

### 3 Calculus

**Problem 3.1.** Calculate the following sum

$$\sum_{i=0}^{\infty} \left( \frac{1}{6^i} + 0.25^i \right) \quad \frac{38}{15}$$

**Problem 3.2.** Find the following limit

$$\lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3} = 6$$

**Problem 3.3.** Find the slope of the function  $f(x) = x^3 - 4$  at  $(-1, -5)$ .  $f'(-1) = 3$

**Problem 3.4.** Find the following derivative

$$\left( \frac{x^2 + 3}{x + 2} \right)' \quad \frac{(x+3)(x-1)}{(x+2)^2}$$

**Problem 3.5.** Find the second derivative of

$$f(x) = x^7 + 4x^2 \quad f''(x) = 42x^5 + 8$$

**Problem 3.6.** Find the derivative of

$$f(x) = \frac{x^4 + 4^x}{\ln(x)} \quad \frac{(4x^3 + 4^x \ln 4) \ln x - \frac{1}{x}(x^4 + 4^x)}{\ln(x)^2}$$

**Problem 3.7.** Consider the following function. Find all of its stationary points and classify them as local minima, local maxima or inflection points.

$$f(x) = 3x^3 - 9x \quad \begin{array}{ll} x = -1 & \text{local maxima} \\ x = 1 & \text{local minima} \end{array}$$

**Problem 3.8.** Let  $f(x, y) = x^2 + 2y^3$ . Calculate  $f(2, 3) = 58$

**Problem 3.9.** Consider the following function:  $f(x, y) = \ln(2x - y)$ . For what combinations of  $x$  and  $y$  is this function defined?  $\frac{y}{x} < 2$

**Problem 3.10.** Find all partial derivatives of the following function:  $\frac{\partial f(x, y)}{\partial x} = e^y \cdot 5x^4 + y^3 \cdot 2x$   
 $f(x, y) = x^5 e^y + x^2 y^3$   
 $\frac{\partial f(x, y)}{\partial y} = x^5 \cdot e^y + x^2 \cdot 3y^2$

**Problem 3.11.** Find the local maxima or minima of the following function:

$$f(x, y) = \sqrt{xy} - 0.7x - 0.7y \quad *y$$

**Problem 3.12.** Solve the following constrained optimization problem using Lagrange's method:  $\max x^2 y^2$   
s.t.  $x + y = 10$

$$x = y = 5 \quad \max x^2 y^2 = 625$$

### 4 Linear algebra

**Problem 4.1.** Take the following matrices:

$$A = \begin{bmatrix} 2 & 3 \\ 4 & 1 \\ 1 & 2 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 4 & 1 \\ 2 & 1 & 2 \end{bmatrix}$$

What is  $A \cdot B$ ?

$$\begin{bmatrix} 8 & 11 & 8 \\ 6 & 17 & 6 \\ 5 & 6 & 5 \end{bmatrix}$$