base 5?
* Is 346802 the lowest Riesel number to base 5?
* Is 174308 the lowest Sierpiński number to base 6?
* Is 1597 a Riesel number to base 6? (Equivalently, is 84687 the lowest Riesel number to base 6?)

(for the generalization of the lowest Sierpiński numbers and the lowest Riesel numbers to other bases, see *CRUS* pages and this article)

other unsolved problems (open problems) about the prime numbers:

* Goldbach conjecture
* Twin prime conjecture
* Polignac's conjecture
* First Hardy–Littlewood conjecture
* Fourth Landau problem
* Bunyakovsky conjecture
* Dickson's conjecture
* Schinzel's hypothesis H
* Are there any odd perfect numbers?
* Are there any almost perfect numbers other than powers of 2?

* Are there any quasiperfect numbers?
* For any given natural number $n \geq 2$, are there infinitely many *n*-perfect numbers?
* For any given natural number $n \geq 2$, are there infinitely many *n*-hyperperfect numbers?
* Find the set of friendly numbers, especially, are 10, 14, 15, 20, 22, 26, 33, 34, … friendly? (they are conjectured to be solitary, i.e. not friendly, but if friendly, their smallest friends are large numbers, like the status for the number 24, although 24 is friendly, its smallest friend is 91963648)
* Are there any odd weird numbers?
* Are there infinitely many amicable numbers?
* Are there any pairs of amicable numbers which have opposite parity?
* Are there any pairs of relatively prime amicable numbers?
* Are there infinitely many betrothed numbers?
* Are there any pairs of betrothed numbers which have the same parity?
* Are there infinitely many sociable number cycles?
* Are there any sociable number cycles with length 3?
* Are there any sociable number cycles such that not all numbers have the same parity?
* Are there any quasi-sociable number cycles with odd length?
* Are there any numbers n such that *eulerphi*(x) = n has exactly one solution?
* Are there any composite numbers n such that *eulerphi*(n) divides $n-1$?
* Artin's conjecture on primitive roots
* For any given integer a which is not a square and does not equal to −1, are there infinitely many primes with a as a primitive root?
* For any given positive integer b which is not a perfect power, are there infinitely many primes with b as smallest positive primitive root?
* For any given negative integer b which is not a perfect power, are there infinitely many primes with b as largest negative primitive root?
* Are there infinitely many Sophie Germain primes (A005384)? (Equivalently, are there infinitely many safe primes (A005385))?
* Are there infinitely many Sophie Germain primes of the second kind (A005382)? (Equivalently, are there infinitely many safe primes of the second kind (A005383))?
* Are there infinitely many Proth primes (A080076)?
* Are there infinitely many Proth primes of the second kind (A112715)?
* Are there infinitely many Pierpont primes (A005109)?
* Are there infinitely many Pierpont primes of the second kind (A005105)?
* Are there infinitely many Cullen primes (primes of the form $n*2^n+1$)?
* Are there infinitely many Woodall primes (primes of the form $n*2^n-1$)?
* Are there any primes p such that $p*2^p+1$ is also prime?
* For any given natural number $b \geq 2$, are there infinitely generalized Cullen primes in base b (primes of the form $n*b^n+1$)?
* For any given natural number $b \geq 2$, are there infinitely generalized Woodall primes in base b (primes of the form $n*b^n-1$)?
* Are there infinitely many Carol primes (primes of the form $(2^n-1)^2-2$)?
* Are there infinitely many Kynea primes (primes of the form $(2^n+1)^2-2$)?
* For any given even natural number $b \geq 2$, are there infinitely generalized Carol primes in base b (primes of the form $n*b^n+1$)? (If b is odd, then all generalized Carol numbers in base b are divisible by 2, and thus there is no generalized Carol prime in base b)

* For any given even natural number $b \geq 2$, are there infinitely generalized Kynea primes in base b (primes of the form $n*b^n-1$)? (If b is odd, then all generalized Kynea numbers in base b are divisible by 2, and thus there is no generalized Kynea prime in base b)

And many hard problems in number theory which are either proved or disproved:

* Fermat's Last Theorem (proved)
** Euler's sum of powers conjecture (disproved)
* Catalan's conjecture (proved)
* Dirichlet's theorem (proved)
* length of primes in arithmetic progression has no upper bound (proved)

Notation

In what follows, if x is a string of symbols over the alphabet Σ_b := {0, 1, ..., b − 1} (the set of the base-b digits) we let $[x]_b$ denote the evaluation of x in the positional numeral system with base (or radix) b (starting with the most significant digit), and $[\lambda]_b$:= 0 where λ is the empty string. This is extended to languages as follows: $[L]_b$:= { $[x]_b$: $x \in L$ }. We use the convention that A := 10, B := 11, C := 12, ..., Z := 35, to conveniently represent strings of symbols in base b > 10. We let $(x)_b$ be the canonical representation of x in base b, that is, the representation without leading zeros. Finally, as usual, for a language L we let L^n := $LLL...LLL$ with n Ls and L^* := $\cup_{i\geq 0} L^i$.

Besides, we use $M(S)$ to denote the the minimal set (the set of the minimal elements) of the set S of strings for the subsequence ordering, and we use L_b to denote the language of base-b representations of the prime numbers which are > b (thus, L_b is a set of strings), this is a list for L_b for bases 2 ≤ b ≤ 36:

b	L_b (using A−Z to represent digit values 10 to 35)
2	11, 101, 111, 1011, 1101, 10001, 10011, 10111, 11101, 11111, 100101, 101001, 101011, 101111, 110101, 111011, 111101, 1000011, 1000111, 1001001, 1001111, 1010011, 1011001, 1100001, 1100101, 1100111, 1101011, 1101101, 1110001, 1111111, 10000011, 10001001, 10001011, 10010101, 10010111, 10011101, 10100011, 10100111, 10101101, 10110011, 10110101, 10111111, 11000001, 11000101, 11000111, 11010011, 11011111, 11100011, 11100101, 11101001, 11101111, 11110001, 11111011, 100000001, 100000111, 100001101, 100001111, 100010101, 100011001, 100011011, 100100101, 100110011, 100110111, 100111001, 100111101, 101001011, 101010001, 101011011, 101011101, 101100001, 101100111, 101101111, 101110101, 101111011, 101111111, 110000101, 110001101, 110010001, 110011001, 110100011, 110100101, 110101111, 110110001, 110110111, 110111011, 111000001, 111001001, 111001101, 111001111, 111010011, 111011111, 111100111, 111101011, 111110011, 111110111, 111111101, 1000001001, 1000001011, 1000011101, 1000100011, ...
3	12, 21, 102, 111, 122, 201, 212, 1002, 1011, 1101, 1112, 1121, 1202, 1222, 2012, 2021, 2111, 2122, 2201, 2221, 10002, 10022, 10121, 10202, 10211, 10222, 11001, 11012, 11201, 11212, 12002, 12011, 12112, 12121, 12211,

	20001, 20012, 20102, 20122, 20201, 21002, 21011, 21022, 21101, 21211, 22021, 22102, 22111, 22122, 22212, 22221, 100022, 100112, 100202, 100222, 101001, 101021, 101102, 101111, 101212, 102101, 102112, 102121, 102202, 110021, 110111, 110212, 110221, 111002, 111022, 111121, 111211, 112001, 112012, 112102, 112201, 112212, 120011, 120112, 120121, 120222, 121001, 121021, 121102, 121122, 121221, 122002, 122011, 122022, 122202, 200001, 200012, 200111, 200122, 200212, 201022, 201101, 202001, 202021, 202122, ...
4	11, 13, 23, 31, 101, 103, 113, 131, 133, 211, 221, 223, 233, 311, 323, 331, 1003, 1013, 1021, 1033, 1103, 1121, 1201, 1211, 1213, 1223, 1231, 1301, 1333, 2003, 2021, 2023, 2111, 2113, 2131, 2203, 2213, 2231, 2303, 2311, 2333, 3001, 3011, 3013, 3103, 3133, 3203, 3211, 3221, 3233, 3301, 3323, 10001, 10013, 10031, 10033, 10111, 10121, 10123, 10211, 10303, 10313, 10321, 10331, 11023, 11101, 11123, 11131, 11201, 11213, 11233, 11311, 11323, 11333, 12011, 12031, 12101, 12121, 12203, 12211, 12233, 12301, 12313, 12323, 13001, 13021, 13031, 13033, 13103, 13133, 13213, 13223, 13303, 13313, 13331, 20021, 20023, 20131, 20203, 20231, ...
5	12, 21, 23, 32, 34, 43, 104, 111, 122, 131, 133, 142, 203, 214, 221, 232, 241, 243, 304, 313, 324, 342, 401, 403, 412, 414, 423, 1002, 1011, 1022, 1024, 1044, 1101, 1112, 1123, 1132, 1143, 1204, 1211, 1231, 1233, 1242, 1244, 1321, 1343, 1402, 1404, 1413, 1424, 1431, 2001, 2012, 2023, 2034, 2041, 2102, 2111, 2113, 2133, 2212, 2221, 2223, 2232, 2311, 2322, 2342, 2344, 2403, 2414, 2432, 2443, 3004, 3013, 3024, 3042, 3101, 3114, 3134, 3141, 3211, 3213, 3224, 3233, 3244, 3312, 3321, 3323, 3332, 3404, 3422, 3431, 3444, 4003, 4014, 4041, 4043, 4131, 4142, 4212, 4223, ...
6	11, 15, 21, 25, 31, 35, 45, 51, 101, 105, 111, 115, 125, 135, 141, 151, 155, 201, 211, 215, 225, 241, 245, 251, 255, 301, 305, 331, 335, 345, 351, 405, 411, 421, 431, 435, 445, 455, 501, 515, 521, 525, 531, 551, 1011, 1015, 1021, 1025, 1035, 1041, 1055, 1105, 1115, 1125, 1131, 1141, 1145, 1151, 1205, 1231, 1235, 1241, 1245, 1311, 1321, 1335, 1341, 1345, 1355, 1411, 1421, 1431, 1435, 1445, 1501, 1505, 1521, 1535, 1541, 1555, 2001, 2011, 2015, 2025, 2041, 2045, 2051, 2055, 2115, 2131, 2135, 2151, 2155, 2205, 2225, 2231, 2301, 2311, 2325, 2335, ...
7	14, 16, 23, 25, 32, 41, 43, 52, 56, 61, 65, 104, 113, 115, 124, 131, 133, 142, 146, 155, 166, 203, 205, 212, 214, 221, 241, 245, 254, 256, 302, 304, 313, 322, 326, 335, 344, 346, 362, 364, 401, 403, 421, 436, 443, 445, 452, 461, 463, 506, 515, 524, 533, 535, 544, 551, 553, 566, 616, 623, 625, 632, 652, 661, 1004, 1006, 1013, 1022, 1033, 1042, 1051, 1055, 1064, 1105, 1112, 1123, 1136, 1141, 1154, 1156, 1165, 1202, 1211, 1222, 1226, 1231, 1235, 1253, 1264, 1301, 1312, 1316, 1325, 1343, 1345, 1402, 1411, 1424, 1433, 1442, ...
8	13, 15, 21, 23, 27, 35, 37, 45, 51, 53, 57, 65, 73, 75, 103, 107, 111, 117, 123, 131, 141, 145, 147, 153, 155, 161, 177, 203, 211, 213, 225, 227, 235, 243, 247, 255, 263, 265, 277, 301, 305, 307, 323, 337, 343, 345, 351, 357, 361, 373, 401, 407, 415, 417, 425, 431, 433, 445, 463, 467, 471, 475, 513, 521, 533, 535, 541, 547, 557, 565, 573, 577, 605, 615, 621, 631, 643, 645, 657, 661, 667, 673, 701, 711, 715, 717, 723, 737, 747, 753, 763, 767, 775, 1011, 1013, 1035, 1043, 1055, 1063, 1071, ...

9	12, 14, 18, 21, 25, 32, 34, 41, 45, 47, 52, 58, 65, 67, 74, 78, 81, 87, 102, 108, 117, 122, 124, 128, 131, 135, 151, 155, 162, 164, 175, 177, 184, 201, 205, 212, 218, 221, 232, 234, 238, 241, 254, 267, 272, 274, 278, 285, 287, 308, 315, 322, 328, 331, 337, 342, 344, 355, 371, 375, 377, 382, 407, 414, 425, 427, 432, 438, 447, 454, 461, 465, 472, 481, 485, 504, 515, 517, 528, 531, 537, 542, 548, 557, 562, 564, 568, 582, 601, 605, 614, 618, 625, 638, 641, 661, 667, 678, 685, 702, ...
10	11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107, 109, 113, 127, 131, 137, 139, 149, 151, 157, 163, 167, 173, 179, 181, 191, 193, 197, 199, 211, 223, 227, 229, 233, 239, 241, 251, 257, 263, 269, 271, 277, 281, 283, 293, 307, 311, 313, 317, 331, 337, 347, 349, 353, 359, 367, 373, 379, 383, 389, 397, 401, 409, 419, 421, 431, 433, 439, 443, 449, 457, 461, 463, 467, 479, 487, 491, 499, 503, 509, 521, 523, 541, 547, 557, 563, 569, ...
11	12, 16, 18, 21, 27, 29, 34, 38, 3A, 43, 49, 54, 56, 61, 65, 67, 72, 76, 81, 89, 92, 94, 98, 9A, A3, 106, 10A, 115, 117, 126, 128, 133, 139, 142, 148, 153, 155, 164, 166, 16A, 171, 182, 193, 197, 199, 1A2, 1A8, 1AA, 209, 214, 21A, 225, 227, 232, 236, 238, 247, 25A, 263, 265, 269, 281, 287, 296, 298, 2A1, 2A7, 304, 30A, 315, 319, 324, 331, 335, 342, 351, 353, 362, 364, 36A, 373, 379, 386, 38A, 391, 395, 3A6, 403, 407, 414, 418, 423, 434, 436, 452, 458, 467, 472, 478, 47A, ...
12	11, 15, 17, 1B, 25, 27, 31, 35, 37, 3B, 45, 4B, 51, 57, 5B, 61, 67, 6B, 75, 81, 85, 87, 8B, 91, 95, A7, AB, B5, B7, 105, 107, 111, 117, 11B, 125, 12B, 131, 13B, 141, 145, 147, 157, 167, 16B, 171, 175, 17B, 181, 18B, 195, 19B, 1A5, 1A7, 1B1, 1B5, 1B7, 205, 217, 21B, 221, 225, 237, 241, 24B, 251, 255, 25B, 267, 271, 277, 27B, 285, 291, 295, 2A1, 2AB, 2B1, 2BB, 301, 307, 30B, 315, 321, 325, 327, 32B, 33B, 347, 34B, 357, 35B, 365, 375, 377, 391, 397, 3A5, 3AB, 3B5, 3B7, ...
13	14, 16, 1A, 23, 25, 2B, 32, 34, 38, 41, 47, 49, 52, 56, 58, 61, 65, 6B, 76, 7A, 7C, 83, 85, 89, 9A, A1, A7, A9, B6, B8, C1, C7, CB, 104, 10A, 10C, 119, 11B, 122, 124, 133, 142, 146, 148, 14C, 155, 157, 164, 16A, 173, 179, 17B, 184, 188, 18A, 197, 1A8, 1AC, 1B1, 1B5, 1C6, 1CC, 209, 20B, 212, 218, 223, 229, 232, 236, 23C, 247, 24B, 256, 263, 265, 272, 274, 27A, 281, 287, 292, 296, 298, 29C, 2AB, 2B6, 2BA, 2C5, 2C9, 302, 311, 313, 328, 331, 33B, 344, 34A, 34C, 355, ...
14	13, 15, 19, 21, 23, 29, 2D, 31, 35, 3B, 43, 45, 4B, 51, 53, 59, 5D, 65, 6D, 73, 75, 79, 7B, 81, 91, 95, 9B, 9D, A9, AB, B3, B9, BD, C5, CB, CD, D9, DB, 101, 103, 111, 11D, 123, 125, 129, 131, 133, 13D, 145, 14B, 153, 155, 15B, 161, 163, 16D, 17D, 183, 185, 189, 199, 1A1, 1AB, 1AD, 1B3, 1B9, 1C3, 1C9, 1D1, 1D5, 1DB, 205, 209, 213, 21D, 221, 22B, 22D, 235, 239, 241, 249, 24D, 251, 255, 263, 26B, 271, 279, 27D, 285, 293, 295, 2A9, 2B1, 2BB, 2C3, 2C9, 2CB, 2D3, ...
15	12, 14, 18, 1E, 21, 27, 2B, 2D, 32, 38, 3E, 41, 47, 4B, 4D, 54, 58, 5E, 67, 6B, 6D, 72, 74, 78, 87, 8B, 92, 94, 9E, A1, A7, AD, B2, B8, BE, C1, CB, CD, D2, D4, E1, ED, 102, 104, 108, 10E, 111, 11B, 122, 128, 12E, 131, 137, 13B, 13D, 148, 157, 15B, 15D, 162, 171, 177, 182, 184, 188, 18E, 197, 19D, 1A4, 1A8, 1AE, 1B7, 1BB, 1C4, 1CE, 1D1, 1DB, 1DD, 1E4, 1E8, 1EE, 207, 20B, 20D, 212, 21E, 227, 22B, 234, 238, 23E, 24B, 24D, 261, 267, 272, 278, 27E, 281,

	287, ...
16	11, 13, 17, 1D, 1F, 25, 29, 2B, 2F, 35, 3B, 3D, 43, 47, 49, 4F, 53, 59, 61, 65, 67, 6B, 6D, 71, 7F, 83, 89, 8B, 95, 97, 9D, A3, A7, AD, B3, B5, BF, C1, C5, C7, D3, DF, E3, E5, E9, EF, F1, FB, 101, 107, 10D, 10F, 115, 119, 11B, 125, 133, 137, 139, 13D, 14B, 151, 15B, 15D, 161, 167, 16F, 175, 17B, 17F, 185, 18D, 191, 199, 1A3, 1A5, 1AF, 1B1, 1B7, 1BB, 1C1, 1C9, 1CD, 1CF, 1D3, 1DF, 1E7, 1EB, 1F3, 1F7, 1FD, 209, 20B, 21D, 223, 22D, 233, 239, 23B, 241, ...
17	12, 16, 1C, 1E, 23, 27, 29, 2D, 32, 38, 3A, 3G, 43, 45, 4B, 4F, 54, 5C, 5G, 61, 65, 67, 6B, 78, 7C, 81, 83, 8D, 8F, 94, 9A, 9E, A3, A9, AB, B4, B6, BA, BC, C7, D2, D6, D8, DC, E1, E3, ED, F2, F8, FE, FG, G5, G9, GB, 104, 111, 115, 117, 11B, 128, 12E, 137, 139, 13D, 142, 14A, 14G, 155, 159, 15F, 166, 16A, 171, 17B, 17D, 186, 188, 18E, 191, 197, 19F, 1A2, 1A4, 1A8, 1B3, 1BB, 1BF, 1C6, 1CA, 1CG, 1DB, 1DD, 1EE, 1F3, 1FD, 1G2, 1G8, 1GA, 1GG, 209, ...
18	11, 15, 1B, 1D, 21, 25, 27, 2B, 2H, 35, 37, 3D, 3H, 41, 47, 4B, 4H, 57, 5B, 5D, 5H, 61, 65, 71, 75, 7B, 7D, 85, 87, 8D, 91, 95, 9B, 9H, A1, AB, AD, AH, B1, BD, C7, CB, CD, CH, D5, D7, DH, E5, EB, EH, F1, F7, FB, FD, G5, H1, H5, H7, HB, 107, 10D, 115, 117, 11B, 11H, 127, 12D, 131, 135, 13B, 141, 145, 14D, 155, 157, 15H, 161, 167, 16B, 16H, 177, 17B, 17D, 17H, 18B, 191, 195, 19D, 19H, 1A5, 1AH, 1B1, 1C1, 1C7, 1CH, 1D5, 1DB, 1DD, 1E1, 1EB, ...
19	14, 1A, 1C, 1I, 23, 25, 29, 2F, 32, 34, 3A, 3E, 3G, 43, 47, 4D, 52, 56, 58, 5C, 5E, 5I, 6D, 6H, 74, 76, 7G, 7I, 85, 8B, 8F, 92, 98, 9A, A1, A3, A7, A9, B2, BE, BI, C1, C5, CB, CD, D4, DA, DG, E3, E5, EB, EF, EH, F8, G3, G7, G9, GD, H8, HE, I5, I7, IB, IH, 106, 10C, 10I, 113, 119, 11H, 122, 12A, 131, 133, 13D, 13F, 142, 146, 14C, 151, 155, 157, 15B, 164, 16C, 16G, 175, 179, 17F, 188, 18A, 199, 19F, 1A6, 1AC, 1AI, 1B1, 1B7, 1BH, 1C4, ...
20	13, 19, 1B, 1H, 21, 23, 27, 2D, 2J, 31, 37, 3B, 3D, 3J, 43, 49, 4H, 51, 53, 57, 59, 5D, 67, 6B, 6H, 6J, 79, 7B, 7H, 83, 87, 8D, 8J, 91, 9B, 9D, 9H, 9J, AB, B3, B7, B9, BD, BJ, C1, CB, CH, D3, D9, DB, DH, E1, E3, ED, F7, FB, FD, FH, GB, GH, H7, H9, HD, HJ, I7, ID, IJ, J3, J9, JH, 101, 109, 10J, 111, 11B, 11D, 11J, 123, 129, 12H, 131, 133, 137, 13J, 147, 14B, 14J, 153, 159, 161, 163, 171, 177, 17H, 183, 189, 18B, 18H, 197, 19D, ...
21	12, 18, 1A, 1G, 1K, 21, 25, 2B, 2H, 2J, 34, 38, 3A, 3G, 3K, 45, 4D, 4H, 4J, 52, 54, 58, 61, 65, 6B, 6D, 72, 74, 7A, 7G, 7K, 85, 8B, 8D, 92, 94, 98, 9A, A1, AD, AH, AJ, B2, B8, BA, BK, C5, CB, CH, CJ, D4, D8, DA, DK, ED, EH, EJ, F2, FG, G1, GB, GD, GH, H2, HA, HG, I1, I5, IB, IJ, J2, JA, JK, K1, KB, KD, KJ, 102, 108, 10G, 10K, 111, 115, 11H, 124, 128, 12G, 12K, 135, 13H, 13J, 14G, 151, 15B, 15H, 162, 164, 16A, 16K, 175, ...
22	11, 17, 19, 1F, 1J, 1L, 23, 29, 2F, 2H, 31, 35, 37, 3D, 3H, 41, 49, 4D, 4F, 4J, 4L, 53, 5H, 5L, 65, 67, 6H, 6J, 73, 79, 7D, 7J, 83, 85, 8F, 8H, 8L, 91, 9D, A3, A7, A9, AD, AJ, AL, B9, BF, BL, C5, C7, CD, CH, CJ, D7, DL, E3, E5, E9, F1, F7, FH, FJ, G1, G7, GF, GL, H5, H9, HF, I1, I5, ID, J1, J3, JD, JF, JL, K3, K9, KH, KL, L1, L5, LH, 103, 107, 10F, 10J, 113, 11F, 11H, 12D, 12J, 137, 13D, 13J, 13L, 145, 14F, 14L, ...
23	16, 18, 1E, 1I, 1K, 21, 27, 2D, 2F, 2L, 32, 34, 3A, 3E, 3K, 45, 49, 4B, 4F, 4H, 4L, 5C, 5G, 5M, 61, 6B, 6D, 6J, 72, 76, 7C, 7I, 7K, 87, 89, 8D, 8F, 94, 9G, 9K, 9M, A3, A9, AB, AL, B4, BA, BG, BI, C1, C5, C7, CH, D8, DC, DE, DI, E9, EF, F2, F4, F8, FE, FM, G5, GB, GF, GL, H6, HA, HI, I5, I7, IH, IJ, J2, J6, JC, JK,

	K1, K3, K7, KJ, L4, L8, LG, LK, M3, MF, MH, 10C, 10I, 115, 11B, 11H, 11J, 122, 12C, 12I, 131, ...
24	15, 17, 1D, 1H, 1J, 1N, 25, 2B, 2D, 2J, 2N, 31, 37, 3B, 3H, 41, 45, 47, 4B, 4D, 4H, 57, 5B, 5H, 5J, 65, 67, 6D, 6J, 6N, 75, 7B, 7D, 7N, 81, 85, 87, 8J, 97, 9B, 9D, 9H, 9N, A1, AB, AH, AN, B5, B7, BD, BH, BJ, C5, CJ, CN, D1, D5, DJ, E1, EB, ED, EH, EN, F7, FD, FJ, FN, G5, GD, GH, H1, HB, HD, HN, I1, I7, IB, IH, J1, J5, J7, JB, JN, K7, KB, KJ, KN, L5, LH, LJ, MD, MJ, N5, NB, NH, NJ, 101, 10B, 10H, 10N, ...
25	14, 16, 1C, 1G, 1I, 1M, 23, 29, 2B, 2H, 2L, 2N, 34, 38, 3E, 3M, 41, 43, 47, 49, 4D, 52, 56, 5C, 5E, 5O, 61, 67, 6D, 6H, 6N, 74, 76, 7G, 7I, 7M, 7O, 8B, 8N, 92, 94, 98, 9E, 9G, A1, A7, AD, AJ, AL, B2, B6, B8, BI, C7, CB, CD, CH, D6, DC, DM, DO, E3, E9, EH, EN, F4, F8, FE, FM, G1, G9, GJ, GL, H6, H8, HE, HI, HO, I7, IB, ID, IH, J4, JC, JG, JO, K3, K9, KL, KN, LG, LM, M7, MD, MJ, ML, N2, NC, NI, NO, ...
26	13, 15, 1B, 1F, 1H, 1L, 21, 27, 29, 2F, 2J, 2L, 31, 35, 3B, 3J, 3N, 3P, 43, 45, 49, 4N, 51, 57, 59, 5J, 5L, 61, 67, 6B, 6H, 6N, 6P, 79, 7B, 7F, 7H, 83, 8F, 8J, 8L, 8P, 95, 97, 9H, 9N, A3, A9, AB, AH, AL, AN, B7, BL, BP, C1, C5, CJ, CP, D9, DB, DF, DL, E3, E9, EF, EJ, EP, F7, FB, FJ, G3, G5, GF, GH, GN, H1, H7, HF, HJ, HL, HP, IB, IJ, IN, J5, J9, JF, K1, K3, KL, L1, LB, LH, LN, LP, M5, MF, ML, N1, ...
27	12, 14, 1A, 1E, 1G, 1K, 1Q, 25, 27, 2D, 2H, 2J, 2P, 32, 38, 3G, 3K, 3M, 3Q, 41, 45, 4J, 4N, 52, 54, 5E, 5G, 5M, 61, 65, 6B, 6H, 6J, 72, 74, 78, 7A, 7M, 87, 8B, 8D, 8H, 8N, 8P, 98, 9E, 9K, 9Q, A1, A7, AB, AD, AN, BA, BE, BG, BK, C7, CD, CN, CP, D2, D8, DG, DM, E1, E5, EB, EJ, EN, F4, FE, FG, FQ, G1, G7, GB, GH, GP, H2, H4, H8, HK, I1, I5, ID, IH, IN, J8, JA, K1, K7, KH, KN, L2, L4, LA, LK, LQ, M5, ...
28	11, 13, 19, 1D, 1F, 1J, 1P, 23, 25, 2B, 2F, 2H, 2N, 2R, 35, 3D, 3H, 3J, 3N, 3P, 41, 4F, 4J, 4P, 4R, 59, 5B, 5H, 5N, 5R, 65, 6B, 6D, 6N, 6P, 71, 73, 7F, 7R, 83, 85, 89, 8F, 8H, 8R, 95, 9B, 9H, 9J, 9P, A1, A3, AD, AR, B3, B5, B9, BN, C1, CB, CD, CH, CN, D3, D9, DF, DJ, DP, E5, E9, EH, ER, F1, FB, FD, FJ, FN, G1, G9, GD, GF, GJ, H3, HB, HF, HN, HR, I5, IH, IJ, J9, JF, JP, K3, K9, KB, KH, KR, L5, LB, ...
29	12, 18, 1C, 1E, 1I, 1O, 21, 23, 29, 2D, 2F, 2L, 2P, 32, 3A, 3E, 3G, 3K, 3M, 3Q, 4B, 4F, 4L, 4N, 54, 56, 5C, 5I, 5M, 5S, 65, 67, 6H, 6J, 6N, 6P, 78, 7K, 7O, 7Q, 81, 87, 89, 8J, 8P, 92, 98, 9A, 9G, 9K, 9M, A3, AH, AL, AN, AR, BC, BI, BS, C1, C5, CB, CJ, CP, D2, D6, DC, DK, DO, E3, ED, EF, EP, ER, F4, F8, FE, FM, FQ, FS, G3, GF, GN, GR, H6, HA, HG, HS, I1, IJ, IP, J6, JC, JI, JK, JQ, K7, KD, KJ, KL, ...
30	11, 17, 1B, 1D, 1H, 1N, 1T, 21, 27, 2B, 2D, 2J, 2N, 2T, 37, 3B, 3D, 3H, 3J, 3N, 47, 4B, 4H, 4J, 4T, 51, 57, 5D, 5H, 5N, 5T, 61, 6B, 6D, 6H, 6J, 71, 7D, 7H, 7J, 7N, 7T, 81, 8B, 8H, 8N, 8T, 91, 97, 9B, 9D, 9N, A7, AB, AD, AH, B1, B7, BH, BJ, BN, BT, C7, CD, CJ, CN, CT, D7, DB, DJ, DT, E1, EB, ED, EJ, EN, ET, F7, FB, FD, FH, FT, G7, GB, GJ, GN, GT, HB, HD, I1, I7, IH, IN, IT, J1, J7, JH, JN, JT, K1, ...
31	16, 1A, 1C, 1G, 1M, 1S, 1U, 25, 29, 2B, 2H, 2L, 2R, 34, 38, 3A, 3E, 3G, 3K, 43, 47, 4D, 4F, 4P, 4R, 52, 58, 5C, 5I, 5O, 5Q, 65, 67, 6B, 6D, 6P, 76, 7A, 7C, 7G, 7M, 7O, 83, 89, 8F, 8L, 8N, 8T, 92, 94, 9E, 9S, A1, A3, A7, AL, AR, B6, B8,

	BC, BI, BQ, C1, C7, CB, CH, CP, CT, D6, DG, DI, DS, DU, E5, E9, EF, EN, ER, ET, F2, FE, FM, FQ, G3, G7, GD, GP, GR, HE, HK, HU, I5, IB, ID, IJ, IT, J4, JA, JC, JI, ...
[32](#)	15, 19, 1B, 1F, 1L, 1R, 1T, 23, 27, 29, 2F, 2J, 2P, 31, 35, 37, 3B, 3D, 3H, 3V, 43, 49, 4B, 4L, 4N, 4T, 53, 57, 5D, 5J, 5L, 5V, 61, 65, 67, 6J, 6V, 73, 75, 79, 7F, 7H, 7R, 81, 87, 8D, 8F, 8L, 8P, 8R, 95, 9J, 9N, 9P, 9T, AB, AH, AR, AT, B1, B7, BF, BL, BR, BV, C5, CD, CH, CP, D3, D5, DF, DH, DN, DR, E1, E9, ED, EF, EJ, EV, F7, FB, FJ, FN, FT, G9, GB, GT, H3, HD, HJ, HP, HR, I1, IB, IH, IN, IP, IV, ...
33	14, 18, 1A, 1E, 1K, 1Q, 1S, 21, 25, 27, 2D, 2H, 2N, 2V, 32, 34, 38, 3A, 3E, 3S, 3W, 45, 47, 4H, 4J, 4P, 4V, 52, 58, 5E, 5G, 5Q, 5S, 5W, 61, 6D, 6P, 6T, 6V, 72, 78, 7A, 7K, 7Q, 7W, 85, 87, 8D, 8H, 8J, 8T, 9A, 9E, 9G, 9K, A1, A7, AH, AJ, AN, AT, B4, BA, BG, BK, BQ, C1, C5, CD, CN, CP, D2, D4, DA, DE, DK, DS, DW, E1, E5, EH, EP, ET, F4, F8, FE, FQ, FS, GD, GJ, GT, H2, H8, HA, HG, HQ, HW, I5, I7, ID, ...
34	13, 17, 19, 1D, 1J, 1P, 1R, 1X, 23, 25, 2B, 2F, 2L, 2T, 2X, 31, 35, 37, 3B, 3P, 3T, 41, 43, 4D, 4F, 4L, 4R, 4V, 53, 59, 5B, 5L, 5N, 5R, 5T, 67, 6J, 6N, 6P, 6T, 71, 73, 7D, 7J, 7P, 7V, 7X, 85, 89, 8B, 8L, 91, 95, 97, 9B, 9P, 9V, A7, A9, AD, AJ, AR, AX, B5, B9, BF, BN, BR, C1, CB, CD, CN, CP, CV, D1, D7, DF, DJ, DL, DP, E3, EB, EF, EN, ER, EX, FB, FD, FV, G3, GD, GJ, GP, GR, GX, H9, HF, HL, HN, HT, ...
35	12, 16, 18, 1C, 1I, 1O, 1Q, 1W, 21, 23, 29, 2D, 2J, 2R, 2V, 2X, 32, 34, 38, 3M, 3Q, 3W, 3Y, 49, 4B, 4H, 4N, 4R, 4X, 54, 56, 5G, 5I, 5M, 5O, 61, 6D, 6H, 6J, 6N, 6T, 6V, 76, 7C, 7I, 7O, 7Q, 7W, 81, 83, 8D, 8R, 8V, 8X, 92, 9G, 9M, 9W, 9Y, A3, A9, AH, AN, AT, AX, B4, BC, BG, BO, BY, C1, CB, CD, CJ, CN, CT, D2, D6, D8, DC, DO, DW, E1, E9, ED, EJ, EV, EX, FG, FM, FW, G3, G9, GB, GH, GR, GX, H4, H6, HC, ...
[36](#)	11, 15, 17, 1B, 1H, 1N, 1P, 1V, 1Z, 21, 27, 2B, 2H, 2P, 2T, 2V, 2Z, 31, 35, 3J, 3N, 3T, 3V, 45, 47, 4D, 4J, 4N, 4T, 4Z, 51, 5B, 5D, 5H, 5J, 5V, 67, 6B, 6D, 6H, 6N, 6P, 6Z, 75, 7B, 7H, 7J, 7P, 7T, 7V, 85, 8J, 8N, 8P, 8T, 97, 9D, 9N, 9P, 9T, 9Z, A7, AD, AJ, AN, AT, B1, B5, BD, BN, BP, BZ, C1, C7, CB, CH, CP, CT, CV, CZ, DB, DJ, DN, DV, DZ, E5, EH, EJ, F1, F7, FH, FN, FT, FV, G1, GB, GH, GN, GP, GV, ...

The primes in $M(L_b)$ are called **minimal prime base *b*** in this article, although in fact this name should be used for L_b is the language of base-*b* representations of the prime numbers, where primes > *b* is not required (reference), this problem is an extension of the original minimal prime problem to include *Conjectures 'R Us* Sierpinski/Riesel conjectures base *b* with *k*-values < *b*, i.e. the smallest prime of the form $k*b^n+1$ and $k*b^n-1$ for all $k < b$. The original minimal prime base *b* puzzle does not cover CRUS Sierpinski/Riesel conjectures base *b* with CK < *b*, since in Riesel side, the prime is not minimal prime in original definition if either *k*−1 or *b*−1 (or both) is prime, and in Sierpinski side, the prime is not minimal prime in original definition if *k* is prime (e.g. $25*30^{34205}-1$ is not minimal prime in base 30 in original definition, since it is OT^{34205} in base 30, and T (= 29 in decimal) is prime, but it is minimal prime in base 30 if only primes > base are counted), but this extended version of minimal prime base *b* problem does, this requires a restriction of prime > *b*, and the primes ≤ *b* (including the *k*−1, *b*−1, *k*) are not allowed (i.e. only counting the primes > *b*, and we want to find the minimal set of “the primes > *b*” in base *b*), in fact, to include these conjectures, we only need to exclude the single-digit primes (i.e. the primes < *b*), also, in fact, I create this problem because I think that the single-digit primes are trivial (like strictly non-palindromic number, single-digit numbers are trivially palindromic, thus to test whether a number *n* is strictly non-palindromic, we do not consider the bases *b* > *n*, since in these bases, *n* is a single-digit number, thus trivially palindromic, note that all strictly non-palindromic numbers > 6 are primes), thus I do not count them (also see this forum post, there is someone else who also exclude the single-digit primes, but his research is about substring instead of subsequence), however, including the base (*b*) itself results in automatic elimination of all possible extension numbers with “0 after 1” from the set (when the base is prime, if the base is composite, then there is no difference to include the base (*b*) itself or not), which is quite restrictive (since when the base is prime, then the base (*b*) itself is the only prime ending with 0, i.e. having trailing zero, since in any base, all numbers ending with 0 (i.e. having trailing zero) are divisible by the base (*b*), thus cannot be prime unless it is equal the base (*b*), i.e. “10” in base *b*, note that the numbers cannot have leading zero, since typically this is not the way we write numbers (in any base), thus for all primes in our sets (i.e. all primes > base (*b*)), all zero digits must be “between” other digits) (see this forum post, there is someone else who also excludes the prime = base), thus, we also exclude the prime = *b* (i.e. the prime “10”) (you may ask me why we do not exclude the prime = *b*+1? Since *b*+1 is “11” in base *b*, this is a generalized repunit number base *b*, if we exclude it (“11” in base b), then we have the next question: should we exclude “111”, “1111”, “11111”, etc. in base *b*? This is hard to answer, and if we exclude them all, the result will not be “primes > *m*” for some integer *m*, thus we do not exclude “11” in base *b* but exclude “10” in base *b*, we also exclude the single-digit primes (i.e. the primes < *b*) in base *b*), besides, this problem is better than the original minimal prime problem since this problem is regardless whether 1 is considered as prime or not, i.e. no matter 1 is considered as prime or not prime (in the beginning of the 20th century, 1 is regarded as prime) (reference of why 1 is not prime), the sets $M(L_b)$ in this problem are the same, while the sets $M(L_b)$ in the original minimal prime problem are different, e.g. in base 10, if 1 is considered as prime, then the set $M(L_b)$ in the original minimal prime problem is {1, 2, 3, 5, 7, 89, 409, 449, 499, 6469, 6949, 9049, 9649, 9949, 60649, 666649, 946669, 60000049, 66000049, 66600049}, while if 1 is not considered as prime, then the set $M(L_b)$ in the original minimal prime problem is {2, 3, 5, 7, 11, 19, 41, 61, 89, 409, 449, 499, 881, 991, 6469, 6949, 9001, 9049, 9649, 9949, 60649, 666649, 946669, 60000049, 66000049, 66600049}, however, in base 10, the set $M(L_b)$ in this problem is always {11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 227, 251, 257, 277, 281, 349, 409, 449, 499, 521, 557, 577, 587, 727, 757, 787, 821, 827, 857, 877, 881, 887, 991, 2087, 2221, 5051, 5081, 5501, 5581, 5801, 5851, 6469, 6949, 8501, 9001, 9049, 9221, 9551, 9649, 9851, 9949, 20021, 20201, 50207, 60649, 80051, 666649, 946669, 5200007, 22000001, 60000049, 66000049, 66600049, 80555551, 555555555551, 5000000000000000000000000000027}, no matter 1 is considered as prime or not prime. The third reason for excluding the primes ≤ *b* is that starting with *b*+1 makes the formula of the number of possible (first digit,last digit) combo of a minimal prime in base *b* more simple and smooth number, since if start with b, then when *b* is prime, there is an additional possible (first digit,last digit) combo: (1,0), and hence the formula will be (*b*−1)**eulerphi*(*b*)+1 if *b* is prime, or (*b*−1)**eulerphi*(*b*) if *b* is composite (the fully formula will be (*b*−1)**eulerphi*(*b*)+*isprime*(*b*) or (*b*−1)**eulerphi*(*b*)+*floor*((*b*−*eulerphi*(*b*)) / (*b*−1))), which is more complex, and if start with 1 (i.e. the original minimal prime problem), the formula is much more complex, since the prime digits (i.e. the single-digit primes) should be excluded, and (for such prime >b) the first digit has b−1−*primepi*(b) choices, and the last digit has A048864(b) choices, by the rule of product, there are (b−1−*primepi*(b))*(A048864(b)) choices of the (first digit,last digit) combo (if for such prime ≥b instead of >b, then the formula will be (b−1−*primepi*(b))*(A048864(b))+1 if b is prime, or (b−1−*primepi*(b))*(A048864(b)) if b is composite), which is much more complex, (also, the possible (first digit,last digit) combo for a prime >b in base b are exactly the (first digit,last digit) combos which there are infinitely many primes have, while this is not true when the requiring is prime ≥ b or prime ≥ 2 instead of prime > b, since this will contain the prime factors of b, which are not coprime to b and hence there is only this prime (and not infinitely many primes) have this (first digit,last digit) combo) thus the problem in this forum (i.e. the minimal prime (start with b+1) problem) is much better than the original minimal prime problem.

For example, 857 is a minimal prime in decimal because there is no prime > 10 among the shorter subsequences of the digits: 8, 5, 7, 85, 87, 57. The subsequence does not have to consist of consecutive digits, so 149 is not a minimal prime in decimal (because 19 is prime and 19 > 10). But it does have to be in the same order; so, for example, 991 is still a minimal prime in decimal even though a subset of the digits can form the shorter prime 19 > 10 by changing the order.

A summary of the results of our algorithm is presented in the table in the next section, I completely solved all bases up to 16 except for bases 14, 16, and the odd bases >6 (the proofs are at the end of this article), for bases 14, 16, and the odd bases >6, I only found all minimal primes up to certain limit (about 2^{32}) and some larger minimal primes (such as $3^{16}1$ in base 7 and 54^{11} in base 9). I left as a challenge to readers the task of solving (finding all minimal primes and proving that these are all such primes) bases 7, 9, 11, 13, 14, 15, 16, and bases 17 through 36 (this will be a hard problem, e.g. base 23 has a minimal prime $9E^{800873}$, and base 30 has a minimal prime OT^{34205}).

Prime numbers are central in number theory because of the fundamental theorem of arithmetic: every natural number greater than 1 is either a prime itself or can be factorized as a product of primes that is unique up to their order (sociology is applied psychology, psychology is applied biology, biology is applied chemistry, chemistry is applied physics, physics is applied mathematics, the basics of mathematics is the numbers, the basics of the numbers is the natural numbers, the researching of the natural numbers is number theory). Also, for a completely multiplicative function $f(x)$ (i.e. an arithmetic function (i.e. a function whose domain is the natural numbers), such that $f(1) = 1$ and $f(x*y) = f(x)*f(y)$ holds for all positive integers x and y), all $f(n)$ are completely determined by $f(p)$ with prime p (i.e. a completely multiplicative function is completely determined by its values at the prime numbers). Also many functions in number theory are highly related to prime numbers, such as Liouville function, Möbius function, Euler's totient function, Carmichael function, Dedekind psi function, and divisor function (all of them are multiplicative functions, although only Liouville function is a completely multiplicative function). Also see The Prime Pages (a website about the prime numbers). Also see Primegrid. Also see the set of the primes (**warning: the related link "The n-1 and n+1 primality tests by Curtis Bright, INTP (2013-10-09)" in this article is wrong, the correct link is this**) and factoring into primes.

addition	multiplication

subtraction	division
0	1
negation	reciprocal
the set {1}	the set of the prime numbers
less than	divides
1 + 1 + 1 + … + 1 with n 1's	the prime factorization of n (e.g. $360 = 2^3 * 3^2 * 5$)

Addition and multiplication are the basic operations of arithmetic (which is also the basics of mathematics). In the addition operation, the identity element is 0, and all natural numbers > 0 can be written as the sum of many 1's, and the number 1 cannot be broken up; in the multiplication operation, the identity element is 1, and all natural numbers > 1 can be written as the product of many prime numbers, and the prime numbers cannot be broken up. Also, primes are the natural numbers $n \geq 1$ such that if n divides $x*y$ (x and y are natural numbers), then n divides either x or y (or both). Also, prime numbers are the numbers n such that the ring of integers modulo n (Z_n) is a field (also is an integral domain, also is a division ring, also has no zero divisors other than 0 (for the special case that n = 1, see zero ring)). Also, many famous problems in number theory are related to the prime numbers, such as the Goldbach's conjecture, the twin prime conjecture, the , etc. and also many famous problems in number theory, although they do not have "prime number" in their , but solving them needs to using the prime numbers, such as the Fermat's Last Theorem, the Riemann hypothesis, the *abc* conjecture, etc. Besides, "the set $M(L_b)$" to "the set of the prime numbers (except b itself) digit strings with length > 1 in base b" to "the partially ordered binary relation by subsequence" is "the set of the prime numbers" to "the set of the integers > 1" to "the partially ordered binary relation by divisibility" (and indeed, the "> 1" in "the prime numbers (except b itself) digit strings with length > 1 in base b" can be corresponded to the "> 1" in "the integers > 1") (for the reason why b itself is excluded, see the sections above and this forum post), thus the problem in this article is very important and beautiful.

subsequence ordering	divisibility ordering
the prime numbers > b digit strings in base b	the integers > 1
the set $M(L_b)$	the set of the prime numbers

Recreations involving the decimal digits of primes have a long history. To give just a few examples, without trying to be exhaustive, Yates studied the "repunits", which are primes of the form 111...111. Caldwell and Dubner studied the "near-repdigits", which are primes with all like or repeated digits but one (e.g. 7877 and 333337). Card introduced prime numbers such as 37337999, in which every nonempty prefix is also a prime; he called them "snowball" primes. These were later studied by Angell & Godwin and Caldwell, who called them "right-truncatable" primes. They also studied the "left-truncatable" primes, such as 4632647, in which every nonempty suffix is prime (the left-truncatable primes are called "Russian doll primes" like that the right-truncatable primes are called "snowball primes", see this page). Kahan and Weintraub gave a list of all the left-truncatable primes (The list of all left-truncatable primes and right-truncatable primes are in http://primerecords.dk/left-truncatable.txt and http://primerecords.dk/right-truncatable.txt, respectively, also see OEIS sequences A024785 and A024770). Huestis introduced the "recursively laminar primes". In this note, I discuss an apparently new problem on the decimal digits of primes, but one inspired from a classical theorem in formal language theory, i.e. there are only finitely many minimal elements for the subsequence ordering of any given set of strings (in fact, every set of pairwise incomparable strings (for the subsequence ordering) is finite).

(Important note: suffix, prefix ⊂ substring ⊂ subsequence, but subsequence ⊄ substring ⊄ suffix, prefix)

However, there is no reason to only study these classes of primes in decimal, since the number 10 is not special in mathematics, decimal (base 10) is not special in mathematics, we use decimal (base 10) only because humans have 10 fingers, this fact does not have any mathematical significance, and if humans have 12 fingers instead of 10 fingers, we will use duodecimal (base 12) instead of decimal (base 10), e.g. in base 10 there are "full reptend primes", the primes *p* which the repeating decimal of *k*/*p* for integers 1 ≤ *k* ≤ *p*−1 are the cyclic permutation of a (*p*−1)-digit number (e.g. *p* = 7, the repeating decimal of *k*/7 for integers 1 ≤ *k* ≤ 6 are the cyclic permutation of the 6-digit number 142857: 142857, 285714, 428571, 571428, 714285, 857142), such number is called cyclic number, a prime *p* is a full reptend prime if and only if the period length of 1/*p* in decimal is *p*−1 (by Fermat's little theorem, for all primes *p* not dividing 10, the period length of 1/*p* in decimal always divides *p*−1, and if *p* divides 10, then 1/*p* terminates in decimal and does not give a repeating decimal), i.e. 10 is a primitive root mod *p*, and this can be generalized to other bases *b*, full reptend primes in base *b* are the primes *p* which the "repeating base *b*" of *k*/*p* for integers 1 ≤ *k* ≤ *p*−1 are the cyclic permutation of a (*p*−1)-digit number in base *b*, a prime *p* is a full reptend prime in base *b* if and only if the period length of 1/*p* in base *b* is *p*−1 (by Fermat's little theorem, for all primes *p* not dividing *b*, the period length of 1/*p* in base *b* always divides *p*−1, and if *p* divides *b*, then 1/*p* terminates in base *b* and does not give a "repeating base *b*"), i.e. *b* is a primitive root mod *p*, if *b* is an even square, then such prime *p* does not exist, and if *b* is an odd square, then the only such prime *p* is 2, and the natural density of the primes *p* (over the set of the primes) such that *b* is a primitive root mod *p* for given base *b* is conjectured to be 0.373955813619..., if *b* is neither perfect power and *sf*(*b*) is not == 1 mod 4 (i.e. *b* is in A085397), this is Artin's conjecture on primitive roots, if *b* is a perfect *r*-th power with *r* prime (i.e. *r* divides A052409(*b*)), then the natural density should be multiplied by $\frac{r(r-2)}{r^2-r-1}$, and if *sf*(*b*) is == 1 mod 4, then the natural density should be multiplied by $1 - \prod_{p\ prime, p\ |\ sf(b)} \frac{1}{1+p-p^2}$, see this reference.

The smallest full reptend primes in base *b* for *b* = 2, 3, 4, … 36 are (0 if no full reptend primes exist for this base *b*) 3, 2, 0, 2, 11, 2, 3, 2, 7, 2, 5, 2, 3, 2, 0, 2, 5, 2, 3, 2, 5, 2, 7, 2, 3, 2, 5, 2, 11, 2, 3, 2, 19, 2, 0 (*OEIS* sequence A056619)

The smallest base such that *p* is a full reptend prime for the first 100 primes *p* (i.e. *p* = 2, 3, 5, 7, …, 541) are 3, 2, 2, 3, 2, 2, 3, 2, 5, 2, 3, 2, 6, 3, 5, 2, 2, 2, 2, 7, 5, 3, 2, 3, 5, 2, 5, 2, 6, 3, 3, 2, 3, 2, 2, 6, 5, 2, 5, 2, 2, 2, 19, 5, 2, 3, 2, 3, 2, 6, 3, 7, 7, 6, 3, 5, 2, 6, 5, 3, 3, 2, 5, 17, 10, 2, 3, 10, 2, 2, 3, 7, 6, 2, 2, 5, 2, 5, 3, 21, 2, 2, 7, 5, 15, 2, 3, 13, 2, 3, 2, 13, 3, 2, 7, 5, 2, 3, 2, 2 (*OEIS* sequence A001918)

The smallest prime *p* such that *b* is the smallest base such that *p* is a full reptend prime for *b* = 2, 3, 4, … 36 are (0 if no such primes exist for this base *b*) 3, 7, 0, 23, 41, 71, 0, 0, 313, 643, 4111, 457, 1031, 439, 0, 311, 53173, 191, 107227, 409, 3361, 2161, 533821, 0, 12391, 0, 133321, 15791, 124153, 5881, 0, 268969, 48889, 64609, 0 (*OEIS* sequence A023048)

b	full reptend primes in base *b* (written in base 10)	*OEIS* sequence
2	3, 5, 11, 13, 19, 29, 37, 53, 59, 61, 67, 83, 101, 107, 131, 139, 149, 163, 173, 179, 181, 197, 211, 227, 269, 293, 317, 347, 349, 373, 379, 389, 419, 421, 443, 461, 467, 491, 509, 523, 541, ...	A001122
3	2, 5, 7, 17, 19, 29, 31, 43, 53, 79, 89, 101, 113, 127, 137, 139, 149, 163, 173, 197, 199, 211, 223, 233, 257, 269, 281, 283, 293, 317, 331, 353, 379, 389, 401, 449, 461, 463, 487, 509, 521, ...	A019334
4	(not exist, since 4 is square number, thus 4 is quadratic residue mod all primes and cannot be primitive root mod any odd primes, this only remains to check the prime 2, but 2 divides 4)	A000000 (the empty sequence)
5	2, 3, 7, 17, 23, 37, 43, 47, 53, 73, 83, 97, 103, 107, 113, 137, 157, 167, 173, 193, 197, 223, 227, 233, 257, 263, 277, 283, 293, 307, 317, 347, 353, 373, 383, 397, 433, 443, 463, 467, 503, 523, ...	A019335
6	11, 13, 17, 41, 59, 61, 79, 83, 89, 103, 107, 109, 113, 127, 131, 137, 151, 157, 179, 199, 223, 227, 229, 233, 251, 257, 271, 277, 347, 367, 373, 397, 401, 419, 443, 449, 467, 487, 491, 521, ...	A019336
7	2, 5, 11, 13, 17, 23, 41, 61, 67, 71, 79, 89, 97, 101, 107, 127, 151, 163, 173, 179, 211, 229, 239, 241, 257, 263, 269, 293, 347, 349, 359, 379, 397, 431, 433, 443, 461, 491, 499, 509, 521, ...	A019337
8	3, 5, 11, 29, 53, 59, 83, 101, 107, 131, 149, 173, 179, 197, 227, 269, 293, 317, 347, 389, 419, 443, 461, 467, 491, 509, ...	A019338
9	2 (this is all, since 9 is square number, thus 9 is quadratic residue mod all primes and cannot be primitive root mod any odd primes)	
10	7, 17, 19, 23, 29, 47, 59, 61, 97, 109, 113, 131, 149, 167, 179, 181, 193, 223, 229, 233, 257, 263, 269, 313, 337, 367, 379, 383, 389, 419, 433, 461, 487, 491, 499, 503, 509, 541, ...	A001913
11	2, 3, 13, 17, 23, 29, 31, 41, 47, 59, 67, 71, 73, 101, 103, 109, 149, 163, 173, 179, 197, 223, 233, 251, 277, 281, 293, 331, 367, 373, 383, 419, 443, 461, 463, 467, 487, 499, ...	A019339
12	5, 7, 17, 31, 41, 43, 53, 67, 101, 103, 113, 127, 137, 139, 149, 151, 163, 173, 197, 223, 257, 269, 281, 283, 293, 317, 353, 367, 379, 389, 401, 449, 461, 509, 523, ...	A019340

13	2, 5, 11, 19, 31, 37, 41, 47, 59, 67, 71, 73, 83, 89, 97, 109, 137, 149, 151, 167, 197, 227, 239, 241, 281, 293, 307, 317, 349, 353, 359, 379, 383, 397, 401, 431, 449, 457, 479, 487, 509, 541, ...	A019341
14	3, 17, 19, 23, 29, 53, 59, 73, 83, 89, 97, 109, 127, 131, 149, 151, 227, 239, 241, 251, 257, 263, 277, 283, 307, 313, 317, 353, 359, 373, 389, 419, 421, 431, 433, 467, 487, 521, 541, ...	A019342
15	2, 13, 19, 23, 29, 37, 41, 47, 73, 83, 89, 97, 101, 107, 139, 149, 151, 157, 167, 193, 199, 227, 263, 269, 271, 281, 313, 337, 347, 373, 379, 383, 389, 401, 433, 439, 449, 457, 461, 467, 499, 503, 509, 521, ...	A019343
16	(not exist, since 16 is square number, thus 16 is quadratic residue mod all primes and cannot be primitive root mod any odd primes, this only remains to check the prime 2, but 2 divides 16)	A000000 (the empty sequence)
17	2, 3, 5, 7, 11, 23, 31, 37, 41, 61, 97, 107, 113, 131, 139, 167, 173, 193, 197, 211, 227, 233, 269, 277, 283, 311, 313, 317, 347, 367, 379, 401, 419, 431, 439, 449, 479, 487, 499, 503, 521, …	A019344
18	5, 11, 29, 37, 43, 53, 59, 61, 67, 83, 101, 107, 109, 139, 149, 157, 163, 173, 179, 181, 197, 227, 251, 269, 277, 283, 293, 317, 347, 349, 379, 389, 397, 419, 421, 461, 467, 491, 509, 523, 541, …	A019345
19	2, 7, 11, 13, 23, 29, 37, 41, 43, 47, 53, 83, 89, 113, 139, 163, 173, 191, 193, 239, 251, 257, 263, 269, 281, 293, 311, 317, 337, 347, 359, 367, 401, 419, 433, 443, 449, 463, 467, 479, 491, 499, 503, 509, 521, …	A019346
20	3, 13, 17, 23, 37, 43, 47, 53, 67, 73, 83, 103, 107, 113, 137, 157, 163, 167, 173, 223, 227, 233, 257, 263, 277, 283, 293, 313, 317, 337, 347, 353, 367, 383, 397, 433, 443, 463, 467, 487, 503, …	A019347
21	2, 19, 23, 29, 31, 53, 71, 97, 103, 107, 113, 137, 139, 149, 157, 179, 181, 191, 197, 223, 233, 239, 263, 271, 281, 307, 313, 317, 347, 359, 389, 397, 401, 409, 431, 443, 449, 491, 523, …	A019348
22	5, 17, 19, 31, 37, 41, 47, 53, 71, 83, 107, 131, 139, 191, 193, 199, 211, 223, 227, 233, 269, 281, 283, 307, 311, 317, 337, 347, 367, 383, 389, 397, 409, 421, 487, 491, 509, 523, …	A019349
23	2, 3, 5, 17, 47, 59, 89, 97, 113, 127, 131, 137, 149, 167, 179, 181, 223, 229, 281, 293, 307, 311, 337, 347, 389, 401, 421, 433, 439, 443, 457, 487, 491, 499, 521, …	A019350
24	7, 11, 13, 17, 31, 37, 41, 59, 83, 89, 107, 109, 113, 137, 157, 179, 181, 223, 227, 229, 233, 251, 257, 277, 281, 347, 353, 373, 397, 401, 419, 421, 443, 463, 467, 487, 491, 541, …	A019351
25	2 (this is all, since 25 is square number, thus 25 is quadratic residue mod all primes and cannot be primitive root mod any odd primes)	

26	3, 7, 29, 41, 43, 47, 53, 61, 73, 89, 97, 101, 107, 131, 137, 139, 157, 167, 173, 179, 193, 239, 251, 269, 271, 281, 283, 347, 353, 359, 373, 383, 389, 409, 419, 443, 449, 457, 463, 467, 479, 491, …	A019352
27	2, 5, 17, 29, 53, 89, 101, 113, 137, 149, 173, 197, 233, 257, 269, 281, 293, 317, 353, 389, 401, 449, 461, 509, 521, …	A019353
28	5, 11, 13, 17, 23, 41, 43, 67, 71, 73, 79, 89, 101, 107, 173, 179, 181, 191, 229, 257, 263, 269, 293, 313, 331, 347, 353, 359, 379, 397, 409, 431, 433, 443, 461, 463, 487, 491, 499, 509, 521, …	A019354
29	2, 3, 11, 17, 19, 41, 43, 47, 73, 79, 89, 97, 101, 113, 127, 131, 137, 163, 191, 211, 229, 251, 263, 269, 293, 307, 311, 317, 331, 337, 359, 389, 409, 433, 443, 449, 461, 467, 479, 491, 503, …	A019355
30	11, 23, 41, 43, 47, 59, 61, 79, 89, 109, 131, 151, 167, 173, 179, 193, 197, 199, 251, 263, 281, 293, 307, 317, 349, 383, 419, 421, 433, 439, 449, 457, 491, 503, 521, 523, 541, …	A019356
31	2, 7, 17, 29, 47, 53, 59, 61, 67, 71, 73, 89, 107, 131, 137, 197, 227, 229, 241, 269, 277, 283, 307, 311, 313, 337, 353, 359, 379, 389, 401, 419, 431, 433, 439, 443, 449, 457, 461, 467, 479, 503, 509, …	A019357
32	3, 5, 13, 19, 29, 37, 53, 59, 67, 83, 107, 139, 149, 163, 173, 179, 197, 227, 269, 293, 317, 347, 349, 373, 379, 389, 419, 443, 467, 509, 523, …	A019358
33	2, 5, 7, 13, 19, 23, 43, 47, 53, 59, 71, 73, 89, 113, 137, 179, 191, 251, 257, 269, 311, 317, 337, 349, 353, 383, 389, 409, 419, 439, 443, 449, 457, 467, 509, …	A019359
34	19, 23, 31, 41, 43, 53, 59, 67, 73, 79, 83, 101, 113, 149, 157, 167, 179, 193, 199, 233, 241, 251, 293, 311, 313, 337, 349, 367, 373, 389, 401, 431, 439, 449, 461, 467, 479, 491, 503, 509, 523, …	A019360
35	2, 3, 11, 37, 41, 47, 53, 61, 71, 79, 83, 89, 101, 103, 137, 151, 167, 179, 191, 197, 211, 223, 227, 229, 233, 239, 241, 269, 283, 317, 331, 359, 373, 379, 383, 409, 431, 457, 461, 467, 499, 503, 509, 521, …	A019361
36	(not exist, since 36 is square number, thus 36 is quadratic residue mod all primes and cannot be primitive root mod any odd primes, this only remains to check the prime 2, but 2 divides 36)	A000000 (the empty sequence)

Another example is in base 10 there are unique primes, the primes *p* such that there is no other prime *q* such that the period length of the decimal expansion of its reciprocal, 1/*p*, is equal to the period length of the reciprocal of *q*, 1/*q*, a number *n* is a unique period (i.e. there is only one prime *p* such that the decimal expansion of 1/*p* has period length *n*) if and only if the Zsigmondy number *Zs*(*n*,10,1) (see Zsigmondy's theorem) is a prime power p^r, and hence *p* is the unique prime with period length *n*, and this can be generalized to other bases *b*, a number *n* is a unique period (i.e. there is only one prime *p* such that the decimal expansion of 1/p has period length n) if and only if the Zsigmondy number $Zs(n,b,1)$ (see Zsigmondy's theorem) is a prime power p^r, and hence p is the unique prime with period length n in base b, if $Zs(n,b,1)$ is a true power of a prime (i.e. p^r with $r > 1$), then the prime p is a generalized Wieferich prime base b (reference: list of generalized Wieferich primes ≤ $1.202*10^{12}$ base b for 2 ≤ b ≤ 10125, b is not a perfect power). All generalized repunit primes base b (list for bases b ≤ 1000) and all generalized Fermat primes (list for bases b ≤ 1000) are generalized unique primes base b, and there is a list of top 20 known generalized unique primes (with period length ≥ 3, since period lengths 1 and 2 are trivial).

