

HYPERPERFECT NUMBERS WITH FIVE AND SIX DIFFERENT PRIME FACTORS

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ABSTRACT

A natural number N is *hyperperfect* if there exists an integer k such that $N - 1 = k[\sigma(N) - N]$, where $\sigma(N)$ is the sum of the positive divisors of N . The classical perfect numbers are hyperperfect numbers corresponding to $k = 1$. In this paper we exhibit several hyperperfect numbers with five different prime factors and the first known hyperperfect number with six different prime factors.

A natural number N is said to be *hyperperfect* if there exists an integer k such that $N - 1 = k[\sigma(N) - N]$, where $\sigma(N)$ is the sum of the positive divisors of N . The ordinary perfect numbers, for which $\sigma(N) = 2N$, correspond to the case where $k = 1$.

Hyperperfect numbers have been studied by Minoli [2], [3], [4], Bear [2], de Riele [6], [7], [8], McCranie, [1], and Nash [5]. Several examples have been found of hyperperfect numbers with two, three and four different prime factors and one such number with five different prime factors was discovered by de Riele [8].

In this paper we include some new hyperperfect numbers with five different prime factors and the first known example with six different prime factors as well. These numbers were found with the aid of Rules 1, 2 and 3 that appear in [8] and a new Rule found by the author. For convenience, we state these four rules below.

First, corresponding to the positive integer k , we define M_k as the set of all natural numbers N satisfying the equation $N - 1 = k[\sigma(N) - N]$ and M_k as the set of all hyperperfect numbers for that value of k . We also write \bar{a} for $\sigma(a)$.

Rule 1: If $a \in M_k$ and p is a prime $\equiv k\bar{a} + 1 \pmod{k}$, then $ap \in M_k$.

Rule 2: If $a \in M_k$ and p and q are distinct primes such that $(p - k\bar{a})(q - k\bar{a}) = 1 + k + k\bar{a} + k^2\bar{a}^2$, then $apq \in M_k$.

Rule 3: If $a \in M_k$ and p and q are distinct primes such that $(p - k\bar{a})(q - k\bar{a}) = 1 + k\bar{a} + k^2\bar{a}^2$, then $apq \in M_k$.

(New) Rule A: Corresponding to a natural number a , if p is prime and k is a positive integer such that $[(\bar{a} - a)p + \bar{a}][a - (\bar{a} - a)k] = \bar{a} - a + a\bar{a}$, then $ap \in M_k$.

The proofs of all four rules follow directly from the definitions of the sets M_k and M_k .

The first three of the examples that follow were obtained by starting with the product of two primes, using Rule A to obtain the product of three primes as a member of M_k and using Rule 2 to obtain the product of five primes as a member of M_k .

The next four examples were found by starting with a prime, using Rule A to obtain the product of two primes as a member of M_k , using Rule 3 to obtain the product of four primes as a member of M_k and using Rule 1 to obtain the product of five primes as a member of M_k .

The example consisting of the product of six primes was obtained by starting with a fixed value of k , using Rule 3 twice and then using Rule 2.

Hyperperfect numbers with five different prime factors:

$(k = 1248)$	1291 37501 476132479 28791173123859572047 520060488238717511603772559
$(k = 1950)$	3203 4987 34208591 1066077464194829831 1102348360488921030326118050798021
$(k = 2430)$	2689 25537 2157247 360118565294860859 2198057306271677000602725577428569
$(k = 10614)$	10957 339091 39439240306867 27734632534386560971 43139874781820825169656707227912245469451468171
$(k = 26772)$	36523 100279 98055842567377 492140464742929022592433 4731905104999413492854312609911804722484433193580909
$(k = 293400)$	295411 43099891 3735634901757104587 248172206527617130489282964323 3463235230118690455327796482112090145545311176157791- 276882091789801
$(k = 297330)$	298999 53266429 4735474581938100751 29859812937658890188853684723636751 6695979299326579123964088700700805499318701609602720- 59720831606502102671

Hyperperfect number M with six different prime factors:

$$(k = 22998384)$$

$$M = p \cdot q \cdot r \cdot s \cdot t \cdot u, \text{ where}$$

$$p = 22998427$$

$$q = 12300620431171$$

$$r = 6506126308645398457840655623$$

$$s = 13747866042237024565058771024703857840557127659936183$$

$$t = 58194276398238994797319319270186821750600718607846222519-$$

$$9595249210644189053083234331415971905873398089973847$$

$$u = 18962384608661284895373626306450232703958904749109475-$$

$$1670519427585107487959535355483837058005247913558036-$$

$$650457536917039612851105938350412346368896787464071102-$$

$$12975936960656330834440627247005979243282572792663$$

The number M has 418 digits.

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