# Tables of Record Gaps Between Prime Constellations

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#### Abstract

We present tables of record gaps between densest prime constellations, or k-tuplets. The tables contain all maximal gaps between prime k-tuplets up to  $10^{15}$ , for each  $k \le 7$ .

Introduction. Mathematicians have long known that there are infinitely many primes. We also know that the average gaps between primes near p are about  $\log p$  (this follows from the prime number theorem). However, this knowledge has not helped prove or disprove Cramér's conjecture [1] that a record prime gap near p is  $O(\log^2 p)$ . Computations strongly suggest that  $O(\log^2 p)$  is a realistic upper bound for the size of prime gaps; moreover, it even seems possible that the prime gap ending at p is always less than  $\log^2 p$  [8, 9, 12, 13, A005250]. Still, the  $O(\log^2 p)$  upper bound for prime gaps remains just a plausible conjecture.

For prime k-tuples, the situation is somewhat similar — yet even more uncertain. Zhang's theorem [16] implies that at least some types of prime k-tuples do occur infinitely often. The Hardy-Littlewood k-tuple conjecture [6, 10, p. 62–68] suggests that admissible prime k-tuples near x occur with average gaps  $a \approx C_k \log^k x$ ; the coefficients  $C_k$  are reciprocal to the Hardy-Littlewood k-tuple constants<sup>1</sup> [2, 5]. Consider admissible prime k-tuples with a given pattern, and denote by  $G_k(x)$  the largest gap between k-tuples below x. Then the following heuristic formula expresses the expected size of record gaps in terms of the average gap a, as  $x \to \infty$ :

$$G_k(x) \sim a \log \frac{x}{a} - ba \lesssim C_k \log^{k+1} x$$
, with  $a = C_k \log^k x$ ,  $b \approx \frac{2}{k}$  [7, p. 5–7].

In the absence of rigorous upper bounds for gaps between prime constellations, it is important to accumulate extensive numerical evidence on actual record gaps. To that end, these tables contain all maximal gaps between prime k-tuplets (the densest admissible clusters of k primes) up to  $10^{15}$ , for each  $k \leq 7$ . Tables for k = 1, 2, 4 have been previously published by several authors [3, 7, 8, 9, 11, 13]. For k = 1 the data extends up to  $4 \times 10^{18}$  [9]. Tables for  $3 \leq k \leq 7$  have been independently obtained as part of my own computation [7, 13]. So is it **always** true that  $G_k(x) < C_k \log^{k+1} x$ ? It seems likely; no counterexamples thus far!

The prime k-tuplets with  $k \le 7$ , the numerical values are  $C_2 \approx 0.757392$ ,  $C_3 \approx 0.349864$ ,  $C_4 \approx 0.240895$ ,  $C_5 \approx 0.0986992$ ,  $C_6 \approx 0.0578081$ ,  $C_7 \approx 0.0185281$ . For definitions and further discussion, see also [7].

# 1 Record gaps between primes

TABLE 1 Maximal gaps between primes below  $4\times10^{18}~[8,\,9,\,13,\,\text{OEIS A005250}]$ 