b	unique periods in base *b* (≤ 4096) (written in base 10)
2	2, 3, 4, 5, 7, 8, 9, 10, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 24, 26, 27, 30, 31, 32, 33, 34, 38, 40, 42, 46, 49, 54, 56, 61, 62, 65, 69, 77, 78, 80, 85, 86, 89, 90, 93, 98, 107, 120, 122, 126, 127, 129, 133, 145, 147, 150, 158, 165, 170, 174, 184, 192, 195, 202, 208, 234, 254, 261, 280, 296, 312, 322, 334, 342, 345, 366, 374, 382, 398, 410, 414, 425, 447, 471, 507, 521, 550, 567, 579, 590, 600, 602, 607, 626, 690, 694, 712, 745, 795, 816, 889, 897, 909, 954, 990, 1106, 1192, 1224, 1230, 1279, 1384, 1386, 1402, 1464, 1512, 1554, 1562, 1600, 1670, 1683, 1727, 1781, 1834, 1904, 1990, 1992, 2008, 2037, 2203, 2281, 2298, 2353, 2406, 2456, 2499, 2536, 2838, 3006, 3074, 3217, 3415, 3418, 3481, 3766, 3817, 3927, ...
3	1, 3, 4, 5, 6, 7, 8, 9, 10, 12, 13, 14, 15, 20, 21, 24, 26, 32, 33, 36, 40, 46, 60, 63, 64, 70, 71, 72, 86, 103, 108, 128, 130, 132, 143, 145, 154, 161, 236, 255, 261, 276, 279, 287, 304, 364, 430, 464, 513, 528, 541, 562, 665, 672, 680, 707, 718, 747, 760, 782, 828, 875, 892, 974, 984, 987, 1037, 1058, 1070, 1073, 1080, 1091, 1154, 1248, 1367, 1426, 1440, 1462, 1524, 1598, 1623, 1627, 1863, 1985, 2132, 2188, 2196, 2340, 2460, 2508, 2626, 2640, 2739, 2856, 3092, 3158, 3262, 3315, 3326, 3482, 3638, 3982, 4018, 4036, ...
4	1, 2, 3, 4, 6, 8, 10, 12, 16, 20, 28, 40, 60, 92, 96, 104, 140, 148, 156, 300, 356, 408, 596, 612, 692, 732, 756, 800, 952, 996, 1004, 1228, 1268, 2240, 2532, 3060, 3796, 3824, 3944, ...
5	1, 2, 3, 4, 6, 7, 8, 10, 11, 12, 13, 18, 24, 28, 47, 48, 49, 56, 57, 88, 90, 92, 108, 110, 116, 120, 127, 134, 141, 149, 161, 171, 181, 198, 202, 206, 236, 248, 288, 357, 384, 420, 458, 500, 530, 536, 619, 620, 694, 798, 897, 929, 981, 992, 1064, 1134, 1230, 1670, 1807, 2094, 2162, 2369, 2516, 2649, 2988, 3407, 3888, ...
6	1, 2, 3, 4, 5, 6, 7, 8, 18, 21, 22, 24, 29, 30, 42, 50, 62, 71, 86, 90, 94, 118, 124, 127, 129, 144, 154, 186, 192, 214, 271, 354, 360, 411, 480, 509, 558, 575, 663, 764, 814, 825, 874, 1028, 1049, 1050, 1102, 1113, 1131, 1158, 1376, 1464, 1468, 1535, 1622, 1782, 1834, 1924, 2096, 2176, 2409, 2464, 2816, 3013, 3438, 3453, 3663, ...
7	3, 5, 6, 8, 13, 18, 21, 28, 30, 34, 36, 46, 48, 50, 54, 55, 58, 63, 76, 84, 94, 105, 122, 131, 148, 149, 224, 280, 288, 296, 332, 352, 456, 528, 531, 581, 650, 654, 730, 740, 759, 1026, 1047, 1065, 1460, 1660, 1699, 1959, 2067, 2260, 2380, 2665, 2890, 3238, 4020, ...
8	1, 2, 3, 6, 9, 18, 30, 42, 78, 87, 114, 138, 189, 303, 318, 330, 408, 462, 504, 561, 1002, 1389, 1746, 1794, 2040, 2418, 2790, 3894, 4077, ...
9	1, 2, 4, 6, 10, 12, 16, 18, 20, 30, 32, 36, 54, 64, 66, 118, 138, 152, 182, 232, 264,

	336, 340, 380, 414, 446, 492, 540, 624, 720, 762, 1066, 1094, 1098, 1170, 1230, 1254, 1320, 1428, 1546, 2018, 2574, 2724, 2804, 2920, 3074, 3316, 3646, ...
10	1, 2, 3, 4, 9, 10, 12, 14, 19, 23, 24, 36, 38, 39, 48, 62, 93, 106, 120, 134, 150, 196, 294, 317, 320, 385, 586, 597, 654, 738, 945, 1031, 1172, 1282, 1404, 1426, 1452, 1521, 1752, 1812, 1836, 1844, 1862, 2134, 2232, 2264, 2667, 3750, 3903, 3927, ...
11	2, 4, 5, 6, 8, 9, 10, 14, 15, 17, 18, 19, 20, 27, 36, 42, 45, 52, 60, 73, 91, 104, 139, 205, 234, 246, 318, 358, 388, 403, 458, 552, 810, 855, 878, 907, 1114, 1131, 1220, 1272, 1431, 1470, 1568, 1614, 1688, 1696, 1907, 2029, 2136, 2288, 2535, 2577, ...
12	1, 2, 3, 5, 10, 12, 19, 20, 21, 22, 56, 60, 63, 70, 80, 84, 92, 97, 109, 111, 123, 164, 189, 218, 276, 317, 353, 364, 386, 405, 456, 511, 636, 675, 701, 793, 945, 1090, 1268, 1272, 1971, 2088, 2368, 2482, 2893, 2966, 3290, ...
13	2, 3, 5, 6, 7, 8, 9, 12, 16, 22, 24, 28, 33, 34, 38, 78, 80, 102, 137, 140, 147, 224, 230, 283, 304, 341, 360, 372, 384, 418, 420, 436, 483, 568, 570, 594, 737, 744, 855, 883, 991, 1021, 1193, 1222, 1615, 1628, 1838, 2032, 2146, 2302, 2530, 2830, 2958, 3030, 3528, 3671, 3885, ...
14	1, 3, 4, 6, 7, 14, 19, 24, 31, 33, 35, 36, 41, 55, 60, 106, 114, 129, 152, 153, 172, 222, 265, 286, 400, 448, 560, 584, 864, 1006, 1335, 1363, 1520, 1536, 1659, 1862, 1925, 2332, 2458, 2687, 3381, 3512, 3870, 3976, ...
15	3, 4, 6, 7, 14, 24, 43, 54, 58, 73, 85, 93, 102, 184, 220, 221, 228, 232, 247, 291, 305, 486, 487, 505, 551, 552, 590, 1029, 1194, 1274, 1406, 1444, 1532, 1548, 1748, 1986, 2093, 2182, 2202, 2579, 2781, 3054, 3239, 3696, ...
16	2, 4, 6, 8, 10, 14, 20, 30, 46, 48, 52, 70, 74, 78, 150, 178, 204, 298, 306, 346, 366, 378, 400, 476, 498, 502, 614, 634, 1120, 1266, 1530, 1898, 1912, 1972, 2548, 2770, 3738, 3850, ...
17	1, 2, 3, 5, 7, 8, 11, 12, 14, 15, 34, 42, 46, 47, 48, 50, 71, 77, 94, 110, 114, 147, 154, 176, 228, 235, 258, 275, 338, 350, 419, 450, 480, 515, 589, 624, 666, 716, 724, 810, 815, 1232, 1490, 1934, 2106, 2391, 2732, 2904, 3462, 3912, 4053, ...
18	1, 2, 3, 6, 14, 17, 21, 24, 30, 33, 38, 45, 46, 72, 78, 114, 146, 168, 288, 414, 440, 448, 665, 792, 801, 816, 975, 1165, 1176, 1267, 1466, 1513, 1882, 1920, 1998, 2194, 2272, 2643, 2800, 2946, 3434, 3504, 3813, 3866, 3957, ...
19	2, 3, 4, 6, 19, 20, 31, 34, 47, 56, 59, 61, 70, 74, 91, 92, 96, 98, 107, 120, 145, 156, 168, 242, 276, 314, 326, 337, 387, 565, 602, 612, 892, 984, 1061, 1067, 1079, 1262, 1328, 2356, 3033, 3419, 3501, 3963, ...
20	1, 3, 4, 6, 8, 9, 10, 11, 17, 30, 98, 100, 110, 126, 154, 158, 160, 168, 178, 182, 228, 266, 270, 280, 340, 416, 480, 574, 774, 980, 1052, 1139, 1338, 1418, 1474, 1487, 1594, 1902, 2326, 3112, 3520, 3808, 3830, ...
21	2, 3, 5, 6, 8, 9, 10, 11, 14, 17, 26, 43, 64, 74, 81, 104, 192, 271, 321, 335, 348, 404, 437, 445, 516, 671, 694, 788, 1788, 1943, 2343, 2742, 3031, 3135, ...
22	2, 5, 6, 7, 10, 21, 25, 26, 69, 79, 86, 93, 100, 101, 154, 158, 161, 171, 202, 214, 294, 354, 359, 424, 454, 602, 687, 706, 744, 857, 1028, 1074, 1136, 1150, 1345,

	1408, 1525, 1572, 1578, 1988, 2142, 2665, ...
23	2, 5, 8, 11, 15, 22, 26, 39, 42, 45, 54, 56, 132, 134, 145, 147, 196, 212, 218, 252, 343, 580, 662, 816, 820, 846, 1078, 1092, 1174, 1189, 1548, 1632, 2040, 2180, 2348, 2732, 3100, 3181, 4010, ...
24	1, 2, 3, 4, 5, 8, 14, 19, 22, 38, 45, 53, 54, 70, 71, 117, 140, 144, 169, 186, 192, 195, 196, 430, 653, 661, 744, 834, 855, 870, 927, 1128, 1158, 1390, 1516, 1555, 1617, 1844, 2022, 2060, 2208, 2812, 3153, 3952, ...
25	2, 4, 6, 12, 14, 24, 28, 44, 46, 54, 58, 60, 118, 124, 144, 192, 210, 250, 268, 310, 496, 532, 1258, 1494, 1944, 2050, 2498, 2728, 3324, 3418, 3646, 3862, 4014, ...
26	1, 2, 4, 7, 9, 18, 20, 22, 24, 30, 43, 69, 132, 140, 186, 200, 210, 218, 267, 347, 454, 495, 554, 585, 645, 694, 980, 1028, 1060, 1098, 1432, 1714, 1828, 3513, 3786, ...
27	2, 3, 12, 21, 24, 36, 87, 93, 171, 249, 276, 360, 480, 621, 732, 780, 1716, 3843, ...
28	1, 2, 3, 5, 6, 8, 17, 21, 38, 81, 91, 96, 102, 132, 148, 156, 240, 258, 260, 276, 457, 464, 465, 500, 506, 535, 684, 746, 838, 930, 982, 1015, 1189, 1296, 1335, 1345, 1390, 1423, 2062, 2723, 2893, 3078, ...
29	4, 5, 6, 7, 8, 14, 30, 32, 39, 45, 50, 76, 116, 151, 222, 357, 402, 462, 570, 588, 636, 671, 695, 844, 1498, 1650, 1770, 3175, 3195, 3312, 3538, 3719, ...
30	1, 2, 5, 9, 11, 12, 21, 36, 51, 64, 91, 163, 174, 195, 230, 278, 318, 342, 346, 424, 530, 569, 578, 795, 984, 1094, 1167, 1335, 1564, 1605, 1658, 1789, 2159, 2204, 2225, 3366, 3458, 3615, ...
31	3, 7, 12, 17, 24, 30, 31, 33, 40, 176, 218, 308, 404, 420, 630, 693, 890, 915, 922, 1475, 2122, 2185, 2487, 2541, 2907, 3387, 4055, ...
32	1, 6, 30, 85, 110, 120, 320, 1050, 1065, 1385, 2490, 3080, 3920, ...
33	1, 2, 3, 10, 16, 25, 28, 30, 35, 36, 45, 56, 76, 87, 110, 134, 135, 197, 200, 220, 228, 314, 324, 330, 396, 498, 583, 624, 725, 806, 940, 1145, 1240, 1644, 1750, 2171, 2268, 2675, 2781, 2790, 2808, 3581, ...
34	3, 6, 8, 10, 13, 20, 24, 56, 87, 154, 164, 196, 282, 363, 428, 652, 744, 780, 860, 902, 952, 1178, 1493, 1540, 1643, 1904, 2184, 2277, 2468, 2943, ...
35	2, 4, 6, 8, 18, 21, 22, 26, 42, 128, 154, 158, 170, 180, 184, 192, 254, 313, 450, 624, 737, 762, 798, 874, 912, 1002, 1006, 1098, 1234, 1297, 1418, 1714, 1926, 2325, 2343, 2368, 2998, 3567, 4064, ...
36	2, 4, 12, 62, 72, 96, 180, 240, 382, 514, 688, 732, 734, 962, 1048, 1088, 1232, 1408, 2088, 2176, 2248, 2724, 3180, ...

Another example is in base 10 there are automorphic numbers, the natural numbers *n* whose square “ends” in the same digits as *n* itself, and this can be generalized to other bases *b*. Given a number base *b*, a natural number *n* with *k* digits is an automorphic number if *n* is a fixed point of the polynomial function $f(x) = x^2$ over Z/b^kZ, the ring of integers modulo b^k. As the inverse limit of Z/b^kZ is Z_b, the ring of *b*-adic integers, automorphic numbers are used to find the numerical representations of the fixed points of $f(x) = x^2$ over Z_b. A fixed point of $f(x)$ is a zero of the function $g(x) = f(x) - x$. In the ring of integers modulo b, there are $2^{omega(b)}$ zeroes to $g(x) = x^2 - x$, where *omega*(b) is the number of distinct prime factors in b. An element x in Z/bZ is a zero of $g(x) = x^2 - x$ if and only if $x == 0 \bmod p^{valuation(b,p)}$ or $x == 1 \bmod p^{valuation(b,p)}$ for all primes p dividing b (for the examples of *valuation*(b,p) for primes p = 2, 3, 5, 7, see the OEIS sequences A007814, A007949, A112765, A214411, respectively). Since there are two possible values in the set {0,1}, and there are *omega*(b) such p dividing b, there are $2^{omega(b)}$ zeroes of $g(x) = x^2 - x$, and thus there are $2^{omega(b)}$ fixed points of $f(x) = x^2$. According to Hensel's lemma, if there are k zeroes or fixed points of a polynomial function modulo b, then there are k corresponding zeroes or fixed points of the same function modulo any power of b, and this remains true in the inverse limit. Thus, in any given base b there are $2^{omega(b)}$ b-adic fixed points of $f(x) = x^2$.

As 0 is always a zero divisor, 0 and 1 are always fixed points of $f(x) = x^2$, and 0 and 1 are automorphic numbers in every base. These solutions are called trivial automorphic numbers. If b is a prime power, then the ring of b-adic numbers has no zero divisors other than 0, so the only fixed points of $f(x) = x^2$ are 0 and 1. As a result, nontrivial automorphic numbers, those other than 0 and 1, only exist when the base b has at least two distinct prime factors.

b	nontrivial automorphic numbers in base *b*
6	…4155152221350213, …1400403334205344
10	…6259918212890625, …3740081787109376
12	…B3452B21B61B3854, …0876909A05A08369
14	…A8CBA57337AA0C37, …3512386AA633D1A8
15	…CE8570624D4BDA86, …20697E8CA1A3146A
18	…01G4C968DA4E1249, …HG1D58B947D3GFDA
20	…9F1B657D121AB6B5, …A4I8DEC6IHI98D8G
21	…J03D7HID8J86H7G7, …1KH7D327C1CE3D4F
22	…A1F0E7IGDI8D185B, …BK6L7E3583D8KDGC
24	…KK4L76I751E4D0L9, …33J2GH5GIM9JAN2G
26	…NODPBN31MM3H1G6D, …21C0E2MO33M8O9JE
28	…E2ILKR7QB6IDAAQ8, …DP9670K1GL9EHH1L
30	…GQ881C8LBQ6LB2J6, …R2230RO2307OH13A, …G1JIRJR6F3FE1Q7F, …DSAB2A2NEQEFS3MG, …2RRQT25RQTM5CSQL, …D3LLSHL8I3N8IRAP
33	…BE9LG6LOKN0BVC7C, …LINBGQB8C9WL1KPM
34	…HVLAS5K7H4HI248H, …G2CN5SDQGTGFVTPI
35	…S7AV6H8SIPXWTC1F, …6RO3SHQ6G9125MXL

36	…PNZH5ZDJPZBEDN29, …AC0IU0MGA0OLMCXS

Thus, we had better study about the base b digits of primes for other bases b. For the repunit primes, there are a list of repunit primes or PRPs in all bases $2 \le b \le 160$ and length ≤ 32803, and a list of repunit primes or PRPs in all bases $2 \le b \le 999$ and length ≤ 3571, also see OEIS sequences A084740 and A084738 for the smallest repunit (probable) primes in base b; for the near-repdigit primes, there was no list of the smallest such primes (only a list of factorization of such numbers in decimal (base 10)), but recently I built a list of the smallest primes or PRPs (searched to length 5000, lists 0 if no primes or PRPs in this form with length ≤ 5000) in given near-repdigit form $x\{y\}$ (i.e. *xyyy...yyy*) or $\{x\}y$ (i.e. *xxx...xxxy*) (where x and y are digits in base b) in bases $2 \le b \le 36$ (I stop at base 36 since this base is a maximum base for which it is possible to write the numbers with the symbols 0, 1, ..., 9 (the 10 Arabic numerals) and A, B, ..., Z (the 26 Latin letters) of the Latin alphabet, references: http://www.tonymarston.net/php-mysql/converter.html https://www.dcode.fr/base-36-cipher http://www.urticator.net/essay/5/567.html http://www.urticator.net/essay/6/624.html https://docs.python.org/3/library/functions.html#int https://reference.wolfram.com/language/ref/BaseForm.html https://baseconvert.com/ https://www.calculand.com/unit-converter/zahlen.php?og=Base+2-36&ug=1 https://linesegment.web.fc2.com/application/math/numbers/RadixConversion.html (in Japanese)); for the left-truncatable primes, there is a list for primes $\le 10^6$ for bases $2 \le b \le 20$, and there is a graph of the actual values and estimation formulas for bases $3 \le b \le 120$ (no such prime exists for $b = 2$), also there is a page for find largest such prime in a given base b, also see OEIS sequences A103443 and A103463 and A076623 for the largest left-truncatable primes in base b and the total number of left-truncatable primes in base b; for the right-truncatable primes, there is a list for bases $2 \le b \le 20$, and there is data for bases $3 \le b \le 90$ (no such prime exists for $b = 2$), also see OEIS sequences A023107 and A103483 and A076586 for the largest right-truncatable primes in base b and the total number of right-truncatable primes in base b. Thus, this new problem on the digits of primes (i.e. the problem on the digits of primes inspired from a classical theorem in formal language theory) should also be generalized to other bases, and this problem in various bases is exactly the target of this article (in this article we aim to solve this problem in bases $2 \le b \le 36$ (I stop at base 36 since this base is a maximum base for which it is possible to write the numbers with the symbols 0, 1, ..., 9 (the 10 Arabic numerals) and A, B, ..., Z (the 26 Latin letters) of the Latin alphabet, references: http://www.tonymarston.net/php-mysql/converter.html https://www.dcode.fr/base-36-cipher http://www.urticator.net/essay/5/567.html http://www.urticator.net/essay/6/624.html https://docs.python.org/3/library/functions.html#int https://reference.wolfram.com/language/ref/BaseForm.html https://baseconvert.com/ https://www.calculand.com/unit-converter/zahlen.php?og=Base+2-36&ug=1 https://linesegment.web.fc2.com/application/math/numbers/RadixConversion.html (in Japanese)), but since this problem (finding all minimal primes) is much harder than finding all left-truncatable primes or all right-truncatable primes for the same base, in this article we only solve this problem in bases $2 \le b \le 16$, and I left as a challenge to readers the task of solving this problem in bases $17 \le b \le 36$, of course, you can also try to solve this problem in bases $2 \le b \le 120$ as the same problem for the left-truncatable primes, but this will be extremely difficult).

There is a conjecture that there are infinitely many repunit primes in all bases b which are not perfect powers (if b is a perfect power, then it can be shown that there is at most one repunit prime in base b, since the numbers with all digits 1 in these bases can be factored algebraically (this strategy (algebraic factorization) will also be used in our problem, to show that some families (such as 10^n1 in base 8 and 38^n in base 9) contains no primes > base)), and it is also conjectured that there are also infinitely many primes in any given near-repdigit form $x\{y\}$ (i.e. *xyyy...yyy*) or $\{x\}y$ (i.e. *xxx...xxxy*) (where x and y are digits in base b) if this form cannot be proven as only contain composites or only contain finitely many primes, also, it is conjectured that there are finitely many left-truncatable primes and finitely many right-truncatable primes in any given base b, however, unlike minimal primes (which can be proven to be finite in any given base b by using the theorem that there are no infinite antichains for the subsequence ordering), none of these conjectures are proven.

These classes of primes are related to the class of primes in this article (i.e. minimal primes) and hence related to the problem in this article (i.e. finding $M(L_b)$ for bases $2 \le b \le 36$), since the smallest repunit prime (if exists) is always a minimal prime to the same base b, and the smallest near-repdigit prime with a given form $x\{y\}$ (i.e. *xyyy...yyy*) or $\{x\}y$ (i.e. *xxx...xxxy*) (where x and y are digits in base b) (if exists) is also always a minimal prime to the same base b unless the "repeating digit" (i.e. y for $x\{y\}$, x for $\{x\}y$) is 1 (also, many (but not all, if b > 4, this condition (i.e. b > 4) regards "repunit prime" as a special situation of "near-repdigit prime") minimal primes are also near-repdigit primes in the same base b, may of the form $x\{y\}$ (i.e. *xyyy...yyy*) or $\{x\}y$ (i.e. *xxx...xxxy*) (where x and y are digits in base b) or neither of these two forms, such as the minimal prime 555555555555525 in base 8), also, since all suffixes and all prefixes are also substrings, hence also subsequences, a left-truncatable prime or right-truncatable prime with length ≥ 3 cannot be a minimal prime to the same base b, and left-truncatable primes or right-truncatable primes can be regarded the opposite of minimal primes (reference).

Problems about the digits of prime numbers have a long history, and many of them are still unsolved. For example, are there infinitely many primes, all of whose base-10 digits are 1? Currently, there are only five such "repunits" known, corresponding to $(10^p - 1)/9$ for $p \in \{2, 19, 23, 317, 1031, 49081\}$ (references for recently proven prime with p = 49081: https://mersenneforum.org/showpost.php?p=602219&postcount=35 https://primes.utm.edu/bios/page.php?id=579 https://primes.utm.edu/top20/page.php?id=57 https://primes.utm.edu/top20/page.php?id=27). It seems likely that four more are given by $p \in \{86453, 109297, 270343, 5794777, 8177207\}$, but this has not yet been rigorously proven. This problem also exists for other bases, e.g. for base 12, there are only nine proven such numbers, corresponding to $(12^p - 1)/11$ for $p \in \{2, 3, 5, 19, 97, 109, 317, 353, 701\}$. It seems likely that five more are given by $p \in \{9739, 14951, 37573, 46889, 769543\}$, but this has not yet been rigorously proven. However, for some bases there exists no such primes, these bases are 9, 25, 32, 49, 64, 81, 121, 125, 144, ..., (https://oeis.org/A096059) this is because the numbers with all digits 1 in these bases can be factored algebraically (this strategy (algebraic factorization) will also be used in our problem, to show that some families (such as 10^n1 in base 8 and 38^n in base 9) contains no primes > base). Some positive integers n are repunit in some base $2 \le b \le n-2$ (every integer $n \ge 3$ are trivially repunit in base $b = n-1$ since n is written "11" in base $b = n-1$, but every integer $n \ge 2$ are not repunit in any base $b \ge n$ since n is written "10" in base $b = n$ and n is single-digit number (and this digit is not 1) in any base $b > n$), they are called Brazilian numbers, all integers >6 which are neither primes nor squares of primes are Brazilian numbers, but it is unknown whether there are infinitely many primes which are also Brazilian numbers (however, it is known that every squares of primes except 121 = “11111” in base 3 are not Brazilian numbers, see https://oeis.org/A190300).

Any repunit in any base *b* having a composite number of digits is necessarily composite. Only repunits (in any base *b*) having a prime number of digits might be prime. This is a necessary but not sufficient condition, e.g. 11111111111111111111111111111111111 (the repunit with 35 (= 5 * 7, which is composite) digits) = 11111 * 1000010000100001000010000100001 = 1111111 * 10000001000000100000010000001, since 35 = 5 * 7 = 7 * 5, and this repunit factorization does not depend on the base *b* in which the repunit is expressed. (note that the value of the repunit (in any base *b*) having 1 digit is 1, and 1 is not prime)

A repunit (in any base *b*) with length *n* can be prime only if *n* is prime, since otherwise $b^{k*m}-1$ is a binomial number which can be factored algebraically. In fact, if *n* = 2**m* is even, then $b^{2*m}-1 = (b^m-1) * (b^m+1)$.

The lengths of the smallest repunit primes in base *b* for *b* = 2, 3, 4, … 36 are (0 if no repunit primes exist for this base *b*) 2, 3, 2, 3, 2, 5, 3, 0, 2, 17, 2, 5, 3, 3, 2, 3, 2, 19, 3, 3, 2, 5, 3, 0, 7, 3, 2, 5, 2, 7, 0, 3, 13, 313, 2 (*OEIS* sequence A084740)

The smallest base such that the repunit with length *p* is prime for the first 100 primes *p* (i.e. *p* = 2, 3, 5, 7, …, 541) are 2, 2, 2, 2, 5, 2, 2, 2, 10, 6, 2, 61, 14, 15, 5, 24, 19, 2, 46, 3, 11, 22, 41, 2, 12, 22, 3, 2, 12, 86, 2, 7, 13, 11, 5, 29, 56, 30, 44, 60, 304, 5, 74, 118, 33, 156, 46, 183, 72, 606, 602, 223, 115, 37, 52, 104, 41, 6, 338, 217, 13, 136, 220, 162, 35, 10, 218, 19, 26, 39, 12, 22, 67, 120, 195, 48, 54, 463, 38, 41, 17, 808, 404, 46, 76, 793, 38, 28, 215, 37, 236, 59, 15, 514, 260, 498, 6, 2, 95, 3 (*OEIS* sequence A066180)

b	lengths of repunit primes in base *b* (written in base 10) (such lengths must be primes, since if *m* divides *n*, then the repunit with length *m* divides the repunit with length *n*, in the same base *b*) (*Italic* for unproven probable primes) (with link of the factorization (≥33.3333% factored) of *N*−1 or *N*+1, or the *Primo* primality certificate, for known definitely primes > 10^{299})	*OEIS* sequence
2	2, 3, 5, 7, 13, 17, 19, 31, 61, 89, 107, 127, 521, 607, 1279, 2203, 2281, 3217, 4253, 4423, 9689, 9941, 11213, 19937, 21701, 23209, 44497, 86243, 110503, 132049, 216091, 756839, 859433, 1257787, 1398269, 2976221, 3021377, 6972593, 13466917, 20996011, 24036583, 25964951, 30402457, 32582657, 37156667, 42643801, 43112609, 57885161, …, 74207281, …, 77232917, …, 82589933, … (the Mersenne primes, see https://www.mersenne.org/primes/ and https://primes.utm.edu/mersenne/, all are proven primes, since these primes can be proven prime using the Lucas–Lehmer primality test)	A000043

3	3, 7, 13, 71, 103, 541, 1091, 1367, 1627, 4177, 9011, 9551, *36913*, *43063*, *49681*, *57917*, *483611*, *877843*, *2215303*, *2704981*, *3598867*, …	A028491
4	2 (this is all, since $(4^n-1)/3 = (2^n-1) * (2^n+1)/3$ for prime $n \neq 2$ (and both factors are > 1, thus this factorization is not trivial))	
5	3, 7, 11, 13, 47, 127, 149, 181, 619, 929, 3407, 10949, *13241*, *13873*, *16519*, *201359*, *396413*, *1888279*, *3300593*, …	A004061
6	2, 3, 7, 29, 71, 127, 271, 509, 1049, 6389, 6883, *10613*, *19889*, *79987*, *608099*, *1365019*, …	A004062
7	5, 13, 131, 149, 1699, *14221*, *35201*, *126037*, *371669*, *1264699*, …	A004063
8	3 (this is all, since $(8^n-1)/7 = (2^n-1) * (4^n+2^n+1)/7$ for prime $n \neq 3$ (and both factors are > 1, thus this factorization is not trivial))	
9	not exist (since $(9^n-1)/8 = (3^n-1)/2 * (3^n+1)/4$ for prime $n \neq 2$ (and both factors are > 1, thus this factorization is not trivial), it only remains to check the case $n = 2$, but $(9^2-1)/8 = 10 = 2 * 5$ is not a prime)	A000000 (the empty sequence)
10	2, 19, 23, 317, 1031, 49081, *86453*, *109297*, *270343*, *5794777*, *8177207*, …	A004023
11	17, 19, 73, 139, 907, 1907, 2029, 4801, 5153, *10867*, *20161*, *293831*, *1868983*, …	A005808
12	2, 3, 5, 19, 97, 109, 317, 353, 701, *9739*, *14951*, *37573*, *46889*, *769543*, …	A004064
13	5, 7, 137, 283, 883, 991, 1021, 1193, 3671, *18743*, *31751*, *101089*, *1503503*, …	A016054
14	3, 7, 19, 31, 41, 2687, *19697*, *59693*, *67421*, *441697*, …	A006032
15	3, 43, 73, 487, 2579, *8741*, *37441*, *89009*, *505117*, *639833*, …	A006033
16	2 (this is all, since $(16^n-1)/15 = (4^n-1)/3 * (4^n+1)/5$ for prime $n \neq 2$ (and both factors are > 1, thus this factorization is not trivial))	
17	3, 5, 7, 11, 47, 71, 419, 4799, *35149*, *54919*, *74509*, *1990523*, …	A006034
18	2, *25667*, *28807*, *142031*, *157051*, *180181*, *414269*, *1270141*, …	A133857
19	19, 31, 47, 59, 61, 107, 337, 1061, *9511*, *22051*, *209359*, …	A006035

20	3, 11, 17, 1487, *31013*, *48859*, *61403*, *472709*, *984349*, …	A127995
21	3, 11, 17, 43, 271, *156217*, *328129*, …	A127996
22	2, 5, 79, 101, 359, 857, 4463, *9029*, *27823*, …	A127997
23	5, 3181, *61441*, *91943*, *121949*, *221411*, …	A204940
24	3, 5, 19, 53, 71, 653, 661, *10343*, *49307*, *115597*, *152783*, …	A127998
25	not exist (since $(25^n-1)/24 = (5^n-1)/4 * (5^n+1)/6$ for prime $n \neq 2$ (and both factors are > 1, thus this factorization is not trivial), it only remains to check the case $n = 2$, but $(25^2-1)/24 = 26 = 2 * 13$ is not a prime)	A000000 (the empty sequence)
26	7, 43, 347, *12421*, *12473*, *26717*, …	A127999
27	3 (this is all, since $(27^n-1)/26 = (3^n-1)/2 * (9^n+3^n+1)/13$ for prime $n \neq 3$ (and both factors are > 1, thus this factorization is not trivial))	
28	2, 5, 17, 457, 1423, *115877*, …	A128000
29	5, 151, 3719, *49211*, *77237*, …	A181979
30	2, 5, 11, 163, 569, 1789, *8447*, *72871*, *78857*, *82883*, …	A098438
31	7, 17, 31, 5581, *9973*, *101111*, *535571*, …	A128002
32	not exist (since $(32^n-1)/31 = (2^n-1) * (16^n+8^n+4^n+2^n+1)/31$ for prime $n \neq 5$ (and both factors are > 1, thus this factorization is not trivial), it only remains to check the case $n = 5$, but $(32^5-1)/31 = 1082401 = 601 * 1801$ is not a prime)	A000000 (the empty sequence)
33	3, 197, 3581, *6871*, *183661*, …	A209120
34	13, 1493, *5851*, *6379*, *125101*, …	A185073
35	313, 1297, *568453*, …	A348170
36	2 (this is all, since $(36^n-1)/35 = (6^n-1)/5 * (6^n+1)/7$ for prime $n \neq 2$ (and both factors are > 1, thus this factorization is not trivial))	

Another unsolved problem about the digits of prime numbers is whether there are infinitely many palindromic primes (primes which remain the same when their digits are reversed, such as 151 and 94849) in base 10? So far, the largest known such prime is $10^{1888529} - 10^{944264} - 1$, this number has 1888529 digits, can also be written as $9^{944264}89^{944264}$, and the largest 20 known such primes are listed in this page. Of course, this problem also exists for other bases, there is no single bases for which it is known whether there are infinitely many palindromic primes. Some positive integers n are not palindromic in any base $2 \leq b \leq n-2$ (the reasons for the upper limit of $n-2$ on the base b are: every integer $n \geq 3$ are trivially palindromic in base $b = n-1$ since n is written "11" in base $b = n-1$, also every positive integer n are trivially palindromic in any base $b > n$ since n is single-digit number in any base $b > n$, but every integer $n \geq 2$ are not palindromic in base $b = n$ since n is written "10" in base $b = n$), they are called strictly non-palindromic numbers, the first such numbers n are 0, 1, 2, 3, 4, 6, 11, 19, 47, 53, 79, 103, 137, 139, 149, 163, 167, 179, 223, 263, 269, 283, 293, 311, 317, 347, 359, 367, 389, 439, 491, 563, 569, 593, 607, 659, 739, 827, 853, 877, 977, 983, 997, 1019, 1049, 1061, 1187 (for $n < 4$, the range of bases b is empty, so these numbers are strictly non-palindromic in a trivial way), all such integers > 6 are primes, since all composites $n > 6$ is either "product of two numbers k and m with $m-k \geq 2$" (in this case, n is written "kk" in base $b = m-1$) or "square of prime p" (in this case, n is written "121" in base $b = p-1$ if $p > 3$, or written "1001" in base $b = 2$ if $p = 3$), it is also unknown whether there are infinitely many such integers, but it is known that in every base b, almost all palindromic numbers are composite (neither 1 nor prime), see this reference.

Table

$|x|$ is the length of x, and in the "$max(x, x \in L_b)$" column, xy^nz means $xyyy...yyyz$ with n y's (the n-value is written in decimal), not y to the nth power.

b	$\|M(L_b)\|$	$max(x, x \in M(L_b))$	$max(\|x\|, x \in M(L_b))$	Algebraic form of $max(x, x \in M(L_b))$
2	1	11	2	3
3	3	111	3	13
4	5	221	3	41
5	22	$10^{93}13$	96	$5^{95}+8$
6	11	40041	5	5209
7	71	$3^{16}1$	17	$(7^{17}-5)/2$
8	75	$4^{220}7$	221	$(4*8^{221}+17)/7$
9①	≥149	$30^{1158}11$	1161	$3*9^{1160}+10$
10	77	$50^{28}27$	31	$5*10^{30}+27$
11①	≥914	55^{1011} or 57^n with n>50000	1013	$(607*11^{1011}-17)/10$
12	106	$40^{39}77$	42	$4*12^{41}+91$
13①②	≥2497	$80^{32017}111$ or 95^n with n>50000 or $A3^nA$ with n>50000	32021	$8*13^{32020}+183$
14①	≥606	$4D^{19698}$	19699	$5*14^{19698}-1$
15①	≥1212	$7^{155}97$	157	$(15^{157}+59)/2$
16①②	≥2045	DB^{32234}	32235	$(206*16^{32234}-11)/15$

Notes:

① I have not proved these bases, these are the largest elements in $M(L_b)$ known to me, and they are just the lower bounds.

② Data based on results of strong probable primality tests, i.e. at least one element in the set $M(L_b)$ is only strong probable prime (i.e. numbers which passed the Miller–Rabin primality tests to first few prime bases, for the smallest *composite* number which passed the Miller–Rabin primality test to first *n* prime bases, see https://oeis.org/A014233) and not definitely prime, since we cannot definitely say that they are primes, thus we cannot definitely say that they are elements in $M(L_b)$, and we cannot definitely say that the $|M(L_b)|$ and $max(x, x\in M(L_b))$ are these numbers, and we cannot definitely say that the corresponding families can be removed from the list of unsolved families, and we cannot definitely compute this part of the set $M(L_b)$, e.g. since $80^{32017}111$ (base 13) is only strong probable prime and it is the smallest (probable) prime in family 8{0}111 in base 13, we cannot definitely say that the family 8{0}111 (base 13) can be removed from the list of unsolved families, and since DB^{32234} (base 16) is only strong probable prime and it is the smallest (probable) prime in family D{B} in base 16, we cannot definitely say that the family D{B} (base 16) can be removed from the list of unsolved families.

It is found that both $|M(L_b)|$ and $max(|x|, x\in M(L_b))$ are roughly $e^{\gamma*(b-1)*eulerphi(b)}$, the value $(b-1)*$*eulerphi*(b) is the number of possible (first digit,last digit) (also called (initial digit,final digit)) combos (ordered pair) of a minimal prime in base *b* (these (first digit,last digit) combos are also all possible (first digit,last digit) combos (ordered pair) of a prime > *b* in base *b*) (these (first digit,last digit) combos for decimal (base *b* = 10) are listed in A085820, except the single-digit numbers (i.e. 1, 3, 7, 9) (i.e. first digit is 0, and hence the number has leading zeros) in this sequence, the smallest primes with these (first digit,last digit) combos listed in A085820 (except the single-digit numbers (i.e. 1, 3, 7, 9) in this sequence) are (*italic* for primes which are not minimal primes): 11, 13, 17, 19, *211*, 23, 227, 29, 31, *313*, 37, 349, 41, 43, 47, 409, 521, 53, *547*, 59, 61, *613*, 67, *619*, 71, 73, 727, 79, *811*, 83, 827, 89, *911*, *953*, 97, *919*, and the smallest minimal primes with these (first digit,last digit) combos listed in A085820 (except the single-digit numbers (i.e. 1, 3, 7, 9) in this sequence) are (0 if no such minimal prime exists): 11, 13, 17, 19, 251, 23, 227, 29, 31, 0, 37, 349, 41, 43, 47, 409, 521, 53, 557, 59, 61, 0, 67, 6469, 71, 73, 727, 79, 821, 83, 827, 89, 991, 0, 97, 9049) (they are only all “possible” (first digit,last digit) combos (ordered pair) of a minimal prime in base *b*, this does not mean that they must be realized, e.g. there are no minimal primes with (first digit,last digit) = (2,2) in base 3, and there are no minimal primes with (first digit,last digit) = (3,3), (6,3), or (9,3) in base 10, but it is conjectured that there are only finitely many such examples (i.e. for every enough large base *b*, for any given such (first digit,last digit) combo, there is a minimal prime with this (first digit,last digit) combo), also, it is conjectured that all such examples have *gcd*(first digit, last digit, $b-1$) > 1 (i.e. there is a prime number which divides first digit, last digit, and $b-1$ simultaneously), since the first digit has $b-1$ choices (all digits except 0 can be the first digit), and the last digit has *eulerphi*(*b*) choices (only digits coprime to *b* (i.e. the digits in the reduced residue system mod *b*) can be the last digit), by the rule of product, there are $(b-1)*$*eulerphi*(b) choices of the (first digit,last digit) combo. (the set of these (first digit,last digit) combos is exactly the Cartesian product of the set of the possible first digits of a prime number > *b* in base *b* and the set of the possible last digits of a prime number > *b* in base *b*, i.e. $\{d \mid d$ is integer, $1 \le d \le b-1\} \times \{d \mid d$ is integer, $1 \le d \le b-1$, gcd(d,b) = 1}, or (Z/bZ − {0}) × ((Z/bZ)$^\times$)) Thus, (b−1)*eulerphi(b) is also the relative hardness for (finding and proving the set $M(L_b)$ in) base b, there is exactly a sequence of (b−1)*eulerphi(b) in OEIS: A062955, for these (b−1)*eulerphi(b) possible (first digit,last digit) combos, we want to find all minimal primes with such (first digit,last digit) combo, if the string “first digit, last digit” represents a prime in base b, then this prime will be the only minimal prime with this (first digit,last digit) combo (since the string “first digit, last digit” is a subsequence of all numbers with this (first digit,last digit) combo), otherwise, we should find all digits which can be inserted to this (first digit,last digit) combo, i.e. the string “first digit, such digit, last digit” is neither prime nor have a subsequence which represents a prime, then do this repeatedly (find the possible (first digit,last digit) combos for the string which inserted to the starting (first digit,last digit) combo, etc.), then do program loops, these program loops must be finite by the theorem that there are no infinite antichains for the subsequence ordering, see the “proof” section and this forum post and this article.

base (b)	number of possible first digits of a prime > b in base b (equal b−1, since all digits except 0 can be the first digit)	number of possible last digits of a prime > b in base b (equal eulerphi(b), since only digits coprime to b (i.e. the digits in the reduced residue system mod b) can be the last digit)	number of possible (first digit,last digit) combos of a prime > b in base b (equal (b−1)*eulerphi(b), by the rule of product), also the relative hardness for base b
2	1	1	1
3	2	2	4
4	3	2	6
5	4	4	16
6	5	2	10
7	6	6	36
8	7	4	28
9	8	6	48
10	9	4	36
11	10	10	100
12	11	4	44
13	12	12	144
14	13	6	78
15	14	8	112
16	15	8	120

17	16	16	256
18	17	6	102
19	18	18	324
20	19	8	152
21	20	12	240
22	21	10	210
23	22	22	484
24	23	8	184
25	24	20	480
26	25	12	300
27	26	18	468
28	27	12	324
29	28	28	784
30	29	8	232
31	30	30	900
32	31	16	496
33	32	20	640
34	33	16	528
35	34	24	816
36	35	12	420

(Note: not all (first digit,last digit) combos must be realized for a minimal prime base *b*, e.g. there are no minimal primes with (first digit,last digit) = (2,2) in base 3, and there are no minimal primes with (first digit,last digit) = (3,3), (6,3), or (9,3) in base 10, for more such examples, see this post)

The probability for a random prime to have a given (first digit,last digit) combo (ordered pair) which is a possible (first digit,last digit) combo (ordered pair) of a prime > *b* in base *b* (i.e. “first digit” is not 0, and “last digit” is coprime to *b*) are all the same (for the example of decimal (base *b* = 10), there are *OEIS* sequences A077648 (first digit), A007652 (last digit), A138840 ((first digit,last digit) combo (ordered pair)), A137589 (results after deletion of all digits of primes, except the first digit and the last digit, this is the same as A138840 except the single-digit primes, and this is indeed another reason for why we exclude the single-digit primes from our minimal prime problem)), i.e. they are all 1/((*b*−1)**eulerphi*(*b*)) no matter which (first digit,last digit) combo (ordered pair) is given, the only condition is that “first digit” is not 0, and “last digit” is coprime to b (however, there is a hard problem: for any given base b and given (first digit,last digit) combo (ordered pair) satisfying this condition (i.e. “first digit” is not 0, and “last digit” is coprime to b), is there always an integer N such that for the set of the primes > base (b) and ≤ N, the number of primes with this (first digit,last digit) combo is more than the number of primes with any other given (first digit,last digit) combo? (i.e. the number of primes p with A138840 = A137589 (their analogs in other bases b) = any given n such that $b < n < b^2$ and n is coprime to b, is more than the number of primes p with A138840 = A137589 (their analogs in other bases b) = any other given m ($m \neq n$) such that $b < m < b^2$ and m is coprime to b?)), for the first digit, there is a reference about this, the primes do not follow the Benford’s law (see this reference) (reference of Benford’s law to other bases) (only the prime factors of the numbers with exponential growth (such as the repunits and the Fibonacci numbers) follow, also the primes p such that $(b^n-1)/(b-1)$ is prime for non-perfectpower b (e.g. A004023 for b = 10, and A000043 for b = 2) follow), instead, all nonzero digits have the same probability (i.e. probability $1/(b-1)$) for a random prime in base b, just like a positive integer in base b, for the last digit, by the prime number theorem (extended to arithmetic progression), all digits coprime to b have the same probability (i.e. probability 1/*eulerphi*(b)) for a random prime in base b, however, according to Chebyshev's bias, if d_1 is a quadratic residue mod b, d_2 is a quadratic nonresidue mod b (i.e. d_1 can be the last digit of a square number, while d_2 cannot be), then for the primes ≤ N for a random positive integer N, the probability for the number of primes end with d_2 in base b is more than the number of primes end with d_1 in base b is larger than 50%, e.g. the smallest N such that the number of primes end with 1 in base 4 is more than the number of primes end with 3 in base 4 is 12203231 (26861 in decimal), and the smallest N such that the number of primes end with 1 in base 3 is more than the number of primes end with 2 in base 3 is 2011012212222201102200001 (608981813029 in decimal), references: https://oeis.org/A007350 https://oeis.org/A007352 https://oeis.org/A199547 https://oeis.org/A306891 https://oeis.org/A038698 https://oeis.org/A112632 https://oeis.org/A275939 https://oeis.org/A306499 https://oeis.org/A306500, this is a classic example of the strong law of small numbers (Prime Glossary page), another classic example is it appears that the sum of the Liouville function (which is an important function in number theory, defined as $(-1)^{\mathit{bigomega}(n)}$, which is A008836($n$)) of the positive integers ≤ N is ≤ 0 if N > 1, is it always true? (the Pólya conjecture), the smallest N such that this conjecture is false is 906150257 (this conjecture is important in number theory since if this conjecture is true, then the Riemann hypothesis can be proved, and hence many conjectures in number theory can also be proved, e.g. Mills’ primes will be known to be 2, 11, 1361, 2521008887, 16022236204009818131831320183, 411310114921510480003052953791595317048613962353975993313594999488277040407 4832568499, ... https://oeis.org/A051254, and the Mills’ constant will be known to be 1.30637788386308069046861449260260571291678... https://oeis.org/A051021, which (let this constant be A) floor(A^{3^n}) are primes for all positive integers n, and this formula will be the first known formula for primes which only use exponential functions and floor functions (and not use factorial), thus can be easily to calculate, and there will not be “the largest known prime number”! (since floor(A^{3^n}) contains infinitely many numbers), currently, the largest known Mills’ prime is (((((((((1361^3+6)^3+80)^3+12)^3+450)^3+894)^3+3636)^3+70756)^3+97220)^3+66768)^3 +300840)^3+1623568, which has 555154 digits, see PRP top), for more examples of the strong law of small numbers, see https://primes.utm.edu/glossary/xpage/LawOfSmall.html and https://oeis.org/A005165/a005165.pdf, and there are also examples of the strong law of small numbers which are related to the research in this article: Are the base 10 numbers 527, 5027, 50027, 500027, 5000027, 50000027, ..., all composite? (which is corresponding to the largest minimal prime in base 10: $50^{28}27$) Are the base 8 numbers 47, 447, 4447, 44447, 444447, 4444447, ..., all composite? (which is corresponding to the largest minimal prime in base 8: $4^{220}7$) Are the base 16 numbers DB, DBB, DBBB, DBBBB, DBBBBB, DBBBBBB, ..., all composite? (which is corresponding to the minimal prime in base 16: DB^{32234} (it is not known whether this is the largest minimal prime in base 16 or not (the families {3}AF and {4}DD may have larger smallest primes), nor the primality of this prime (i.e. this prime is only a probable prime, not a definitely prime) etc.), a paradox related to the strong law of small numbers is interesting number paradox (Prime Curios! page), this paradox is a humorous paradox which arises from the attempt to classify every natural number as either “interesting” or “uninteresting”, this paradox states that every natural number is interesting, i.e. every natural number has an interesting property, the “proof” (okay, a joke proof) is by contradiction: if there exists a non-empty set of uninteresting natural numbers, there would be a smallest uninteresting number – but the smallest uninteresting number is itself interesting because it is the smallest uninteresting number, thus producing a contradiction, there are examples of the interesting properties which are related to the research in this article: 5000000000000000000000000000027 is the largest minimal prime in base 10, 21870014779720278736374332149114462520188534743847615898363462279537144492484599310778624146468224150373895489844303219383829573677353011540369291867378470695590964880740521967077028064041941947533607 is the largest minimal prime in base 8, 705490352625161496279722666407220454094798939 is the largest minimal prime in base 12, etc. and there are also other paradoxes related to this paradox: the Berry paradox, the Richard's paradox, they are related to Cantor's diagonal argument to prove that the set of the real numbers is uncountable (this is also related to Gödel's incompleteness theorems, these theorems are widely, but not universally, interpreted as showing that Hilbert’s program to find a complete and consistent set of axioms for all mathematics is impossible, we can use a simple proposition to show this: **This proposition has no formal proof**, and consider whether this proposition is true or not), but it can be proved that the set of the rational numbers, the set of the algebraic numbers, the set of the computable numbers, the set of the definable numbers, are all countable, i.e. *card*(these sets) are all equal to *card*(*N*), where *N* is the set of the natural numbers, but *card*(*R*) (*R* is the set of the real numbers) is larger than *card*(*N*), and the continuum hypothesis is that *card*(*R*) = $2^{card(N)}$, references: Prime Curios! (the smallest number with no curios is 326) What's Special About This Number? (the smallest number not has a property in this page is 391) Properties of the First 5000 Integers (the smallest number not in this page is 291) my website for “What is special about this number?” (the smallest number not in this page is) (this page has many (most number-theory-related) interesting properties of nonnegative integers, to show the nonnegative integer is unique, you can combine them with this list (for the smallest prime or PRP with given form in base *b*, when *b* itself has unique interesting property, and when such prime or PRP is large, e.g. 72 is the smallest Achilles number, and the smallest prime of the form 3{z} in base *b* = 72 has length 1119850, and 276 is the smallest number whose aliquot sequence has not yet been fully determined (see https://oeis.org/A131884), and the smallest prime of the form 1{z} in base *b* = 276 has length 2485, and 836 is the smallest weird number which is also an untouchable number (also, 836 is twice 418, and 418 is the smallest non-primepower *k* such that binomial(2**k*, *k*) == 2 (mod *k*) (besides, 418 is also the only known such even non-primepower *k*) (see https://oeis.org/A328497 and https://oeis.org/A082180 and https://oeis.org/A228562 and https://oeis.org/A136327)), and for base *b* = 836, there are no known primes of the form 1{0}1, 2{0}1, 7{z} (thus, these three families are unsolved families in base *b* = 836) and the smallest prime of the form 7{0}1 has length 5701) and with Sierpinski conjecture / Riesel conjecture / extended Sierpinski conjecture / extended Riesel conjecture in base *b* (when *b* itself has unique interesting property, and when such prime or PRP is large), and not only base *b*, but also the number-theory functions (Euler phi function, sigma function, Carmichael lambda function, etc.) (or their inverse functions) at *b*), also, currently the smallest number without its own article is Wikipedia is 262, also, currently the smallest number not in OEIS is 20067.

Excluding the primes ≤ *b* (i.e. only counting the primes > *b*) makes the formula of the number of possible (first digit,last digit) combo of a minimal prime in base *b* more simple and smooth number, since if only excluding the primes < *b* (i.e. counting the primes ≥ *b*), then when *b* is prime, there is an additional possible (first digit,last digit) combo: (1,0), and hence the formula will be (*b*−1)**eulerphi*(*b*)+1 if *b* is prime, or (*b*−1)**eulerphi*(*b*) if *b* is composite (the fully formula will be (*b*−1)**eulerphi*(*b*)+*isprime*(*b*) or (*b*−1)**eulerphi*(*b*)+*floor*((*b*−*eulerphi*(*b*)) / (*b*−1))), which is more complex, and if start with 1 (i.e. the original minimal prime problem), the formula is much more complex, since the prime digits (i.e. the single-digit primes) should be excluded (thus, e.g. for decimal (base *b*=10), the primes are limited in A034844), and (for such prime > *b*) the first digit has *b*−1−*pi*(*b*) (i.e. A065855(*b*)) choices, and the last digit has A048864(*b*) choices, by the rule of product, there are (*b*−1−*pi*(*b*))*(A048864(*b*)) choices of the (first digit,last digit) combo (for such prime ≥ *b* instead of > *b*, the formula will be (*b*−1−*pi*(*b*))*(A048864(*b*))+1 if *b* is prime, or (*b*−1−*pi*(*b*))*(A048864(*b*)) if *b* is composite, and for all such primes, the formula will be (*b*−1−*pi*(*b*))*(A048864(*b*))+*omega*(*b*)), which is much more complex, (also, the possible (first digit,last digit) combo for a prime > *b* in base *b* are exactly the (first digit,last digit) combos which there are infinitely many primes have, while this is not true when the requiring of the prime is ≥ *b* or ≥ 2 instead of > *b*, since this will contain the prime factors of *b*, which are not coprime to *b* and hence there is only this prime (and not infinitely many primes) have this (first digit,last digit) combo), thus this problem is much better than the original minimal prime problem (another reason is that this problem is regardless whether 1 is considered as prime or not, i.e. no matter 1 is considered as prime or not prime (in the beginning of the 20th century, 1 is regarded as prime) (reference of why 1 is not prime), the sets $M(L_b)$ in this problem are the same, while the sets $M(L_b)$ in the original minimal prime problem are different, e.g. in base 10, if 1 is considered as prime, then the set $M(L_b)$ in the original minimal prime problem is {1, 2, 3, 5, 7, 89, 409, 449, 499, 6469, 6949, 9049, 9649, 9949, 60649, 666649, 946669, 60000049, 66000049, 66600049}, while if 1 is not considered as prime, then the set $M(L_b)$ in the original minimal prime problem is {2, 3, 5, 7, 11, 19, 41, 61, 89, 409, 449, 499, 881, 991, 6469, 6949, 9001, 9049, 9649, 9949, 60649, 666649, 946669, 60000049, 66000049, 66600049}, however, in base 10, the set $M(L_b)$ in this problem is always {11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 227, 251, 257, 277, 281, 349, 409, 449, 499, 521, 557, 577, 587, 727, 757, 787, 821, 827, 857, 877, 881, 887, 991, 2087, 2221, 5051, 5081, 5501, 5581, 5801, 5851, 6469, 6949, 8501, 9001, 9049, 9221, 9551, 9649, 9851, 9949, 20021, 20201, 50207, 60649, 80051, 666649, 946669, 5200007, 22000001, 60000049, 66000049, 66600049, 80555551, 555555555551, 5000000000000000000000000000027}, no matter 1 is considered as prime or not prime) (another reason is that if we include the prime = *b* (i.e. the

prime “10”) or the primes < *b* (i.e. the single-digit primes), then some properties in this post will be incorrect), thus, start with *b*+1 (instead of *b*, 2, 1, b^2, b^2+1, *b*+2, 2*b, 2*b+1, ...) makes this minimal prime problem most beautiful (prime = *b* (i.e. the prime “10”) and primes < *b* (i.e. single-digit primes) need to be excluded, while the prime = *b*+1 (i.e. the prime “11”) and other two-digit primes and other repunit primes do not need).
), reference: https://mersenneforum.org/showpost.php?p=562832&postcount=52.

Inclusion of the primes	Formula of the number of possible (first digit,last digit) combo of a minimal prime in base *b*
primes > *b*	(*b*−1)**eulerphi*(*b*)
primes ≥ *b*	(*b*−1)**eulerphi*(*b*)+*isprime*(*b*)
all primes	(*b*−1−*pi*(*b*))*(A048864(*b*))+*omega*(*b*))

Data

The data of bases 14, 16, and the odd bases >8 are possibly not complete, only tested to the test limit in the discussion of these bases and found the smallest (probable) prime in some unsolved families of these bases, but there may be more unsolved families not found by me.

Our results assume that a number which has passed Miller–Rabin primality tests to all prime bases *p* ≤ 64 (i.e. the first 18 prime bases, bases 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, and 61, for the composite numbers which pass this test to the first *n* prime bases (i.e. numbers which are strong pseudoprimes to the first *n* prime bases), see https://oeis.org/A014233, we use *n* = 18 for the primality tests) and the strong Lucas primality test with parameters (*P*, *Q*) defined by Selfridge's Method *A* (for the composite numbers which pass this test (i.e. numbers which are strong Lucas pseudoprimes with parameters (*P*, *Q*) defined by Selfridge's Method *A*), see https://oeis.org/A217255) is in fact prime, since in some cases (e.g. *b* = 13 and *b* = 16) some candidate elements of $M(L_b)$ are too long to be proven prime rigorously (and neither *N*−1 nor *N*+1 can be ≥33.3333% factored), and the probability that such a number is in fact composite is very low, e.g. for such a number with 5000 decimal digits, the probability is less than $7.6*10^{-680}$, and for such a number with 100000 decimal digits, the probability is less than $1.3*10^{-10584}$, both of them are “almost” zero, i.e. we can “almost surely” (99.9999…% (with more than 10000 9’s) surely, but not 100% surely) that they are primes, and the numbers which currently cannot be proven prime rigorously are usually very large (usually > 10^{5000}, see top 20 ECPP proving page and top 20 Primo proving page, the largest prime which is proven by ECPP is the repunit $(10^{49081}-1)/9$, this number has 49081 decimal digits (all digits are 1), and this number is the largest known ordinary prime, i.e. none of $p^n \pm 1$ (for small *n*) factor enough to make the number easily provable using the classical methods of primality proof), and if such a number is larger, then probability that this number is in fact composite is lower, thus the probability is much less than $7.6*10^{-680}$, see this page, also, our tests (combine of the Miller–Rabin primality tests to the first 13 prime bases and the strong Lucas primality test with parameters (*P*, *Q*) defined by Selfridge's Method *A*) cover the Baillie–PSW primality test (which is only combine of the Miller–Rabin primality tests to base 2 and the strong Lucas primality test with parameters (P, Q) defined by Selfridge's Method A, i.e. (let D be the first number in the sequence 5, −7, 9, −11, 13, −15 ... such that $\left(\frac{D}{N}\right) = -1$ (N is the number which we want to test primality), where $\left(\frac{m}{n}\right)$ is the Jacobi symbol), set P = 1 and Q = (1−D)/4), and no known composites which pass the Baillie–PSW test, and no composites < 2^{64} pass the Baillie–PSW test (reference and reference) (a number which passes both a strong Fermat test and a strong Lucas test is very likely to be prime, since Fermat pseudoprimes tend to fall into the residue class 1 (mod m) for many small m, i.e. N−1 has many divisors (i.e. *bigomega*(N−1) is large), while Lucas pseudoprimes tend to fall into the residue class −1 (mod m) for many small m, i.e. N+1 has many divisors (i.e. *bigomega*(N+1) is large), thus a composite which passes both a strong Fermat test and a strong Lucas test must satisfy many conditions (both N−1 and N+1 must have many divisors, and such N is very hard to exist, since N−1 and N+1 cannot be both divisible by 4, also N−1 and N+1 cannot be both divisible by 3), thus such a composite is very unlikely to exist (like odd perfect numbers and quasiperfect numbers, such numbers must satisfy many conditions, thus very unlikely to exist)), although it is still conjectured that there exist infinitely many “Baillie–PSW pseudoprimes”, i.e. composites which pass the Baillie–PSW test, thus if a such number is in fact composite, it will be a pseudoprime to the Baillie–PSW test, which currently no single example is known!

