

Final exam

1. Find an equation of the tangent line to the parametric curve given by

$$x = \cos(t) + t, \quad y = \sin(t) - t^2,$$

at $t = 0$.

2. Determine an equation of the tangent line to the polar curve

$$r = \sin(2\theta),$$

at $\theta = \pi/4$.

3. Does the sequence given by

$$a_n = \frac{3n^3 - 4n}{7n^3 + 9},$$

converge or diverge? In case of convergence, calculate the limit.

4. Determine if the following series converge or diverge.

a. $\sum_{n=1}^{\infty} \frac{4n \sin(n\pi/2)}{n^4 - 5}$

b. $\sum_{n=1}^{\infty} \frac{3^n}{n!}$

c. $\sum_{n=1}^{\infty} \frac{n^n}{3^{n^2}}$

d. $\sum_{n=1}^{\infty} \frac{1}{n^{\frac{9}{10}}}$

5. Compute the interval and radius of convergence for the power series

$$\sum_{n=0}^{\infty} \frac{(x-1)^n}{4^n}.$$

Good Luck!