

Minimal elements for the base b representations of the primes which are $> b$

Introduction

A string x is a subsequence of another string y , if x can be obtained from y by deleting zero or more of the characters in y . For example, 514 is a substring of 352148. The empty string λ is a subsequence of every string. There are 2^n substrings of a string with length n , e.g. the substrings of 123456 are (totally $2^6 = 64$ substrings)

λ , 1, 2, 3, 4, 5, 6, 12, 13, 14, 15, 16, 23, 24, 25, 26, 34, 35, 36, 45, 46, 56, 123, 124, 125, 126, 134, 135, 136, 145, 146, 156, 234, 235, 236, 245, 246, 256, 345, 346, 356, 456, 1234, 1235, 1236, 1245, 1246, 1256, 1345, 1346, 1356, 1456, 2345, 2346, 2356, 2456, 3456, 12345, 12346, 12356, 12456, 13456, 23456, 123456

(in this article, we only consider the substrings with length ≥ 2 , e.g. for the string 123456, we have these substrings: 12, 13, 14, 15, 16, 23, 24, 25, 26, 34, 35, 36, 45, 46, 56, 123, 124, 125, 126, 134, 135, 136, 145, 146, 156, 234, 235, 236, 245, 246, 256, 345, 346, 356, 456, 1234, 1235, 1236, 1245, 1246, 1256, 1345, 1346, 1356, 1456, 2345, 2346, 2356, 2456, 3456, 12345, 12346, 12356, 12456, 13456, 23456, 123456, totally 57 substrings, and for a string with length n , there are $2^n - n - 1$ substrings)

Two strings x and y are comparable if either x is a substring of y , or y is a substring of x . A surprising result from formal language theory is that every set of pairwise incomparable strings is finite. This means that from any set of strings we can find its minimal elements. A string x in a set of strings S is a minimal string if whenever y (an element of S) is a substring of x , we have $y = x$.

The set of all minimal strings of S is denoted $M(S)$, the set $M(S)$ must be finite! Even if S is an infinite set, such as the set of prime strings in decimal.

Equivalently, a string x in a set of strings S is a minimal string if and only if any proper substring of x (substring of x which is unequal to x) is not in S .

Although the set $M(S)$ of minimal strings is necessarily finite, determining it explicitly for a given S can be a difficult computational problem. We use some numbertheoretic heuristics to compute $M(L_b)$, where L_b is the language of base- b representations of the prime numbers which are $> b$, for $2 \leq b \leq 16$.

This is a list for L_b for all bases $2 \leq b \leq 16$ and selected bases $17 \leq b \leq 36$, using A-Z to represent digit values 10 to 35.

| b | L_b |
|-----|--|
| 2 | 11, 101, 111, 1011, 1101, 10001, 10011, 10111, 11101, 11111, 100101, 101001, 101011, 101111, 110101, 111011, 111101, 1000011, 1000111, 1001001, 1001111, 1010011, 1011001, 1100001, 1100101, 1100111, 1101011, 1101101, 1110001, 1111111, 10000011, 10001001, 10001011, 10010101, 10010111, 10011101, 10100011, 10100111, 10101101, 10110011, 10110101, 10111111, 11000001, 11000101, 11000111, 11010011, 11011111, 11100011, 11100101, 11101001, 11101111, 11110001, 11111011, 100000001, 100000111, 100001101, 100001111, 100010101, 100011001, 100011011, 100100101, 100110011, 100110111, 100111001, 100111101, 101001011, 101010001, 101011011, 101011101, 101100001, 101100111, 101101111, 101110101, 101111011, 101111111, 110000101, 110001101, 110010001, 110011001, 110100011, 110100101, 110101111, 110110001, 110110111, 110111011, 111000001, 111001001, 111001101, 111001111, 111010011, 111011111, 111100111, 111101011, 111110011, 111110111, 111111101, 1000001001, 1000001011, 1000011101, 1000100011, ... |
| 3 | 12, 21, 102, 111, 122, 201, 212, 1002, 1011, 1101, 1112, 1121, 1202, 1222, 2012, 2021, 2111, 2122, 2201, 2221, 10002, 10022, 10121, 10202, 10211, 10222, 11001, 11012, 11201, 11212, 12002, 12011, 12112, 12121, 12211, 20001, 20012, 20102, 20122, 20201, 21002, 21011, 21022, 21101, 21211, 22021, 22102, 22111, 22122, 22212, 22221, 100022, 100112, 100202, 100222, 101001, 101021, 101102, 101111, 101212, 102101, 102112, 102121, 102202, 110021, 110111, 110212, 110221, 111002, 111022, 111121, 111211, 112001, 112012, 112102, 112201, 112212, 120011, 120112, 120121, 120222, 121001, 121021, 121102, 121122, 121221, 122002, 122011, 122022, 122202, 200001, 200012, 200111, 200122, 200212, 201022, 201101, 202001, 202021, 202122, ... |
| 4 | 11, 13, 23, 31, 101, 103, 113, 131, 133, 211, 221, 223, 233, 311, 323, 331, 1003, 1013, 1021, 1033, 1103, 1121, 1201, 1211, 1213, 1223, 1231, 1301, 1333, 2003, 2021, 2023, 2111, 2113, 2131, 2203, 2213, 2231, 2303, 2311, 2333, 3001, 3011, 3013, 3103, 3133, 3203, 3211, 3221, 3233, 3301, 3323, 10001, 10013, 10031, 10033, 10111, 10121, 10123, 10211, 10303, 10313, 10321, 10331, 11023, 11101, 11123, 11131, 11201, 11213, 11233, 11311, 11323, 11333, 12011, 12031, 12101, 12121, 12203, 12211, 12233, 12301, 12313, 12323, 13001, 13021, 13031, 13033, 13103, 13133, 13213, 13223, 13303, 13313, 13331, 20021, 20023, 20131, 20203, 20231, ... |
| 5 | 12, 21, 23, 32, 34, 43, 104, 111, 122, 131, 133, 142, 203, 214, 221, 232, 241, 243, 304, 313, 324, 342, 401, 403, 412, 414, 423, 1002, 1011, 1022, 1024, 1044, 1101, 1112, 1123, 1132, 1143, 1204, 1211, 1231, 1233, 1242, 1244, 1321, 1343, 1402, 1404, 1413, 1424, 1431, 2001, 2012, 2023, 2034, 2041, 2102, 2111, 2113, 2133, 2212, 2221, 2223, 2232, 2311, 2322, 2342, 2344, 2403, 2414, 2432, 2443, 3004, 3013, 3024, 3042, 3101, 3114, 3134, 3141, 3211, 3213, 3224, 3233, 3244, 3312, 3321, 3323, 3332, 3404, 3422, 3431, 3444, 4003, 4014, 4041, 4043, 4131, 4142, 4212, 4223, ... |
| 6 | 11, 15, 21, 25, 31, 35, 45, 51, 101, 105, 111, 115, 125, 135, 141, 151, 155, 201, 211, 215, 225, 241, 245, 251, 255, 301, 305, 331, 335, 345, 351, 405, 411, 421, 431, 435, 445, 455, 501, 515, 521, 525, 531, 551, 1011, 1015, 1021, |

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| | 1025, 1035, 1041, 1055, 1105, 1115, 1125, 1131, 1141, 1145, 1151, 1205, 1231, 1235, 1241, 1245, 1311, 1321, 1335, 1341, 1345, 1355, 1411, 1421, 1431, 1435, 1445, 1501, 1505, 1521, 1535, 1541, 1555, 2001, 2011, 2015, 2025, 2041, 2045, 2051, 2055, 2115, 2131, 2135, 2151, 2155, 2205, 2225, 2231, 2301, 2311, 2325, 2335, ... |
| 7 | 14, 16, 23, 25, 32, 41, 43, 52, 56, 61, 65, 104, 113, 115, 124, 131, 133, 142, 146, 155, 166, 203, 205, 212, 214, 221, 241, 245, 254, 256, 302, 304, 313, 322, 326, 335, 344, 346, 362, 364, 401, 403, 421, 436, 443, 445, 452, 461, 463, 506, 515, 524, 533, 535, 544, 551, 553, 566, 616, 623, 625, 632, 652, 661, 1004, 1006, 1013, 1022, 1033, 1042, 1051, 1055, 1064, 1105, 1112, 1123, 1136, 1141, 1154, 1156, 1165, 1202, 1211, 1222, 1226, 1231, 1235, 1253, 1264, 1301, 1312, 1316, 1325, 1343, 1345, 1402, 1411, 1424, 1433, 1442, ... |
| 8 | 13, 15, 21, 23, 27, 35, 37, 45, 51, 53, 57, 65, 73, 75, 103, 107, 111, 117, 123, 131, 141, 145, 147, 153, 155, 161, 177, 203, 211, 213, 225, 227, 235, 243, 247, 255, 263, 265, 277, 301, 305, 307, 323, 337, 343, 345, 351, 357, 361, 373, 401, 407, 415, 417, 425, 431, 433, 445, 463, 467, 471, 475, 513, 521, 533, 535, 541, 547, 557, 565, 573, 577, 605, 615, 621, 631, 643, 645, 657, 661, 667, 673, 701, 711, 715, 717, 723, 737, 747, 753, 763, 767, 775, 1011, 1013, 1035, 1043, 1055, 1063, 1071, ... |
| 9 | 12, 14, 18, 21, 25, 32, 34, 41, 45, 47, 52, 58, 65, 67, 74, 78, 81, 87, 102, 108, 117, 122, 124, 128, 131, 135, 151, 155, 162, 164, 175, 177, 184, 201, 205, 212, 218, 221, 232, 234, 238, 241, 254, 267, 272, 274, 278, 285, 287, 308, 315, 322, 328, 331, 337, 342, 344, 355, 371, 375, 377, 382, 407, 414, 425, 427, 432, 438, 447, 454, 461, 465, 472, 481, 485, 504, 515, 517, 528, 531, 537, 542, 548, 557, 562, 564, 568, 582, 601, 605, 614, 618, 625, 638, 641, 661, 667, 678, 685, 702, ... |
| 10 | 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107, 109, 113, 127, 131, 137, 139, 149, 151, 157, 163, 167, 173, 179, 181, 191, 193, 197, 199, 211, 223, 227, 229, 233, 239, 241, 251, 257, 263, 269, 271, 277, 281, 283, 293, 307, 311, 313, 317, 331, 337, 347, 349, 353, 359, 367, 373, 379, 383, 389, 397, 401, 409, 419, 421, 431, 433, 439, 443, 449, 457, 461, 463, 467, 479, 487, 491, 499, 503, 509, 521, 523, 541, 547, 557, 563, 569, ... |
| 11 | 12, 16, 18, 21, 27, 29, 34, 38, 3A, 43, 49, 54, 56, 61, 65, 67, 72, 76, 81, 89, 92, 94, 98, 9A, A3, 106, 10A, 115, 117, 126, 128, 133, 139, 142, 148, 153, 155, 164, 166, 16A, 171, 182, 193, 197, 199, 1A2, 1A8, 1AA, 209, 214, 21A, 225, 227, 232, 236, 238, 247, 25A, 263, 265, 269, 281, 287, 296, 298, 2A1, 2A7, 304, 30A, 315, 319, 324, 331, 335, 342, 351, 353, 362, 364, 36A, 373, 379, 386, 38A, 391, 395, 3A6, 403, 407, 414, 418, 423, 434, 436, 452, 458, 467, 472, 478, 47A, ... |
| 12 | 11, 15, 17, 1B, 25, 27, 31, 35, 37, 3B, 45, 4B, 51, 57, 5B, 61, 67, 6B, 75, 81, 85, 87, 8B, 91, 95, A7, AB, B5, B7, 105, 107, 111, 117, 11B, 125, 12B, 131, 13B, 141, 145, 147, 157, 167, 16B, 171, 175, 17B, 181, 18B, 195, 19B, 1A5, 1A7, 1B1, 1B5, 1B7, 205, 217, 21B, 221, 225, 237, 241, 24B, 251, 255, 25B, 267, 271, 277, 27B, 285, 291, 295, 2A1, 2AB, 2B1, 2BB, 301, 307, 30B, 315, 321, 325, 327, 32B, 33B, 347, 34B, 357, 35B, 365, 375, 377, 391, 397, 3A5, 3AB, 3B5, 3B7, ... |

