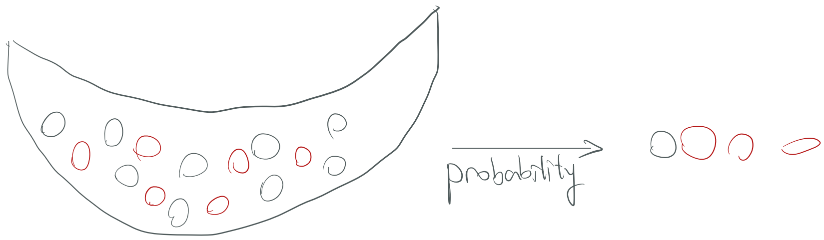




Example: Coin toss

$\Omega : \{H, T\}$

$\mathcal{F} : \{\emptyset, \{H\}, \{H,T\}\}$

$P : \mathcal{F} \to [0, 1]$

$(\Omega, \mathcal{F}, P)$ is the probability space

- $P(\Omega) = 1$
- $P(A) \geq 0$ , $A \in \mathcal{F}$
- $P(\bigcup_{i=1}^{n} A_i) = \sum_{i=1}^{n} P(A_i)$ if $A_i \cap A_j = \emptyset$

- $P(A^c) = 1 - P(A)$
- $P(A) \leq 1$

- $P(\emptyset) = 0$
- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- $P(A|B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{P(B|A)P(A)}{P(B)}$ ← Bayes' Rule
- $P(A \cap B) = P(A) \cdot P(B)$ if $A, B$ are independent

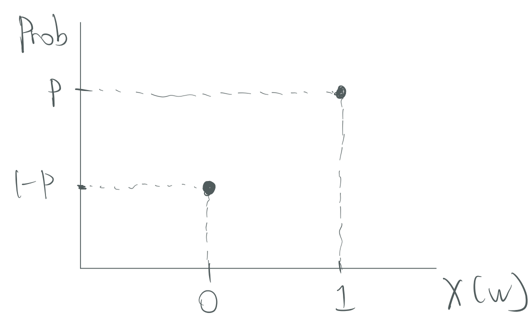
  $P(A|B) = P(A)$ if independent.

A random variable $X : \Omega \to \mathbb{R}$

$$X(w) = \begin{cases} 1 & \text{if } w = \{H\} \\ 0 & \text{if } w = \{T\} \end{cases}$$

$$\begin{aligned} P_x(1) &= P(X=1) \\ &= P(\{w \mid X(w) = 1\}) \\ &= P(\{H\}) \end{aligned}$$

Distribution of our coin toss:




Bernoulli Trial

$X_1, X_2, \ldots\ldots$

Probability that it takes K tosses to get the first H.

$$P_{X_1}(0)\, P_{X_2}(0)\, P_{X_3}(0), \cdots P_{X_k}(1)$$

$$= (1-p)^{k-1}\, p$$

$\uparrow$

Geometric distribution

Probability of getting K H's in N toss:

$$P_n(k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$\uparrow$ Binomial Distribution.

Flip a coin over a large period of time, on average 60 H's/hour

Poisson Distribution:

$$P(k) = \frac{\lambda^k e^{-\lambda}}{k!} \quad \text{where } \lambda \text{ is the rate}$$

With a continuum of possible outcomes,

$P(X<t) \leftarrow$ continuous dist

P(time between consecutive H's)

$$= e^{-\mu t}$$

Probability Density function

$$P(t \in [t_1, t_2]) = \int_{t_1}^{t_2} \mu e^{-\mu t}\, dt$$

$$= P(t \geq t_1,\ t \leq t_2)$$