Tables of Fibonacci and Lucas Factorizations

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Dedicated to Dov Jarden

Abstract. We list the known prime factors of the Fibonacci numbers F_n for $n \leq 999$ and Lucas numbers L_n for $n \leq 500$. We discuss the various methods used to obtain these factorizations, and primality tests, and give some history of the subject.

1. Introduction. In the Supplements section at the end of this issue we give in two tables the known prime factors of the Fibonacci numbers F_n, $3 \leq n \leq 999$, n odd, and the Lucas numbers L_n, $2 \leq n \leq 500$. The sequences F_n and L_n are defined recursively by the formulas

$$
\begin{aligned}
F_{n+2} &= F_{n+1} + F_n, & F_0 &= 0, & F_1 &= 1, \\
L_{n+2} &= L_{n+1} + L_n, & L_0 &= 2, & L_1 &= 1.
\end{aligned}
\tag{1.1}
$$

The use of a different subscripting destroys the divisibility properties of these numbers.

We also have the formulas

$$
F_n = \frac{\alpha^n - \beta^n}{\alpha - \beta}, \qquad L_n = \alpha^n + \beta^n,
\tag{1.2}
$$

where $\alpha = (1 + \sqrt{5})/2$ and $\beta = (1 - \sqrt{5})/2$. This paper is concerned with the multiplicative structure of F_n and L_n. It includes both theoretical and numerical results.

2. Multiplicative Structure of F_n and L_n. The identity

$$
F_{2n} = F_n L_n
\tag{2.1}
$$

follows directly from (1.2). Although the Fibonacci and Lucas numbers are defined additively, this is one of many multiplicative identities relating these sequences. The identities in this paper are derived from the familiar polynomial factorization

$$
x^n - y^n = \prod_{d|n} \Phi_d(x, y), \qquad n \geq 1,
\tag{2.2}
$$

where $\Phi_d(x, y)$ is the dth cyclotomic polynomial in homogeneous form.

Define the *primitive part* F_d^* of F_d to be

$$
F_d^* = \begin{cases} 1, & d = 1, \\ \Phi_d(\alpha, \beta), & d \geq 2. \end{cases}
\tag{2.3}
$$

Then we have the factorization

$$(2.4) \qquad\qquad F_n = \prod_{d|n} F_d^*, \qquad n \geq 1.$$

Here the F_d^* are rational integers, computable by the inverse formula

$$(2.5) \qquad\qquad F_d^* = \prod_{\delta|d} F_\delta^{\mu(d/\delta)}, \qquad d \geq 1,$$

where μ is the Möbius function. The ratio $F_n' = F_n/F_n^*$ is called the *algebraic part* of F_n.

Formula (2.4) reduces factoring F_n to factoring the F_d^*'s. Formula (2.5) shows that the primitive part can be obtained without factoring.

A prime factor of F_n (resp. L_n) is called *primitive* if it does not divide F_k (resp. L_k) for $1 \leq k < n$; otherwise it is called *algebraic*. A composite factor of F_n is also called *algebraic* if it is a product of prime algebraic factors. Any prime divisor of F_n' (resp. L_n') is necessarily algebraic, but under certain circumstances a prime divisor of F_n^* (resp. L_n^*) is not primitive. Such an algebraic prime factor p of F_n^* (resp. L_n^*) is called *intrinsic* and is listed as p^* in these tables. This occurs exactly when $n = p^r m$, $r \geq 1$, where p is a primitive factor of F_m (resp. L_m). In this case p always divides F_n^* (resp. L_n^*) to just the first power.

Example. The factorization of F_{105}, given by (2.4), is

$$F_{105} = \prod_{d|105} F_d^* = F_1^* F_3^* F_5^* F_7^* F_{15}^* F_{21}^* F_{35}^* F_{105}^*.$$

This factorization is abbreviated in Table 2 as

$$105 \ (3,5,7,15,21,35) \ 8288823481.$$

Here the numbers within the parentheses are the subscripts of the algebraic factors F_d^*, $1 < d < 105$. (The factor $F_1^* = 1$ is omitted.) The primitive part $F_{105}^* = 8288823481$ is given after the parentheses. The lines in Table 2 corresponding to the numbers inside the parentheses are:

$$3 \ \underline{2}$$
$$5 \ \underline{5}$$
$$7 \ \underline{13}$$
$$15 \ (3,5) \ \underline{61}$$
$$21 \ (3,7) \ \underline{421}$$
$$35 \ (5,7) \ \underline{141961}$$

The factorization of F_{105} is then obtained by collecting the primitive prime factors from their respective lines. These follow the parentheses (if any) on the seven lines and are underlined above for emphasis. Thus,

$$F_{105} = 2 \cdot 5 \cdot 13 \cdot 61 \cdot 421 \cdot 141961 \cdot 8288823481.$$

Because of (2.1), the algebraic multiplicative structure for L_n can be derived directly from that of F_{2n}. Let $n = 2^s m$, where m is odd. Then

$$(2.6) \qquad\qquad L_n = \prod_{d|m} L_{2^s d}^*, \qquad n \geq 1,$$

where

$$(2.7) \qquad L^*_{2^s d} = F^*_{2^{s+1} d} = \prod_{\delta | d} L^{\mu(d/\delta)}_{2^s \delta}, \qquad d \geq 1.$$

The *primitive part* of L_n is $L^*_n = F^*_{2n}$. The *algebraic part* of L_n is

$$(2.8) \qquad L'_n = L_n/L^*_n.$$

Furthermore, as a result of a generalization by Lucas of a special identity discovered by Aurifeuille, we also have for odd n

$$\begin{aligned}
\frac{L_{5n}}{L_n} = \frac{\alpha^{5n} + \beta^{5n}}{\alpha^n + \beta^n} &= \alpha^{4n} - \alpha^{3n}\beta^n + \alpha^{2n}\beta^{2n} - \alpha^n\beta^{3n} + \beta^{4n} \\
&= (\alpha^{2n} - 3\alpha^n\beta^n + \beta^{2n})^2 + 5\alpha^n\beta^n(\alpha^n - \beta^n)^2 \\
&= (5F_n^2 + 1)^2 - 25F_n^2 \\
&= (5F_n^2 + 5F_n + 1)(5F_n^2 - 5F_n + 1) \text{ (using } \alpha\beta = -1 \text{ and } \alpha - \beta = \sqrt{5}).
\end{aligned}$$

Consequently, we have the special Aurifeuillian factorization

$$(2.9) \qquad L_{5n} = L_n A_{5n} B_{5n}, \qquad n \text{ odd,}$$

where

$$A_{5n} = 5F_n^2 - 5F_n + 1, \qquad B_{5n} = 5F_n^2 + 5F_n + 1.$$

This decomposition means that these L_{5n}'s have two different algebraic factorizations. For example, from (2.6) and (2.9)

$$L_{105} = \prod_{d|105} L^*_d = L^*_1 L^*_3 L^*_5 L^*_7 L^*_{15} L^*_{21} L^*_{35} L^*_{105}$$

and

$$L_{105} = L_{21} A_{105} B_{105}.$$

Primitive parts A^*_n and B^*_n can also be defined for A_n and B_n. Let $n \geq 1$ be odd and set $n = 5^s m, s \geq 0, 5 \nmid m$. Let $\varepsilon_d = \frac{1}{2}\left(1 + \left(\frac{d}{5}\right)\right)$, where $\left(\frac{d}{5}\right)$ is the Legendre symbol. Let

$$(2.10) \qquad \begin{aligned}
A^*_{5n} &= \prod_{d|m} [(A_{5n/d})^{\varepsilon_d}(B_{5n/d})^{1-\varepsilon_d}]^{\mu(d)}, \\
B^*_{5n} &= \prod_{d|m} [(A_{5n/d})^{1-\varepsilon_d}(B_{5n/d})^{\varepsilon_d}]^{\mu(d)}.
\end{aligned}$$

(Here A^*_{5n} and B^*_{5n} are rational integers such that $(A^*_{5n}, B^*_{5n}) = 1$ and $L^*_{5n} = A^*_{5n} B^*_{5n}$.) Then

$$(2.11) \qquad \begin{aligned}
A_{5n} &= \prod_{d|m} (A^*_{5n/d})^{\varepsilon_d}(B^*_{5n/d})^{1-\varepsilon_d}, \\
B_{5n} &= \prod_{d|m} (A^*_{5n/d})^{1-\varepsilon_d}(B^*_{5n/d})^{\varepsilon_d}.
\end{aligned}$$

Thus, in the above example we have

$$A_{105} = A^*_5 B^*_{15} B^*_{35} A^*_{105}, \qquad B_{105} = B^*_5 A^*_{15} A^*_{35} B^*_{105}.$$

Since $A_5^* = A_{15}^* = 1$, these are omitted in Table 3, while B_5^* is written as L_5^* and B_{15}^* as L_{15}^*.