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| 13 | 14, 16, 1A, 23, 25, 2B, 32, 34, 38, 41, 47, 49, 52, 56, 58, 61, 65, 6B, 76, 7A, 7C, 83, 85, 89, 9A, A1, A7, A9, B6, B8, C1, C7, CB, 104, 10A, 10C, 119, 11B, 122, 124, 133, 142, 146, 148, 14C, 155, 157, 164, 16A, 173, 179, 17B, 184, 188, 18A, 197, 1A8, 1AC, 1B1, 1B5, 1C6, 1CC, 209, 20B, 212, 218, 223, 229, 232, 236, 23C, 247, 24B, 256, 263, 265, 272, 274, 27A, 281, 287, 292, 296, 298, 29C, 2AB, 2B6, 2BA, 2C5, 2C9, 302, 311, 313, 328, 331, 33B, 344, 34A, 34C, 355, ... |
| 14 | 13, 15, 19, 21, 23, 29, 2D, 31, 35, 3B, 43, 45, 4B, 51, 53, 59, 5D, 65, 6D, 73, 75, 79, 7B, 81, 91, 95, 9B, 9D, A9, AB, B3, B9, BD, C5, CB, CD, D9, DB, 101, 103, 111, 11D, 123, 125, 129, 131, 133, 13D, 145, 14B, 153, 155, 15B, 161, 163, 16D, 17D, 183, 185, 189, 199, 1A1, 1AB, 1AD, 1B3, 1B9, 1C3, 1C9, 1D1, 1D5, 1DB, 205, 209, 213, 21D, 221, 22B, 22D, 235, 239, 241, 249, 24D, 251, 255, 263, 26B, 271, 279, 27D, 285, 293, 295, 2A9, 2B1, 2BB, 2C3, 2C9, 2CB, 2D3, ... |
| 15 | 12, 14, 18, 1E, 21, 27, 2B, 2D, 32, 38, 3E, 41, 47, 4B, 4D, 54, 58, 5E, 67, 6B, 6D, 72, 74, 78, 87, 8B, 92, 94, 9E, A1, A7, AD, B2, B8, BE, C1, CB, CD, D2, D4, E1, ED, 102, 104, 108, 10E, 111, 11B, 122, 128, 12E, 131, 137, 13B, 13D, 148, 157, 15B, 15D, 162, 171, 177, 182, 184, 188, 18E, 197, 19D, 1A4, 1A8, 1AE, 1B7, 1BB, 1C4, 1CE, 1D1, 1DB, 1DD, 1E4, 1E8, 1EE, 207, 20B, 20D, 212, 21E, 227, 22B, 234, 238, 23E, 24B, 24D, 261, 267, 272, 278, 27E, 281, 287, ... |
| 16 | 11, 13, 17, 1D, 1F, 25, 29, 2B, 2F, 35, 3B, 3D, 43, 47, 49, 4F, 53, 59, 61, 65, 67, 6B, 6D, 71, 7F, 83, 89, 8B, 95, 97, 9D, A3, A7, AD, B3, B5, BF, C1, C5, C7, D3, DF, E3, E5, E9, EF, F1, FB, 101, 107, 10D, 10F, 115, 119, 11B, 125, 133, 137, 139, 13D, 14B, 151, 15B, 15D, 161, 167, 16F, 175, 17B, 17F, 185, 18D, 191, 199, 1A3, 1A5, 1AF, 1B1, 1B7, 1BB, 1C1, 1C9, 1CD, 1CF, 1D3, 1DF, 1E7, 1EB, 1F3, 1F7, 1FD, 209, 20B, 21D, 223, 22D, 233, 239, 23B, 241, ... |
| 18 | 11, 15, 1B, 1D, 21, 25, 27, 2B, 2H, 35, 37, 3D, 3H, 41, 47, 4B, 4H, 57, 5B, 5D, 5H, 61, 65, 71, 75, 7B, 7D, 85, 87, 8D, 91, 95, 9B, 9H, A1, AB, AD, AH, B1, BD, C7, CB, CD, CH, D5, D7, DH, E5, EB, EH, F1, F7, FB, FD, G5, H1, H5, H7, HB, 107, 10D, 115, 117, 11B, 11H, 127, 12D, 131, 135, 13B, 141, 145, 14D, 155, 157, 15H, 161, 167, 16B, 16H, 177, 17B, 17D, 17H, 18B, 191, 195, 19D, 19H, 1A5, 1AH, 1B1, 1C1, 1C7, 1CH, 1D5, 1DB, 1DD, 1E1, 1EB, ... |
| 20 | 13, 19, 1B, 1H, 21, 23, 27, 2D, 2J, 31, 37, 3B, 3D, 3J, 43, 49, 4H, 51, 53, 57, 59, 5D, 67, 6B, 6H, 6J, 79, 7B, 7H, 83, 87, 8D, 8J, 91, 9B, 9D, 9H, 9J, AB, B3, B7, B9, BD, BJ, C1, CB, CH, D3, D9, DB, DH, E1, E3, ED, F7, FB, FD, FH, GB, GH, H7, H9, HD, HJ, I7, ID, IJ, J3, J9, JH, 101, 109, 10J, 111, 11B, 11D, 11J, 123, 129, 12H, 131, 133, 137, 13J, 147, 14B, 14J, 153, 159, 161, 163, 171, 177, 17H, 183, 189, 18B, 18H, 197, 19D, ... |
| 24 | 15, 17, 1D, 1H, 1J, 1N, 25, 2B, 2D, 2J, 2N, 31, 37, 3B, 3H, 41, 45, 47, 4B, 4D, 4H, 57, 5B, 5H, 5J, 65, 67, 6D, 6J, 6N, 75, 7B, 7D, 7N, 81, 85, 87, 8J, 97, 9B, 9D, 9H, 9N, A1, AB, AH, AN, B5, B7, BD, BH, BJ, C5, CJ, CN, D1, D5, DJ, E1, EB, ED, EH, EN, F7, FD, FJ, FN, G5, GD, GH, H1, HB, HD, HN, I1, I7, IB, IH, J1, J5, J7, JB, JN, K7, KB, KJ, KN, L5, LH, LJ, MD, MJ, N5, NB, NH, NJ, 101, 10B, 10H, 10N, ... |
| 30 | 11, 17, 1B, 1D, 1H, 1N, 1T, 21, 27, 2B, 2D, 2J, 2N, 2T, 37, 3B, 3D, 3H, 3J, 3N, 47, 4B, 4H, 4J, 4T, 51, 57, 5D, 5H, 5N, 5T, 61, 6B, 6D, 6H, 6J, 71, 7D, 7H, 7J, 7N, 7T, 81, 8B, 8H, 8N, 8T, 91, 97, 9B, 9D, 9N, A7, AB, AD, AH, B1, B7, BH, |

| | |
|----|---|
| | BJ, BN, BT, C7, CD, CJ, CN, CT, D7, DB, DJ, DT, E1, EB, ED, EJ, EN, ET, F7, FB, FD, FH, FT, G7, GB, GJ, GN, GT, HB, HD, I1, I7, IH, IN, IT, J1, J7, JH, JN, JT, K1, ... |
| 32 | 15, 19, 1B, 1F, 1L, 1R, 1T, 23, 27, 29, 2F, 2J, 2P, 31, 35, 37, 3B, 3D, 3H, 3V, 43, 49, 4B, 4L, 4N, 4T, 53, 57, 5D, 5J, 5L, 5V, 61, 65, 67, 6J, 6V, 73, 75, 79, 7F, 7H, 7R, 81, 87, 8D, 8F, 8L, 8P, 8R, 95, 9J, 9N, 9P, 9T, AB, AH, AR, AT, B1, B7, BF, BL, BR, BV, C5, CD, CH, CP, D3, D5, DF, DH, DN, DR, E1, E9, ED, EF, EJ, EV, F7, FB, FJ, FN, FT, G9, GB, GT, H3, HD, HJ, HP, HR, I1, IB, IH, IN, IP, IV, ... |
| 36 | 11, 15, 17, 1B, 1H, 1N, 1P, 1V, 1Z, 21, 27, 2B, 2H, 2P, 2T, 2V, 2Z, 31, 35, 3J, 3N, 3T, 3V, 45, 47, 4D, 4J, 4N, 4T, 4Z, 51, 5B, 5D, 5H, 5J, 5V, 67, 6B, 6D, 6H, 6N, 6P, 6Z, 75, 7B, 7H, 7J, 7P, 7T, 7V, 85, 8J, 8N, 8P, 8T, 97, 9D, 9N, 9P, 9T, 9Z, A7, AD, AJ, AN, AT, B1, B5, BD, BN, BP, BZ, C1, C7, CB, CH, CP, CT, CV, CZ, DB, DJ, DN, DV, DZ, E5, EH, EJ, F1, F7, FH, FN, FT, FV, G1, GB, GH, GN, GP, GV, ... |

The primes in $M(L_b)$ are called **minimal prime base b** in this article, although in fact this name should be used for L_b is the language of base- b representations of the prime numbers, where primes $> b$ is not required, this problem an extension of the original minimal prime problem to include the generalized Sierpinski conjecture base b and the generalized Riesel conjecture base b , for all k -values $< b$. For example, 857 is a quasi-minimal prime in decimal because there is no prime > 10 among the shorter subsequences of the digits: 8, 5, 7, 85, 87, 57. The subsequence does not have to consist of consecutive digits, so 149 is not a quasi-minimal prime in decimal (because 19 is prime and $19 > 10$). But it does have to be in the same order; so, for example, 991 is still a quasi-minimal prime in decimal even though a subset of the digits can form the shorter prime $19 > 10$ by changing the order.

A summary of the results of our algorithm is presented in the table in the next section; I completely solved all bases up to 16 except for bases 14, 16, and the odd bases > 6 (the proofs are at the end of this article), for bases 14, 16, and the odd bases > 6 , I only found all minimal primes up to certain limit (about 2^{32}) and some larger minimal primes (such as 3^{161} in base 7). I leave the reader to completely solve all bases up to 36 (this will be a hard problem, e.g. base 23 has a minimal prime $9E^{800873}$, and base 30 has a minimal prime OT^{34205}).

Problems about the digits of prime numbers have a long history, and many of them are still unsolved. For example, are there infinitely many primes, all of whose base-10 digits are 1? Currently, there are only five such “repunits” known, corresponding to $(10^p - 1)/9$ for $p \in \{2, 19, 23, 317, 1031\}$. It seems likely that four more are given by $p \in \{49081, 86453, 109297, 270343\}$, but this has not yet been rigorously proven. This problem also exists for other bases, e.g. for base 12, there are only nine proven such numbers, corresponding to $(12^p - 1)/11$ for $p \in \{2, 3, 5, 19, 97, 109, 317, 353, 701\}$. It seems likely that five more are given by $p \in \{9739, 14951, 37573, 46889, 769543\}$, but this has not yet been rigorously proven. However, for some bases there exists no such primes, these bases are 9, 25, 32, 49, 64, 81, 121, 125, 144, ..., this is because the numbers with all digits 1 in these bases can be factored algebraically.