Conse	Consecutive primes		Cons	Gap $g$	
2	3	$\frac{\text{Gap } g}{1}$	25056082087	25056082543	456
3	5 5	$\frac{1}{2}$	42652618343	42652618807	464
7	11	$\frac{2}{4}$	127976334671	127976335139	468
23	29	6	182226896239	182226896713	474
89	97	8	241160624143	241160624629	486
113	127	14	297501075799	297501076289	490
523	541	18	303371455241	303371455741	500
887	907	20	304599508537	304599509051	514
1129	1151	$\frac{20}{22}$	416608695821	416608696337	514
1327	1361	$\frac{22}{34}$	461690510011	461690510543	532
9551	9587	36	614487453523	614487454057	532
15683	15727	44	738832927927	738832928467	540
19609	19661	52	1346294310749	1346294311331	582
31397	31469	$\frac{32}{72}$	1408695493609	1408695494197	588
155921	156007	86	1968188556461	1968188557063	602
360653	360749	96	2614941710599	2614941711251	652
370261	370373	112	7177162611713	7177162612387	674
492113	492227	112 $114$	13829048559701	13829048560417	716
1349533	1349651	114	19581334192423	19581334193189	716 766
		132	42842283925351	42842283926129	700 778
1357201 2010733	$   \begin{array}{c}     1357333 \\     2010881   \end{array} $	132	90874329411493	90874329412297	804
4652353		148 $154$	171231342420521	171231342421327	806
	4652507				906
17051707	17051887	180	218209405436543	218209405437449	
20831323	20831533	210	1189459969825483	1189459969826399	916
47326693	47326913	220	1686994940955803	1686994940956727	924
122164747	122164969	222	1693182318746371	1693182318747503	1132
189695659	189695893	234	43841547845541059	43841547845542243	1184
191912783	191913031	248	55350776431903243	55350776431904441	1198
387096133	387096383	250	80873624627234849	80873624627236069	1220
436273009	436273291	282	203986478517455989	203986478517457213	1224
1294268491	1294268779	288	218034721194214273	218034721194215521	1248
1453168141	1453168433	292	305405826521087869	305405826521089141	1272
2300942549	2300942869	320	352521223451364323	352521223451365651	1328
3842610773	3842611109	336	401429925999153707	401429925999155063	1356
4302407359	4302407713	354	418032645936712127	418032645936713497	1370
10726904659	10726905041	382	804212830686677669	804212830686679111	1442
20678048297	20678048681	384	1425172824437699411	1425172824437700887	1476
22367084959	22367085353	394			

# 2 Record gaps between twin primes

TABLE 2  $\mbox{Maximal gaps between twin primes } \{p,\,p+2\} \ \ [3,\,7,\,11,\,13,\,{\rm OEIS~A113274}]$ 

	0 1	-	62 / 2 9 [ / /		-
Initial primes	s in twin pairs	Gap $g_2$	Initial primes in twin pairs		Gap $g_2$
3	5	2	24857578817	24857585369	6552
5	11	6	40253418059	40253424707	6648
17	29	12	42441715487	42441722537	7050
41	59	18	43725662621	43725670601	7980
71	101	30	65095731749	65095739789	8040
311	347	36	134037421667	134037430661	8994
347	419	72	198311685749	198311695061	9312
659	809	150	223093059731	223093069049	9318
2381	2549	168	353503437239	353503447439	10200
5879	6089	210	484797803249	484797813587	10338
13397	13679	282	638432376191	638432386859	10668
18539	18911	372	784468515221	784468525931	10710
24419	24917	498	794623899269	794623910657	11388
62297	62927	630	1246446371789	1246446383771	11982
187907	188831	924	1344856591289	1344856603427	12138
687521	688451	930	1496875686461	1496875698749	12288
688451	689459	1008	2156652267611	2156652280241	12630
850349	851801	1452	2435613754109	2435613767159	13050
2868959	2870471	1512	4491437003327	4491437017589	14262
4869911	4871441	1530	13104143169251	13104143183687	14436
9923987	9925709	1722	14437327538267	14437327553219	14952
14656517	14658419	1902	18306891187511	18306891202907	15396
17382479	17384669	2190	18853633225211	18853633240931	15720
30752231	30754487	2256	23275487664899	23275487681261	16362
32822369	32825201	2832	23634280586867	23634280603289	16422
96894041	96896909	2868	38533601831027	38533601847617	16590
136283429	136286441	3012	43697538391391	43697538408287	16896
234966929	234970031	3102	56484333976919	56484333994001	17082
248641037	248644217	3180	74668675816277	74668675834661	18384
255949949	255953429	3480	116741875898981	116741875918727	19746
390817727	390821531	3804	136391104728629	136391104748621	19992
698542487	698547257	4770	221346439666109	221346439686641	20532
2466641069	2466646361	5292	353971046703347	353971046725277	21930
4289385521	4289391551	6030	450811253543219	450811253565767	22548
19181736269	19181742551	6282	742914612256169	742914612279527	23358
24215097497	24215103971	6474	1121784847637957	1121784847661339	23382