Those Lucas numbers which do not have an Aurifeuillian factorization appear in the tables in the same format as the Fibonacci factorizations. However, the Aurifeuillian factorizations appear in an expanded format. For example, the above factorization appears as:

$$105\ (3,7,21)\ A \cdot B$$
$$A\ (15, 35B)\ 21211$$
$$B\ (5, 35A)\ 767131.$$

The list of numbers immediately after the index 105 indicate that L_{105} has the algebraic factors L_3^*, L_7^*, and L_{21}^*. Furthermore, A_{105}^* has algebraic factors L_{15}^* and B_{35}^*, while B_{105}^* has algebraic factors L_5^* and A_{35}^*. In computing A_n^* and B_n^*, the following result is sometimes useful [9, p. 16]:

THEOREM 1 (CROSSOVER THEOREM). *For odd* $k, n \geq 1$ *where* $(5, k) = 1$ *and* $\left(\frac{k}{5}\right)$ *is the Jacobi symbol,*

$$if\ \left(\frac{k}{5}\right) = 1, \quad then\ A_{5n} \mid A_{5kn}\ and\ B_{5n} \mid B_{5kn};$$

$$if\ \left(\frac{k}{5}\right) = -1, \quad then\ A_{5n} \mid B_{5kn}\ and\ B_{5n} \mid A_{5kn}.$$

The tables are organized using formulas (2.4) and (2.6). As a result, no prime factor appears explicitly more than once in the tables (except intrinsic factors and the repeated factor 2 of L_3). Where space permits, we list the known factors in their entirety on a single line. We list all prime factors of 25 digits or less, carrying over to a second line, without breaking the factor, when necessary. All other factors are listed as either Pxx or Cxx, indicating respectively a prime or a composite cofactor of xx digits. When a factorization is incomplete, we leave space on the line for new factors to be inserted by hand.

3. Factorization Methods. A variety of methods have been used to effect the factorizations given herein. These include the Pollard $p - 1$ and Brent-Pollard Rho methods [13], the analogous $p + 1$ method [19], the Continued Fraction (CFRAC) method of Morrison and Brillhart [14], Pomerance's Quadratic Sieve (QS) method [8], along with its extensions and improvements (MP-QS) [17], [18], and Lenstra's Elliptic Curve Method (ECM) [11], [13]. Of course, many of the smaller prime factors are quite old, and were originally found by trial division or the difference of squares method.

Some of the methods utilize the form of the prime divisors given by the following theorems [9, p. 11].

THEOREM 2. *Let* n *be odd and let* p *be an odd, primitive prime divisor of* F_n. *Then*

(i) $p \equiv 1 \bmod 4$.

(ii) *if* $p \equiv \pm 1 \bmod 10$, *then* $p \equiv 1 \bmod 4n$.

(iii) *if* $p \equiv \pm 3 \bmod 10$, *then* $p \equiv 2n - 1 \bmod 4n$.

THEOREM 3. *Let n be positive and let p be an odd, primitive prime divisor of L_n. Then*

(i) *if $p \equiv \pm 1 \bmod 10$, then $p \equiv 1 \bmod 2n$.*

(ii) *if $p \equiv \pm 3 \bmod 10$, then $p \equiv -1 \bmod 2n$.*

4. Primality Testing. In [9, p. 36], Brillhart gave the following results of primality tests on the Fibonacci and Lucas numbers: F_n, $3 \leq n < 1000$, is prime if and only if $n = 3, 4, 5, 7, 11, 13, 17, 23, 29, 43, 47, 83, 131, 137, 359, 431, 433, 449, 509, 569, 571$; L_n, $0 \leq n \leq 500$, is prime if and only if $n = 0, 2, 4, 5, 7, 8, 11, 13, 16, 17, 19, 31, 37, 41, 47, 53, 61, 71, 79, 113, 313, 353$. More recently, H. C. Williams has discovered that $F_{2971}, L_{503}, L_{613}, L_{617}$ and L_{863} are also prime. Williams also states that F_{4723} and F_{5387} are probable primes [21].

For F_n to be prime, $n \geq 5$, it is necessary, but not sufficient, that n be prime. Similarly, L_n can be prime only when n is prime or a power of 2. There are several identities that can be used for primality proofs if one should find either F_n or L_n or their primitive parts to be probable primes. These identities are useful because in proving N prime, the methods of [5] depend upon auxiliary factorizations of $N \pm 1$. For the Fibonacci numbers we have [9, p. 95]:

$$(4.1) \qquad F_{4k+1} - 1 = F_k L_k L_{2k+1}, \qquad F_{4k+3} - 1 = F_{k+1} L_{k+1} L_{2k+1}$$

and

$$(4.2) \qquad F_{4k+1} + 1 = F_{2k+1} L_{2k}, \qquad F_{4k+3} + 1 = F_{2k+1} L_{2k+2}.$$

For the Lucas numbers we have

$$(4.3) \qquad L_{4k} - 1 = L_{6k}/L_{2k}, \qquad L_{4k} + 1 = (L_{2k} - 1)(L_{2k} + 1)$$

and

$$(4.4) \qquad \begin{array}{ll} L_{4k+1} - 1 = 5 F_k L_k F_{2k+1}, & L_{4k+3} - 1 = L_{2k+1} L_{2k+2}, \\ L_{4k+1} + 1 = L_{2k} L_{2k+1}, & L_{4k+3} + 1 = 5 L_{k+1} F_{k+1} F_{2k+1}. \end{array}$$

For the Lucas Aurifeuillians we have

$$(4.5) \qquad \begin{array}{ll} A_{5k} - 1 = 5 F_k (F_k - 1), & B_{5k} - 1 = 5 F_k (F_k + 1), \\ A_{5k} + 1 = (L_{k-1} - 1)(L_{k+1} - 1), & B_{5k} + 1 = (L_{k-1} + 1)(L_{k+1} + 1). \end{array}$$

The use of these formulas is apparent. They break the factorizations of $F_n \pm 1$ and $L_n \pm 1$ into factorizations of smaller F_n's and L_n's and thus facilitate the primality test. There are a number of additional formulas of a similar kind for $F_n^* \pm 1$ and $L_n^* \pm 1$.

All factors and cofactors in Tables 2 and 3 with fewer than 85 digits, and not labelled as Cxx, have been proved prime by Silverman using the methods presented in [5, Section 3] and [20]. These methods depend upon auxiliary factorizations of $p - 1$, $p + 1$, $p^2 + 1$, $p^2 + p + 1$, and $p^2 - p + 1$. If these cyclotomic polynomials have enough small prime factors, then the methods produce very fast proofs of primality along with a compact certificate which can later be used to verify the proof. Andrew Odlyzko has proved all of the remaining probable prime cofactors to be prime using an implementation of the Cohen-Lenstra algorithm [6].

5. History of Tables. Brillhart found many small factors (up to 10 digits) by a direct search program, using Theorems 2 and 3 to restrict the search range for trial division [1], [2]. He later programmed a difference of squares method with modular exclusion to factor $F_{169}, L_{131}, L_{133}, L_{134}, L_{158}, L_{173}$, and L_{237}.

In 1968 Brillhart used D. H. Lehmer's delay line sieve DLS 127 at U. C. Berkeley [10] to factor $F_{255}, L_{166}, L_{214}, L_{252}$, and L_{258}, again using a difference of squares with modular exclusion. The most remarkable of these factorizations,

$$F_{255}^* = 20778644396941 \cdot 20862774425341,$$

was found in just 40 seconds. Although these two factors are very close, there is no known formula which can account for this factorization.