Table

In the " $\max(x, x \in L_b)$ " column, $xy^n z$ means $xyyy...yyy z$ with n y 's (the n -value is written in decimal), not y to the n th power.

| b | $ M(L_b) $ | $\max(x, x \in M(L_b))$ | $\max(x , x \in M(L_b))$ | Algebraic form of $\max(x, x \in M(L_b))$ |
|------|-------------|--|---------------------------|---|
| 2 | 1 | 11 | 2 | 3 |
| 3 | 3 | 111 | 3 | 13 |
| 4 | 5 | 221 | 3 | 41 |
| 5 | 22 | $10^{93}13$ | 96 | $5^{95}+8$ |
| 6 | 11 | 40041 | 5 | 5209 |
| 7** | ≥ 71 | $3^{16}1$ | 17 | $\frac{7^{17} - 5}{2}$ |
| 8 | 75 | $4^{220}7$ | 221 | $\frac{4 \cdot 8^{221} + 17}{7}$ |
| 9** | ≥ 147 | $30^{1158}11$ | 1161 | $3 \cdot 9^{1160} + 10$ |
| 10 | 77 | $50^{28}27$ | 31 | $5 \cdot 10^{30} + 27$ |
| 11** | ≥ 903 | 557^{1011} or 57^n with $n > 25000$ | 1013 | $\frac{607 \cdot 11^{1011} - 7}{10}$ |
| 12 | 106 | $40^{39}77$ | 42 | $4 \cdot 12^{41} + 91$ |
| 13** | ≥ 2452 | $80^{32017}111$ or 95^n with $n > 25000$ | 32021 | $8 \cdot 13^{32020} + 183$ |
| 14** | ≥ 596 | $4D^{19698}$ | 19699 | $5 \cdot 14^{19698} - 1$ |
| 15** | ≥ 1151 | $7^{155}97$ | 157 | $\frac{15^{157} + 59}{2}$ |
| 16** | ≥ 1877 | DB^{32234} | 32235 | $\frac{206 \cdot 16^{32234} - 11}{15}$ |

** I have not proved these bases, these are the largest elements in $M(L_b)$ known to me, and they are just the lower bounds.

Lists

The sets of bases 14, 16, and the odd bases >6 are possible not complete, only tested to the test limit in the discussion of these bases and found the smallest (probable) prime in some unsolved families of these bases, but there may be more unsolved families not found by me.

There are two unsolved families found by me and tested to length 25000 with no (probable) prime found, they are 57^n in base 11 and 95^n in base 13.

base 2

11

base 3

12, 21, 111

base 4

11, 13, 23, 31, 221

base 5

[illegible]

base 6

11, 15, 21, 25, 31, 35, 45, 51, 4401, 4441, 40041

base 7 (not proved, only checked to the prime 5100000001)

14, 16, 23, 25, 32, 41, 43, 52, 56, 61, 65, 113, 115, 131, 133, 155, 212, 221, 304, 313, 335, 344, 346, 364, 445, 515, 533, 535, 544, 551, 553, 1022, 1051, 1112, 1202, 1211, 1222, 2111, 3031, 3055, 3334, 3503, 3505, 3545, 4504, 4555, 5011, 5455, 5545, 5554, 6034, 6634, 11111, 11201, 30011, 30101, 31001, 31111, 33001, 33311, 35555, 40054, 100121, 150001, 300053, 351101, 531101, 1100021, 33333301, 5100000001, ..., 33333333333333331, ...

base 8

[illegible]

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base 15 (not proved, only checked to the prime 55555557)

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Proof

The following is a “semi-algorithm” that is guaranteed to produce $M(L)$, but it is not so easy to implement:

- In practice, for arbitrary L , we cannot feasibly carry out step (5). Instead, we work with L' , some regular overapproximation to L , until we can show $L' = \emptyset$ (which implies $L = \emptyset$). In

practice, L' is usually chosen to be a finite union of sets of the form $L_1 L_2^* L_3$, where each of L_1, L_2, L_3 is finite. In the case we consider in this paper, we then have to determine whether such a language contains a prime or not.

However, it is not even known if the following simpler decision problem is recursively solvable:

Problem: Given strings x, y, z , and a base b , does there exist a prime number whose base- b expansion is of the form $xy^n z$ for some $n \geq 0$?

It will be necessary for our algorithm to determine if families of the form $(xy^n z)_b$ contain a prime $> b$ or not. We use two different heuristic strategies to show that such families contain no primes for numbers $> b$.

In the first strategy, we mimic the well-known technique of “covering congruences”, by finding some finite set S of primes p such that every number in a given family is divisible by some element of S . In the second strategy, we attempt to find a difference-of-squares or difference-of-cubes factorization.

Examples of first strategy: (we can show that the corresponding numbers are $>$ all elements in S , if n makes corresponding numbers $>$ base (i.e. $n \geq 1$ for 51^n in base 9 and 25^n in base 11 and $4^n D$ in base 16, $n \geq 0$ for other examples), thus these factorizations are nontrivial)

- * In base 10, all numbers of the form $46^n 9$ are divisible by 7
- * In base 4, all numbers of the form $20^n 1$ are divisible by 3
- * In base 15, all numbers of the form $96^n 8$ are divisible by 11
- * In base 9, all numbers of the form 51^n are divisible by some element of $\{2, 5\}$
- * In base 11, all numbers of the form 25^n are divisible by some element of $\{2, 3\}$
- * In base 8, all numbers of the form $64^n 7$ are divisible by some element of $\{3, 5, 13\}$
- * In base 14, all numbers of the form $B0^n 1$ are divisible by some element of $\{3, 5\}$
- * In base 16, all numbers of the form $4^n D$ are divisible by some element of $\{3, 7, 13\}$

Example of second strategy: (we can show that both factors are > 1 , if n makes corresponding numbers $>$ base (i.e. $n \geq 2$ for 1^n in base 9, $n \geq 0$ for $B^n 9B$ in base 12, $n \geq 1$ for other examples), thus these factorizations are nontrivial)

- * In base 9, all numbers of the form 1^n factored as $(3^n - 1) * (3^n + 1) / 8$
- * In base 8, all numbers of the form $10^n 1$ factored as $(2^{n+1} + 1) * (4^{n+1} - 2^{n+1} + 1)$
- * In base 9, all numbers of the form 38^n factored as $(2 * 3^n - 1) * (2 * 3^n + 1)$
- * In base 16, all numbers of the form $8F^n$ factored as $(3 * 4^n - 1) * (3 * 4^n + 1)$
- * In base 16, all numbers of the form $4^n 1$ factored as $(2 * 4^{n+1} - 7) * (2 * 4^{n+1} + 7) / 15$
- * In base 16, all numbers of the form $C^n D$ factored as $(2 * 4^{n+1} - 2 * 2^{n+1} + 1) * (2 * 4^{n+1} + 2 * 2^{n+1} + 1) / 5$
- * In base 14, numbers of the form $8D^n$ are divisible by 5 if n is odd and factored as $(3 * 14^{n/2} - 1) * (3 * 14^{n/2} + 1)$ if n is even
- * In base 12, numbers of the form $B^n 9B$ are divisible by 13 if n is odd and factored as $(12^{(n+2)/2} - 5) * (12^{(n+2)/2} + 5)$ if n is even

As previously mentioned, in practice to compute $M(L_b)$ one works with an underapproximation M of $M(L_b)$ and an overapproximation L of $L_b - \text{sup}(M)$. One then refines such approximations until $L = \emptyset$ from which it follows that $M = M(L_b)$.

For the initial approximation, note that every minimal prime in base b with at least 4 digits is of the form xY^*z , where $x \in \{x \mid x \text{ is base-}b \text{ digit, } x \neq 0\}$, $z \in \{z \mid z \text{ is base-}b \text{ digit, } \gcd(z, b) = 1\}$, and Y^* (for this (x, z) pair) = $\{y \mid xy, xz, yz, xyz \text{ are all composites}\}$. (Of course, if xz is prime, then the Y^* set for this (x, z) pair is \emptyset)

Making use of this, our algorithm sets M to be the set of base- b representations of the minimal primes with at most 3 digits (which can be found simply by brute force) and L to be $\bigcup_{x,z} (xY^*z)$ as described above.

All remaining minimal primes are members of L , so to find them we explore the families in L . During this process, each family will be decomposed into possibly multiple other families. For example, a simple way of exploring the family xY^*z where $Y = \{y_1, \dots, y_n\}$ is to decompose it into the families $xY^*y_1z, \dots, xY^*y_nz$. If the smallest member (say xy_iz) of any such family happens to be prime, it can be added to M and the family xY^*y_iz removed from consideration. Furthermore, once M has been updated it may be possible to simplify some families in L . In this case, xY^*y_jz (for $j \neq i$) can be simplified to $x(Y - y_i)^*y_jz$ since no minimal prime contains xy_iz as a proper subword.

At the conclusion of the algorithm described, L will consist of simple families (of the form xy^*z) which have not yet yielded a prime, but for which there is no obvious reason why there can't be a prime of such a form. In such a case, the only way to proceed is to test the primality of larger and larger numbers of such form and hope a prime is eventually discovered.

We call families of the form xy^*z (where $x, z \in \Sigma_b^+$ and $y \in \Sigma_b$) *simple families*. Our algorithm then proceeds as follows:

1.

$M := \{\text{minimal primes in base } b \text{ of length } \leq 3\}$

$L :=$

S

$x, z \in \Sigma_b^+$

xY^*

z , where $x \neq 0$ and Y is the set of digits y such that xyz has no subword in M

2. While L contains non-simple families:

(a) Explore each family of L by applying Lemma 19, and update L .

(b) Examine each family of L :

i. Let w be the shortest string in the family. If w has a subword in M , then remove the family from L . If w represents a prime, then add w to M and remove the family from L .

ii. If possible, simplify the family by applying Lemma 21 with $k = 1$.

- iii. Using the techniques of Section 4, check if the family can be proven to only contain composites for numbers $> \text{base}$, and if so then remove the family from L .
 (c) Apply Lemmas 21, 23, 25, and 27 to the families of L as much as possible and update L ; after each split examine the new families as in (b).

The numbers in simple families are of the form $\frac{a \cdot b^{n+c}}{\gcd(a+c, b-1)}$ for some fixed integers a, b, c where $a \geq 1, b \geq 2$ (b is the base), $c \neq 0, \gcd(a, c) = 1, \gcd(b, c) = 1$. Except in the special case $c = \pm 1$ and $\gcd(a+c, b-1) = 1$, when n is large the known primality tests for such a number are too inefficient to run. In this case one must resort to a probable primality test such as a Miller–Rabin test or a Baillie–PSW test, unless a divisor of the number can be found. Since we are testing many numbers in an exponential sequence, it is possible to use a sieving process (such as *srsieve* software) to find divisors rather than using trial division.

Some families xy^*z could not be ruled out as only containing composites for numbers $> \text{base}$ and no primes $> \text{base}$ could be found in the family, even after searching through numbers with over 10000 digits. Many xy^*z families contain no small primes even though they do contain very large primes, for example:

- * In base 5, the smallest prime in the family $1\{0\}13$ is $10^{93}13$
- * In base 8, the smallest prime in the family $\{4\}7$ is $4^{220}7$
- * In base 9, the smallest prime in the family $3\{0\}11$ is $30^{1158}11$
- * In base 9, the smallest prime in the family $7\{6\}2$ is $76^{329}2$
- * In base 11, family $5\{7\}$ could not be ruled out as only containing composites and no primes found in the family after searching to length 10000
- * In base 13, the smallest prime in the family $8\{0\}111$ is $80^{32017}111$ (this prime is only a probable prime, i.e. not proven prime)
- * In base 13, the smallest prime in the family $2B3\{0\}1$ is $2B3^{15197}1$
- * In base 13, the smallest prime in the family $39\{0\}1$ is $390^{6266}1$
- * In base 13, family $9\{5\}$ could not be ruled out as only containing composites and no primes found in the family after searching to length 10000
- * In base 14, the smallest prime in the family $4\{D\}$ is $4D^{19698}$
- * In base 16, the smallest prime in the family $D\{B\}$ is DB^{32234} (this prime is only a probable prime, i.e. not proven prime)

For any given base b , we find all (x, z) digits-pair such that $x \neq 0$ and $\gcd(z, b) = 1$, and find the corresponding sets Y^* , see below.

