Mathematics problems

1 Elementary algebra

Problem 1.1. Simplify

$$\frac{y^{58}}{y^4 \cdot y^{12}} \quad = y^{42}$$

Problem 1.2. Solve for x:

$$8^2 \cdot 2^x = 2^9 \quad x=3$$

Problem 1.3. Calculate the missing value. If $\frac{x}{y}$ is 3, then $x^{-2}y^2 = \frac{1}{9}$

Problem 1.4. Calculate

$$\frac{\sqrt{2^{13}}}{\sqrt{8^3}} = 4$$

Problem 1.5. True or False (x and y and z are real numbers):

(a) $x + y = y + x$ T

(b) $x(y + z) = xy + xz$ T

(c) $x^{y+z} = x^y x^z$ T

(d) $\frac{x^y}{x^z} = x^{y-z}$ T

Problem 1.6. Find the solution for the equality below:

$$\frac{x^2 - 25}{x - 5} = 3 \quad x=-2$$

2 Functions of one variable

Problem 2.1 (Based on SYD 2.5.6). The relationship between temperatures measured in Kelvin and Fahrenheit is linear. 0 K is equivalent to -460°F and 1000 K is the same as 1340°F. Which temperature is measured by the same number on both scales? 575K = 575F

Problem 2.2. Take the following function $f(x) = 2x + 3$. Find y if $f(y) = 17$. $y=7$

Problem 2.3. Find all values of x that satisfy:

$$3^{2x^2-4x+3} = 27 \quad x_1 = 0, \; x_2 = 2$$

Problem 2.4. Solve the following problem. If the annual GDP growth of a country is 1%, how long does it take the economy to double its GDP? $\log_{1.01}(2) \approx 69.66$ years

Problem 2.5. Calculate the following value

$$\ln\left(\frac{e^2}{e^3}\right) = -1$$