Between 1970 and 1973, Brillhart and Morrison found a large number of complete factorizations using the continued fraction method, CFRAC, on an IBM 360/91 at UCLA [9], [14].

Starting in 1974, J. L. Selfridge and Marvin C. Wunderlich used an improved version of the UCLA program on an IBM 360/65 at NIU in Dekalb, Illinois to factor many 37-41 digit cofactors. They also implemented the first stage of Pollard's just-discovered $p - 1$ method, and found many new factors. Earl Ecklund and Brillhart programmed and used the first stage of the $p + 1$ method as well [5, p. xlii].

H. C. Williams [19] applied the $p \pm 1$ methods to 174 composite Fibonacci and Lucas cofactors which had at most 80 digits.

Thorkil Naur ran the $p - 1$ and Pollard Rho methods on F_n for odd n, $1 \le n \le 399$, and on L_n for $0 \le n \le 500$. When a factor was at most 53 digits, he completed it via CFRAC. His book [15] and paper [16] list several new factorizations which are included herein.

Montgomery, between 1983 and 1986, applied the methods of [13] to all composite table entries, using idle time on a VAX/780, two VAX/750's and a CDC 7600. He found about 200 previously unknown factors of 11 to 36 digits. Over half of these were found by ECM. He used 10 elliptic curves with limits of 10^4 and $6 \cdot 10^5$, another ten curves with limits of $1.6 \cdot 10^4$ and 10^6, and a third set of ten curves with limits of $3.2 \cdot 10^4$ and $2 \cdot 10^6$. Often he used four, five or more sets, but the work is uneven (many more curves were used on the Lucas numbers than on the Fibonacci numbers). Montgomery [13, Section 6] also ran $p + 1$ with an initial value (seed) of 15/8 mod N using limits of $3 \cdot 10^5$ and 10^7, and again with a seed of 23/11 mod N using limits of $2 \cdot 10^6$ and 10^8. If $P \equiv 15/8$ mod N, then $P^2 - 4 \equiv -31/64$ mod N will be a quadratic residue precisely when -31 is a quadratic residue, so this will find a factor of p if $p - \left(\frac{-31}{p}\right)$ is highly composite; this includes cases where 31 divides whichever of $p \pm 1$ is highly composite. The seed of 23/11 mod N catches cases where $p - \left(\frac{5}{p}\right)$ is highly composite. By Theorems 2 and 3, if $p \mid F_n^*$ (n odd) or $p \mid L_n^*$, then $p - \left(\frac{5}{p}\right)$ is divisible by $2n$, so the latter case occurs frequently. However, these runs did miss some primes p for which $p + 1$ is highly composite, such as the factor

$$2170208701449020077201 = 2 \cdot 7 \cdot 12583 \cdot 55807 \cdot 424267 \cdot 520309 - 1$$

of F_{795} (found by MP-QS; -31 is a nonresidue, but the limits were not high enough on that run).

Davis and Holdridge [7], in 1984, completed the factorizations of four cofactors $(F_{277}, L_{362}, L_{370},$ and $L_{471})$ of 57 to 58 digits, using QS on a CRAY 1S.

Silverman, between 1983 and 1986, ran $p - 1$ with limits of $3 \cdot 10^6$ and $5 \cdot 10^7$ on the entire Lucas table and on the Fibonacci table to F_{499}. He also ran $p - 1$ with limits of $2 \cdot 10^5$ and $3 \cdot 10^6$ on the Fibonacci table from F_{501} to F_{999}. This work was accomplished on a Micro-VAX/1 and found about 80 new factors. Some runs with ECM on the Lucas table using the same machine revealed no new factors. Silverman also completed the factorizations of all cofactors below 73 digits, and several larger ones, using either CFRAC or MP-QS [17], [18] on a combination of VAX/780's and SUN-3/75's. The larger factorizations were accomplished using a parallel implementation of MP-QS on a network of SUN's.

6. Accuracy and Completeness of Tables.
Montgomery and Silverman independently verified each entry in the main tables. They checked that

- Each listed factor divides the number and is a prime or probable prime.
- The proper list of algebraic (including intrinsic) factors appears
- The primitive prime factors appear in ascending order.
- If no cofactor is given, the list of factors is complete.
- If a cofactor is labelled as Cxx, then it is indeed composite and has xx digits.
- If a cofactor is labelled as Pxx, then it is a prime or probable prime and has xx digits.
- No odd primitive prime factor of F_n or L_n was found to divide twice, further strengthening the conjecture that no such prime exists.

Earlier versions of these tables were checked on computers by Michael Morrison and Tim Korb.

As of August 1987 there remain 140 composite Fibonacci cofactors and 10 composite Lucas cofactors in the tables. During 1986 Silverman and Montgomery found numerous factors greater than 20 digits, but none smaller. Based upon numerous runs with ECM, the authors are confident that there are at most 3 undetected factors less than 20 digits.

7. Discussion of Methods.
It is still an open question what the best method is to attack a large arbitrary composite number. The authors' experience suggests that the following procedure is perhaps the most reasonable.

As long as the remaining cofactor N is not a probable prime, do the following in order:

(1) Trial division up to some small limit, perhaps $(\ln N)^2$.

(2) ECM is generally more effective than $p \pm 1$, but $p \pm 1$ is so much faster that trying it first is worthwhile. A good first set of starting limits is about 10^4 and 10^5. This should perhaps take a couple of minutes on a typical mainframe for (say) an 80-digit number.

(3) ECM should now be tried, using about 5 curves and limits of 10^4 and $5 \cdot 10^5$.

(4) If the remaining cofactor is sufficiently small (say up to 60 digits), it should be finished with MP-QS. If the number is larger than this, it is worthwhile devoting more ECM trials with higher limits to it.

(5) If ECM fails and the number is less than about 70 digits, then MP-QS should now be applied. Seventy digits will take about a day on a typical modern mainframe. One can of course attempt larger numbers with a supercomputer or special hardware. The largest number ever factored with MP-QS, as of December 1986, was an 87-digit cofactor of $5^{128} + 1$ using a parallel implementation on a SUN network. That factorization took 3950 total CPU hours, divided among 10 SUN-3's over a period of about 5 weeks.

(6) Finally, if the cofactor is still too large, one can keep trying ECM with higher limits or set the number aside.