Bold for minimal primes in base b , i.e. elements of the set $M(L_b)$

base 2

The possible (first digit, last digit) for an element with ≥ 3 digits in the minimal set of the strings for primes with at least two digits are:

(1, 1)

- * Case (1, 1):

**** 11** is prime, and thus the only minimal prime in this family.

base 3

The possible (first digit,last digit) for an element with ≥ 3 digits in the minimal set of the strings for primes with at least two digits are:

(1,1), (1,2), (2,1), (2,2)

* Case (1,1):

**** Since 12, 21, 111 are primes, we only need to consider the family $1\{0\}1$ (since any digits 1, 2 between them will produce smaller primes)**

***** All numbers of the form $1\{0\}1$ are divisible by 2, thus cannot be prime.**

* Case (1,2):

**** 12** is prime, and thus the only minimal prime in this family.

* Case (2,1):

**** 21** is prime, and thus the only minimal prime in this family.

* Case (2,2):

**** Since 21, 12 are primes, we only need to consider the family $2\{0,2\}2$ (since any digits 1 between them will produce smaller primes)**

***** All numbers of the form $2\{0,2\}2$ are divisible by 2, thus cannot be prime.**

base 4

The possible (first digit,last digit) for an element with ≥ 3 digits in the minimal set of the strings for primes with at least two digits are:

(1,1), (1,3), (2,1), (2,3), (3,1), (3,3)

* Case (1,1):

**** 11** is prime, and thus the only minimal prime in this family.

* Case (1,3):

**** 13** is prime, and thus the only minimal prime in this family.

* Case (2,1):

** Since 23, 11, 31, **221** are primes, we only need to consider the family $2\{0\}1$ (since any digits 1, 2, 3 between them will produce smaller primes)

*** All numbers of the form $2\{0\}1$ are divisible by 3, thus cannot be prime.

* Case (2,3):

** **23** is prime, and thus the only minimal prime in this family.

* Case (3,1):

** **31** is prime, and thus the only minimal prime in this family.

* Case (3,3):

** Since 31, 13, 23 are primes, we only need to consider the family $3\{0,3\}3$ (since any digits 1, 2 between them will produce smaller primes)

*** All numbers of the form $3\{0,3\}3$ are divisible by 3, thus cannot be prime.

base 5

The possible (first digit,last digit) for an element with ≥ 3 digits in the minimal set of the strings for primes with at least two digits are:

(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (3,4), (4,1), (4,2), (4,3), (4,4)

* Case (1,1):

** Since 12, 21, **111**, **131** are primes, we only need to consider the family $1\{0,4\}1$ (since any digits 1, 2, 3 between them will produce smaller primes)

*** All numbers of the form $1\{0,4\}1$ are divisible by 2, thus cannot be prime.

* Case (1,2):

** **12** is prime, and thus the only minimal prime in this family.

* Case (1,3):

** Since 12, 23, 43, **133** are primes, we only need to consider the family $1\{0,1\}3$ (since any digits 2, 3, 4 between them will produce smaller primes)

*** Since 111 is prime, we only need to consider the families $1\{0\}3$ and $1\{0\}1\{0\}3$ (since any digit combo 11 between (1,3) will produce smaller primes)

**** All numbers of the form $1\{0\}3$ are divisible by 2, thus cannot be prime.

**** For the $1\{0\}1\{0\}3$ family, since **10103** is prime, we only need to consider the families $1\{0\}13$ and $11\{0\}3$ (since any digit combo 010 between (1,3) will produce smaller primes)

***** The smallest prime of the form $10^{13} + 1$ is

[illegible]

***** All numbers of the form $11\{0\}3$ are divisible by 3, thus cannot be prime.

* Case (1,4):

** Since 12, 34, **104** are primes, we only need to consider the families 1{1,4}4 (since any digits 0, 2, 3 between them will produce smaller primes)

*** Since 111, 414 are primes, we only need to consider the family $1\{4\}4$ and $11\{4\}4$ (since any digit combo 11 or 41 between them will produce smaller primes)

**** The smallest prime of the form $1\{4\}4$ is **14444**.

**** All numbers of the form $11\{4\}4$ are divisible by 2, thus cannot be prime.

* Case (2,1):

**** 21** is prime, and thus the only minimal prime in this family.

* Case (2,2):

** Since 21, 23, 12, 32 are primes, we only need to consider the family $2\{0,2,4\}2$ (since any digits 1, 3 between them will produce smaller primes)

*** All numbers of the form $2\{0,2,4\}2$ are divisible by 2, thus cannot be prime.

* Case (2,3):

**** 23** is prime, and thus the only minimal prime in this family.

* Case (2,4):

** Since 21, 23, 34 are primes, we only need to consider the family $2\{0,2,4\}4$ (since any digits 1, 3 between them will produce smaller primes)

*** All numbers of the form $2\{0,2,4\}$ are divisible by 2, thus cannot be prime.

* Case (3,1):

** Since 32, 34, 21 are primes, we only need to consider the family $3\{0,1,3\}1$ (since any digits 2, 4 between them will produce smaller primes)

*** Since 313, 111, 131, **3101** are primes, we only need to consider the families $3\{0,3\}1$ and $3\{0,3\}11$ (since any digit combo 10, 11, 13 between (3,1) will produce smaller primes)

**** For the 3{0,3}1 family, we can separate this family to four families:

***** For the $30\{0,3\}01$ family, we have the prime **30301**, and the remain case is the family $30\{0\}01$.

***** All numbers of the form $30\{0\}01$ are divisible by 2, thus cannot be prime.

***** For the $30\{0,3\}31$ family, note that there must be an even number of 3's between (30,31), or the result number will be divisible by 2 and cannot be prime.

***** Since 33331 is prime, any digit combo 33 between (30,31) will produce smaller primes.

***** Thus, the only possible prime is the smallest prime in the family $30\{0\}31$, and this prime is **300031**.

***** For the $33\{0,3\}01$ family, note that there must be an even number of 3's between (33,01), or the result number will be divisible by 2 and cannot be prime.

***** Since 33331 is prime, any digit combo 33 between (33,01) will produce smaller primes.

***** Thus, the only possible prime is the smallest prime in the family $33\{0\}01$, and this prime is **33001**.

***** For the $33\{0,3\}31$ family, we have the prime **33331**, and the remain case is the family $33\{0\}31$.

***** All numbers of the form $33\{0\}31$ are divisible by 2, thus cannot be prime.

* Case (3,2):

** **32** is prime, and thus the only minimal prime in this family.

* Case (3,3):

** Since 32, 34, 23, 43, **313** are primes, we only need to consider the family $3\{0,3\}3$ (since any digits 1, 2, 4 between them will produce smaller primes)

*** All numbers of the form $3\{0,3\}3$ are divisible by 3, thus cannot be prime.

* Case (3,4):

** **34** is prime, and thus the only minimal prime in this family.

* Case (4,1):

** Since 43, 21, **401** are primes, we only need to consider the family $4\{1,4\}1$ (since any digits 0, 2, 3 between them will produce smaller primes)

*** Since 414, 111 are primes, we only need to consider the family $4\{4\}1$ and $4\{4\}11$ (since any digit combo 14 or 11 between them will produce smaller primes)

**** The smallest prime of the form $4\{4\}1$ is **44441**.

**** All numbers of the form $4\{4\}11$ are divisible by 2, thus cannot be prime.

* Case (4,2):

** Since 43, 12, 32 are primes, we only need to consider the family $4\{0,2,4\}^2$ (since any digits 1, 3 between them will produce smaller primes)

*** All numbers of the form $4\{0,2,4\}^2$ are divisible by 2, thus cannot be prime.

* Case (4,3):

** **43** is prime, and thus the only minimal prime in this family.

* Case (4,4):

** Since 43, 34, **414** are primes, we only need to consider the family $4\{0,2,4\}^4$ (since any digits 1, 3 between them will produce smaller primes)

*** All numbers of the form $4\{0,2,4\}^4$ are divisible by 2, thus cannot be prime.

base 6

The possible (first digit,last digit) for an element with ≥ 3 digits in the minimal set of the strings for primes with at least two digits are:

(1,1), (1,5), (2,1), (2,5), (3,1), (3,5), (4,1), (4,5), (5,1), (5,5)

* Case (1,1):

** **11** is prime, and thus the only minimal prime in this family.

* Case (1,5):

** **15** is prime, and thus the only minimal prime in this family.

* Case (2,1):

** **21** is prime, and thus the only minimal prime in this family.

* Case (2,5):

** **25** is prime, and thus the only minimal prime in this family.

* Case (3,1):

** **31** is prime, and thus the only minimal prime in this family.

* Case (3,5):

** **35** is prime, and thus the only minimal prime in this family.

* Case (4,1):

** Since 45, 11, 21, 31, 51 are primes, we only need to consider the family $4\{0,4\}1$ (since any digits 1, 2, 3, 5 between them will produce smaller primes)

*** Since **4401** and **4441** are primes, we only need to consider the families $4\{0\}1$ and $4\{0\}41$ (since any digits combo 40 and 44 between them will produce smaller primes)

**** All numbers of the form $4\{0\}1$ are divisible by 5, thus cannot be prime.

**** The smallest prime of the form $4\{0\}41$ is **40041**

* Case (4,5):

** **45** is prime, and thus the only minimal prime in this family.

* Case (5,1):

** **51** is prime, and thus the only minimal prime in this family.

* Case (5,5):

** Since 51, 15, 25, 35, 45 are primes, we only need to consider the family $5\{0,5\}5$ (since any digits 1, 2, 3, 4 between them will produce smaller primes)

*** All numbers of the form $5\{0,5\}5$ are divisible by 5, thus cannot be prime.

base 8

The possible (first digit,last digit) for an element with ≥ 3 digits in the minimal set of the strings for primes with at least two digits are:

(1,1), (1,3), (1,5), (1,7), (2,1), (2,3), (2,5), (2,7), (3,1), (3,3), (3,5), (3,7), (4,1), (4,3), (4,5), (4,7), (5,1), (5,3), (5,5), (5,7), (6,1), (6,3), (6,5), (6,7), (7,1), (7,3), (7,5), (7,7)

* Case (1,1):

** Since 13, 15, 21, 51, **111**, **141**, **161** are primes, we only need to consider the family $1\{0,7\}1$ (since any digits 1, 2, 3, 4, 5, 6 between them will produce smaller primes)

*** Since 107, 177, 701 are primes, we only need to consider the number 171 and the family $1\{0\}1$ (since any digits combo 07, 70, 77 between them will produce smaller primes)

**** 171 is not prime.

**** All numbers of the form $1\{0\}1$ factored as $10^{n+1} = (2^{n+1}) * (4^n - 2^{n+1})$, thus cannot be prime.

* Case (1,3):

** **13** is prime, and thus the only minimal prime in this family.

* Case (1,5):

**** 15** is prime, and thus the only minimal prime in this family.

* Case (1,7):

**** Since 13, 15, 27, 37, 57, 107, 117, 147, 177** are primes, we only need to consider the family $1\{6\}7$ (since any digits 0, 1, 2, 3, 4, 5, 7 between them will produce smaller primes)

***** The smallest prime of the form $1\{6\}7$ is 16667** (not minimal prime, since 667 is prime)

* Case (2,1):

**** 21** is prime, and thus the only minimal prime in this family.

* Case (2,3):

**** 23** is prime, and thus the only minimal prime in this family.

* Case (2,5):

