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Sierpinshi

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divisible by $M_{101}$; hence the result that $M_{101}$ is a composite number.

In order to verify that the number $M_{3217}$ having 969 digits is a prime, it has been necessary to show that the number $r_{3216}$ (corresponding to $M_{3217}$) is divisible by $M_{3217}$. It has required about several thousands squaring operations and then dividing the squares obtained, having not more than 969 digits, by the number $M_{3217}$, which the existing electronic computers have made possible.

We write here only the first and the last few digits of this number. It is $M_{3217} = 259,117,0...09,315,071$. All the 969 digits of this number are given in the journal *Mathematical Tables and other Aids to Computation* 12(1958), p. 60. It is also mentioned there that verification that the number $M_{3217}$ is a prime, took the Swedish electronic computer BESK $5\frac{1}{2}$ hours.

The conjecture has been advanced that if the Mersenne number $M_n$ is a prime then the number $M_{M_n}$ is also a prime. This is true for the four smallest Mersenne numbers, but for the fifth Mersenne prime number, i.e. for the number $M_{13} = 8191$, as D. H. Wheeler calculated in 1953, it is not true, because $N_{M_{13}} = 2^{8191}-1$ (having 2466 digits) is composite (see Robinson [1]). The verification of this (with the help of the theorem of Lucas–Lehmer) required one hundred hours work on an electronic computer. We do not know any of the prime divisors of this composite number. However, in 1957 it was discovered (see Robinson [2]) that although the number $M_{17}$ is prime, the number $M_{M_{17}}$ is composite, being divisible by $1768(2^{17}-1)+1$, and also that, although the number $M_{19}$ is prime, the number $M_{M_{19}}$ is composite, being divisible by $120(2^{19}-1)+1$.

The conjecture has also been advanced (and so far not disproved) that the numbers $q_0, q_1, q_2, ...$ where $q_0 = 2$ and $q_{n+1} = 2^{q_n}-1$ for $n = 0, 1, 2, ...$ are all prime. This is true for the numbers $q_n$ where $n \leq 4$, but the number $q_5$ has, as is easy to calculate, more than $10^{37}$ digits, and so we are not able to write it, let alone verifying whether or not it is prime.

We remember in connection with Mersenne numbers the

finding of even perfect number. Originally Euclid gave the following method of obtaining even perfect numbers: we calculate the sum of the successsive terms of the geometric progression $1, 2, 2^2, 2^3, \ldots$ If such a sum is a prime number, we multiply it by the last term. We have thus obtained a perfect number. Euler proved that this method allows us to get all even perfect numbers.

In other words, this shows that *all even perfect numbers are of the form* $2^{p-1}M_p$, *where* $M_p$ *is a prime number.*

It follows that we know as many even perfect numbers as there are known Mersenne primes, namely at present 20.

The smallest perfect number is $2M_2 = 6$, the greatest known perfect number is $2^{4422}(2^{4423}-1)$. We do not know odd perfect numbers, we only know (Kanold [1]) that, if they exist, they are very large (greater than $10^{20}$).

As for Mersenne numbers, we further mention that F. Jakóbczyk has advanced the conjecture that, if $p$ is a prime number, then the number $M_p$ is not divisible by any square of a prime number. A. Schinzel has put the question whether there exist infinitely many Mersenne numbers which are the products of different prime numbers.

### 26. Prime numbers in several infinite sequences

The question whether or not a given infinite sequence, defined in even a simple way, will contain infinitely many primes is in general very difficult. As we have already said, we do not know if sequences like $n^2+1$, $n!+1$, $n!-1$, $2^n+1$, $2^n-1$ (for $n = 1, 2, \ldots$) contain infinitely many primes. We also do not know if the infinite sequence 1, 11, 111, 1111, ... contains an infinity of primes. Similar is the case of the so-called *Fibonacci numbers* $u_n$ ($n = 1, 2, \ldots$), defined by the conditions

$$u_1 = u_2 = 1 \quad \text{and} \quad u_{n+2} = u_n + u_{n+1} \quad (\text{for} \quad n = 1, 2, \ldots).$$

The first few terms of this sequence are the numbers

$$u_1 = 1, \, u_2 = 1, \, u_3 = 2, \, u_4 = 3, \, u_5 = 5, \, u_6 = 8,$$
$$u_7 = 13, \, u_8 = 21, \ldots$$