TABLE 1

Prime Factors With More Than 25 Digits

N	Factor	Discoverer	Method	Machine
L_{386}	1024502971279512003440504 3	Montgomery	ECM	CDC 7600
F_{563}	1215877129695937786329413 3	Montgomery	ECM	CDC 7600
L_{431}	1378049553112721035601842 1	Silverman	$p-1$	UVAX/1
F_{425}	1418795434530356438839000 1	Silverman	MP-QS	VAX/780
F_{507}	1734088919521289239979717 3	Silverman	MP-QS	VAX/8600
L_{406}	2367069891188086575898038 7	Silverman	MP-QS	VAX/780
L_{371}	3566879698948480066612280 9	Silverman	MP-QS	VAX/780
L_{422}	3630268919283211904258986 7	Silverman	MP-QS	SUN-3/75
L_{467}	4738105317478219139589703 1	Montgomery	ECM	CDC 7600
L_{320}	6237955583180309986727296 1	Naur	CFRAC	Mathilda
F_{837}	1362997727025444376796603 33	Silverman	MP-QS	SUN-3/75
F_{445}	1565252892825484140817990 81	Silverman	MP-QS	VAX/780
L_{471}	4783302581233605541998691 69	Davis	QS	CRAY 1S
F_{277}	5054710057406915248532936 21	Davis	QS	CRAY 1S
F_{517}	6414661243496076970162380 97	Silverman	MP-QS	SUN-3/75
F_{741}	6696520722710512716984361 13	Silverman	MP-QS	SUN-3/75
F_{597}	12262448164949728997664039 49	Silverman	MP-QS	SUN-3/75
F_{503}	24300147477009994230170175 01	Silverman	MP-QS	SUN-3/75
F_{869}	58904308212046650885354699 13	Montgomery	ECM	CDC 7600
L_{479}	163726493049495886839207254 89	Silverman	MP-QS	VAX/780
F_{559}	260938370570172472695312215 21	Silverman	MP-QS	SUN-3/75
F_{317}	503546330165333805042385219 09	Silverman	MP-QS	VAX/780
F_{461}	579073653337871288861411261 77	Silverman	MP-QS	SUN-3/75
F_{633}	1923474742854608312004939200 89	Silverman	MP-QS	SUN-3/75
L_{326}	5730056809961208559007838719 63	Silverman	MP-QS	SUN-3/75
F_{971}	6198026072595145833302356937 29	Montgomery	$p-(5/p)$	CDC 7600
L_{412}	10904143353831684635611451676 23	Montgomery	ECM	CDC 7600
L_{344}	14039810997233210293799139486 41	Silverman	MP-QS	VAX/780
L_{482}	53734303291224688218836710121 69	Montgomery	ECM	CDC 7600
L_{377}	92204072437237199421543178883 99	Silverman	MP-QS	SUN-3/75
F_{489}	550104833504084870524855707442 97	Silverman	MP-QS	SUN-3/75
F_{663}	542202788462733966380018208818 089	Silverman	MP-QS	SUN-3/75
F_{681}	131653446329084721859009751356 4513	Silverman	MP-QS	SUN-3/75
L_{430}	151741654463971917564526438024 7161	Silverman	MP-QS	SUN-3/75
F_{383}	153185084438107746146196036434 86769	Silverman	MP-QS	SUN-3/75
F_{427}	249495868964998482871252356673 56281	Silverman	MP-QS	SUN-3/75
L_{464}	227693725298545340302283668318 476481	Montgomery	ECM	CDC 7600

The present practical limit of technology seems to be about 16 digits for prime factors found by Pollard Rho, 18 digits for Brent's variation of Pollard Rho, and 25 digits for ECM. The $p \pm 1$ methods occasionally have huge successes where a factor over 25 digits is found; for example, these methods could have found the 29-digit factor of L_{479} with a little more effort. However, factors of 18 to 20 digits are more typical. The CFRAC method has been demonstrated for products up to 10^{64}, QS for products up to 10^{71}, and MP-QS for products up to 10^{87}. This comparison is not quite fair, however, because the CFRAC and QS results were achieved either on a supercomputer or on special purpose hardware, while the MP-QS results were achieved on a network of SUN's [17], [18].

Table 1 lists all of the known nonlargest primitive prime factors of F_n or L_n having more than 25 digits. The cofactor of each of these, when it is composite, is assumed to have at least one prime factor exceeding the factor listed. Each entry includes the discoverer, the method of discovery, and the machine used. In the "machine" column the notation "UVAX/1" is an abbreviation for Micro-VAX/1.

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Supplement to
Tables of Fibonacci and Lucas Factorizations

By John Brillhart, Peter L. Montgomery and Robert D. Silverman

TABLE 2   FIBONACCI FACTORIZATIONS $2 < n < 1000$, n odd

n	Prime Factors
3	2
5	5
7	13
9	(3) 17
11	89
13	233
15	(3,5) 61
17	1597
19	37.113
21	(3,7) 421
23	28657
25	(5) 5*.3001
27	(3,9) 53.109
29	514229
31	557.2417
33	(3,11) 19801
35	(5,7) 141961
37	73.149.2221
39	(3,13) 135721
41	2789.59369
43	433494437
45	(3,5,9,15) 109441
47	2971215073
49	(7) 97.6168709
51	(3,17) 6376021
53	953.55945741
55	(5,11) 661.474541
57	(3,19) 797.54833
59	353.2710260697
61	4513.555003497
63	(3,7,9,21) 35239681
65	(5,13) 14736206161
67	269.116849.1429913
69	(3,23) 137.829.18077
71	6673.46165371073
73	9375829.86020717
75	(3,5,15,25) 230686501
77	(7,11) 988681.4832521

79 157.9218047149753
81 (3,9,27) 2269.4373.19441
83 9919485309475497
85 (5,17) 9521.3415914041
87 (3,29) 173.3821263937
89 1069.1665088321800481
91 (7,13) 13*.741469.159607993
93 (3,31) 453110055090901
95 (5,19) 761.29641.67735001
97 193.389.3084989.361040209
99 (3,9,11,33) 197.1854680 5133
101 743519377.770857978613
103 519121.5644193.512119709
105 (3,5,7,15,21,35) 828882348 1
107 1247833.8242065050061761
109 827728777.3252967548 8417
111 (3,37) 1459000305513721
113 677.272602401466814027129
115 (5,23) 1381.2441738887963981
117 (3,9,13,39) 29717.39589685693
119 (7,17) 159512939815855788121
121 (11) 9741581346638144559 6089
123 (3,41) 6854195773394970 1
125 (5,25) 5*.158414167964045700001
127 27941.556805304822773221 0073
129 (3,43) 257.5417.8513.39639893
131 1066340417491711059581457 2169
133 (7,19) 3457.42293.351301301 942501
135 (3,5,9,15,27,45) 1114769954 3 67361
137 1913470240009327808144942 3917
139 277.2114537501.8552672293 7689093
141 (3,47) 108289.1435097.14201773 7
143 (11,13) 8581.192958415375685 0496621
145 (5,29) 3496199969307370789 0201
147 (3,7,21,49) 293.3529.347502052673
149 110557.162709.4000949.8560764659457 7
151 5737.2811666624525581164699 15877
153 (3,9,17,51) 17*.7175323114950564593
155 (5,31) 21701.1237053388 1.61182778621
157 313.11617.7636481.10424204306491346737
159 (3,53) 317.97639037.229602768949
161 (7,23) 8693.61260610775505899 7066597

163 977.4892609.33365519393.32566222308133
165 (3,5,11,15,33,55) 86461.518101.900241
167 18104700793.19663443186934560856572 1
169 (13) 337.89909.104600155609.126213229732669
171 (3,9,19,57) 6841.5741461760879844361
173 1639343785721.38967874900762927153273 3
175 (5,7,25,35) 701.17231203730201189308301
177 (3,59) 2191261.805134061.1297027681
179 21481.156089.341881664090389892953461376 9
181 8689.422453.8175789237238547745514610 93
183 (3,61) 1097.142973479719757757800833
185 (5,37) 17029455131913055556907097618161
187 (11,17) 373.10157807305963434099105034917037
189 (3,7,9,21,27,63) 38933.9559219503167350 37
191 4870723671313.7578108062569891284399757 93
193 9465278929.102093043203232693397682600849 7
195 (3,5,13,15,39,65) 8899925083749987768 1
197 15761.25795969.22715026569 7.7171851071258886549
199 397.4367821692010020482611713785500552696 33
201 (3,67) 5050260704396247169315999021
203 (7,29) 1217.56470541.2586982700065673399465953 3
205 (5,41) 821.125598581.364481178578913215364 01
207 (3,9,23,69) 4072353155773627601222196481
209 (11,19) 573141209550512977366791653799982620 01
211 22504837.38490197.80097288 1.804754238584495930 21
213 (3,71) 1277.185790722054921374395775013
215 (5,43) 2607553541.6771281736158080495201162 1
217 (7,31) 433.44269.21722177 3.2191174861.6274653314021
219 (3,73) 123953.4139537.31692512459458437 61
221 (13,17) 2035724124 97.9065749871802464532639294019 3
223 4013.108377.2515341 89.16434461004641013889615607081 3
225 (3,5,9,15,25,45,75) 1198166198205095705361600 1
227 23609.521953413798302515907884711361194672857273 77
229 457.2749.40487201.1326054499 01.4783156029762036179855 3
231 (3,7,11,21,33,77) 9164259601748159235188401
233 139801.250477390419633.6314840895836931495578295471 41
235 (5,47) 5737.281166662755939867526629556036931 14561
237 (3,79) 1668481.40762577.7698999052751136773
239 10037.62141.2228536579597318057.285469086229614923336 9
241 11042621.7005329677.1342874889289644763267952824739273
243 (3,9,27,81) 448607550257.16000411124306440307056 1
245 (5,7,35,49) 128955073914024460192651484831956 41