**** Since 21, 23, 27, 15, 35, 45, 65, 75, 225, 255** are primes, we only need to consider the family $2\{0\}5$ (since any digits 1, 2, 3, 4, 5, 6, 7 between them will produce smaller primes)

***** All numbers of the form $2\{0\}5$ are divisible by 7, thus cannot be prime.**

* Case (2,7):

**** 27** is prime, and thus the only minimal prime in this family.

* Case (3,1):

**** Since 35, 37, 21, 51, 301, 361** are primes, we only need to consider the family $3\{1,3,4\}1$ (since any digits 0, 2, 5, 6, 7 between them will produce smaller primes)

***** Since 13, 343, 111, 131, 141, 431, 3331, 3411** are primes, we only need to consider the families $3\{3\}11$, $33\{1,4\}1$, $3\{3,4\}4\{4\}1$ (since any digits combo 11, 13, 14, 33, 41, 43 between them will produce smaller primes)

****** All numbers of the form $3\{3\}11$ are divisible by 3, thus cannot be prime.**

****** For the $33\{1,4\}1$ family, since 111 and 141 are primes, we only need to consider the families $33\{4\}1$ and $33\{4\}11$ (since any digits combo 11, 14 between them will produce smaller primes)**

******* The smallest prime of the form $33\{4\}1$ is 3344441**

******* All numbers of the form $33\{4\}11$ are divisible by 301, thus cannot be prime.**

****** For the $3\{3,4\}4\{4\}1$ family, since 3331 and 3344441 are primes, we only need to consider the families $3\{4\}1$, $3\{4\}31$, $3\{4\}341$, $3\{4\}3441$, $3\{4\}34441$ (since any digits combo 33 or 34444 between (3,1) will produce smaller primes)**

******* All numbers of the form $3\{4\}1$ are divisible by 31, thus cannot be prime.**

***** Since 4443 is prime, we only need to consider the numbers 3431, 34431, 34341, 344341, 343441, 3443441, 3434441, 34434441 (since any digit combo 444 between (3,3{4}1) will produce smaller primes)

***** None of 3431, 34431, 34341, 344341, 343441, 3443441, 3434441, 34434441 are primes.

* Case (3,3):

** Since 35, 37, 13, 23, 53, 73, **343** are primes, we only need to consider the family $3\{0,3,6\}3$ (since any digits 1, 2, 4, 5, 7 between them will produce smaller primes)

*** All numbers of the form $3\{0,3,6\}3$ are divisible by 3, thus cannot be prime.

* Case (3,5):

** **35** is prime, and thus the only minimal prime in this family.

* Case (3,7):

** **37** is prime, and thus the only minimal prime in this family.

* Case (4,1):

** Since 45, 21, 51, **401**, **431**, **471** are primes, we only need to consider the family $4\{1,4,6\}1$ (since any digits 0, 2, 3, 5, 7 between them will produce smaller primes)

*** Since 111, 141, 161, 661, **4611** are primes, we only need to consider the families $4\{4\}11$, $4\{4,6\}4\{1,4,6\}1$, $4\{4\}6\{4\}1$ (since any digits combo 11, 14, 16, 61, 66 between them will produce smaller primes)

**** The smallest prime of the form $4\{4\}11$ is 4444444444444411 (not minimal prime, since 444444441 is prime)

**** For the $4\{4,6\}4\{1,4,6\}1$ family, we can separate this family to $4\{4,6\}41$, $4\{4,6\}411$, $4\{4,6\}461$

***** For the $4\{4,6\}41$ family, since 661 and 6441 are primes, we only need to consider the families $4\{4\}41$ and $4\{4\}641$ (since any digits combo 64 or 66 between (4,41) will produce smaller primes)

***** The smallest prime of the form $4\{4\}41$ is **444444441**

***** The smallest prime of the form $4\{4\}641$ is **444641**

***** For the $4\{4,6\}411$ family, since 661 and 6441 are primes, we only need to consider the families $4\{4\}411$ and $4\{4\}6411$ (since any digits combo 64 or 66 between (4,411) will produce smaller primes)

***** The smallest prime of the form $4\{4\}411$ is **444444441**

***** The smallest prime of the form $4\{4\}6411$ is 4444444444444446411 (not minimal prime, since 444444441 and 444641 are primes)

***** For the $4\{4,6\}461$ family, since 661 is prime, we only need to consider the family $4\{4\}461$

***** The smallest prime of the form $4\{4\}461$ is 4444444461 (not minimal prime, since 444444441 is prime)

**** For the 4{4}6{4}1 family, since 6441 is prime, we only need to consider the families 4{4}61 and 4{4}641 (since any digits combo 44 between (4{4}6,1) will produce smaller primes)

***** The smallest prime of the form $4\{4\}61$ is 4444444461 (not minimal prime, since 444444441 is prime)

***** The smallest prime of the form $4\{4\}641$ is **444641**

* Case (4,3):

** Since 45, 13, 23, 53, 73, **433**, **463** are primes, we only need to consider the family $4\{0,4\}3$ (since any digits 1, 2, 3, 5, 6, 7 between them will produce smaller primes)

*** Since **4043** and **4443** are primes, we only need to consider the families 4{0}3 and 44{0}3 (since any digits combo 04, 44 between them will produce smaller primes)

**** All numbers of the form $4\{0\}3$ are divisible by 7, thus cannot be prime.

**** All numbers of the form $44\{0\}3$ are divisible by 3, thus cannot be prime.

* Case (4,5):

**** 45** is prime, and thus the only minimal prime in this family.

* Case (4,7):

** Since 45, 27, 37, 57, **407**, **417**, **467** are primes, we only need to consider the family $4\{4,7\}7$ (since any digits 0, 1, 2, 3, 5, 6 between them will produce smaller primes)

*** Since 747 is prime, we only need to consider the families 4{4}7, 4{4}77, 4{7}7, 44{7}7 (since any digits combo 74 between (4,7) will produce smaller primes)

[illegible]

**** The smallest prime of the form $4\{4\}77$ is **4444477**

**** The smallest prime of the form $4\{7\}7$ is **47777**

[illegible]

* Case (6,1):

** Since 65, 21, 51, 631, 661 are primes, we only need to consider the family $6\{0,1,4,7\}1$ (since any digits 2, 3, 5, 6 between them will produce smaller primes)

*** Since 111, 141, 401, 471, 701, 711, 6101, 6441 are primes, we only need to consider the families $6\{0\}0\{0,1,4,7\}1$, $6\{0,4\}1\{7\}1$, $6\{0,7\}4\{1\}1$, $6\{0,1,7\}7\{4,7\}1$ (since any digits combo 11, 14, 40, 47, 70, 71, 10, 44 between them will produce smaller primes)

**** For the $6\{0\}0\{0,1,4,7\}1$ family, since 6007 is prime, we only need to consider the families $6\{0\}0\{0,1,4\}1$ and $60\{1,4,7\}7\{0,1,4,7\}1$ (since any digits combo 1007 between (6,1) will produce smaller primes)

***** For the $6\{0\}0\{0,1,4\}1$ family, since 111, 141, 401, 6101, 6441, 60411 are primes, we only need to consider the families $6\{0\}1$, $6\{0\}11$, $6\{0\}41$ (since any digits combo 10, 11, 14, 40, 41, 44 between $(6\{0\}0,1)$ will produce smaller primes)

***** All numbers of the form $6\{0\}1$ are divisible by 7, thus cannot be prime.

***** All numbers of the form $6\{0\}11$ are divisible by 3, thus cannot be prime.

***** All numbers of the form $6\{0\}41$ are divisible by 3, thus cannot be prime.

***** For the $60\{1,4,7\}7\{0,1,4,7\}1$ family, since 701, 711, 60741 are primes, we only need to consider the family $60\{1,4,7\}7\{7\}1$ (since any digits 0, 1, 4 between $(60\{1,4,7\}7,1)$ will produce smaller primes)

***** Since 471, 60171 is prime, we only need to consider the family $60\{7\}1$ (since any digits 1, 4 between $(60,7\{7\}1)$ will produce smaller primes)

***** All numbers of the form $60\{7\}1$ are divisible by 7, thus cannot be prime.

**** For the $6\{0,4\}1\{7\}1$ family, since 417, 471 are primes, we only need to consider the families $6\{0\}1\{7\}1$ and $6\{0,4\}11$

***** For the $6\{0\}1\{7\}1$ family, since 60171 is prime, and thus the only minimal prime in the family $6\{0\}1\{7\}1$.

***** For the $6\{0,4\}11$ family, since 401, 6441, 60411 are primes, we only need to consider the number 6411 and the family $6\{0\}11$

***** 6411 is not prime.

***** All numbers of the form $6\{0\}11$ are divisible by 3, thus cannot be prime.

**** For the $6\{0,7\}4\{1\}1$ family, since 60411 is prime, we only need to consider the families $6\{7\}4\{1\}1$ and $6\{0,7\}41$

***** For the $6\{7\}4\{1\}1$ family, since 111, 6777 are primes, we only need to consider the numbers 641, 6411, 6741, 67411, 67741, 677411

***** None of 641, 6411, 6741, 67411, 67741, 677411 are primes.

***** For the $6\{0,7\}41$ family, since 701, 6777, 60741 are primes, we only need to consider the families $6\{0\}41$ and the numbers 6741, 67741 (since any digits combo 07, 70, 777 between (6,41) will produce smaller primes)

***** All numbers of the form $6\{0\}41$ are divisible by 3, thus cannot be prime.

***** Neither of 6741, 67741 are primes.

***** For the $6\{0,1,7\}7\{4,7\}1$ family, since 747 is prime, we only need to consider the families $6\{0,1,7\}7\{4\}1$, $6\{0,1,7\}7\{7\}1$, $6\{0,1,7\}7\{7\}\{4\}1$ (since any digits combo 47 between $(6\{0,1,7\}7,1)$ will produce smaller primes)

***** For the $6\{0,1,7\}7\{4\}1$ family, since 6441 is prime, we only need to consider the families $6\{0,1,7\}71$ and $6\{0,1,7\}741$ (since any digits combo 44 between $(6\{0,1,7\}7,1)$ will produce smaller primes)

***** For the $6\{0,1,7\}71$ family, since all numbers of the form $6\{0,7\}71$ are divisible by 7 and cannot be prime, and 111 is prime (thus, any digits combo 11 between (6,71) will produce smaller primes), we only need to consider the family $6\{0,7\}1\{0,7\}71$

***** Since 717 and 60171 are primes, we only need to consider the family $61\{0,7\}71$ (since any digit combo 0, 7 between $(6,1\{0,7\}71)$ will produce smaller primes)

***** Since 177 and 6101 are primes, we only need to consider the number 6171 (since any digit combo 0, 7 between $(61,71)$ will produce smaller primes)

***** 6171 is not prime.

***** All numbers in the $6\{0,1,7\}7\{7\}1$ or $6\{0,1,7\}7\{7\}\{4\}1$ families are also in the $6\{0,1,7\}7\{4\}1$ family, thus these two families cannot have more minimal primes.

