Network Analysis:

Example Networks Internet:
- What will internet traffic through Belgium look like today?
- Anomalous traffic patterns
- Model of the internet

Biology:
- Are certain patterns of interactions among genes more common than expected?
- Which regions of the brain communicate for a given task?

Social:
- Who is friends with whom?
- Who are the influencers?
- What social groups are present?
- How does information flow through the network?

Graphs:
We use Graphs and Graph Theory to model / represent and analyse Networks.
A Graph is comprised of Nodes / Vertices connected by Edges. These Edges can be undirected (where edges are symmetrical connections between Nodes - A is connected to B then B is connected to A) or directed

The degree of a Node is the number of edges connected to it.
A path between two Nodes is a sequence of edges that joins these two Nodes.
A graph is called complete if there is an edge between every pair of Nodes.

Network Characteristics:

Distribution of edges / node degrees:
- Anomaly detection
- Ranking / Recommendation
- Describe flow through the network

Centrality of a node:
- Identify influencers
- Discover groups / clusterings
- How nodes affect connectivity / flow

Network Analysis:

Networks / Graphs are generated by processes or functions on its nodes / edges. For example: creating a new account (adding a node), making friends / following / connecting (adding an edge), etc.

The state of a Network / Graph at a given point in time is the stochastic result of these processes.

One way we can model the characteristics from the previous slide is by
modeling the state of the Graph (i.e. finding the random process that
generated the given Graph)

Random Graph Model:
Both methods are related in that: G(N,p) conditioned on the event that it has M edges, is equal in distribution to G(N, M).
Proof:

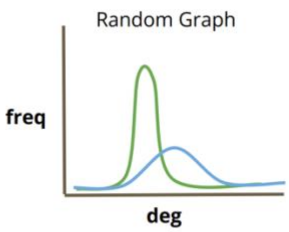
$$
\begin{aligned}
P(G(N,p) \,|\, |E_{G(N,p)}| = M) &= \frac{P(G(N,p), |E_{G(N,p)}| = M)}{P(\,|E_{G(N,p)}| = M)} \\
&= \frac{p^M (1-p)^{\binom{N}{2}-M}}{\binom{\binom{N}{2}}{M} p^M (1-p)^{\binom{N}{2}-M}} \\
&= \binom{\binom{N}{2}}{M}^{-1}
\end{aligned}
$$

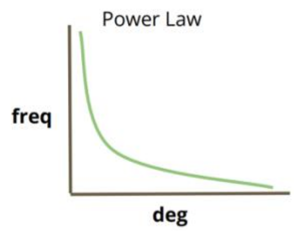
Random Graph Model:
What is the distribution of the degree of Nodes?

$$P(deg(v) = k) = \binom{N-1}{k} p^k (1-p)^{N-1-k}$$

Power Law
Most real-life social networks follow have a degree distribution following a power law of the form

$\mathbf{P(k) = Ck^{-\alpha}}$ for some constants C & α





Metrics on Graphs:
Diameter Let dij be the shortest path between node i and node j. The diameter of G is defined as

$$\mathbf{Diam(G) = max_{ij}\, d_{ij}}$$

Density
Let N = # Nodes, M = # Edges Density = 2M / N(N-1)

Degree Centrality
The more central a node is, the higher its number of connections

$$C_{deg}(v) = Deg(v)$$

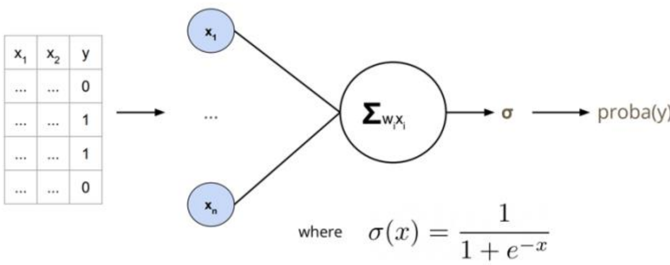
Closeness Centrality
The more central a node is, the closer it is to all other nodes

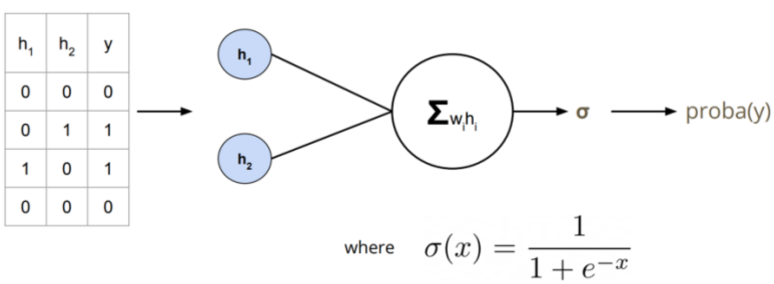
$$C_{close}(v) = \frac{1}{\sum_u d(u, v)}$$

Given m Rankings $w_1, \dots, w_m$, we can generate an aggregate ranking w^*

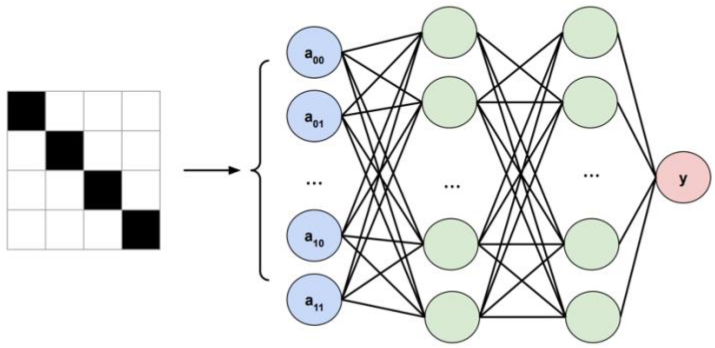
$$w^* = \arg\max_w \sum_{i=1}^{m} d_\tau(w, w_i)$$

Neural Networks:
Logistic Regression Re-Revisited





Neural Networks - Convolutional Neural Networks



Creating such a filter allows us to:
1. Reduce the number of weights
2. Capture features all over the image
The process of applying a filter (or kernel) is called a convolution

Recurrent Neural Networks
Handling sequences of input.
Intuition: What a word is / might be in a sentence is easier to figure out if you know the words around it.
Applications:
1. Predicting the next word
2. Translation
3. Speech Recognition
4. Video Tagging