There are five unsolved families (families for which not even a probable prime is known nor can be ruled out as only contain composites (only count the numbers > base (b)), i.e. whether these families contain a prime > base (b) are open problems) for bases 2 ≤ b ≤ 16 found by me and searched to length 50000 with no prime or strong probable prime found:

b	Unsolved family	Algebraic form
11	57^n	$(57*11^n−7)/10$
13	95^n	$(113*13^n−5)/12$
13	$A3^nA$	$(41*13^{n+1}+27)/4$
16	3^nAF	$(16^{n+2}+619)/5$
16	4^nDD	$(4*16^{n+2}+2291)/15$

base 2

11

base 3

12, 21, 111

base 4

11, 13, 23, 31, 221

base 5

12, 21, 23, 32, 34, 43, 104, 111, 131, 133, 313, 401, 414, 3101, 10103, 14444, 30301, 33001, 33331, 44441, 300031, 1000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000013

base 6

11, 15, 21, 25, 31, 35, 45, 51, 4401, 4441, 40041

base 7

14, 16, 23, 25, 32, 41, 43, 52, 56, 61, 65, 113, 115, 131, 133, 155, 212, 221, 304, 313, 335, 344, 346, 364, 445, 515, 533, 535, 544, 551, 553, 1022, 1051, 1112, 1202, 1211, 1222, 2111, 3031, 3055, 3334, 3503, 3505, 3545, 4504, 4555, 5011, 5455, 5545, 5554, 6034, 6634, 11111, 11201, 30011, 30101, 31001, 31111, 33001, 33311, 35555, 40054, 100121, 150001, 300053, 351101, 531101, 1100021, 33333301, 5100000001, 33333333333333331

base 8

13, 15, 21, 23, 27, 35, 37, 45, 51, 53, 57, 65, 73, 75, 107, 111, 117, 141, 147, 161, 177, 225, 255, 301, 343, 361, 401, 407, 417, 431, 433, 463, 467, 471, 631, 643, 661, 667, 701, 711, 717, 747, 767, 3331, 3411, 4043, 4443, 4611, 5205, 6007, 6101, 6441, 6477, 6707, 6777, 7461, 7641, 47777, 60171, 60411, 60741, 444641, 500025, 505525, 3344441, 4444477, 5500525, 5550525, 55555025, 444444441, 744444441, 77774444441, 7777777777771, 555555555555525, 4447

base 9 (not proved, only checked to the prime 8333333335)

12, 14, 18, 21, 25, 32, 34, 41, 45, 47, 52, 58, 65, 67, 74, 78, 81, 87, 117, 131, 135, 151, 155, 175, 177, 238, 272, 308, 315, 331, 337, 355, 371, 375, 377, 438, 504, 515, 517, 531, 537, 557, 564, 601, 638, 661, 702, 711, 722, 735, 737, 751, 755, 757, 771, 805, 838, 1011, 1015, 1101, 1701, 2027, 2207, 3017, 3057, 3101, 3501, 3561, 3611, 3688, 3868, 5035, 5051, 5071, 5101, 5501, 5554, 5705, 5707, 7017, 7075, 7105, 7301, 8535, 8544, 8555, 8854, 20777, 22227, 22777, 30161, 33388, 50161, 50611, 53335, 55111, 55535, 55551, 57061, 57775, 70631, 71007, 77207, 100037, 100071, 100761, 105007, 270707, 301111, 305111, 333035, 333385, 333835, 338885, 350007, 500075, 530005, 555611, 631111, 720707, 2770007, 3030335, 7776662, 30300005, 30333335, 38333335, 51116111, 70000361, 300030005, 300033305, 351111111, 1300000007, 5161111111, 8333333335, ..., 300000000035, ..., 311111111161, ..., 544444444444, ..., 2000000000007, ..., 5700000000001, ..., 88888888833335, ..., 100000000000507, ..., 5111111111111161, ..., 8888888888888888888335, ..., 30000000000000000000051, ..., 1000000000000000000000000057, ..., 56111111111111111111111111111111111111, ..., 7662, ..., 277707, ..., 300011, ...

base 10

11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 227, 251, 257, 277, 281, 349, 409, 449, 499, 521, 557, 577, 587, 727, 757, 787, 821, 827, 857, 877, 881, 887, 991, 2087, 2221, 5051, 5081, 5501, 5581, 5801, 5851, 6469, 6949, 8501, 9001, 9049, 9221, 9551, 9649, 9851, 9949, 20021, 20201, 50207, 60649, 80051, 666649, 946669, 5200007, 22000001, 60000049, 66000049, 66600049, 80555551, 555555555551, 5000000000000000000000000000027 base 11 (not proved, only checked to the prime 1500000001)

12, 16, 18, 21, 27, 29, 34, 38, 3A, 43, 49, 54, 56, 61, 65, 67, 72, 76, 81, 89, 92, 94, 98, 9A, A3, 10A, 115, 117, 133, 139, 153, 155, 171, 193, 197, 199, 1AA, 225, 232, 236, 25A, 263, 315, 319, 331, 335, 351, 353, 362, 373, 379, 391, 395, 407, 414, 452, 458, 478, 47A, 485, 4A5, 4A7, 502, 508, 511, 513, 533, 535, 539, 551, 571, 579, 588, 595, 623, 632, 70A, 711, 715, 731, 733, 737, 755, 759, 775, 791, 797, 7AA, 803, 847, 858, 85A, 874, 885, 887, 913, 919, 931, 937, 957, 959, 975, 995, A07, A1A, A25, A45, A74, A7A, A85, AA1, AA7, 1101, 11A9, 1305, 1451, 1457, 15A7, 175A, 17A5, 17A9, 2023, 2045, 2052, 2083, 20A5, 2333, 2A05, 2A52, 3013, 3026, 3059, 3097, 3206, 3222, 3233, 3307, 3332, 3505, 4025, 4151, 4157, 4175, 4405, 4445, 4487, 450A, 4575, 5017, 5031, 5059, 5075, 5097, 5099, 5105, 515A, 517A, 520A, 5301, 5583, 5705, 577A, 5853, 5873, 5909, 5A17, 5A57, 5A77, 5A8A, 6683, 66A9, 7019, 7073, 7079, 7088, 7093, 7095, 7309, 7451, 7501, 7507, 7578, 757A, 75A7, 7787, 7804, 7844, 7848, 7853, 7877, 78A4, 7A04, 7A57, 7A79, 7A95, 8078, 8245, 8333, 8355, 8366, 8375, 8425, 8553, 8663, 8708, 8777, 878A, 8A05, 9053, 9101, 9107, 9305, 9505, 9703, A052, A119, A151, A175, A515, A517, A575, A577, A5A8, A719, A779, A911, AAA9, 10011, 10075, 10091, 10109, 10411, 10444, 10705, 10709, 10774, 10901, 11104, 11131, 11144, 11191, 1141A, 114A1, 13757, 1411A, 14477, 144A4, 14A04, 14A11, 17045, 17704, 1774A, 17777, 177A4, 17A47, 1A091, 1A109, 1A114, 1A404, 1A411, 1A709, 20005, 20555, 22203, 25228, 25282, 25552, 25822, 28522, 30037, 30701, 30707, 31113, 33777, 35009, 35757, 39997, 40045, 4041A, 40441, 4045A, 404A1, 4111A, 411A1, 42005, 44401, 44474, 444A1, 44555, 44577, 445AA, 44744, 44A01, 47471, 47477, 47701, 5057A, 50903, 5228A, 52A22, 52A55, 52A82, 55007, 550A9, 55205, 55522, 55557, 55593, 55805, 57007, 57573, 57773, 57807, 5822A, 58307, 58505, 58A22, 59773, 59917, 59973, 59977, 59999, 5A015, 5A2A2, 5AA99, 60836, 60863, 68636, 6A609, 6A669, 6A696, 6A906, 6A966, 70048, 70103, 70471, 70583, 70714, 71474, 717A4, 71A09, 74084, 74444, 74448, 74477, 744A8, 74747, 74774, 7488A, 74A48, 75773, 77144, 77401, 77447, 77799, 77A09, 78008, 78783, 7884A, 78888, 788A8, 79939, 79993, 79999, 7A051, 7A444, 7A471, 80005, 80252, 80405, 80522, 80757, 80AA5, 83002, 84045, 85307, 86883, 88863, 8A788, 90073, 90707, 90901, 95003, 97779, 97939, 99111, 99177, 99973, A0111, A0669, A0966, A0999, A0A09, A1404, A4177, A4401, A4717, A5228, A52AA, A5558, A580A, A5822, A58AA, A5A59, A5AA2, A6096, A6966, A6999, A7051, A7778, A7808, A9055, A9091, A9699, A9969, AA52A, AA58A, 100019, 100079, 101113, 101119, 101911, 107003, 140004, 144011, 144404, 1A0019, 1A0141, 1A5001, 1A7005, 1A9001, 222223, 222823, 300107, 300202, 300323, 303203, 307577, 310007, 332003, 370777, 400555, 401A11, 404001, 404111, 405AAA, 41A011, 440A41, 441011, 451777, 455555, 470051, 470444, 474404, 4A0401, 4A4041, 500015, 500053, 500077, 500507, 505577, 522A2A, 525223, 528A2A, 531707, 550777, 553707, 5555A9, 555A99, 557707, 55A559, 5807A7, 580A0A, 580A55, 58A0AA, 590007, 599907, 5A2228, 5A2822, 5A2AAA, 5A552A, 5AA22A, 5AAA22, 60A069, 683006, 6A0096, 6A0A96, 6A9099, 6A9909, 700778, 701074, 701777, 704408, 704417, 704457, 704484, 707041, 707441, 707708, 707744, 707784, 710777, 717044, 717077, 740008, 74484A, 770441, 770744, 770748, 770771, 777017, 777071, 777448, 777484, 777701, 7778A8, 777A19, 777A48, 778883, 78A808, 790003, 7A1009, 7A4408, 7A7708, 80A555, 828283, 828883, 840555, 850505, 868306, 873005, 883202, 900701, 909739, 909979, 909991, 970771, 977701, 979909, 990739, 990777, 990793, 997099, 999709, 999901, A00009, A00599, A01901, A05509, A0A058, A0A955, A10114, A555A2, A55999, A59991,

A5A222, A5A22A, A60609, A66069, A66906, A69006, A79005, A87888, A90099, A90996, A96006, A96666, A97177, A97771, AA0A58, AA5A22, AAA522, 1000501, 1011141, 1030007, 1070047, 111114A, 1111A14, 1111A41, 1144441, 14A4444, 1700005, 1700474, 1A44444, 2555505, 2845055, 3030023, 3100003, 3333397, 4000111, 4011111, 41A1111, 4411111, 444441A, 4444771, 4470004, 4505005, 4744417, 4774441, 4777404, 4777417, 4777747, 4A11111, 4A40001, 5000093, 50005A7, 5005777, 5050553, 5055503, 5070777, 5222222, 5222AAA, 52AAAA2, 52AAAAA, 5505053, 5552AAA, 5555599, 5555A58, 5558A0A, 5558A55, 5558AAA, 55A0009, 55AAA52, 580000A, 5822222, 58AAAAA, 5A2222A, 5AA2222, 6000A69, 6000A96, 6A00069, 7000417, 7000741, 7000835, 7000857, 7007177, 7008305, 7014447, 7017444, 7074177, 7077477, 7077741, 7077747, 7100447, 7174404, 717444A, 7400404, 7700717, 7701077, 7701707, 7707778, 7774004, 7777104, 777741A, 7777441, 777774A, 7777A47, 7779003, 777A008, 777A778, 777A808, 77A4777, 7900399, 8305007, 8500707, 8555707, 8883022, 8AA5222, 9000035, 9007999, 9009717, 9009777, 9009997, 9090997, 9099907, 9355555, 9790099, 9900991, 9900997, 9907909, 9909079, 9979009, 9990079, 9990091, 9990907, 9999771, 9999799, 9999979, A000696, A000991, A001091, A006906, A010044, A040041, A0AAA58, A141111, A5222A2, A600A69, A906606, A909009, A990009, A997701, AA55A52, AAA5552, AAAAA52, 10004747, 10005007, 17000744, 22888823, 28888223, 30010111, 30555777, 31011111, 33000023, 40A00041, 45000055, 47040004, 50377777, 50555553, 5282AAA2, 55505003, 555A5A52, 555AAA2A, 55A5A552, 5AAAAA2A, 60A99999, 70000057, 70070474, 70074704, 70174004, 70700078, 70700474, 70704704, 70710707, 70771007, 70777177, 71074004, 74470001, 77000177, 77070477, 77100077, 77470004, 77700404, 77710007, 77717707, 77748808, 7774A888, 77770078, 77770474, 77774704, 77777008, 77777404, 77777778, 80555055, 88828823, 88888326, 88888823, 8A522222, 90097909, 90700999, 90977777, 97000001, 97000717, 97770007, 99000001, 99000771, 99077001, 99090097, 99777707, 99900097, 99970717, 99999097, 99999707, A0000058, A0004041, A00055A9, A000A559, A1900001, A5555009, A5A55552, A9700001, A9909006, A9990006, A9990606, A9999917, A9999966, 100000507, 100035077, 100050777, 100057707, 101111114, 15A000001, 170000447, 300577777, 40000A401, 447771777, 44A444441, 474000004, 477700004, 477777774, 505000003, 55555AA2A, 5555A5A2A, 700000147, 700017004, 700044004, 700077774, 700170004, 701000047, 701700004, 704000044, 704040004, 707070774, 707077704, 707770704, 707777004, 717000004, 717700007, 770000078, 770004704, 770070747, 770070774, 770700008, 770700084, 770707074, 777000044, 777000774, 777717007, 777770477, 777770747, 7777777A4, 77A700008, 888888302, 900000091, 900090799, 970009099, 990990007, 997000077, 999999997, A0000AA58, A00990001, A05555559, A44444111, A44444777, A44477777, A66666669, A90000606, A99999006, A99999099, 1000007447, 1005000007, 1500000001, ..., 3700000001, ..., 4000000005, ..., 600000A999, ..., A000144444, ..., A900000066, ..., 33333333337, ..., 44444444447, ..., A0000000001, ..., A0014444444, ..., 100000000057, ..., 40000000A0041, ..., A000000014444, ..., A044444444441, ..., A144444444411, ..., 40000000000401, ..., A0000044444441, ..., A00000000444441, ..., 11111111111111111, ..., 14444444444441111, ..., 44444444444444111, ..., 70000000000000004, ..., A1444444444444444, ..., A9999999999999996, ..., 888888888888888883, ..., 1444444444444444444, ..., 7777777777777777771, ..., 4000000000000000A041, ..., 45AAAAAAAAAAAAAAAAAAAA, ..., 9777777777777777777707, ..., A999999999999999999999, ..., 10000000000000000000747, ..., 3577777777777777777777777, ..., 10000000000000000000000044, ..., 77700000000000000000000008, ..., A44444444444444444444444441, ...,

1500000000000000000000000007, ..., 40000000000000000000000000041, ...,
440000000000000000000000000001, ..., 99999999999999999999999999999999 1, ...,
1900000000000000000000000000000000001, ...,
7777777777777777777777777777777777704, ...,
A477777777777777777777777777777777777777777, ...,
444444444444444444444444444444444444444444441, ...,
805555555555555555555555555555555555555555555555555555555555555, ...,
4477777777777777777777777777777777777777777777777777777777777777777, ...,
9977777777777777777777777777777777777777777777777777777777777777777, ...,
1000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000051, ...,
5555555555555555555555555555555555555555555555555555555555555555555555555
5555555555555555555555555555555555555555555555555555555555555555555555555
5555555555555552A, ...,
5077777777777777777777777777777777777777777777777777777777777777777777777
7777777777777777777777777777777777777777777777777777777777777777777777777
77777777777777777777777777777777777777777777777777777777777777, ...,
7777777777777777777777777777777777777777777777777777777777777777777777777
7777777777777777777777777777777777777777777777777777777777777777777777777
7777777777777777777777777777777777777777777777777777777777777777777777777
7777777777777777777777777777777777777777777777777777777777777777777777777
7777777777777777777777777777777777777777777777777777777777777777777777777
7777777777777777777777777777777777777777777777777777777777777777777777777
7777777777777777777777777777777777777777777777777777777777777777777777777
7777777777777777777777777777777777777777777777777777777777777777777777777
7777777777777777777777777777777777777777777777777777777777777777777777777
7777777777777777777777777777777777777777777777777777777777777777777777777
7777777777777777777777777777744, ...,
5577777777777777777777777777777777777777777777777777777777777777777777777
7777777777777777777777777777777777777777777777777777777777777777777777777
7777777777777777777777777777777777777777777777777777777777777777777777777
7777777777777777777777777777777777777777777777777777777777777777777777777
7777777777777777777777777777777777777777777777777777777777777777777777777
7777777777777777777777777777777777777777777777777777777777777777777777777
7777777777777777777777777777777777777777777777777777777777777777777777777
7777777777777777777777777777777777777777777777777777777777777777777777777
7777777777777777777777777777777777777777777777777777777777777777777777777
7777777777777777777777777777777777777777777777777777777777777777777777777
7777777777777777777777777777777777777777777777777777777777777777777777777
7777777777777777777777777777777777777777777777777777777777777777777777777
7777777777777777777777777777777777777777777777777777777777777777777777777
7777777777777777777777777777777777777777777777777777777777777777, ...

base 12

11, 15, 17, 1B, 25, 27, 31, 35, 37, 3B, 45, 4B, 51, 57, 5B, 61, 67, 6B, 75, 81, 85, 87, 8B, 91, 95, A7, AB, B5, B7, 221, 241, 2A1, 2B1, 2BB, 401, 421, 447, 471, 497, 565, 655, 665, 701, 70B, 721, 747, 771, 77B, 797, 7A1, 7BB, 907, 90B, 9BB, A41, B21, B2B, 2001, 200B, 202B, 222B, 229B, 292B, 299B, 4441, 4707, 4777, 6A05, 6AA5, 729B, 7441, 7B41, 929B, 9777, 992B, 9947, 997B, 9997, A0A1, A201, A605, A6A5, AA65, B001, B0B1, BB01, BB41, 600A5, 7999B, 9999B, AAAA1, B04A1, B0B9B, BAA01, BAAA1, BB09B, BBBB1, 44AAA1, A00065, BBBAA1, AAA0001, B00099B, AA000001, BBBBBB99B, B0000000000000000000000000009B, 4000000000000000000000000000000000000000077

base 13 (not proved, only checked to the prime 1010008001, also the numbers $B0^{6540}BBA$ and $80^{32017}111$ are only probable primes, i.e. not definitely primes)

14, 16, 1A, 23, 25, 2B, 32, 34, 38, 41, 47, 49, 52, 56, 58, 61, 65, 6B, 76, 7A, 7C, 83, 85, 89, 9A, A1, A7, A9, B6, B8, C1, C7, CB, 10C, 119, 11B, 122, 133, 155, 157, 173, 179, 17B, 188, 197, 1B1, 1B5, 1CC, 209, 212, 218, 229, 272, 274, 281, 287, 292, 296, 298, 29C, 2C9, 311, 313, 331, 33B, 355, 371, 373, 379, 397, 3A6, 3AA, 3B3, 3B9, 3BB, 3CA, 43C, 445, 44B, 45A, 463, 4A3, 4A5, 4B2, 4B4, 4BA, 50C, 511, 515, 533, 54A, 551, 559, 571, 575, 57B, 595, 599, 5B3, 5B9, 5CC, 607, 629, 63A, 643, 674, 704, 715, 724, 728, 731, 737, 739, 742, 751, 75B, 773, 775, 779, 782, 784, 791, 793, 797, 7B1, 812, 818, 874, 878, 8AB, 8B4, 902, 919, 922, 926, 92C, 937, 93B, 946, 95B, 962, 968, 971, 977, 979, 982, 98C, 9B3, 9B5, A03, A3C, A45, A4B, A54, AA3, AAB, B02, B0C, B11, B15, B17, B24, B33, B39, B42, B57, B59, B71, B93, B9B, BA4, BAA, BB1, BB9, BC2, BCC, C29, C43, C98, CA3, 1013, 1031, 1037, 105B, 1075, 10B7, 10BB, 1105, 1112, 1121, 1127, 113C, 1172, 1187, 1208, 1211, 1277, 12C8, 1307, 1309, 131C, 139C, 151C, 1721, 1727, 1787, 1901, 1909, 1912, 1918, 193C, 1981, 198B, 199C, 19B2, 19C3, 1B29, 1BB2, 1BBC, 1C28, 1C39, 2021, 2078, 2117, 2201, 2221, 2267, 2278, 2627, 2678, 2711, 2771, 2788, 3037, 3053, 306A, 3077, 3091, 309B, 30AC, 3305, 353C, 35AB, 35BA, 35BC, 3677, 3905, 390B, 39C5, 3A0C, 3AB5, 3B5C, 3C35, 3C59, 3C95, 403A, 40AB, 4333, 435B, 4403, 44C3, 4535, 4544, 454C, 45B5, 45BB, 480B, 4B35, 4B5B, 4C36, 5057, 5077, 509B, 50A4, 5107, 5305, 530B, 539C, 53AB, 53C9, 5444, 5455, 54C4, 5503, 5545, 55AB, 5774, 5794, 590B, 594B, 5974, 59B4, 5A4C, 5A53, 5AA4, 5AB5, 5ABB, 5ACA, 5B4B, 5B5A, 5BA5, 5CA4, 6227, 6278, 6667, 6698, 6733, 6872, 6928, 6944, 694C, 6973, 6986, 6997, 69C8, 6AC3, 6C92, 6C94, 7019, 7057, 70B5, 7103, 710B, 7118, 7127, 7129, 7172, 7178, 7192, 7211, 7217, 7219, 7271, 7303, 7408, 7433, 7444, 7505, 7507, 7574, 770B, 7774, 7778, 7787, 7871, 7877, 7888, 794B, 7994, 79B4, 7B43, 7B74, 7B94, 7BB2, 8027, 8072, 8081, 80BA, 8171, 8207, 821C, 848B, 8687, 8711, 8722, 87BB, 8867, 88B2, 88BA, 8B22, 8B2A, 8BAC, 9004, 9017, 9031, 9053, 9055, 9073, 9091, 90BB, 90C8, 9107, 9118, 913C, 9181, 91C3, 9284, 935C, 93C5, 9424, 9428, 9448, 9509, 959C, 96C4, 9703, 9743, 9745, 974B, 97B2, 9811, 981B, 987B, 98B1, 991C, 9967, 9998, 9B12, 9B74, 9B92, 9BBC, 9C55, 9C86, 9CC4, A0BA, A306, A436, A535, A5B5, A636, A6C3, A80B, AB04, AB22, AB35, AB3B, AB4C, AB55, ABAC, ABB5, AC36, ACA5, B044, B04A, B0B7, B129, B1B2, B219, B222, B291, B299, B2CA, B35A, B3A5, B404, B44C, B45B, B4B3, B501, B51C, B55A, B5A5, B5AB, B5C3, B707, B792, B794, B905, B912, B9C5, BA5B, BAB3, BB03, BB45, BB72, BBA5, BBB2, BC44, BC53, BC95, BC99, C30A, C36A, C395, C454, C535, C553, C593, C944, C953, C964, CC94, 10015, 10051, 10099, 10118, 10291, 10712, 10772, 10811, 10877, 10921, 10B92, 11111, 11135, 11171, 111C8, 11531, 11C03, 13001, 13177, 13777, 13915, 13951, 13991, 159BB, 17018, 17102, 17111, 17117, 17171, 17177, 17708, 17711, 17801, 18071, 18101, 18271, 18B27, 19003, 19153, 19315, 19351, 19591, 19913, 19951, 1C099, 20171, 20177, 20207, 20227, 20777, 21011, 21077, 2111C, 21707, 22207, 30017, 300B5, 301C9, 3033A, 303A3, 303C5, 3050B, 305C9, 3095C, 30B05, 31007, 3159B, 31999, 31C09, 3330A, 33353, 33593, 33757, 33C5C, 33CC5, 35003, 3591B, 39353, 39539, 39935, 39995, 3ACCC, 3C5C3, 3CC53, 40043, 40306, 405C4, 408BC, 40BBB, 40C54, 43066, 4366A, 4443A, 45055, 45505, 45554, 4555C, 455BC, 455C3, 45C04, 488BC, 4B03B, 4B0B5, 4B55C, 4BB0B, 4C003, 4C054, 4C5C4, 50053, 500B1, 5035A, 504B5, 5053A, 50554, 505B4, 50A35, 50B07, 50BBA, 5139B, 519BB, 51BB7, 535AC, 53A5C, 53AC5, 53BAC, 54004, 54035, 5403B, 545C3, 54B05, 54B5C, 54BBC, 54C53, 55357, 5535B, 553AC, 554BC, 55537, 55544, 5554C, 55577, 555A4, 555BB, 55A5C, 55B04, 55B55, 55B77, 55BB5, 55BC4, 55C54, 55C5A, 57403, 591BB, 59443, 59BB7, 5A044, 5AC04, 5AC35, 5B001, 5B007, 5B0AB, 5B0B4, 5B4C5, 5B544, 5B555, 5B5BB, 5B744, 5B777, 5BA0B, 5BB44, 5BB55, 5BBC4, 5BC54, 5C039, 5C35A, 5C53A, 60098, 60964, 60988, 60A63, 66094, 66377, 66692, 66694, 669C2, 669C4, 66A36, 67022, 67099, 67222, 67277, 67772, 68627, 69088, 690C4, 69808, 69994, 6A663, 7007B, 70181, 70222, 70277, 70772, 70808, 70B0B, 70B29, 71113, 71711, 71908, 71999, 7199B, 71BB7, 71BBB, 74035, 74305, 7430B, 74503, 75443, 75454, 75535, 77072, 77108, 77177, 77717, 77BBB, 78011, 79BBB, 7B007, 7B7B7, 7B7BB, 7BBB3, 7BBB7, 80117, 80221, 80771, 80777, 80807, 8084B, 80B7B, 80BBB, 81107, 8400B, 86267, 87107, 87277, 87727, 87B27, 88111, 88201, 88702, 88771, 8888B, 88B77, 88BBB, 8B1BC, 8B727, 90035, 90059, 90088, 90095, 9009B, 90101, 90103, 90305, 90488, 904CC, 90574, 90644, 9064C, 90806, 908B7, 9090B, 90994, 90B09, 90C35, 90C59, 90C64, 91111, 91135, 91315, 9180B, 92008, 92408, 92488, 93359, 93395, 944C2, 944CC, 94505, 9455C, 94804, 94888, 94C0C, 94C33, 94C4C, 95045, 95504, 95573, 955C4, 95C54, 96044, 97BBB, 98066, 98408, 98444, 98804, 98848, 99001, 99005, 9900B, 99074, 990BC, 99113, 99175, 99278, 99335, 99454, 994C3, 99517, 99593, 9984B, 99881, 99904, 99917, 99935, 99955, 99973, 999BB, 999C2, 999C4, 99B99, 9B00B, 9B04B, 9B0B4, 9B1BB, 9BB04, 9C059, 9C244, 9C404, 9C44C, 9C488, 9C503, 9C5C9, 9C644, 9C664, 9CC88, 9CCC2, A00B4, A05BB, A08B2, A08BC, A0BC4, A3336, A3633, A443A, A4443, A50BB, A55C5, A5AAC, A5BBA, A5C53, A5C55, AACC5, AB05B, AB0BB, AB40A, ABBBC, ABC4A, ACC5A, ACCC3, B0053, B0075, B010B, B0455, B0743, B0774, B0909, B0BB4, B2277, B2A2C, B3005, B351B, B37B5, B3A0B, B3ABC, B3B0A, B400A, B4035, B403B, B4053, B4305, B4BC5, B4C0A, B504B, B50BA, B530A, B5454, B54BC, B54C5, B5544, B55B5, B5B44, B5B4C, B5BB5, B7403, B7535, B77BB, B7955, B7B7B, B9207, B9504, B9999, BA055, BA305, BABC5, BAC35, BB054, BB05A, BB207, BB3B5, BB4C3, BB504, BB544, BB54C, BB5B5, BB753, BB7B7, BBABC, BBB04, BBB4C, BBB55, BBBAC, BC035, BC455, C0353, C0359, C03AC, C0904, C0959, C0A5A, C0CC5, C3059, C335C, C5A0A, C5A44, C5AAC, C6692, C69C2, C904C, C9305, C9905, C995C, C99C5, C9C04, C9C59, C9CC2, CA50A, CA5AC, CAA05, CAA5A, CC335, CC544, CC5AA, CC935, CC955, 100039, 100178, 100718, 100903, 101177, 101708, 101711, 101777, 102017, 102071, 103999, 107081, 107777, 108217, 109111, 109151, 110078, 110108, 110717, 111017, 111103, 1111C3, 111301, 111707, 113501, 115103, 117017, 117107, 117181, 117701, 120701, 13C999, 159103, 170717, 177002, 177707, 180002, 187001, 18C002, 19111C, 199903, 1B0007, 1BB077, 1BBB07, 1C0903, 1C8002, 1C9993, 200027, 207107, 217777, 219991, 220027, 222227, 270008, 271007, 277777, 290444, 300059, 300509, 303359, 303995, 309959, 30B50A, 3336AC, 333707, 33395C, 335707, 3360A3, 350009, 36660A, 3666AC, 370007, 377B07, 39001C, 399503, 3BC005, 400366, 400555, 400B3B, 400B53, 400BB5, 400CC3, 4030B5, 40B053, 40B30B, 40B505, 43600A, 450004, 4A088B, 4B0503, 4B5C05, 4BBBB5, 4BC505, 500039, 50045B, 50405B, 504B0B, 50555B, 5055B5, 505B0A, 509003, 50A50B, 50B045, 50B054, 539B01, 550054, 5500BA, 55040B, 553BC5, 5553C5, 55550B, 5555C3, 555C04, 55B00A, 55BB0B, 570007, 5A500B, 5A555B, 5AC505, 5B055B, 5B0B5B, 5B5B5C, 5B5BC5, 5BB05B, 5BBB0B, 5BBB54, 5BBBB4, 5BBC0A, 5BC405, 5C5A5A, 5CA5A5, 600694, 6060A3, 609992, 637777, 6606A3, 6660A3, 667727, 667808, 668777, 669664, 670088, 679988, 696064, 69C064, 6A6333, 700727, 700811, 700909, 70098B, 700B92, 701117, 701171, 701717, 707027, 707111, 707171, 707201, 707801, 70788B, 7080BB, 708101, 70881B, 70887B, 70B227, 710012, 710177, 711002, 711017, 711071, 717707, 718001, 718111, 720077, 722002, 727777, 74BB3B, 74BB53, 770102, 770171, 770801, 777112, 777202, 777727, 777772, 778801, 77B772, 780008, 78087B, 781001, 788B07, 79088B, 794555, 7B000B, 7B0535, 7B077B, 7B2777, 7B4BBB, 7BB4BB, 800021, 800717, 801077, 80BB07, 811117, 870077, 8777B7, 877B77, 880177, 88071B, 88077B, 8808BC, 887017, 88707B, 888227, 88877B, 8887B7, 888821, 888827, 888BB7, 8B001B, 8B00BB, 8BBB77, 8BBBB7, 900097, 900BC9, 901115, 903935, 904033, 90440C, 908008, 908866, 909359, 909C05, 90B944, 90C95C, 90CC95, 91008B, 91115C, 911503, 920888, 930335, 933503, 935903, 940033, 94040C, 940808, 94CCCC, 950005, 950744, 95555C, 9555C5, 95C003, 95C005, 96400C, 96440C, 96664C, 966664, 966994, 969094, 969964, 97008B, 97080B, 975554, 97800B, 97880B, 980006, 980864, 980B07, 984884, 986006, 986606, 986644, 988006, 988088, 988664, 988817, 988886, 988B0B, 98B007, 990115, 990151, 990694, 990B44, 990C5C, 991501, 993059, 99408B, 994555, 995404, 995435, 996694, 9978BB, 998087, 999097, 999103, 99944C, 999503, 9995C3, 999754, 999901, 99990B, 999B09, 99B4C4, 99C0C5, 99C539, 99CC05, 9B9444, 9B9909, 9C0484, 9C0808, 9C2888, 9C400C, 9C4CCC, 9C6994, 9C90C5, 9C9C5C, 9CC008, 9CC5C3, 9CC905, 9CCC08, A0055B, A005AC, A0088B, A00B2C, A00BBB, A0555C, A05CAA, A0A5AC, A0A5CA, A0AC05, A0AC5A, A0B50B, A0BB0B, A0BBB4, A0C5AC, A3660A, A5050B, A555AC, A5B00B, AA0C05, AAA05C, AAA0C5, AAC05C, AB4444, ABB00B, AC050A, AC333A, B0001B, B00099, B0030B, B004B5, B00A35, B00B54, B030BA, B05043, B0555B, B05B0A, B05B5B, B07B53, B09074, B09755, B09975, B09995, B0AB0B, B0B04B, B0B535, B0BB53, B4C055, B50003, B5003A, B500A3, B50504, B50B04, B53BC5, B54BBB, B550BB, B555BC, B55C55, B5B004, B5B0BB, B5B50B, B5B554, B5B55C, B5B5B4, B5BBB4, B5BBBC, B5BC0A, B5C045, B5C054, B70995, B70B3B, B74555, B74B55, B99921, B99945, BAC505, BB0555, BB077B, BB0B5B, BB0BB5, BB500A, BB53BC, BB53C5, BB5505, BB55BC, BB5BBA, BB5C0A, BB7BB4, BBB00A, BBB74B, BBBB54, BBBBAB, BC5054, BC5504, C00094, C00694, C009C4, C00C05, C03035, C050AA, C05309, C05404, C0544C, C05AC4, C05C39, C06092, C06694, C09035, C094CC, C09992, C09994, C09C4C, C09C95, C0CC3A, C0CC92, C33539, C35009, C4C555, C50309, C50AAA, C53009, C550A5, C555CA, C55A5A, C55CA5, C5AC55, C60094, C60694, C93335, C95405, C99094, CA05CA, CA0AC5, CA555C, CAC5CA, CC05A4, CC0AA5, CC0C05, CC3509, CC4555, CC5039, CC5554, CC555A, CC6092, CCC0C5, CCC353, CCC959, CCC9C2, 1000271, 1000802, 1000871, 1001771, 1001801, 1007078, 1008002, 1008107, 1008701, 1010117, 1027001, 1070771, 1077107, 1077701, 1080107, 1101077, 1110008, 1111078, 1115003, 1117777, 1170008, 1170101, 1700078, 1700777, 1800017, 1877017, 18B7772, 18BBB0B, 1999391, 1999931, 1BBBB3B, 2011001, 2107001, 2110001, 2700017, 2700707, 300000A, 3000019, 3000A33, 3003335, 3003395, 3009335, 300A05B, 3010009, 30A3333, 3335C09, 3339359, 3353777, 336A333, 3393959, 33AC333, 3537007, 3577777, 3636337, 3757777, 395C903, 3AC3333, 40003B5, 400B0B3, 400BBC3, 403B005, 405050B, 40B5555, 40BB555, 40CC555, 4436606, 4444306, 45C5555, 4BC5555, 4C55555, 4CC5004, 4CCC0C3, 500001B, 50003A5, 50005BA, 500B55B, 501000B, 505004B, 505B05B, 50B50B5, 50B550B, 50BB004, 5300009, 5400B0B, 54B000B, 5500BBB, 550B05B, 553000A, 5537777, 555054B, 55505BA, 5550B74, 5555054, 5555BAC, 5555C05, 555B005, 555C00A, 555CA55, 55AC005, 55AC555, 55B005B, 55CA0A5, 5A00004, 5AA5C05, 5B05B05, 5B50B05, 5B5C004, 5BBBBB5, 5BBBBCA, 5C00093, 5C003A5, 5C00A0A, 5C0A055, 5C505AA, 5C5555A, 6000692, 600A333, 606A333, 6363337, 6720002, 6906664, 7000112, 7000712, 7001201, 7001777, 7005553, 70088B7, 7009555, 7010771, 7070881, 7088107, 709800B, 70B9992, 7100021, 7100081, 7100087, 7101107, 7110101, 7120001, 7170077, 7200202, 7270007, 74BBB05, 7700027, 7700201, 7700221, 7700881, 7701017, 7701101, 7707101, 7707701, 7711001, 7770101, 7771201, 7777001, 7777021, 7777102, 77777B7, 777B207, 777B777, 7780001, 77881BB, 788001B, 798000B, 7B00955, 7B00995, 7B55553, 7B55555, 7B77722, 7BB777B, 7BBB40B, 800000B, 8000BB7, 8001B0B, 8010011, 8010101, 8020111, 80B100B, 81B000B, 8677777, 8770001, 8777071, 8801B07, 88040BC, 8822177, 8880007, 8882777, 8887772, 8888087, 8888801, 888B07B, 888B10B, 8B0B00B, 8B777B2, 8BB000B, 9000008, 9000013, 9001151, 9086666, 9088864, 9094003, 9097808, 9099905, 90B99C9, 9151003, 9170008, 91BBBB7, 9244444, 9290111, 940C444, 9430003, 944404C, 94444C4, 944C044, 944C444, 9555005, 9555557, 9644404, 964444C, 96640CC, 9800008, 98800B7, 98884BB, 9888844, 9888884, 98BBB0B, 990888B, 9909C95, 990C94C, 9939953, 9944443, 9955555, 9988807, 998BB07, 99905C9, 9990C95, 9991115, 9994033, 9996644, 9997B44, 999B201, 999CC95, 99CCC5C, 9B20001, 9BBBB44, 9C03335, 9C04444, 9C08888, 9C640CC, 9C80008, 9C99994, 9CC9959, A00AA5C, A00AAC5, A00C50A, A00C555, A00C5AA, A05C00A, A0C005A, A0C0555, A0C555A, A30000A, A33500A, A55553A, A55555C, A5C00AA, A5CAAAA, A8BBB0A, AA00AC5, AA00C5A, AA05C0A, AA5CAAA, AAAC5AA, AAC0555, AC005AA, AC0555A, AC5000A, AC5505A, AC5550A, AC66663, ACC0555, B00007B, B0003AB, B000435, B0004BB, B000A3B, B000B5A, B000BA3, B003777, B005054, B005504, B0055BB, B00777B, B007B3B, B00A0BB, B00AB05, B00B0BA, B00B555, B00B55B, B00BB5B, B00BBB3, B040B0B, B04B00B, B050054, B0500B4, B0554BB, B05B055, B070005, B073B05, B0B00AB, B0B0A0B, B0B50BB, B0B550B, B0B554B, B0BABBB, B0BB305, B1BBB3B, B30000B, B377B77, B400B0B, B4C5005, B5000B4, B5003B5, B505505, B550004, B550055, B555555, B555C05, B5B005B, B5C5505, B70000B, B7B300B, B7BB777, B7BBBBB, B920001, B99545C, B99954C, B999744, BA000BB, BABBB0B, BB000AB, BB0055B, BB05B0B, BB074BB, BB0BABB, BB4000B, BB4430A, BB500BB, BB540BB, BB5555B, BB5BBBB, BB74B0B, BB77B44, BB7B40B, BBB005B, BBB0077, BBB00B5, BBB3007, BBB4444, BBB4B0B, BBB500B, BBB7B3B, BBB7BB5, BBBAB0B, BBBB375, BBBB3B7, BBBBB7B, BBBC40A, BC05045, C000092, C0000C5, C0005A4, C000C5C, C005AAA, C009095, C00940C, C00955C, C00C5A4, C050039, C0505A5, C050A55, C055555, C05AA55, C05C044, C05C554, C05CAAA, C0C5A04, C300035, C33333A, C3333C5, C550555, C55500A, C555505, C555A55, C5A0055, C5A0505, C5C0044, C995554, C999992, C9C0C95, C9C40CC, C9C9995, C9CCC35, CA05055, CA055A5, CA0A555, CA50505, CAAC555, CC00005, CC00995, CC00C3A, CC00C5C, CC5A004, CC5A505, CC69992, CCA0C5A, CCA5A55, CCAC555, CCC005C, CCC0539, CCC5309, CCC5A55, CCC5C39, CCC9095, CCCAAC5, CCCC692, CCCCC3A, 10001081, 10002107, 10007717, 10107781, 10210007, 10500001, 11000177, 11000771, 11117008, 12000071, 12700001, 18001007, 18010007, 1C000082, 20007017, 27070007, 30003935, 30333935, 40000036, 40000553, 4000503B, 4050003B, 40BC0055, 40CCCCC3, 44300006, 44366666, 4B0000B3, 4B050005, 4CC0C555, 4CCCC555, 4CCCCC03, 50000035, 50000A5B, 50005BBB, 5000B454, 5000BBB5, 50050BBB, 500B0BB5, 500BB0B5, 50B0BB05, 5350000A, 5400005B, 5500B50B, 5505005B, 5550005B, 55555004, 55555B05, 55555B07, 55555B5C, 555A350A, 555C0505, 55B000BB, 55B0500B, 55C00A05,

55C50505, 5A00005B, 5AAA5AC5, 5B005004, 5B0B00BB, 5B5000B5, 5BB00B05, 5BB5000B, 5BBB0005, 5BBBC005, 5BC00045, 5C0050A5, 5C050555, 5C05500A, 5C055505, 5C0A000A, 5C0AAAAA, 5C5000A5, 5C5A0555, 5CA05005, 5CA0A00A, 5CAA000A, 5CAAA0AA, 60000092, 600066A3, 60009C04, 66666A63, 67999009, 7000001B, 70001087, 70007771, 70010102, 70011101, 70017071, 70070021, 70077701, 7008BBBB, 70177777, 701B7777, 70700021, 70707071, 70710002, 70801007, 7090008B, 70955555, 71007071, 71110007, 71170001, 71770001, 74BB5555, 75555554, 77000021, 77771011, 77777071, 77777101, 77777701, 7900800B, 7BBBBB4B, 800004BC, 80000887, 8008080B, 80088887, 80170007, 80211001, 80700017, 8080080B, 87700007, 8777771B, 8800001C, 88000087, 8808000B, 88100077, 88222777, 88271777, 8870001B, 888001B7, 8880B01B, 88881017, 88881707, 8888881C, 9000018B, 90000866, 904C4444, 90888808, 90900007, 90999959, 90999C5C, 90C44444, 90C9CCC5, 91BBBB0B, 92999111, 9440000C, 95555543, 95555554, 96664444, 99094433, 99099959, 9918BBB7, 99964444, 999664CC, 99990995, 99999121, 99999433, 9999953C, 99999644, 99999943, 99999B21, 99999BC9, 99C0940C, 99C9994C, 9BBBBBB4, 9BC00009, 9C888808, 9CCC095C, 9CCCCC95, A000B5BC, A0CC5055, A0CC5505, A0CCC555, A350000B, A5C0A00A, A5CA000A, AA000A5C, AA5C000A, AC666333, ACC55555, B000003B, B0000095, B0000974, B0000ABB, B0000BAB, B000540B, B0050B55, B0055055, B005BB0B, B0099545, B00A5555, B00BBABB, B00BBB05, B00BBB5A, B00BBBBA, B04BBB05, B0500555, B0505B0B, B0555054, B05B5005, B07B40BB, B07BBBB5, B0B005BB, B0B00B0A, B0B55BBB, B0B5B00B, B2900007, B40000B5, B4000505, B4BBB005, B5000B55, B505BBBB, B50BB00B, B550005B, B550050B, B5505005, B555543B, B74B00BB, B777B277, B7B400BB, B7BB400B, BA0B0005, BAB0000B, BB007B4B, BB05005B, BB3B0007, BB755554, BBA0000B, BBBB4443, BBBB7B05, BBBBB075, BBBBB50B, BBBBB53A, BC005405, BC055554, BC540005, C000333A, C00033C5, C000A0A5, C000AAA5, C0040555, C00455C5, C0045C55, C005055A, C0055504, C00AAA5C, C00AC555, C00C55A5, C00C5A55, C00CA555, C055050A, C05CA505, C094000C, C0A00A5C, C0A50055, C0C0005C, C0C00692, C0C0333A, C505050A, C555055A, C5555504, C55C5055, C5A55055, CA000A5C, CA0C5505, CA555AAA, CA5AAA55, CACC5505, CC0009C2, CC055AC5, CC5005A5, CC555055, CC5A5555, CC5C0A55, CCA50055, CCC0003A, CCC0A555, CCC55AC5, CCC5C5A5, CCCA0555, CCCAC05A, CCCC0995, CCCC35C9, CCCCC05C, 100000082, 100008017, 100077071, 101070071, 101700071, 108100007, 110007101, 110080001, 110700071, 118000001, 120001007, 170007071, 170070701, 177010007, 177070001, 181770007, 200001101, 300000035, 30000005B, 30333335C, 333333067, 3333336A3, 333333995, 33333AC33, 3366666A3, 377777777, 4000000C3, 40000BC55, 40005C053, 4000BC055, 40B000055, 444444443, 4B0000505, 4BBC00005, 500000B54, 5000400BB, 50005B50B, 5000B4005, 500B0B0BB, 500B40005, 500BB00BB, 50540000B, 505B0BBBB, 50B0005B5, 50B00B0B5, 50B500004, 50BB0B005, 50BB0BBBB, 5400000B5, 5400000BB, 55000055B, 5500005B5, 5500050B5, 555555553, 555555A3A, 5555A300A, 5555AAAC5, 555CAAAAA, 55AAAAAC5, 55C000555, 55C550005, 59C000003, 5B0000504, 5B000B0B5, 5C0000AAA, 5C000550A, 5C0055A55, 5C00A5555, 5C050A005, 5C500005A, 5C500050A, 5C500500A, 5C5050505, 5C5055005, 5C50A5505, 5C5550005, 5C55AAAAA, 5C9000003, 5CA000505, 5CA000A55, 5CA00AAAA, 5CA055AAA, 5CA0AAA0A, 5CA55AAAA, 5CAAAAA55, 6000000A3, 60A366666, 6A3666666, 7000000B7, 700001021, 700007221, 700077101, 700080107, 700700012, 700B77777, 701001101, 701007077, 701700701, 702000002, 707100017, 707200007, 707710001, 710000117, 710100011, 711100777, 717100007, 74B000003, 770001011, 770077771, 770200001, 770700071, 771000011, 771000107, 771070001, 777000701, 777070771, 788001007, 799090999, 799099909, 799909099, 7B0000005, 80000877B, 800011001, 800110001, 870007001, 877700002, 8800700B7, 881070007, 887000B07, 8880001BB, 909990007, 90999995C, 909CCCCC5, 911500001, 928888888, 940444444, 944444044, 955500007, 988666666, 990944444, 99909995C, 9990999C5, 999929444, 99999095C, 99999640C, 99999664C, 99999994C, 999999B44, 99B290007, 99C909995, 99CC99995, 9BBBBB40B, 9CC999995, 9CCC9CCC5, 9CCCCC53C, A000005CA, A00000A5C, A00000C5A, A0A00005C, A0C500055, A0C500505, A3333335A, A8BBBBBBB, AAAC55555, AAC555AAA, AC000005A, AC0005505, AC0055005, AC0550005, AC05AAAAA, AC5005555, AC5500005, AC5550055, ACCC50505, ACCCCC555, B000000B3, B000005BA, B0000A0B5, B0000BB3B, B00050BBB, B00500405, B00555005, B00B3000A, B0400B005, B0540000B, B09555554, B0AB00005, B0BBB05BB, B0BBBBBA3, B40050005, B44444444, B45000005, B50000BBB, B500BBBBB, B505B000B, B50B00055, B50B00505, B5BB0000B, B5BC00505, B5C400005, B90000009, BA000B005, BB00000BA, BB00050BB, BB7BBB005, BBBBBB443, BBBBBBBB3, BC0000545, BC5000045, C0000003A, C000005AA, C00050044, C000555AA, C000555C4, C00055AC5, C0005A505, C0005C55A, C000A5505, C0550005A, C0555AAAA, C055A0005, C055AAAA5, C05C0055A, C05C0505A, C0A005505, C0A0AAAA5, C0AAAA555, C0AC00555, C50000404, C5000550A, C550005AA, C555555C5, C55555AAA, C55C55555, C5A500005, C5A5555AA, C5A5AA555, C5A5AAAA5, C5AAAA55, C5C505004, CACCC5055, CC0005A55, CC000A555, CC005A055, CC00A0555, CC00A5505, CC00C0692, CC0A55005, CCA550005, CCAAAA555, CCC00333A, CCC55C555, CCCC0C092, CCCC333AC, CCCC9CC95, CCCCC5AC4, CCCCCA5CA, 1000000091, 1010008001, ..., 4000000054, ..., 400000008B, ..., 7100000009, ..., 7900000009, ..., 8777777777, ..., 9000000011, ..., 900000011C, ..., 9044444444, ..., 9088888888, ..., AB0000000A, ..., C99940000C, ..., C999400CCC, ..., 21700000001, ..., 5BBBBBBBBBB, ..., 9444444400C, ..., 98888888888, ..., 99999969664, ..., CCAAAAAAAA5, ..., CCCCCCCCCC5, ..., 100000000028, ..., 1C9999999999, ..., 810000010001, ..., 94444444444C, ..., 94C444444444, ..., ABBBBBBBBBBB, ..., 1500000000001, ..., 1B0000000000B, ..., 777777777777B, ..., 8880000000001, ..., 8888888888881, ..., 9100000000001, ..., 9664444444444, ..., 9994444444444, ..., 9999940000CCC, ..., 9999999999694, ..., 9999999999911, ..., 88000000000001, ..., B0000000000077, ..., B540000000000B, ..., 310000000000009, ..., 90400000000000C, ..., 944444444444444, ..., 1100000000000801, ..., 5555555555555554, ..., CCCCCCCCCCCCCA5A, ..., 5555555555555555B, ..., 59000000000000003, ..., 9999940000000000C, ..., 430000000000000006, ..., 4A000000000000000B, ..., 700000000000000013, ..., 800000000000000111C, ..., 9400000000000000CCC, ..., C000000000000000544, ..., 60000000000000000094, ..., 67777777777777777777, ..., 87000000000000000002, ..., 9BBBBBBBBBBBBBBBBBBBB, ..., BBBBBBBBBBBBBBBBBBB44, ..., 7700000000000000000111, ..., 91000000000000000000003, ..., 181000000000000000000007, ..., 1100000000000000000000003, ..., 999400000000000000000000C, ..., 99999999999999999999999994, ..., 8000000000000000000000001011, ..., 9866666666666666666666666666, ..., 19999999999999999999999999999, ..., 7B77777777777777777777777777777, ..., 15000000000000000000000000000000B, ..., 27700000000000000000000000000000007, ..., B10000000000000000000000000000000000B, ..., C0AAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAA5, ..., 9400000000000000000000000000000000000C, ...,

9CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC53, ...,
88888888888888888888888888888888888888888888888888887, ...,
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB5, ...,
455555555555555555555555555555555555555555555555555555555, ...,
820000000000000000000000000000000000000000000000000000001, ...,
2000000000000000000000000000000000000000000000000000000000000000111, ...,
177777777777777777777777777777777777777777777777777777777777777777777777, ...,
72222222222222222222222222222222222222222222222222222222222222222222222222222, ...,
9000000000000000000000000000000000000000000000000000000000000000000000000000000000000000B9, ...,
C5AAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAA, ...,
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBA, ...,
C0000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000005, ...,
666666666666666666666666666666666666666666666666666666666666666666666666666666666666666666666666666666666666666666666666666666666666666A3, ...,
800000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000001101, ...,
3333333333333333333333333333333333333333333333333333333333333333333333333333333333333333333333333333333333333333333333333333333333333333333333333333333333333333333333333333335, ...,
4000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000003, ...,
5777, ...,
50044, ...,
9991, ...,
8BBB, ...,

9B000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000009, ...,
10000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000093, ...,
81100000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
1, ...,
B7777777777777777777777777777777777777777777777777777777777777777777777777777777
77777777777777777777777777777777777777777777777777777777777777777777777777777777
77777777777777777777777777777777777777777777777777777777777777777777777777777777
77777777777777777777777777777777777777777777777777777777777777777777777777777777
77777777777777777777777777777777777777777777777777777777777777777777777777777777
77777777777777777777777777777777777777777777777777777777777777777777777777777777
77777777777777777777777777777777777777777777777777777777, ...,
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBC, ...,
1BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBB, ...,
80000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000

000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000087, ...,
CC5555555555555555555555555555555555555555555555555555555555555555555555555555555
555555555555555555555555555555555555555555555555555555555555555555555555555555555
555555555555555555555555555555555555555555555555555555555555555555555555555555555
555555555555555555555555555555555555555555555555555555555555555555555555555555555
555555555555555555555555555555555555555555555555555555555555555555555555555555555
555555555555555555555555555555555555555555555555555555555555555555555555555555555
555555555555555555555555555555555555555555555555555555555555555555555555555555555
555555555555555555555555555555555555555555555555555555555555555555555555555555555
555555555555555555555555555555555555555555555555555555555555555555555555555555555
555555555555555555555555555555555555555555555555555555555555555, ...,
999999999999999999999999999999999999999999999999999999999999999999999999999999999
999999999999999999999999999999999999999999999999999999999999999999999999999999999
999999999999999999999999999999999999999999999999999999999999999999999999999999999
999999999999999999999999999999999999999999999999999999999999999999999999999999999
999999999999999999999999999999999999999999999999999999999999999999999999999999999
999999999999999999999999999999999999999999999999999999999999999999999999999999999
999999999999999999999999999999999999999999999999999999999999999999999999999999999
999999999999999999999999999999999999999999999999999999999999999999999999999999999
999999999999999999999999999999999999999999999999999999999999999999999999999999999
999999999999999999999999999999999999999999999999999999999999999999999999999999999
999999999999999999999999999999999999999999999999999999999999999999999999999999999
999999999999999999999999999999999999999999999999999999999999999999999999999999999
999999999999999999999999999999999999999999999999999999999999999999999999999999999
9999999999999999999B, ...,
100000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000

00000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000181, ...,
99999999999999999999999999999999999999999999999999999999999999999999999999999999
99999999999999999999999999999999999999999999999999999999999999999999999999999999
99999999999999999999999999999999999999999999999999999999999999999999999999999999
99999999999999999999999999999999999999999999999999999999999999999999999999999999
99999999999999999999999999999999999999999999999999999999999999999999999999999999
99999999999999999999999999999999999999999999999999999999999999999999999999999999
99999999999999999999999999999999999999999999999999999999999999999999999999999999
99999999999999999999999999999999999999999999999999999999999999999999999999999999
99999999999999999999999999999999999999999999999999999999999999999999999999999999
99999999999999999999999999999999999999999999999999999999999999999999999999999999
99999999999999999999999999999999999999999999999999999999999999999999999999999999
99999999999999999999999999999999999999999999999999999999999999999999999999999999
99999999999999999999999999999999999999999999999999999999999999999999999999999999
99999999999999999999999999999999999999999999999999999999999999999999999999999999
99999999999999999999999999999999999999999999999999999999999999999999999999999999
99999999999999999999999999999999999999999999999999999999999999999999999999999999
99999999999999999999999999999999999999999999999999999999999999999999999999999999
99999999999999999999999999999999999999999999999999999999999999999999999999999999
999999999999999999999999999999999999999999999999999995, ...,
77777777777777777777777777777777777777777777777777777777777777777777777777777777
77777777777777777777777777777777777777777777777777777777777777777777777777777777
77777777777777777777777777777777777777777777777777777777777777777777777777777777
77777777777777777777777777777777777777777777777777777777777777777777777777777777
77777777777777777777777777777777777777777777777777777777777777777777777777777777
77777777777777777777777777777777777777777777777777777777777777777777777777777777
77777777777777777777777777777777777777777777777777777777777777777777777777777777
77777777777777777777777777777777777777777777777777777777777777777777777777777777
77777777777777777777777777777777777777777777777777777777777777777777777777777777
77777777777777777777777777777777777777777777777777777777777777777777777777777777
77777777777777777777777777777777777777777777777777777777777777777777777777777777
77777777777777777777777777777777777777777777777777777777777777777777777777777777
77777777777777777777777777777777777777777777777777777777777777777777777777777777
77777777777777777777777777777777777777777777777777777777777777777777777777777777
77777777777777777777777777777777777777777777777777777777777777777777777777777777
77777777777777777777777777777777777777777777777777777777777777777777777777777777
77777777777777777777777777777777777777777777777777777777777777777777777777777777
77777777777777777777777777777777777777777777777777777777777777777777777777777777
77777777777777777777777777777777777777777777777777777777777777777777777777777777
77777777777777777777777777777777777777777777777777777777777777777777777777777777
7777777777777777777777777777777777777777777777771, ...,
93000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000

00000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000001, ...,
72000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000002, ...,

39000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000

00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000001, ...,
B0000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000

00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000

0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000BBA, ...,
2B30000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000

00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000

0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000

00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000

000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000

00000000000000001, ...,
8000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000

0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000

00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000

00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000

000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000

00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000

000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000000

00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000

00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000

0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000111, ...

base 14 (not proved, only checked to the prime 108000000D)

13, 15, 19, 21, 23, 29, 2D, 31, 35, 3B, 43, 45, 4B, 51, 53, 59, 5D, 65, 6D, 73, 75, 79, 7B, 81, 91, 95, 9B, 9D, A9, AB, B3, B9, BD, C5, CB, CD, D9, DB, 101, 111, 11D, 161, 17D, 1A1, 1AD, 1D1, 205, 22B, 255, 26B, 285, 2BB, 30D, 33D, 349, 389, 3D3, 40D, 41D, 44D, 469, 471, 499, 4AD, 4C1, 4D1, 50B, 525, 52B, 55B, 585, 58B, 60B, 61B, 683, 689, 6A3, 701, 71D, 741, 771, 77D, 7DD, 803, 80B, 825, 82B, 833, 839, 84D, 86B, 88D, 893, 8AD, 8BB, 8D3, 983, 9A3, A0D, A1D, A25, A41, A4D, AAD, AC1, AC3, AD1, B05, B41, B5B, B6B, B85, BA1, BB1, C49, C61, C83, C89, CC1, D01, D03, D33, D4D, D61, D71, D7D, D83, DA1, DA5, DC3, DD1, 10BB, 10DD, 128B, 18DD, 1B71, 1B8B, 1C41, 1D8D, 2BA5, 33A3, 347D, 3863, 3A7D, 40A1, 478D, 4809, 48C9, 48DD, 4C09, 4D8D, 56BB, 6049, 60C1, 6171, 61C1, 628B, 6409, 6461, 64A1, 6711, 6761, 67A1, 6A11, 6A71, 6B71, 6B8B, 708D, 748D, 7611, 780D, 7CA1, 8009, 8055, 807D, 8089, 80C9, 80DD, 837D, 8505, 88A3, 89C9, 8A05, 8A85, 8C63, 8C99, 8CC9, 9489, 94C9, 9869, 9899, A063, A071, A0A1, A0A3, A303, A603, A611, A633, A663, A83D, A883, A8A5, AA01, AD8D, B02B, B061, B08B, B10B, BC01, C0A3, C141, C171, C401, C441, CCA3, D005, D055, D08D, D18D, D1C1, D225, D80D, D885, DC11, 1062B, 11BBB, 1668B, 1B00B, 1BBBB, 1D00D, 1DD0D, 1DDDD, 2B225, 30083, 308A3, 33383, 338C3, 37A8D, 38883, 38AA3, 38DDD, 3A033, 3A8DD, 3AA83, 3AAA3, 3CA63, 40061, 400C9, 40601, 40641, 44141, 444C9, 44601, 44661, 44849, 44A01, 44AA1, 46061, 46411, 48489, 5B555, 5BA55, 5BBB5, 60A01, 60AA1, 64401, 66411, 66601, 66649, 6666B, 666B1, 66949, 66B11, 6BC11, 766C1, 7A661, 7AA11, 80649, 80669, 80699, 80885, 80949, 80AA5, 84409, 84849, 84889, 85A55, 86099, 86449, 86609, 86999, 86C09, 8700D, 884C9, 88805, 88809, 88899, 88B55, 89069, 89099, 89449, 89609, 89889, 89999, 8A5A5, 8AA55, 8AAA3, 8B555, 8BAA5, 8CAA3, 908C9, 90989, 94449, 98C09, 99089, 99409, 99949, A0085, A0A85, A7A11, A7A61, A8005, AA383, AA711, AA7A1, AA855, ADDD5, B011B, B07C1, B0C71, B11BB, B2225, B5555, B5AA5, B67C1, B76C1, B7C11, BB2B5, BB88B, BBB55, C04A1, C0A01, C0AA1, C3A03, D0ADD, D3DDD, DA8DD, DD38D, DDA63, DDD25, DDD55, DDDAD, 10006B, 11088B, 116B2B, 166B2B, 20008B, 300A33, 30A363, 3CA003, 400041, 400489, 401441, 404001, 404089, 404411, 404441, 404CC9, 406611, 40CCC9, 440001, 440409, 444041, 444611, 444641, 460011, 460041, 466401, 4A0001, 4A6AA1, 5BAAA5, 600411, 604041, 640011, 660441, 6666C1, 666A61, 6A0061, 6A0601, 6A6061, 6AAA61, 76A6A1, 8000A5, 85B5A5, 869669, 884049, 8885A5, 888669, 8886C3, 888BA5, 888C69, 889849, 896669, 898049, 900049, 900649, 908449, 940009, 969649, 988849, 990649, A08555, A33333, A3A333, A3A363, A6A6A1, A6AAA1, A88855, AAA085, AAA3A3, ADAAA3, ADD085, B0001B, B000C1, B00711, B2000B, B2AAA5, B60071, B66011, B66071, B666C1, B66C11, BA5A55, BAA5A5, BAAA55, C00A11, C00A71, C3A333, CA0333, CA3AA3, CAAA03, CAAA11, CAAAA1, D1000D, D3DA8D, DDAAA3, 100008B, 100020B, 3000A03, 3000CA3, 308CCC3, 38CCCC3, 4000011,

4000449, 4040449, 4400089, 4440009, 4440011, 4440449, 4440889, 4444441, 4664441, 4666AA1, 46AAAA1, 4A66A61, 4CCCCC9, 6000001, 6000141, 6000441, 6000A61, 60A6661, 6666441, 6666661, 66A0001, 66A0661, 6AA6661, 6AA6AA1, 6B60001, 6B66661, 8884449, 8888849, 88888C3, 888CCC3, 9008409, 9008849, 9088049, A000001, A000383, A006601, A600601, A660661, A766AA1, A7AAAA1, AA6AA61, AAA66A1, AAAA661, AAAAAA1, ADD8555, BBB2AA5, BBBB20B, CA00011, CAA3A33, D144441, DADDDDD, DDDD0D5, DDDD8DD, 1000002B, 1000800D, 1102000B, 1688888B, 30000A63, 40008849, 40400009, 444446A1, 46144441, 46666611, 4AA6A661, 60066141, 66614441, 666BBB2B, 6A600001, 80008005, 84444449, 866666C3, 90008889, 99999809, 999998C9, A8DD5555, AA6A6661, AAAAA003, AD555505, C0000411, CA000033, DADDDAA3, 10000080D, 11888888B, 300A00003, 3DDDDDD8D, 400000409, 400088889, 400444409, 440448889, 4AA666661, 600006661, 601444441, 606644441, 80000D805, 8D000000D, 8DD555555, 8DDDDD00D, A00066661, A88888885, AAAAAA805, AAAAAAA85, C00000711, CAAA33363, CAAAAA363, D00000DAD, DD8555555, DDDDDDD3D, 100000004D, 108000000D, ..., 8C66666669, ..., D555555555, ..., 300000000A3, ..., 40888888889, ..., 800000000085, ..., 800000000D85, ..., 808000000005, ..., 866666666C69, ..., 8666666666699, ..., 44444444444049, ..., 404444444444009, ..., 644444444444449, ..., 1000000000000000D, ..., 40444444444488889, ..., 9888888888888888C9, ..., 112000000000000000B, ..., 4000000000000000889, ..., 4044444444444444889, ..., D0000000000000000AD, ..., 66666666666666666B2B, ..., 99999999999998888889, ..., 4000000000000000000000849, ..., 44448888888888888888888889, ..., 99998888888888888888888889, ..., 4444444444444444444444444489, ..., 4444444444888888888888888889, ..., 9999999888888888888888888889, ..., 888888888888888888888888888889, ..., 4444444444444444444444444488888889, ..., 40444444444444444444444444444444409, ..., 8555555555555555555555555555555555555, ..., 99999999999999999999999999999999999989, ..., 3A000000000000000000000000000000000000003, ..., CA0000000000000000000000000000000000000000000000000003, ..., AAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAA3, ..., 44444444444444444444444444444444444444444444444444444444444444409, ..., 6BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBB2B, ...,
1888888888888888888888888888888888888888888888888888888888888888888888888
8888888B, ...,
8CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCC3, ...,
4000000000000000000000000000000000000000000000000000000000000000000000000
0000000000049, ...,
8888888888888888888888888888888888888888888888888888888888888888888888888
8888888888888B, ...,
34DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD

DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD, ...,
4DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD

DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD

DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD

DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD

DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD

DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD

DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD

DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD, ...

base 15 (not proved, only checked to the prime 555555557)