* Case (6,3):

** Since 65, 13, 23, 53, 73, **643** are primes, we only need to consider the family $6\{0,3,6\}3$ (since any digits 1, 2, 4, 5, 7 between them will produce smaller primes)

*** All numbers of the form $6\{0,3,6\}3$ are divisible by 3, thus cannot be prime.

* Case (6,5):

** **65** is prime, and thus the only minimal prime in this family.

* Case (6,7):

** Since 65, 27, 37, 57, **667** are primes, we only need to consider the family $6\{0,1,4,7\}7$ (since any digits 2, 3, 5, 6 between them will produce smaller primes)

*** Since 107, 117, 147, 177, 407, 417, 717, 747, **6007**, **6477**, **6707**, **6777** are primes, we only need to consider the families $60\{1,4,7\}7$, $6\{0\}17$, $6\{0,4\}4\{4\}7$, $6\{0\}77$ (since any digits combo 00, 10, 11, 14, 17, 40, 41, 47, 70, 71, 74, 77 between them will produce smaller primes)

**** All numbers of the form $6\{0\}17$ or $6\{0\}77$ are divisible by 3, thus cannot be prime.


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***** The smallest prime of the form 7{7}4444441 is 77774444441
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***** Since this prime has just 4 7's, we only need to consider the families with ≤ 3 7's

***** The smallest prime of the form $7\{4\}1$ is **744444441**

***** All numbers of the form $77\{4\}1$ are divisible by 5, thus cannot be prime.

***** The smallest prime of the form $777\{4\}1$ is 77744444444441 (not minimal prime, since 444444441 and 744444441 are primes)

* Case (7,3):

** **73** is prime, and thus the only minimal prime in this family.

* Case (7,5):

** **75** is prime, and thus the only minimal prime in this family.

* Case (7,7):

** Since 73, 75, 27, 37, 57, **717**, **747**, **767** are primes, we only need to consider the family $7\{0,7\}7$ (since any digits 1, 2, 3, 4, 5, 6 between them will produce smaller primes)

*** All numbers of the form $7\{0,7\}7$ are divisible by 7, thus cannot be prime.

base 10

The possible (first digit,last digit) for an element with ≥ 3 digits in the minimal set of the strings for primes with at least two digits are:

(1,1), (1,3), (1,7), (1,9), (2,1), (2,3), (2,7), (2,9), (3,1), (3,3), (3,7), (3,9), (4,1), (4,3), (4,7), (4,9), (5,1), (5,3), (5,7), (5,9), (6,1), (6,3), (6,7), (6,9), (7,1), (7,3), (7,7), (7,9), (8,1), (8,3), (8,7), (8,9), (9,1), (9,3), (9,7), (9,9)

* Case (1,1):

** **11** is prime, and thus the only minimal prime in this family.

* Case (1,3):

** **13** is prime, and thus the only minimal prime in this family.

* Case (1,7):

** **17** is prime, and thus the only minimal prime in this family.

* Case (1,9):

** **19** is prime, and thus the only minimal prime in this family.

* Case (2,1):

** Since 23, 29, 11, 31, 41, 61, 71, **251**, **281** are primes, we only need to consider the family $2\{0,2\}1$ (since any digits 1, 3, 4, 5, 6, 7, 8, 9 between them will produce smaller primes)

*** Since **2221** and **20201** are primes, we only need to consider the families $2\{0\}1$, $2\{0\}21$, $22\{0\}1$ (since any digits combo 22 or 020 between them will produce smaller primes)

**** All numbers of the form $2\{0\}1$ are divisible by 3, thus cannot be prime.

**** The smallest prime of the form $2\{0\}21$ is **20021**

**** The smallest prime of the form $22\{0\}1$ is **22000001**

* Case (2,3):

** **23** is prime, and thus the only minimal prime in this family.

* Case (2,7):

** Since 23, 29, 17, 37, 47, 67, 97 **227**, **257**, **277** are primes, we only need to consider the family $2\{0,8\}7$ (since any digits 1, 2, 3, 4, 5, 6, 7, 9 between them will produce smaller primes)

*** Since 887 and **2087** are primes, we only need to consider the families $2\{0\}7$ and $28\{0\}7$ (since any digit combo 08 or 88 between them will produce smaller primes)

**** All numbers of the form $2\{0\}7$ are divisible by 3, thus cannot be prime.

**** All numbers of the form $28\{0\}7$ are divisible by 7, thus cannot be prime.

* Case (2,9):

** **29** is prime, and thus the only minimal prime in this family.

* Case (3,1):

** **31** is prime, and thus the only minimal prime in this family.

* Case (3,3):

** Since 31, 37, 13, 23, 43, 53, 73, 83 are primes, we only need to consider the family $3\{0,3,6,9\}3$ (since any digits 1, 2, 4, 5, 7, 8 between them will produce smaller primes)

*** All numbers of the form $3\{0,3,6,9\}3$ are divisible by 3, thus cannot be prime.

* Case (3,7):

** **37** is prime, and thus the only minimal prime in this family.

* Case (3,9):

** Since 31, 37, 19, 29, 59, 79, 89, **349** are primes, we only need to consider the family $3\{0,3,6,9\}9$ (since any digits 1, 2, 4, 5, 7, 8 between them will produce smaller primes)

*** All numbers of the form $3\{0,3,6,9\}9$ are divisible by 3, thus cannot be prime.

* Case (4,1):

** **41** is prime, and thus the only minimal prime in this family.

* Case (4,3):

** **43** is prime, and thus the only minimal prime in this family.

* Case (4,7):

** **47** is prime, and thus the only minimal prime in this family.

* Case (4,9):

** Since 41, 43, 47, 19, 29, 59, 79, 89, **409**, **449**, **499** are primes, we only need to consider the family $4\{6\}9$ (since any digits 0, 1, 2, 3, 4, 5, 7, 8, 9 between them will produce smaller primes)

*** All numbers of the form $4\{6\}9$ are divisible by 7, thus cannot be prime.

* Case (5,1):

** Since 53, 59, 11, 31, 41, 61, 71, **521** are primes, we only need to consider the family $5\{0,5,8\}1$ (since any digits 1, 2, 3, 4, 6, 7, 9 between them will produce smaller primes)

*** Since 881 is prime, we only need to consider the families $5\{0,5\}1$ and $5\{0,5\}8\{0,5\}1$ (since any digit combo 88 between them will produce smaller primes)

**** For the $5\{0,5\}1$ family, since **5051** and **5501** are primes, we only need to consider the families $5\{0\}1$ and $5\{5\}1$ (since any digit combo 05 or 50 between them will produce smaller primes)

***** All numbers of the form $5\{0\}1$ are divisible by 3, thus cannot be prime.

***** The smallest prime of the form $5\{5\}1$ is **55555555551**

**** For the $5\{0,5\}8\{0,5\}1$ family, since **5081**, **5581**, **5801**, **5851** are primes, we only need to consider the number 581

***** 581 is not prime.

* Case (5,3):

** **53** is prime, and thus the only minimal prime in this family.

* Case (5,7):

** Since 53, 59, 17, 37, 47, 67, 97, **557**, **577**, **587** are primes, we only need to consider the family $5\{0,2\}7$ (since any digits 1, 3, 4, 5, 6, 7, 8, 9 between them will produce smaller primes)

*** Since 227 and **50207** are primes, we only need to consider the families $5\{0\}7$, $5\{0\}27$, $52\{0\}7$ (since any digits combo 22 or 020 between them will produce smaller primes)

**** All numbers of the form $5\{0\}7$ are divisible by 3, thus cannot be prime.

**** The smallest prime of the form $5\{0\}27$ is **50000000000000000000000000027**

**** The smallest prime of the form $52\{0\}7$ is **5200007**

* Case (5,9):

** **59** is prime, and thus the only minimal prime in this family.

* Case (6,1):

** **61** is prime, and thus the only minimal prime in this family.

* Case (6,3):

** Since 61, 67, 13, 23, 43, 53, 73, 83 are primes, we only need to consider the family $6\{0,3,6,9\}3$ (since any digits 1, 2, 4, 5, 7, 8 between them will produce smaller primes)

*** All numbers of the form $6\{0,3,6,9\}3$ are divisible by 3, thus cannot be prime.

* Case (6,7):

** **67** is prime, and thus the only minimal prime in this family.

* Case (6,9):

** Since 61, 67, 19, 29, 59, 79, 89 are primes, we only need to consider the family $6\{0,3,4,6,9\}9$ (since any digits 1, 2, 5, 7, 8 between them will produce smaller primes)

*** Since 449 is prime, we only need to consider the families $6\{0,3,6,9\}9$ and $6\{0,3,6,9\}4\{0,3,6,9\}9$ (since any digit combo 44 between them will produce smaller primes)

**** All numbers of the form $6\{0,3,6,9\}9$ are divisible by 3, thus cannot be prime.

**** For the $6\{0,3,6,9\}4\{0,3,6,9\}9$ family, since 409, 43, **6469**, 499 are primes, we only need to consider the family $6\{0,3,6,9\}49$

***** Since 349, **6949** are primes, we only need to consider the family $6\{0,6\}49$

***** Since **60649** is prime, we only need to consider the family $6\{6\}\{0\}49$ (since any digits combo 06 between $\{6,49\}$ will produce smaller primes)

***** The smallest prime of the form $6\{6\}49$ is **666649**

***** Since this prime has just 4 6's, we only need to consider the families with ≤ 3 6's

***** The smallest prime of the form $6\{0\}49$ is **60000049**

***** The smallest prime of the form $66\{0\}49$ is **66000049**

***** The smallest prime of the form $666\{0\}49$ is **66600049**

* Case (7,1):

** **71** is prime, and thus the only minimal prime in this family.

* Case (7,3):

** **73** is prime, and thus the only minimal prime in this family.

* Case (7,7):

** Since 71, 73, 79, 17, 37, 47, 67, 97, **727**, **757**, **787** are primes, we only need to consider the family $7\{0,7\}7$ (since any digits 1, 2, 3, 4, 5, 6, 8, 9 between them will produce smaller primes)

*** All numbers of the form $7\{0,7\}7$ are divisible by 7, thus cannot be prime.

* Case (7,9):

** **79** is prime, and thus the only minimal prime in this family.

* Case (8,1):

** Since 83, 89, 11, 31, 41, 61, 71, **821**, **881** are primes, we only need to consider the family $8\{0,5\}1$ (since any digits 1, 2, 3, 4, 6, 7, 8, 9 between them will produce smaller primes)

*** Since **8501** is prime, we only need to consider the family $8\{0\}\{5\}1$ (since any digits combo 50 between them will produce smaller primes)

**** Since **80051** is prime, we only need to consider the families $8\{0\}1$, $8\{5\}1$, $80\{5\}1$ (since any digits combo 005 between them will produce smaller primes)

***** All numbers of the form $8\{0\}1$ are divisible by 3, thus cannot be prime.

***** The smallest prime of the form $8\{5\}1$ is 85555555555555555551 (not minimal prime, since 555555555551 is prime)

***** The smallest prime of the form $80\{5\}1$ is **80555551**

* Case (8,3):

** **83** is prime, and thus the only minimal prime in this family.

* Case (8,7):

** Since 83, 89, 17, 37, 47, 67, 97, **827**, **857**, **877**, **887** are primes, we only need to consider the family $8\{0\}7$ (since any digits 1, 2, 3, 4, 5, 6, 7, 8, 9 between them will produce smaller primes)

*** All numbers of the form $8\{0\}7$ are divisible by 3, thus cannot be prime.

* Case (8,9):

** **89** is prime, and thus the only minimal prime in this family.

* Case (9,1):

** Since 97, 11, 31, 41, 61, 71, **991** are primes, we only need to consider the family $9\{0,2,5,8\}1$ (since any digits 1, 3, 4, 6, 7, 9 between them will produce smaller primes)

*** Since 251, 281, 521, 821, 881, **9001**, **9221**, **9551**, **9851** are primes, we only need to consider the families $9\{2,5,8\}0\{2,5,8\}1$, $9\{0\}2\{0\}1$, $9\{0\}5\{0,8\}1$, $9\{0,5\}8\{0\}1$ (since any digits combo 00, 22, 25, 28, 52, 55, 82, 85, 88 between them will produce smaller primes)

**** For the $9\{2,5,8\}0\{2,5,8\}1$ family, since any digits combo 22, 25, 28, 52, 55, 82, 85, 88 between (9,1) will produce smaller primes, we only need to consider the numbers 901, 9021, 9051, 9081, 9201, 9501, 9801, 90581, 95081, 95801

***** 95801 is the only prime among 901, 9021, 9051, 9081, 9201, 9501, 9801, 90581, 95081, 95801, but it is not minimal prime since 5801 is prime.