247 (13,19) 409100738617.46773060043367904676926312147328153
249 (3,83) 10330432052554409.23812215284009787769
251 58241677475027 3.21937080329465122026615412418990661913
253 (11,23) 4322114369.22012288236641589.13784970303338047612061
255 (3,5,15,17,51,85) 20778644396941.20862774425341
257 5653.329719786716459055645 21.12300267217119313471360714649
259 (7,37) 1553.4046567737 93.3041266742295771985148799223649
261 (3,9,29,87) 2089.20357.36017.40193.322073.6857029027549
263 4733.93629.9283622296 46390194235291216984425664630 89390281
265 (5,53) 15901.2741218753 681.9269185994574681 25920827581
267 (3,89) 122887425153289.6445588773497030342 877309
269 5381.251797518266 9813.321709447478106 41.169360443982964 8789853
271 449187076348 273.43026721252586712195174061909035949380 58573
273 (3,7,13,21,39,91) 640457.14835473303 43905886515273
275 (5,11,25,55) 7239101.158067979101.5527278404454199535 3821801
277 50547100574069 15248532936 21.686112130818733090898632 8104917
279 (3,9,13,93) 11717.5949600585 08093.627983032252706321
281 174221.1194682731.142057352004178426204 9438829321530369 3455057
283 10753.825229.15791401.444111888848805843163235784 2986308326 4881
285 (3,5,15,19,57,95) 9567346167150463285024 8330601
287 (7,41) 1981600710018533267967006925074901845705010648 32201
289 (17) 577.1733.98837.101232653.10620051943 57.6580786582777254448438 4541
291 (3,97) 76674415738994497 73.22799311775497587 0677
293 64390759997.118863916349728525229520098964471 6155238134997314729
295 (5,59) 1181.35401.75521.16048 1.737501.1120969250625390660846 9121
297 (3,9,11,27,33,99) 593.4157.1360418597.123692430 68750242280033
299 (13,23) 2056992877234272520846845 34202713928205600284860517 1521
301 (7,43) 6380692745271140473407781 5684636922789694353596672 8521
303 (3,101) 855022438967448 1.960496579172798748513 69421
305 (5,61) 2441.6101.204152539662 47698801.64727767017179982409 43861
307 613.9143689.530702786773 8937.2169138415139980143903 9258352068114 71857
309 (3,103) 617.318889.3238614229 7.883364563627459323040861
311 837833.6872477.60371175 53.127223270401321860892580102952310478 01838093
313 1877.5009.79013461238 03597.11558583251423132840655438799726119705 876273
315 (3,5,7,9,15,21,35,45,63,105) 9761221.12057700 2874549237021 7501
317 1307309.503546330165 3338050423852109.120553346546982 45346994276837
319 (11,29) 1913.578029.143552 2969.1535414556000361 3.186262431846834334 28323529
321 (3,107) 2644387026552261 9375245858112105515141492 8921
323 (17,19) 1109581873.85542 64644357.22469617512 7497245934985432764 2081
325 (5,13,25,65) 1301.4235 401.60541650 1.880262501.49284706967787 569058301
327 (3,109) 653.1746181.158 95461414272767943483 64363643680457
329 (7,47) 1973.26321.127 391874411097592672469891 375644477141948573 020237

331 29129.22966686648 632120276391228028485 2000841318497622533370 591664502461
333 (3,9,37,111) 12653.12413484893 3957.93050773155759022676 7593761
335 (5,67) 20404106545895102 0261541412419965696 1913
337 673.15229266824729 11712664446222833267 85140960410433121183406 7048447153001
339 (3,113) 149161.25831 7.220987865057977688872 2153486901420033
341 (11,31) 76122766534291 3.197907695243868721 4558282384863830955384 8674337
343 (7,49) 46649.54490387 5662050509.1089441709440098759 78306751482234414 702393
345 (3,5,15,23,69,115) 18630 1.25013864044961447973 15281604981
347 324097.1434497.334 5860598013.342016737990237623 17333.P26
349 1358309.2663569.27 52093037677787 7058673.P38
351 (3,9,13,27,39,117) 262 3373.8023861.6579032167 974040371744098034257
353 736357.35980201012 573915498603609023562625 25975949247991832187 257385201689
355 (5,71) 4261.7530970 1.309273161.920760926139808 1.49279722643391864 192801
357 (3,7,17,21,51,119) 1429.258469.2765386 2393863258463881623 092961
359 47542043773469822 0747368027166749382927701417016557193662268716376 935476241
361 (19) 6567762529.11 96762644057.3150927 8278169308781415 97.P26
363 (3,11,33,121) 94905 5960433596379608184769903538 5001836615801
365 (5,73) 210241.2758 3781.7582750806268 01.4810862617792334756 259918335342941
367 733.17969789.75991 755.5648966761.43397676601.11415 0315493.P27
369 (3,9,41,123) 8117.199 261.847387933193.9382 5995206691118469915 49277702112864025685 32457966113
371 (7,53) 20701710668 9145433069236091111846 99154927770211 28640256853245796 6113
373 2237.9697.371509.2 0580649.P58   C78
375 (3,5,15,25,75,125) 9001.169 501.41510105455501 .990629934069446 53501
377 (13,29) 10426425175 3.3615755575741.P48
379 757.11889989.64273 8983137.P58
381 (3,127) 18995897.3185 450213669826968428 420712039093396176 57693
383 1639241.1531850 44381077461461960364 3486769.P40
385 (5,7,11,35,55,77) 3780700 1.75954341.138376484 41.26387109578002673 148692221
387 (3,9,43,129) 773.116 101.1427967283 3.380740013616392 4598586714500108 609
389 2333.   C83
391 (17,23) 149356 6753.P65
393 (3,131) 20066 57.14166370809 4656392797852009838 7072442306082819 3993
395 (5,79) 1472561.14 89536056442840431762 3565634381855333948 6582362797645 241
397   C83
399 (3,7,19,21,57,133) 10590 0957340012552950416 609459864426267 30201
401 1388582 9.P77
403 (13,31) 15313.50 68933.42136290591640 129.P48
405 (3,5,9,15,27,45,81,135) 23692 245912756822601.5 8441864401139481683 7041
407 (11,37) 14099718640983 6049132841.P53
409 4909.25357.809742 9243052573.P62
411 (3,137) 544986 1.972663078773.568718248580824312 9.303625618559802 05333
413 (7,59) 45651 2029.P65

415 (5,83) 84541766583283064860 1.2000966632406982580545 01.P25
417 (3,139) 9173.68394723839902999250337117682947175404952197622 45783337
419 11623061.41725948237.P70
421 11789.644049169.45688564527041.P62
423 (3,9,47,141) 5059011062939973161728794842994673444156809986180 713505281
425 (5,17,25,85) 52815601.1418795434530356438390001.P35
427 (7,61) 853.24949586896499848287125235667356281.P39
429 (3,11,13,33,39,143) 857.48049.2663339613830198847157242280783003329461657
431 P90
433 P91
435 (3,5,15,29,87,145) 6961.7196661390635764407916551386458559471906041
437 (19,23)   C83
439 877.163309.   C84
441 (3,7,9,21,49,63,147) 135829.34032420633346148283190550609393982544473 17949
443 8861.104549.15170977.1303309310797189261.P58
445 (5,89) 1801361.6877921.156525289282548414081799081.P35
447 (3,149) 46489.204143987934854374977239155143074091000488165524965 7303909
     .12049668613888885141
449 P94
451 (11,41) 855997.928157.102772929982202372929846693.P48
453 (3,151) 6504851101245855642075184444892863218188442910121507308788 74501
455 (5,7,13,35,65,91) 202021.36768087721.40281313801.31262954473 11401.
457 6397.1685613424 1.P82
459 (3,9,17,51,153) 2753.204311803636 9.13095384194065076117.P26
461 26737.21176476637.92081548928843 57.57907365333787128886141126177.P37
463 2777.69783361.P86
465 (3,5,15,31,93,155) 5581.76261.693648841170 1.5966638725450 1.627655040817361
467   C98
469 (7,67) 937.52529.976457.32924737.2935480 37.104712482697806353.P37
471 (3,157) 9947521.407290125830008994401.516975898656776821074144595127483817001
473 (11,43) P88
475 (5,19,25,95) 1901.5701.36309 01.P62
477 (3,9,53,159) 155845052765866 69.P51
479 2637373.   C94
481 (13,37) 6733.1529581.278574997.P72
483 (3,7,21,23,69,161) 179522067706 9.634145290536014605627260029639586638 4176869
485 (5,97) 3881.5821.168923041923 01.5117158577735 21.41735927244227 21.P30
487 1949.94477.1694761.P87
489 (3,163) 55010483350408487052485570744297.P37
491 141600090215093.1621212105820049.P73
493 (17,29)   C94
495 (3,5,9,11,15,33,45,55,99,165) 1250839826281.P39