12, 14, 18, 1E, 21, 27, 2B, 2D, 32, 38, 3E, 41, 47, 4B, 4D, 54, 58, 5E, 67, 6B, 6D, 72, 74, 78, 87, 8B, 92, 94, 9E, A1, A7, AD, B2, B8, BE, C1, CB, CD, D2, D4, E1, ED, 111, 11B, 131, 137, 13B, 13D, 157, 15B, 15D, 171, 177, 197, 19D, 1B7, 1BB, 1D1, 1DB, 1DD, 234, 298, 311, 31B, 337, 33D, 344, 351, 357, 35B, 364, 377, 391, 39B, 39D, 3A4, 3BD, 3C4, 3D7, 3DB, 3DD, 452, 51B, 51D, 531, 53B, 551, 55D, 562, 571, 577, 5A2, 5B1, 5B7, 5BB, 5BD, 5C2, 5D1, 5D7, 634, 652, 681, 698, 717, 71B, 731, 737, 757, 75D, 77D, 79B, 79D, 7B1, 7B7, 7BD, 7D7, 7DD, 801, 852, 88D, 8D8, 91D, 93B, 93D, 95B, 95D, 971, 977, 97B, 97D, 988, 991, 9BD, 9C8, 9D1, A98, AAB, B1D, B31, B3B, B44, B51, B57, B7B, B7D, B97, B9B, BB7, BC4, BD1, BD7, BDD, C07, C34, C52, C7E, C98, CC7, CE7, D0E, D1D, D31, D51, D5B, D68, D77, D7B, D91, D97, DA8, DAE, DCE, DD1, EB4, EEB, 107B, 1091, 10B1, 1107, 110D, 1561, 1651, 1691, 1B01, 2052, 2502, 2522, 303B, 307D, 3097, 30BB, 30D1, 3107, 3361, 3701, 3907, 3B01, 3B0B, 3C97, 4434, 4498, 4834, 4898, 49A8, 4E34, 5037, 507D, 5091, 509B, 5107, 5161, 5202, 53C7, 5552, 570B, 590B, 590D, 59C7, 5A5B, 5C97, 5D0D, 5DAB, 6061, 6151, 6191, 6511, 6601, 6911, 707B, 7091, 7097, 70AE, 70BB, 70CE, 70DB, 7561, 760E, 7691, 76CE, 7907, 7961, 7A0E, 7A3B, 7AEE, 7B0B, 7BAB, 7C0E, 7C77, 7CAE, 7D0B, 7D61, 7DAB, 7E5B, 7E6E, 7E7B, 7EBB, 8098, 811D, 8191, 835D, 853D, 8881, 8908, 8951, 8968, 899D, 8D3D, 8D5D, 8D6E, 8DDD, 8E98, 9011, 9037, 9097, 90D7, 9301, 93C7, 95C7, 9611, 9631, 96A8, 9811, 9851, 989D, 990B, 990D, 998D, 99AB, 99C7, 99D8, 9A08, 9A9B, 9AA8, 9ABB, 9B61, 9BC7, 9D0B, 9DAB, 9DC7, 9DD8, A052, A304, A502, A55B, A9BB, AB04, AB64, B09D, B107, B10B, B161, B1AB, B1C7, B30D, B3C7, B50B, B664, B691, B6A4, B707, B761, B90D, B961, BA5B, BABB, BBAB, BBB4, BC37, BC77, C777, C937, C997, D011, D03D, D05D, D09B, D0B1, D0BD, D101, D10B, D30D,

D3AB, D507, D50D, D66E, D761, D7DE, D811, D85D, D86E, D89D, D8C8, D8E8, D9AB, D9D8, DA3B, DA9B, DABB, DB01, DB61, DBAB, DC88, DD07, DD0B, DD7E, DD8D, DDE7, DE6E, E252, E33B, E522, E57B, E7AE, E7CE, E898, E997, E9A8, E9BB, EA34, EB5B, EE98, EEC7, 10017, 10B0D, 170AB, 17A0B, 19001, 19601, 1A09B, 1D0C7, 22E52, 2EA52, 30017, 3001D, 300B1, 301C7, 30334, 30631, 307AB, 3300B, 3333B, 36031, 36301, 37A0B, 37BBB, 39997, 3A30B, 3B0C7, 3D001, 3D601, 40034, 40968, 43334, 49668, 49998, 50022, 5009D, 501C7, 50222, 50507, 505C7, 50611, 50C57, 53007, 53997, 55537, 5555B, 5557B, 5599B, 56101, 56691, 56961, 5700D, 5755B, 59001, 59557, 59997, 5999D, 599DB, 59DDD, 5D99B, 5DD3D, 5DD9D, 60931, 63031, 65691, 66951, 69031, 69361, 69561, 70011, 70051, 7005B, 7006E, 7030D, 703AB, 70501, 70701, 707C7, 71601, 71951, 7300D, 7333B, 75001, 7555B, 75911, 76011, 76051, 766EE, 76EEE, 7700B, 77191, 77661, 7776E, 77771, 777BB, 77911, 77BBB, 79001, 7A05B, 7A66E, 7AA6E, 7AAAE, 7ACCE, 7C6EE, 7CCEE, 7CECE, 7CEEE, 7D3BB, 7E7C7, 7EECE, 80034, 80304, 80434, 809DD, 80A34, 84A34, 850DD, 85961, 86661, 88151, 88331, 88511, 88591, 88898, 890DD, 89998, 89D0D, 8D90D, 8E434, 90017, 90051, 900A8, 900DB, 901C7, 90C57, 90D8D, 91007, 91061, 9199B, 95997, 96068, 96561, 99397, 99537, 9999B, 999B7, 999D7, 999DB, 999DD, 99BBB, 99DBB, 99DD7, 99DDD, 9B007, 9B00B, 9B0AB, 9BB11, 9BBBB, 9D007, 9D08D, 9D537, 9D9BB, 9D9DB, 9DD57, 9DDB7, 9DDDB, 9DDDD, A0A34, A0B5B, A0BBB, A0E34, A2E52, A330B, A8434, A8834, A8E34, A909B, AAA34, AAE52, AB0BB, AB334, ABB34, AE034, AE834, AE99B, AEA52, AEE52, B0011, B0071, B0077, B00B1, B0611, B0A64, B500D, B599D, B6101, B7771, B7911, BA064, BAAA4, BAB34, BB061, BB304, BB53D, BB601, BBB91, BBB9D, BBBBD, BDA0B, BDBBB, D0088, D00D7, D0307, D05C7, D070D, D0888, D0B07, D0BC7, D0C08, D0DC7, D0DD8, D1661, D59DD, D5D3D, D5DDD, D6611, D700D, D8D0D, D900B, D9908, D999D, D9BBB, D9D9D, D9DDB, DB007, DB00D, DB1B1, DB53D, DB59D, DB99D, DBBB1, DD0D8, DD33B, DD3B7, DD3BB, DD57D, DD898, DD9DD, DDB37, DDBDB, DDD08, DDD3D, DDD5D, DDD7D, DDD88, DDD9D, DDDB7, DDDC8, DDDD7, DDE98, DE037, DE998, DEB07, E0098, E00C7, E0537, E0557, E077B, E0834, E0968, E3334, E37AB, E39C7, E4034, E5307, E55AB, E705B, E750B, E766E, E76EE, E8304, E8434, E9608, E9C37, EAE52, EBB0B, EC557, EC597, EC957, 1000BD, 1009AB, 10A90B, 1900AB, 190661, 19099B, 190A0B, 1A900B, 222A52, 2AAA52, 31000D, 330331, 333334, 3733AB, 373ABB, 3BBB61, 430004, 490068, 490608, 5000DB, 500D0B, 505557, 505A0B, 50D00B, 50DDDB, 50DDDD, 522222, 5500AB, 5500C7, 550957, 550A0B, 555A9B, 559057, 560011, 590661, 633331, 666331, 666591, 666661, 7050AB, 705A0B, 706101, 70A50B, 7300AB, 761661, 76666E, 777011, 777101, 77750B, 777A5B, 777CEE, 779051, 791501, 7E7797, 7ECCCE, 7EEE97, 800D9D, 808834, 836631, 83D661, 843004, 856611, 884034, 884304, 888E34, 88A434, 88AE34, 8A4034, 8AEE34, 8E8034, 8E8E34, 8EEE34, 9000BB, 9001AB, 900B07, 900D98, 903661, 905661, 906651, 9080DD, 9099A8, 909D9B, 90A668, 90DD9B, 90DDBB, 910001, 9100AB, 91A00B, 930007, 950001, 956661, 9909A8, 995907, 999068, 999507, 999907, 9B0B1B, 9B0BB1, 9BB01B, 9C5597, 9C5957, 9D09DD, 9D0D9D, 9D800D, 9DB307, 9DD09D, A00034, A0033B, A033B4, A2A252, AAAA52, ABBBBB, B00004, B0001B, B0003D, B00A04, B0555B, B07191, B07711, B07777, B0B911, B0BDBB, B77011, B777C7, BB0001, BB0034, BB035D, BB055B, BB0BDB, BB9101, BBB0DB, BBB50D, BBBB01, BBD0BB, C55397, C55557, C55597, D0003B, D00057, D0007D, D000B7, D000C8, D008DD, D00DAB, D0333B, D05537, D099DD, D09DDD, D0DDBB, D555C7, D5C537, D88008, D88088, D888EE, D909DD, D9D0DD, D9DD0D, DB0BBB, DBBB0B, DBBB0D, DC0008, DC5537, DDDDD8, DDDEBB, DDE99B, DE0808, DE0C57, DE300B, DE5537, DE8888, DEE088, DEE307, DEE888, DEEE37, DEEE57, DEEEC8, E0000B, E007BB, E00A52, E03BC7, E07ABB, E09B07,

E0A99B, E0C397, E0E76E, E50057, E55007, E55597, E55937, E730AB, E73A0B, E80E34, E88834, E8E034, E90008, E95557, EA099B, EE4304, EE5057, EE5507, EE8E34, EE9307, EEE434, 100001D, 1000A9B, 1000DC7, 22AA252, 3000BC7, 3033301, 3076661, 333B304, 33B3034, 3B33304, 3D66661, 50007AB, 5005957, 5500597, 5550057, 5559007, 5559597, 5595007, 5966661, 5DDDDDB, 6366631, 7010001, 7066651, 7100061, 733BBBB, 766A6AE, 77505AB, 7776501, 777775B, 777AACE, 777ECCE, 777EEAE, 7CCCCCE, 7E30A0B, 7EEEEAE, 8300004, 8363331, 8693331, 880E834, 8833304, 8888034, 8888434, 888A034, 88A3334, 88E8834, 88EE034, 88EE304, 8AA3334, 8D0009D, 8EE8834, 9000361, 9000668, 9003331, 9005557, 9006008, 9008D0D, 9083331, 9090968, 90BBB01, 90D0908, 9500661, 9555597, 9555957, 9660008, 9900968, 9995597, 9996008, 9999557, 9999597, 9999908, 9A66668, A003B34, A003BB4, AA22252, B00B034, B00B35D, B033334, B0B6661, B0BB01B, B100001, B333304, B777777, B99999D, BA60004, BAA0334, BBB001B, BBB6611, BBBBB11, BBBD00B, BD000AB, D0000DB, D009098, D00CCC8, D00D908, D00D99D, D03000B, D0BB0BB, D0D9008, D0D9998, D1000C7, D800008, D8DDEEE, D90080D, DBBBBBB, DD09998, DDD5557, DDDDBBB, DDDDDBD, DDDE8EE, DECC008, DECCCC8, DEE0CC8, DEEC0C8, E000397, E0003BB, E000434, E00076E, E000937, E007A5B, E00909B, E0090B7, E009307, E00B077, E00E434, E00E797, E00E937, E05999B, E09009B, E0900B7, E0E0937, E0E7E97, E0EAA52, E0EEA52, E555057, E5555C7, E7777C7, E77E797, E88EE34, E999998, EA5999B, EB000BB, EB0BBBB, EE00434, EE0E797, EEE076E, EEE706E, EEE8834, EEEE557, EEEE797, 30333331, 30B66661, 33000034, 33030004, 33B33004, 500575AB, 55000007, 5500075B, 55500907, 55555057, 55555907, 55559507, 60003301, 60033001, 60330001, 7000003D, 70106661, 70666611, 77000001, 7777770B, 777777C7, 77777ACE, 77777EAE, 777E30AB, 777E3A0B, 7CCCC66E, 800005DD, 88AA0834, 90000008, 900008DD, 90099668, 90500557, 90555007, 90666668, 90909998, 90990998, 90996668, 9099999D, 90D00098, 90D90998, 95500057, 99099098, 99555057, 99900998, 99966608, 99966668, 99999668, 99999998, 9D009008, 9D090998, A0803334, A2222252, AAA52222, B00005AB, B000B55B, B0BBBB5B, B3330034, BB0BBB1B, BBAA3334, BBB0BB1B, BBB0BB5B, BBDB000B, D000BBBB, D00100C7, D8888888, D900008D, D9000098, DBB000BB, DC0CCCC8, DCC0CCC8, DCCCC008, DD000908, DD09009D, DDDDDDAB, DDDDDEEE, DDDEEE8E, DDDEEEE8, DEE80008, E0777E97, E0E0E397, E0E77797, E0EE0397, E7777797, E9066668, EE00E397, EE077797, EE0E0397, EEE00797, EEE07E97, EEE0AA52, EEE55397, EEE55557, EEEAAA52, EEEEE834, EEEEEA52, 300003331, 300007661, 300330031, 333000004, 333300001, 333B00034, 3700000AB, 3B3300034, 500000057, 555555007, 555555557, 5DDDDDDDD, 600000331, 7500000AB, 75000A00B, 75A00000B, 761000001, 77000E0C7, 777700EC7, 7777730AB, 7777777AE, 77777EE97, 7777E7E97, 777999997, 7A500000B, 7BBBBBB5B, 88888A834, 900000031, 900666608, 909990098, 90D009998, 950000557, 966666008, 990000007, 990555507, 999999997, A000000B4, A0005999B, AAEEEEE34, B000AA334, BBBBB005B, BBBBBBB5B, D09999998, D0D90009D, D800000DD, D90009998, DCCCC0CC8, DE88EEEEE, DEEEEEE88, E000B7777, E000BBBBB, E003ABBBB, EE0000797, EE0EEE397, EE5555557, EE777EE97, EEEEEE537, EEEEEE937, ..., 2222222252, ..., 3000000071, ..., 500000007B, ..., 7000000071, ..., 900000009B, ..., 900000009D, ..., 9000099998, ..., D300000007, ..., DDDDDDDDDE, ..., DDDEEEEEEE, ..., 9999999999D, ..., 5000000000DD, ..., 9000000000B7, ..., D000000001C7, ..., DCCCCCCCCCC8, ..., 3333333333331, ..., 3BBBBBBBBBBBB, ..., 71000000000001, ..., 77777777777777B, ..., DEEEEEEEEEEEEEE, ..., DDDDDDDDDDDDDDDDB, ..., E9666666666666668, ..., 3330000000000000031, ..., 7BBBBBBBBBBBBBBBBBBBB, ..., BBBBBBBBBBBBBBBBBBBBBB1, ..., B0000000000000000000005B, ..., B7000000000000000000000001, ..., 6330000000000000000000000001, ..., EBBBBBBBBBBBBBBBBBBBBBBBBBBBB, ..., 500000000000000000000000000000017, ..., 9666666666666666666666666666666666666666666666666666666666666666666666666666666666666666666608, ..., EEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEE397, ..., 77777777777777777777777777777777777777777777777777777777777777777777777777777777777777777777777777777777777777777777777777777777777777777777777777797, ...

base 16 (not proved, only checked to 1000000000, also the number DB^{32234} is only a probable prime, i.e. not definitely prime)

11, 13, 17, 1D, 1F, 25, 29, 2B, 2F, 35, 3B, 3D, 43, 47, 49, 4F, 53, 59, 61, 65, 67, 6B, 6D, 71, 7F, 83, 89, 8B, 95, 97, 9D, A3, A7, AD, B3, B5, BF, C1, C5, C7, D3, DF, E3, E5, E9, EF, F1, FB, 14B, 15B, 185, 199, 1A5, 1BB, 1C9, 1EB, 223, 22D, 233, 241, 277, 281, 287, 28D, 2A1, 2D7, 2DD, 2E7, 301, 337, 373, 377, 38F, 3A1, 3A9, 41B, 42D, 445, 455, 45D, 481, 4B1, 4BD, 4CD, 4D5, 4E1, 4EB, 50B, 515, 51B, 527, 551, 557, 55D, 577, 581, 58F, 5AB, 5CB, 5CF, 5D1, 5D5, 5DB, 5E7, 623, 709, 727, 737, 745, 74B, 755, 757, 773, 779, 78D, 7BB, 7C3, 7C9, 7CD, 7DB, 7EB, 7ED, 805, 80F, 815, 821, 827, 841, 851, 85D, 85F, 8A5, 8DD, 8E1, 8F5, 923, 98F, 99B, 9A9, 9EB, A21, A6F, A81, A85, A99, A9F, AA9, AAB, ACF, B1B, B2D, B7B, B8D, B99, B9B, BB7, BB9, BCB, BDD, BE1, C0B, CB9, CBB, CEB, D01, D21, D2D, D55, D69, D79, D81, D85, D87, D8D, DAB, DB7, DBD, DC9, DCD, DD5, DDB, DE7, E21, E27, E4B, E7D, E87, EB1, EB7, ED1, EDB, EED, F07, F0D, F4D, FD9, FFD, 1069, 1505, 1609, 1669, 16A9, 19AB, 1A69, 1AB9, 2027, 204D, 2063, 207D, 20C3, 20ED, 2221, 22E1, 2327, 244D, 26C3, 274D, 2E01, 2E0D, 2ECD, 3023, 3079, 3109, 3263, 3341, 36AF, 3941, 3991, 39AF, 3E41, 3E81, 3EE1, 3EE7, 3F79, 4021, 40DB, 440B, 444B, 44A1, 44AB, 44DB, 4541, 45BB, 4A41, 4B0B, 4BBB, 4C4B, 4D41, 4DED, 5045, 50A1, 50ED, 540D, 5441, 555B, 556F, 5585, 560F, 56FF, 5705, 574D, 580D, 582D, 5855, 588D, 5A01, 5AA1, 5B01, 5B4B, 5B87, 5BB1, 5BEB, 5C4D, 5CDD, 5CED, 5DD7, 5DDD, 5E0D, 5E2D, 5EBB, 68FF, 6A69, 6AC9, 6C8F, 6CA9, 6CAF, 6F8F, 6FAF, 7033, 7063, 7075, 7087, 70A5, 70AB, 7303, 7393, 74DD, 754D, 7603, 7633, 7663, 7669, 7705, 772D, 775D, 77D5, 7807, 7877, 7885, 7939, 7969, 7993, 79AB, 7A05, 7A69, 7A9B, 7AA5, 7B77, 7BA9, 7D4D, 7D75, 7D77, 8077, 808D, 80D7, 80E7, 8587, 86CF, 8777, 8785, 8885, 88CF, 88ED, 88FD, 8C6F, 8C8F, 8E8D, 8EE7, 8F2D, 8F8D, 9031, 9041, 90AF, 90B9, 9221, 9319, 9401, 944B, 9881, 9931, 9941, 9991, 99AF, 9A0F, 9A1B, 9A4B, 9AFF, 9BA1, 9BB1, 9CAF, 9E81, 9EA1, 9FAF, A001, A05B, A0C9, A105, A10B, A4CB, A55B, A6C9, A88F, A91B, A9B1, A9BB, AA15, AB01, AB0B, AB19, ABBB, AC09, AF09, B041, B04B, B069, B07D, B087, B0B1, B0ED, B1A9, B201, B40B, B40D, B609, B70D, B7A9, B807, B9A1, BA41, BAA1, BB4B, BBB1, BBDB, BBED, BD19, BD41, BDBB, BDEB, BE07, BEE7, C0D9, C203, C24D, C6A9, C88D, C88F, C8CF, C8ED, C9AF, C9CB, CA09, CA4B, CA69, CAC9, CC0D, CC23, CC4D, CC9B, CD09, CDD9, CE4D, CEDD, CFA9, CFCD, D04B, D099, D405, D415, D44B, D4A5, D4DD, D50D, D70B, D74D, D77B, D7CB, D91B, D991, DA05, DA09, DA15, DA51, DB91, DBEB, DD7D, DDA1, DDED, DE0B, DE41, DE4D, DEA1, E02D, E07B, E0D7, E1CB, E2CD, E401, E801, EABB, EACB, EAEB, EBAB, EC4D, ECDD, ED07, EDD7, EE7B, EE81, EEAB, EEE1,

F08F, F0A9, F227, F2ED, F3AF, F485, F58D, F72D, F763, F769, F787, F7A5, F7E7, F82D, F86F, F877, F88D, F8D7, F8E7, F8FF, FCCD, FED7, FF85, FF8F, FFA9, 100AB, 10BA9, 1A0CB, 1BA09, 200E1, 2C603, 2CC03, 30227, 303AF, 30AAF, 32003, 32207, 32CC3, 330AF, 33169, 33221, 33391, 33881, 33AFF, 38807, 38887, 3AFFF, 3F203, 3F887, 3FAFF, 400BB, 4084D, 40A4B, 42001, 44221, 44401, 444D1, 4480D, 4488D, 44CCB, 44D4D, 44E8D, 4804D, 4840D, 4A0CB, 4A54B, 4CACB, 4D0DD, 4D40D, 4D44D, 5004D, 50075, 502CD, 5044D, 50887, 50EE1, 5448D, 548ED, 55A45, 55F45, 5844D, 5A4A5, 5AE41, 5B0CD, 5B44D, 5BBCD, 5D4ED, 5E0E1, 5EB4D, 5EC8D, 5ECCD, 5EE41, 5F06F, 5F7DD, 5F885, 5F8CD, 5FC8D, 5FF75, 6088F, 60AFF, 630AF, 633AF, 660A9, 668CF, 669AF, 66A09, 66A0F, 66FA9, 6886F, 6A00F, 6A0FF, 6A8AF, 6AFFF, 7002D, 7024D, 70B0D, 70B7D, 7200D, 73363, 73999, 7444D, 770B7, 777D7, 77B07, 77D7D, 77DD7, 79003, 79999, 7B00D, 7D05D, 7D7DD, 8007D, 800D1, 8074D, 82CCD, 82E4D, 8448D, 8484D, 8704D, 8724D, 87887, 88001, 8800D, 880CD, 88507, 88555, 8866F, 8872D, 8877D, 888D1, 888D7, 88AA1, 88C2D, 88D57, 88D75, 88D77, 8AFAF, 8C2CD, 8C40D, 8C8CD, 8CCED, 8CE2D, 8CFED, 8E007, 8E20D, 8E24D, 8F6FF, 8FAAF, 900CB, 901AB, 90901, 909A1, 90AB1, 90AE1, 90EE1, 910AB, 93331, 940AB, 963AF, 966AF, 99019, 99109, 99A01, 9AAE1, 9B00B, 9B0AB, 9B441, 9BABB, 9BBBB, 9E441, A00BB, A0405, A044B, A08AF, A0A51, A0B91, A0C4B, A1B09, A54A5, A5B41, A6609, A904B, A94A1, A9C4B, A9E01, A9E41, AA0A1, AA441, AA501, AA8AF, AAEE1, AAF45, AAF8F, ABBA1, ACC69, AE0BB, AE0EB, AEAE1, AEE0B, AEEA1, AEECB, AF045, AF4A5, AFA8F, B00A1, B00D7, B044D, B0777, B0A0B, B0A91, B0BBD, B0BCD, B0C09, B0DA9, B0EAB, B2207, B4001, B6669, B7707, B7D07, B8081, B9021, BA091, BA109, BA4BB, BB001, BB0EB, BB8A1, BBBEB, BBE0B, BBEBB, BC009, BCECD, BD0A9, BE44D, BEB0D, BEBBB, BEEBB, C0263, C02C3, C02ED, C040D, C0CA9, C0CCD, C2663, C2CED, C32C3, C3323, C400D, C40ED, C44CB, C44ED, C480D, C484D, C4CAB, C60AF, C686F, C6A0F, C86FF, C8C2D, CAA0F, CAFAF, CBCED, CC0AF, CC44B, CC82D, CC8FF, CCAF9, CCAFF, CCCFD, CCFAF, CD00D, CD4CB, CD4ED, CDDDD, CF2C3, CFC8F, CFE8D, D0045, D07DD, D09BB, D0D4D, D0DD7, D0EBB, D0EEB, D1009, D1045, D10B9, D1BA9, D54BB, D54ED, D5AE1, D5D07, D5EE1, D70DD, D7707, D7777, D77DD, D7DD7, D9441, D9AE1, D9B0B, DA9A1, DA9E1, DAA41, DAAA1, DBB0B, DBBA1, DC4CB, DD227, DD44D, DDDD7, E0081, E00E1, E010B, E088D, E08CD, E0B0D, E0BBD, E100B, E4D0D, E777B, E77AB, E7CCB, E844D, E848D, E884D, E88A1, EB0BB, EBB4D, EBBEB, EBEEB, EC8CD, ECBCD, ECC8D, ED04D, EE001, EE0EB, EE4A1, EEEBB, F0085, F09AF, F0C23, F0CAF, F2663, F2C03, F3799, F3887, F4A05, F4AA5, F506F, F5845, F5885, F5C2D, F5ECD, F5F45, F66A9, F688F, F6AFF, F7399, F777D, F8545, F8555, F8AAF, F8F87, F9AAF, FA0F9, FA405, FA669, FAFF9, FC263, FCA0F, FCAFF, FCE8D, FCF23, FD777, FDDDD, FDEDD, FEC2D, FEC8D, FF545, FF6AF, FF739, FF775, FF9AF, FFC23, 100055, 100555, 10A9CB, 1A090B, 1A900B, 1CACCB, 1CCACB, 20DEE1, 266003, 3000AF, 300A0F, 300AFF, 308087, 308E07, 3323E1, 333A0F, 339331, 33CA0F, 33CF23, 33CFAF, 33F323, 380087, 3A00AF, 3A0F0F, 3AA0FF, 3AAF0F, 3C33AF, 3C3A0F, 3C3FAF, 3CCAAF, 3F0FAF, 3F32C3, 3FF0AF, 3FFAAF, 4004CB, 400A05, 4048ED, 404DDD, 40AA05, 40D04D, 40DD4D, 40E0DD, 40E48D, 440041, 44008D, 44044D, 4404DD, 44440D, 4448ED, 4484ED, 448E4D, 44E44D, 48888D, 4AA005, 4DD00D, 4DD04D, 4DDD0D, 4E048D, 4E448D, 4E880D, 5000DD, 500201, 50066F, 5008CD, 500C2D, 500D7D, 50C20D, 520C0D, 544EDD, 54AA05, 54AAA5, 54ED4D, 566AAF, 57D00D, 580087, 5A5545, 5C20CD, 5C8CCD, 5CC2CD, 5D000D, 5D070D, 5F666F, 5FAA45, 5FFF45, 60008F, 600A0F, 603AAF, 6060AF, 6066AF, 60A0AF, 63AA0F, 6663AF, 66668F, 666AAF, 668A8F, 66AFF9, 68888F, 693AAF, 7007B7, 70404D, 70770B, 70770D, 707BE7, 70DD0D, 733339, 733699, 74004D, 74040D, 77007B, 770CCB, 777B4D, 777BE7, 777CCB, 77ACCB, 77B74D, 77D0DD, 7A0CCB, 7B744D, 7CACCB, 7DDD99, 80044D, 800807, 80200D, 8044ED, 80C04D, 80CC2D, 80E44D, 8404ED, 84888D, 84E04D, 84E40D, 86686F, 8668AF, 8686AF, 86F66F, 86FFFF, 87000D, 87744D, 880807, 886AFF, 88824D, 88870D, 888787, 88884D, 88886F, 88887D, 88888D, 888C4D, 888FAF, 88AA8F, 88CC8D, 88F6AF, 88F8AF, 88FA8F, 88FF6F, 88FF87, 88FFAF, 8A8FFF, 8C0C2D, 8C802D, 8CCFFF, 8CE00D, 8CE0CD, 8CFCCF, 8E00CD, 8E044D, 8E0CCD, 8EC0CD, 8F68AF, 8F88F7, 8FCFCF, 8FF887, 8FFCCF, 8FFF6F, 9002E1, 9004AB, 9008A1, 900919, 900ABB, 900B21, 90B801, 90CCCB, 9332E1, 944441, 94ACCB, 990001, 9900A1, 9A4441, 9A4AA1, 9AA4A1, 9AAA41, 9AAAAF, 9B66C9, 9BBA0B, 9BC0C9, 9BC669, 9BC6C9, 9C4ACB, A0094B, A00ECB, A09441, A0A08F, A0E0CB, A0ECCB, A0F669, A40A05, A4AAA5, A50E41, A5AA45, A60069, A8FAFF, A9AA41, AA5E41, AAA4A5, AAA545, AC6669, ACCC4B, ACCCC9, AEAA41, AFF405, AFF669, AFFA45, AFFFF9, B00921, B00BEB, B00CC9, B00D91, B08801, B0D077, B70077, B70E77, B77E77, B88877, B88881, B94421, BAE00B, BB00AB, BB0DA1, BB444D, BB44D1, BB8881, BBBBBD, BBBC4D, BBCCCD, BC0CC9, BC66C9, BCC669, BCC6C9, BCCC09, BE000D, BE00BD, BE0B4D, BE0CCD, BEA00B, BECCCD, C0084D, C00A0F, C0608F, C0668F, C0844D, C0A0FF, C0AFF9, C0C3AF, C0C68F, C0CAAF, C0CDED, C0D0ED, C0E80D, C0EC2D, C0EC8D, C0FA0F, C0FAAF, C2CC63, C30CAF, C333AF, C3CAAF, C3CCAF, C4048D, C40D4D, C4404D, C4408D, C4440D, C44DDD, C4ACCB, C4DCCB, C4DD4D, C6068F, C66AAF, C68AAF, C6AA8F, C8044D, C8440D, C8666F, CA00FF, CA0FFF, CAAAAF, CAAFFF, CAFF0F, CBE0CD, CC008F, CC0C8F, CC3CAF, CC4ACB, CC608F, CC66AF, CCBECD, CCC4AB, CCCA0F, CCCC8F, CCCE8D, CE0C8D, CF0F23, CF0FAF, CFAFFF, CFCAAF, CFFAFF, D0005D, D00BA9, D05EDD, D077D7, D10CCB, D22207, D4000B, D4040D, D4044D, D40CCB, D70077, D7D00D, D90009, D900BB, DB00BB, DB4441, DD400D, DDD109, DDD1A9, DDD919, DDD941, DED00D, E00D4D, E00EEB, E0AAE1, E0AE41, E0AEA1, E0B44D, E0BCCD, E0BEBB, E0D0DD, E0E441, E4048D, E4448D, E800CD, E8200D, EA0E41, EAA0E1, EBB00B, ECCCAB, EDDDDD, EEBE0B, F00263, F0056F, F00A45, F02C63, F03F23, F05405, F060AF, F08585, F0A4A5, F0F2C3, F0F323, F2CCC3, F33203, F33C23, F5F66F, F5FF6F, F68CCF, F6AA8F, F888AF, FA0F45, FAA045, FAA545, FAFC69, FC0AAF, FC66AF, FCCCAF, FCFFAF, FF0323, FF056F, FF3203, FF7903, FFA045, FFA4A5, FFAA45, FFC0AF, FFF4A5, FFF575, FFFA45, FFFCAF, 10A009B, 20000D1, 2CCC663, 30A00FF, 30CCCAF, 30FA00F, 30FCCAF, 3333C23, 333C2C3, 33C3AAF, 33FCAAF, 33FFFAF, 3A0A00F, 3AAAA0F, 3AF000F, 3AFAAAF, 3C0CA0F, 3CCC3AF, 3CFF323, 3F33F23, 3FAA00F, 3FF3323, 4004441, 400DDD1, 400E00D, 400ED0D, 404404D, 404448D, 404E4DD, 440EDDD, 4440EDD, 44444ED, 4444E4D, 44DDDDD, 4A000A5, 4CCCCAB, 4D0CCCB, 4E4404D, 4E4444D, 4E4DDDD, 5000021, 5004221, 5006AAF, 500FF6F, 5042201, 508CCCD, 5400005, 5400AA5, 5555405, 5808007, 5AA4005, 5C0008D, 5CCC8CD, 5D4444D, 5EEEEEB, 5F40005, 5F554A5, 5F6AAAF, 60000AF, 60006A9, 600866F, 6008AAF, 600AA8F, 600F6A9, 606608F, 606686F, 608666F, 60AA08F, 60AAA8F, 66000AF, 66666A9, 6666AF9, 6866A8F, 6AAAAAF, 70070D7, 70077DD, 700DDDD, 707077D, 707D007, 70D00DD, 770077D, 770400D, 770740D, 7777775, 77777B7, 77777DD, 7777ACB, 77B88E7, 77DD00D, 77DDDDD, 7D0D00D, 7DD0D07, 7DDD00D, 800002D, 8000CED, 80C0E0D, 80CECCD, 840400D, 844000D, 844E00D, 868688F, 880444D, 884404D, 887D007, 8888801, 8888881, 8888E07, 8888F77, 8888FE7, 88A8AFF, 88AAAFF, 88FAFFF, 8A8AAAF, 8A8AAFF, 8AAA8FF, 8C00ECD, 8C8444D, 8E4400D, 8FCCCCF, 900BBAB, 90CC4AB, 9908AA1, 99E0E01, 9B00801, 9B6CCC9, A000FF9, A006069, A00A8FF, A01CCCB, A05F545, A0BEEEB, A0E4AA1,

AA0008F, AA08FFF, AA40AA5, AA8FFFF, AAAA405, AE04AA1, AE44441, AE4AAA1, AECCCCB, AF40005, AFA5A45, AFFFC69, B000BAB, B000EBB, B0D0007, B222227, B6CCCC9, B8880A1, BA000EB, BA0BEEB, BAEEEEB, BB000CD, BB00C0D, BB0B00D, BC6CC69, BC6CCC9, BCCCC69, BCCCCED, C0000A9, C00068F, C000CFD, C000E2D, C000FAF, C004D4D, C00E20D, C00E8CD, C00F68F, C033A0F, C0802CD, C086AAF, C0A00AF, C0AFFFF, C0C086F, C0C0F8F, C0CA00F, C0CC08F, C0D044D, C0F0AFF, C0FF023, C0FFFAF, C33FA0F, C33FAAF, C3CA00F, C3FFCAF, C8002CD, C8200CD, CCC668F, CCCAA8F, CCCC0A9, CCCC3AF, CCCCCA9, CCCDC4B, CE0008D, CE2000D, CE8CCCD, CF000AF, CFF0AAF, CFFF0AF, D0000EB, D0005EB, D000775, D000EDD, D007077, D00DDD9, D00ED0D, D0AAA45, D0AAAA5, D0EDDDD, D19000B, D4404ED, D4440ED, D5BBBBB, DCCCC4B, DD00DD9, DD07077, DD0DD09, DD0DDD9, DD99999, DDD0D09, DDDD0D9, DDDD9E1, DDDDD09, DDDDD99, DE0DDDD, DEEEEEB, E00001B, E0004A1, E000CAB, E00A041, E00BB0B, E00BBBB, E00C80D, E00CCCB, E044DDD, E0AA4A1, E0AAA41, E0BBB0B, E0D444D, E40444D, E4DDD4D, E88CCCD, E8C000D, E8CCCCD, EA04441, EA0A4A1, EBB000D, EBCCCCD, ED0D00D, EEAAA01, EEBBBBB, EEE000B, F0002C3, F002CC3, F003323, F005545, F00F4A5, F033323, F0400A5, F0A5545, F333323, F333F23, F6660AF, F733333, FA00009, FA004A5, FAAAA45, FC6668F, FCC668F, FD00AA5, FEE7777, FF0F263, FF26003, FF3F323, FF5F887, FFAFF45, FFFF263, FFFF379, 2CCCCC63, 30CCA00F, 33333319, 3333FCAF, 3333FFAF, 33FFA00F, 3C00CCAF, 3C00FCAF, 3CF3FF23, 40000441, 40000CAB, 4000DAA1, 400440DD, 400ACCCB, 400CCCAB, 400E44DD, 4040D00D, 404400DD, 40444EDD, 4044D00D, 40ACCCCB, 40DDDDDD, 440000D1, 44000DDD, 4400DD0D, 44E400DD, 4A00004B, 4A0AAAA5, 5000C08D, 52000CCD, 555400A5, 55540A05, 58800007, 58888087, 5A540005, 5C00020D, 5F5400A5, 5F888887, 60006AAF, 600093AF, 600AAAAF, 608CCCCF, 6600686F, 6606866F, 6688AAAF, 7000077D, 70000D5D, 7000707B, 7000707D, 7000740D, 70500D0D, 7070040D, 707007DD, 7070777B, 7077744D, 7077777B, 77007D0D, 7700B44D, 7707000B, 7707D00D, 7770700D, 7770777B, 7777740D, 7777770B, 7777777D, 77777CAB, 7777B887, 778888E7, 788888E7, 79333333, 7ACCCCCB, 7D0000DD, 7D00D0DD, 7DD00D0D, 7DDDDDA9, 80000081, 80000087, 8000E0CD, 80400E4D, 80A0AAA1, 80EC000D, 84000E4D, 8404444D, 84400E4D, 868AAAAF, 86AAAA8F, 8884044D, 88FFFE77, 8C44444D, 8CCCCAAF, 8E40004D, 900000BB, 90000B0B, 90100009, 90800AA1, 93333AAF, 94AAAAA1, 980000A1, 998AAAA1, A00000F9, A0000EEB, A0005A45, A0055545, A00AAA45, A0666669, A0AAA045, A0AAAA45, A0AAE4A1, A0B44441, A4A00005, A6066669, A8AAFFFF, AA055545, AA0AA045, AAA00A45, AAAAA045, B00000AB, B000EEEB, B00EEE0B, B0900081, B0BBBBAB, B7777787, B9000081, B9008001, B9800001, BA00000B, BBBB0ABB, BCCCCCC9, C000004D, C000086F, C0000AFF, C0000E8D, C0000FDD, C00033AF, C0003CAF, C000448D, C000AFFF, C000CF8F, C004444D, C00663AF, C00F00AF, C00FCCAF, C0FFCCAF, C844444D, CC3A000F, CCCCCBED, CCCCCE2D, CCCCD999, CCDCCC4B, CD44444D, CFAF000F, CFFFF023, D00400ED, D004404D, D00777A5, D00E00DD, D0444E0D, D40000ED, D444E00D, D7DDDDDD, DD00D007, DD0D0077, DD0D0707, DDD0040D, DDDDDD19, DDDDDDD1, E0000CCB, E0044441, E00A4AA1, E888820D, E8888CCD, E888C80D, E8AAAAA1, EB00C0CD, EBBC00CD, ECCCCCCB, F00006AF, F00040A5, F00066AF, F06666AF, F0F004A5, F33FFF23, F60006AF, F6AAA0AF, F88888F7, FE777777, FF33F2C3, FF3FFF23, FF588887, FFFF02C3, FFFF5F6F, FFFFF887, FFFFFF79, 10CCCCCAB, 266666603, 333333AAF, 333333F23, 3333FF2C3, 333CCCCAF, 333FFCCAF, 3A000000F, 3FA00000F, 40000048D, 4000004DD, 4000040D1, 40000ACCB, 4000400D1, 4040000DD, 404D0000D, 40A000005, 40E00444D, 40ED0000D, 444E000DD, 444ED000D, 48444444D, 4A0000005, 4AAAAAAA5, 500000C8D, 500000F8D, 50CCCCC8D, 50FFFFF6F, 5AAAAAA45, 5C020000D, 5E444444D, 666666AFF, 70000044D, 70000440D, 700007CCB, 700007D07, 70044000D, 70070007D, 77070007D, 77700040D, 77700070D, 77707044D, 77770000D, 77777777B, 777888887, 7D0DDDDDD, 7DD0000D7, 8008880A1, 800888A01, 800C000ED, 888800087, 88888AF8F, 888CCCCCD, 88CCCCCCD, 8AAAAAFFF, 8AAFFFFFF, 8CECCCCCD, 8CFFFFCFF, 8EC00000D, 900010009, 908A0AAA1, 9800AAAA1, 9B0CCCCC9, A00000669, A00005545, A0000A545, A000FFF45, A0AAAAA8F, A4000004B, A55540005, A5F554005, AA0A0AA45, AA0AAA8FF, AA4000005, AAA0AA8FF, AAAA0A8FF, AAAA0AA8F, B00000881, B00009801, B00090081, B00BBBABB, B0EB0000B, B4444444D, B77777777, B7E777777, BB00000BD, BB0C0000D, BBBBBA00B, BBBBBBABB, BE0EEEE0B, BE7777777, C00000CAF, C00006AAF, C000082CD, C00063AFF, C000820CD, C00F00023, C0444444D, C66666AFF, CCCD99999, CF0000023, CF66666AF, D00000009, D0000044D, D0044000D, D040E000D, D0440000D, D0DD000D9, DAAAAAA45, E004044DD, E004444DD, E044400DD, E0C00008D, E0C08000D, E0EAAAAA1, E2000000D, E400044DD, EAAA4AAA1, EAAAAEAA1, EAAAEA041, EBBBBC00D, EEEE00CCB, F00000545, F02600003, F066AAAAF, F0FF5666F, F3FFF3F23, F60AAAA0F, F77777777, FFEEEEEE7, FFFF33323, FFFF5666F, FFFFF2CC3, FFFFF7777, FFFFFEEE7, FFFFFFF77, ..., 2666666663, ..., 400000000D, ..., 500000006F, ..., 700000077B, ..., 8000000AA1, ..., 800AAAAA01, ..., 8886888AAF, ..., 88888888AF, ..., 8888888A8F, ..., 888AAFFFFF, ..., 9000000019, ..., 9000000109, ..., 908AAAAA01, ..., AAAAAAAAA1, ..., AAAAAAAE41, ..., C000CC866F, ..., C00CCCCCAF, ..., C6666666AF, ..., CCCCCCCAAF, ..., CFFFFFFAAF, ..., E444444441, ..., E4444444DD, ..., EAAAAAA4A1, ..., F260000003, ..., FEEEEEEEE7, ..., FFFFFFF56F, ..., 22000000007, ..., 4000000004B, ..., 400000000A5, ..., 52CCCCCCCCD, ..., 80AAAAAAA01, ..., 87000000007, ..., A0444444441, ..., A0AAAAAEA41, ..., BEEEEEEEEEB, ..., C0006666AFF, ..., C000CCCC6AF, ..., C0AF000000F, ..., EAAAEAAAAA1, ..., FAAAAAAAA8F, ..., 588888888887, ..., 800AAAAAAAA1, ..., 888888AFFFFF, ..., 88AFFFFFFFFF, ..., 8CCCCCCCCFCF, ..., A0000000AA8F, ..., A40000000005, ..., A44044444441, ..., AAAAAAA00A8F, ..., C00000000C8F, ..., CA0F0000000F, ..., CCCCCCCCC6AF, ..., D10000000005, ..., E0A04AAAAAA1, ..., 1A000000000B, ..., 5BBBBBBBBBBB, ..., 66666666006AF, ..., 7A0000000000B, ..., 88888888888FF, ..., 88888888FFFFF, ..., 888888F88888F, ..., 88F888888888F, ..., A000000000A8F, ..., A0FFFFFFFFF45, ..., C000000000023, ..., 86666666666F6F, ..., C00000000000AF, ..., C0000006666AF, ..., C0A000000000F, ..., C444444444444D, ..., CFF0A0000000F, ..., D00000000007B, ..., 686666666666666F, ..., 68CCCCCCCCCCCF, ..., 77700000000007D, ..., 8000000000000A1, ..., 888888AAAAAAAAF, ..., 9B0000000000009, ..., AAAAAAAAAAAAAA45, ..., CFFFFFFFFFA000F, ..., DDDDDDDDDDDDDD9, ..., 58CCCCCCCCCCCCCD, ..., 8666666666666666F, ..., 8ECCCCCCCCCCCCCD, ..., A000000000000009, ..., 8CFFFFFFFFFFFFFCF, ..., 5C200000000000000D, ..., B0000000000000981, ..., CFFFFFFFFFFFFFA00F, ..., AAAAAAAAAAAAAA008FF, ..., BBBBBBBBBBBBBBBBAB, ..., A000000000000000045, ..., CD9999999999999999, ..., CFFA000000000000000F, ..., 700000000000000007D7, ..., E0000000000000000441, ..., CFFFFFFFFA00000000000F, ..., 400000000000000000000085, ..., 8AAAAAAAAAAAAAAAAAAAAFF, ..., 8D00000000000000000000007, ..., 3333333333333333333333331, ..., 666666666666666666666666AF, ..., EB000000000000000000000D, ..., 7DDDDDDDDDDDDDDDDDDDDDDDD, ...,

844444444444444444444444444444D, ...,
8CCCCCCCCCCCCCCCCCCCCCCCCCCCCCFF, ...,
A8AAAAAAAAAAAAAAAAAAAAAAAAAAAAAF, ...,
AAAAAAAAAAAAAAAAAAAAAAAAAAAAA8FF, ...,
22222222222222222222222222222227, ..., CFA0000000000000000000000000000F, ...,
8AAAAAAAAAAAAAAAAAAAAAAAAAAAAAAA8F, ...,
CFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFAF, ...,
40444444444444444444444444444444441, ...,
EEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEB, ...,
910000000000000000000000000000000009, ...,
2C00000000000000000000000000000000003, ...,
C0000000000000000000000000000000000DD, ...,
26000000000000000000000000000000000003, ...,
7700000000000000000000000000000000007D, ...,
D00000000000000000000000000000000000A5, ...,
4DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD, ...,
AAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAA8F, ...,
CAF0000000000000000000000000000000000000F, ...,
FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF323, ...,
88AAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAF, ...,
C0CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCAF, ...,
B0000000000000000000000000000000000000000000000C9, ...,
BEB000000000000000000000000000000000000000000000B, ...,
FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF2C3, ...,
AF666666666666666666666666666666666666666666666669, ...,
888888888888888888888888888888888888888888888888888887, ...,
CA000000000000000000000000000000000000000000000000000F, ...,
E44444444444444444444444444444444444444444444444444444444D, ...,
CAFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF, ...,
A8AFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF, ...,
500000000000000000000000000000000000000000000000000000000000000008D, ...,
F8CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCF, ...,
5200000000000000000000000000000000000000000000000000000000000000000000000
1, ...,
E000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000041, ...,
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCAF, ...,
C000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000CD, ...,
EBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBB, ...,
3FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF
FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF
FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF23, ...,

8888FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF
FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF
FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF
FFFF, ...,
5444444444444444444444444444444444444444444444444444444444444444444444444444
444444444444444444444444444444444444444444444444444444444444444444444444444
444444444444444444444444444444444444444444444444444444444444444444444444444
44444444444444444444444D, ...,
880000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000007, ...,
D44444444444444444444444444444444444444444444444444444444444444444444444444
444444444444444444444444444444444444444444444444444444444444444444444444444
444444444444444444444444444444444444444444444444444444444444444444444444444
4444444444444444444444444444444444444444444444D, ...,
5FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF
FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF
FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF
FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF
FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF
FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF
FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF
FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF
FFFFFFFFF6F, ...,
88FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF
FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF
FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF
FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF
FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF
FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF
FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF
FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF
FFFFFFFFFFF, ...,
D99999999999999999999999999999999999999999999999999999999999999999999999999
999999999999999999999999999999999999999999999999999999999999999999999999999
999999999999999999999999999999999999999999999999999999999999999999999999999
999999999999999999999999999999999999999999999999999999999999999999999999999
999999999999999999999999999999999999999999999999999999999999999999999999999
999999999999999999999999999999999999999999999999999999999999999999999999999
999999999999999999999999999999999999999999999999999999999999999999999999999
999999999999999999999999999999999999999999999999999999999999999999999999999
999999999999999999999999999999999999999999999999999999999999999999999999999
999999999999999999999999999999999999999999999999999999999999999999999999999
999999999999999999999999999999999999999999999999999999999999999999999999999
999999999999999999999999999999999999999999999999999999999999999999999999999
999999999999999999999999999999999999999999999999999999999999999999999999999
999999999999999999999999999999999999999999999999999999999999999999999999999

9999999999999999999999999999999, ...,
F888888888888888888888888888888888888888888888888888888888888888888888888
8888888888888888888888888888888888888888888888888888888888888888888888888
8888888888888888888888888888888888888888888888888888888888888888888888888
8888888888888888888888888888888888888888888888888888888888888888888888888
8888888888888888888888888888888888888888888888888888888888888888888888888
8888888888888888888888888888888888888888888888888888888888888888888888888
8888888888888888888888888888888888888888888888888888888888888888888888888
8888888888888888888888888888888888888888888888888888888888888888888888888
8888888888888888888888888888888888888888888888888888888888888888888888888
8888888888888888888888888888888888888888888888888888888888888888888888888
8888888888888888888888888888888888888888888888888888888888888888888888888
8888888888888888888888888888888888888888888888888888888888888888888888888
8888888888888888888888888888888888888888888888888888888888888888888888888
8888888888888888888888888888888888888888888888888888888888888888888888888
8888888888888888888888888888888888888888888888888888888888888888888888888
8888888888888888888888888888888888888888888888888888888888888888888888888
8888888888888888888888888888888888888888888888888888888888888888888888888
8888888888888888888888888888888888888888888888888888888888888888888888888
8888888888888888888888888888888888888888888888888888888888888888888888888
8888888888888888888888888888888888888888888888888888888888888888888888888
8888888888888888888888888888888888888888888888888888888888F, ...,
9000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000

0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000091, ...,
5BCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC

CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCD, ...,
DBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB

BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB

BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB

BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB

BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB

BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB

BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB

BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB

BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB

BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB

BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB

BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBBB
BBBBBBBBBBBBBBBBBBBBBBBBBBB, ...

Proof

There are lemmas, corollaries, theorems, proofs, conjectures, hypotheses, open problems, heuristic arguments, for this problem about the sets of the primes with no proper subsequence whose value is also prime in the positional numeral system with base (or radix) b for $2 \le b \le 36$.

Proving $M(L_b)$ = the set S is equivalent to:

* Prove that all elements in S, when read as base b representation, are primes > b.

* Prove that all proper subsequence of all elements in S, when read as base b representation, which are > b, are composite.

* Prove that all primes > b, when written in base b, contain at least one element in S as subsequence (equivalently, prove that all strings not containing any element in S as subsequence, when read as base b representation, which are > b, are composite).

($M(L_b) = S$ is proved if and only if all these three problems are proved, i.e. $M(L_b) = S$ is a theorem if and only if all these three “conjectures” are theorems)

e.g. proving $M(L_{10})$ = {11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 227, 251, 257, 277, 281, 349, 409, 449, 499, 521, 557, 577, 587, 727, 757, 787, 821, 827, 857, 877, 881, 887, 991, 2087, 2221, 5051, 5081, 5501, 5581, 5801, 5851, 6469, 6949, 8501, 9001, 9049, 9221, 9551, 9649, 9851, 9949, 20021, 20201, 50207, 60649, 80051, 666649, 946669, 5200007, 22000001, 60000049, 66000049, 66600049, 80555551, 555555555551, 5000000000000000000000000000027}, is equivalent to:

* Prove that all of 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 227, 251, 257, 277, 281, 349, 409, 449, 499, 521, 557, 577, 587, 727, 757, 787, 821, 827, 857, 877, 881, 887, 991, 2087, 2221, 5051, 5081, 5501, 5581, 5801, 5851, 6469, 6949, 8501, 9001, 9049, 9221, 9551, 9649, 9851, 9949, 20021, 20201, 50207, 60649, 80051, 666649, 946669, 5200007, 22000001, 60000049, 66000049, 66600049, 80555551, 555555555551, 5000000000000000000000000000027 are primes > 10.

* Prove that all proper subsequence of all elements in {11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 227, 251, 257, 277, 281, 349, 409, 449, 499, 521, 557, 577, 587, 727, 757, 787, 821, 827, 857, 877, 881, 887, 991, 2087, 2221, 5051, 5081, 5501, 5581, 5801, 5851, 6469, 6949, 8501, 9001, 9049, 9221, 9551, 9649, 9851, 9949, 20021, 20201, 50207, 60649, 80051, 666649, 946669, 5200007, 22000001, 60000049, 66000049, 66600049, 80555551, 555555555551, 5000000000000000000000000000027} which are > 10 are composite.

* Prove that all primes > 10 contain at least one element in {11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 227, 251, 257, 277, 281, 349, 409, 449, 499, 521, 557, 577, 587, 727, 757, 787, 821, 827, 857, 877, 881, 887, 991, 2087, 2221, 5051, 5081, 5501, 5581, 5801, 5851, 6469, 6949, 8501, 9001, 9049, 9221, 9551, 9649,

9851, 9949, 20021, 20201, 50207, 60649, 80051, 666649, 946669, 5200007, 22000001, 60000049, 66000049, 66600049, 80555551, 555555555551, 5000000000000000000000000000027} as subsequence (equivalently, prove that all numbers > 10 not containing any element in {11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 227, 251, 257, 277, 281, 349, 409, 449, 499, 521, 557, 577, 587, 727, 757, 787, 821, 827, 857, 877, 881, 887, 991, 2087, 2221, 5051, 5081, 5501, 5581, 5801, 5851, 6469, 6949, 8501, 9001, 9049, 9221, 9551, 9649, 9851, 9949, 20021, 20201, 50207, 60649, 80051, 666649, 946669, 5200007, 22000001, 60000049, 66000049, 66600049, 80555551, 555555555551, 5000000000000000000000000000027} as subsequence are composite).

(since for base b = 10, all these three problems are proved, i.e. all they are theorems, thus, $M(L_{10})$ = {11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 227, 251, 257, 277, 281, 349, 409, 449, 499, 521, 557, 577, 587, 727, 757, 787, 821, 827, 857, 877, 881, 887, 991, 2087, 2221, 5051, 5081, 5501, 5581, 5801, 5851, 6469, 6949, 8501, 9001, 9049, 9221, 9551, 9649, 9851, 9949, 20021, 20201, 50207, 60649, 80051, 666649, 946669, 5200007, 22000001, 60000049, 66000049, 66600049, 80555551, 555555555551, 5000000000000000000000000000027} is also proved, i.e. $M(L_{10})$ = {11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 227, 251, 257, 277, 281, 349, 409, 449, 499, 521, 557, 577, 587, 727, 757, 787, 821, 827, 857, 877, 881, 887, 991, 2087, 2221, 5051, 5081, 5501, 5581, 5801, 5851, 6469, 6949, 8501, 9001, 9049, 9221, 9551, 9649, 9851, 9949, 20021, 20201, 50207, 60649, 80051, 666649, 946669, 5200007, 22000001, 60000049, 66000049, 66600049, 80555551, 555555555551, 5000000000000000000000000000027} is also a theorem)

For the first part, since the numbers are clearly > b, thus we only need to prove that they are primes, we can use *ECPP* (such as *Primo*) to prove that these 77 numbers are definitely primes (i.e. not merely probable primes), in this case of base 10, the largest number has only 31 digits and can be proved primality in <1 second, but in other case, such as base 13, 14, and 16, there are numbers > 10^{10000} in the sets, thus *ECPP* (or *N*−1, *N*+1, if this prime −1 or +1 can be trivially factored, such as the case of base 14, the large prime $5*14^{19698}-1$ in this set) is need to prove their primality; the second part is the easiest part of these three parts, as we can use either trial division or Fermat test to prove their compositeness (if these numbers have small prime divisors, or these numbers fails the Fermat primality tests, then they are defined composite), unless the numbers are Fermat pseudoprimes to many bases (such as the Carmichael numbers and the numbers of the form $p*q$ with p, q primes and q = $2*p-1$ (https://oeis.org/A129521)) (reference of pseudoprimes) with no small prime factors (say < 2^{32}), in this case, we need to run either Miller–Rabin primality test or Lucas primality test to prove their compositeness (the worst case is that the number is a Carmichael number which is strong pseudoprime to several bases, see this article, this article gives a 397 digit such number, another example is this 23707 digit number), the combine of these two tests is Baillie–PSW primality test, and there is no known composites which pass this test, also it is known that no composites ≤ 2^{64} which pass this test, this is because strong Fermat pseudoprimes to base 2 (https://oeis.org/A001262) tend to fall into the residue class +1 (mod m) for many small m, whereas strong Lucas pseudoprimes (https://oeis.org/A217255) tend to fall into the residue class −1 (mod m) for many small m. As a result, a number which passes both a strong Fermat and a strong Lucas test is very likely to be prime.

Determining $M(L)$ for arbitrary L is in general unsolvable, and can be difficult even when L is relatively simple, also, determining $M(L)$ for arbitrary L may be an open problem or NP-complete or an undecidable problem, or an example of Gödel's incompleteness theorems (like the continuum hypothesis and the halting problem), or as hard as the unsolved problems in mathematics, such as the Riemann hypothesis and the *abc* conjecture, which are the two famous hard problems in number theory.

The following is a "semi-algorithm" that is guaranteed to produce $M(L)$, but it is not so easy to implement:

(1) $M = \emptyset$
(2) while ($L \neq \emptyset$) do
(3) choose x, a shortest string in L
(4) $M = M \cup \{x\}$
(5) $L = L - sup(\{x\})$

In practice, for arbitrary L, we cannot feasibly carry out step (5). Instead, we work with L', some regular overapproximation to L, until we can show $L' = \emptyset$ (which implies $L = \emptyset$). In practice, L' is usually chosen to be a finite union of sets of the form $L_1L_2^*L_3$, where each of L_1, L_2, L_3 is finite. In the case we consider in this paper, we then have to determine whether such a language contains a prime or not.

However, it is not even known if the following simpler decision problem is recursively solvable:

Problem: Given strings x, y, z, and a base b, does there exist a prime number whose base-b expansion is of the form xy^nz for some $n \geq 0$? (If we say "yes", then we should find such a prime (the smallest such prime may be very large, e.g. $> 2^{65536}$, and if so, then we should use primality testing programs such as *PFGW* or *LLR* to find it, and before using these programs, we should use sieving programs such as *srsieve* (or *sr1/2/5sieve*) to remove the numbers either having small prime factors or having algebraic factors) and prove its primality (and if we want to solve the problem in this article, we should check whether this prime is the smallest such prime or not, i.e. prove all smaller numbers of the form xy^nz with $n \geq 0$ are composite, usually by trial division or Fermat primality test), and if we say "no", then we should prove that such prime does not exist, may by covering congruence, algebraic factorization, or combine of them)

An algorithm to solve this problem, for example, would allow us to decide if there are any additional Fermat primes (of the form $2^{2^n}+1$) other than the known ones (corresponding to $n = 0, 1, 2, 3, 4$). To see this, take $b = 2$, $x = 1$, $y = 0$, and $z = 0^{16}1$, or take $b = 2$, $x = 10^{16}$, $y = 0$, and $z = 1$. Since if 2^n+1 is prime then n must be a power of two, a prime of the form $(xy^*z)_b$ must be a new Fermat prime. Besides, it would allow us to decide if there are infinitely many Mersenne primes (of the form 2^p-1 with prime p). To see this, take $b = 2$, $x = \lambda$ (the empty string), $y = 1$, and $z = 1^{n+1}$, or take $b = 2$, $x = 1^{n+1}$, $y = 1$, and $z = \lambda$ (the empty string), where n is the exponent of the Mersenne prime which we want to know whether it is the largest Mersenne prime or not. Since if 2^n-1 is prime then n must be a prime, a prime of the form $(xy^*z)_b$ must be a new Mersenne prime. Also, it would allow us to decide if 21181 is a

Sierpinski number (take b = 2, x = 101001010111101, y = 0, and z = 1) and if 23669 is a Riesel number (take b = 2, x = 101110001110100, y = 1, and z = λ (the empty string)). Also, it would allow us to solve the numbers n with unknown status (i.e. n = 603, 1244, 1861) in this page (take b = 10, x = 6031, 12441, 18611, respectively, y = 1, and z = λ (the empty string), or take b = 10, x = 603, 1244, 1861, respectively, y = 1, and z = 1).

Therefore, in practice, we are forced to try to rule out prime representations based on heuristics such as modular techniques and factorizations.

It will be necessary for our algorithm to determine if families of the form $(xy^*z)_b$ contain a prime > b or not. We use two different heuristic strategies to show that such families contain no primes > b.