**** For the $9\{0\}2\{0\}1$ family, since 9001 is prime, we only need to consider the numbers 921, 9201, 9021

***** None of 921, 9201, 9021 are primes.

**** For the $9\{0\}5\{0,8\}1$ family, since 9001 and 881 are primes, we only need to consider the numbers 951, 9051, 9501, 9581, 90581, 95081, 95801

***** 95801 is the only prime among 951, 9051, 9501, 9581, 90581, 95081, 95801, but it is not minimal prime since 5801 is prime.

**** For the $9\{0,5\}8\{0\}1$ family, since 9001 and 5581 are primes, we only need to consider the numbers 981, 9081, 9581, 9801, 90581, 95081, 95801

***** 95801 is the only prime among 981, 9081, 9581, 9801, 90581, 95081, 95801, but it is not minimal prime since 5801 is prime.

* Case (9,3):

** Since 97, 13, 23, 43, 53, 73, 83 are primes, we only need to consider the family $9\{0,3,6,9\}3$ (since any digits 1, 2, 4, 5, 7, 8 between them will produce smaller primes)

*** All numbers of the form $9\{0,3,6,9\}3$ are divisible by 3, thus cannot be prime.

* Case (9,7):

** **97** is prime, and thus the only minimal prime in this family.

* Case (9,9):

** Since 97, 19, 29, 59, 79, 89 are primes, we only need to consider the family $9\{0,3,4,6,9\}9$ (since any digits 1, 2, 5, 7, 8 between them will produce smaller primes)

*** Since 449 is prime, we only need to consider the families $9\{0,3,6,9\}9$ and $9\{0,3,6,9\}4\{0,3,6,9\}9$ (since any digit combo 44 between them will produce smaller primes)

**** All numbers of the form $9\{0,3,6,9\}9$ are divisible by 3, thus cannot be prime.

**** For the $9\{0,3,6,9\}4\{0,3,6,9\}9$ family, since **9049**, 349, **9649**, **9949** are primes, we only need to consider the family $94\{0,3,6,9\}9$

***** Since 409, 43, 499 are primes, we only need to consider the family $94\{6\}9$ (since any digits 0, 3, 9 between (94,9) will produce smaller primes)

***** The smallest prime of the form $94\{6\}9$ is **946669**

base 12

The possible (first digit,last digit) for an element with ≥ 3 digits in the minimal set of the strings for primes with at least two digits are:

(1,1), (1,5), (1,7), (1,B), (2,1), (2,5), (2,7), (2,B), (3,1), (3,5), (3,7), (3,B), (4,1), (4,5), (4,7), (4,B), (5,1), (5,5), (5,7), (5,B), (6,1), (6,5), (6,7), (6,B), (7,1), (7,5), (7,7), (7,B), (8,1), (8,5), (8,7), (8,B), (9,1), (9,5), (9,7), (9,B), (A,1), (A,5), (A,7), (A,B), (B,1), (B,5), (B,7), (B,B)

* Case (1,1):

** **11** is prime, and thus the only minimal prime in this family.

* Case (1,5):

** **15** is prime, and thus the only minimal prime in this family.

* Case (1,7):

** **17** is prime, and thus the only minimal prime in this family.

* Case (1,B):

** **1B** is prime, and thus the only minimal prime in this family.

* Case (2,1):

** Since 25, 27, 11, 31, 51, 61, 81, 91, **221**, **241**, **2A1**, **2B1** are primes, we only need to consider the family $2\{0\}1$ (since any digits 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B between them will produce smaller primes)

*** The smallest prime of the form $2\{0\}1$ is **2001**

* Case (2,5):

** **25** is prime, and thus the only minimal prime in this family.

* Case (2,7):

** **27** is prime, and thus the only minimal prime in this family.

* Case (2,B):

** Since 25, 27, 1B, 3B, 4B, 5B, 6B, 8B, AB, **2BB** are primes, we only need to consider the family $2\{0,2,9\}B$ (since any digits 1, 3, 4, 5, 6, 7, 8, A, B between them will produce smaller primes)

*** Since 90B, **200B**, **202B**, **222B**, **229B**, **292B**, **299B** are primes, we only need to consider the numbers 20B, 22B, 29B, 209B, 220B (since any digits combo 00, 02, 22, 29, 90, 92, 99 between them will produce smaller primes)

**** None of 20B, 22B, 29B, 209B, 220B are primes.

* Case (3,1):

** **31** is prime, and thus the only minimal prime in this family.

* Case (3,5):

** **35** is prime, and thus the only minimal prime in this family.

* Case (3,7):

** **37** is prime, and thus the only minimal prime in this family.

* Case (3,B):

** **3B** is prime, and thus the only minimal prime in this family.

* Case (4,1):

** Since 45, 4B, 11, 31, 51, 61, 81, 91, **401**, **421**, **471** are primes, we only need to consider the family $4\{4,A\}1$ (since any digit 0, 1, 2, 3, 5, 6, 7, 8, 9, B between them will produce smaller primes)

*** Since A41 and **4441** are primes, we only need to consider the families $4\{A\}1$ and $44\{A\}1$ (since any digit combo 44, A4 between them will produce smaller primes)

**** All numbers of the form $4\{A\}1$ are divisible by 5, thus cannot be prime.

**** The smallest prime of the form $44\{A\}1$ is **44AAA1**

* Case (4,5):

** **45** is prime, and thus the only minimal prime in this family.

* Case (4,7):

** Since 45, 4B, 17, 27, 37, 57, 67, 87, A7, B7, **447**, **497** are primes, we only need to consider the family $4\{0,7\}7$ (since any digit 1, 2, 3, 4, 5, 6, 8, 9, A, B between them will produce smaller primes)

*** Since **4707** and **4777** are primes, we only need to consider the families $4\{0\}7$ and $4\{0\}77$ (since any digit combo 70, 77 between them will produce smaller primes)

**** All numbers of the form $4\{0\}7$ are divisible by B, thus cannot be prime.

****** The smallest prime of the form $4\{0\}77$ is $\textbf{400000000000000000000000000000000000000077}$**

* Case (4,B):

**** 4B** is prime, and thus the only minimal prime in this family.

* Case (5,1):

**** 51** is prime, and thus the only minimal prime in this family.

* Case (5,5):

** Since 51, 57, 5B, 15, 25, 35, 45, 75, 85, 95, B5, **565** are primes, we only need to consider the family 5{0,5,A}5 (since any digits 1, 2, 3, 4, 6, 7, 8, 9, B between them will produce smaller primes)

*** All numbers of the form $5\{0,5,A\}5$ are divisible by 5, thus cannot be prime.

* Case (5,7):

**** 57** is prime, and thus the only minimal prime in this family.

* Case (5,B):

**** 5B** is prime, and thus the only minimal prime in this family.

* Case (6,1):

**** 61** is prime, and thus the only minimal prime in this family.

* Case (6,5):

** Since 61, 67, 6B, 15, 25, 35, 45, 75, 85, 95, B5, **655**, **665** are primes, we only need to consider the family 6{0,A}5 (since any digits 1, 2, 3, 4, 5, 6, 7, 8, 9, B between them will produce smaller primes)

*** Since **6A05** and **6AA5** are primes, we only need to consider the families 6{0}5 and 6{0}A5 (since any digit combo A0, AA between them will produce smaller primes)

**** All numbers of the form $6\{0\}5$ are divisible by B, thus cannot be prime.

**** The smallest prime of the form $60A5$ is **600A5**

* Case (6,7):

**** 67** is prime, and thus the only minimal prime in this family.

* Case (6,B):

**** 6B** is prime, and thus the only minimal prime in this family.

* Case (7,1):

**** Since 75, 11, 31, 51, 61, 81, 91, 701, 721, 771, 7A1** are primes, we only need to consider the family $7\{4,B\}1$ (since any digits 0, 1, 2, 3, 5, 6, 7, 8, 9, A between them will produce smaller primes)

***** Since 7BB, 7441 and 7B41** are primes, we only need to consider the numbers 741, 7B1, 74B1

****** None of 741, 7B1, 74B1** are primes.

* Case (7,5):

**** 75** is prime, and thus the only minimal prime in this family.

* Case (7,7):

**** Since 75, 17, 27, 37, 57, 67, 87, A7, B7, 747, 797** are primes, we only need to consider the family $7\{0,7\}7$ (since any digits 1, 2, 3, 4, 5, 6, 8, 9, A, B between them will produce smaller primes)

***** All numbers of the form $7\{0,7\}7$** are divisible by 7, thus cannot be prime.

* Case (7,B):

**** Since 75, 1B, 3B, 4B, 5B, 6B, 8B, AB, 70B, 77B, 7BB** are primes, we only need to consider the family $7\{2,9\}B$ (since any digits 0, 1, 3, 4, 5, 6, 7, 8, A, B between them will produce smaller primes)

***** Since 222B, 729B** is prime, we only need to consider the families $7\{9\}B$, $7\{9\}2B$, $7\{9\}22B$ (since any digits combo 222, 29 between them will produce smaller primes)

****** The smallest prime of the form $7\{9\}B$ is 7999B**

****** The smallest prime of the form $7\{9\}2B$ is 79992B** (not minimal prime, since 992B and 7999B are primes)

****** The smallest prime of the form $7\{9\}22B$ is 79922B** (not minimal prime, since 992B is prime)

* Case (8,1):

**** 81** is prime, and thus the only minimal prime in this family.

* Case (8,5):

**** 85** is prime, and thus the only minimal prime in this family.

* Case (8,7):

**** 87** is prime, and thus the only minimal prime in this family.

* Case (8,B):

**** 8B** is prime, and thus the only minimal prime in this family.

* Case (9,1):

** **91** is prime, and thus the only minimal prime in this family.

* Case (9,5):

** **95** is prime, and thus the only minimal prime in this family.

* Case (9,7):

** Since 91, 95, 17, 27, 37, 57, 67, 87, A7, B7, **907** are primes, we only need to consider the family $9\{4,7,9\}7$ (since any digit 0, 1, 2, 3, 5, 6, 8, A, B between them will produce smaller primes)

*** Since 447, 497, 747, 797, **9777**, **9947**, **9997** are primes, we only need to consider the numbers 947, 977, 997, 9477, 9977 (since any digits combo 44, 49, 74, 77, 79, 94, 99 between them will produce smaller primes)

**** None of 947, 977, 997, 9477, 9977 are primes.

* Case (9,B):

** Since 91, 95, 1B, 3B, 4B, 5B, 6B, 8B, AB, **90B**, **9BB** are primes, we only need to consider the family $9\{2,7,9\}B$ (since any digit 0, 1, 3, 4, 5, 6, 8, A, B between them will produce smaller primes)

*** Since 27, 77B, **929B**, **992B**, **997B** are primes, we only need to consider the families $9\{2,7\}2\{2\}B$, $97\{2,9\}B$, $9\{7,9\}9\{9\}B$ (since any digits combo 27, 29, 77, 92, 97 between them will produce smaller primes)

**** For the $9\{2,7\}2\{2\}B$ family, since 27 and 77B are primes, we only need to consider the families $9\{2\}2\{2\}B$ and $97\{2\}2\{2\}B$ (since any digits combo 27, 77 between $(9,2\{2\}B)$ will produce smaller primes)

***** The smallest prime of the form $9\{2\}2\{2\}B$ is 9222B (not minimal prime, since 222B is prime)

***** The smallest prime of the form $97\{2\}2\{2\}B$ is 972222222222B (not minimal prime, since 222B is prime)

**** For the $97\{2,9\}B$ family, since 729B and 929B are primes, we only need to consider the family $97\{9\}2\{2\}B$ (since any digits combo 29 between $(97,B)$ will produce smaller primes)

***** Since 222B is prime, we only need to consider the families $97\{9\}B$, $97\{9\}2B$, $97\{9\}22B$ (since any digit combo 222 between $(97,B)$ will produce smaller primes)