# 3 Record gaps between prime triplets

TABLE 3.1 Maximal gaps between prime triplets  $\{p,\,p+2,\,p+6\}$  [OEIS A201598]

				. , , ,	-
Initial prim	nes in tuples	Gap $g_3$	Initial primes in tuples		Gap $g_3$
5	11	6	242419361	242454281	34920
17	41	24	913183487	913222307	38820
41	101	60	1139296721	1139336111	39390
107	191	84	2146630637	2146672391	41754
347	461	114	2188525331	2188568351	43020
461	641	180	3207540881	3207585191	44310
881	1091	210	3577586921	3577639421	52500
1607	1871	264	7274246711	7274318057	71346
2267	2657	390	33115389407	33115467521	78114
2687	3251	564	97128744521	97128825371	80850
6197	6827	630	99216417017	99216500057	83040
6827	7877	1050	103205810327	103205893751	83424
39227	40427	1200	133645751381	133645853711	102330
46181	47711	1530	373845384527	373845494147	109620
56891	58907	2016	412647825677	412647937127	111450
83267	86111	2844	413307596957	413307728921	131964
167621	171047	3426	1368748574441	1368748707197	132756
375251	379007	3756	1862944563707	1862944700711	137004
381527	385391	3864	2368150202501	2368150349687	147186
549161	553097	3936	2370801522107	2370801671081	148974
741677	745751	4074	3710432509181	3710432675231	166050
805031	809141	4110	5235737405807	5235737580317	174510
931571	937661	6090	8615518909601	8615519100521	190920
2095361	2103611	8250	10423696470287	10423696665227	194940
2428451	2437691	9240	10660256412977	10660256613551	200574
4769111	4778381	9270	11602981439237	11602981647011	207774
4938287	4948631	10344	21824373608561	21824373830087	221526
12300641	12311147	10506	36385356561077	36385356802337	241260
12652457	12663191	10734	81232357111331	81232357386611	275280
13430171	13441091	10920	186584419495421	186584419772321	276900
14094797	14107727	12930	297164678680151	297164678975621	295470
18074027	18089231	15204	428204300934581	428204301233081	298500
29480651	29500841	20190	450907041535541	450907041850547	315006
107379731	107400017	20286	464151342563471	464151342898121	334650
138778301	138799517	21216	484860391301771	484860391645037	343266
156377861	156403607	25746	666901733009921	666901733361947	352026

TABLE 3.2 Maximal gaps between prime triplets  $\{p,\,p+4,\,p+6\}$  [OEIS A201596]