(Reference: the divisibility rule for base b:

* For prime p dividing b, the number is divisible by p if and only if the last digit of this number is divisible by p.
* For prime p dividing b−1, the number is divisible by p if and only if the sum of the digits of this number is divisible by p.
* For prime p dividing b+1, the number is divisible by p if and only if the alternating sum of the digits of this number is divisible by p. (this can also show that all palindromic primes in any base b have an odd number of digits, the only possible exception is “11” in base b (i.e. b+1 itself))
* The section “Divisibility Rules in Lotsa Various Bases” in its talk page
* Divisibility rules in other bases b
* The divisibility rule of b^2-b+1 in base b
)

In the first strategy, we mimic the well-known technique of “covering congruences”, by finding some finite set S of primes p such that every number in a given family is divisible by some element of S (this is equivalent to finding an integer N such that all numbers in a given family are not coprime to N, e.g. all numbers in the family 2{5} in base 11 are not coprime to 6, *gcd*$((5*11^n-1)/2, 6)$ can only be 2 or 3, and cannot be 1) (the primes in S must be prime factors of $(b^n-1)/(b-1)$ (i.e. the generalized repunit number in base b with length n), where n is the period, e.g. for b = 10, the primes in S must be prime factors of $(10^n-1)/9$ (A002275(n)), and for b = 2, the primes in S must be prime factors of 2^n-1 (A000225(n)), for the list of values of $(b^n-1)/(b-1)$, see https://oeis.org/A055129) (examples: the conjectured smallest Sierpinski number 78557 and the conjectured smallest Riesel number 509203, which have covering sets {3, 5, 7, 13, 19, 37, 73} and {3, 5, 7, 13, 17, 241}, respectively, and their periods are 36 and 24, respectively, see https://oeis.org/A244562, https://oeis.org/A244561, https://oeis.org/A257647, https://oeis.org/A244071, https://oeis.org/A244070, https://oeis.org/A258154, another examples are the families 9{1}3 and 9{4}9 and 9{5}9 in base b = 10 (all these three families have covering set {3, 7, 11, 13}, and their periods are all 6), see https://stdkmd.net/nrr/9/91113.htm#prime_period, https://stdkmd.net/nrr/9/94449.htm#prime_period, https://stdkmd.net/nrr/9/95559.htm#prime_period, http://www.worldofnumbers.com/deplat.htm), another examples are the families 37{1}, 176{1}, 209{1}, 407{1}, 936{1}, 1023{1}, 4070{3}, 891{7}, 10175{9} in base 10 (these families

have different covering sets, all these sets are subsets of {3, 7, 11, 13, 37}, and their periods are all 6), see http://www.worldofnumbers.com/Appending%201s%20to%20n.txt and http://www.worldofnumbers.com/em197.htm and https://www.rose-hulman.edu/~rickert/Compositeseq/, also there are examples for the non-$\frac{a\cdot b^{n}+c}{gcd(a+c,b-1)}$ (i.e. non-*x*{*y*}*z* families): the family {1}37{1} in base 10 (where the two {1} have the same number of 1's) (this family has covering set {3, 13, 37}, and its period is 3), see http://gladhoboexpress.blogspot.com/2019/05/prime-sandwiches-made-with-one-derbread.html; the family {76}7 = 7{67} in base 10 (in fact, this family is mathematically equivalent to the family 7:{67} in base 100, where "67" is the base 100 digit with digit value 67), see http://www.worldofnumbers.com/undulat.htm (by this page, 3:{43} in base 100 and 7:{17} in base 100 are unsolved families) (reference of covering sets). In the second strategy, we attempt to find an algebraic factorization, such as difference-of-two-squares factorization, sum/difference-of-two-cubes factorization, and Aurifeuillian factorization of x^4+4y^4 (examples of Aurifeuillian factorizations), if *a*, *b*, *c* are all *r*-th powers for some $r > 1$ (i.e. *gcd*(A052409(*a*), A052409(*b*), A052409(*c*)) > 1), then $\frac{a\cdot b^{n}+c}{gcd(a+c,b-1)}$ ($a\geq1$, $b\geq2$ (*b* is the base), $c\neq0$, $gcd(a,c)=1$, $gcd(b,c)=1$) is always composite, with only a possible exception of very small *n*, the same holds for the situation when *b* and 4**a***c* are both 4th powers, such examples of *x*{*y*}*z* families only exist in perfect power bases *b* (thus not exist in base 10 and base 2, since neither 10 nor 2 is perfect power), e.g. families {1}, 3{1}, 3{8}, 3{8}35, {8}5 in base 9 (all have difference-of-two-squares factorization), family 1{0}1 in base 8 (has sum-of-two-cubes factorization), families 10{5}, 1{5}, {4}1, 7{3}, 8{5}, 8{F}, B{4}1, {F}7 in base 16 (all have difference-of-two-squares factorization), families {C}D, {C}DD in base 16 (both have Aurifeuillian factorization for x^4+4y^4), also there are examples for the non-$\frac{a\cdot b^{n}+c}{gcd(a+c,b-1)}$ (i.e. non-*x*{*y*}*z* families): the families {1}0{1} in base 10 (where the two {1} have the same number of 1's), {1}2{1} in base 10 (where the two {1} have the same number of 1's), {3}2{3} in base 10 (where the two {3} have the same number of 3's), {3}4{3} in base 10 (where the two {3} have the same number of 3's), all of them have special algebraic factorizations, i.e. algebraic factorizations which are neither sum/difference-of-two-*r*-th-powers factorization (i.e. factorization of binomial numbers) nor Aurifeuillian factorization of x^4+4y^4, see http://www.worldofnumbers.com/wing.htm (however, no *x*{*y*}*z* families in any base *b* can have algebraic factorizations which are neither sum/difference-of-two-*r*-th-powers factorization (i.e. factorization of binomial numbers) nor Aurifeuillian factorization, no matter full algebraic factorizations or partial algebraic factorizations). For the examples of combine of the two strategies (i.e. combine of covering congruences and algebraic factorization), see https://arxiv.org/pdf/1110.4671.pdf and http://www.fq.math.ca/Scanned/33-3/izotov.pdf and https://oeis.org/A213353, $4008735125781478102999926000625*2^n+1$ ($=44745755^4*2^n+1$) and
$151854033223239213153630592218944998133269330577330714408611445716011170576987377001403174164964 81*2^n-1$
($=38968453038738811751593146208088870460669724698 09^2*2^n-1$) are examples, and the family 38{1} in base $b = 10$ is also an example, see http://www.worldofnumbers.com/em197.htm and http://www.worldofnumbers.com/Appending%201s%20to%20n.txt and https://oeis.org/A069568 and https://archive.fo/vKSJO, also there are examples in the *Conjectures 'R Us* pages: Sierpinski side ($k*b^n+1$) (see bases b = 55, 63, 200, 225, …) and Riesel side ($k*b^n-1$) (see bases b = 12, 19, 24, 28, 30, 33, …).

Examples of the first strategy: (we can show that the corresponding numbers are > all elements in S, if n makes corresponding numbers > b (i.e. n≥1 for 51^n in base 9 and 25^n in base 11 and 4^nD in base 16 and 8^nF in base 16, n≥0 for other examples), thus these factorizations are nontrivial)

* In base 10, all numbers of the form 46^n9 (algebraic form: $(14*10^{n+1}+7)/3$) (n≥0) are divisible by 7, and no numbers of the form 46^n9 (base 10) with n≥0 is equal to 7, thus no number of the form 46^n9 (base 10) with n≥0 is prime (factordb)
* In base 6, all numbers of the form 40^n1 (algebraic form: $4*6^{n+1}+1$) (n≥0) are divisible by 5, and no numbers of the form 40^n1 (base 6) with n≥0 is equal to 5, thus no number of the form 40^n1 (base 6) with n≥0 is prime (factordb)
* In base 15, all numbers of the form 96^n8 (algebraic form: $(66*15^{n+1}+11)/7$) (n≥0) are divisible by 11, and no numbers of the form 96^n8 (base 15) with n≥0 is equal to 11, thus no number of the form 96^n8 (base 15) with n≥0 is prime (factordb)
* In base 9, all numbers of the form 51^n (algebraic form: $(41*9^n-1)/8$) (n≥1) are divisible by some element of {2,5}, and no numbers of the form 51^n (base 9) with n≥1 is equal to 2 or 5, thus no number of the form 51^n (base 9) with n≥1 is prime (note: the prime 5 (i.e. n = 0) is not allowed since the prime must be > base) (factordb)
* In base 11, all numbers of the form 25^n (algebraic form: $(5*11^n-1)/2$) (n≥1) are divisible by some element of {2,3}, and no numbers of the form 25^n (base 11) with n≥1 is equal to 2 or 3, thus no number of the form 25^n (base 11) with n≥1 is prime (note: the prime 2 (i.e. n = 0) is not allowed since the prime must be > base) (factordb)
* In base 14, all numbers of the form $B0^n1$ (algebraic form: $11*14^{n+1}+1$) (n≥0) are divisible by some element of {3,5}, and no numbers of the form $B0^n1$ (base 14) with n≥0 is equal to 3 or 5, thus no number of the form $B0^n1$ (base 14) with n≥0 is prime (factordb)
* In base 8, all numbers of the form 64^n7 (algebraic form: $(46*8^{n+1}+17)/7$) (n≥0) are divisible by some element of {3,5,13}, and no numbers of the form 64^n7 (base 8) with n≥0 is equal to 3, 5, or 13, thus no number of the form 64^n7 (base 8) with n≥0 is prime (factordb)
* In base 13, all numbers of the form 30^n95 (algebraic form: $3*13^{n+2}+122$) (n≥0) are divisible by some element of {5,7,17}, and no numbers of the form 30^n95 (base 13) with n≥0 is equal to 5, 7, or 17, thus no number of the form 30^n95 (base 13) with n≥0 is prime (factordb)
* In base 16, all numbers of the form 4^nD (algebraic form: $(4*16^{n+1}+131)/15$) (n≥1) are divisible by some element of {3,7,13}, and no numbers of the form 4^nD (base 16) with n≥1 is equal to 3, 7, or 13, thus no number of the form 4^nD (base 16) with n≥1 is prime (note: the prime D (i.e. n = 0) is not allowed since the prime must be > base) (factordb)
* In base 16, all numbers of the form 8^nF (algebraic form: $(8*16^{n+1}+97)/15$) (n≥1) are divisible by some element of {3,7,13}, and no numbers of the form 8^nF (base 16) with n≥1 is equal to 3, 7, or 13, thus no number of the form 8^nF (base 16) with n≥1 is prime (factordb)

Examples of the second strategy: (we can show that both factors are > 1, if n makes corresponding numbers > b (i.e. n≥2 for 1^n in base 9, n≥0 for 10^n1 in base 8 and $B4^n1$ in base 16, n≥1 for other examples), thus these factorizations are nontrivial)

* In base 9, all numbers of the form 1^n (algebraic form: $(9^n-1)/8$) (n≥2) factored as (3^n-1) * (3^n+1) / 8, and since if n≥3, $3^n-1 \ge 3^3-1 = 26 > 8$, $3^n+1 \ge 3^3+1 = 28 > 8$, this factorization is nontrivial if n≥3, and this only remains to check the case n=2, but for n=2, $(9^n-1)/8 = 10$ and 10 is not prime, thus no number of the form 1^n (base 9) with n≥2 is prime (factordb)

* In base 8, all numbers of the form 10^n1 (algebraic form: $8^{n+1}+1$) ($n\ge0$) factored as $(2^{n+1}+1) * (4^{n+1}-2^{n+1}+1)$, and since if $n\ge0$, $2^{n+1}+1 \ge 2^1+1 = 3 > 1$, $4^{n+1}-2^{n+1}+1 \ge 4^1-2^1+1 = 3 > 1$, this factorization is nontrivial, thus no number of the form 10^n1 (base 8) with $n\ge0$ is prime (factordb)
* In base 9, all numbers of the form 38^n (algebraic form: $4*9^n-1$) ($n\ge1$) factored as $(2*3^n-1) * (2*3^n+1)$, and since if $n\ge1$, $2*3^n-1 \ge 2*3^1-1 = 5 > 1$, $2*3^n+1 \ge 2*3^1+1 = 7 > 1$, this factorization is nontrivial, thus no number of the form 38^n (base 9) with $n\ge1$ is prime (note: the prime 3 (i.e. $n = 0$) is not allowed since the prime must be > base) (factordb)
* In base 16, all numbers of the form $8F^n$ (algebraic form: $9*16^n-1$) ($n\ge1$) factored as $(3*4^n-1) * (3*4^n+1)$, and since if $n\ge1$, $3*4^n-1 \ge 3*4^1-1 = 11 > 1$, $3*4^n+1 \ge 3*4^1+1 = 13 > 1$, this factorization is nontrivial, thus no number of the form $8F^n$ (base 16) with $n\ge1$ is prime (factordb)
* In base 16, all numbers of the form F^n7 (algebraic form: $16^{n+1}-9$) ($n\ge1$) factored as $(4^{n+1}-3) * (4^{n+1}+3)$, and since if $n\ge1$, $4^{n+1}-3 \ge 4^2-3 = 13 > 1$, $4^{n+1}+3 \ge 4^2+3 = 19 > 1$, this factorization is nontrivial, thus no number of the form F^n7 (base 16) with $n\ge1$ is prime (note: the prime 7 (i.e. $n = 0$) is not allowed since the prime must be > base) (factordb)
* In base 9, all numbers of the form 31^n (algebraic form: $(25*9^n-1)/8$) ($n\ge1$) factored as $(5*3^n-1) * (5*3^n+1) / 8$, and since if $n\ge1$, $5*3^n-1 \ge 5*3^1-1 = 14 > 8$, $5*3^n+1 \ge 5*3^1+1 = 16 > 8$, this factorization is nontrivial, thus no number of the form 31^n (base 9) with $n\ge1$ is prime (note: the prime 3 (i.e. $n = 0$) is not allowed since the prime must be > base) (factordb)
* In base 16, all numbers of the form 4^n1 (algebraic form: $(4*16^{n+1}-49)/15$) ($n\ge1$) factored as $(2*4^{n+1}-7) * (2*4^{n+1}+7) / 15$, and since if $n\ge1$, $2*4^{n+1}-7 \ge 2*4^2-7 = 25 > 15$, $2*4^{n+1}+7 \ge 2*4^2+7 = 39 > 15$, this factorization is nontrivial, thus no number of the form 4^n1 (base 16) with $n\ge1$ is prime (factordb)
* In base 16, all numbers of the form 15^n (algebraic form: $(4*16^n-1)/3$) ($n\ge1$) factored as $(2*4^n-1) * (2*4^n+1) / 3$, and since if $n\ge1$, $2*4^n-1 \ge 2*4^1-1 = 7 > 3$, $2*4^n+1 \ge 2*4^1+1 = 9 > 3$, this factorization is nontrivial, thus no number of the form 15^n (base 16) with $n\ge1$ is prime (factordb)
* In base 16, all numbers of the from C^nD (algebraic form: $(4*16^{n+1}+1)/5$) ($n\ge1$) factored as $(2*4^{n+1}-2*2^{n+1}+1) * (2*4^{n+1}+2*2^{n+1}+1) / 5$, and since if $n\ge1$, $2*4^{n+1}-2*2^{n+1}+1 \ge 2*4^2-2*2^2+1 = 25 > 5$, $2*4^{n+1}+2*2^{n+1}+1 \ge 2*4^2+2*2^2+1 = 41 > 5$, this factorization is nontrivial, thus no number of the form C^nD (base 16) with $n\ge1$ is prime (note: the prime D (i.e. $n = 0$) is not allowed since the prime must be > base) (factordb)
* In base 16, all numbers of the form $B4^n1$ (algebraic form: $(169*16^{n+1}-49)/15$) ($n\ge0$) factored as $(13*4^{n+1}-7) * (13*4^{n+1}+7) / 15$, and since if $n\ge0$, $13*4^{n+1}-7 \ge 13*4^1-7 = 45 > 15$, $13*4^{n+1}+7 \ge 13*4^1+7 = 59 > 15$, this factorization is nontrivial, thus no number of the form $B4^n1$ (base 16) with $n\ge0$ is prime (factordb)

Examples of combine of the two strategies: (we can show that for the part of the first strategy, the corresponding numbers are > all elements in S, and for the part of the second strategy, both factors are > 1, if n makes corresponding numbers > b (i.e. $n\ge0$ for B^n9B in base 12, $n\ge1$ for other examples), thus these factorizations are nontrivial)

* In base 14, numbers of the form $8D^n$ (algebraic form: $9*14^n-1$) ($n\ge1$) are divisible by 5 if n is odd and factored as $(3*14^{n/2}-1) * (3*14^{n/2}+1)$ if n is even, and no numbers of the form $8D^n$ (base 14) with $n\ge1$ is equal to 5, and since if $n\ge2$ (if $n\ge1$ and n is even, then $n\ge2$), $3*14^{n/2}-1 \ge 3*14^1-1 = 41 > 1$, $3*14^{n/2}+1 \ge 3*14^1+1 = 43 > 1$, this factorization is nontrivial, thus no number of the form $8D^n$ (base 14) with $n\ge1$ is prime (factordb)

* In base 12, numbers of the form B^{n}9B (algebraic form: $12^{n+2}-25$) ($n\geq0$) are divisible by 13 if n is odd and factored as $(12^{(n+2)/2}-5) * (12^{(n+2)/2}+5)$ if n is even, and no numbers of the form B^{n}9B (base 12) with $n\geq0$ is equal to 13, and since if $n\geq0$, $12^{(n+2)/2}-5 \geq 12^1-5 = 7 > 1$, $12^{(n+2)/2}+5 \geq 12^1+5 = 17 > 1$, this factorization is nontrivial, thus no number of the form B^{n}9B (base 12) with $n\geq0$ is prime (factordb)
* In base 14, numbers of the form D^{n}5 (algebraic form: $14^{n+1}-9$) ($n\geq1$) are divisible by 5 if n is even and factored as $(14^{(n+1)/2}-3) * (14^{(n+1)/2}+3)$ if n is odd, and no numbers of the form D^{n}5 (base 14) with $n\geq1$ is equal to 5, and since if $n\geq1$, $14^{(n+1)/2}-3 \geq 14^1-3 = 11 > 1$, $14^{(n+1)/2}+3 \geq 14^1+3 = 17 > 1$, this factorization is nontrivial, thus no number of the form D^{n}5 (base 14) with $n\geq1$ is prime (note: the prime 5 (i.e. n = 0) is not allowed since the prime must be > base) (factordb)
* In base 17, numbers of the form 19^n (algebraic form: $(25*17^n-9)/16$) ($n\geq1$) are divisible by 2 if n is odd and factored as $(5*17^{n/2}-3) * (5*17^{n/2}+3) / 16$ if n is even, and no numbers of the form 19^n (base 17) with $n\geq1$ is equal to 2, and since if $n\geq2$ (if $n\geq1$ and n is even, then $n\geq2$), $5*17^{n/2}-3 \geq 5*17^1-3 = 82 > 16$, $5*17^{n/2}+3 \geq 5*17^1+3 = 88 > 16$, this factorization is nontrivial, thus no number of the form 19^n (base 17) with $n\geq1$ is prime (factordb)
* In base 19, numbers of the from 16^n (algebraic form: $(4*19^n-1)/3$) ($n\geq1$) are divisible by 5 if n is odd and factored as $(2*19^{n/2}-1) * (2*19^{n/2}+1) / 3$ if n is even, and no numbers of the form 16^n (base 19) with $n\geq1$ is equal to 5, and since if $n\geq2$ (if $n\geq1$ and n is even, then $n\geq2$), $2*19^{n/2}-1 \geq 2*19^1-1 = 37 > 3$, $2*19^{n/2}+1 \geq 2*19^1+1 = 39 > 3$, this factorization is nontrivial, thus no number of the form 16^n (base 19) with $n\geq1$ is prime (factordb)

(for the base b forms xy*z converted to the algebraic forms $\frac{a\cdot b^{n+r}+c}{gcd(a+c,b-1)}$ (b is the base, r is the length of z), using: https://stdkmd.net/nrr/exprgen.htm (only for base 10 forms) and https://www.numberempire.com/simplifyexpression.php (enter the obvious algebraic forms, e.g. for base 8 family 64^{n}7, enter "6*8^(n+1)+4*8*(8^n-1)/7+7", this website will return "(23*2^(3*n+4)+17)/7", and this form can be easily converted to (46*8^(n+1)+17)/7) (b is given in its factorized form), also for the examples see page 16 of https://cs.uwaterloo.ca/~cbright/reports/mepn.pdf (all unsolved families in the original minimal prime problem (i.e. prime > base (b) is not required) for bases $2 \leq b \leq 30$) (a and c are given in their prime factorization form, e.g. if a or c is 360, then this table writes "$2^3 * 3^2 * 5$" rather than "360") and the excel file https://docs.google.com/spreadsheets/d/e/2PACX-1vRCn7Ytp1_Jbgi2b0MkjPxWE6yk3Eq81Wa3kWUUmRY8odQWJzGFBL1RZ4nqks3RJXuqlUoWm37HO6pu/pubhtml (all unsolved families in the original minimal prime problem (i.e. prime > base (b) is not required) for bases $2 \leq b \leq 50$ except b = 43, 47, 49) (there is also a zipped file https://mersenneforum.org/attachment.php?attachmentid=25078&d=1623428406 for them))

(Note: the factors only shown the algebraic forms, if you want the base b forms, see this post)

As previously mentioned, in practice to compute $M(L_b)$ one works with an underapproximation M of $M(L_b)$ and an overapproximation L of $L_b - sup(M)$. One then refines such approximations until $L = \emptyset$ from which it follows that $M = M(L_b)$.

For the initial approximation, note that every minimal prime in base b with at least 4 digits is of the form xY*z, where $x \in \{x \mid x$ is base-b digit, $x \neq 0\}$, z $\in \{z \mid z$ is base-b digit, $gcd(z,b)$ =

1}, and Y^* (for this (x,z) pair) = {y | xy, xz, yz, xyz are all composites}. (Of course, if xz is prime, then the Y^* set for this (x,z) pair is ∅)

Making use of this, our algorithm sets M to be the set of base-b representations of the minimal primes with at most 3 digits (which can be found simply by brute force) and L to be $\cup_{x,z} (xY^*z)$ as described above.

All remaining minimal primes are members of L, so to find them we explore the families in L. During this process, each family will be decomposed into possibly multiple other families. For example, a simple way of exploring the family xY^*z where $Y = \{y_1, ..., y_n\}$ is to decompose it into the families xY^*y_1z, ..., xY^*y_nz. If the smallest member (say xy_iz) of any such family happens to be prime, it can be added to M and the family xY^*y_iz removed from consideration. Furthermore, once M has been updated it may be possible to simplify some families in L. In this case, xY^*y_jz (for $j \neq i$) can be simplified to $x(Y-y_i)^*y_jz$ since no minimal prime contains xy_iz as a proper subsequence.

We call families of the form xy^*z (where x, z ∈ Σ_b^* and y ∈ Σ_b) *simple* families. Our algorithm then proceeds as follows:

1. Let

M := {minimal primes in base b of length ≤ 3}
$L := \cup_{x,z \in \Sigma b} (xY^*z)$
where $x \neq 0$ and Y is the set of digits y such that xyz has no subword in M.

2. While L contains non-simple families:

(a) Explore each family of L, and update L.
(b) Examine each family of L:

i. Let w be the shortest string in the family. If w has a subword in M, then remove the family from L. If w represents a prime, then add w to M and remove the family from L.
ii. If possible, simplify the family.
iii. Check if the family can be proven to contain no primes > base, and if so then remove the family from L.

(c) As much as possible and update L; after each split examine the new families as in (b).

At the conclusion of the algorithm described, L will consist of simple families (of the form xy^*z) which have not yet yielded a prime, but for which there is no obvious reason why there can't be a prime of such a form. In such a case, the only way to proceed is to test the primality of larger and larger numbers of such form and hope a prime is eventually discovered (we usually conjecture that there must be a prime > base (b) at some point if it cannot be proven to contain no primes > base (b), by covering congruence, algebraic factorization, or combine of them, since there is a heuristic argument that there are infinitely many such primes (reference), since by the prime number theorem, the chance that a random n-digit base b number is prime is approximately $1/n$ (reference reference) (also see

this page and this page, the chance is approximately $\frac{b-1}{ln(b)} \cdot \frac{b^{n-1}}{n}$, where *ln* is the natural logarithm). If one conjectures the numbers *xy*z* behave similarly (i.e. "*N* of the form *xy*z*" and "*N* is prime" are independent events) you would expect $\sum_{n=2}^{\infty} \frac{1}{n} = \infty$ (harmonic series is divergent) primes of the form *xy*z*, of course, this does not always happen, since some *xy*z* families can be proven to contain no primes > base (*b*), and every *xy*z* family has its own Nash weight (or difficulty), *xy*z* families which can be proven to contain no primes > base have Nash weight (or difficulty) 0, thus *xy*z* families are not "completely" random (but we still conjectured that for a *xy*z* families which cannot be proven to contain no primes or only finitely primes, using covering congruence, algebra factorization, or combine of them, the number of primes with ≤ *n* digit is roughly *c*ln*(*n*) for some positive constant *c*, the constant *c* varies with family *xy*z*). They are random enough that the prime number theorem can be used to predict their primality, but divisibility by small primes is not as random and can easily be predicted: Once one candidate is found to be divisible by a prime *p* or to have an algebraic factorization (e.g. difference-of-two-squares factorization, sum/difference-of-two-cubes factorization, Aurifeuillian factorization for x^4+4y^4), another predictable candidate will also be divisible by *p* or also have the same algebraic factorization. This decreases the probability of expected primes. Sometimes though, the candidates will never be divisible by a prime *p*, which increases the probability of expected primes. However, it is at least a reasonable conjecture in the absence of evidence to the contrary, the numbers in simple families are of the form $\frac{a \cdot b^n + c}{gcd(a+c,b-1)}$ for some fixed integer triple (*a*, *b*, *c*), where *a*≥1, *b*≥2 (*b* is the base), *c*≠0, *gcd*(*a*,*c*)=1, *gcd*(*b*,*c*)=1, this is an exponential sequence, there is also a similar conjecture for polynomial sequence: the Bunyakovsky conjecture, the condition is similar to our conjecture in this article, both are the small prime factors and the algebraic factors, the main difference is that polynomial sequence cannot have a covering set with >1 primes, however, unlike our conjecture (the analog of Bunyakovsky conjecture for exponential sequences), the analog of Dickson's conjecture and Schinzel's hypothesis H for exponential sequences is widely believed to be false, e.g. for all integer *k* divisible by 3, it is widely believed that there are only finitely many integers *n*≥1 such that $k*2^n±1$ are twin primes (see this page and this page and this page, the conjecture that 237 is the smallest odd number *k* divisible by 3 such that $k*2^n±1$ are never twin primes will never be proven, the smaller odd numbers *k* divisible by 3 with no known such twin primes (and unlikely any exist) are {111, 123, 153, 159, 171, 183, 189, 219, 225}), another example is that it is widely believed that 127 is the largest number *n* such that the Mersenne number 2^n-1 and the Wagstaff number $(2^n+1)/3$ are both primes (see New Mersenne Conjecture and its status page, the known such *n* are {3, 5, 7, 13, 17, 19, 31, 61, 127}, and they are listed in https://oeis.org/A107360) (in fact, if *n* is even number, then $(2^n+1)/3$ is not integer, thus we only need to consider odd *n*, and for odd number $n = 2*m+1$, $(2^n+1)/3 = (2*4^m+1)/3$, thus it can be written as the form $\frac{a \cdot b^n + c}{gcd(a+c,b-1)}$, with (*a*, *b*, *c*) = (2, 4, 1), thus is included in this conjecture, also, if *n* is odd composite, then 2^n-1 and $(2^n+1)/3$ are both composites, thus we only need to consider odd prime *n*), another example is that it is widely believed that there are only finitely many integers *n* such that *n* and *n*±1 all have primitive roots (the known such *n* are {2, 3, 4, 5, 6, 10, 18, 26, 82, 242,
13261687909436368734633837029995090067109312758094815943451355684192470326832804768010205770069260168834737042384420000006022058158963387968160292916287523165029802832132330561775181299908212255315879210032138211709801

267978611718212818248251166441580761640 2}), the last number is equal to $3^{541}-1$, and $3^{541}-1$ may be the largest such n, since it is widely believed that there are only finitely many integers $n \geq 1$ such that the given pair of exponential sequences both produce primes: $(2*3^n-1, 2*3^n+1)$, $((3^n+1)/2, 3^n+2)$, $((3^n-1)/2, 3^n-2)$, see https://oeis.org/A305237, also it is widely believed that for any polynomial sequence and any exponential sequence, there are only finitely many n such that both sequences produce primes, e.g. it is widely believed that only finitely many Mersenne exponents (i.e. primes p such that 2^p-1 is also prime) are Sophie Germain primes (such primes p are listed in https://oeis.org/A065406), i.e. the number of primes p such that $2*p+1$ and 2^p-1 are both prime is expected to be finite, also it is widely believed that only finitely many Mersenne exponents (i.e. primes p such that 2^p-1 is also prime) are members of twin primes pair (such primes p are listed in https://oeis.org/A346645), see this post and this thread). For example, the base 11 family 57^n, this family have already been searched to length 50000 with no prime or PRP found, however (we use the sense of http://www.iakovlev.org/zip/riesel2.pdf, https://stdkmd.net/nrr/1/10003.htm#prime_period, https://stdkmd.net/nrr/3/30001.htm#prime_period, https://stdkmd.net/nrr/1/13333.htm#prime_period, https://stdkmd.net/nrr/3/33331.htm#prime_period, https://stdkmd.net/nrr/1/11113.htm#prime_period, https://stdkmd.net/nrr/3/31111.htm#prime_period, https://mersenneforum.org/showpost.php?p=138737&postcount=24, https://mersenneforum.org/showpost.php?p=153508&postcount=147, to show this, i.e. this family (the base 11 family 57^n) cannot be ruled out as contain no primes > base, by covering congruence, algebraic factorization, or combine of them) the algebraic form of this family is $(57*11^n-7)/10$, and there is no n satisfying that $57*11^n$ and 7 are both r-th powers for some $r>1$ to make this number have difference-of-two-r-th-powers factorization (i.e. factorization of binomial numbers) (since 7 is not perfect power), nor there is n satisfying that $57*11^n$ and −7 are (one is 4th power, another is of the form $4*m^4$) to make this number have Aurifeuillian factorization for x^4+4y^4 (since −7 is neither 4th power nor of the form 4*m^4), thus, base 11 family 57^n has no algebraic factorization for any n, thus 57^n eventually should yield a prime unless it can be proven to contain no primes > base using covering congruence, and we have:

57^n is divisible by 2 for n == 1 mod 2
57^n is divisible by 13 for n == 2 mod 12
57^n is divisible by 17 for n == 4 mod 16
57^n is divisible by 5 for n == 0 mod 5
57^n is divisible by 23 for n == 6 mod 22
57^n is divisible by 601 for n == 8 mod 600
57^n is divisible by 97 for n == 12 mod 48
57^n is divisible by 1279 for n == 16 mod 426
…

and it does not appear to be any covering set of primes (and its Nash weight (or difficulty) is positive, and it has prime candidate), so there must be a prime at some point. If there is a covering set of primes of the base 11 family 57^n, then the period must be at least 107016229864 3200, which is extremely impossible, since according to the factordb page:

57^8 = 601 * 2033021, and *znorder*(*mod*(11,601)) = 600, *znorder*(*mod*(11,2033021)) = 101651, thus the period must be divisible by either 600 or 101651 (or both).
57^{18} = 285023 * 111189373092367, and *znorder*(*mod*(11,285023)) = 142511, *znorder*(*mod*(11,111189373092367)) = 37063124364122, thus the period must be divisible by either 142511 or 37063124364122 (or both).
57^{24} = 100124417 * 560737110230598229, and *znorder*(*mod*(11,100124417)) = 100124416, *znorder*(*mod*(11,560737110230598229)) = 140184277557649557, thus the period must be divisible by either 100124416 or 140184277557649557 (or both).

Thus, if there is a covering set of primes of the base 11 family 57^n, then the period must be at least *lcm*(600, 142511, 100124416) = 1070162298643200 (>10^{15}), and hence the base 11 family 57^n is extremely impossible to have a full covering set of primes.

If a form can be proven as only contain composite numbers by covering congruence, then every number of this form has small prime factors (usually < 10^6), and if a form can be proven as only contain composite numbers by algebraic factorization, then every number of this form has two factors with near size (for the case for difference-of-two-squares factorization and Aurifeuillian factorization of x^4+4y^4) or a factor with near double the size of the other (for the case for sum/difference-of-two-cubes factorization), if a for can be proven as only contain composite numbers by combine of them, then every number of this form meet at least one of these two conditions, but see the factorizations for *n* = 24 and *n* = 48 and *n* = 92, they do not meet any of these two conditions, thus this form cannot be ruled as composite for all *n*, and hence there must be a prime at some point.

The multiplicative order of *b* mod the primes is important in this problem, since if a prime *p* divides the number with *n* digits in a family in base *b*, then *p* also divides the number with *k**r*+*n* digits in the same family in base *b* for all nonnegative integer *k*, where *r* is the multiplicative order of *b* mod *p* (unless the multiplicative order of *b* mod *p* is 1, i.e. *p* divides *b*−1, in this case *p* also divides the number with *k**p*+*n* digits in the same family in base *b* for all nonnegative integer *k*), the primes *p* such that the multiplicative order of *b* mod *p* is *n* are exactly the primes *p* dividing *Zs*(*n*,*b*,1), where *Zs* is the Zsigmondy number, i.e. *Zs*(*n*,*b*,1) is the greatest divisor of b^n-1 that is coprime to b^m-1 for all positive integers *m* < *n*, with *b* ≥ 2 and *n* ≥ 1, if (and only if) there is only one such prime, then this prime is unique prime in base *b*, see list of the multiplicative order of *b* mod *p* for *b*≤128 and primes *p*≤4096, list of primes *p* such that the multiplicative order of *b* mod *p* is *n* for 2≤*b*≤64 and 1≤*n*≤64 (the same lists in factorizations of $b^n\pm1$: only primitive prime factors all prime factors all prime factors, separate Aurifeuillian L, M's), smallest prime *p* such that znorder(Mod(*m*,*p*)) = (*p*−1)/*n* for 2≤*m*≤128 and 1≤*n*≤128, bases *b* such that Phi(*n*,*b*) (where Phi is cyclotomic polynomial) has algebra factors or small prime factors, bases *b* such that there is unique prime with period length *n*, unique period length in base *b*, also see factorization of $b^n\pm1$ (which is equivalent to factorization of *Zs*(*n*,*b*,1)) with *b*≤12 13≤*b*≤99 *b*=10 *b* is prime *b*=*n* and *b* is prime any *b* any *b*, also see this page and this page and this page.

(these references only include the multiplicative order of the base (*b*) mod the primes (i.e. *znorder*(Mod(*b*,*p*)) with prime *p*), if you want to calculate the multiplicative order of the base (*b*) mod a composite number *c* coprime to *b*, factor *c* to product of distinct prime powers, and calculate the multiplicative order of *b* mod p^e (i.e. *znorder*(Mod(*b*,p^e))) for all these prime powers p^e, and *znorder*(Mod(b,p^e)) = $p^{max(e-r(b,p),0)}$**znorder*(Mod(b,p)), where $r(b,p)$ is the largest integer s such that p^s divides $b^{p-1}-1$, the primes p such that $r(b,p) > 1$ are called generalized Wieferich prime base b, and if $r(p,q)$ and $r(q,p)$ are both > 1 for primes p and q, then (p,q) are called Wieferich pair, there are currently only 7 known such (p,q) pairs: (2, 1093), (3, 1006003), (5, 1645333507), (5, 188748146801), (83, 4871), (911, 318917), (2903, 18787), primes p such that $r(b,p) > 1$ for b = the smallest primitive root mod p (A001918(n), if p is the n-th prime) are called non-generous primes (https://oeis.org/A055578), there are currently only 3 known such primes p: 2, 40487, 6692367337, generalized Wieferich primes and Wieferich pairs are related to Fermat Last Theorem and *abc* conjecture and Catalan conjecture, and for the values of $r(b,p)$ see http://www.fermatquotient.com/FermatQuotienten/FermQ_Sort.txt and http://www.fermatquotient.com/FermatQuotienten/FermQ_Sorg.txt and http://www.asahi-net.or.jp/~KC2H-MSM/mathland/math11/fer_quo.htm and http://www.urticator.net/essay/6/624.html and https://archive.fo/Hd9Rr and http://www.sci.kobe-u.ac.jp/old/seminar/pdf/2008_yamazaki.pdf, data is available for primes $p \le$ search limit in these pages, for a base b, if p is not list here then $r(b,p) = 1$, if p is list here with no exponent given then $r(b,p) = 2$, if p is list here with an exponent given then $r(b,p)$ = this exponent, perfect power bases are not listed in these two pages, and $r(b^m,p) = p^s*r(b,p)$ if p is odd prime, where s is the largest nonnegative integer such that p^s divides m, $r(b^m,2)$ = largest nonnegative integer s such that 2^s divides b^m-1, finally, calculate the least common multiple of these multiplicative orders of b mod p^e) (references: http://go.helms-net.de/math/expdioph/fermatquot_ge2_table1.htm http://go.helms-net.de/math/expdioph/fermatquotients.pdf)

The smallest Wieferich primes in base b for b = 2, 3, 4, … 36 are 1093, 11, 1093, 2, 66161, 5, 3, 2, 3, 71, 2693, 2, 29, 29131, 1093, 2, 5, 3, 281, 2, 13, 13, 5, 2, 3, 11, 3, 2, 7, 7, 5, 2, 46145917691, 3, 66161 (*OEIS* sequence A039951)

The smallest base such that p is a Wieferich prime for the first 100 primes p (i.e. p = 2, 3, 5, 7, …, 541) are 5, 8, 7, 18, 3, 19, 38, 28, 28, 14, 115, 18, 51, 19, 53, 338, 53, 264, 143, 11, 306, 31, 99, 184, 53, 181, 43, 164, 96, 68, 38, 58, 19, 328, 313, 78, 226, 65, 253, 259, 532, 78, 176, 276, 143, 174, 165, 69, 330, 44, 33, 332, 94, 263, 48, 79, 171, 747, 731, 20, 147, 91, 40, 1260, 104, 707, 18, 476, 75, 223, 14, 257, 159, 242, 174, 1259, 632, 175, 280, 751, 369, 251, 867, 349, 194, 590, 210, 735, 52, 255, 863, 583, 10, 753, 346, 1449, 93, 308, 241, 555 (*OEIS* sequence A039678)

b	known generalized Wieferich primes base b (written in base 10) (search limit: $6*10^{17}$ for b = 2 (and hence b = 4, 8, 16, 32), $1.2*10^{15}$ for b = 3, 5, 7 (and hence b = 9, 25, 27), $1.479*10^{14}$ for other b)	*OEIS* sequence
2	1093, 3511	A001220
3	11, 1006003	A014127
4	1093, 3511	Essentially the same as A001220, since $4 = 2^2$ (2 divides

		2, thus no need to add this prime)
5	2, 20771, 40487, 53471161, 1645333507, 6692367337, 188748146801	A123692
6	66161, 534851, 3152573	A212583
7	5, 491531	A123693
8	3, 1093, 3511	Essentially the same as A001220 plus the prime 3, since 8 = 2^3
9	2 (order 2), 11, 1006003	Essentially the same as A014127 plus the prime 2, since 9 = 3^2
10	3, 487, 56598313	A045616
11	71	
12	2693, 123653	A111027
13	2, 863, 1747591	A128667
14	29, 353, 7596952219	A234810
15	29131, 119327070011	A242741
16	1093, 3511	Essentially the same as A001220, since 16 = 2^4 (2 divides 2, thus no need to add this prime)
17	2 (order 3), 3, 46021, 48947, 478225523351	A128668
18	5, 7 (order 2), 37, 331, 33923, 1284043	A244260
19	3, 7 (order 2), 13, 43, 137, 63061489	A090968
20	281, 46457, 9377747, 122959073	A242982
21	2	
22	13, 673, 1595813, 492366587, 9809862296159	A298951
23	13, 2481757, 13703077, 15546404183, 2549536629329	A128669

24	5, 25633	
25	2 (order 2), 20771, 40487, 53471161, 1645333507, 6692367337, 188748146801	Essentially the same as A123692, since 25 = 5^2 (2 is already a Wieferich prime base 5)
26	3 (order 2), 5, 71, 486999673, 6695256707	A306255
27	11, 1006003	Essentially the same as A014127, since 27 = 3^3 (3 divides 3, thus no need to add this prime)
28	3 (order 2), 19, 23	
29	2	
30	7, 160541, 94727075783	A306256
31	7, 79, 6451, 2806861	A331424
32	5, 1093, 3511	Essentially the same as A001220 plus the prime 5, since 32 = 2^5
33	2 (order 4), 233, 47441, 9639595369	
34	46145917691	
35	3, 1613, 3571	
36	66161, 534851, 3152573	Essentially the same as A212583, since 36 = 6^2 (2 divides 6, thus no need to add this prime)

The numbers in simple families are of the form $\frac{a \cdot b^n + c}{gcd(a+c, b-1)}$ for some fixed integers *a*, *b*, *c* where *a*≥1, *b*≥2 (*b* is the base), *c*≠0, *gcd*(*a*,*c*)=1, *gcd*(*b*,*c*)=1 (thus, all large minimal primes base *b* (but possible not all minimal primes base *b* if *b* is large, e.g. *b* = 25, 29, 31, 35) have a nice short algebraic description (see this page and this page, the prime numbers in these two pages do *not* have nice short algebraic descriptions, also see this page) and have simple expression (expression with ≤ 40 characters, all taken from “0” “1” “2” “3” “4” “5” “6”

"7" "8" "9" "+" "-" "*" "/" "^" "(" ")"), factorial (!), double factorial (!!), and primorial (#) are not allowed since they can be used to ensure many small factors, see this page). Except in the special case $c = \pm 1$ and $gcd(a+c,b-1) = 1$, when n is large the known primality tests for such a number are too inefficient to run (since this special case $c = \pm 1$ and $gcd(a+c,b-1) = 1$ is the only case which $N-1$ and/or $N+1$ is smooth, i.e. the case $c = 1$ and $gcd(a+c,b-1) = 1$ (corresponding to generalized Proth prime base b: $a*b^n+1$, they are related to generalized Sierpinski conjecture base b) can be easily proven prime using Pocklington $N-1$ method, and the case $c = -1$ and $gcd(a+c,b-1) = 1$ (corresponding to generalized Riesel prime base b: $a*b^n-1$, they are related to generalized Riesel conjecture base b) can be easily proven prime using Morrison $N+1$ method) (see the $N-1$ and $N+1$ primality tests and A variant $N+1$ primality test and Wikipedia page of Pocklington primality test and Brillhart-Lehmer-Selfridge primality test). In this case one must resort to a probable primality test such as a Miller–Rabin primality test or a Baillie–PSW primality test, unless a divisor of the number can be found, and thus these numbers cannot be definitely primes and can only be probable primes, and we cannot definitely say that the corresponding families can be removed from the list of unsolved families, and we cannot definitely compute this part of the set $M(L_b)$. Since we are testing many numbers in an exponential sequence, it is possible to use a sieving process (such as *srsieve* software) to find divisors rather than using trial division, i.e. we will remove the integers n such that $\frac{a \cdot b^n+c}{gcd(a+c,b-1)}$ either has a prime factor less than certain limit (say 2^{32}) or has algebraic factorization, and test the primality of $\frac{a \cdot b^n+c}{gcd(a+c,b-1)}$ for other integers n.

To do this, we made use of Geoffrey Reynolds' *srsieve* software. This program uses the baby-step giant-step algorithm to find all primes p which divide $a*b^n+c$ where p and n lie in a specified range. Since this program cannot handle the general case $\frac{a \cdot b^n+c}{gcd(a+c,b-1)}$ when $gcd(a+c,b-1) > 1$ we only used it to sieve the sequence $a*b^n+c$ for primes p not dividing $gcd(a+c,b-1)$, and initialized the list of candidates to not include n for which there is some prime p dividing $gcd(a+c,b-1)$ for which p divides $\frac{a \cdot b^n+c}{gcd(a+c,b-1)}$. The program had to be modified slightly to remove a check which would prevent it from running in the case when a, b, and c were all odd (since then 2 divides $a*b^n+c$, but 2 may not divide $\frac{a \cdot b^n+c}{gcd(a+c,b-1)}$).

Once the numbers with small divisors had been removed, it remained to test the remaining numbers using a probable primality test. For this we used the software *LLR* by Jean Penné. Although undocumented, it is possible to run this program on numbers of the form $\frac{a \cdot b^n+c}{gcd(a+c,b-1)}$ when $gcd(a+c,b-1) > 1$, so this program required no modifications (also, *LLR* can do a proven primality test (i.e. prove the primality) for numbers of the form $a*b^n\pm 1$ (i.e. the special case $c = \pm 1$ and $gcd(a+c,b-1) = 1$) with $b^n > a$). A script was also written which allowed one to run *srsieve* while *LLR* was testing the remaining candidates, so that when a divisor was found by *srsieve* on a number which had not yet been tested by *LLR* it would be removed from the list of candidates. In the cases where the elements of $M(L_b)$ could be proven prime rigorously, we employed *PRIMO* by Marcel Martin, an elliptic curve primality proving implementation (for the primes of the form $\frac{a \cdot b^n+c}{gcd(a+c,b-1)}$, with $c \neq \pm 1$ and/or $gcd(a+c,b-1) \neq 1$, we cannot use the classical tests (including the tests of $N-1$, $N+1$, N^2+1, N^2+N+1, N^2-N+1 (all such polynomials are cyclotomic polynomials of N, and such tests are called cyclotomy proofs, see this page), and the combined tests), since for these primes, none of them is at least 1/3 factorable (Brillhart-Lehmer-Selfridge primality test) (see this page) (if we want to use the classical tests to prove the primality of N, then we must factor at least one of $N-1$, $N+1$, N^2+1, N^2+N+1, N^2-N+1 to the factored part ≥33.3333% (i.e. product of known prime factors ≥ the cube root of N), and except trial division with the primes up to certain limit (say 2^{64}) and the algebra factors (e.g. difference-of-two-squares factorization, sum/difference-of-two-cubes factorization, Aurifeuillian factorization, and algebra factors of the Cunningham number $b^n \pm 1$ (b^n-1 can be factored to product of all $\Phi_d(b)$ with d dividing n, and b^n+1 can be factored to product of all $\Phi_d(b)$ with d dividing $2*n$ but not dividing n, where Φ is the cyclotomic polynomial, the nth cyclotomic polynomial (Φ_n) has degree *eulerphi*(n), and its *eulerphi*(n) roots (by the fundamental theorem of algebra, it has *eulerphi*(n) roots, counted with multiplicity) are all nth primitive roots of unity), see this page and this page) (sometimes non-Cunningham numbers can also have algebra factors (e.g. difference-of-two-squares factorization, sum/difference-of-two-cubes factorization, Aurifeuillian factorization), such as $k*b^n-1$ when k is a square and n is even and $k*b^n-1$ when k is a cube and n is divisible by 3 and $k*b^n+1$ when k is a cube and n is divisible by 3 and 54^n in base $b = 10$ when n is even and 5^n2 in base $b = 10$ when n is either even or == 1 mod 3 (or both) and $3773*88^n-1$ when n == 2 mod 3 and $80^{298}C$ in base $b = 18$, and the examples of families which can be ruled out as contain no primes > b by all or partial algebraic factors), we can use elliptic-curve factorization method (ECM) (reference: *ECMNET* and its record page and its top 50 table), Pollard $P-1$ method, Williams $P+1$ method, Pollard rho method, Fermat method (which is best when there is a factor near the square root of n, and is impractically for general number n), special number field sieve (SNFS), general number field sieve (GNFS), etc. to factor the numbers (see this reference), however, all these factorization algorithms take long time, i.e. they cannot be done in polynomial time (unlike primality proving, when the numbers are sufficiently large, no efficient, non-quantum integer factorization algorithm is known, i.e. integer factorization may be P-complete and NP-complete and NP-hard (thus, factor a large integer is much harder than determining whether an integer of the same size is prime (determining whether an integer is prime and factor an integer are two completely different problems, we can quickly use Fermat primality test to prove that an integer is composite, although the most ancient trial division and sieve of Eratosthenes can solving these two problems simultaneously), there are many numbers with 500 digits to 10000 digits which are known to be composite but do not have any known factors other than 1 and themselves). However, it has not been proven that no efficient algorithm exists (this is an unsolved problem in computer science). Also, many OEIS sequences need factors, see https://oeis.org/wiki/OEIS_sequences_needing_factors. Besides, the current integer factorization record of largest penultimate prime factor (i.e. factor other than the largest one, not count the algebraic factors) is 151-digit 1383935292384841005422034635844427018156982031199817979611378169173761867125492953158940839353699757587741707731483357994575596276075222709507199980369 (factordb entry), which is a factor of the Aurifeuillian M-part of $7^{889}+1$ (factordb entry) and found by special number field sieve (SNFS), see https://homes.cerias.purdue.edu/~ssw/cun/champ.txt, and the current record of elliptic-curve factorization method (ECM) is 83-digit 16559819925107279963180573885975861071762981898238616724384425798932514688349020287 (factordb entry), which is a factor of $7^{337}+1$ (factordb entry), and its *B1/B2* is 7600000000, and its *sigma* is 3882127693, see ECM top 50 table and factordb list of all prime factors ($>10^{24}$) found by the ECM method. The presumed difficulty of this problem is at the heart of widely used algorithms in cryptography such as RSA, there are many large semiprimes, called RSA numbers, which are very hard to factor and are part of the RSA Factoring Challenge. Besides, integer factorization can be used for public-key cryptography is because it has no known polynomial time algorithm. Many areas of mathematics and computer science have been brought to bear on the problem, including elliptic curves, algebraic number theory, and quantum computing), and hence to do this is impractically), i.e. they are ordinary primes, and if the prime is not large (say less than 10^{25000}), we can use elliptic curve primality proving (ECPP) to proof (see *PRIMO* top 20 records and elliptic curve primality proving top 20 records and top primes proven by Francois Morain's programs) and make primality certificate, but if the prime is very large (say > 10^{25000}), the known primality tests for such a number are too inefficient to run (although there is AKS primality test, which can prove the primality of any general prime in polynomial time, but since its time complexity is $O(ln(N)^{12})$, and if we can do 10^9 bitwise operations per second, use this test to prove the primality of a 5000-digit (in decimal) prime need 5.422859049×10^{39} seconds, or 1.719577324×10^{32} years, much longer than the age of the universe, thus to do this test is still impractically), thus we can only resort to a probable primality test such as Miller–Rabin primality test and Baillie–PSW primality test, unless a divisor of the number can be found, and hence we cannot prove the primality of this number, thus we cannot definitely say that the corresponding families can be removed from the list of unsolved families, and we cannot definitely compute this part of the sets $M(L_b)$.

If we assume a number which has passed the Fermat primality tests to many bases is in fact prime, our list for base 16 minimal primes would wrongly include the composite 15^{63} (its value is $(4*16^{63}-1)/3$), and our list for base 9 minimal primes would wrongly include the composite 1^{13} (its value is $(9^{13}-1)/8$) (and hence would wrongly exclude the prime 561^{36}, since this prime has 1^{13} as subsequence), although their corresponding families (1{5} in base 16, {1} in base 9, respectively) can be ruled out as only contain composite numbers (only count the numbers > base), and our data will be wrong for these bases (thus, for this minimal prime problem in base *b*, especially for square base *b*, we should not assume a number which has passed the Fermat primality tests to many bases is in fact prime, we need to combine with Lucas primality tests, to do Baillie–PSW primality test), see this page and this page for the examples for Fermat pseudoprimes in related problems (Sierpinski problems and Riesel problems and generalized repunit primes problems, all are related to the problem in this article), also see this page for Fermat pseudoprimes in the original minimal prime problem. (reference: the danger of relying only on Fermat tests) (reference of pseudoprimes) (also https://oeis.org/A014233: Smallest number which is strong pseudoprime to all the first *n* prime bases (i.e. base 2, base 3, base 5, base 7, base 11, …, base "*n*-th prime")) (also references for datas for pseudoprimes: http://ntheory.org/pseudoprimes.html http://www.cecm.sfu.ca/Pseudoprimes/index-2-to-64.html, datas: Fermat pseudoprimes base 2 strong pseudoprimes base 2 Lucas pseudoprimes strong Lucas pseudoprimes Fermat pseudoprimes base 2 < 2^{64} Fermat pseudoprimes base 2 < 2^{64} with strong pseudoprimes and Carmichael number marked Fermat pseudoprime base 2 < 2^{64} with prime factorizations)

Number	Bases $b \le 64$ such that this number is Fermat pseudoprime (called "Fermat liars")	count

15^{63} (base b = 16)	2, 3, 4, 6, 8, 9, 11, 12, 13, 16, 17, 18, 19, 22, 24, 26, 27, 29, 31, 32, 33, 34, 36, 37, 38, 39, 41, 44, 47, 48, 51, 52, 54, 57, 58, 59, 61, 62, 64	39 (61.90%)
1^{13} (base b = 9)	2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 13, 14, 15, 16, 18, 19, 20, 21, 23, 24, 25, 26, 27, 28, 29, 30, 32, 35, 36, 37, 38, 39, 40, 41, 42, 45, 46, 48, 49, 50, 52, 54, 56, 57, 58, 59, 60, 61, 63, 64	50 (79.37%)
85^{36} (base b = 16)	3, 5, 8, 9, 13, 15, 17, 22, 24, 25, 27, 28, 29, 39, 40, 41, 45, 46, 47, 51, 53, 62, 64	23 (36.51%)
28462346 $*3^7+1$ (see this page)	3, 4, 7, 9, 10, 11, 12, 13, 16, 17, 19, 21, 25, 27, 28, 29, 30, 33, 36, 39, 40, 41, 44, 46, 47, 48, 49, 51, 52, 53, 57, 59, 61, 62, 63, 64	36 (57.14%)
I0901 (base b = 26) (see this page)	2, 3, 4, 6, 7, 8, 9, 11, 12, 13, 14, 16, 18, 21, 22, 23, 24, 25, 26, 27, 28, 32, 33, 36, 39, 41, 42, 43, 44, 46, 48, 49, 50, 52, 53, 54, 56, 59, 63, 64	40 (63.49%)

The 10 largest known primes which are proven primes using elliptic curve primality proving (they are also the 10 largest known ordinary primes (i.e. neither *N*−1 nor *N*+1 is ≥33.3333% factorable) are:

Prime	Number of digits	Primality certificate
$(10^{49081}-1)/9$ (a repunit prime)	49081 (1111111111…1111111111)	factordb entry all decimal digits primality certificate
partition(1289844341) (where *partition* is the partition function)	40000 (1008370026…2253769461)	factordb entry all decimal digits primality certificate
$2^{116224}-15905$ (a dual Riesel prime, although not the smallest dual Riesel prime for k = 15905 (i.e. prime of the form $2^n-15905$), $2^n-15905$ is already prime for n = 14 and 22 and 28)	34987 (8132349794…5583993311)	factordb entry all decimal digits primality certificate
$(14665*10^{34110}-56641)/9999$ (a palindromic prime)	34111 (1466646664…4666466641)	factordb entry all decimal digits primality certificate
A number with picture "57885161"	34093 (1000000000…0000532669)	factordb entry all decimal digits primality certificate
Lucas(148091) (the 148091st Lucas number)	30950 (1543946543…5102253799	factordb entry all decimal digits primality certificate

	)	
Fibonacci(148091) (the 148091st Fibonacci number)	30949 (6904738850…7109274809)	factordb entry all decimal digits primality certificate
−*tau*(331^{2128}), where *tau* is Ramanujan tau function)	29492 (4272870686…2041256991)	factordb entry all decimal digits primality certificate
Lucas(140057) (the 140057th Lucas number)	29271 (1520304185…6629047971)	factordb entry all decimal digits primality certificate
$(2^{95369}+1)/3$ (a Wagstaff prime)	28709 (2834883547…8961702571)	factordb entry all decimal digits primality certificate

Fermat pseudoprime (to base *b* = 2: https://oeis.org/A001567, and see this data)	Lucas pseudoprime (to parameters (*P*, *Q*) = (1, −1): https://oeis.org/A081264 union https://oeis.org/A141137, and see this data) (to parameters (*P*, *Q*) defined by Selfridge's Method *A*: https://oeis.org/A217120, and see this data)
Strong Fermat pseudoprime (to base *b* = 2: https://oeis.org/A001262, and see this data)	Strong Lucas pseudoprime (to parameters (*P*, *Q*) defined by Selfridge's Method *A*: https://oeis.org/A217255, and see this data)
Over Fermat pseudoprime (to base *b* = 2: composite factors of A019320(*n*) / *gcd*(A019320(*n*), *n*) = A064078(*n*) for some *n*, there is an OEIS sequence: https://oeis.org/A141232)	Over Lucas pseudoprime (to parameters (*P*, *Q*) = (1, −1): composite factors of A061446(*n*) / *gcd*(A061446(*n*), *n*) = A178763(*n*) for some *n*)
Smallest *n* such that a given prime *p* divides 2^n-1: https://oeis.org/A014664	Smallest *n* such that a given prime *p* divides *Fibonacci*(*n*): https://oeis.org/A001177
Numbers *n* such that 2^n-1 is prime: https://oeis.org/A000043	Numbers *n* such that *Fibonacci*(*n*) is prime: https://oeis.org/A001605
Numbers *n* such that $(2^n+1)/3$ is prime: https://oeis.org/A000978	Numbers *n* such that *Lucas*(*n*) is prime: https://oeis.org/A001606
Numbers *n* such that 2^n-1 and $(2^n+1)/3$ are both primes: https://oeis.org/A107360	Numbers *n* such that *Fibonacci*(*n*) and *Lucas*(*n*) are both primes: https://oeis.org/A080327
Numbers *n* such that A019320(*n*) / *gcd*(A019320(*n*), *n*) = A064078(*n*) is prime: https://oeis.org/A161508	Numbers *n* such that A061446(*n*) / *gcd*(A061446(*n*), *n*) = A178763(*n*) is prime: https://oeis.org/A152012
Unique primes in base 2:	Prime Fibonacci integers:

https://oeis.org/A144755 (exactly the primes dividing no over Fermat pseudoprime (to base $b = 2$)	https://oeis.org/A178762 (exactly the primes dividing no over Lucas pseudoprime (to parameters $(P, Q) = (1, -1)$)
Primes with primitive root 2: https://oeis.org/A001122	Primes with Fibonacci primitive root: https://oeis.org/A214029
Cyclotomic polynomial (A019320(n) = *Phi*(n,2), A019321(n) = *Phi*(n,3), …)	Fibcyclotomic polynomial (A061446(n) = *FibPhi*(n,1), A008555(n) = *FibPhi*(n,2), …)
Fermat quotient to base 2: https://oeis.org/A007663	Fibonacci quotient: https://oeis.org/A092330
Wieferich prime (to base $b = 2$)	Wall–Sun–Sun prime (to parameters $(P, Q) = (1, -1)$)
Baillie–PSW pseudoprime (none are known, none < 2^{64} exist)	
Carmichael number (https://oeis.org/A002997)	Lucas–Carmichael number (https://oeis.org/A006972)
Euler's totient function (https://oeis.org/A000010)	Dedekind psi function (https://oeis.org/A001615)
Range of Euler's totient function (https://oeis.org/A002202), also even nontotient numbers (https://oeis.org/A005277)	Range of Dedekind psi function (https://oeis.org/A203444), also even non-Dedekind numbers (https://oeis.org/A307055)
Pépin primality test (for Fermat numbers, i.e. numbers of the form 2^n+1 (https://oeis.org/A000051), if 2^n+1 is prime, then n must be power of 2, such primes are https://oeis.org/A019434)	Lucas–Lehmer primality test (for Mersenne numbers, i.e. numbers of the form 2^n-1 (https://oeis.org/A000225), if 2^n-1 is prime, then n must be prime, such primes are https://oeis.org/A000668)
https://oeis.org/A060377 (Pépin primality test numbers)	https://oeis.org/A003010 (Lucas–Lehmer primality test numbers)
https://oeis.org/A152153 (Residues of Pépin primality test for Fermat numbers)	https://oeis.org/A095847 (Residues of Lucas–Lehmer primality test for Mersenne numbers)
https://oeis.org/A129802 (Possible bases for Pépin primality test for Fermat numbers, the original base for Pépin primality test is 3)	https://oeis.org/A018844 (Possible starting values for Lucas–Lehmer primality test for Mersenne numbers, the original starting value for Lucas–Lehmer primality test is 4)
Proth primality test (for numbers of the form $k*2^n+1$ with k odd and $k<2^n$, i.e. Proth numbers, https://oeis.org/A080075, such primes are https://oeis.org/A080076, also there is a list of such primes sorted by k)	Lucas–Lehmer–Riesel primality test (for numbers of the form $k*2^n-1$ with k odd and $k<2^n$, i.e. Proth numbers of the second kind, https://oeis.org/A112714, such primes are https://oeis.org/A112715, also there is a list of such primes sorted by k)
Sierpiński problem (finding and proving the	Riesel problem (finding and proving the

smallest odd k such that $k*2^n+1$ is composite for all $n \ge 1$, the smallest such k is conjectured to be 78557, such k are called Sierpiński numbers, see https://oeis.org/A076336, also there is a list of primes of the form $k*2^n+1$ for odd k)	smallest odd k such that $k*2^n-1$ is composite for all $n \ge 1$, the smallest such k is conjectured to be 509203, such k are called Riesel numbers, see https://oeis.org/A101036, also there is a list of primes of the form $k*2^n-1$ for odd k)
Pocklington $N-1$ primality test (for numbers n such that $n-1$ can be trivially fully factored) (factordb list of primes proven by this primality test) (factordb list of large primes (≥100000 digits) proven by this primality test)	Morrison $N+1$ primality test (for numbers n such that $n+1$ can be trivially fully factored) (factordb list of primes proven by this primality test) (factordb list of large primes (≥100000 digits) proven by this primality test)
Generalized Sierpiński problems to bases $b > 2$ (finding and proving the smallest k such that $k*b^n+1$ is composite for all $n \ge 1$)	Generalized Riesel problems to bases $b > 2$ (finding and proving the smallest k such that $k*b^n-1$ is composite for all $n \ge 1$)
Combined $N-1$ / $N+1$ primality test (and other cyclotomy tests, i.e. $\Phi_r(N)$ for small r (where Φ is the cyclotomic polynomial), including N^2+1, N^2+N+1, N^2-N+1) (factordb list of primes proven by this primality test)	
Pollard $P-1$ integer factorization method (factordb list of prime factors found by this method)	Williams $P+1$ integer factorization method (factordb list of prime factors found by this method)

No matter we want to check whether a given family $xy*z$ in given base b can be ruled out as containing no primes > base, or to factor $N-1$ or/and $N+1$ for a large minimal prime in base b to prove that this number is really prime, we need to factor the numbers of the form $xy*z$ (at first, we find all algebraic factors of $N-1$ or/and $N+1$ (e.g. difference-of-two-squares factorization, sum/difference-of-two-cubes factorization, Aurifeuillian factorization, and algebra factors of the Cunningham number $b^n\pm1$ (b^n-1 can be factored to product of all $\Phi_d(b)$ with d dividing n, and b^n+1 can be factored to product of all $\Phi_d(b)$ with d dividing $2*n$ but not dividing n, where Φ is the cyclotomic polynomial, the nth cyclotomic polynomial (Φ_n) has degree *eulerphi*(n), and its *eulerphi*(n) roots (by the fundamental theorem of algebra, it has *eulerphi*(n) roots, counted with multiplicity) are all nth primitive roots of unity), see this page and this page and this page and this page) (sometimes non-Cunningham numbers can also have algebra factors (e.g. difference-of-two-squares factorization, sum/difference-of-two-cubes factorization, Aurifeuillian factorization), such as 54^n in base b = 10 when n is even and $3773*88^n-1$ when n == 2 mod 3 and 80^{298}C in base b = 18, and the examples of families which can be ruled out as contain no primes > b by all or partial algebraic factors)), for the factorization of the Cunningham numbers $b^n\pm1$ (which is equivalent to factor the numbers in the families {1} and 1{0}1 in base b) see: b≤12 13≤b≤99 b=10 b is prime b=n and b is prime any b any b, and for the factorization of numbers in the families $xy*z$ in base b other then the Cunningham numbers $b^n\pm1$ see: b=10, families {x}y b=10, families x{y} b=10, families {x}yx b=10, families xy{x} b=10, families x{y}x b=10, families x{y}z b=2, families 11{0}1, 101{0}1, 111{0}1, 1001{0}1, 1011{0}1, 1101{0}1, 1111{0}1, 10{1}, 100{1}, 110{1}, 1000{1}, 1010{1}, 1100{1}, 1110{1} b=3, family {2}1

