$$a^2 + b^2 = \{1, 1\}^2 - 2ab$$

$$\frac{a^3 + b^3}{a + b} = \{1, 1\}^2 - 3ab$$

$$\frac{a^5 - b^5}{a - b} = \{1, 3, 1\}^2 - 5ab \{1, 1\}^2$$

$$\frac{a^6 + b^6}{a^2 + b^2} = \{1, 3, 1\}^2 - 6ab \{1, 1\}^2$$

$$\frac{a^7 + b^7}{a + b} = \{1, 3, 3, 1\}^2 - 7ab \{1, 1, 1\}^2$$

$$\frac{a^{10} + b^{10}}{a^2 + b^2} = \{1, 5, 7, 5, 1\}^2 - 10ab\{1, 2, 2, 1\}^2$$

$$\frac{a^{11} + b^{11}}{a + b} = \{1, 5, -1, -1, 5, 1\}^2 - 11ab \{1, 1, -1, 1, 1\}^2$$

$$\frac{a^{13} - b^{13}}{a - b} = \{1, 7, 15, 19, 15, 7, 1\}^2 - 13ab \{1, 3, 5, 5, 3, 1\}^2$$

$$\frac{a^{14} + b^{14}}{a^2 + b^2} = \{1, 7, 3, -7, 3, 7, 1\}^2 - 14ab \{1, 2, -1, -1, 2, 1\}^2$$

$$\frac{a^{15} + b^{15}}{(a^5 + b^5)(a^3 + b^3)(a + b)^{-1}} = \{1, 8, 13, 8, 1\}^2 - 15ab\{1, 3, 3, 1\}^2$$

$$\frac{a^{17} - b^{17}}{a - b} = \{1, 9, 11, -5, -15, -5, 11, 9, 1\}^2 - 17ab \{1, 3, 1, -3, -3, 1, 3, 1\}^2$$

$$\frac{a^{19} + b^{19}}{a + b} = \{1, 9, 17, 27, 31, 31, 27, 17, 9, 1\}^2 - 19ab \{1, 3, 5, 7, 7, 7, 5, 3, 1\}^2$$

$$\frac{a^{21}-b^{21}}{(a^7-b^7)(a^3-b^3)(a-b)^{-1}} = \{1, 10, 13, 7, 13, 10, 1\}^2 - 21ab\{1, 3, 2, 2, 3, 1\}^2$$

$$\frac{a^{22} + b^{22}}{a^2 + b^2} = \{1, 11, 27, 33, 21, 11, 21, 33, 27, 11, 1\}^2 - 22ab \{1, 4, 7, 6, 3, 3, 6, 7, 4, 1\}^2$$

$$\frac{a^{23} + b^{23}}{a + b} = \{1, 11, 9, -19, -15, 25, 25, -15, -19, 9, 11, 1\}^2 - 23ab\{1, 3, -1, -5, 1, 7, 1, -5, -1, 3, 1\}^2$$

$$\frac{a^{26} + b^{26}}{a^2 + b^2} = \{1, 13, 19, -13, -11, 13, 7, 13, -11, -13, 19, 13, 1\}^2 - 26ab\{1, 4, 1, -4, 1, 2, 2, 1, -4, 1, 4, 1\}^2$$

$$\frac{a^{29} - b^{29}}{a - b} = \{1, 15, 33, 13, 15, 57, 45, 19, 45, 57, 15, 13, 33, 15, 1\}^2 - 29ab \{1, 5, 5, 1, 7, 11, 5, 5, 11, 7, 1, 5, 5, 1\}^2$$

$$\frac{a^{30}+b^{30}}{(a^{10}+b^{10})(a^6+b^6)(a^2+b^2)^{-1}}=\{1,\,15,\,38,\,45,\,43,\,45,\,38,\,15,\,1\}^2-30ab\,\{1,\,5,\,8,\,8,\,8,\,8,\,5,\,1\}^2$$