			-		=
Initial p	rimes in tuples	Gap $g_3$	Initial primes in tuples		Gap $g_3$
	7 13	6	2562574867	2562620653	45786
15	3	24	2985876133	2985923323	47190
3'	7 67	30	4760009587	4760057833	48246
103	3 193	90	5557217797	5557277653	59856
307	7 457	150	10481744677	10481806897	62220
457	7 613	156	19587414277	19587476563	62286
613	823	210	25302582667	25302648457	65790
2137	7 2377	240	30944120407	30944191387	70980
237	7 2683	306	37638900283	37638972667	72384
2797	7 3163	366	49356265723	49356340387	74664
3463	3847	384	49428907933	49428989167	81234
4783	5227	444	70192637737	70192720303	82566
5737	7 6547	810	74734558567	74734648657	90090
9433	10267	834	111228311647	111228407113	95466
14557	7 15643	1086	134100150127	134100250717	100590
24103	3   25303	1200	195126585733	195126688957	103224
45817	7 47143	1326	239527477753	239527584553	106800
52177	7 54493	2316	415890988417	415891106857	118440
126487	7 130363	3876	688823669533	688823797237	127704
317587	7 321817	4230	906056631937	906056767327	135390
580687	7 585037	4350	926175746857	926175884923	138066
715873	724117	8244	1157745737047	1157745878893	141846
2719663	3   2728543	8880	1208782895053	1208783041927	146874
6227563	6237013	9450	2124064384483	2124064533817	149334
8114857	7 8125543	10686	2543551885573	2543552039053	153480
10085623	3  10096573	10950	4321372168453	4321372359523	191070
10137493	$3 \qquad 10149277$	11784	6136808604343	6136808803753	199410
18773137	7 18785953	12816	18292411110217	18292411310077	199860
21297553	3   21311107	13554	19057076066317	19057076286553	220236
25291363	3  25306867	15504	21794613251773	21794613477097	225324
43472497	43488073	15576	35806145634613	35806145873077	238464
52645423	52661677	16254	75359307977293	75359308223467	246174
69718147	7 69734653	16506	89903831167897	89903831419687	251790
80002627	7 80019223	16596	125428917151957	125428917432697	280740
89776327	7 89795773	19446	194629563521143	194629563808363	287220
90338953	90358897	19944	367947033766573	367947034079923	313350
109060027	7 109081543	21516	376957618687747	376957619020813	333066
148770907		38340	483633763994653	483633764339287	344634
1060162843		39990	539785800105313	539785800491887	386574
1327914037	7 1327955593	41556			
	·		·	·	

# 4 Record gaps between prime quadruplets

TABLE 4 Maximal gaps between prime quadruplets  $\{p,\,p+2,\,p+6,\,p+8\}$  [OEIS A113404]

Initial prin	Initial primes in tuples		Initial primes in tuples		Gap $g_4$
5	11	6	3043111031	3043668371	557340
11	101	90	3593321651	3593956781	635130
191	821	630	5675642501	5676488561	846060
821	1481	660	25346635661	25347516191	880530
2081	3251	1170	27329170151	27330084401	914250
3461	5651	2190	35643379901	35644302761	922860
5651	9431	3780	56390149631	56391153821	1004190
25301	31721	6420	60368686121	60369756611	1070490
34841	43781	8940	71335575131	71336662541	1087410
88811	97841	9030	76427973101	76429066451	1093350
122201	135461	13260	87995596391	87996794651	1198260
171161	187631	16470	96616771961	96618108401	1336440
301991	326141	24150	151023350501	151024686971	1336470
739391	768191	28800	164550390671	164551739111	1348440
1410971	1440581	29610	171577885181	171579255431	1370250
1468631	1508621	39990	210999769991	211001269931	1499940
2990831	3047411	56580	260522319641	260523870281	1550640
3741161	3798071	56910	342611795411	342614346161	2550750
5074871	5146481	71610	1970587668521	1970590230311	2561790
5527001	5610461	83460	4231588103921	4231591019861	2915940
8926451	9020981	94530	5314235268731	5314238192771	2924040
17186591	17301041	114450	7002440794001	7002443749661	2955660
21872441	22030271	157830	8547351574961	8547354997451	3422490
47615831	47774891	159060	15114108020021	15114111476741	3456720
66714671	66885851	171180	16837633318811	16837637203481	3884670
76384661	76562021	177360	30709975578251	30709979806601	4228350
87607361	87797861	190500	43785651890171	43785656428091	4537920
122033201	122231111	197910	47998980412211	47998985015621	4603410
132574061	132842111	268050	55341128536691	55341133421591	4884900
204335771	204651611	315840	92944027480721	92944033332041	5851320
628246181	628641701	395520	412724560672211	412724567171921	6499710
1749443741	1749878981	435240	473020890377921	473020896922661	6544740
2115383651	2115824561	440910	885441677887301	885441684455891	6568590
2128346411	2128859981	513570	947465687782631	947465694532961	6750330
2625166541	2625702551	536010	979876637827721	979876644811451	6983730
2932936421	2933475731	539310			