Some families *xy*z* could not be ruled out as containing no primes > base, but no primes > base could be found in the family, even after searching through numbers with over 50000 digits. Many *xy*z* families contain no small primes even though they do contain very large primes, for example: (show the factordb link for the list of the factors of numbers in these families, like https://stdkmd.net/nrr/1/10003.htm#prime_period, https://stdkmd.net/nrr/3/30001.htm#prime_period, https://stdkmd.net/nrr/1/13333.htm#prime_period, https://stdkmd.net/nrr/3/33331.htm#prime_period, https://stdkmd.net/nrr/1/11113.htm#prime_period, https://stdkmd.net/nrr/3/31111.htm#prime_period)

* In base 5, the smallest prime in the family 10^n13 (algebraic form: $5^{n+2}+8$) ($n\geq0$) is $10^{93}13$ (algebraic form: $5^{95}+8$) (factordb list of the factorization of the numbers of this form) (factordb entry of this prime) (this prime written in base 5)
* In base 8, the smallest prime in the family 4^n7 (algebraic form: $(4*8^{n+1}+17)/7$) ($n\geq1$) is $4^{220}7$ (algebraic form: $(4*8^{221}+17)/7$) (the prime 7 (i.e. n = 0) is not counted since the prime must be > base) (factordb list of the factorization of the numbers of this form) (factordb entry of this prime) (this prime written in base 8)
* In base 9, the smallest prime in the family 30^n11 (algebraic form: $3*9^{n+2}+10$) ($n\geq0$) is $30^{1158}11$ (algebraic form: $3*9^{1160}+10$) (factordb list of the factorization of the numbers of this form) (factordb entry of this prime) (primality certificate of this prime) (this prime written in base 9)
* In base 9, the smallest prime in the family 27^n07 (algebraic form: $(23*9^{n+2}-511)/8$) ($n\geq0$) is $27^{686}07$ (algebraic form: $(23*9^{688}-511)/8$) (factordb list of the factorization of the numbers of this form) (factordb entry of this prime) (primality certificate of this prime) (this prime written in base 9)
* In base 9, the smallest prime in the family 76^n2 (algebraic form: $(31*9^{n+1}-19)/4$) ($n\geq0$) is $76^{329}2$ (algebraic form: $(31*9^{330}-19)/4$) (factordb list of the factorization of the numbers of this form) (factordb entry of this prime) (primality certificate of this prime) (this prime written in base 9)
* In base 10, the smallest prime in the family 50^n27 (algebraic form: $5*10^{n+2}+27$) ($n\geq0$) is $50^{28}27$ (algebraic form: $5*10^{30}+27$) (factordb list of the factorization of the numbers of this form) (factordb entry of this prime) (this prime written in base 10)
* In base 11, family 57^n (algebraic form: $(57*11^n-7)/10$) ($n\geq1$) cannot be ruled out as containing no primes > base (using covering congruence, algebra factorization, or combine of them) but no primes > base found in the family after searching to length 50000 (the prime 5 (i.e. n = 0) is not counted since the prime must be > base) (factordb list of the factorization of the numbers of this form)
* In base 12, the smallest prime in the family 40^n77 (algebraic form: $4*12^{n+2}+91$) ($n\geq0$) is $40^{39}77$ (algebraic form: $4*12^{41}+91$) (factordb list of the factorization of the numbers of this form) (factordb entry of this prime) (this prime written in base 12)
* In base 12, the smallest prime in the family $B0^n9B$ (algebraic form: $11*12^{n+2}+119$) ($n\geq0$) is $B0^{27}9B$ (algebraic form: $11*12^{29}+119$) (factordb list of the factorization of the numbers of this form) (factordb entry of this prime) (this prime written in base 12)
* In base 13, the smallest prime in the family 80^n111 (algebraic form: $8*13^{n+3}+183$) ($n\geq0$) is $80^{32017}111$ (algebraic form: $8*13^{32020}+183$) (this prime is only a probable prime, i.e. not definitely prime) (factordb list of the factorization of the numbers of this form) (factordb entry

of this PRP) (factorization of $n-1$ for this PRP) (factorization of $n+1$ for this PRP) (this PRP written in base 13)
* In base 13, the smallest prime in the family 2B30^{n}1 (algebraic form: $484*13^{n+1}+1$) ($n\geq0$) is 2B30^{15197}1 (algebraic form: $484*13^{15198}+1$) (factordb list of the factorization of the numbers of this form) (factordb entry of this prime) (this prime can be easily proven prime using the $n-1$ test, since $n-1$ = $2^2 * 11^2 * 13^{15198}$ is trivially 100% factored) (this prime written in base 13)
* In base 13, the smallest prime in the family B0^{n}BBA (algebraic form: $11*13^{n+3}+2012$) ($n\geq0$) is B0^{6540}BBA (algebraic form: $11*13^{6543}+2012$) (this prime is only a probable prime, i.e. not definitely prime) (factordb list of the factorization of the numbers of this form) (factordb entry of this PRP) (factorization of $n-1$ for this PRP) (factorization of $n+1$ for this PRP) (this PRP written in base 13)
* In base 13, the smallest prime in the family 390^{n}1 (algebraic form: $48*13^{n+1}+1$) ($n\geq0$) is 390^{6266}1 (algebraic form: $48*13^{6267}+1$) (factordb list of the factorization of the numbers of this form) (factordb entry of this prime) (this prime can be easily proven prime using the $n-1$ test, since $n-1$ = $2^4 * 3 * 13^{6267}$ is trivially 100% factored) (this prime written in base 13)
* In base 13, the smallest prime in the family 720^{n}2 (algebraic form: $93*13^{n+1}+2$) ($n\geq0$) is 720^{2297}2 (algebraic form: $93*13^{2298}+2$) (factordb list of the factorization of the numbers of this form) (factordb entry of this prime) (since this prime has only 2562 digits, it should be able to proven prime easily, but currently there is no primality certificate available in factordb)
* In base 13, family 95^n (algebraic form: $(113*13^n-5)/12$) ($n\geq1$) cannot be ruled out as containing no primes > base (using covering congruence, algebra factorization, or combine of them) but no primes > base found in the family after searching to length 50000 (factordb list of the factorization of the numbers of this form)
* In base 13, family A3^{n}A (algebraic form: $(41*13^{n+1}+27)/4$) ($n\geq0$) cannot be ruled out as containing no primes > base but no primes > base found in the family after searching to length 50000 (factordb list of the factorization of the numbers of this form)
* In base 14, the smallest prime in the family 4D^n (algebraic form: $5*14^n-1$) ($n\geq1$) is 4D^{19698} (algebraic form: $5*14^{19698}-1$) (factordb list of the factorization of the numbers of this form) (factordb entry of this prime) (this prime can be easily proven prime using the $n+1$ test, since $n+1$ = $2^{19698} * 5 * 7^{19698}$ is trivially 100% factored) (this prime written in base 14)
* In base 16, family 3^{n}AF (algebraic form: $(16^{n+2}+619)/5$) ($n\geq0$) cannot be ruled out as containing no primes > base (using covering congruence, algebra factorization, or combine of them) but no primes > base found in the family after searching to length 50000 (factordb list of the factorization of the numbers of this form)
* In base 16, family 4^{n}DD (algebraic form: $(4*16^{n+2}+2291)/15$) ($n\geq0$) cannot be ruled out as containing no primes > base (using covering congruence, algebra factorization, or combine of them) but no primes > base found in the family after searching to length 50000 (factordb list of the factorization of the numbers of this form)
* In base 16, the smallest prime in the family DB^n (algebraic form: $(206*16^n-11)/15$) ($n\geq1$) is DB^{32234} (algebraic form: $(206*16^{32234}-11)/15$) (this prime is only a probable prime, i.e. not definitely prime) (the prime D (i.e. n = 0) is not counted since the prime must be > base) (factordb list of the factorization of the numbers of this form) (factordb entry of this PRP) (factorization of $n-1$ for this PRP) (factorization of $n+1$ for this PRP) (this PRP written in base 16)
* In base 16, the smallest prime in the family 5BC^{n}D (algebraic form: $(459*16^{n+1}+1)/5$) ($n\geq0$) is 5BC^{3700}D (algebraic form: $(459*16^{3701}+1)/5$) (factordb list of the factorization of the numbers of this form) (factordb entry of this prime) (primality certificate of this prime) (this prime written in base 16)

* In base 16, the smallest prime in the family 90^n91 (algebraic form: $9*16^{n+2}+145$) ($n≥0$) is $90^{3542}91$ (algebraic form: $9*16^{3544}+145$) (factordb list of the factorization of the numbers of this form) (factordb entry of this prime) (primality certificate of this prime) (this prime written in base 16)
* In base 16, the smallest prime in the family $F8^nF$ (algebraic form: $(233*16^{n+1}+97)/15$) ($n≥0$) is $F8^{1517}F$ (algebraic form: $(233*16^{1518}+97)/15$) (factordb list of the factorization of the numbers of this form) (factordb entry of this prime) (primality certificate of this prime) (this prime written in base 16)
* In base 16, the smallest prime in the family $D9^n$ (algebraic form: $(68*16^n-3)/5$) ($n≥1$) is $D9^{1052}$ (algebraic form: $(68*16^{1052}-3)/5$) (factordb list of the factorization of the numbers of this form) (factordb entry of this prime) (primality certificate of this prime) (this prime written in base 16)
* In base 16, the smallest prime in the family $88F^n$ (algebraic form: $137*16^n-1$) ($n≥0$) is $88F^{545}$ (algebraic form: $137*16^{545}-1$) (factordb list of the factorization of the numbers of this form) (factordb entry of this prime) (this prime can be easily proven prime using the *n*+1 test, since *n*+1 = 2^{2180} * 137 is trivially 100% factored) (this prime written in base 16)
* In base 16, the smallest prime in the family $5F^n6F$ (algebraic form: $6*16^{n+2}-145$) ($n≥0$) is $5F^{544}6F$ (algebraic form: $6*16^{546}-145$) (factordb list of the factorization of the numbers of this form) (factordb entry of this prime) (primality certificate of this prime) (this prime written in base 16)

For any given base *b*, we find all (*x*,*z*) digits-pair such that *x* ≠ 0 and *gcd*(*z*,*b*) = 1, and find the corresponding sets *Y**, see below.

Bold for minimal primes in base *b*, i.e. elements of the set $M(L_b)$

base 2

The possible (first digit,last digit) for an element with >=3 digits in the minimal set of the strings for primes with at least two digits are:

(1,1)

* Case (1,1):

** **11** is prime, and thus the only minimal prime in this family.

base 3

The possible (first digit,last digit) for an element with >=3 digits in the minimal set of the strings for primes with at least two digits are:

(1,1), (1,2), (2,1), (2,2)

* Case (1,1):

** Since 12, 21, **111** are primes, we only need to consider the family 1{0}1 (since any digits 1, 2 between them will produce smaller primes)

*** All numbers of the form 1{0}1 are divisible by 2, thus cannot be prime.

* Case (1,2):

** **12** is prime, and thus the only minimal prime in this family.

* Case (2,1):

** **21** is prime, and thus the only minimal prime in this family.

* Case (2,2):

** Since 21, 12 are primes, we only need to consider the family 2{0,2}2 (since any digits 1 between them will produce smaller primes)

*** All numbers of the form 2{0,2}2 are divisible by 2, thus cannot be prime.

base 4

The possible (first digit,last digit) for an element with >=3 digits in the minimal set of the strings for primes with at least two digits are:

(1,1), (1,3), (2,1), (2,3), (3,1), (3,3)

* Case (1,1):

** **11** is prime, and thus the only minimal prime in this family.

* Case (1,3):

** **13** is prime, and thus the only minimal prime in this family.

* Case (2,1):

** Since 23, 11, 31, **221** are primes, we only need to consider the family 2{0}1 (since any digits 1, 2, 3 between them will produce smaller primes)

*** All numbers of the form 2{0}1 are divisible by 3, thus cannot be prime.

* Case (2,3):

** **23** is prime, and thus the only minimal prime in this family.

* Case (3,1):

** **31** is prime, and thus the only minimal prime in this family.

* Case (3,3):

** Since 31, 13, 23 are primes, we only need to consider the family 3{0,3}3 (since any digits 1, 2 between them will produce smaller primes)

*** All numbers of the form 3{0,3}3 are divisible by 3, thus cannot be prime.

base 5

The possible (first digit,last digit) for an element with >=3 digits in the minimal set of the strings for primes with at least two digits are:

(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (3,4), (4,1), (4,2), (4,3), (4,4)

* Case (1,1):

** Since 12, 21, **111**, **131** are primes, we only need to consider the family 1{0,4}1 (since any digits 1, 2, 3 between them will produce smaller primes)

*** All numbers of the form 1{0,4}1 are divisible by 2, thus cannot be prime.

* Case (1,2):

** **12** is prime, and thus the only minimal prime in this family.

* Case (1,3):

** Since 12, 23, 43, **133** are primes, we only need to consider the family 1{0,1}3 (since any digits 2, 3, 4 between them will produce smaller primes)

*** Since 111 is prime, we only need to consider the families 1{0}3 and 1{0}1{0}3 (since any digit combo 11 between (1,3) will produce smaller primes)

**** All numbers of the form 1{0}3 are divisible by 2, thus cannot be prime.

**** For the 1{0}1{0}3 family, since **10103** is prime, we only need to consider the families 1{0}13 and 11{0}3 (since any digit combo 010 between (1,3) will produce smaller primes)

***** The smallest prime of the form 1{0}13 is **1000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000013**, which can be written as $10^{93}13$ and equal the prime $5^{95}+8$ (factordb entry) (shown in base 5) (factorization of the numbers of this form)

***** All numbers of the form 11{0}3 are divisible by 3, thus cannot be prime.

* Case (1,4):

** Since 12, 34, **104** are primes, we only need to consider the family 1{1,4}4 (since any digits 0, 2, 3 between them will produce smaller primes)

*** Since 111, 414 are primes, we only need to consider the families 1{4}4 and 11{4}4 (since any digit combo 11 or 41 between them will produce smaller primes)

**** The smallest prime of the form 1{4}4 is **14444**.

**** All numbers of the form 11{4}4 are divisible by 2, thus cannot be prime.

* Case (2,1):

** **21** is prime, and thus the only minimal prime in this family.

* Case (2,2):

** Since 21, 23, 12, 32 are primes, we only need to consider the family 2{0,2,4}2 (since any digits 1, 3 between them will produce smaller primes)

*** All numbers of the form 2{0,2,4}2 are divisible by 2, thus cannot be prime.

* Case (2,3):

** **23** is prime, and thus the only minimal prime in this family.

* Case (2,4):

** Since 21, 23, 34 are primes, we only need to consider the family 2{0,2,4}4 (since any digits 1, 3 between them will produce smaller primes)

*** All numbers of the form 2{0,2,4}4 are divisible by 2, thus cannot be prime.

* Case (3,1):

** Since 32, 34, 21 are primes, we only need to consider the family 3{0,1,3}1 (since any digits 2, 4 between them will produce smaller primes)

*** Since 313, 111, 131, **3101** are primes, we only need to consider the families 3{0,3}1 and 3{0,3}11 (since any digit combo 10, 11, 13 between (3,1) will produce smaller primes)

**** For the 3{0,3}1 family, we can separate this family to four families:

***** For the 30{0,3}01 family, we have the prime **30301**, and the remain case is the family 30{0}01.

****** All numbers of the form 30{0}01 are divisible by 2, thus cannot be prime.

***** For the 30{0,3}31 family, note that there must be an even number of 3's between (30,31), or the result number will be divisible by 2 and cannot be prime.

****** Since 33331 is prime, any digit combo 33 between (30,31) will produce smaller primes.

******* Thus, the only possible prime is the smallest prime in the family 30{0}31, and this prime is **300031**.

***** For the 33{0,3}01 family, note that there must be an even number of 3's between (33,01), or the result number will be divisible by 2 and cannot be prime.

****** Since 33331 is prime, any digit combo 33 between (33,01) will produce smaller primes.

******* Thus, the only possible prime is the smallest prime in the family 33{0}01, and this prime is **33001**.

***** For the 33{0,3}31 family, we have the prime **33331**, and the remain case is the family 33{0}31.

****** All numbers of the form 33{0}31 are divisible by 2, thus cannot be prime.

**** All numbers of the form 3{0,3}11 are divisible by 3, thus cannot be prime.

* Case (3,2):

** **32** is prime, and thus the only minimal prime in this family.

* Case (3,3):

** Since 32, 34, 23, 43, **313** are primes, we only need to consider the family 3{0,3}3 (since any digits 1, 2, 4 between them will produce smaller primes)

*** All numbers of the form 3{0,3}3 are divisible by 3, thus cannot be prime.

* Case (3,4):

** **34** is prime, and thus the only minimal prime in this family.

* Case (4,1):

** Since 43, 21, **401** are primes, we only need to consider the family 4{1,4}1 (since any digits 0, 2, 3 between them will produce smaller primes)

*** Since 414, 111 are primes, we only need to consider the families 4{4}1 and 4{4}11 (since any digit combo 14 or 11 between them will produce smaller primes)

**** The smallest prime of the form 4{4}1 is **44441**.

**** All numbers of the form 4{4}11 are divisible by 2, thus cannot be prime.

* Case (4,2):

** Since 43, 12, 32 are primes, we only need to consider the family 4{0,2,4}2 (since any digits 1, 3 between them will produce smaller primes)

*** All numbers of the form 4{0,2,4}2 are divisible by 2, thus cannot be prime.

* Case (4,3):

** **43** is prime, and thus the only minimal prime in this family.

* Case (4,4):

** Since 43, 34, **414** are primes, we only need to consider the family 4{0,2,4}4 (since any digits 1, 3 between them will produce smaller primes)

*** All numbers of the form 4{0,2,4}4 are divisible by 2, thus cannot be prime.

base 6

The possible (first digit,last digit) for an element with >=3 digits in the minimal set of the strings for primes with at least two digits are:

(1,1), (1,5), (2,1), (2,5), (3,1), (3,5), (4,1), (4,5), (5,1), (5,5)

* Case (1,1):

** **11** is prime, and thus the only minimal prime in this family.

* Case (1,5):

** **15** is prime, and thus the only minimal prime in this family.

* Case (2,1):

** **21** is prime, and thus the only minimal prime in this family.

* Case (2,5):

** **25** is prime, and thus the only minimal prime in this family.

* Case (3,1):

** **31** is prime, and thus the only minimal prime in this family.

* Case (3,5):

** **35** is prime, and thus the only minimal prime in this family.

* Case (4,1):

** Since 45, 11, 21, 31, 51 are primes, we only need to consider the family 4{0,4}1 (since any digits 1, 2, 3, 5 between them will produce smaller primes)

*** Since **4401** and **4441** are primes, we only need to consider the families 4{0}1 and 4{0}41 (since any digits combo 40 and 44 between them will produce smaller primes)

**** All numbers of the form 4{0}1 are divisible by 5, thus cannot be prime.

**** The smallest prime of the form 4{0}41 is **40041**

* Case (4,5):

** **45** is prime, and thus the only minimal prime in this family.

* Case (5,1):

** **51** is prime, and thus the only minimal prime in this family.

* Case (5,5):

** Since 51, 15, 25, 35, 45 are primes, we only need to consider the family 5{0,5}5 (since any digits 1, 2, 3, 4 between them will produce smaller primes)

*** All numbers of the form 5{0,5}5 are divisible by 5, thus cannot be prime.

base 7

The possible (first digit,last digit) for an element with >=3 digits in the minimal set of the strings for primes with at least two digits are:

(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)

* Case (1,1):

** Since 14, 16, 41, 61, **131** are primes, we only need to consider the family 1{0,1,2,5}1 (since any digits 3, 4, 6 between them will produce smaller primes)

*** Since the digit sum of primes must be odd (otherwise the number will be divisible by 2, thus cannot be prime), there is an odd total number of 1 and 5 in the {}

**** If there are >=3 number of 1 and 5 in the {}:

***** If there is 111 in the {}, then we have the prime **11111**

***** If there is 115 in the {}, then the prime 115 is a subsequence

***** If there is 151 in the {}, then the prime 115 is a subsequence

***** If there is 155 in the {}, then the prime 155 is a subsequence

***** If there is 511 in the {}, then the current number is 15111, which has digit sum = 12, but digit sum divisible by 3 will cause the number divisible by 3 and cannot be prime, and we cannot add more 1 or 5 to this number (to avoid 11111, 155, 515, 551 as subsequence), thus we must add at least one 2 to this number, but then the number has both 2 and 5, and will have either 25 or 52 as subsequence, thus cannot be minimal prime

***** If there is 515 in the {}, then the prime 515 is a subsequence

***** If there is 551 in the {}, then the prime 551 is a subsequence

***** If there is 555 in the {}, then the prime 551 is a subsequence

**** Thus there is only one 1 (and no 5) or only one 5 (and no 1) in the {}, i.e. we only need to consider the families 1{0,2}1{0,2}1 and 1{0,2}5{0,2}1

***** For the 1{0,2}1{0,2}1 family, since **1211** is prime, we only need to consider the family 1{0}1{0,2}1

****** Since all numbers of the form 1{0}1{0}1 are divisible by 3 and cannot be prime, we only need to consider the family 1{0}1{0}2{0}1

******* Since **11201** is prime, we only need to consider the family 1{0}1{0}21

******** The smallest prime of the form 11{0}21 is **1100021**

******** All numbers of the form 101{0}21 are divisible by 5, thus cannot be prime

******** The smallest prime of the form 1001{0}21 is **100121**

********* Since this prime has no 0 between 1{0}1 and 21, we do not need to consider more families

***** For the 1{0,2}5{0,2}1 family, since 25 and 52 are primes, we only need to consider the family 1{0}5{0}1

****** Since **1051** is prime, we only need to consider the family 15{0}1

******* The smallest prime of the form 15{0}1 is **150001**

* Case (1,2):

** Since 14, 16, 32, 52 are primes, we only need to consider the family 1{0,1,2}2 (since any digits 3, 4, 5, 6 between them will produce smaller primes)

*** Since **1112** and **1222** are primes, there is at most one 1 and at most one 2 in {}

**** If there are one 1 and one 2 in {}, then the digit sum is 6, and the number will be divisible by 6 and cannot be prime.

**** If there is one 1 but no 2 in {}, then the digit sum is 4, and the number will be divisible by 2 and cannot be prime.

**** If there is no 1 but one 2 in {}, then the form is 1{0}2{0}2

***** Since **1022** and **1202** are primes, we only need to consider the number 122

****** 122 is not prime.

**** If there is no 1 and no 2 in {}, then the digit sum is 3, and the number will be divisible by 3 and cannot be prime.

* Case (1,3):

** Since 14, 16, 23, 43, **113**, **133** are primes, we only need to consider the family 1{0,5}3 (since any digits 1, 2, 3, 4, 6 between them will produce smaller primes)

*** Since 155 is prime, we only need to consider the family 1{0}3 and 1{0}5{0}3

**** All numbers of the form 1{0}3 are divisible by 2, thus cannot be prime.

**** All numbers of the form 1{0}5{0}3 are divisible by 3, thus cannot be prime.

* Case (1,4):

** **14** is prime, and thus the only minimal prime in this family.

* Case (1,5):

** Since 14, 16, 25, 65, **115**, **155** are primes, we only need to consider the family 1{0,3}5 (since any digits 1, 2, 4, 5, 6 between them will produce smaller primes)

*** All numbers of the form 1{0,3}5 are divisible by 3, thus cannot be prime.

* Case (1,6):

** **16** is prime, and thus the only minimal prime in this family.

* Case (2,1):

** Since 23, 25, 41, 61, **221** are primes, we only need to consider the family 2{0,1}1 (since any digits 2, 3, 4, 5, 6 between them will produce smaller primes)

*** Since **2111** is prime, we only need to consider the families 2{0}1 and 2{0}1{0}1

**** All numbers of the form 2{0}1 are divisible by 3, thus cannot be prime.

**** All numbers of the form 2{0}1{0}1 are divisible by 2, thus cannot be prime.

* Case (2,2):

** Since 23, 25, 32, 52, **212** are primes, we only need to consider the family 2{0,2,4,6}2 (since any digits 1, 3, 5 between them will produce smaller primes)

*** All numbers of the form 2{0,2,4,6}2 are divisible by 2, thus cannot be prime.

* Case (2,3):

** **23** is prime, and thus the only minimal prime in this family.

* Case (2,4):

** Since 23, 25, 14 are primes, we only need to consider the family 2{0,2,4,6}4 (since any digits 1, 3, 5 between them will produce smaller primes)

*** All numbers of the form 2{0,2,4,6}4 are divisible by 2, thus cannot be prime.

* Case (2,5):

** **25** is prime, and thus the only minimal prime in this family.

* Case (2,6):

** Since 23, 25, 16, 56 are primes, we only need to consider the family 2{0,2,4,6}6 (since any digits 1, 3, 5 between them will produce smaller primes)

*** All numbers of the form 2{0,2,4,6}6 are divisible by 2, thus cannot be prime.

* Case (3,1):

** Since 32, 41, 61 are primes, we only need to consider the family 3{0,1,3,5}1 (since any digits 2, 4, 6 between them will produce smaller primes)

*** Since 551 is prime, we only need to consider the family 3{0,1,3}1 and 3{0,1,3}5{0,1,3}1 (since any digits combo 55 between (3,1) will produce smaller primes)

**** For the 3{0,1,3}1 family, since **3031** and 131 are primes, we only need to consider the families 3{0,1}1 and 3{3}3{0,1}1 (since any digits combo 03, 13 between (3,1) will produce smaller primes, thus for the digits between (3,1), all 3's must be before all 0's and 1's, and thus we can let the red 3 in 3{3}3{0,1}1 be the rightmost 3 between (3,1), all digits before this 3 must be 3's, and all digits after this 3 must be either 0's or 1's)

***** For the 3{0,1}1 family:

****** If there are >=2 0's and >=1 1's between (3,1), then at least one of **30011**, **30101**, **31001** will be a subsequence.

****** If there are no 1's between (3,1), then the form will be 3{0}1

******* All numbers of the form 3{0}1 are divisible by 2, thus cannot be prime.

****** If there are no 0's between (3,1), then the form will be 3{1}1

******* The smallest prime of the form 3{1}1 is **31111**

****** If there are exactly 1 0's between (3,1), then there must be <3 1's between (3,1), or **31111** will be a subsequence.

******* If there are 2 1's between (3,1), then the digit sum is 6, thus the number is divisible by 6 and cannot be prime.

******* If there are 1 1's between (3,1), then the number can only be either 3101 or 3011

******** Neither 3101 nor 3011 is prime.

******* If there are no 1's between (3,1), then the number must be 301

******** 301 is not prime.

***** For the 3{3}3{0,1}1 family:

****** If there are at least one 3 between (3,3{0,1}1) and at least one 1 between (3{3}3,1), then **33311** will be a subsequence.

****** If there are no 3 between (3,3{0,1}1), then the form will be 33{0,1}1

******* If there are at least 3 1's between (33,1), then 31111 will be a subsequence.

******* If there are exactly 2 1's between (33,1), then the digit sum is 12, thus the number is divisible by 3 and cannot be prime.

******* If there are exactly 1 1's between (33,1), then the digit sum is 11, thus the number is divisible by 2 and cannot be prime.

******* If there are no 1's between (33,1), then the form will be 33{0}1

******** The smallest prime of the form 33{0}1 is **33001**

****** If there are no 1 between (3{3}3,1), then the form will be 3{3}3{0}1

******* If there are at least 2 0's between (3{3}3,1), then 33001 will be a subsequence.

******* If there are exactly 1 0's between (3{3}3,1), then the form is 3{3}301

******** The smallest prime of the form 3{3}301 is **33333301**

******* If there are no 0's between (3{3}3,1), then the form is 3{3}31

******** The smallest prime of the form 3{3}31 is **33333333333333331**

**** For the 3{0,1,3}5{0,1,3}1 family, since 335 is prime, we only need to consider the family 3{0,1}5{0,1,3}1

***** Numbers containing 3 between (3{0,1}5,1):

****** The form is 3{0,1}5{0,1,3}3{0,1,3}1

******* Since 3031 and 131 are primes, we only need to consider the family 35{3}3{0,1,3}1 (since any digits combo 03, 13 between (3,1) will produce smaller primes)

******** Since 533 is prime, we only need to consider the family 353{0,1}1 (since any digits combo 33 between (35,1) will produce smaller primes)

********* Since 5011 is prime, we only need to consider the family 353{1}{0}1 (since any digits combo 01 between (353,1) will produce smaller primes)

********** If there are at least 3 1's between (353,{0}1), then 31111 will be a subsequence.

********** If there are exactly 2 1's between (353,{0}1), then the digit sum is 20, thus the number is divisible by 2 and cannot be prime.

********** If there are exactly 1 1's between (353,{0}1), then the form is 3531{0}1

*********** The smallest prime of the form 3531{0}1 is 3531001, but it is not minimal prime since 31001 is prime.

********** If there are no 1's between (353,{0}1), then the digit sum is 15, thus the number is divisible by 6 and cannot be prime.

***** Numbers not containing 3 between (3{0,1}5,1):

****** The form is 3{0,1}5{0,1}1

******* If there are >=2 0's and >=1 1's between (3,1), then at least one of 30011, 30101, 31001 will be a subsequence.

******* If there are no 1's between (3,1), then the form will be 3{0}5{0}1

******** All numbers of the form 3{0}5{0}1 are divisible by 3, thus cannot be prime.

******* If there are no 0's between (3,1), then the form will be 3{1}5{1}1

******** If there are >=3 1's between (3,1), then 31111 will be a subsequence.

******** If there are exactly 2 1's between (3,1), then the number can only be 31151, 31511, 35111

********* None of 31151, 31511, 35111 are primes.

******** If there are exactly 1 1's between (3,1), then the digit sum is 13, thus the number is divisible by 2 and cannot be prime.

******** If there are no 1's between (3,1), then the number is 351

********* 351 is not prime.

******* If there are exactly 1 0's between (3,1), then the form will be 3{1}0{1}5{1}1 or 3{1}5{1}0{1}1

******** No matter 3{1}0{1}5{1}1 or 3{1}5{1}0{1}1, if there are >=3 1's between (3,1), then 31111 will be a subsequence.

******** If there are exactly 2 1's between (3,1), then the number can only be 311051, 310151, 310511, 301151, 301511, 305111, 311501, 315101, 315011, 351101, 351011, 350111

********* Of these numbers, 311051, 301151, 311501, 351101, 350111 are primes.

********** However, 311051, 301151, 311501 have 115 as subsequence, and 350111 has 5011 as subsequence, thus only **351101** is minimal prime.

******** No matter 3{1}0{1}5{1}1 or 3{1}5{1}0{1}1, if there are exactly 1 1's between (3,1), then the digit sum is 13, thus the number is divisible by 2 and cannot be prime.

******** If there are no 1's between (3,1), then the number is 3051 for 3{1}0{1}5{1}1 or 3501 for 3{1}5{1}0{1}1

********* Neither 3051 nor 3501 is prime.

* Case (3,2):

** **32** is prime, and thus the only minimal prime in this family.

* Case (3,3):

** Since 32, 23, 43, **313** are primes, we only need to consider the family 3{0,3,5,6}3 (since any digits 1, 2, 4 between them will produce smaller primes)

*** If there are >=2 5's in {}, then 553 will be a subsequence.

*** If there are no 5's in {}, then the family will be 3{0,3,6}3

**** All numbers of the form 3{0,3,6}3 are divisible by 3, thus cannot be prime.

*** If there are exactly 1 5's in {}, then the family will be 3{0,3,6}5{0,3,6}3

**** Since 335, 65, **3503**, 533, 56 are primes, we only need to consider the family 3{0}53 (since any digit 3, 6 between (3,5{0,3,6}3) and any digit 0, 3, 6 between (3{0,3,6}5,3) will produce smaller primes)

***** The smallest prime of the form 3{0}53 is **300053**

* Case (3,4):

** Since 32, 14, **304**, **344**, **364** are primes, we only need to consider the family 3{3,5}4 (since any digits 0, 1, 2, 4, 6 between them will produce smaller primes)

*** Since **3334** and 335 are primes, we only need to consider the family 3{5}4 and 3{5}34 (since any digits combo 33, 35 between them will produce smaller primes)

**** The smallest prime of the form 3{5}4 is
3555555555555555555555555555555555555555555555555555555555555555555555555
5555555555555555555555555555555555555555555555555555555555555555555555555
5555555555555555555555555555555555555555555555555555555555555555555555555
5555555555555555555555555555555555555555555555555555555555555555555555555
5555555555555555555555555555555555555555555555555555555555555555555555555
5555555555555555555555555555555555555555555555555555555555555555555555555
5555555555555555555555555555555555555555555555555555555555555555555555555
5555555555555555555555555555555555555555555555555555555555555555555555555
5555555555555555555555555555555555555555555555555555555555555555555555555
5555555555555555555555555555555555555555555555555555555555555555555555555
5555555555555555555555555555555555555555555555555555555555555555555555555
5555555555555555555555555555555555555555555555555555555555555555555555555
5555555555555555555555555555555555555555555555555555555555555555555555555
5555555555555555555555555555555555555555555555555555555555555555555555555
5555555555555555555555555555555555555555555555555555555555555555555555555
5555555555555555555555555555555555555555555555555555555555555555555555555

5555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555
5555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555
5555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555
5555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555
5555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555
5555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555
5555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555
5555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555
5555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555
5555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555
5555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555
5555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555
5555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555
5555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555
5555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555
5555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555
5555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555
5555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555
5555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555
5555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555
5555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555
5555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555
5555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555
5555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555
5555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555
5555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555
5555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555
5555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555
5555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555
5555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555
5555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555
5555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555
5555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555
5555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555
5555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555
5555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555
5555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555
5555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555
5555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555
5555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555
5555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555
5555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555
5555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555
5555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555
5555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555
5555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555
5555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555
5555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555
5555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555
5555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555
5555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555
5555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555

555555555555555555555555555555555555555555555555555555555555555555555555555555555
555555555555555555555555555555555555555555555555555555555555555555555555555555555
555555555555555555555555555555555555555555555555555555555555555555555555555555555
555555555555555555555555555555555555555555555555555555555555555555555555555555555
555555555555555555555555555555555555555555555555555555555555555555555555555555555
555555555555555555555555555555555555555555555555555555555555555555555555555555555
555555555555555555555555555555555555555555555555555555555555555555555555555555555
555555555555555555555555555555555555555555555555555555555555555555555555555555555
555555555555555555555555555555555555555555555555555555555555555555555555555555555
555555555555555555555555555555555555555555555555555555555555555555555555555555555
555555555555555555555555555555555555555555555555555555555555555555555555555555555
555555555555555555555555555555555555555555555555555555555555555555555555555555555
555555555555555555555555555555555555555555555555555555555555555555555555555555555
555555555555555555555555555555555555555555555555555555555555555555555555555555555
555555555555555555555555555555555555555555555555555555555555555555555555555555555
555555555555555555555555555555555555555555555555555555555555555555555555555555555
555555555555555555555555555555555555555555555555555555555555555555555555555555555
555555555555555555555555555555555555555555555555555555555555555555555555555555555
555555555555555555555555555555555555555555555555555555555555555555555555555555555
555555555555555555555555555555555555555555555555555555555555555555555555555555555
555555555555555555555555555555555555555555555555555555555555555555555555555555555
555555555555555555555555555555555555555555555555555555555555555555555555555555555
555555555555555555555555555555555555555555555555555555555555555555555555555555555
555555555555555555555555555555555555555555555555555555555555555555555555555555555
555555555555555555555555555555555555555555555555555555555555555555555555555555555
555555555555555555555555555555555555555555555555555555555555555555555555555555555
555555555555555555555555555555555555555555555555555555555555555555555555555555555
555555555555555555555555555555555555555555555555555555555555555555555555555555555
555555555555555555555555555555555555555555555555555555555555555555555555555555555
555555555555555555555555555555555555555555555555555555555555555555555555555555555
555555555555555555555555555555555555555555555555555555555555555555555555555555555
555555555555555555555555555555555555555555555555555555555555555555555555555555555
555555555555555555555555555555555555555555555555555555555555555555555555555555555
555555555555555555555555555555555555555555555555555555555555555555555555555555555
555555555555555555555555555555555555555555555555555555555555555555555555555555555
555555555555555555555555555555555555555555555555555555555555555555555555555555555
555555555555555555555555555555555555555555555555555555555555555555555555555555555
555555555555555555555555555555555555555555555555555555555555555555555555555555555
555555555555555555555555555555555555555555555555555555555555555555555555555555555
555555555555555555555555555555555555555555555555555555555555555555555555555555555
555555555555555555555555555555555555555555555555555555555555555555555555555555555
555555555555555555555555555555555555555555555555555555555555555555555555555555555
555555555555555555555555555555555555555555555555555555555555555555555555555555555
555555555555555555555555555555555555555555555555555555555555555555555555555555555
555555555555555555555555555555555555555555555555555555555555555555555555555555555
555555555555555555555555555555555555555555555555555555555555555555555555555555555
54 with 9234 5's, which can be written as $35^{9234}4$ and equal the prime $(23*7^{9235}-11)/6$ (factordb entry) (shown in base 7) (factorization of the numbers of this form) (this number is only probable prime, i.e. not definitely prime) (not minimal prime, since 35555 and 5554 are primes)

**** The smallest prime of the form 3{5}34 is
3555555555555555555555555555555555555555555555555555555555555555534 with 63 5's, which

can be written as $35^{63}34$ and equal the prime $(23*7^{65}-95)/6$ (factordb entry) (shown in base 7) (factorization of the numbers of this form) (not minimal prime, since 35555, 553, and 5554 are primes)

* Case (3,5):

** Since 32, 25, 65, **335** are primes, we only need to consider the family 3{0,1,4,5}5 (since any digits 2, 3, 6 between them will produce smaller primes)

*** If there are at least one 1's and at least one 5's in {}, then either 155 or 515 will be a subsequence.

*** If there are at least one 1's and at least one 4's in {}, then either 14 or 41 will be a subsequence.

*** If there are at least two 1's in {}, then 115 will be a subsequence.

*** If there are exactly one 1's and no 4's or 5's in {}, then the family will be 3{0}1{0}5

**** All numbers of the form 3{0}1{0}5 are divisible by 3, thus cannot be prime.

*** If there is no 1's in {}, then the family will be 3{0,4,5}5

**** If there are at least to 4's in {}, then 344 and 445 will be subsequences.

**** If there is no 4's in {}, then the family will be 3{0,5}5

***** Since **3055** and **3505** are primes, we only need to consider the families 3{0}5 and 3{5}5

****** All numbers of the form 3{0}5 are divisible by 2, thus cannot be prime.

****** The smallest prime of the form 3{5}5 is **35555**

**** If there is exactly one 4's in {}, then the family will be 3{0,5}4{0,5}5

***** Since 304, **3545** are primes, we only need to consider the families 34{0,5}5 (since any digits 0 or 5 between (3,4{0,5}5) will produce small primes)

****** All numbers of the form 34{0,5}5 are divisible by 5, thus cannot be prime.

* Case (3,6):

** Since 32, 16, 56, **346** are primes, we only need to consider the family 3{0,3,6}6 (since any digits 1, 2, 4, 5 between them will produce smaller primes)

*** All numbers of the form 3{0,3,6}6 are divisible by 3, thus cannot be prime.

* Case (4,1):

** **41** is prime, and thus the only minimal prime in this family.

* Case (4,2):

** Since 41, 43, 32, 52 are primes, we only need to consider the family 4{0,2,4,6}2 (since any digits 1, 3, 5 between them will produce smaller primes)

*** All numbers of the form 4{0,2,4,6}2 are divisible by 2, thus cannot be prime.

* Case (4,3):

** **43** is prime, and thus the only minimal prime in this family.

* Case (4,4):

** Since 41, 43, 14 are primes, we only need to consider the family 4{0,2,4,5,6}4 (since any digits 1, 3 between them will produce smaller primes)

*** If there is no 5's in {}, then the family will be 4{0,2,4,6}4

**** All numbers of the form 4{0,2,4,6}4 are divisible by 2, thus cannot be prime.

*** If there is at least one 5's in {}, then there cannot be 2 in {} (since if so, then either 25 or 52 will be a subsequence) and there cannot be 6 in {} (since if so, then either 65 or 56 will be a subsequence), thus the family is 4{0,4,5}5{0,4,5}4

**** Since 445, **4504**, 544 are primes, we only need to consider the family 4{0,5}5{5}4 (since any digit 4 between (4,5{0,4,5}4) and any digit 0, 4 between (4{0,4,5}5,4) will produce smaller primes)

***** If there are at least two 0's between (4,5{0,4,5}4), then **40054** will be a subsequence.

***** If there is no 0's between (4,5{0,4,5}4), then the family will be 4{5}5{5}4, which is equivalent to 4{5}4

****** The smallest prime of the form 4{5}4 is 45555555555555554 (not minimal prime, since 4555 and 5554 are primes)

***** If there is exactly one 0's between (4,5{0,4,5}4), then the family will be 4{5}0{5}5{5}4

****** Since 4504 is prime, we only need to consider the family 40{5}5{5}4 (since any digit 5 between (4,0{5}5{5}4) will produce small primes), which is equivalent to 40{5}4

******* The smallest prime of the form 40{5}4 is 405555555555555554 (not minimal prime, since 4555 and 5554 are primes)

* Case (4,5):

** Since 41, 43, 25, 65, **445** are primes, we only need to consider the family 4{0,5}5 (since any digits 1, 2, 3, 4, 6 between them will produce smaller primes)

*** If there are at least two 5's in {}, then **4555** will be a subsequence.

*** If there is exactly one 5's in {}, then the digit sum is 20, and the number will be divisible by 2 and cannot be prime.

*** If there is no 5's in {}, then the family will be 4{0}5

**** All numbers of the form 4{0}5 are divisible by 3, thus cannot be prime.

* Case (4,6):

** Since 41, 43, 16, 56 are primes, we only need to consider the family 4{0,2,4,6}6 (since any digits 1, 3, 5 between them will produce smaller primes)

*** All numbers of the form 4{0,2,4,6}6 are divisible by 2, thus cannot be prime.

* Case (5,1):

** Since 52, 56, 41, 61, **551** are primes, we only need to consider the family 5{0,1,3}1 (since any digits 2, 4, 5, 6 between them will produce smaller primes)

*** If there are at least two 3's in {}, then 533 will be a subsequence.

*** If there is no 3's in {}, then the family will be 5{0,1}1

**** Since **5011** is prime, we only need to consider the family 5{1}{0}1

***** Since 11111 is prime, we only need to consider the families 5{0}1, 51{0}1, 511{0}1, 5111{0}1 (since any digits combo 1111 between (5,1) will produce small primes)

****** All numbers of the form 5{0}1 are divisible by 6, thus cannot be prime.

****** The smallest prime of the form 51{0}1 is **5100000001**

****** All numbers of the form 511{0}1 are divisible by 2, thus cannot be prime.

****** All numbers of the form 5111{0}1 are divisible by 3, thus cannot be prime.

*** If there is exactly one 3's in {}, then the family will be 5{0,1}3{0,1}1

**** If there is at least one 1's between (5,3{0,1}1), then 131 will be a subsequence.

***** Thus we only need to consider the family 5{0}3{0,1}1

****** If there are no 1's between (5{0}3,1), then the digit sum is 12, and the number will be divisible by 3 and cannot be prime.

****** If there are exactly one 1's between (5{0}3,1), then the digit sum is 13, and the number will be divisible by 2 and cannot be prime.

****** If there are exactly three 1's between (5{0}3,1), then the digit sum is 15, and the number will be divisible by 6 and cannot be prime.

****** If there are at least four 1's between (5{0}3,1), then 11111 will be a subsequence.

****** If there are exactly two 1's between (5{0}3,1), then the family will be 5{0}3{0}1{0}1{0}1

******* Since 5011 is prime, we only need to consider the family 5311{0}1 (since any digit 0 between (5,1{0}1) will produce small primes, this includes the leftmost three {} in 5{0}3{0}1{0}1{0}1, and thus only the rightmost {} can contain 0)

******** The smallest prime of the form 5311{0}1 is **531101**

* Case (5,2):

** **52** is prime, and thus the only minimal prime in this family.

* Case (5,3):

** Since 52, 56, 23, 43, **533**, **553** are primes, we only need to consider the family 5{0,1}3 (since any digits 2, 3, 4, 5, 6 between them will produce smaller primes)

*** If there are at least two 1's in {}, then 113 will be a subsequence.

*** If there is exactly one 1's in {}, then the digit sum is 12, and the number will be divisible by 3 and cannot be prime.

*** If there is no 1's in {}, then the digit sum is 11, and the number will be divisible by 2 and cannot be prime.

* Case (5,4):

** Since 52, 56, 14, **544** are primes, we only need to consider the family 5{0,3,5}4 (since any digits 1, 2, 4, 6 between them will produce smaller primes)

*** If there are no 5's in {}, then the family will be 5{0,3}4

**** All numbers of the form 5{0,3}4 are divisible by 3, thus cannot be prime.

*** If there are at least one 5's and at least one 3's in {}, then either 535 or 553 will be a subsequence.

*** If there are exactly one 5's and no 3's in {}, then the digit sum is 20, and the number will be divisible by 2 and cannot be prime.

*** If there are at least two 5's in {}, then **5554** will be a subsequence.

* Case (5,5):

** Since 52, 56, 25, 65, **515**, **535** are primes, we only need to consider the family 5{0,4,5}5 (since any digits 1, 2, 3, 6 between them will produce smaller primes)

*** If there are no 4's in {}, then the family will be 5{0,5}5

**** All numbers of the form 5{0,5}5 are divisible by 5, thus cannot be prime.

*** If there are no 5's in {}, then the family will be 5{0,4}5

**** All numbers of the form 5{0,4}5 are divisible by 2, thus cannot be prime.

*** If there are at least one 4's and at least one 5's in {}, then either **5455** or **5545** will be a subsequence.

* Case (5,6):

** **56** is prime, and thus the only minimal prime in this family.

* Case (6,1):

** **61** is prime, and thus the only minimal prime in this family.

* Case (6,2):

** Since 61, 65, 32, 52 are primes, we only need to consider the family 6{0,2,4,6}2 (since any digits 1, 3, 5 between them will produce smaller primes)

*** All numbers of the form 6{0,2,4,6}2 are divisible by 2, thus cannot be prime.

* Case (6,3):

** Since 61, 65, 23, 43 are primes, we only need to consider the family 6{0,3,6}3 (since any digits 1, 2, 4, 5 between them will produce smaller primes)

*** All numbers of the form 6{0,3,6}3 are divisible by 3, thus cannot be prime.

* Case (6,4):

** Since 61, 65, 14 are primes, we only need to consider the family 6{0,2,3,4,6}4 (since any digits 1, 5 between them will produce smaller primes)

*** If there is no 3's in {}, then the family will be 6{0,2,4,6}4

**** All numbers of the form 6{0,2,4,6}4 are divisible by 2, thus cannot be prime.

*** If there are exactly two 3's in {}, then the family will be 6{0,2,4,6}3{0,2,4,6}3{0,2,4,6}4

**** All numbers of the form 6{0,2,4,6}3{0,2,4,6}3{0,2,4,6}4 are divisible by 2, thus cannot be prime.

*** If there are at least three 3's in {}, then 3334 will be a subsequence.

*** If there is exactly one 3's in {}, then the family will be 6{0,2,4,6}3{0,2,4,6}4

**** If there is 0 between (6,3{0,2,4,6}4), then **6034** will be a subsequence.

**** If there is 2 between (6,3{0,2,4,6}4), then 23 will be a subsequence.

**** If there is 4 between (6,3{0,2,4,6}4), then 43 will be a subsequence.

**** If there is 6 between (6,3{0,2,4,6}4), then **6634** will be a subsequence.

**** If there is 0 between (6{0,2,4,6}3,4), then 304 will be a subsequence.

**** If there is 2 between (6{0,2,4,6}3,4), then 32 will be a subsequence.

**** If there is 4 between (6{0,2,4,6}3,4), then 344 will be a subsequence.

**** If there is 6 between (6{0,2,4,6}3,4), then 364 will be a subsequence.

**** Thus the number can only be 634

***** 634 is not prime.

* Case (6,5):

** **65** is prime, and thus the only minimal prime in this family.

* Case (6,6):

** Since 61, 65, 16, 56 are primes, we only need to consider the family 6{0,2,3,4,6}6 (since any digits 1, 5 between them will produce smaller primes)

*** If there is no 3's in {}, then the family will be 6{0,2,4,6}6

**** All numbers of the form 6{0,2,4,6}6 are divisible by 2, thus cannot be prime.

*** If there is no 2's and no 4's in {}, then the family will be 6{0,3,6}6

**** All numbers of the form 6{0,3,6}6 are divisible by 3, thus cannot be prime.

*** If there is at least one 3's and at least one 2's in {}, then either 32 or 23 will be a subsequence.

*** If there is at least one 3's and at least one 4's in {}, then either 346 or 43 will be a subsequence.

base 8

The possible (first digit,last digit) for an element with >=3 digits in the minimal set of the strings for primes with at least two digits are:

(1,1), (1,3), (1,5), (1,7), (2,1), (2,3), (2,5), (2,7), (3,1), (3,3), (3,5), (3,7), (4,1), (4,3), (4,5), (4,7), (5,1), (5,3), (5,5), (5,7), (6,1), (6,3), (6,5), (6,7), (7,1), (7,3), (7,5), (7,7)

* Case (1,1):

** Since 13, 15, 21, 51, **111**, **141**, **161** are primes, we only need to consider the family 1{0,7}1 (since any digits 1, 2, 3, 4, 5, 6 between them will produce smaller primes)

*** Since 107, 177, 701 are primes, we only need to consider the number 171 and the family 1{0}1 (since any digits combo 07, 70, 77 between them will produce smaller primes)

**** 171 is not prime.

**** All numbers of the form 1{0}1 factored as $10^n+1 = (2^n+1) * (4^n-2^n+1)$ ($n \geq 1$) (and since if $n \geq 1$, $2^n+1 \geq 2^1+1 = 3 > 1$, $4^n-2^n+1 \geq 4^1-2^1+1 = 3 > 1$, this factorization is nontrivial), thus cannot be prime.

* Case (1,3):

** **13** is prime, and thus the only minimal prime in this family.

* Case (1,5):

** **15** is prime, and thus the only minimal prime in this family.

* Case (1,7):

** Since 13, 15, 27, 37, 57, **107**, **117**, **147**, **177** are primes, we only need to consider the family 1{6}7 (since any digits 0, 1, 2, 3, 4, 5, 7 between them will produce smaller primes)

*** The smallest prime of the form 1{6}7 is 16667 (not minimal prime, since 667 is prime)

* Case (2,1):

** **21** is prime, and thus the only minimal prime in this family.

* Case (2,3):

** **23** is prime, and thus the only minimal prime in this family.

* Case (2,5):

** Since 21, 23, 27, 15, 35, 45, 65, 75, **225**, **255** are primes, we only need to consider the family 2{0}5 (since any digits 1, 2, 3, 4, 5, 6, 7 between them will produce smaller primes)

*** All numbers of the form 2{0}5 are divisible by 7, thus cannot be prime.

* Case (2,7):

** **27** is prime, and thus the only minimal prime in this family.

* Case (3,1):

** Since 35, 37, 21, 51, **301**, **361** are primes, we only need to consider the family 3{1,3,4}1 (since any digits 0, 2, 5, 6, 7 between them will produce smaller primes)

*** Since 13, 343, 111, 131, 141, 431, **3331**, **3411** are primes, we only need to consider the families 3{3}11, 33{1,4}1, 3{3,4}4{4}1 (since any digits combo 11, 13, 14, 33, 41, 43 between them will produce smaller primes)

**** All numbers of the form 3{3}11 are divisible by 3, thus cannot be prime.

**** For the 33{1,4}1 family, since 111 and 141 are primes, we only need to consider the families 33{4}1 and 33{4}11 (since any digits combo 11, 14 between them will produce smaller primes)

***** The smallest prime of the form 33{4}1 is **3344441**

***** All numbers of the form 33{4}11 are divisible by 301, thus cannot be prime.

**** For the 3{3,4}4{4}1 family, since 3331 and 3344441 are primes, we only need to consider the families 3{4}1, 3{4}31, 3{4}341, 3{4}3441, 3{4}34441 (since any digits combo 33 or 34444 between (3,1) will produce smaller primes)

***** All numbers of the form 3{4}1 are divisible by 31, thus cannot be prime.

***** Since 4443 is prime, we only need to consider the numbers 3431, 34431, 34341, 344341, 343441, 3443441, 3434441, 34434441 (since any digit combo 444 between (3,3{4}1) will produce smaller primes)

****** None of 3431, 34431, 34341, 344341, 343441, 3443441, 3434441, 34434441 are primes.

* Case (3,3):

** Since 35, 37, 13, 23, 53, 73, **343** are primes, we only need to consider the family 3{0,3,6}3 (since any digits 1, 2, 4, 5, 7 between them will produce smaller primes)

*** All numbers of the form 3{0,3,6}3 are divisible by 3, thus cannot be prime.

* Case (3,5):

** **35** is prime, and thus the only minimal prime in this family.

* Case (3,7):

** **37** is prime, and thus the only minimal prime in this family.

* Case (4,1):

** Since 45, 21, 51, **401**, **431**, **471** are primes, we only need to consider the family 4{1,4,6}1 (since any digits 0, 2, 3, 5, 7 between them will produce smaller primes)

*** Since 111, 141, 161, 661, **4611** are primes, we only need to consider the families 4{4}11, 4{4,6}4{1,4,6}1, 4{4}6{4}1 (since any digits combo 11, 14, 16, 61, 66 between them will produce smaller primes)

**** The smallest prime of the form 4{4}11 is 44444444444444411 (not minimal prime, since 444444441 is prime)

**** For the 4{4,6}4{1,4,6}1 family, we can separate this family to 4{4,6}41, 4{4,6}411, 4{4,6}461

***** For the 4{4,6}41 family, since 661 and 6441 are primes, we only need to consider the families 4{4}41 and 4{4}641 (since any digits combo 64 or 66 between (4,41) will produce smaller primes)

****** The smallest prime of the form 4{4}41 is **444444441**

****** The smallest prime of the form 4{4}641 is **444641**

***** For the 4{4,6}411 family, since 661 and 6441 are primes, we only need to consider the families 4{4}411 and 4{4}6411 (since any digits combo 64 or 66 between (4,411) will produce smaller primes)

****** The smallest prime of the form 4{4}411 is **444444441**

****** The smallest prime of the form 4{4}6411 is 4444444444444446411 (not minimal prime, since 444444441 and 444641 are primes)

***** For the 4{4,6}461 family, since 661 is prime, we only need to consider the family 4{4}461

****** The smallest prime of the form 4{4}461 is 4444444461 (not minimal prime, since 444444441 is prime)

**** For the 4{4}6{4}1 family, since 6441 is prime, we only need to consider the families 4{4}61 and 4{4}641 (since any digits combo 44 between (4{4}6,1) will produce smaller primes)

***** The smallest prime of the form 4{4}61 is 4444444461 (not minimal prime, since 444444441 is prime)

***** The smallest prime of the form 4{4}641 is **444641**

* Case (4,3):

** Since 45, 13, 23, 53, 73, **433**, **463** are primes, we only need to consider the family 4{0,4}3 (since any digits 1, 2, 3, 5, 6, 7 between them will produce smaller primes)

*** Since **4043** and **4443** are primes, we only need to consider the families 4{0}3 and 44{0}3 (since any digits combo 04, 44 between them will produce smaller primes)

**** All numbers of the form 4{0}3 are divisible by 7, thus cannot be prime.

**** All numbers of the form 44{0}3 are divisible by 3, thus cannot be prime.

* Case (4,5):

** **45** is prime, and thus the only minimal prime in this family.

* Case (4,7):

** Since 45, 27, 37, 57, **407**, **417**, **467** are primes, we only need to consider the family 4{4,7}7 (since any digits 0, 1, 2, 3, 5, 6 between them will produce smaller primes)

*** Since 747 is prime, we only need to consider the families 4{4}7, 4{4}77, 4{7}7, 44{7}7 (since any digits combo 74 between (4,7) will produce smaller primes)

**** The smallest prime of the form 4{4}7 is **447**, with 220 4's, which can be written as $4^{220}7$ and equal the prime $(4*8^{221}+17)/7$ (factordb entry) (shown in base 8) (factorization of the numbers of this form)

**** The smallest prime of the form 4{4}77 is **4444477**

**** The smallest prime of the form 4{7}7 is **47777**

**** The smallest prime of the form 44{7}7 is 4477, with 851 7's, which can be written as 447^{851} and equal the prime $37*8^{851}-1$ (factordb entry) (shown in base 8) (factorization of the numbers of this form) (not minimal prime, since 47777 is prime)

* Case (5,1):

** **51** is prime, and thus the only minimal prime in this family.

* Case (5,3):

** **53** is prime, and thus the only minimal prime in this family.

* Case (5,5):

** Since 51, 53, 57, 15, 35, 45, 65, 75 are primes, we only need to consider the family 5{0,2,5}5 (since any digits 1, 3, 4, 6, 7 between them will produce smaller primes)

*** Since 225, 255, **5205** are primes, we only need to consider the families 5{0,5}5 and 5{0,5}25 (since any digits combo 20, 22, 25 between them will produce smaller primes)

**** All numbers of the form 5{0,5}5 are divisible by 5, thus cannot be prime.

**** For the 5{0,5}25 family, since **500025** and **505525** are primes, we only need to consider the number 500525 the families 5{5}25, 5{5}025, 5{5}0025, 5{5}0525, 5{5}00525, 5{5}05025 (since any digits combo 000, 055 between (5,25) will produce smaller primes)

***** 500525 is not prime.

***** The smallest prime of the form 5{5}25 is **555555555555525**

***** The smallest prime of the form 5{5}025 is **55555025**

***** The smallest prime of the form 5{5}0025 is 5555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555555550025, with 184 5's, which can be written as $5^{183}0025$ and equal the prime $(5*8^{187}-20333)/7$ (factordb entry) (shown in base 8) (factorization of the numbers of this form) (not minimal prime, since 55555025 and 555555555555525 are primes)

***** The smallest prime of the form 5{5}0525 is **5550525**

***** The smallest prime of the form 5{5}00525 is **5500525**

***** The smallest prime of the form 5{5}05025 is 5555555555555555555555555505025, with 25 5's, which can be written as $5^{23}05025$ and equal the prime $(5*8^{28}-145773)/7$ (factordb entry) (shown in base 8) (factorization of the numbers of this form) (not minimal prime, since 5550525, 55555025, and 555555555555525 are primes)

* Case (5,7):

** **57** is prime, and thus the only minimal prime in this family.

* Case (6,1):

** Since 65, 21, 51, **631**, **661** are primes, we only need to consider the family 6{0,1,4,7}1 (since any digits 2, 3, 5, 6 between them will produce smaller primes)

*** Numbers containing 4: (note that the number cannot contain two or more 4's, or **6441** will be a subsequence)

**** The form is 6{0,1,7}4{0,1,7}1

***** Since 141, 401, 471 are primes, we only need to consider the family 6{0,7}4{1}1

****** Since 111 is prime, we only need to consider the families 6{0,7}41 and 6{0,7}411

******* For the 6{0,7}41 family, since **60741** is prime, we only need to consider the family 6{7}{0}41

******** Since 6777 is prime, we only need to consider the families 6{0}41, 67{0}41, 677{0}41

********* All numbers of the form 6{0}41 are divisible by 3, thus cannot be prime.

********* All numbers of the form 67{0}41 are divisible by 13, thus cannot be prime.

********* All numbers of the form 677{0}41 are divisible by 3, thus cannot be prime.

******* For the 6{0,7}411 family, since **60411** is prime, we only need to consider the family 6{7}411

******** The smallest prime of the form 6{7}411 is 67777411 (not minimal prime, since 6777 is prime)

*** Numbers not containing 4:

**** The form is 6{0,1,7}1

***** Since 111 is prime, we only need to consider the families 6{0,7}1 and 6{0,7}1{0,7}1

****** All numbers of the form 6{0,7}1 are divisible by 7, thus cannot be prime.

****** For the 6{0,7}1{0,7}1 family, since 711 and **6101** are primes, we only need to consider the family 6{0}1{7}1

******* Since **60171** is prime, we only need to consider the families 6{0}11 and 61{7}1

******** All numbers of the form 6{0}11 are divisible by 3, thus cannot be prime.

******** The smallest prime of the form 61{7}1 is 617771 (not minimal prime, since 6777 is prime)

* Case (6,3):

** Since 65, 13, 23, 53, 73, **643** are primes, we only need to consider the family 6{0,3,6}3 (since any digits 1, 2, 4, 5, 7 between them will produce smaller primes)

*** All numbers of the form 6{0,3,6}3 are divisible by 3, thus cannot be prime.

* Case (6,5):

** **65** is prime, and thus the only minimal prime in this family.

* Case (6,7):

** Since 65, 27, 37, 57, **667** are primes, we only need to consider the family 6{0,1,4,7}7 (since any digits 2, 3, 5, 6 between them will produce smaller primes)

*** Since 107, 117, 147, 177, 407, 417, 717, 747, **6007**, **6477**, **6707**, **6777** are primes, we only need to consider the families 60{1,4,7}7, 6{0}17, 6{0,4}4{4}7, 6{0}77 (since any digits combo 00, 10, 11, 14, 17, 40, 41, 47, 70, 71, 74, 77 between them will produce smaller primes)