497 (7,71)   C88
499 997.492013.3074837.P89
501 (3,167) 9024686010889754273.P51
503 10061.59726221.3956577789.5195875313232444620 9.24300147477009994230 17017501.
     .196745364145020787213452779187514784417
505 (5,101) 44614641121.96070038904 1.P62
507 (3,13,39,169) 1013.1006914877.173408891952128923997917 3.P27
509 P107
511 (7,73) 20441.3545317.78439756057054169.P63
513 (3,9,19,27,57,171) 8209.33857.347813.593029.25355537.P41
515 (5,103) 8438893382141.P72
517 (11,47) 1033.58937.242882935400038849127521.641466124396076970162380 97.P38
519 (3,173) 6229.1174427453.3231124141939501.P44
521 9377.   C105
523 8369.3537386857.   C96
525 (3,5,7,15,21,25,35,75,105,175) 4201.823201.2553601.23169301.82061511001.
     .8481116649425701
527 (17,31) 471137.1635458366076 53.P81
529 (23) 45981737.30621190365773979605100 1.
531 (3,9,59,177) 7433.2043289.2192843129417.P51
533 (13,41) 25647961.14821985227373.1077776264469061.P65
535 (5,107) 2141.84898675907138979050 1.P64
537 (3,179) 11813.142240444249423907190721.P48
539 (7,11,49,77) 6469.18253437603966181.P68
541 54101.12364910977.P98
543 (3,181) 35837.16094521.P64
545 (5,109) 2398001.   C85
547 1093.20266379551129.   C98
549 (3,9,61,183) 6209036 29.58830104 33.1042408369 7.8025631995861.P33
551 (19,29) 3236574 1.1116312758369.P86
553 (7,79) 152629.221201.7998082133.   C78
555 (3,5,15,37,111,185) 40719241.4964932064922 1.299262832090182161.
     .196447231496394995461
557 7532497142267303 09.   C99
559 (13,43) 1117.14533.23417629.433586113.13631732633.
     .2609388705701724726953122152 1.P44
561 (3,11,17,33,51,187) 32083440797931137.P51   C82
563 10133.6281953.1215877129695937786329413 3.
565 (5,113) 6781.66199696261.435832069347090810 01.P60
567 (3,7,9,21,27,63,81,189) 13043111509.4911491214 1.393650430012 1.
     .737066046375289.277193936879118907 21
569 P119

571 P119
573 (3,191) 2670181.4817807925924421.P58
575 (5,23,25,115) 66701.   C88
577 1153.2309.492757.1698689.1240154177.P93
579 (3,193)   C81
581 (7,83) 11621.P99
583 (11,53)   C109
585 (3,5,9,13,15,39,45,65,117,195) 2341.P57
587 3080261369.P113
589 (19,31) 3533.23561.14842491364401.2746348619173417.881811033467161969.P60
591 (3,197) 4729.22221540969737.6556208367360005292317.P44
593 3557.3164852861.1143800237963593361.4315428929594011898689.
.3659508747983000594301.P49
595 (5,7,17,35,85,119) 2381.8310112721.9022425301.2535918135079561.
.61859474392640261.4651115729702571326164661
597 (3,199) 1193.55092353.1226624481649497289976403949.P45   C128
599 4304994428485397.P110
601 22549039789436761.P109
603 (3,9,67,201) 3617.16054441098650821.P64
605 (5,11,55,121) `109981741.P84
607 1213.123829.582721.10223093.28549637.P99
609 (3,7,21,29,87,203) 369282510197.2139244501969.P47
611 (13,47) 157637.25557953333678066429.P92
613
615 (3,5,15,41,123,205) 19681.299259001.1846858344247612322281.P33   C102
617 234461.6643248296130757140737.
619 1237.3354287957.P117
621 (3,9,23,27,69,207) 10656361.17642247580301401.P60
623 (7,89) 115877.92579793601.P95
625 (5,25,125) 5*.532501.   C99
627 (3,11,19,33,57,209) 406191917.7116439969.2126105960876701042477457.P33
629 (17,37) 26417.666111001.31886344849.1405898788412057400553.P76
631 593141.P126
633 (3,211) 155717.9320165401.41773163881.192347474285460831200493920089.P33 C111
635 (5,127) 2553105939466879921.P88   C128
637 (7,13,49,91) 2549.170301425972639233.P85
639 (3,9,71,213) 7669.66490765817401.P71
641 149993.146899717877919281.
643 5077129.
645 (3,5,15,43,129,215) 19148761.72846749180048315217661.P40
647 4381535761.   C126
649 (11,59) 1297.468577.100213776846657651262073.   C90

651 (3,7,21,31,93,217) 5209.23143579913.16312246063516015073.P42   C137
653
655 (5,131) 149341.2901110281.480405122406661.P80   C138
657 (3,9,73,219) 290393.30300841.613192553.2424917505169.22700849209613. C129
.9456946117108451513.7204599162311576189300389
659
661 25117.115013.
663 (3,13,17,39,51,221) 340399154629.54220778462733966380018208818089.P36
665 (5,7,19,35,95,133) 7123481.1655770201.127654132789883268521.P54
667 (23,29) 9337.108713769721.5636517080386800396781.P93   C83
669 (3,223) 13381.46871477.524888033.404275734463249277.P55   C93
671 (11,61) 13421.93941.197273.575717.844117.12239041.   C119
673 282661.47010552106184753.
675 (3,5,9,15,25,27,45,75,135,225) 394201.6641555895901.P57   C124
677 212741833.970100381.   C121
679 (7,97)
681 (3,227) 208109513.17505440236343865677.13165344632908472185900097513564513.P34
683 258173.34482885205518361.205151568022959109.393867451788289513.P86
685 (5,137) 2741.3516376261.12120059685497061.162302467515721218821.P64   C83
687 (3,229) 1373.3331306969.   C93
689 (13,53) 4133.3955824601.8663131853.582233977115909.   C132
691 83466682124689.
693 (3,7,9,11,21,33,63,77,99,231) P76
695 (5,139) 97536301.183060221.3038624652573481.371710122688368821.P66
697 (17,41)
699 (3,233) 13953397457.245701220509.P76   C134
701 42061.96737.242836213.274479353.8302568897206778357.   C101
703 (19,37) 37*.25309.63246097.138259218919774258197.   C102
705 (3,5,15,47,141,235) 28201.P73
707 (7,101) P126
709 63428557.228680861.
711 (3,9,79,237) 212375701.8771441328469.461740953705414265853.   C132
.737688450468098159248993.P32
713 (23,31)   C139
715 (5,11,13,55,65,143) 7096612381.   C91
717 (3,239) 1433.7099733.   C90
719   C150
721 (7,103) 173040421686336917.   C111
723 (3,241) 5303337419059397.   C85
725 (5,25,29,145) 374166701.   C109
727 1453.2909.10177.326233981.   C133
729 (3,9,27,81,243)   C102