## 5 Record gaps between prime quintuplets

 $\begin{tabular}{l} TABLE 5.1 \\ Maximal gaps between prime quintuplets $\{p,\,p+2,\,p+6,\,p+8,\,p+12\}$ \\ \end{tabular}$ 

Initial pr	imes in tuples	Gap $g_5$	Initial p	rimes in tuples	Gap $g_5$
5	11	6	107604759671	107616100511	11340840
11	101	90	140760439991	140772689501	12249510
101	1481	1380	162661360481	162673773671	12413190
1481	16061	14580	187735329491	187749510491	14181000
22271	43781	21510	327978626531	327994719461	16092930
55331	144161	88830	508259311991	508275672341	16360350
536441	633461	97020	620537349191	620554105931	16756740
661091	768191	107100	667672901711	667689883031	16981320
1461401	1573541	112140	1079628551621	1079646141851	17590230
1615841	1917731	301890	1104604933841	1104624218981	19285140
5527001	5928821	401820	1182148717481	1182168243071	19525590
11086841	11664551	577710	1197151034531	1197173264711	22230180
35240321	35930171	689850	2286697462781	2286720012251	22549470
53266391	54112601	846210	2435950632251	2435980618781	29986530
72610121	73467131	857010	3276773115431	3276805283951	32168520
92202821	93188981	986160	5229301162991	5229337555061	36392070
117458981	119114111	1655130	9196865051651	9196903746881	38695230
196091171	198126911	2035740	14660925945221	14660966101421	40156200
636118781	638385101	2266320	21006417451961	21006458070461	40618500
975348161	977815451	2467290	22175175736991	22175216733491	40996500
1156096301	1158711011	2614710	22726966063091	22727007515411	41452320
1277816921	1281122231	3305310	22931291089451	22931338667591	47578140
1347962381	1351492601	3530220	31060723328351	31060771959221	48630870
2195593481	2199473531	3880050	85489258071311	85489313115881	55044570
3128295551	3132180971	3885420	90913430825291	90913489290971	58465680
4015046591	4020337031	5290440	96730325054171	96730390102391	65048220
8280668651	8286382451	5713800	199672700175071	199672765913051	65737980
9027127091	9033176981	6049890	275444947505591	275445018294491	70788900
15686967971	15693096311	6128340	331992774272981	331992848243801	73970820
18901038971	18908988011	7949040	465968834865971	465968914851101	79985130
21785624291	21793595561	7971270	686535413263871	686535495684161	82420290
30310287431	30321057581	10770150	761914822198961	761914910291531	88092570

[OEIS A201073]

TABLE 5.2 Maximal gaps between prime quintuplets  $\{p,\,p+4,\,p+6,\,p+10,\,p+12\}$ 