**** All numbers of the form 6{0}17 or 6{0}77 are divisible by 3, thus cannot be prime.

**** For the 60{1,4,7}7 family, since 117, 147, 177, 417, 6477, 717, 747, 6777 are primes, we only need to consider the numbers 6017, 6047, 6077 and the family 60{4}7 (since any digit combo 11, 14, 17, 41, 47, 71, 74, 77 between (60,7) will produce smaller primes)

***** None of 6017, 6047, 6077 are primes.

**** All numbers of the form 60{4}7 are divisible by 21, thus cannot be prime.

**** For the 6{0,4}4{4}7 family, since 6007 and 407 are primes, we only need to consider the families 6{4}7 and 60{4}7 (since any digit combo 00, 40 between (6,4{4}7) will produce smaller primes)

***** All numbers of the form 6{4}7 are divisible by 3, 5, or 15, thus cannot be prime.

***** All numbers of the form 60{4}7 are divisible by 21, thus cannot be prime.

* Case (7,1):

** Since 73, 75, 21, 51, **701**, **711** are primes, we only need to consider the family 7{4,6,7}1 (since any digits 0, 1, 2, 3, 5 between them will produce smaller primes)

*** Since 747, 767, 471, 661, **7461**, **7641** are primes, we only need to consider the families 7{4,7}4{4}1, 7{7}61, 7{7}7{4,6,7}1 (since any digits combo 46, 47, 64, 66, 67 between them will produce smaller primes)

**** For the 7{4,7}4{4}1 family, since 747, 471 are primes, we only need to consider the family 7{7}{4}1 (since any digits combo 47 between (7,4{4}1) will produce smaller primes)

***** The smallest prime of the form 7{7}1 is **7777777777771**

***** The smallest prime of the form 7{7}41 is 777777777777777777777777777777777777777777777777777777777777777777777777777777741, with 79 7's, which can be written as $7^{79}41$ and equal the prime $8^{81}-31$ (factordb entry) (shown in base 8) (factorization of the numbers of this form) (not minimal prime, since 7777777777771 is prime)

***** The smallest prime of the form 7{7}441 is 777777777777777777777777777777777777777777777777777777777777777777777777777777777777441, with 84 7's, which can be written as $7^{84}441$ and equal the prime $8^{87}-223$ (factordb entry) (shown in base 8) (factorization of the numbers of this form) (not minimal prime, since 7777777777771 is prime)

***** The smallest prime of the form 7{7}4441 is 7774441, with 233 7's, which can be written as $7^{233}4441$ and equal the prime $8^{237}-1759$ (factordb entry) (shown in base 8) (factorization of the numbers of this form) (not minimal prime, since 7777777777771 is prime)

***** The smallest prime of the form 7{7}44441 is 7777777777777777777777777777777777777777777777777777777744441, with 56 7's, which can be written as $7^{56}44441$ and equal the prime $8^{61}-14047$ (factordb entry) (shown in base 8) (factorization of the numbers of this form) (not minimal prime, since 7777777777771 is prime)

***** All numbers of the form 7{7}444441 are divisible by 7, thus cannot be prime.

***** The smallest prime of the form 7{7}4444441 is **77774444441**

****** Since this prime has just 4 7's, we only need to consider the families with <=3 7's

******* The smallest prime of the form 7{4}1 is **744444441**

******* All numbers of the form 77{4}1 are divisible by 5, thus cannot be prime.

******* The smallest prime of the form 777{4}1 is 7774444444444441 (not minimal prime, since 444444441 and 744444441 are primes)

* Case (7,3):

** **73** is prime, and thus the only minimal prime in this family.

* Case (7,5):

** **75** is prime, and thus the only minimal prime in this family.

* Case (7,7):

** Since 73, 75, 27, 37, 57, **717**, **747**, **767** are primes, we only need to consider the family 7{0,7}7 (since any digits 1, 2, 3, 4, 5, 6 between them will produce smaller primes)

*** All numbers of the form 7{0,7}7 are divisible by 7, thus cannot be prime.

base 10

The possible (first digit,last digit) for an element with >=3 digits in the minimal set of the strings for primes with at least two digits are:

(1,1), (1,3), (1,7), (1,9), (2,1), (2,3), (2,7), (2,9), (3,1), (3,3), (3,7), (3,9), (4,1), (4,3), (4,7), (4,9), (5,1), (5,3), (5,7), (5,9), (6,1), (6,3), (6,7), (6,9), (7,1), (7,3), (7,7), (7,9), (8,1), (8,3), (8,7), (8,9), (9,1), (9,3), (9,7), (9,9)

* Case (1,1):

** **11** is prime, and thus the only minimal prime in this family.

* Case (1,3):

** **13** is prime, and thus the only minimal prime in this family.

* Case (1,7):

** **17** is prime, and thus the only minimal prime in this family.

* Case (1,9):

** **19** is prime, and thus the only minimal prime in this family.

* Case (2,1):

** Since 23, 29, 11, 31, 41, 61, 71, **251**, **281** are primes, we only need to consider the family 2{0,2}1 (since any digits 1, 3, 4, 5, 6, 7, 8, 9 between them will produce smaller primes)

*** Since **2221** and **20201** are primes, we only need to consider the families 2{0}1, 2{0}21, 22{0}1 (since any digits combo 22 or 020 between them will produce smaller primes)

**** All numbers of the form 2{0}1 are divisible by 3, thus cannot be prime.

**** The smallest prime of the form 2{0}21 is **20021**

**** The smallest prime of the form 22{0}1 is **22000001**

* Case (2,3):

** **23** is prime, and thus the only minimal prime in this family.

* Case (2,7):

** Since 23, 29, 17, 37, 47, 67, 97, **227**, **257**, **277** are primes, we only need to consider the family 2{0,8}7 (since any digits 1, 2, 3, 4, 5, 6, 7, 9 between them will produce smaller primes)

*** Since 887 and **2087** are primes, we only need to consider the families 2{0}7 and 28{0}7 (since any digit combo 08 or 88 between them will produce smaller primes)

**** All numbers of the form 2{0}7 are divisible by 3, thus cannot be prime.

**** All numbers of the form 28{0}7 are divisible by 7, thus cannot be prime.

* Case (2,9):

** **29** is prime, and thus the only minimal prime in this family.

* Case (3,1):

** **31** is prime, and thus the only minimal prime in this family.

* Case (3,3):

** Since 31, 37, 13, 23, 43, 53, 73, 83 are primes, we only need to consider the family 3{0,3,6,9}3 (since any digits 1, 2, 4, 5, 7, 8 between them will produce smaller primes)

*** All numbers of the form 3{0,3,6,9}3 are divisible by 3, thus cannot be prime.

* Case (3,7):

** **37** is prime, and thus the only minimal prime in this family.

* Case (3,9):

** Since 31, 37, 19, 29, 59, 79, 89, **349** are primes, we only need to consider the family 3{0,3,6,9}9 (since any digits 1, 2, 4, 5, 7, 8 between them will produce smaller primes)

*** All numbers of the form 3{0,3,6,9}9 are divisible by 3, thus cannot be prime.

* Case (4,1):

** **41** is prime, and thus the only minimal prime in this family.

* Case (4,3):

** **43** is prime, and thus the only minimal prime in this family.

* Case (4,7):

** **47** is prime, and thus the only minimal prime in this family.

* Case (4,9):

** Since 41, 43, 47, 19, 29, 59, 79, 89, **409**, **449**, **499** are primes, we only need to consider the family 4{6}9 (since any digits 0, 1, 2, 3, 4, 5, 7, 8, 9 between them will produce smaller primes)

*** All numbers of the form 4{6}9 are divisible by 7, thus cannot be prime.

* Case (5,1):

** Since 53, 59, 11, 31, 41, 61, 71, **521** are primes, we only need to consider the family 5{0,5,8}1 (since any digits 1, 2, 3, 4, 6, 7, 9 between them will produce smaller primes)

*** Since 881 is prime, we only need to consider the families 5{0,5}1 and 5{0,5}8{0,5}1 (since any digit combo 88 between them will produce smaller primes)

**** For the 5{0,5}1 family, since **5051** and **5501** are primes, we only need to consider the families 5{0}1 and 5{5}1 (since any digit combo 05 or 50 between them will produce smaller primes)

***** All numbers of the form 5{0}1 are divisible by 3, thus cannot be prime.

***** The smallest prime of the form 5{5}1 is **555555555551**

**** For the 5{0,5}8{0,5}1 family, since **5081**, **5581**, **5801**, **5851** are primes, we only need to consider the number 581

***** 581 is not prime.

* Case (5,3):

** **53** is prime, and thus the only minimal prime in this family.

* Case (5,7):

** Since 53, 59, 17, 37, 47, 67, 97, **557**, **577**, **587** are primes, we only need to consider the family 5{0,2}7 (since any digits 1, 3, 4, 5, 6, 7, 8, 9 between them will produce smaller primes)

*** Since 227 and **50207** are primes, we only need to consider the families 5{0}7, 5{0}27, 52{0}7 (since any digits combo 22 or 020 between them will produce smaller primes)

**** All numbers of the form 5{0}7 are divisible by 3, thus cannot be prime.

**** The smallest prime of the form 5{0}27 is **500000000000000000000000000000027**, with 28 0's, which can be written as $50^{28}27$ and equal the prime $5*10^{30}+27$ (factordb entry) (shown in base 10) (factorization of the numbers of this form)

**** The smallest prime of the form 52{0}7 is **5200007**

* Case (5,9):

** **59** is prime, and thus the only minimal prime in this family.

* Case (6,1):

** **61** is prime, and thus the only minimal prime in this family.

* Case (6,3):

** Since 61, 67, 13, 23, 43, 53, 73, 83 are primes, we only need to consider the family 6{0,3,6,9}3 (since any digits 1, 2, 4, 5, 7, 8 between them will produce smaller primes)

*** All numbers of the form 6{0,3,6,9}3 are divisible by 3, thus cannot be prime.

* Case (6,7):

** **67** is prime, and thus the only minimal prime in this family.

* Case (6,9):

** Since 61, 67, 19, 29, 59, 79, 89 are primes, we only need to consider the family 6{0,3,4,6,9}9 (since any digits 1, 2, 5, 7, 8 between them will produce smaller primes)

*** Since 449 is prime, we only need to consider the families 6{0,3,6,9}9 and 6{0,3,6,9}4{0,3,6,9}9 (since any digit combo 44 between them will produce smaller primes)

**** All numbers of the form 6{0,3,6,9}9 are divisible by 3, thus cannot be prime.

**** For the 6{0,3,6,9}4{0,3,6,9}9 family, since 409, 43, **6469**, 499 are primes, we only need to consider the family 6{0,3,6,9}49

***** Since 349, **6949** are primes, we only need to consider the family 6{0,6}49

****** Since **60649** is prime, we only need to consider the family 6{6}{0}49 (since any digits combo 06 between {6,49} will produce smaller primes)

******* The smallest prime of the form 6{6}49 is **666649**

******** Since this prime has just 4 6's, we only need to consider the families with <=3 6's

********* The smallest prime of the form 6{0}49 is **60000049**

********* The smallest prime of the form 66{0}49 is **66000049**

********* The smallest prime of the form 666{0}49 is **66600049**

* Case (7,1):

** **71** is prime, and thus the only minimal prime in this family.

* Case (7,3):

** **73** is prime, and thus the only minimal prime in this family.

* Case (7,7):

** Since 71, 73, 79, 17, 37, 47, 67, 97, **727**, **757**, **787** are primes, we only need to consider the family 7{0,7}7 (since any digits 1, 2, 3, 4, 5, 6, 8, 9 between them will produce smaller primes)

*** All numbers of the form 7{0,7}7 are divisible by 7, thus cannot be prime.

* Case (7,9):

** **79** is prime, and thus the only minimal prime in this family.

* Case (8,1):

** Since 83, 89, 11, 31, 41, 61, 71, **821**, **881** are primes, we only need to consider the family 8{0,5}1 (since any digits 1, 2, 3, 4, 6, 7, 8, 9 between them will produce smaller primes)

*** Since **8501** is prime, we only need to consider the family 8{0}{5}1 (since any digits combo 50 between them will produce smaller primes)

**** Since **80051** is prime, we only need to consider the families 8{0}1, 8{5}1, 80{5}1 (since any digits combo 005 between them will produce smaller primes)

***** All numbers of the form 8{0}1 are divisible by 3, thus cannot be prime.

***** The smallest prime of the form 8{5}1 is 8555555555555555555551 (not minimal prime, since 555555555551 is prime)

***** The smallest prime of the form 80{5}1 is **80555551**

* Case (8,3):

** **83** is prime, and thus the only minimal prime in this family.

* Case (8,7):

** Since 83, 89, 17, 37, 47, 67, 97, **827**, **857**, **877**, **887** are primes, we only need to consider the family 8{0}7 (since any digits 1, 2, 3, 4, 5, 6, 7, 8, 9 between them will produce smaller primes)

*** All numbers of the form 8{0}7 are divisible by 3, thus cannot be prime.

* Case (8,9):

** **89** is prime, and thus the only minimal prime in this family.

* Case (9,1):

** Since 97, 11, 31, 41, 61, 71, **991** are primes, we only need to consider the family 9{0,2,5,8}1 (since any digits 1, 3, 4, 6, 7, 9 between them will produce smaller primes)

*** Since 251, 281, 521, 821, 881, **9001**, **9221**, **9551**, **9851** are primes, we only need to consider the families 9{2,5,8}0{2,5,8}1, 9{0}2{0}1, 9{0}5{0,8}1, 9{0,5}8{0}1 (since any digits combo 00, 22, 25, 28, 52, 55, 82, 85, 88 between them will produce smaller primes)

**** For the 9{2,5,8}0{2,5,8}1 family, since any digits combo 22, 25, 28, 52, 55, 82, 85, 88 between (9,1) will produce smaller primes, we only need to consider the numbers 901, 9021, 9051, 9081, 9201, 9501, 9801, 90581, 95081, 95801

***** 95801 is the only prime among 901, 9021, 9051, 9081, 9201, 9501, 9801, 90581, 95081, 95801, but it is not minimal prime since 5801 is prime.

**** For the 9{0}2{0}1 family, since 9001 is prime, we only need to consider the numbers 921, 9201, 9021

***** None of 921, 9201, 9021 are primes.

**** For the 9{0}5{0,8}1 family, since 9001 and 881 are primes, we only need to consider the numbers 951, 9051, 9501, 9581, 90581, 95081, 95801

***** 95801 is the only prime among 951, 9051, 9501, 9581, 90581, 95081, 95801, but it is not minimal prime since 5801 is prime.

**** For the 9{0,5}8{0}1 family, since 9001 and 5581 are primes, we only need to consider the numbers 981, 9081, 9581, 9801, 90581, 95081, 95801

***** 95801 is the only prime among 981, 9081, 9581, 9801, 90581, 95081, 95801, but it is not minimal prime since 5801 is prime.

* Case (9,3):

** Since 97, 13, 23, 43, 53, 73, 83 are primes, we only need to consider the family 9{0,3,6,9}3 (since any digits 1, 2, 4, 5, 7, 8 between them will produce smaller primes)

*** All numbers of the form 9{0,3,6,9}3 are divisible by 3, thus cannot be prime.

* Case (9,7):

** **97** is prime, and thus the only minimal prime in this family.

* Case (9,9):

** Since 97, 19, 29, 59, 79, 89 are primes, we only need to consider the family 9{0,3,4,6,9}9 (since any digits 1, 2, 5, 7, 8 between them will produce smaller primes)

*** Since 449 is prime, we only need to consider the families 9{0,3,6,9}9 and 9{0,3,6,9}4{0,3,6,9}9 (since any digit combo 44 between them will produce smaller primes)

**** All numbers of the form 9{0,3,6,9}9 are divisible by 3, thus cannot be prime.

**** For the 9{0,3,6,9}4{0,3,6,9}9 family, since **9049**, 349, **9649**, **9949** are primes, we only need to consider the family 94{0,3,6,9}9

***** Since 409, 43, 499 are primes, we only need to consider the family 94{6}9 (since any digits 0, 3, 9 between (94,9) will produce smaller primes)

****** The smallest prime of the form 94{6}9 is **946669**

base 12

The possible (first digit,last digit) for an element with >=3 digits in the minimal set of the strings for primes with at least two digits are:

(1,1), (1,5), (1,7), (1,B), (2,1), (2,5), (2,7), (2,B), (3,1), (3,5), (3,7), (3,B), (4,1), (4,5), (4,7), (4,B), (5,1), (5,5), (5,7), (5,B), (6,1), (6,5), (6,7), (6,B), (7,1), (7,5), (7,7), (7,B), (8,1), (8,5), (8,7), (8,B), (9,1), (9,5), (9,7), (9,B), (A,1), (A,5), (A,7), (A,B), (B,1), (B,5), (B,7), (B,B)

* Case (1,1):

** **11** is prime, and thus the only minimal prime in this family.

* Case (1,5):

** **15** is prime, and thus the only minimal prime in this family.

* Case (1,7):

** **17** is prime, and thus the only minimal prime in this family.

* Case (1,B):

** **1B** is prime, and thus the only minimal prime in this family.

* Case (2,1):

** Since 25, 27, 11, 31, 51, 61, 81, 91, **221**, **241**, **2A1**, **2B1** are primes, we only need to consider the family 2{0}1 (since any digits 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B between them will produce smaller primes)

*** The smallest prime of the form 2{0}1 is **2001**

* Case (2,5):

** **25** is prime, and thus the only minimal prime in this family.

* Case (2,7):

** **27** is prime, and thus the only minimal prime in this family.

* Case (2,B):

** Since 25, 27, 1B, 3B, 4B, 5B, 6B, 8B, AB, **2BB** are primes, we only need to consider the family 2{0,2,9}B (since any digits 1, 3, 4, 5, 6, 7, 8, A, B between them will produce smaller primes)

*** Since 90B, **200B**, **202B**, **222B**, **229B**, **292B**, **299B** are primes, we only need to consider the numbers 20B, 22B, 29B, 209B, 220B (since any digits combo 00, 02, 22, 29, 90, 92, 99 between them will produce smaller primes)

**** None of 20B, 22B, 29B, 209B, 220B are primes.

* Case (3,1):

** **31** is prime, and thus the only minimal prime in this family.

* Case (3,5):

** **35** is prime, and thus the only minimal prime in this family.

* Case (3,7):

** **37** is prime, and thus the only minimal prime in this family.

* Case (3,B):

** **3B** is prime, and thus the only minimal prime in this family.

* Case (4,1):

** Since 45, 4B, 11, 31, 51, 61, 81, 91, **401**, **421**, **471** are primes, we only need to consider the family 4{4,A}1 (since any digit 0, 1, 2, 3, 5, 6, 7, 8, 9, B between them will produce smaller primes)

*** Since A41 and **4441** are primes, we only need to consider the families 4{A}1 and 44{A}1 (since any digit combo 44, A4 between them will produce smaller primes)

**** All numbers of the form 4{A}1 are divisible by 5, thus cannot be prime.

**** The smallest prime of the form 44{A}1 is **44AAA1**

* Case (4,5):

** **45** is prime, and thus the only minimal prime in this family.

* Case (4,7):

** Since 45, 4B, 17, 27, 37, 57, 67, 87, A7, B7, **447**, **497** are primes, we only need to consider the family 4{0,7}7 (since any digit 1, 2, 3, 4, 5, 6, 8, 9, A, B between them will produce smaller primes)

*** Since **4707** and **4777** are primes, we only need to consider the families 4{0}7 and 4{0}77 (since any digit combo 70, 77 between them will produce smaller primes)

**** All numbers of the form 4{0}7 are divisible by B, thus cannot be prime.

**** The smallest prime of the form 4{0}77 is **400000000000000000000000000000000000000077**, with 39 0's, which can be written as $40^{39}77$ and equal the prime $4*12^{41}+91$ (factordb entry) (shown in base 12) (factorization of the numbers of this form)

* Case (4,B):

** **4B** is prime, and thus the only minimal prime in this family.

* Case (5,1):

** **51** is prime, and thus the only minimal prime in this family.

* Case (5,5):

** Since 51, 57, 5B, 15, 25, 35, 45, 75, 85, 95, B5, **565** are primes, we only need to consider the family 5{0,5,A}5 (since any digits 1, 2, 3, 4, 6, 7, 8, 9, B between them will produce smaller primes)

*** All numbers of the form 5{0,5,A}5 are divisible by 5, thus cannot be prime.

* Case (5,7):

** **57** is prime, and thus the only minimal prime in this family.

* Case (5,B):

** **5B** is prime, and thus the only minimal prime in this family.

* Case (6,1):

** **61** is prime, and thus the only minimal prime in this family.

* Case (6,5):

** Since 61, 67, 6B, 15, 25, 35, 45, 75, 85, 95, B5, **655**, **665** are primes, we only need to consider the family 6{0,A}5 (since any digits 1, 2, 3, 4, 5, 6, 7, 8, 9, B between them will produce smaller primes)

*** Since **6A05** and **6AA5** are primes, we only need to consider the families 6{0}5 and 6{0}A5 (since any digit combo A0, AA between them will produce smaller primes)

**** All numbers of the form 6{0}5 are divisible by B, thus cannot be prime.

**** The smallest prime of the form 6{0}A5 is **600A5**

* Case (6,7):

** **67** is prime, and thus the only minimal prime in this family.

* Case (6,B):

** **6B** is prime, and thus the only minimal prime in this family.

* Case (7,1):

** Since 75, 11, 31, 51, 61, 81, 91, **701**, **721**, **771**, **7A1** are primes, we only need to consider the family 7{4,B}1 (since any digits 0, 1, 2, 3, 5, 6, 7, 8, 9, A between them will produce smaller primes)

*** Since 7BB, **7441** and **7B41** are primes, we only need to consider the numbers 741, 7B1, 74B1

**** None of 741, 7B1, 74B1 are primes.

* Case (7,5):

** **75** is prime, and thus the only minimal prime in this family.

* Case (7,7):

** Since 75, 17, 27, 37, 57, 67, 87, A7, B7, **747**, **797** are primes, we only need to consider the family 7{0,7}7 (since any digits 1, 2, 3, 4, 5, 6, 8, 9, A, B between them will produce smaller primes)

*** All numbers of the form 7{0,7}7 are divisible by 7, thus cannot be prime.

* Case (7,B):

** Since 75, 1B, 3B, 4B, 5B, 6B, 8B, AB, **70B**, **77B**, **7BB** are primes, we only need to consider the family 7{2,9}B (since any digits 0, 1, 3, 4, 5, 6, 7, 8, A, B between them will produce smaller primes)

*** Since 222B, 729B is prime, we only need to consider the families 7{9}B, 7{9}2B, 7{9}22B (since any digits combo 222, 29 between them will produce smaller primes)

**** The smallest prime of the form 7{9}B is **7999B**

**** The smallest prime of the form 7{9}2B is 79992B (not minimal prime, since 992B and 7999B are primes)

**** The smallest prime of the form 7{9}22B is 79922B (not minimal prime, since 992B is prime)

* Case (8,1):

** **81** is prime, and thus the only minimal prime in this family.

* Case (8,5):

** **85** is prime, and thus the only minimal prime in this family.

* Case (8,7):

** **87** is prime, and thus the only minimal prime in this family.

* Case (8,B):

** **8B** is prime, and thus the only minimal prime in this family.

* Case (9,1):

** **91** is prime, and thus the only minimal prime in this family.

* Case (9,5):

** **95** is prime, and thus the only minimal prime in this family.

* Case (9,7):

** Since 91, 95, 17, 27, 37, 57, 67, 87, A7, B7, **907** are primes, we only need to consider the family 9{4,7,9}7 (since any digit 0, 1, 2, 3, 5, 6, 8, A, B between them will produce smaller primes)

*** Since 447, 497, 747, 797, **9777**, **9947**, **9997** are primes, we only need to consider the numbers 947, 977, 997, 9477, 9977 (since any digits combo 44, 49, 74, 77, 79, 94, 99 between them will produce smaller primes)

**** None of 947, 977, 997, 9477, 9977 are primes.

* Case (9,B):

** Since 91, 95, 1B, 3B, 4B, 5B, 6B, 8B, AB, **90B**, **9BB** are primes, we only need to consider the family 9{2,7,9}B (since any digit 0, 1, 3, 4, 5, 6, 8, A, B between them will produce smaller primes)

*** Since 27, 77B, **929B**, **992B**, **997B** are primes, we only need to consider the families 9{2,7}2{2}B, 97{2,9}B, 9{7,9}9{9}B (since any digits combo 27, 29, 77, 92, 97 between them will produce smaller primes)

**** For the 9{2,7}2{2}B family, since 27 and 77B are primes, we only need to consider the families 9{2}2{2}B and 97{2}2{2}B (since any digits combo 27, 77 between (9,2{2}B) will produce smaller primes)

***** The smallest prime of the form 9{2}2{2}B is 9222B (not minimal prime, since 222B is prime)

***** The smallest prime of the form 97{2}2{2}B is 9722222222222B (not minimal prime, since 222B is prime)

**** For the 97{2,9}B family, since 729B and 929B are primes, we only need to consider the family 97{9}{2}B (since any digits combo 29 between (97,B) will produce smaller primes)

***** Since 222B is prime, we only need to consider the families 97{9}B, 97{9}2B, 97{9}22B (since any digit combo 222 between (97,B) will produce smaller primes)

****** All numbers of the form 97{9}B are divisible by 11, thus cannot be prime.

****** The smallest prime of the form 97{9}2B is 979999992B (not minimal prime, since 9999B is prime)

****** All numbers of the form 97{9}22B are divisible by 11, thus cannot be prime.

**** For the 9{7,9}9{9}B family, since 77B and 9999B are primes, we only need to consider the numbers 99B, 999B, 979B, 9799B, 9979B

***** None of 99B, 999B, 979B, 9799B, 9979B are primes.

* Case (A,1):

** Since A7, AB, 11, 31, 51, 61, 81, 91, **A41** are primes, we only need to consider the family A{0,2,A}1 (since any digits 1, 3, 4, 5, 6, 7, 8, 9, B between them will produce smaller primes)

*** Since 221, 2A1, **A0A1**, **A201** are primes, we only need to consider the families A{A}{0}1 and A{A}{0}21 (since any digits combo 0A, 20, 22, 2A between them will produce smaller primes)

**** For the A{A}{0}1 family:

***** All numbers of the form A{0}1 are divisible by B, thus cannot be prime.

***** The smallest prime of the form AA{0}1 is **AA000001**

***** The smallest prime of the form AAA{0}1 is **AAA0001**

***** The smallest prime of the form AAAA{0}1 is **AAAA1**

****** Since this prime has no 0's, we do not need to consider the families {A}1, {A}01, {A}001, etc.

**** All numbers of the form A{A}{0}21 are divisible by 5, thus cannot be prime.

* Case (A,5):

** Since A7, AB, 15, 25, 35, 45, 75, 85, 95, B5 are primes, we only need to consider the family A{0,5,6,A}5 (since any digits 1, 2, 3, 4, 7, 8, 9, B between them will produce smaller primes)

*** Since 565, 655, 665, **A605**, **A6A5**, **AA65** are primes, we only need to consider the families A{0,5,A}5 and A{0}65 (since any digits combo 56, 60, 65, 66, 6A, A6 between them will produce smaller primes)

**** All numbers of the form A{0,5,A}5 are divisible by 5, thus cannot be prime.

**** The smallest prime of the form A{0}65 is **A00065**

* Case (A,7):

** **A7** is prime, and thus the only minimal prime in this family.

* Case (A,B):

** **AB** is prime, and thus the only minimal prime in this family.

* Case (B,1):

** Since B5, B7, 11, 31, 51, 61, 81, 91, **B21** are primes, we only need to consider the family B{0,4,A,B}1 (since any digits 1, 2, 3, 5, 6, 7, 8, 9 between them will produce smaller primes)

*** Since 4B, AB, 401, A41, **B001**, **B0B1**, **BB01**, **BB41** are primes, we only need to consider the families B{A}0{4,A}1, B{0,4}4{4,A}1, B{0,4,A,B}A{0,A}1, B{B}B{A,B}1 (since any digits combo 00, 0B, 40, 4B, A4, AB, B0, B4 between them will produce smaller primes)

**** For the B{A}0{4,A}1 family, since A41 is prime, we only need consider the families B0{4}{A}1 and B{A}0{A}1

***** For the B0{4}{A}1 family, since **B04A1** is prime, we only need to consider the families B0{4}1 and B0{A}1

****** The smallest prime of the form B0{4}1 is B04441 (not minimal prime, since 4441 is prime)

****** The smallest prime of the form B0{A}1 is B0AAAA1 (not minimal prime, since AAAA1 is prime)

***** For the B{A}0{A}1 family, since A0A1 is prime, we only need to consider the families B{A}01 and B0{A}1

****** The smallest prime of the form B{A}01 is **BAA01**

****** The smallest prime of the form B0{A}1 is B0AAAAA1 (not minimal prime, since AAAA1 is prime)

**** For the B{0,4}4{4,A}1 family, since 4441 is prime, we only need to consider the families B{0}4{4,A}1 and B{0,4}4{A}1

***** For the B{0}4{4,A}1 family, since B001 is prime, we only need to consider the families B4{4,A}1 and B04{4,A}1

****** For the B4{4,A}1 family, since A41 is prime, we only need to consider the family B4{4}{A}1

******* Since 4441 and BAAA1 are primes, we only need to consider the numbers B41, B441, B4A1, B44A1, B4AA1, B44AA1

******** None of B41, B441, B4A1, B44A1, B4AA1, B44AA1 are primes.

****** For the B04{4,A}1 family, since **B04A1** is prime, we only need to consider the family B04{4}1

******* The smallest prime of the form B04{4}1 is B04441 (not minimal prime, since 4441 is prime)

***** For the B{0,4}4{A}1 family, since 401, 4441, B001 are primes, we only need to consider the families B4{A}1, B04{A}1, B44{A}1, B044{A}1 (since any digits combo 00, 40, 44 between (B,4{A}1) will produce smaller primes)

****** The smallest prime of the form B4{A}1 is B4AAA1 (not minimal prime, since BAAA1 is prime)

****** The smallest prime of the form B04{A}1 is **B04A1**

****** The smallest prime of the form B44{A}1 is B44AAAAAAA1 (not minimal prime, since BAAA1 is prime)

****** The smallest prime of the form B044{A}1 is B044A1 (not minimal prime, since B04A1 is prime)

**** For the B{0,4,A,B}A{0,A}1 family, since all numbers in this family with 0 between (B,1) are in the B{A}0{4,A}1 family, and all numbers in this family with 4 between (B,1) are in the B{0,4}4{4,A}1 family, we only need to consider the family B{A,B}A{A}1

***** Since **BAAA1** is prime, we only need to consider the families B{A,B}A1 and B{A,B}AA1

****** For the B{A,B}A1 family, since AB and **BAAA1** are primes, we only need to consider the families B{B}A1 and B{B}AA1

******* All numbers of the form B{B}A1 are divisible by B, thus cannot be prime.

******* The smallest prime of the form B{B}AA1 is **BBBAA1**

****** For the B{A,B}AA1 family, since **BAAA1** is prime, we only need to consider the families B{B}AA1

******* The smallest prime of the form B{B}AA1 is **BBBAA1**

**** For the B{B}B{A,B}1 family, since AB and BAAA1 are primes, we only need to consider the families B{B}B1, B{B}BA1, B{B}BAA1 (since any digits combo AB or AAA between (B{B}B,1) will produce smaller primes)

***** The smallest prime of the form B{B}B1 is **BBBB1**

***** All numbers of the form B{B}BA1 are divisible by B, thus cannot be prime.

***** The smallest prime of the form B{B}BAA1 is **BBBAA1**

* Case (B,5):

** **B5** is prime, and thus the only minimal prime in this family.

* Case (B,7):

** **B7** is prime, and thus the only minimal prime in this family.

* Case (B,B):

** Since B5, B7, 1B, 3B, 4B, 5B, 6B, 8B, AB, **B2B** are primes, we only need to consider the family B{0,9,B}B (since any digits 1, 2, 3, 4, 5, 6, 7, 8, A between them will produce smaller primes)

*** Since 90B and 9BB are primes, we only need to consider the families B{0,B}{9}B

**** Since 9999B is prime, we only need to consider the families B{0,B}B, B{0,B}9B, B{0,B}99B, B{0,B}999B

***** All numbers of the form B{0,B}B are divisible by B, thus cannot be prime.

***** For the B{0,B}9B family:

****** Since **B0B9B** and **BB09B** are primes, we only need to consider the families B{0}9B and B{B}9B (since any digits combo 0B, B0 between (B,9B) will produce smaller primes)

******* The smallest prime of the form B{0}9B is **B0000000000000000000000000009B**, with 27 0's, which can be written as $B0^{27}9B$ and equal the prime $11*12^{29}+119$ (factordb entry) (shown in base 12) (factorization of the numbers of this form)

******* All numbers of the from B{B}9B is either divisible by 11 (if totally number of B's is even) or factored as 10^(2*n)-21 = (10^n-5) * (10^n+5) (if totally number of B's is odd number 2*n-1 (n≥1)) (and since if n≥1, 10^n-5 ≥ 10^1-5 = 7 > 1, 10^n+5 ≥ 10^1+5 = 15 > 1, this factorization is nontrivial), thus cannot be prime.

***** For the B{0,B}99B family:

****** Since B0B9B and BB09B are primes, we only need to consider the families B{0}99B and B{B}99B (since any digits combo 0B, B0 between (B,99B) will produce smaller primes)

******* The smallest prime of the form B{0}99B is **B00099B**

******* The smallest prime of the form B{B}99B is **BBBBBB99B**

***** For the B{0,B}999B family:

****** Since B0B9B and BB09B are primes, we only need to consider the families B{0}999B and B{B}999B (since any digits combo 0B, B0 between (B,999B) will produce smaller primes)

******* The smallest prime of the form B{0}999B is B00999B, which can be written as $B0^{1765}999B$ and equal the prime $11*12^{1769}+16967$ (factordb entry) (primality certificate of this prime) (shown in base 12) (factorization of the numbers of this form) (not minimal prime, since B00099B and B00000000000000000000000000009B are primes)

******* The smallest prime of the form B{B}999B is BB999B, with 245 B's, which can be written as $B^{244}999B$ and equal the prime $12^{248}-3769$ (factordb entry) (shown in base 12) (factorization of the numbers of this form) (not minimal prime, since BBBBBB99B is prime)

Conclusion and perspectives

References

Main reference for this article: The Mersenneforum thread https://mersenneforum.org/showthread.php?t=24972 (which is the entry of the researching in this article in Mersenneforum)

Other references:

[1] http://primes.utm.edu/glossary/xpage/MinimalPrime.html (article “minimal prime” in The Prime Glossary)

[2] https://en.wikipedia.org/wiki/Minimal_prime_(recreational_mathematics) (article “minimal prime” in Wikipedia)

[3] https://www.primepuzzles.net/puzzles/puzz_178.htm (the puzzle of minimal primes (when the restriction of prime>base is not required) in The Prime Puzzles & Problems Connection, **warning: the solutions for the minimal $4k+1$ and $4k-1$ primes given by Andrew Rupinsiki have many errors, the list wrongly including many primes which are not minimal primes, including the largest “minimal $4k+1$ prime” in the list: $9^{630}493 = 10^{633}-507$ (factordb entry) (primality certificate of this prime) (shown in base 10), this prime is not a minimal $4k+1$ prime since 9949 is also a prime == 1 mod 4, and 9949 is a subsequence of $9^{630}493$, there are 146 (instead of 173) minimal $4k+1$ primes and 113 (instead of 138) minimal $4k-1$ primes, and the largest minimal $4k+1$ prime is $8^{77}33 = (8*10^{79}-503)/9$ (factordb entry) (shown in base 10) instead of $9^{630}493 = 10^{633}-507$ (factordb entry) (primality certificate of this prime) (shown in base 10), for the correct solution see https://raw.githubusercontent.com/curtisbright/mepn-data/master/data/primes1mod4/minimal.10.txt (minimal $4k+1$ primes) and https://raw.githubusercontent.com/curtisbright/mepn-data/master/data/primes3mod4/minimal.10.txt (minimal $4k-1$ primes) (or https://oeis.org/A111055/b111055.txt (minimal $4k+1$ primes) and https://oeis.org/A111056/b111056.txt (minimal $4k-1$ primes), note: since the limit of the numbers in OEIS b-file is $10^{1000}-1$, the list https://oeis.org/A111056/b111056.txt does not include the large prime $2^{19151}99 = (2*10^{19153}+691)/9$), respectively**) (factordb entry of the largest minimal $4k-1$ prime: $2^{19151}99$) (primality certificate of the largest minimal $4k-1$ prime: $2^{19151}99$) (the largest minimal $4k-1$ prime ($2^{19151}99$) shown in base 10)

[4] https://www.primepuzzles.net/problems/prob_083.htm (the problem of minimal primes in The Prime Puzzles & Problems Connection)

[5] https://github.com/xayahrainie4793/non-single-digit-primes (my data for these $M(L_b)$ sets for $2 \le b \le 36$, file “minimal *b*” (for $2 \le b \le 18$) is the data of all known minimal primes or PRPs in base *b* (format: “base *b* representation”=decimal representation), and file “kernel *b*.txt” (for $17 \le b \le 36$) is the data of minimal primes < 2^{32} in base *b* (format: “base *b* representation”=decimal representation), and file “unsolved *b*” (for $2 \le b \le 16$) is the list of all known unsolved families (families for which not even a probable prime is known nor can be ruled out as only contain composites) in base *b*, the format of the unsolved families is *x*{*y*}*z* for *xyyy*...*yyyz* (format: “base *b* form”=algebraic form), and file “unprovenPRP *b*” is the list of all unproven probable primes such that if their primalities are proven, then they will be minimal primes in base *b* (format: “base *b* representation”=decimal representation))

[6] http://recursed.blogspot.com/2006/12/prime-game.html (Shallit’s The Prime Game page) (also file of cards, and the cards for the case when the restriction of prime>base is required (i.e. the problem in this article) is “Ask a friend to write down a prime number > 10. Bet them that you can always strike out 0 or more digits to get a prime on this card. {11, 13, 17, 19,

23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 227, 251, 257, 277, 281, 349, 409, 449, 499, 521, 557, 577, 587, 727, 757, 787, 821, 827, 857, 877, 881, 887, 991, 2087, 2221, 5051, 5081, 5501, 5581, 5801, 5851, 6469, 6949, 8501, 9001, 9049, 9221, 9551, 9649, 9851, 9949, 20021, 20201, 50207, 60649, 80051, 666649, 946669, 5200007, 22000001, 60000049, 66000049, 66600049, 80555551, 555555555551, 5000000000000000000000000000027})

[7] http://www.cs.uwaterloo.ca/~shallit/Papers/minimal5.pdf (Shallit's proof of base 10 minimal primes, when the restriction of prime>base is not required) (the same pdf files: http://www.wiskundemeisjes.nl/wp-content/uploads/2007/02/minimal.pdf and http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.7.5686&rep=rep1&type=pdf)

[8] https://scholar.colorado.edu/downloads/hh63sw661 (proofs of minimal primes in bases $b \le 10$, when the restriction of prime>base is not required, **warning: the sets of $M(L_b)$ have errors for $b = 8$ and $b = 10$, $b = 8$ misses the prime 6101 and $b = 10$ misses the primes 9001 and 9049 and instead wrongly including the primes 90001, 90469, and 9000049, thus the correct values of $|M(L_b)|$ for $b = 8$ and $b = 10$ are 15 and 26 (instead of 14 and 27), respectively**) (scanned copy version in GitHub) (there is also a talking for minimal primes in bases $b \le 10$, when the restriction of prime>base is not required, **but also have error in base 8, this talking misses the prime 111 in base 8**)

[9] https://cs.uwaterloo.ca/~cbright/reports/mepn.pdf (the article for this minimal prime problem in bases $b \le 30$, when the restriction of prime>base is not required, **warning: this article incorrectly uses "subword" or "substring" for subsequence**) (scanned copy in GitHub) (a similar pdf file: https://cs.uwaterloo.ca/~shallit/Papers/br10.pdf) (scanned copy version in GitHub) (this article also has its own entry in https://cs.uwaterloo.ca/~cbright/extended-research-statement.pdf, see section 3.1)

[10] https://cs.uwaterloo.ca/~cbright/talks/minimal-slides.pdf (the article for this minimal prime problem in bases $b \le 30$, when the restriction of prime>base is not required, **warning: this article incorrectly uses "subword" for subsequence**) (scanned copy version in GitHub)

[11] https://doi.org/10.1080/10586458.2015.1064048 (the article for this minimal prime problem in bases $b \le 30$, when the restriction of prime>base is not required, **warning: this article incorrectly uses "substring" for subsequence**) (the same article in ResearchGate: https://www.researchgate.net/publication/297608030_Minimal_Elements_for_the_Prime_Numbers) (the article report file: https://raw.githubusercontent.com/curtisbright/mepn-data/master/report/report.tex) (scanned copy version in GitHub)

[12] https://github.com/curtisbright/mepn-data (data for these $M(L_b)$ sets and unsolved families for $2 \le b \le 30$, when the restriction of prime>base is not required, file "minimal.*b*.txt" is the data of all known minimal primes or PRPs in base *b* (only base *b* representation, no decimal representation (unless the base *b* is exactly 10, of course)), and file "unsolved.*b*.txt" is the list of all unsolved families (families for which not even a probable prime is known nor can be ruled out as only contain composites) in base *b*, the format of the unsolved families is *xy**z* for *xyyy*...*yyyz*, for bases $2 \le b \le 16$ and b = 18, 20, 22, 23, 24, 30 are completely

solved, except the largest element in $M(L_{13})$ and largest 9 elements in $M(L_{23})$ (except the second-largest element in $M(L_{23})$, it can be proven prime using $N-1$ primality test, since $n-1$ can be trivially fully factored for this number n) are only probable primes, i.e. not definitely primes, thus we cannot definitely say that the corresponding families can be removed from the list of unsolved families, and we cannot definitely compute this part of the sets $M(L_b)$, search limits of lengths for the unsolved families (families for which not even a probable prime is known nor can be ruled out as only contain composites) in base *b*: 1000000 for *b* = 17, 707000 for *b* = 19, 506000 for *b* = 21, 292000 for *b* = 25, 486000 for *b* = 26, 368000 for *b* = 27, 543000 for *b* = 28, 233000 for *b* = 29, and file "sieve.*b*.txt" is the *LLR* sieving file for the unsolved families in base *b*, which is computed by *srsieve* (the *srsieve* program should be updated to allow sieving sequences $a*b^n+c$ with *a*, *b*, *c* all odd)) (article about the largest element in $M(L_{13})$)

[13] https://github.com/RaymondDevillers/primes (data for these $M(L_b)$ sets and unsolved families for $28 \le b \le 50$, when the restriction of prime>base is not required, using lowercase letters a−n to represent digit values 36 to 49 for bases *b* > 36, file "kernel *b*" is the data of all known minimal primes or PRPs in base *b* (format: "base *b* representation"=decimal representation), and file "left *b*" is the list of all unsolved families (families for which not even a probable prime is known nor can be ruled out as only contain composites) in base *b*, the format of the unsolved families is *x*{*y*}*z* for *xyyy*...*yyyz*, only bases *b* = 30 and *b* = 42 are completely solved, search limits of lengths for the unsolved families (families for which not even a probable prime is known nor can be ruled out as only contain composites) in base *b*: 10000 for all *b*)

(the lower bound of $|M(L_b)|$ is "Number of known minimal primes or PRPs (when the restriction of prime>base is not required)" + number of "Additional minimal primes or PRPs (when the restriction of prime>base is not required) not in the lists", and the upper bound of $|M(L_b)|$ is "Number of known minimal primes or PRPs (when the restriction of prime>base is not required)" + number of "Additional minimal primes or PRPs (when the restriction of prime>base is not required) not in the lists" + "Number of unsolved families (when the restriction of prime>base is not required)")

b	Number of known minimal primes or PRPs (when the restriction of prime>base is not required)	Number of unsolved families (when the restriction of prime>base is not requi	Additional minimal primes or PRPs (when the restriction of prime>base is not required) not in the lists	Unneeded families (when the restriction of prime>base is not required)	Search limit higher then the lists

		red)			
2	2	0			
3	3	0			
4	3	0			
5	8	0			
6	7	0			
7	9	0			
8	15	0			
9	12	0			
10	26	0			
11	152	0			
12	17	0			
13	228	0			
14	240	0			
15	100	0			
16	483	0			
17	1279	1			
18	50	0			
19	3462	1			
20	651	0			
21	2600	1			
22	1242	0			
23	6021	0			
24	306	0			
25	17597	12			
26	5662	2			
27	17210	5			
28	5783	1			
29	57283	14			

30	220	0			
31	79189	14	E8U^{21866}P = 443*31^{21867}−6 (factordb entry) IEL^{29787} = (5727*31^{29787}−7)/10 (factordb entry) LF^{21052}G = (43*31^{21053}+1)/2 (factordb entry) MIO^{10747}L = (3504*31^{10748}−19)/5 (factordb entry) PEO0^{22367}Q = 24483*31^{22368}+26 (factordb entry) L^{10012}9G = (7*31^{10014}−3777)/10 (factordb entry) R^{22137}1R = (9*31^{22139}−8069)/10 (factordb entry)	ILE{L} (no primes or PRPs up to ILEL^{30000}, and IEL^{29787} is PRP) L0{F}G (no primes or PRPs up to L0F^{23000}G, and LF^{21052}G is PRP) {L}9IG (no primes or PRPs up to L^{13000}9IG, and L^{10012}9G is PRP)	M{P} (searched to length 41962) P{F}G (searched to length 37061) SP{0}K (searched to length 28000) {F}G (searched to length 4194303) {F}KO (searched to length 35000) {F}RA (searched to length 34000) {L}CE (searched to length 21000) {L}G (searched to length 30000) {L}IS (searched to length 25000) {L}SO (searched to length 22000) {P}I (searched to length 32000) {R}1 (searched to length 27000) {R}8 (searched to length 33000)

					{U}P8K (searched to length 30000)
32	45205	78			4{0}1 (searched to length 1717986918) G{0}1 (searched to length 3435973836) UG{0}1 (searched to length 560002)
33	57676	33			
34	56457	33			
35	182378	15			
36	6296	1	$P^{81993}SZ = (5*36^{81995}+821)/7$ (factordb entry)		O{L}Z (searched to length 100000)
37	314988	275	$FYa^{22021} = 590*37^{22021}-1$ (factordb entry) $R8a^{20895} = 1008*37^{20895}-1$ (factordb entry)		
38	106838	77			1{0}1 (searched to length 16777216)
39	230317	43			
40	37773	1	$QaU^{12380}X = (13998*40^{12381}+29)/13$ (factordb entry)		S{Q}d (searched to length 100000)
41	689061	335			
42	4551	0			
43	900795	536			
44	255911	103			

45	323437	47	$O0^{18521}1 = 24*45^{18522}+1$ (factordb entry)	AO{0}1 (the smallest prime is $AO0^{44790}1 = 474*45^{44791}+1$ (factordb entry), but $O0^{18521}1$ is prime) (Note: O{0}1F1 and O{0}ZZ1 are still needed, since they are only searched to length 10000)	9W1{0}1 (searched to length 100003)
46	399012	113			d4{0}1 (searched to length 500002)
47	1436289	994			
48	29103	6			a{0}1 (searched to length 500001)
49	4365269	1183	$11c0^{29736}1 = 2488*49^{29737}+1$ (factordb entry) $Fd0^{18340}1 = 774*49^{18341}+1$ (factordb entry) $SLm^{52698} = 1394*49^{52698}-1$ (factordb entry) $Ydm^{16337} = 1706*49^{16337}-1$ (factordb entry)	(Note: S6L{m}, YUUd{m}, YUd{m} are still needed, since they are only searched to length 10000)	
50	189914	62			1{0}1 (searched to length 16777216) a{n} (searched to length 121290)

[14] http://www.bitman.name/math/article/730 (article for minimal primes, when the restriction of prime>base is not required)

[15] http://www.bitman.name/math/table/497 (data for minimal primes in bases 2 ≤ b ≤ 16, when the restriction of prime>base is not required) (also data for b = 17 b = 18 b = 19 b = 20)

[16] https://oeis.org/A071071/a071071.pdf (research of minimal sets of powers of 2, when the restriction of >base is not required) (also this related article for the number 65536)

[17] http://nntdm.net/papers/nntdm-25/NNTDM-25-1-036-047.pdf (research of minimal set of totients+n in base b = 10 for 0≤n≤5, when the restriction of >base is not required) (this is from the article: https://arxiv.org/pdf/1607.01548.pdf (the same article in ResearchGate: https://www.researchgate.net/publication/304964965_Deleting_digits), which is research of minimal set of the range of Euler phi function and the range of Dedekind psi function, both in base b = 10)

(this list include the minimal set of sets S which either are researched in at least one articles above or have *OEIS* sequence, for the minimal set of other sets S (e.g. primes == 1 mod 3, primes == 2 mod 3, semiprimes, prime powers, …, see https://mersenneforum.org/showpost.php?p=572102&postcount=119 and https://mersenneforum.org/showpost.php?p=572225&postcount=122)

S	the minimal set of S (in base b = 10) (unlike the research of the minimal primes in this article, the restriction of >base is not required)
primes (A071062)	{2, 3, 5, 7, 11, 19, 41, 61, 89, 409, 449, 499, 881, 991, 6469, 6949, 9001, 9049, 9649, 9949, 60649, 666649, 946669, 60000049, 66000049, 66600049}
composites (A071070)	{4, 6, 8, 9, 10, 12, 15, 20, 21, 22, 25, 27, 30, 32, 33, 35, 50, 51, 52, 55, 57, 70, 72, 75, 77, 111, 117, 171, 371, 711, 713, 731}
squares (A130448)	{1, 4, 9, 25, 36, 576, 676, 7056, 80656, 665856, 2027776, 2802276, 22282727076, 77770707876, 78807087076, 7888885568656, 8782782707776, 72822772707876, 555006880085056, 782280288087076, 827702888070276, 888288787822276, 2282820800707876, 7880082008070276, 80077778877070276, 88778000807227876, 782828878078078276, 872727072820287876, 2707700770820007076, 7078287780880770276, 7808287827720727876, 8008002202002207876, 27282772777702807876, 70880800720008787876, 72887222220777087876, 80028077888770207876, 80880700827207270276, 87078270070088278276, 88002002000028027076, 288227827888822880787 6, 87707777808882288707 6, 77700027222828822007876, 702087807788807888287876, 7887080878820072808088278 76, 8800700080778088770000022 76, 8880002270870707078808270 76, 8880770272272282770877870 76, 8885888865555050858885555 56, 7770000800780088788282227 776, 7782727788888878708800870 276,

	5000060065066660656065066555556, 8070008800822880080708800087876, 8078787080888880827207777227076, 8000080880708208708700777788278 76, 8228227222200808888780788208878 76, ...} (currently not known, and might be extremely difficult to found)
cubes	{1, 8, 27, 64, 343, 729, 3375, 4096, 35937, 39304, 46656, 50653, 79507, 97336, 300763, 405224, 456533, 474552, 493039, 636056, 704969, 3307949, 4330747, 5545233, 5639752, 5735339, 6539203, 9663597, 23393656, 23639903, 29503629, 37933056, 40353607, 45499293, 50243409, 54439939, 57066625, 57960603, 70444997, 70957944, 73560059, 76765625, 95443993, 202262003, 236029032, 350402625, 377933067, 379503424, 445943744, 454756609, 537367797, 549353259, 563559976, 567663552, 773620632, 907039232, ...} (currently not known, and might be extremely difficult to found)
primes == 1 mod 4 ([A111055](A111055))	{5, 13, 17, 29, 37, 41, 61, 73, 89, 97, 101, 109, 149, 181, 233, 277, 281, 349, 409, 433, 449, 677, 701, 709, 769, 821, 877, 881, 1669, 2221, 3001, 3121, 3169, 3221, 3301, 3833, 4969, 4993, 6469, 6833, 6949, 7121, 7477, 7949, 9001, 9049, 9221, 9649, 9833, 9901, 9949, 11969, 19121, 20021, 20201, 21121, 23021, 23201, 43669, 44777, 47777, 60493, 60649, 66749, 80833, 90121, 91121, 91921, 91969, 94693, 111121, 112121, 119921, 199921, 220301, 466369, 470077, 666493, 666649, 772721, 777221, 777781, 779981, 799921, 800333, 803333, 806033, 833033, 833633, 860333, 863633, 901169, 946369, 946669, 999169, 1111169, 1999969, 4007077, 4044077, 4400477, 4666693, 8000033, 8000633, 8006633, 8600633, 8660033, 8830033, 8863333, 8866633, 22000001, 40400077, 44040077, 60000049, 66000049, 66600049, 79999981, 80666633, 83333333, 86606633, 86666633, 88600033, 88883033, 88886033, 400000477, 400444477, 444000077, 444044477, 836666333, 866663333, 888803633, 888806333, 888880633, 888886333, 8888800033, 8888888033, 88888883333, 440444444477, 7777777777921, 8888888888333, 40000000000777, 44444444400077, 404444444444477, 4444444444444477, 88888888888888633, 999999999999999121, 88888888888888888888888888888888888888888888888888888888888888888888888833}
primes == 3 mod 4 ([A111056](A111056))	{3, 7, 11, 19, 59, 251, 491, 499, 691, 991, 2099, 2699, 2999, 4051, 4451, 4651, 5051, 5651, 5851, 6299, 6451, 6551, 6899, 8291, 8699, 8951, 8999, 9551, 9851, 22091, 22291, 66851, 80051, 80651, 84551, 85451, 86851, 88651, 92899, 98299, 98899, 200891, 208891, 228299, 282299, 545551, 608851, 686051, 822299, 828899, 848851, 866051, 880091, 885551, 888091, 888451, 902299, 909299, 909899, 2000291, 2888299, 2888891, 8000099, 8000891, 8000899, 8028299, 8808299, 8808551, 8880551, 8888851, 9000451, 9000899, 9908099, 9980099, 9990899, 9998099, 9999299, 60000851, 60008651, 60086651, 60866651, 68666651, 80088299, 80555551, 80888299, 88808099, 88808899, 88880899, 90000299, 90080099, 222222899, 800888899, 808802899, 808880099, 808888099, 888800299, 888822899,

	992222299, 2222288899, 8808888899, 8888800099, 8888888299, 8888888891, 48555555551, 555555555551, 999999999899, 88888888888099, 2228888888888899, 9222222222222299, 2288888888888888888888899, 888888888888888888888888888888888888888888899, 86666666666666666666666666666666666666666666666651, 222222222222222222222222222222222222222222222222222222

	222222222222222222222222222222222222222222222222222222222222 222222222222222222222222222222222222222222222222222222222222

	2222222222222222222222222222222222222222222222222222222222222 2222222222222222222222222222222222222222222222222222222222222

	222222222222222222222222222222222222222222222222222222 222222222222222222222222222222222222222222222222222222

	222222222222222222222222222222222222222222222222222222 222222222222222222222222222222222222222222222222222222

	222222222222222222222222222222222222222222222222222222 222222222222222222222222222222222222222222222222222222

	222222222222222222222222222222222222222222222222222222 222222222222222222222222222222222222222222222222222222 2222222222222222222222222222222222299 (with 19151 2's, which can be written as $2^{19151}99$ and equal the prime $(2*10^{19153}+691)/9$ (factordb entry) (shown in base 10) (primality certificate of this prime) (Prime Curios! entry)}
palindromic primes (A114835) (the b-file of this sequence is not complete, it only has the smallest 86 terms, and at least one prime $994^{34019}99$ = $(895*10^{34021}+491)/9$ (found by me) is missed in the b-file, although this prime	{2, 3, 5, 7, 11, 919, 94049, 94649, 94849, 94949, 96469, 98689, 9809089, 9888889, 9889889, 9908099, 9980899, 9989899, 900808009, 906686609, 906989609, 908000809, 908444809, 908808809, 909848909, 960898069, 968999869, 988000889, 989040989, 996686699, 996989699, 999686999, 90689098609, 90899999809, 90999899909, 96099899069, 96600800669, 96609890669, 98000000089, 98844444889, 9009004009009, 9099094909909, 9600098900069, 9668000008669, 9699998999969, 9844444444489, 9899900099989, 9900004000099, 9900994990099, 900006898600009, 900904444409009, 966666989666669, 966668909866669, 966699989996669, 999090040090999, 999904444409999, 90000006860000009, 90000008480000009, 90000089998000009,

is only a probable prime, i.e. not definitely prime)	90999444444499909, 96000060806000069, 99900944444900999, 99990009490009999, 99999884448899999, 9000090994990900009, 9000094444444900009, 9666666080806666669, 9666666668666666669, 9909999994999999099, 9999444444444449999, 9999909994999099999, 9999990994990999999, 900000000080000000009, 900999994444499999009, 90000000009490000000009, 90909444444444444490909, 98999999444444499999989, 9904444444444444444444099, 999999999844444448999999999, 90944444444444444444444444909, 99999999999944444999999999999, 99999999999999499999999999999, 9999999999990004000999999999999, 900000000999999949999999000000009, 989999999999998444899999999999989, 9000000999999999994999999999990000009, ..., 994444444444444444444444444444444444444444444444444444 444444444444444444444444444444444444444444444444444444

	44444444444444444444444444444444444444444444444444444 44444444444444444444444444444444444444444444444444444

	4444444444444444444444444444444444444444444444444444 4444444444444444444444444444444444444444444444444444

	4444444444444444444444444444444444444444444444444444444 4444444444444444444444444444444444444444444444444444444

	444444444444444444444444444444444444444444444444444444444444 444444444444444444444444444444444444444444444444444444444444

	444444444444444444444444444444444444444444444444444444 444444444444444444444444444444444444444444444444444444

	444444444444444444444444444444444444444444444444444444 444444444444444444444444444444444444444444444444444444

	4444444444444444444444444444444444444444444444444444444444444 4444444444444444444444444444444444444444444444444444444444444

	444444444444444444444444444444444444444444444444444444 444444444444444444444444444444444444444444444444444444

	444444444444444444444444444444444444444444444444444444 444444444444444444444444444444444444444444444444444444

	44444444444444444444444444444444444444444444444444444444444444 44444444444444444444444444444444444444444444444444444444444444

	444444444444444444444444444444444444444444444444444444 444444444444444444444444444444444444444444444444444444 499, ... (with 34019 4's, which can be written as $994^{34019}99$ and

	equal the prime (895*10^{34021}+491)/9 (factordb entry) (shown in base 10) (Prime Curios! entry) (this number is only probable prime, i.e. not definitely prime))} (conjectured, not proven, I did not prove that this set is complete, and none has fully checked whether this set is complete or not)
powers of 2 (A071071)	{1, 2, 4, 8, 65536} (conjectured by Jeffrey Shallit, not proven, however of course, if all powers of 2 except 65536 contain at least one of 1, 2, 4, 8, then this conjecture is true, only powers of 16 can be exceptions)
multiples of 3 (A071073)	{0, 3, 6, 9, 12, 15, 18, 21, 24, 27, 42, 45, 48, 51, 54, 57, 72, 75, 78, 81, 84, 87, 111, 114, 117, 141, 144, 147, 171, 174, 177, 222, 225, 228, 252, 255, 258, 282, 285, 288, 411, 414, 417, 441, 444, 447, 471, 474, 477, 522, 525, 528, 552, 555, 558, 582, 585, 588, 711, 714, 717, 741, 744, 747, 771, 774, 777, 822, 825, 828, 852, 855, 858, 882, 885, 888}
multiples of 4 (A071072)	{0, 4, 8, 12, 16, 32, 36, 52, 56, 72, 76, 92, 96}
Fibonacci numbers	{1, 2, 3, 5, 8} (conjectured by Jeffrey Shallit, not proven, however of course, if all Fibonacci numbers contain at least one of 1, 2, 3, 5, 8, then this conjecture is true)
perfect numbers	{6, 28} (if there are no odd perfect numbers, then this is proven, however, since whether there is any odd perfect number is still an open problem, thus we cannot definitely say that this is the minimal set)
range of Euler phi function	{1, 2, 4, 6, 8, 30, 70, 500, 900, 990, 5590, 9550, 555555555550}
range of Dedekind psi function	{1, 3, 4, 6, 8, 20, 72, 90, 222, 252, 500, 522, 552, 570, 592, 750, 770, 992, 7000, 5555555555555555555555555555555555555555555555555555555555555555555550 (with 69 5's, which can be written as $5^{69}0$ and equal 50*(10^69−1)/9) (factordb entry) (shown in base 10)}
totients+1	{2, 3, 5, 7, 9, 11, 41, 61, 81}
totients+2	{3, 4, 6, 8, 10, 12, 20, 22, 50, 72, 90, 770, 992, 5592, 9552, 555555555552} (conjectured, not proven, this conjecture is true if and only if there are no totients of the form 6{9}8, and such totients are conjectured not exist, since such totients are == 2 mod 12, thus must be of the form $(p-1)*p^n$ with p prime and n odd)
totients+3	{4, 5, 7, 9, 11, 13, 21, 23, 31, 33, 61, 63, 81, 83}
totients+4	{5, 6, 8, 10, 12, 14, 20, 22, 24, 32, 34, 40, 44, 70, 74, 92, 300, 472, 772, 900, 904, 994} (conjectured, not proven, this conjecture is true if and only if there are no totients of the form {3,9}26 or {3,9}86, and such totients are conjectured not exist, since such totients are == 2 mod 12, thus must be of the form $(p-1)*p^n$ with p prime and n odd)

totients+5	{6, 7, 9, 11, 13, 15, 21, 23, 25, 33, 35, 41, 45, 51, 53, 83, 85, 301, 443, 505, 801, 881, 555555555555} (conjectured, not proven, this conjecture is true if and only if there are no totients of the form 3{9}8, and such totients are conjectured not exist, since such totients are == 2 mod 12, thus must be of the form (p−1)*p^n with p prime and n odd)
numbers > 0 which are sum of three nonnegative squares	{1, 2, 3, 4, 5, 6, 8, 9, 70, 77}
quadratic residues mod 6	{1, 3, 4, 6, 7, 9, 22, 25, 28, 52, 55, 58, 82, 85, 88}
quadratic residues mod 7	{1, 2, 4, 7, 8, 9, 30, 35, 36, 50, 53, 56, 60, 63, 65, 333, 555, 666}