$$\frac{a^{31} + b^{31}}{a + b} = \{1, 15, 43, 83, 125, 151, 169, 173, 173, 169, 151, 125, 83, 43, 15, 1\}^2 - 31ab\{1, 5, 11, 19, 25, 29, 31, 31, 31, 29, 25, 19, 11, 5, 1\}^2$$

$$\frac{a^{33} - b^{33}}{(a^{11} - b^{11})(a^3 - b^3)(a - b)^{-1}} = \{1, 16, 37, 19, -32, -59, -32, 19, 37, 16, 1\}^2 - 33ab\{1, 5, 6, -1, -9, -9, -1, 6, 5, 1\}^2$$

$$\frac{a^{34} + b^{34}}{a^2 + b^2} = \{1, 17, 59, 119, 181, 221, 243, 255, 257, 255, 243, 221, 181, 119, 59, 17, 1\}^2 - 34ab \{1, 6, 15, 26, 35, 40, 43, 44, 44, 43, 40, 35, 26, 15, 6, 1\}^2$$

$$\frac{a^{35} + b^{35}}{(a^7 + b^7)(a^5 + b^5)(a + b)^{-1}} = \{1, 18, 48, 11, -55, -11, 47, -11, -55, 11, 48, 18, 1\}^2 - 35ab \{1, 6, 7, -5, -8, 5, 5, -8, -5, 7, 6, 1\}^2$$

$$\frac{a^{37} - b^{37}}{a - b} = \{1, 19, 79, 183, 285, 349, 397, 477, 579, 627, 579, 477, 397, 349, 285, 183, 79, 19, 1\}^2 - 37ab \{1, 7, 21, 39, 53, 61, 71, 87, 101, 101, 87, 71, 61, 53, 39, 21, 7, 1\}^2$$

$$\frac{a^{38} + b^{38}}{a^2 + b^2} = \{1, 19, 47, -19, -135, -57, 179, 209, -83, -285, -83, 209, 179, -57, -135, -19, 47, 19, 1\}^2 - 38ab \{1, 6, 5, -14, -21, 10, 39, 14, -37, -37, 14, 39, 10, -21, -14, 5, 6, 1\}^2$$

$$\frac{a^{39} + b^{39}}{(a^{13} + b^{13})(a^3 + b^3)(a + b)^{-1}} = \{1, 20, 73, 119, 142, 173, 193, 173, 142, 119, 73, 20, 1\}^2 - 39ab\{1, 7, 16, 21, 25, 30, 30, 25, 21, 16, 7, 1\}^2$$

$$\frac{a^{41} - b^{41}}{a - b} = \{1, 21, 67, 49, 7, 35, 15, 11, -23, -65, -31, -65, -23, 11, 15, 35, 7, 49, 67, 21, 1\}^2 - 41ab\{1, 7, 11, 3, 3, 5, 1, 1, -9, -7, -7, -9, 1, 1, 5, 3, 3, 11, 7, 1\}^2$$

$$\frac{a^{42} + b^{42}}{(a^{14} + b^{14})(a^6 + b^6)(a^2 + b^2)^{-1}} = \{1, 21, 74, 105, 55, -42, -91, -42, 55, 105, 74, 21, 11\}^2 - 42ab\{1, 7, 15, 14, 1, -12, -12, 1, 14, 15, 7, 1\}^2$$

$$\frac{a^{43} + b^{43}}{a + b} = \{1, 21, 81, 169, 223, 225, 213, 223, 229, 197, 159, 159, 197, 229, 223, 213, 225, 223, 169, 81, 21, 1\}^2 - 43ab \{1, 7, 19, 31, 35, 33, 33, 35, 33, 27, 23, 27, 33, 35, 33, 35, 33, 35, 31, 19, 7, 1\}^2$$

$$\frac{a^{47} + b^{47}}{a + b} = \{1, 23, 65, -15, -169, -97, 179, 287, -37, -375, -149, 311, 311, -149, -375, -37, 287, 179, -97, -169, -15, 65, 23, 1\}^2 - 47ab\{1, 7, 7, -15, -25, 5, 41, 25, -37, -49, 15, 57, 15, -49, -37, 25, 41, 5, -25, -15, 7, 7, 1\}^2$$