731 (17,43) 5849.14621.
733 379693.19600421.21585122600554804312561. — C133
735 (3,5,7,15,21,35,49,105,147,245) 11456220552559241.P55 — C118
737 (11,67)
739 76566426675034441. — C139
741 (3,13,19,39,57,247) 4207397.14085361533757816.69652072271051271698436113.P42 — C138
743 — C155
745 (5,149) — C124
747 (3,9,83,249) 1493.5301729413. — C90
749 (7,107) 4493.61417.16093097817593.P112 — C151
751 551233. — C87
753 (3,251) 12049.22190361600 3409. — C126
755 (5,151)
757 10597.39742501.40252717.15950033877233.P126
759 (3,11,23,33,69,253)
761 36529.580534937.16880910124 6475049. — C92
763 (7,109) 228901.17075535700033.35913189024510 1.20206082340137 67329. — C128
765 (3,5,9,15,17,45,51,85,153,255) 3061.26030477521.72208475461.P56 — C85
767 (13,59) 254016593.
769 50753.129453018593.16085 13413473.3167 9977770 9229. — C138
771 (3,257) P108 — C118
773 4637.164700018 1.12057391 81320733.58580247203154 0853.P116
775 (5,25,31,155) 102301.3524701.20775318670 1.127026950720812 41539401.P81
777 (3,7,21,37,111,259) 3109.
779 (19,41) 17137.7643549.9586373.235997 09605685 3.2977872740805549 53.P101 — C87
781 (11,71) 54293557.
783 (3,9,27,29,87,261) — C139
785 (5,157) 12018978 1.P123 — C106
787 47221.357144232183190 4937. — C141
789 (3,263) — C110
791 (7,113) 216416017. — C133
793 (13,61) 30133.485538041.3772191484024417.4585002595389 57193. — C105
795 (3,5,15,53,159,265) 3181.12721.9927961.3163171441.21702087014490200 77201.P42 — C131
797 1784390549.1518177708621.327867970 6652741. — C145
799 (17,47) 331802328 1.
801 (3,9,89,267) 23063993.P104 — C151
803 (11,73) — C99
805 (5,7,23,35,115,161) 3221.164967041. — C101
807 (3,269) 1613.3229.114593. — C144
809 48541.8706457.31840675992661. — C165
811 48661.
813 (3,271) 11429153.2117130049.568868945 3.P87

815 (5,163) 6521.472048 1.26681866121.P116 — C144
817 (19,43) 166669.2210448070697.40785018272633.P127 — C84
819 (3,7,9,13,21,39,63,91,117,273) 1637.148950950810490737.P70 — C146
821 39409.394375561.14022918848397. — C159
823 29629.4360974949.94207234159104 1.P143
825 (3,5,11,15,25,33,55,75,165,275) — C125
827 183593.625483894296414733995 7. — C126
829 1657.9949.12982141.
831 (3,277) 47877233.1454976293.4687883122606699500317.P76 — C158
833 (7,17,49,119) 1305217182293766 1.
835 (5,167) 52285211663261.
837 (3,9,27,31,93,279) 45197.4603546943052380929.930486103726758084 7793.
.136299772702544437679660333.P42

839 15422497.1356789353 3.
841 (29) 2294249.550797593 33.2301751898421269953.P135 — C161
843 (3,281) 466266959837.2576582465657.818303948755277.238537777797192381.P63 — C164
845 (5,13,65,169) 6761.586281391556115841.P109 — C179
847 (7,11,77,121) 1693.45372265 3.P127 — C175
849 (3,283) 1697.179068261321.P104
851 (23,37) 91909.
853 13649.6918474869.
855 (3,5,9,15,19,45,57,95,171,285) 11352348 1.65772793278 1.P71 — C144
857 — C104
859 108233. — C124
861 (3,7,21,41,123,287) 2400469.P94 — C161
863 327941.9014767341 7.P164 — C117
865 (5,173) — C126
867 (3,17,51,289) 20809.1180853. — C180
869 (11,79) 144253.756551 3.58904308212046650885354 69913. — C100
871 (13,67) 151553.
873 (3,9,97,291) 5237. — C181
875 (5,7,25,35,125,175)
877 1753.
879 (3,293) 3383288581.153424520919 61. — C165
881 264301.1866056170744477.P164 — C113
883 5297.
885 (3,5,15,59,177,295) 3541.678264 1.101501176368069526561.P67 — C129
887 74509.2498744410754149.
889 (7,127) 1777.5333.37337.195581.P142
891 (3,9,11,27,33,81,99,297)
893 (19,47) 54649813.176722963 6313261.P150
895 (5,179) 3306742181.11376008838 1.

897 (3,13,23,39,69,299) 308569. C105
899 (29,31) C176
901 (17,53) 21230971591957817. C157
903 (3,7,21,43,129,301) 469561.2494887570160189.P85
905 (5,181) 9567661.1596164984379521.P129
907 2069773.153448073.85218882568302661649153.P152
909 (3,9,101,303) 92717.9059093. C114
911 107578169. C183
913 (11,83) 19159039596371703036913.P149
915 (3,5,15,61,183,305) 61*.16594230150241. C86
917 (7,131) 55021.329019611.123643882069.P140
919 23893.36761.231589. C178
921 (3,307) 7369. C125
923 (13,71) 50512097. C168
925 (5,25,37,185) 3701.33301.19531306748486501. C127
927 (3,9,103,309) 3709.63879037028000297. C108
929 11149. C190
931 (7,19,49,133) 12326441.64474965673.7288331127303673.417739114281337726513.P104
933 (3,311) 6923104559934229.417557731140104453. C97
935 (5,11,17,55,85,187) 54323672041.18705221788801.P111
937 1873.449180933.7643696695096956649.15970565008944994529.P146
939 (3,313) 54059112661. C120
941 2981089. C190
943 (23,41) C185
945 (3,5,7,9,15,21,27,35,45,63,105,135,189,315) 139861.145150780364101.
.5362483966272446452741.P50

947 93251696081. C187
949 (13,73) 5693.10543705069.75350070457. C157
951 (3,317) 6284209.2954909161. C116
953 148320002521. C188
955 (5,191) 2177401.952879788356681. C138
957 (3,11,29,33,87,319) 932117.P111
959 (7,137) 966236081213127153. C152
961 (31) C195
963 (3,9,107,321) C133
965 (5,193) 640960721.36836894821.3446650156490167901. C123
967 1933.27930829.193208122057.10999042492449833. C164
969 (3,17,19,51,57,323) 40697.335273.796517.2854472449.9858431581.
.4035936389347157.P70
971 52433.104869.38482544680537.17970660075828673.
.619802607259514583330235693729. C134
973 (7,139) 15569.15963037.P162

975 (3,5,13,15,25,39,65,75,195,325) 308569. C101
977 42989.2082966602729.34260977417268029. C171
979 (11,89) 89*.751290473.3619658657. C164
981 (3,9,109,327) 616069.2478834351680013.19639210805443935117. C93
983 5897.2140973.P195
985 (5,197) C164
987 (3,7,21,47,141,329) C116
989 (23,43) 25770975208014221. C177
991 C207
993 (3,331) 7564673. C132
995 (5,199) 83629033601. C155
997 1993.5976017.98102710780517. C184
999 (3,9,27,37,111,333) 1997.16061684237. C122

TABLE 3　LUCAS FACTORIZATIONS　　2 ≤ n ≤ 500

n	Prime Factors		n	Prime Factors
2	3		37	5401521
3	2.2		38	(2) 29134601
4	7		39	(3,13) 79.859
5	11		40	(8) 1601.3041
6	(2) 2*.3*		41	370248451
7	29		42	(2,6,14) 83.1427
8	47		43	6709.144481
9	(3) 19		44	(4) 263.881.967
10	(2) 41		45	(3,9) A.B
11	199		A	(15) 181
12	(4) 2*.23		B	(5) 541
13	521		46	(2) 4969.275449
14	(2) 281		47	6643838879
15	(3,5) 31		48	(16) 2*.769.3167
16	2207		49	(7) 599786069
17	3571		50	(2,10) 401.570601
18	(2,6) 3*.107		51	(3,17) 919.3469
19	9349		52	(4) 103.102193207
20	(4) 2161		53	11921851371
21	(3,7) 211		54	(2,6,18) 3*.11128427
22	(2) 43.307		55	(11) A.B
23	139.461		A	39161
24	(8) 2*.1103		B	(5) 11*.331
25	(5) A.B		56	(8) 10745088481
A	101		57	(3,19) 229.95419
B	151		58	(2) 347.1270083883
26	(2) 90481		59	709.8969.336419
27	(3,9) 5779		60	(4,12,20) 241.20641
28	(4) 7*.14503		61	5600748293801
29	59.19489		62	(2) 3020733700601
30	(2,6,10) 2521		63	(3,7,9,21) 1009.31249
31	3010349		64	127.186812208641
32	1087.4481		65	(13) A.B
33	(3,11) 9901		A	(5) 24571
34	(2) 67.63443		B	131.2081
35	(7) A.B		66	(2,6,22) 261399601
A	(5) 71		67	4021.24994118449
B	911		68	(4) 23230657239121
36	(4,12) 103681		69	(3,23) 691.1485571
			70	(2,10,14) 12317523121
			71	688846502588399
			72	(8,24) 1074957121