Initial pr	imes in tuples	Gap $g_5$	Initial primes in tuples		Gap $g_5$
7	97	90	15434185927	15440743597	6557670
97	1867	1770	17375054227	17381644867	6590640
3457	5647	2190	17537596327	17544955777	7359450
5647	15727	10080	25988605537	25997279377	8673840
19417	43777	24360	66407160637	66416495137	9334500
43777	79687	35910	74862035617	74871605947	9570330
101107	257857	156750	77710388047	77723371717	12983670
1621717	1830337	208620	144124106167	144138703987	14597820
3690517	3995437	304920	210222262087	210238658797	16396710
5425747	5732137	306390	585234882097	585252521167	17639070
8799607	9127627	328020	926017532047	926036335117	18803070
9511417	9933607	422190	986089952917	986113345747	23392830
16388917	16915267	526350	2819808136417	2819832258697	24122280
22678417	23317747	639330	3013422626107	3013449379477	26753370
31875577	32582437	706860	3538026326827	3538053196957	26870130
37162117	38028577	866460	4674635167747	4674662545867	27378120
64210117	65240887	1030770	5757142722757	5757171559957	28837200
119732017	120843637	1111620	7464931087717	7464961813867	30726150
200271517	201418957	1147440	8402871269197	8402904566467	33297270
203169007	204320107	1151100	9292699799017	9292733288557	33489540
241307107	242754637	1447530	10985205390997	10985239010737	33619740
342235627	344005297	1769670	12992848206847	12992884792957	36586110
367358347	369151417	1793070	15589051692667	15589094176627	42483960
378200227	380224837	2024610	24096376903597	24096421071127	44167530
438140947	440461117	2320170	37371241083097	37371285854467	44771370
446609407	448944487	2335080	38728669335607	38728728308527	58972920
711616897	714020467	2403570	91572717670537	91572784840627	67170090
966813007	970371037	3558030	109950817237357	109950886775827	69538470
2044014607	2048210107	4195500	325554440818297	325554513360487	72542190
3510456787	3514919917	4463130	481567288596127	481567361629087	73032960
4700738167	4705340527	4602360	501796510663237	501796584764467	74101230
5798359657	5803569847	5210190	535243109721577	535243185965557	76243980
7896734467	7902065527	5331060	657351798174427	657351876771637	78597210
12654304207	12659672737	5368530	818872754682547	818872840949077	86266530
13890542377	13896088897	5546520	991851356676277	991851464273767	107597490
14662830817	14668797037	5966220			

[OEIS A201062]

## 6 Record gaps between prime sextuplets

TABLE 6 Maximal gaps between prime sextuplets  $\{p,\,p+4,\,p+6,\,p+10,\,p+12,\,p+16\}$ 

Initial p	orimes in tuples	Gap $g_6$	Initial pi	Initial primes in tuples	
7	97	90	422088931207	422248594837	159663630
97	16057	15960	427190088877	427372467157	182378280
19417	43777	24360	610418426197	610613084437	194658240
43777	1091257	1047480	659829553837	660044815597	215261760
3400207	6005887	2605680	660863670277	661094353807	230683530
11664547	14520547	2856000	853633486957	853878823867	245336910
37055647	40660717	3605070	1089611097007	1089869218717	258121710
82984537	87423097	4438560	1247852774797	1248116512537	263737740
89483827	94752727	5268900	1475007144967	1475318162947	311017980
94752727	112710877	17958150	1914335271127	1914657823357	322552230
381674467	403629757	21955290	1953892356667	1954234803877	342447210
1569747997	1593658597	23910600	3428196061177	3428617938787	421877610
2019957337	2057241997	37284660	9367921374937	9368397372277	475997340
5892947647	5933145847	40198200	10254799647007	10255307592697	507945690
6797589427	6860027887	62438460	13786576306957	13787085608827	509301870
14048370097	14112464617	64094520	21016714812547	21017344353277	629540730
23438578897	23504713147	66134250	33157788914347	33158448531067	659616720
24649559647	24720149677	70590030	41348577354307	41349374379487	797025180
29637700987	29715350377	77649390	72702520226377	72703333384387	813158010
29869155847	29952516817	83360970	89165783669857	89166606828697	823158840
45555183127	45645253597	90070470	122421000846367	122421855415957	854569590
52993564567	53086708387	93143820	139864197232927	139865086163977	888931050
58430706067	58528934197	98228130	147693859139077	147694869231727	1010092650
93378527647	93495691687	117164040	186009633998047	186010652137897	1018139850
97236244657	97367556817	131312160	202607131405027	202608270995227	1139590200
240065351077	240216429907	151078830	332396845335547	332397997564807	1152229260
413974098817	414129003637	154904820	424681656944257	424682861904937	1204960680
419322931117	419481585697	158654580	437804272277497	437805730243237	1457965740

[OEIS A200503]