(I left as a challenge to readers the task of finding minimal sets for other subsets of primes, such as "Mersenne primes", "Fermat primes", "primes == 1 mod 3", "primes == 2 mod 3)

[18] http://www.prothsearch.com/sierp.html (the Sierpinski problem) (the *PrimeGrid* page and its status page)

[19] http://www.prothsearch.com/rieselprob.html (the Riesel problem) (the *PrimeGrid* page and its status page)

[20] http://www.primegrid.com/ (with projects for the Sierpinski problem, the Riesel problem, the Prime Sierpinski problem, the Extended Sierpinski problem, Sierpinski/Riesel base 5 problem, generalized Fermat prime search)

[21] http://www.prothsearch.com/ (lists for primes of the form k*2^n+1 for odd k<1200, also factoring status of generalized Fermat numbers of the form $a^{2^n} + b^{2^n}$ for 1≤b<a≤12)

[22] https://archive.fo/VkelU (lists for primes of the form k*2^n−1 for odd k<10000)

[23] https://www.rieselprime.de/default.htm (lists for primes of the form k*2^n±1) (for some k see https://www.rieselprime.de/ziki/Riesel_2_1-300 (k*2^n−1 for odd k<300) and https://www.rieselprime.de/ziki/Riesel_2_300-2000 (k*2^n−1 for odd 300<k<2000) and https://www.rieselprime.de/ziki/Proth_2_1-300 (k*2^n+1 for odd k<300) and https://www.rieselprime.de/ziki/Proth_2_300-2000 (k*2^n+1 for odd 300<k<2000))

[24] http://www.noprimeleftbehind.net/crus/Sierp-conjectures.htm (generalized Sierpinski conjectures in bases b≤1030, some primes found in these conjectures are minimal primes in base b, especially, all primes for k<b (if exist for a (k,b) combo) are always minimal primes in the base b) (also some examples for simple families contain no primes > b) (power-of-2 bases *b* and the reservations page)

[25] http://www.noprimeleftbehind.net/crus/Riesel-conjectures.htm (generalized Riesel conjectures in bases b≤1030, some primes found in these conjectures are minimal primes in

base *b*, especially, all primes for *k*<*b* (if exist for a (*k*,*b*) combo) are always minimal primes in the base *b*) (also some examples for simple families contain no primes > *b*) (power-of-2 bases *b* and the reservations page)

[26] http://www.noprimeleftbehind.net/crus/tab/CRUS_tab.htm (list for the status of the generalized Sierpinski conjectures and the generalized Riesel conjectures in bases *b*≤1030)

[27] https://www.utm.edu/staff/caldwell/preprints/2to100.pdf (article for generalized Sierpinski conjectures in bases *b*≤100)

[28] https://oeis.org/A076336/a076336c.html (the dual Sierpinski problem)

[29] https://mersenneforum.org/showthread.php?t=10761 (list of large (probable) primes for the dual Sierpinski problem) (for the full list see http://www.mit.edu/~kenta/three/prime/dual-sierpinski/ezgxggdm/dualsierp-excerpt.txt and http://mit.edu/kenta/www/three/prime/dual-sierpinski/ezgxggdm/dualsierp.txt.gz)

[30] http://www.kurims.kyoto-u.ac.jp/EMIS/journals/INTEGERS/papers/i61/i61.pdf (article for the mixed (original+dual) Sierpinski problem)

[31] https://mersenneforum.org/showthread.php?t=6545 (research for the mixed (original+dual) Riesel problem)

[32] https://mersenneforum.org/showthread.php?t=26328 (research for the mixed (original+dual) Sierpinski base 5 problem)

[33] http://www.fermatquotient.com/ (generalized repunit primes (primes of the form $(b^n-1)/(b-1)$) in bases *b*≤160, the smallest such prime for base *b* (if exists) is always minimal prime in base *b*) and (generalized half Fermat primes (primes of the form $(b^{2^n}+1)/2$) sorted by *n*, the smallest such prime for base *b* (if exists) is always minimal prime in base *b*)

[34] https://archive.ph/tf7jx (generalized repunit primes (primes of the form $(b^n-1)/(b-1)$) in bases *b*≤1000, the smallest such prime for base *b* (if exists) is always minimal prime in base *b*)

[35] http://jeppesn.dk/generalized-fermat.html (generalized Fermat primes (primes of the form $b^{2^n}+1$) in even bases *b*≤1000, the smallest such prime for base *b* (if exists) is always minimal prime in base *b*)

[36] http://www.noprimeleftbehind.net/crus/GFN-primes.htm (generalized Fermat primes (primes of the form $b^{2^n}+1$) in even bases *b*≤1030, the smallest such prime for base *b* (if exists) is always minimal prime in base *b*)

[37] https://harvey563.tripod.com/wills.txt (primes of the form $(b-1)*b^n-1$ for bases *b*≤2049, the smallest such prime for base *b* (if exists) is always minimal prime in base *b*)

[38] https://www.rieselprime.de/ziki/Williams_prime (primes of the form $(b-1)*b^n-1$ for bases $b\le2049$, the smallest such prime for base b (if exists) is always minimal prime in base b) and (primes of the form $(b-1)*b^n+1$ for bases $b\le1024$, the smallest such prime for base b (if exists) is always minimal prime in base b) and (primes of the form $(b+1)*b^n-1$ for bases $b\le1024$, the smallest such prime for base b (if exists) is minimal prime in base b if and only if there is no smaller prime of the form $2*b^n-1$ for the same base b) and (primes of the form $(b+1)*b^n+1$ for bases $b\le1024$, the smallest such prime for base b (if exists) is minimal prime in base b if and only if there is no smaller prime of the form b^n+1 for the same base b) and (the smallest primes of the form $(b-1)*b^n-1$ for bases $b\le2049$, these primes (if exists) are always minimal primes in base b) and (the smallest primes of the form $(b-1)*b^n+1$ for bases $b\le1024$, these primes (if exists) are always minimal primes in base b) and (the smallest primes of the form $(b+1)*b^n-1$ for bases $b\le1024$, these primes (if exists) are minimal primes in base b if and only if there is no smaller prime of the form $2*b^n-1$ for the same base b) and (the smallest primes of the form $(b+1)*b^n+1$ for bases $b\le1024$, these primes (if exists) are minimal primes in base b if and only if there is no smaller prime of the form b^n+1 for the same base b)

[39] https://sites.google.com/view/williams-primes (primes of the form $(b-1)*b^n-1$ for bases $b\le1024$, the smallest such prime for base b (if exists) is always minimal prime in base b) and (primes of the form $(b-1)*b^n+1$ for bases $b\le1024$, the smallest such prime for base b (if exists) is always minimal prime in base b) and (primes of the form $(b+1)*b^n-1$ for bases $b\le1024$, the smallest such prime for base b (if exists) is minimal prime in base b if and only if there is no smaller prime of the form $2*b^n-1$ for the same base b) and (primes of the form $(b+1)*b^n+1$ for bases $b\le1024$, the smallest such prime for base b (if exists) is minimal prime in base b if and only if there is no smaller prime of the form b^n+1 for the same base b) and (primes of the form $b^n-(b-1)$ for bases $b\le1024$, the smallest such prime for base b with $n \ge 2$ (if exists) is always minimal prime in base b) and (primes of the form $b^n+(b-1)$ for bases $b\le1024$, the smallest such prime for base b (if exists) is always minimal prime in base b) and (primes of the form $b^n-(b+1)$ for bases $b\le1024$, the smallest such prime for base b (if exists) is minimal prime in base b if and only if there is no smaller prime of the form b^n-2 with $n \ge 2$ for the same base b) and (primes of the form $b^n+(b+1)$ for bases $b\le1024$, the smallest such prime for base b with $n \ge 2$ (if exists) is minimal prime in base b if and only if there is no smaller prime of the form b^n+1 for the same base b)

[40] https://www.rieselprime.de/ziki/Riesel_prime_small_bases_least_n (the smallest primes of the form $k*b^n-1$ for $2\le k\le12$ and bases $2\le b\le1024$, these primes (if exists) is always minimal prime in base b if $b>k$)

[41] https://www.rieselprime.de/ziki/Proth_prime_small_bases_least_n (the smallest primes of the form $k*b^n+1$ for $2\le k\le12$ and bases $2\le b\le1024$, these primes (if exists) is always minimal prime in base b if $b>k$)

[42] https://docs.google.com/spreadsheets/d/e/2PACX-1vTKkSNKGVQkUINlp1B3cXe90FWPwiegdA07EE7-U7sqXntKAEQrynoI1sbFvvKriieda3LfkqRwmKME/pubhtml (my list for the smallest primes or PRPs (only primes (or PRPs) > base are considered) in given simple family in bases $2 \le b \le 1024$, including these families:

* Repunit family $(b^n-1)/(b-1)$ (family **{1}**, $n \geq 2$ is needed, since $n = 1$ will produce single-digit numbers, which is not allowed in this research) (search limit of the length: ≥ 100000)
* Fermat family b^n+1 (family **1{0}1**, $n \geq 1$ is needed) (search limit of the length: ≥ 8388608)
* Half Fermat family $(b^n+1)/2$ (family **{#}$**, $n \geq 2$ is needed, since $n = 1$ will produce single-digit numbers, which is not allowed in this research) (search limit of the length: ≥ 262143)
* Wagstaff family $(b^n+1)/(b+1)$ (family **{z0}z1**, $n \geq 3$ is needed, since n must be odd, and n = 1 will produce single-digit numbers, which is not allowed in this research) (search limit of the length: ≥ 17326)
* Proth families $k*b^n+1$ for $2 \leq k \leq 12$ (this includes families **2{0}1**, **3{0}1**, **4{0}1**, **5{0}1**, **6{0}1**, **7{0}1**, **8{0}1**, **9{0}1**, **A{0}1**, **B{0}1**, **C{0}1**, as in the Sierpinski conjectures, $n \geq 1$ is needed) (search limit of the length: ≥ 100000)
* Riesel families $k*b^n-1$ for $2 \leq k \leq 12$ (this includes families **1{z}**, **2{z}**, **3{z}**, **4{z}**, **5{z}**, **6{z}**, **7{z}**, **8{z}**, **9{z}**, **A{z}**, **B{z}**, as in the Riesel conjectures, $n \geq 1$ is needed) (search limit of the length: ≥ 100000)
* b^n+k for $2 \leq k \leq 4$ (this includes families **1{0}2**, **1{0}3**, **1{0}4**, $n \geq 1$ is needed) (search limit of the length: ≥ 5000)
* b^n-k for $2 \leq k \leq 4$ (this includes families **{z}y**, **{z}x**, **{z}w**, $n \geq 2$ is needed, since $n = 1$ will produce single-digit numbers, which is not allowed in this research) (search limit of the length: ≥ 5000)
* Williams families $(b\pm1)*b^n\pm1$ (this includes families **11{0}1** (case "++"), **10{z}** (case "+−"), **z{0}1** (case "−+"), **y{z}** (case "−−"), $n \geq 1$ is needed) (search limit of the length: ≥ 100000)
* Dual Williams families $b^n\pm(b\pm1)$ (this includes families **1{0}11** (case "++", $n \geq 2$ is needed, since $n = 1$ will produce the number "21", which is not in the family), **1{0}z** (case "+−", $n \geq 1$ is needed), **{z}yz** (case "−+", $n \geq 2$ is needed, since $n = 1$ will produce negative numbers), **{z}1** (case "−−", $n \geq 2$ is needed, since $n = 1$ will produce single-digit numbers, which is not allowed in this research) (search limit of the length: ≥ 5000)
* Families ***x*{*y*} and {*x*}*y* with *x*, *y* ≤ 4** (not all done, currently only families 1{*y*} and {1}*y* and *x*{1} and {*x*}1 are in the list) (search limit of the length: ≥ 5000)
* Families ***x*{0}*y* with *x*, *y* ≤ 4** (search limit of the length: ≥ 5000)
* Family $((b-2)*b^n+1)/(b-1)$ (family **{y}z**, $n \geq 2$ is needed, since $n = 1$ will produce single-digit numbers, which is not allowed in this research) (search limit of the length: ≥ 5000)
* Family $(b^n-(2*b-1))/(b-1)$ (family **{1}0z**, $n \geq 3$ is needed, since $n = 1$ will produce negative numbers, and $n = 2$ will produce single-digit numbers, which is not allowed in this research) (search limit of the length: ≥ 5000)

where z means $b-1$, y means $b-2$, x means $b-3$, w means $b-4$, # means $(b-1)/2$ (for odd b), $ means $(b+1)/2$ (for odd b), the format of the families is *x*{*y*}*z* for *xyyy*...*yyyz*, numbers in the list are the lengths of these primes or PRPs in base b, "RC" means this family can be ruled out as only contain composite numbers (only count numbers > base), "NB" means this family is not interpretable in this base (including the case which this family has either leading zeros (leading zeros do not count) or trailing zeros (numbers ending in zero cannot be prime > base) in this base), "unknown" means this family has no known primes or PRPs, the smallest primes in some families in the list may not be minimal primes in the same base b (see the table).

and the smallest primes in other families in the list (if exists) are always minimal primes in the same base b, and since only primes (or PRPs) > base are considered, the smallest allowed length is 2 (i.e. length 1 is not allowed).

Notes:

* The smallest prime in families 1{0}1, 1{0}2, 1{0}3, 1{0}4, 1{0}z, {1}, 1{2}, 1{3}, 1{4}, 1{z}, 2{0}1, 2{0}3, {2}1, 2{z}, 3{0}1, 3{0}2, 3{0}4, {3}1, 3{z}, 4{0}1, 4{0}3, {4}1, 4{z}, 5{0}1, 5{z}, 6{0}1, 6{z}, 7{0}1, 7{z}, 8{0}1, 8{z}, 9{0}1, 9{z}, A{0}1, A{z}, B{0}1, B{z}, C{0}1, {#}$, {y}z, y{z}, z{0}1, {z}1, {z}w, {z}x, {z}y in base b is always a minimal prime in base b, if it exists.
* The smallest prime in families 1{0}11 and 11{0}1 in base b need not be a minimal prime in base b, it is a minimal prime if there is no smaller prime of the form 1{0}1 in the same base b.
* The smallest prime in family 10{z} in base b need not be a minimal prime in base b, it is a minimal prime if there is no smaller prime of the form 1{z} in the same base b.
* The smallest prime in family {1}0z in base b need not be a minimal prime in base b, it is a minimal prime if there is no smaller prime of the form {1} or {1}z ({1}z is not in the list) in the same base b.
* The smallest prime in families {1}2, {1}3, {1}4, 2{1}, 3{1}, 4{1} in base b need not be a minimal prime in base b, it is a minimal prime if there is no smaller prime of the form {1} in the same base b.
* The smallest prime in family {z0}z1 in base b almost cannot be a minimal prime in base b, this family is of interest only because of generalized Wagstaff primes.
* The smallest prime in family {z}yz in base b need not be a minimal prime in base b, it is a minimal prime if there is no smaller prime of the form {z}y in the same base b.
* For the families 1{0}1 and {#}$, only power-of-2 n need to be tested, since all other n have algebraic factorization (sum-of-two-r-th-powers factorization), and thus no need to sieve, instead, we use trial division for the power-of-2 n.
* For the family {1}, only prime n need to be tested, since all other n have algebraic factorization (difference-of-two-r-th-powers factorization, and when n is prime, this factorization is trivial, i.e. one of the two factors is 1).
* For the families 1{0}1, 11{0}1, 2{0}1, 3{0}1, 4{0}1, 5{0}1, 6{0}1, 7{0}1, 8{0}1, 9{0}1, A{0}1, B{0}1, C{0}1, z{0}1, all primes can be proven primes using $N-1$ primality test, since their $N-1$ are the product of a power of b and a number < b, thus trivially 100% factored.
* For the families 10{z}, 1{z}, 2{z}, 3{z}, 4{z}, 5{z}, 6{z}, 7{z}, 8{z}, 9{z}, A{z}, B{z}, y{z}, all primes can be proven primes using $N+1$ primality test, since their $N+1$ are the product of a power of b and a number < b, thus trivially 100% factored.
* For the families 1{0}2, 1{0}11, {1}, {1}2, 1{2}, 1{3}, 1{4}, 2{0}3, 3{0}4, {3}1, {4}1, {#}$, {y}z, {z0}z1, {z}1, their $N-1$ are the product of a Cunningham number base b (i.e. of the form $b^n\pm1$) and a number < b, and Cunningham numbers have algebraic factors to cyclotomic polynomials evaluated at b (b^n-1 can be factored to product of all $\Phi_d(b)$ with d dividing n, and b^n+1 can be factored to product of all $\Phi_d(b)$ with d dividing $2*n$ but not dividing n, where Φ is the cyclotomic polynomial) (see this page), if these algebraic factors have enough factorizations into prime numbers to make the Cunningham number base b (i.e. $b^n\pm1$) (and hence $N-1$) ≥ 33.3333% factored, then we can use $N-1$ primality test to prove the primality of these primes, but if these algebraic factors do not have enough factorizations into prime numbers to make the Cunningham number base b (i.e. $b^n\pm1$) (and hence $N-1$) ≥ 33.3333% factored, then the primality of these primes cannot be proven in polynomial times, and thus these primes are only probable primes and not definitely primes (unless we use *ECPP* primality test to proving their primality, such as *PRIMO*, but this primality test will take a long

time if the primes are large (say > 2^{65536})), and hence we cannot definitely say that they are minimal primes base *b*.
* For the families 1{0}z, {1}0z, 3{0}2, 4{0}3, {z}yz, {z}y, their *N*+1 are the product of a Cunningham number base *b* (i.e. of the form $b^n\pm1$) and a number < *b*, and Cunningham numbers have algebraic factors to cyclotomic polynomials evaluated at *b* (b^n-1 can be factored to product of all $\Phi_d(b)$ with *d* dividing *n*, and b^n+1 can be factored to product of all $\Phi_d(b)$ with *d* dividing 2**n* but not dividing *n*, where Φ is the cyclotomic polynomial) (see this page), if these algebraic factors have enough factorizations into prime numbers to make the Cunningham number base *b* (i.e. $b^n\pm1$) (and hence *N*+1) ≥ 33.3333% factored, then we can use *N*+1 primality test to prove the primality of these primes, but if these algebraic factors do not have enough factorizations into prime numbers to make the Cunningham number base *b* (i.e. $b^n\pm1$) (and hence *N*+1) ≥ 33.3333% factored, then the primality of these primes cannot be proven in polynomial times, and thus these primes are only probable primes and not definitely primes (unless we use *ECPP* primality test to proving their primality, such as *PRIMO*, but this primality test will take a long time if the primes are large (say > 2^{65536})), and hence we cannot definitely say that they are minimal primes base *b*.
* For the family {2}1, their *N*−1 and *N*+1 are the product of a Cunningham number base *b* (i.e. of the form $b^n\pm1$) and a number < *b*, and Cunningham numbers have algebraic factors to cyclotomic polynomials evaluated at *b* (b^n-1 can be factored to product of all $\Phi_d(b)$ with *d* dividing *n*, and b^n+1 can be factored to product of all $\Phi_d(b)$ with *d* dividing 2**n* but not dividing *n*, where Φ is the cyclotomic polynomial) (see this page), if these algebraic factors have enough factorizations into prime numbers to make the Cunningham number base *b* (i.e. $b^n\pm1$) (and hence *N*−1 and/or *N*+1, or neither of them but N^2-1, see cyclotomy primality test) ≥ 33.3333% factored, then we can use *N*−1 primality test or *N*+1 primality test or combine *N*−1 and *N*+1 primality test to prove the primality of these primes, but if these algebraic factors do not have enough factorizations into prime numbers to make the Cunningham number base *b* (i.e. $b^n\pm1$) (and hence both *N*−1 and *N*+1) ≥ 33.3333% factored, then the primality of these primes cannot be proven in polynomial times, and thus these primes are only probable primes and not definitely primes (unless we use *ECPP* primality test to proving their primality, such as *PRIMO*, but this primality test will take a long time if the primes are large (say > 2^{65536})), and hence we cannot definitely say that they are minimal primes base *b*.
* For the families 1{0}3, 1{0}4, {1}3, {1}4, 2{1}, 3{1}, 4{1}, {z}w, {z}x, neither *N*−1 nor *N*+1 are either "the product of a power of *b* and a number < *b*" or "the product of a Cunningham number base *b* (i.e. of the form $b^n\pm1$) and a number < *b*", thus neither *N*−1 nor *N*+1 is easy to factor (at most a few algebraic factors (such as difference-of-two-squares factorization, sum/difference-of-two-cubes factorization, and Aurifeuillian factorization of x^4+4y^4) and a few prime factors < 2^{32} (using trial divisions to found), but these factors usually cannot make either *N*−1 or *N*+1 ≥ 33.3333% factored), and the primality of these primes cannot be proven in polynomial times, and thus these primes are only probable primes and not definitely primes (unless we use *ECPP* primality test to proving their primality, such as *PRIMO*, but this primality test will take a long time if the primes are large (say > 2^{65536})), and hence we cannot definitely say that they are minimal primes base *b*.

Some *OEIS* sequences for the minimal primes (or PRPs) of these forms:

1{0}1: A079706 (the exponents *n*), A084712 (the corresponding primes), A228101 (the log_2 of the exponents *n*), A123669 (length 2 not allowed, the corresponding primes)

1{0}2: A138066 (the exponents *n*), A084713 (the corresponding primes), A138067 (length 2 not allowed, the exponents *n*)

1{0}z: A076845 (the exponents *n*), A076846 (the corresponding primes), A078178 (length 2 not allowed, the exponents *n*), A078179 (length 2 not allowed, the corresponding primes)

1{0}11: A346149 (the exponents *n*), A346154 (the corresponding primes)

{1}: A084740 (the exponents *n*), A084738 (the corresponding primes), A065854 (prime bases *b*, the exponents *n*), A279068 (prime bases *b*, the corresponding primes), A246005 (odd bases *b*, the exponents *n*), A128164 (length 2 not allowed, the exponents *n*), A285642 (length 2 not allowed, the corresponding primes)

1{z}: A119591 (the exponents *n*), A098873 (bases *b* divisible by 6, the exponents *n*)

2{0}1: A119624 (the exponents *n*), A253178 (bases *b* not == 1 mod 3 (as for bases *b* == 1 mod 3, there are no possible primes), the exponents *n*), A098872 (bases *b* divisible by 6, the exponents *n*)

2{z}: A098876 (bases *b* divisible by 6, the exponents *n*)

3{0}1: A098877 (bases *b* divisible by 6, the exponents *n*)

A{0}1: A088782 (the exponents *n*), A088622 (the corresponding primes)

y{z}: A122396 (prime bases *b*, the exponents *n* added by 1)

z{0}1: A305531 (the exponents *n*), A087139 (prime bases *b*, the exponents *n* added by 1)

{z0}z1: A084742 (the exponents *n*), A084741 (the corresponding primes), A065507 (prime bases *b*, the exponents *n*), A126659 (odd bases *b*, the exponents *n*)

{z}yz: A178250 (the exponents *n*)

{z}1: A113516 (the exponents *n*), A343589 (the corresponding primes)

{z}y: A250200 (the exponents *n*), A255707 (length 1 allowed, the exponents *n*), A084714 (length 1 allowed, the corresponding primes), A292201 (length 1 allowed, prime bases *b*, the exponents *n*)

Some large (>100000 base *b* digits) minimal primes (or PRPs) of these forms in top primes (or top PRPs):

12:0^{656920}:1 in base *b* = 68
3:$71^{1119849}$ in base *b* = 72
111:112^{286643} in base *b* = 113
1^{270217} in base *b* = 152 (PRP, not definitely prime)

[$2:0^{333924}:1$ in base b = 218]
[$10:0^{314805}:1$ in base b = 311]
[$5:0^{400784}:1$ in base b = 326]
[$6:0^{369831}:1$ in base b = 409]
[$8:0^{279990}:1$ in base b = 410]
[$5:432^{283918}$ in base b = 433]
[$3:649^{498101}$ in base b = 650]
[$4:0^{269301}:1$ in base b = 737]
[$10:0^{285477}:1$ in base b = 743]
[$4:0^{149138}:1$ in base b = 789]
[$4:0^{468701}:1$ in base b = 797]
[$6:847^{218439}$ in base b = 848]
[$11:0^{227480}:1$ in base b = 878]
[$8:0^{243438}:1$ in base b = 908]

Family	Algebraic form of the family (n is the length)	The smallest allowed base b (if the base b is not allowed, then listed as "NB" in the table)	The smallest allowed length	The smallest prime in this family is a minimal prime if and only if there is no smaller prime of this family(s)	Bases 2 ≤ b ≤ 1024 such that this family is unsolved family	Top 10 primes of this family in bases 2 ≤ b ≤ 1024: base (length)	Bases such that this family can be ruled out as only contain composite numbers (only count numbers > base) (listed "RC" in the table)	Search limit of the lengths (n)
1{0}1	$b^{n-1}+1$	2	2	none (always minimal prime)	{38, 50, 62, 68, 86, 92, 98, 104, 122, 144, 168, 182, 186, 200, 202, 212, 214, 218, 244,	824 (1025) 898 (257) 614 (257) 548 (129) 532 (129) 506 (129) 234 (129) 728	b == 1 mod 2 (trivial factor 2) $b = m^r$ with odd r>1 (sum-of-two-r-th-powers factorization)	≥8388608

					246, 252, 258, 286, 294, 298, 302, 304, 308, 322, 324, 338, 344, 354, 356, 362, 368, 380, 390, 394, 398, 402, 404, 410, 416, 422, 424, 446, 450, 454, 458, 468, 480, 482, 484, 500, 514, 518, 524, 528, 530, 534, 538, 552, 558, 564, 572, 574, 578, 580, 590, 602, 604, 608,	(65) 412 (65) 274 (65)		

					620, 622, 626, 632, 638, 648, 650, 662, 666, 668, 670, 678, 684, 692, 694, 698, 706, 712, 720, 722, 724, 734, 744, 746, 752, 754, 762, 766, 770, 792, 794, 802, 806, 812, 814, 818, 836, 840, 842, 844, 848, 854, 868, 870, 872, 878, 888, 896, 902, 904, 908, 922, 924, 926,			

					932, 938, 942, 944, 948, 954, 958, 964, 968, 974, 978, 980, 988, 994, 998, 1002, 1006, 1014, 1016}			
1{0}2	$b^{n-1}+2$	3	2	none (always minimal prime)	{167, 257, 323, 353, 383, 527, 557, 563, 635, 647, 677, 713, 803, 815, 947, 971, 1013}	719 (2766) 623 (2052) 941 (1870) 791 (1776) 797 (1406) 899 (1252) 551 (1150) 743 (748) 929 (714) 893 (488)	*b* == 0 mod 2 (trivial factor 2) *b* == 1 mod 3 (trivial factor 3)	≥5000
1{0}3	$b^{n-1}+3$	4	2	none (always minimal prime)	{646, 718, 998}	530 (1399) 382 (256) 898 (166) 412 (137) 548 (118) 388 (109) 632	*b* == 1 mod 2 (trivial factor 2) *b* == 0 mod 3 (trivial factor 3)	≥5000

						(88) 442 (41) 292 (40) 802 (37)		
1{0}4	$b^{n-1}+4$	5	2	none (always minimal prime)	{139, 227, 263, 315, 335, 365, 485, 515, 647, 653, 683, 773, 789, 797, 815, 857, 875, 893, 939, 995, 1007}	53 (13403) 113 (10647) 489 (1888) 999 (1708) 563 (1563) 695 (1467) 965 (1415) 413 (1171) 619 (1000) 575 (923)	b == 0 mod 2 (trivial factor 2) b == 1 mod 5 (trivial factor 5) b == 14 mod 15 (covering set {3,5}) $b = m^4$ (Aurifeuillian factorization for x^4+4y^4)	≥5000
1{0}z	$b^{n-1}+(b-1)$	2	2	none (always minimal prime)	{173, 257, 277, 302, 333, 362, 392, 422, 452, 467, 488, 527, 545, 575, 622, 650, 677, 680, 704, 707, 827, 830, 851, 872,	123 (64371) 113 (20089) 512 (4905) 929 (4215) 179 (3357) 904 (3010) 949 (2985) 740 (2795) 614 (2575) 570 (2425)	(none)	≥5000

					886, 887, 902, 908, 932, 942, 947, 962, 1022}			
1{0}11	$b^{n-1}+(b+1)$	2	3 (there is no number in this family with length 2 at all)	1{0}1	{213, 318, 327, 353, 513, 647, 732, 738, 759, 948, 957, 1013}	198 (5198) 1014 (4186) 375 (4015) 951 (3953) 734 (2791) 591 (2566) 452 (1615) 936 (1498) 777 (1379) 648 (974)	*b* == 1 mod 3 (trivial factor 3)	≥5000
10{z}	$(b+1)*b^{n-2}-1$	2	3 (the number with length 2 is 10, whose value is *b* and not > *b*, thus not allowed)	1{z}	{575}	208 (26682) 828 (19659) 607 (11032) 953 (5582) 577 (4622) 503 (2294) 318 (2177) 88 (1706) 316 (1494) 63 (1485)	(none)	≥100000
11{0}1	$(b+1)*b^{n-2}+1$	2	3 (there is no	1{0}1	{813, 863,	327 (13598	*b* == 1 mod 3	≥100000

			number in this family with length 2 at all)		962, 1017}	3) 222 (52727) 717 (37508) 227 (36323) 201 (31276) 710 (24112) 719 (13420) 425 (11231) 683 (6776) 633 (5248)	(trivial factor 3)	
{1}0z	$(b^n-(2*b-1))/(b-1)$	2	3 (the number with length 2 is z, whose value is $b-1$ and not > b, thus not allowed)	{1}, {1}z ({1}z is not in the list)	{167, 217, 229, 232, 253, 317, 325, 337, 347, 355, 375, 403, 411, 421, 427, 457, 483, 505, 507, 537, 547, 577, 597, 601, 627, 631, 632, 641, 643, 649, 657, 679, 688, 697,	161 (9155) 613 (4515) 137 (3953) 599 (3865) 797 (3733) 874 (3393) 843 (3061) 916 (2844) 261 (2663) 479 (2605)	b such that b and $2*b-1$ both squares (difference-of-two-squares factorization) (this includes b = 25, 841)	≥5000

					707, 711, 733, 737, 742, 762, 773, 787, 793, 817, 819, 853, 859, 861, 877, 895, 899, 907, 913, 927, 957, 959, 997, 1003, 1009, 1015, 1017}			
{1}	$(b^n-1)/(b-1)$	2	2	none (always minimal prime)	{185, 269, 281, 380, 384, 385, 394, 452, 465, 511, 574, 601, 631, 632, 636, 711, 713, 759, 771, 795, 861, 866, 881, 938, 948, 951,	152 (270217) 485 (99523) 691 (62903) 649 (43987) 693 (41189) 311 (36497) 752 (32833) 629 (32233) 326 (26713) 331 (25033)	$b = m^r$ with r>1 (difference-of-two-r-th-powers factorization)	≥100000

					956, 963, 1005, 1015}			
{1}2	(b^n+(b−2))/(b−1)	3	2	{1}	{93, 143, 253, 293, 313, 383, 391, 393, 403, 435, 443, 451, 491, 493, 523, 541, 553, 565, 581, 587, 601, 613, 623, 627, 663, 729, 757, 763, 783, 823, 843, 865, 873, 883, 931, 943, 955, 983, 1013, 1015, 1021, 1023}	415 (4690) 527 (3562) 897 (3500) 735 (3384) 877 (3166) 91 (3096) 775 (2958) 537 (2604) 247 (2526) 635 (2436)	b == 0 mod 2 (trivial factor 2)	≥5000
{1}3	(b^n+(2*b−3))/(b−1)	4	2	{1}			b == 0 mod 3 (trivial factor 3)	≥5000

{1}4	(b^n+(3*b−4))/(b−1)	5	2	{1}			b == 0 mod 2 (trivial factor 2)	≥5000
1{2}	(((b+1)/2)*b^n−1)/((b−1)/2)	3	2	none (always minimal prime)			b == 0 mod 2 (trivial factor 2) b such that b and (b+1)/2 both squares (difference-of-two-squares factorization) (this includes b = 49)	≥2500
1{3}	(((b+2)/3)*b^n−1)/((b−1)/3) for b == 1 mod 3 ((b+2)*b^n−3)/(b−1) for b == 2 mod 3	4	2	none (always minimal prime)			b == 0 mod 3 (trivial factor 3) b such that b and (b+2)/3 both squares (difference-of-two-squares factorization) (this includes b = 25 and 361)	≥2500
1{4}	(((b+3)/	5	2	none			b == 0	≥2500

	4)*b^n−1)/((b−1)/4) for *b* == 1 mod 4 (((b+3)/2)*b^n−2)/((b−1)/2) for *b* == 3 mod 4			(always minimal prime)			mod 2 (trivial factor 2) *b* such that *b* and (*b*+3)/4 both squares (difference-of-two-squares factorization) (this does not include any *b*)	
1{z}	2*b^{n-1}−1	2	2	none (always minimal prime)	{581, 992, 1019}	170 (166429) 578 (129469) 698 (127559) 522 (62289) 704 (62035) 515 (58467) 278 (43909) 938 (40423) 303 (40175) 845 (39407)	(none)	≥100000
2{0}1	2*b^{n-1}+1	3	2	none (always minimal prime)	{365, 383, 461, 512, 542, 647, 773,	218 (333926) 101 (192276) 626	*b* == 1 mod 3 (trivial factor 3)	≥100000

					801, 836, 878, 908, 914, 917, 947, 1004}	(174204) 236 (161230) 467 (126776) 695 (94626) 788 (72918) 869 (49150) 206 (46206) 578 (44166)		
2{0}3	2*b^{n-1}+3	4	2	none (always minimal prime)			b == 0 mod 3 (trivial factor 3) b == 1 mod 5 (trivial factor 5)	≥2500
2{1}	((2*b−1)*b^{n-1}−1)/(b−1)	3	2	{1}	{117, 137, 147, 157, 175, 177, 201, 227, 235, 269, 271, 297, 310, 335, 397, 417, 427, 430, 437, 451, 465, 467, 481,	85 (6940) 877 (4980) 947 (4508) 782 (4152) 903 (4006) 955 (3880) 899 (3804) 442 (3172) 109 (3048) 679 (3012)	b such that b and 2*b−1 both squares (difference-of-two-squares factorization) (this includes b = 25, 841)	≥5000

					502, 517, 547, 557, 567, 577, 591, 607, 627, 649, 654, 655, 667, 687, 691, 697, 715, 727, 739, 759, 766, 787, 796, 797, 808, 817, 821, 829, 852, 881, 907, 937, 1007, 1011, 1021}			
{2}1	$(2*b^n-(b+1))/(b-1)$ for even *b* $(b^n-((b+1)/2))/((b-1)/2)$ for odd *b*	3	2	none (always minimal prime)			*b* such that *b* and (*b*+1)/2 both squares (difference-of-two-squares factorization) (this includes *b* = 49)	≥2500
2{z}	$3*b^{n-1}-$	3	2	none	{588,	432	*b* == 1	≥10000

	1			(always minimal prime)	972}	(16003) 446 (4851) 42 (2524) 712 (984) 654 (921) 916 (476) 582 (445) 572 (377) 522 (347) 452 (335)	mod 2 (trivial factor 2)	0
3{0}1	3*b^{n-1}+ 1	4	2	none (always minimal prime)	{718, 912}	358 (9561) 996 (6550) 424 (1106) 648 (647) 652 (621) 690 (358) 314 (281) 108 (271) 654 (217) 730 (199)	*b* == 1 mod 2 (trivial factor 2)	≥100000
3{0}2	3*b^{n-1}+ 2	4	2	none (always minimal prime)			*b* == 0 mod 2 (trivial factor 2) *b* == 1 mod 5 (trivial factor 5)	≥2500
3{0}4	3*b^{n-1}+	5	2	none			*b* == 0	≥2500

	4			(always minimal prime)			mod 2 (trivial factor 2) *b* == 1 mod 7 (trivial factor 7)	
3{1}	((3**b*−2)**b*$^{n-1}$−1)/(*b*−1)	4	2	{1}			*b* such that *b* and 3**b*−2 both squares (difference-of-two-squares factorization) (this includes *b* = 9, 121)	≥5000
{3}1	(3**b*^{n}−(2**b*+1))/(*b*−1) for *b* == 0, 2 mod 3 (*b*^{n}−((2**b*+1)/3))/((*b*−1)/3) for *b* == 1 mod 3	4	2	none (always minimal prime)			*b* such that *b* and (2**b*+1)/3 both squares (difference-of-two-squares factorization) (this includes *b* = 121)	≥2500
3{z}	4**b*$^{n-1}$−1	4	2	none (always minimal prime)	{275, 438, 647, 653, 812, 927, 968}	72 (1119850) 650 (498102) 303 (198358)	*b* == 1 mod 3 (trivial factor 3) *b* == 14 mod 15 (covering set	≥100000

						921 (98668) 480 (93610) 270 (89662) 312 (51566) 527 (46074) 513 (38032) 212 (34414)	{3,5}) *b* == 4 mod 5 (even length: factor 5, odd length: difference-of-two-squares factorization) $b = m^2$ (difference-of-squares factorization)	
4{0}1	$4*b^{n-1}+1$	5	2	none (always minimal prime)	{32, 53, 155, 174, 204, 212, 230, 332, 334, 335, 395, 467, 512, 593, 767, 803, 848, 875, 1024}	797 (468703) 737 (269303) 257 (160423) 789 (149140) 410 (144079) 920 (103687) 934 (101404) 650 (96223) 962 (84235) 679 (69450)	*b* == 1 mod 5 (trivial factor 5) *b* == 14 mod 15 (covering set {3,5}) $b = m^4$ (Aurifeuillian factorization for x^4+4y^4)	≥100000
4{0}3	$4*b^{n-1}+3$	5	2	none (always minimal prime)			*b* == 0 mod 3 (trivial factor 3)	≥2500

							b == 1 mod 7 (trivial factor 7)	
4{1}	$((4*b-3)*b^{n-1}-1)/(b-1)$	5	2	{1}			b such that b and $4*b-3$ both squares (difference-of-two-squares factorization) (this does not include any b)	≥5000
{4}1	$(4*b^n-(3*b+1))/(b-1)$ for even b $(b^n-((3*b+1)/4))/((b-1)/4)$ for b == 1 mod 4 $(2*b^n-((3*b+1)/2))/((b-1)/2)$ for b == 3 mod 4	5	2	none (always minimal prime)			b such that b and $3*b+1$ both squares (difference-of-two-squares factorization) (this includes b = 16, 225)	≥2500
4{z}	$5*b^{n-1}-1$	5	2	none (always minimal prime)	{338, 998}	800 (20509) 14 (19699) 254 (15451) 68 (13575) 196	b == 1 mod 2 (trivial factor 2)	≥100000

						(9850) 986 (5581) 884 (4627) 404 (3435) 1010 (2015) 740 (1595)		
5{0}1	5*b^{n-1}+1	6	2	none (always minimal prime)	{308, 512, 824}	326 (400786) 926 (40036) 350 (20392) 662 (13390) 554 (10630) 536 (8790) 992 (2166) 590 (2152) 626 (2070) 440 (826)	b == 1 mod 2 (trivial factor 2) b == 1 mod 3 (trivial factor 3)	≥100000
5{z}	6*b^{n-1}−1	6	2	none (always minimal prime)	{234, 412, 549, 553, 573, 619, 750, 878, 894, 954}	433 (283919) 258 (212135) 272 (148427) 768 (70214) 299 (64898) 867 (61411) 692 (45447) 678 (40859) 972	b == 1 mod 5 (trivial factor 5) b == 34 mod 35 (covering set {5,7}) b = 6*m^2 with m == 2, 3 mod 5 (odd length: factor 5, even length:	≥100000

						(36703) 635 (36163)	difference-of-squares factorization) (this includes b = 24, 54, 294, 384, 864, 1014)	
6{0}1	$6*b^{n-1}+1$	7	2	none (always minimal prime)	{212, 509, 579, 625, 774, 894, 993, 999}	409 (369833) 643 (164916) 522 (52604) 789 (27297) 587 (24120) 986 (21634) 129 (16797) 108 (16318) 762 (11152) 1018 (9944)	b == 1 mod 7 (trivial factor 7) b == 34 mod 35 (covering set {5,7})	≥100000
6{z}	$7*b^{n-1}-1$	7	2	none (always minimal prime)	{308, 392, 398, 518, 548, 638, 662, 878}	848 (218440) 566 (164828) 362 (146342) 980 (50878) 338 (42868) 488 (33164) 68 (25396)	b == 1 mod 2 (trivial factor 2) b == 1 mod 3 (trivial factor 3)	≥100000

						1016 (23336) 332 (15222) 986 (12506)		
7{0}1	$7*b^{n-1}+1$	8	2	none (always minimal prime)	(none)	1004 (54849) 398 (17473) 632 (8447) 836 (5701) 644 (3379) 500 (1997) 974 (1589) 682 (796) 338 (793) 224 (689)	*b* == 1 mod 2 (trivial factor 2)	(no bases *b* ≤ 1024 have this family as unsolved family, base *b* = 1004 is the last to drop at length *n* = 54849)
7{z}	$8*b^{n-1}-1$	8	2	none (always minimal prime)	{321, 328, 374, 432, 665, 697, 710, 721, 727, 728, 752, 800, 815, 836, 867, 957, 958, 972}	97 (192336) 283 (164769) 202 (155772) 866 (108591) 908 (61797) 655 (53009) 194 (38361) 962 (31841) 811 (31784) 412 (29792)	*b* == 1 mod 7 (trivial factor 7) *b* == 20 mod 21 (covering set {3,7}) *b* == 83, 307 mod 455 (covering set {5,7,13}) (this includes *b* = 83, 307, 538, 762, 993) $b = m^3$ (differe	≥100000

							nce-of-two-cubes factorization)	
8{0}1	8*b^{n-1}+1	9	2	none (always minimal prime)	{86, 140, 182, 263, 353, 368, 389, 395, 422, 426, 428, 434, 443, 488, 497, 558, 572, 575, 593, 606, 698, 710, 746, 758, 770, 773, 785, 824, 828, 866, 911, 930, 953, 957, 983, 993, 1014}	410 (279992) 908 (243440) 53 (227184) 596 (148446) 158 (123476) 23 (119216) 920 (107822) 425 (94662) 641 (87702) 893 (86772)	*b* == 1 mod 3 (trivial factor 3) *b* == 20 mod 21 (covering set {3,7}) *b* == 47, 83 mod 195 (covering set {3,5,13}) *b* = 467 (covering set {3, 5, 7, 19, 37}) *b* = 722 (covering set {3, 5, 13, 73, 109}) *b* = m^3 (sum-of-two-cubes factorization) *b* = 128 (no possible prime since 7*k+3 cannot be power of 2, all powers of 2 are == 1, 2,	≥100000

							4 mod 7 (2^n mod 7 for n = 1, 2, 3, … are 2, 4, 1, 2, 4, 1, 2, 4, 1, 2, 4, 1, …, with period 3), thus 7*k+3 always has a odd factor > 1, and thus this family always have sum-of-two-r-th-powers factorization for some r)	
8{z}	9*b^{n-1}−1	9	2	none (always minimal prime)	{378, 438, 536, 566, 570, 592, 636, 688, 718, 830, 852, 926, 1010}	138 (35686) 990 (23032) 412 (12154) 788 (11326) 808 (6994) 112 (5718) 858 (4170) 188 (3888) 722 (3024) 292 (2928)	b == 1 mod 2 (trivial factor 2) b == 4 mod 5 (even length: factor 5, odd length: difference-of-two-squares factorization) b = m^2 (difference-of-squares	≥100000

							factorization)	
9{0}1	$9*b^{n-1}+1$	10	2	none (always minimal prime)	{724, 884}	592 (96870) 248 (39511) 844 (9688) 544 (4706) 894 (3070) 974 (2016) 244 (1836) 908 (1070) 1004 (840) 848 (544)	*b* == 1 mod 2 (trivial factor 2) *b* == 1 mod 5 (trivial factor 5)	≥100000
9{z}	$10*b^{n-1}-1$	10	2	none (always minimal prime)	{80, 233, 530, 551, 611, 899, 912, 980}	446 (152028) 458 (126262) 284 (112810) 431 (43574) 846 (12781) 599 (11776) 320 (9646) 1020 (6945) 185 (6784) 992 (5434)	*b* == 1 mod 3 (trivial factor 3) *b* == 32 mod 33 (covering set {3,11})	≥100000
A{0}1	$10*b^{n-1}+1$	11	2	none (always minimal prime)	{185, 338, 417, 432, 614, 668,	311 (314807) 743 (285479)	*b* == 1 mod 11 (trivial factor 11) *b* == 32	≥100000

					773, 863, 935, 1000}	173 (264235) 802 (149320) 744 (137056) 977 (125873) 341 (106009) 786 (68169) 986 (48279) 198 (47665)	mod 33 (covering set {3,11})	
A{z}	$11*b^{n-1}-1$	11	2	none (always minimal prime)	{214, 422, 444, 452, 458, 542, 638, 668, 804, 872, 950, 962}	752 (112211) 534 (80328) 978 (14066) 662 (13307) 368 (10867) 488 (10231) 242 (8387) 984 (4522) 692 (3575) 482 (2595)	*b* == 1 mod 2 (trivial factor 2) *b* == 1 mod 5 (trivial factor 5) $b = 11*m^2$ with *m* == 2, 3 mod 5 (odd length: factor 5, even length: difference-of-squares factorization) (this includes *b* = 44, 99, 539, 704)	≥100000

B{0}1	$11*b^{n-1}+1$	12	2	none (always minimal prime)	{560, 770, 968}	878 (227482) 740 (33520) 710 (15272) 908 (9856) 542 (4910) 992 (4414) 68 (3948) 320 (1264) 152 (838) 462 (762)	*b* == 1 mod 2 (trivial factor 2) *b* == 1 mod 3 (trivial factor 3)	≥100000
B{z}	$12*b^{n-1}-1$	12	2	none (always minimal prime)	{263, 615, 912, 978}	186 (112718) 717 (67707) 602 (36518) 153 (21660) 439 (18752) 593 (16064) 707 (10573) 708 (4737) 98 (3600) 692 (3582)	*b* == 1 mod 11 (trivial factor 11) *b* == 142 mod 143 (covering set {11,13}) *b* = 307 (covering set {5, 11, 29}) *b* = 901 (covering set {7, 11, 13, 19})	≥100000
C{0}1	$12*b^{n-1}+1$	13	2	none (always minimal prime)	{163, 207, 354, 362, 368, 480, 620, 692, 697, 736,	68 (656922) 230 (94751) 700 (91953) 334 (83334) 923	*b* == 1 mod 13 (trivial factor 13) *b* == 142 mod 143 (coverin	≥100000

					753, 792, 978, 998, 1019, 1022}	(64365) 359 (61295) 481 (45941) 919 (45359) 593 (42779) 219 (29231)	g set {11,13}) *b* = 296, 901 (covering set {7, 11, 13, 19}) *b* = 562, 828, 900 (covering set {7, 13, 19}) *b* = 563 (covering set {5, 7, 13, 19, 29}) *b* = 597 (covering set {5, 13, 29})	
{#}$	$(b^n+1)/2$	3 (only odd bases are allowed)	2	none (always minimal prime)	{31, 37, 55, 63, 67, 77, 83, 89, 91, 93, 97, 99, 107, 109, 117, 123, 127, 133, 135, 137, 143, 147, 149, 151, 155, 161, 177, 179, 183, 189, 193, 197, 207, 211, 213, 215,	827 (1024) 665 (256) 507 (256) 331 (256) 871 (128) 499 (128) 863 (64) 837 (64) 803 (64) 727 (64)	$b = m^r$ with odd r>1 (sum-of-two-r-th-powers factorization)	≥524287

					217, 223, 225, 227, 233, 235, 241, 247, 249, 255, 257, 263, 265, 269, 273, 277, 281, 283, 285, 287, 291, 293, 297, 303, 307, 311, 319, 327, 347, 351, 355, 357, 359, 361, 367, 369, 377, 381, 383, 385, 387, 389, 393, 397, 401, 407, 411, 413, 417, 421, 423, 437, 439, 443,			

					447, 457, 465, 467, 469, 473, 475, 481, 483, 489, 493, 495, 497, 509, 511, 515, 533, 541, 547, 549, 555, 563, 591, 593, 597, 601, 603, 611, 615, 619, 621, 625, 627, 629, 633, 635, 637, 645, 647, 651, 653, 655, 659, 663, 667, 671, 673, 675, 679, 683, 687, 691, 693, 697,			

					707, 709, 717, 731, 733, 735, 737, 741, 743, 749, 753, 755, 757, 759, 765, 767, 771, 773, 775, 777, 783, 785, 787, 793, 797, 801, 807, 809, 813, 817, 823, 825, 849, 851, 853, 865, 867, 873, 877, 887, 889, 893, 897, 899, 903, 907, 911, 915, 923, 927, 933, 937, 939, 941,			

					943, 945, 947, 953, 957, 961, 967, 975, 977, 983, 987, 993, 999, 1003, 1005, 1009, 1017}			
{y}z	$((b-2)*b^n+1)/(b-1)$	3	2	none (always minimal prime)	{143, 173, 176, 213, 235, 248, 279, 327, 343, 358, 383, 401, 413, 427, 439, 453, 463, 513, 527, 535, 547, 559, 565, 583, 598, 623, 653, 659, 663, 679, 711, 743, 745, 757, 785, 801,	353 (4908) 481 (4730) 1005 (4630) 603 (4532) 416 (4280) 796 (3740) 1021 (3674) 522 (3619) 856 (3299) 373 (3276)	(none)	≥5000

					811, 821, 835, 845, 847, 853, 883, 898, 903, 927, 955, 961, 973, 993, 1013, 1019}			
y{z}	$(b-1)*b^{n-1}-1$	3	2	none (always minimal prime)	{128, 233, 268, 293, 383, 478, 488, 533, 554, 665, 698, 779, 863, 878, 932, 941, 1010}	113 (286644) 38 (136212) 518 (129372) 401 (103670) 638 (74528) 527 (65822) 758 (50564) 938 (50008) 663 (47557) 458 (46900)	(none)	≥100000
z{0}1	$(b-1)*b^{n-1}+1$	2	2	none (always minimal prime)	{123, 342, 362, 422, 438, 479, 487, 512, 542, 602, 757, 767,	363 (142877) 251 (102979) 634 (84823) 452 (71941) 347 (69661)	(none)	≥100000

					817, 830, 872, 893, 932, 992, 997, 1005, 1007}	326 (64757) 953 (60995) 298 (60671) 202 (46774) 564 (38065)		
{z0}z1	$(b^{n+1}+1)/(b+1)$	2	2 (only even lengths exist)	(almost cannot be a minimal prime, this family is of interest only because of generalized Wagstaff primes)	{97, 103, 113, 186, 187, 220, 304, 306, 309, 335, 414, 416, 428, 433, 445, 459, 486, 498, 539, 550, 557, 587, 592, 597, 598, 617, 624, 637, 659, 665, 671, 677, 696, 717, 726, 730, 740, 754, 766, 790, 851, 873, 890,	316 (48538) 175 (31626) 365 (25578) 373 (24006) 188 (22036) 53 (21942) 833 (17116) 124 (16426) 560 (15072) 966 (14820)	$b = m^r$ with odd r>1 (sum-of-two-r-th-powers factorization) $b = 4*m^4$ (Aurifeuillian factorization for x^4+4y^4)	≥17326

					914, 923, 929, 943, 944, 965, 984, 985, 996, 1004, 1005}			
{z}yz	$b^n-(b+1)$	2	2	{z}y	{215, 517, 743, 852, 899}	913 (3773) 353 (2832) 992 (1222) 838 (840) 246 (748) 943 (713) 213 (643) 190 (562) 528 (527) 292 (505)	(none)	≥5000
{z}1	$b^n-(b-1)$	2	2	none (always minimal prime)	{93, 113, 152, 158, 188, 218, 226, 227, 228, 233, 240, 275, 278, 338, 353, 383, 404, 500, 533, 576, 614,	730 (4427) 464 (4421) 918 (4201) 830 (3917) 438 (3436) 293 (3205) 312 (3023) 71 (3019) 88 (2848) 471 (2623)	(none)	≥5000

					641, 653, 704, 723, 728, 758, 779, 791, 878, 881, 899, 908, 929, 944, 953, 965, 968, 978, 983, 986, 1013}			
{z}w	b^n-4	5	2	none (always minimal prime)	{207, 221, 293, 375, 387, 533, 633, 647, 653, 687, 701, 747, 761, 785, 863, 897, 905, 965, 1017}	333 (1977) 251 (1773) 951 (1679) 933 (1641) 695 (1353) 377 (1227) 767 (1199) 797 (905) 303 (741) 335 (715)	*b* == 0 mod 2 (trivial factor 2) *b* == 1 mod 3 (trivial factor 3) *b* == 14 mod 15 (covering set {3,5}) *b* == 4 mod 5 (odd length: factor 5, even length: difference-of-two-squares factorization) $b = m^2$ (difference-of-squares	≥5000

							factorization)	
{z}x	b^n-3	4	2	none (always minimal prime)	(none)	542 (1944) 512 (1600) 478 (1410) 302 (1061) 154 (396) 152 (346) 1000 (330) 698 (306) 1010 (226) 94 (204)	*b* == 1 mod 2 (trivial factor 2)	(no bases *b* ≤ 1024 have this family as unsolved family, base *b* = 542 is the last to drop at length *n* = 1944)
{z}y	b^n-2	3	2	none (always minimal prime)	{305, 353, 397, 535, 539, 597, 641, 731, 739}	317 (13896) 487 (3775) 287 (3410) 485 (3164) 755 (2436) 679 (2175) 809 (1680) 843 (1552) 347 (1122) 551 (864)	*b* == 0 mod 2 (trivial factor 2)	≥5000

[43] https://www.rose-hulman.edu/~rickert/Compositeseq/ (a problem related to this project)

[44] http://www.worldofnumbers.com/em197.htm (a problem related to this project) (for more status page see http://www.worldofnumbers.com/seq197.htm) (for the status page for digit 1 see http://www.worldofnumbers.com/Appending%201s%20to%20n.txt and https://mersenneforum.org/attachment.php?attachmentid=25000&d=1622618552) (for the status page for digit 3 see http://www.worldofnumbers.com/Appending%203s%20to%20n.txt)

[45] http://www.worldofnumbers.com/ (list of special types of primes, including: smoothly undulating palindromic primes http://www.worldofnumbers.com/undulat.htm, palindromic wing primes http://www.worldofnumbers.com/wing.htm, plateau and depression primes http://www.worldofnumbers.com/deplat.htm, palindromic merlon primes http://www.worldofnumbers.com/merlon.htm)

[46]

[47] https://stdkmd.net/nrr/prime/primecount.txt (near- and quasi- repdigit (probable) primes sorted by count)

[48] https://stdkmd.net/nrr/prime/primedifficulty.txt (near- and quasi- repdigit (probable) primes sorted by difficulty)

[49] https://stdkmd.net/nrr/coveringset.htm (covering set of near-repdigit-related sequences)

[50] http://www.numericana.com/answer/primes.htm (the article about the primes and the primality tests) (also http://www.numericana.com/answer/pseudo.htm for pseudoprimes and http://www.numericana.com/answer/factoring.htm for integer factorizations)

[51] http://www.rieselprime.de/dl/CRUS_pack.zip (*srsieve*, *sr1sieve*, *sr2sieve*, *pfgw*, *llr* softwares) (another link: https://www.bc-team.org/app.php/dlext/?cat=3, this link includes *srsieve*, *sr1sieve*, *sr2sieve*, *sr5sieve* softwares)

[52] https://sourceforge.net/projects/openpfgw/ (*pfgw* software)

[53] http://jpenne.free.fr/index2.html (*llr* software)

[54] http://www.ellipsa.eu/public/primo/primo.html (*PRIMO* software)

[55] https://primes.utm.edu/prove/index.html (website for primality proving)

[56] https://primes.utm.edu/notes/prp_prob.html (the probability that a random probable prime is in fact composite)

[57] https://oeis.org/wiki/User:Charles_R_Greathouse_IV/Tables_of_special_primes (expected number of primes in first *n* terms of a given sequence)

[58] https://www.pourlascience.fr/sd/mathematiques/nombres-premiers-inevitables-et-pyramidaux-4744.php (the Scientific American about minimal primes, in French)

[59] https://primes.utm.edu/curios/page.php?curio_id=40841 (the largest base *b* = 10 minimal prime in Prime Curios!) (also for other bases *b*: https://primes.utm.edu/curios/page.php?curio_id=43236 (*b* = 5), https://primes.utm.edu/curios/page.php?curio_id=42961 (*b* = 7), https://primes.utm.edu/curios/page.php?curio_id=42048 (*b* = 16, only the largest known, there may be larger minimal primes)

[60] https://oeis.org/A347819 (*OEIS* sequence for base 10 minimal primes) (for the case when the restriction of prime>base is not required, see https://oeis.org/A071062)

[61] https://oeis.org/A326609 (*OEIS* sequence for the largest base *b* minimal prime, when the restriction of prime>base is not required) (for the length of the largest base *b* minimal prime, see https://oeis.org/A330049, and for the number of base *b* minimal primes, see https://oeis.org/A330048)

[62] https://primes.utm.edu/primes/lists/all.txt (top definitely primes) (search page: https://primes.utm.edu/primes/search.php https://primes.utm.edu/primes/search.php?Advanced=1 https://primes.utm.edu/primes/search_proth.php) (submit page: https://primes.utm.edu/bios/newprover.php https://primes.utm.edu/bios/newcode.php https://primes.utm.edu/bios/index.php)

[63] http://www.primenumbers.net/prptop/prptop.php (top probable primes) (search page: http://www.primenumbers.net/prptop/searchform.php) (submit page: http://www.primenumbers.net/prptop/submit.php)

[64] http://factordb.com (online factor database, including many primes which are minimal primes in a small base) (also factorization of special numbers: https://homes.cerias.purdue.edu/~ssw/cun/index.html (b^n±1 for 2 ≤ *b* ≤ 12, *b* not perfect power) https://maths-people.anu.edu.au/~brent/factors.html (b^n±1 for 13 ≤ *b* ≤ 99, *b* not perfect power) https://mklasson.com/factors/ (k*2^n±1 for odd 3 ≤ *k* ≤ 15) https://stdkmd.net/nrr/ (numbers in families {1}, {*x*}*y*, *x*{*y*}, {*x*}*yx*, *xy*{*x*}, *x*{*y*}*x*, *x*{*y*}*z*, {*x*}*y*{*x*} (where the two {*x*} have the same number of *x*'s)) https://archive.fo/gUdAf (b^n±1 for prime *b*) http://myfactors.mooo.com/ (b^n±1 for 2 ≤ *b* ≤ 9999, *b* not perfect power)

For list of more references, see https://mersenneforum.org/showpost.php?p=571731&postcount=140 and https://mersenneforum.org/showpost.php?p=582061&postcount=154

Also see https://primes.utm.edu/curios/includes/primetest.php and https://www.numberempire.com/primenumbers.php and https://www.bigprimes.net/primalitytest and http://www.proftnj.com/calcprem.htm and https://www.archimedes-lab.org/primOmatic.html and http://www.sonic.net/~undoc/java/PrimeCalc.html for links of prime checkers.

Also see https://www.numberempire.com/numberfactorizer.php and https://www.alpertron.com.ar/ECM.HTM and http://www.javascripter.net/math/calculators/primefactorscalculator.htm and https://primefan.tripod.com/Factorer.html and http://www.se16.info/js/factor.htm and http://math.fau.edu/Richman/mla/factor-f.htm for links of integer factorizers.

Also see https://baseconvert.com/ and https://www.calculand.com/unit-converter/zahlen.php and https://www.cut-the-knot.org/Curriculum/Algorithms/BaseConversion.shtml and https://linesegment.web.fc2.com/application/math/numbers/RadixConversion.html (in Japanese) for links of base converters.

Also see https://primes.utm.edu/lists/small/1000.txt and https://primes.utm.edu/lists/small/millions/ and https://oeis.org/A000040/b000040_1.txt and https://oeis.org/A000040/a000040_1B.7z and https://www2.cs.arizona.edu/icon/oddsends/primes.htm and http://noe-education.org/D11102.php (in French) and https://primefan.tripod.com/500Primes1.html (**warning: this site incorrectly includes 1 as a prime and misses the primes 3229 and 3329**) and https://www.gutenberg.org/files/65/65.txt and http://www.primos.mat.br/indexen.html and https://www.walter-fendt.de/html5/men/primenumbers_en.htm and http://www.rsok.com/~jrm/printprimes.html and https://jocelyn.quizz.chat/np/cache/index.html and https://en.wikipedia.org/wiki/List_of_prime_numbers#The_first_1000_prime_numbers for links of lists of small primes.

Also see http://primefan.tripod.com/500factored.html and http://www.sosmath.com/tables/factor/factor.html and https://en.wikipedia.org/wiki/Table_of_prime_factors for links of lists of factorizations of small integers.

Also see https://en.wikipedia.org/wiki/Table_of_bases for links of lists of small integers in various bases.

(In fact, you can use Wolfram Alpha for prime checker, integer factorizer, and base converter, besides, many mathematical softwares also already have prime checkers, integer factorizers, and base converters, including *Maple*, *wolfram Mathematica*, *PARI/GP*, *Python*, *GMP*, *Magma*, *SageMath*, see the table below, you can download these softwares by clicking the links)

software	*Maple*	*Wolfram Mathematica*	*PARI/GP*	*Python*	*GMP*	*Magma*	*SageMath*
check if a number is probable prime		PrimeQ[*number*]	ispseudoprime(*number*)				
check if a number is definitely prime		ProvablePrimeQ[*number*]	isprime(*number*)				
factor a number		FactorInteger[*nu*	factor(*number*)				

		mber]					
convert a number to base *b*		BaseForm[*number*, *base*] IntegerDigits[*number*, *base*]	digits(*number*, *base*)	int(*number*, *base*)			

Finally, there is a C code for the problem in this article: (need run with *GMP*), see this forum post.