***** All numbers of the form $97\{9\}B$ are divisible by 11, thus cannot be prime.

***** The smallest prime of the form $97\{9\}2B$ is 979999992B (not minimal prime, since 9999B is prime)

***** All numbers of the form $97\{9\}22B$ are divisible by 11, thus cannot be prime.

**** For the $9\{7,9\}9\{9\}B$ family, since 77B and 9999B are primes, we only need to consider the numbers 99B, 999B, 979B, 9799B, 9979B

***** None of 99B, 999B, 979B, 9799B, 9979B are primes.

* Case (A,1):

** Since A7, AB, 11, 31, 51, 61, 81, 91, **A41** are primes, we only need to consider the family $A\{0,2,A\}1$ (since any digits 1, 3, 4, 5, 6, 7, 8, 9, B between them will produce smaller primes)

*** Since 221, 2A1, **A0A1**, **A201** are primes, we only need to consider the families $A\{A\}\{0\}1$ and $A\{A\}\{0\}21$ (since any digits combo 0A, 20, 22, 2A between them will produce smaller primes)

**** For the $A\{A\}\{0\}1$ family:

***** All numbers of the form $A\{0\}1$ are divisible by B, thus cannot be prime.

***** The smallest prime of the form $AA\{0\}1$ is **AA000001**

***** The smallest prime of the form $AAA\{0\}1$ is **AAA0001**

***** The smallest prime of the form $AAAA\{0\}1$ is **AAAA1**

***** Since this prime has no 0's, we do not need to consider the families $\{A\}1$, $\{A\}01$, $\{A\}001$, etc.

**** All numbers of the form $A\{A\}\{0\}21$ are divisible by 5, thus cannot be prime.

* Case (A,5):

** Since A7, AB, 15, 25, 35, 45, 75, 85, 95, B5 are primes, we only need to consider the family $A\{0,5,6,A\}5$ (since any digits 1, 2, 3, 4, 7, 8, 9, B between them will produce smaller primes)

*** Since 565, 655, 665, **A605**, **A6A5**, **AA65** are primes, we only need to consider the families $A\{0,5,A\}5$ and $A\{0\}65$ (since any digits combo 56, 60, 65, 66, 6A, A6 between them will produce smaller primes)

**** All numbers of the form $A\{0,5,A\}5$ are divisible by 5, thus cannot be prime.

**** The smallest prime of the form $A\{0\}65$ is **A00065**

* Case (A,7):

** **A7** is prime, and thus the only minimal prime in this family.

* Case (A,B):

** **AB** is prime, and thus the only minimal prime in this family.

* Case (B,1):

** Since B5, B7, 11, 31, 51, 61, 81, 91, **B21** are primes, we only need to consider the family $B\{0,4,A,B\}1$ (since any digits 1, 2, 3, 5, 6, 7, 8, 9 between them will produce smaller primes)

*** Since 4B, AB, 401, A41, **B001**, **B0B1**, **BB01**, **BB41** are primes, we only need to consider the families $B\{A\}0\{4,A\}1$, $B\{0,4\}4\{4,A\}1$, $B\{0,4,A,B\}A\{0,A\}1$, $B\{B\}B\{A,B\}1$ (since any digits combo 00, 0B, 40, 4B, A4, AB, B0, B4 between them will produce smaller primes)

**** For the $B\{A\}0\{4,A\}1$ family, since A41 is prime, we only need consider the families $B0\{4\}\{A\}1$ and $B\{A\}0\{A\}1$

***** For the $B0\{4\}\{A\}1$ family, since **B04A1** is prime, we only need to consider the families $B0\{4\}1$ and $B0\{A\}1$

***** The smallest prime of the form $B0\{4\}1$ is B04441 (not minimal prime, since 4441 is prime)

***** The smallest prime of the form $B0\{A\}1$ is B0AAAAA1 (not minimal prime, since AAAA1 is prime)

***** For the $B\{A\}0\{A\}1$ family, since A0A1 is prime, we only need to consider the families $B\{A\}01$ and $B0\{A\}1$

***** The smallest prime of the form $B\{A\}01$ is **BAA01**

***** The smallest prime of the form $B0\{A\}1$ is B0AAAAA1 (not minimal prime, since AAAA1 is prime)

**** For the $B\{0,4\}4\{4,A\}1$ family, since 4441 is prime, we only need to consider the families $B\{0\}4\{4,A\}1$ and $B\{0,4\}4\{A\}1$

**** For the $B\{0\}4\{4,A\}1$ family, since B001 is prime, we only need to consider the families $B4\{4,A\}1$ and $B04\{4,A\}1$

***** For the $B4\{4,A\}1$ family, since A41 is prime, we only need to consider the family $B4\{4\}\{A\}1$

***** Since 4441 and BAAA1 are primes, we only need to consider the numbers B41, B441, B4A1, B44A1, B4AA1, B44AA1

***** None of B41, B441, B4A1, B44A1, B4AA1, B44AA1 are primes.

***** For the $B04\{4,A\}1$ family, since **B04A1** is prime, we only need to consider the family $B04\{4\}1$

***** The smallest prime of the form $B04\{4\}1$ is B04441 (not minimal prime, since 4441 is prime)

**** For the $B\{0,4\}4\{A\}1$ family, since 401, 4441, B001 are primes, we only need to consider the families $B4\{A\}1$, $B04\{A\}1$, $B44\{A\}1$, $B044\{A\}1$ (since any digits combo 00, 40, 44 between $(B,4\{A\}1)$ will produce smaller primes)

***** The smallest prime of the form $B4\{A\}1$ is B4AAA1 (not minimal prime, since BAAA1 is prime)

***** The smallest prime of the form $B04\{A\}1$ is **B04A1**

***** The smallest prime of the form $B44\{A\}1$ is B44AAAAAA1 (not minimal prime, since BAAA1 is prime)

***** The smallest prime of the form $B044\{A\}1$ is B044A1 (not minimal prime, since B04A1 is prime)

**** For the $B\{0,4,A,B\}A\{0,A\}1$ family, since all numbers in this family with 0 between (B,1) are in the $B\{A\}0\{4,A\}1$ family, and all numbers in this family with 4 between (B,1) are in the $B\{0,4\}4\{4,A\}1$ family, we only need to consider the family $B\{A,B\}A\{A\}1$

***** Since **BAAA1** is prime, we only need to consider the families $B\{A,B\}A1$ and $B\{A,B\}AA1$

***** For the $B\{A,B\}A1$ family, since AB and **BAAA1** are primes, we only need to consider the families $B\{B\}A1$ and $B\{B\}AA1$

***** All numbers of the form $B\{B\}A1$ are divisible by B, thus cannot be prime.

***** The smallest prime of the form $B\{B\}AA1$ is **BBBAA1**

***** For the $B\{A,B\}AA1$ family, since **BAAA1** is prime, we only need to consider the families $B\{B\}AA1$

***** The smallest prime of the form $B\{B\}AA1$ is **BBBAA1**

**** For the $B\{B\}B\{A,B\}1$ family, since AB and BAAA1 are primes, we only need to consider the families $B\{B\}B1$, $B\{B\}BA1$, $B\{B\}BAA1$ (since any digits combo AB or AAA between ($B\{B\}B,1$) will produce smaller primes)

**** The smallest prime of the form $B\{B\}B1$ is **BBBB1**

**** All numbers of the form $B\{B\}BA1$ are divisible by B, thus cannot be prime.

**** The smallest prime of the form $B\{B\}BAA1$ is **BBBAA1**

* Case (B,5):

** **B5** is prime, and thus the only minimal prime in this family.

* Case (B,7):

** **B7** is prime, and thus the only minimal prime in this family.

* Case (B,B):

** Since B5, B7, 1B, 3B, 4B, 5B, 6B, 8B, AB, **B2B** are primes, we only need to consider the family $B\{0,9,B\}B$ (since any digits 1, 2, 3, 4, 5, 6, 7, 8, A between them will produce smaller primes)

*** Since 90B and 9BB are primes, we only need to consider the families $B\{0,B\}\{9\}B$

**** Since 9999B is prime, we only need to consider the families $B\{0,B\}B$, $B\{0,B\}9B$, $B\{0,B\}99B$, $B\{0,B\}999B$

***** All numbers of the form $B\{0,B\}B$ are divisible by B, thus cannot be prime.

***** For the $B\{0,B\}9B$ family:

***** Since **B0B9B** and **BB09B** are primes, we only need to consider the families $B\{0\}9B$ and $B\{B\}9B$ (since any digits combo 0B, B0 between (B,9B) will produce smaller primes)

BB
BB
BB
BB999B (not minimal prime, since
BBBBBB99B is prime)

References

- [1] <https://primes.utm.edu/glossary/page.php?sort=MinimalPrime> (article “minimal prime” in The Prime Glossary)
- [2] [https://en.wikipedia.org/wiki/Minimal_prime_\(recreational_mathematics\)](https://en.wikipedia.org/wiki/Minimal_prime_(recreational_mathematics)) (article “minimal prime” in Wikipedia)
- [3] <http://www.cs.uwaterloo.ca/~shallit/Papers/minimal5.pdf> (Shallit's proof of base 10 minimal primes, when the restriction of $\text{prime} > \text{base}$ is not required)
- [4] <https://scholar.colorado.edu/downloads/hh63sw661> (proofs of minimal primes in bases up to 10, when the restriction of $\text{prime} > \text{base}$ is not required)
- [5] <http://www.noprimeleftbehind.net/crus/Sierp-conjectures.htm> (generalized Sierpinski conjectures in bases up to 1030, some primes found in these conjectures are minimal primes in the corresponding base, especially, all primes for $k < \text{base}$ (if exist for a (k, base) combo) are always minimal primes in the corresponding base) (also some examples for simple families contain no primes for numbers $> \text{base}$)
- [6] <http://www.noprimeleftbehind.net/crus/Riesel-conjectures.htm> (generalized Riesel conjectures in bases up to 1030, some primes found in these conjectures are minimal primes in the corresponding base, especially, all primes for $k < \text{base}$ (if exist for a (k, base) combo) are always minimal primes in the corresponding base) (also some examples for simple families contain no primes for numbers $> \text{base}$)
- [7] <https://raw.githubusercontent.com/xayahrainie4793/Sierpinski-Riesel-for-fixed-k-and-variable-base/master/Riesel%20k1.txt> (length of the smallest generalized repunit primes in bases up to 1024, these primes (if exist for a base) are always minimal primes in the corresponding base)
- [8] http://www.rieselprime.de/dl/CRUS_pack.zip (*srsieve*, *sr1sieve*, *sr2sieve*, *pfgw*, and *llr* softwares)
- [9] https://primes.utm.edu/curios/page.php?number_id=22380 (the largest base 10 minimal prime in Prime Curios!)
- [10] <https://oeis.org/A071062> (OEIS entry for base 10 minimal primes, when the restriction of $\text{prime} > \text{base}$ is not required)
- [11] <https://harvey563.tripod.com/wills.txt> (primes of the form $(b-1) \cdot b^n - 1$ for bases b up to 2049, the smallest such prime for base b (if exists) is always minimal prime in base b)
- [12] <http://jeppesn.dk/generalized-fermat.html> (generalized Fermat primes in even bases b up to 1000, the smallest such prime for base b (if exists) is always minimal prime in base b)

[13] <http://www.fermatquotient.com/PrimSerien/GenRepu.txt> (generalized repunit primes in bases b up to 160, the smallest such prime for base b (if exists) is always minimal prime in base b)

[14] <https://primes.utm.edu/primes/lists/all.txt> (top proven primes)

[15] <http://www.primenumbers.net/prptop/prptop.php> (top PRPs)

[16] <http://factordb.com> (online factor database, including many primes which are minimal primes in a small base)