73 151549.11899937029
74 (2) 11987.81143477963
75 (3,5,15) A.B
 A (25B) 12301
 B (25A) 18451
76 (4) 10913.6396980401
77 (7,11) 229769.9321929
78 (2,6,26) 12280217041
79 3236112267225 9149
80 (16) 2372514562 6561
81 (3,9,27) 3079.62650261
82 (2) 163.800483.350207569
83 35761381.6202401259
84 (4,12,28) 167.65740583
85 (17) A.B
 A (5) 1158551
 B 12760031
86 (2) 31319571151657828 1
87 (3,29) 349.947104099
88 (8) 9305824 1.562418561
89 179.2223550264098836 9
90 (2,6,10,18,30) 1078334208 1
91 (7,13) 689667151970 161
92 (4) 253367.950637219386 3
93 (3,31) 63799.35510749
94 (2) 563.5641.4632894751907
95 (19) A.B
 A 87382901
 B (5) 19141611
96 (32) 2*.1186257524870 3
97 3299.56678555502141579
98 (2,14) 5881.6102530946904 1
99 (3,9,11,33) 991.2179.1513909
100 (4,20) 9125201.5738108801
101 809.7879.201062946718741
102 (2,6,34) 409.6626511844 9
103 619.1031.525748002643896 1
104 (8) 3329.106513889.325759201

105 (3,7,21) A.B
 A (15,35B) 21211
 B (5,35A) 767131
106 (2) 1483.2969.107601236772040 3
107 47927441.4798364833 12919
108 (4,12,36) 6263.17796216736 7
109 128621.788071.59398511121 1
110 (2,10,22) 5999685492865680 1
111 (3,37) 4441.146521.1121101
112 (16) 223.449.1154149773784223
113 4126704278449210374 70771
114 (2,6,38) 227.26449.212067587
115 (23) A.B
 A (5) 1151.324301
 B 5981.686551
116 (4) 299281.8344284108795067 21
117 (3,9,13,39) 1052645985555841
118 (2) 15247723.10004958719759838 7
119 (7,17) 239.10711.2793273243980 9
120 (8,24,40) 2373590045232 1
121 (11) 9742073320849186904419 9
122 (2) 19763.21291929.248486601 19363
123 (3,41) 4767481.7188487771
124 (4) 743.467729.33758740830460 183
125 (5,25) A.B
 A 2814337800 1
 B 251.112128001
126 (2,6,14,18,42) 1461601.764940961
127 509.5081.487681.13822681.1995424 1
128 119809.469816763452337987558 3
129 (3,43) 308311.76188259140 1
130 (2,10,26) 3121.424264760414508 01
131 1049.41498698461.547733262009 1
132 (4,12,44) 5281.66529.15220444 9
133 (7,19) 10694421739.21529586504 59
134 (2) 6163.20191246924 9.27056226821 63
135 (3,9,27) A.B
 A (5,45B) 271.119611
 B (15,45A) 811.42391
136 (8) 56262783728329194013765488 1
137 541721291.789824878709305828 1
138 (2,6,46) 16561.162563.104376658 7

139 30859.2532791129.14331800109223159
140 (4,20,28) 118021448662479038881
141 (3,47) 79099591.139509555271
142 (2) 283.569.2820403.9799987.35537616083
143 (11,13) 1957099.2120119.178471380021
144 (16,48) 11556157812483852881
145 (29) A.B
   A 13221547510611
   B (5) 12019635394
146 (2) 29201.3712857850184727260788881
147 (3,7,21,49) 65269.620929.8844991
148 (4) 10661921.114087288048701953998401
149 952111.4434539.3263039535803245519
150 (2,6,10,30,50) 601.871295471172401
151 1511.109734721.21753300018483577477 9
152 (8) 562766385967.2206456200865197103
153 (3,9,17,51) 13159.8293976826829399
154 (2,14,22) 1525246790016495022575760603
155 (31) A.B
   A 311.2913888651
   B (5) 823837075741
156 (4,12,52) 1249.9449184218355 1489
157 39980051.16188856575286517818849171
158 (2) 21803.5924683.146298924491847155248 01
159 (3,53) 78546145238192991824 51
160 (32) 641.87813222404439748742016 01
161 (7,23) 1289.1917511.96584086226852 9759
162 (2,6,18,54) 3*.18286203361.6782976947987
163 1043201.66015011686454671192230445929
164 (4) 268457141143002702847905930965201
165 (3,11,33) A.B
   A (5,55B) 1550853481
   B (15,55A) 5116452
166 (2) 6464041.2453296171611034124775964608 1
167 766531.10384992789358454232012732 7909
168 (8,24,56) 115613939510481515041
169 (13) 5961078143640896710403942208493 29
170 (2,10,34) 1361.40801.11614654211954032961
171 (3,9,19,57) 19*.162451.1617661.7038398989
172 (4) 12611771191591164678440459440335 21
173 78889.6248069.16923049609.1712461702613 59
174 (2,6,58) 97787.528295667.563971096 9

175 (5,7,35) A.B
   A (25B) 54601.51636551
   B (25A) 560701.7517651
176 (16) 1409.6086461133983.3197028476422587 83
177 (3,59) 10884439.105117617351706859
178 (2) 52805445356674722912771491192965546201
179 359.10667373847220321.66932254279484647441
180 (4,12,20,36,60) 8641.1337376376598688 1
181 97379.2137326115041977 51.32242356485644069
182 (2,14,26) 232961.6110578634294886534808481
183 (3,61) 1468623970953397571590928 9
184 (8) 367.37309023160481.44172095810038191710 3
185 (37) A.B
   A (5) 265272771839851
   B 2918000731816531
186 (2,6,62) 15917507.85988642159352704 3
187 (11,17) 1871.9056742344085065265097390431
188 (4) 18049.100769.153037630649666194962091443041
189 (3,7,9,21,27,63) 379.85429.912871.12587400 01
190 (2,10,38) 2281.4561.782747561.1747955349080 1
191 22921.39558647250688329211910257559954057439
192 (64) 2*.383.5662847.6803327.19073614849
193 303011.76225351.935527789314618720740315126 1
194 (2) 195163.4501963.5644065667.235011702700054494 7
195 (3,13,39) A.B
   A (15,65B) 2731.86658 1
   B (5,65A) 1951.37928281

196 (4,28) 7*.3016049.61008047911634738722316 29367
197 31498587119911339.470190722289506835024988 9
198 (2,6,18,22,66) 11166702227.10763128994543 63
199 2389.4503769.3603696041481196981078471182 89
200 (8,40) 124001.696600131604388341748173560 01
201 (3,67) 268603942421.9400942999674 91
202 (2) 54749741849614466654316761383509017829700 1
203 (7,29) 29*.2748232098283374889444 28997628 2269
204 (4,12,68) 1223.470039965023902754923207
205 (41) A.B
   A 1231.11135980068237 1
   B (5) 5741.2170732312961
206 (2) 81163.462353921445862223671914407266 72730987
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468 (4,12,36,52,156) P61
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470 (2,10,94) 16921.276222761.P65
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472 (8) 1889.
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474 (2,6,158) 947.10279163.8411395441.592 2309413062354009.P27
475 (5,19,95) A.B
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