## 7 Record gaps between prime septuplets

TABLE 7.1 Maximal gaps between prime 7-tuples  $\{p,\,p+2,\,p+8,\,p+12,\,p+14,\,p+18,\,p+20\}$ 

·					
Initial p	orimes in tuples	Gap $g_7$	Initial primes in tuples		Gap $g_7$
5639	88799	83160	1554893017199	1556874482069	1981464870
88799	284729	195930	2088869793539	2090982626639	2112833100
284729	626609	341880	2104286376329	2106411289049	2124912720
1146779	6560999	5414220	2704298257469	2706823007879	2524750410
8573429	17843459	9270030	3550904257709	3553467600029	2563342320
24001709	42981929	18980220	4438966968419	4442670730019	3703761600
43534019	69156539	25622520	9996858589169	10000866474869	4007885700
87988709	124066079	36077370	21937527068909	21942038052029	4510983120
157131419	208729049	51597630	29984058230039	29988742571309	4684341270
522911099	615095849	92184750	30136375346249	30141383681399	5008335150
706620359	832143449	125523090	32779504324739	32784963061379	5458736640
1590008669	1730416139	140407470	40372176609629	40377635870639	5459261010
2346221399	2488117769	141896370	42762127106969	42767665407989	5538301020
3357195209	3693221669	336026460	54620176867169	54626029928999	5853061830
11768282159	12171651629	403369470	63358011407219	63365153990639	7142583420
30717348029	31152738299	435390270	79763188368959	79770583970249	7395601290
33788417009	34230869579	442452570	109974651670769	109982176374599	7524703830
62923039169	63550891499	627852330	145568747217989	145576919193689	8171975700
68673910169	69428293379	754383210	196317277557209	196325706400709	8428843500
88850237459	89858819579	1008582120	221953318490999	221961886287509	8567796510
163288980299	164310445289	1021464990	249376874266769	249385995968099	9121701330
196782371699	197856064319	1073692620	290608782523049	290618408585369	9626062320
421204876439	422293025249	1088148810	310213774327979	310225023265889	11248937910
427478111309	428623448159	1145336850	471088826892779	471100312066829	11485174050
487635377219	489203880029	1568502810	631565753063879	631578724265759	12971201880
994838839439	996670266659	1831427220	665514714418439	665530090367279	15375948840

[OEIS A201251]

TABLE 7.2 Maximal gaps between prime 7-tuples  $\{p,\,p+2,\,p+6,\,p+8,\,p+12,\,p+18,\,p+20\}$ 

Initial p	Initial primes in tuples Gap $g_7$		Initial primes in tuples		Gap $g_7$
11	165701	165690	382631592641	383960791211	1329198570
165701	1068701	903000	711854781551	714031248641	2176467090
1068701	11900501	10831800	2879574595811	2881987944371	2413348560
25658441	39431921	13773480	3379186846151	3381911721101	2724874950
45002591	67816361	22813770	5102247756491	5105053487531	2805731040
93625991	124716071	31090080	5987254671311	5990491102691	3236431380
257016491	300768311	43751820	7853481899561	7857040317011	3558417450
367438061	428319371	60881310	11824063534091	11828142800471	4079266380
575226131	661972301	86746170	16348094430581	16353374758991	5280328410
1228244651	1346761511	118516860	44226969237161	44233058406611	6089169450
1459270271	1699221521	239951250	54763336591961	54771443197181	8106605220
2923666841	3205239881	281573040	154325181803321	154333374270191	8192466870
10180589591	10540522241	359932650	157436722520921	157445120715341	8398194420
15821203241	16206106991	384903750	281057032201481	281065611322031	8579120550
23393094071	23911479071	518385000	294887168565161	294896169845351	9001280190
37846533071	38749334621	902801550	309902902299701	309914040972071	11138672370
158303571521	159330579041	1027007520	419341934631071	419354153220461	12218589390
350060308511	351146640191	1086331680	854077393259801	854090557643621	13164383820

[OEIS A201051]

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