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Date :	Oct 16 2024	**Board** :	CBSE
Class :	12	**Session # :**	8
Subject :	Mathematics	**Assignment # :**	DeA1
Topic :	Differential Equations	**Subtopic(s) :**	Variable Separable
Lecture #:	1		

Variable Separable:

A **variable separable differential equation** is a first-order differential equation in which the variables can be separated on opposite sides of the equation, allowing the equation to be integrated directly. The general form of a variable separable equation is:

$$\frac{dy}{dx} = g(x)h(y)$$

Example

Consider the equation:

$$\frac{dy}{dx} = 3x^2y$$

Solution:

1. Separate variables:

$$\frac{1}{y}dy = 3x^2dx$$

2. Integrate both sides:

$$\int \frac{1}{y}dy = \int 3x^2dx$$

- The left-hand side becomes $\ln|y|$.
- The right-hand side becomes $x^3 + C$.

Thus, we have:

$$\ln|y| = x^3 + C$$

3. Solve for y:

$$y = e^{x^3+C} = e^C e^{x^3}$$

Let $A = e^C$ (a constant), so:

$$y = Ae^{x^3}$$

This is the general solution.

Session Board:

$$\frac{dy}{dx} + y = 1$$

$$\Rightarrow \frac{dy}{dx} = 1-y$$

$$\Rightarrow dx = \frac{dy}{1-y}$$

$$\Rightarrow \int dx = \int \frac{dy}{1-y} \qquad \frac{1}{1-y}$$

$$\Rightarrow x = -\log|1-y| + C$$

$$\int \frac{1}{x}\,dx = \log|x|$$

$$\int \frac{1}{2x+3}\,dx$$

$mx+C$

$$\int \frac{1}{2x+3}\,dx = \frac{\log|2x+3|}{2}$$

x

Put $2x+3 = t$

$$2dx = dt$$

$$dx = \frac{dt}{2}$$

$$\int \frac{1}{1-y}\,dy$$

$$1-y = t$$

$$-dy = dt$$

$$dy = -dt$$

$$\int -\frac{1}{t}\,dt$$

$$\Rightarrow -\log|t| + C$$

$$\Rightarrow -\log|1-y| + C$$

$$x = -\log|1-y| + C$$

$$x - C = -\log|1-y|$$

$$C - x = \log_e|1-y|$$

$$\downarrow$$

$$e^{(c-x)} = |1-y|$$

$$\log_e m = p$$

$$\boxed{e^p = m}$$

$$e^c \times e^{-x} = |1-y|$$

$$e^c \times e^{-x} = (1-y)$$

$$y = 1 - e^c \times e^{-x} \rightarrow \boxed{1 - C_1 e^{-x}}$$

$$\boxed{y = \frac{e^x - e^c}{e^x}}$$

$$C_1$$

$$e^c = C_1$$

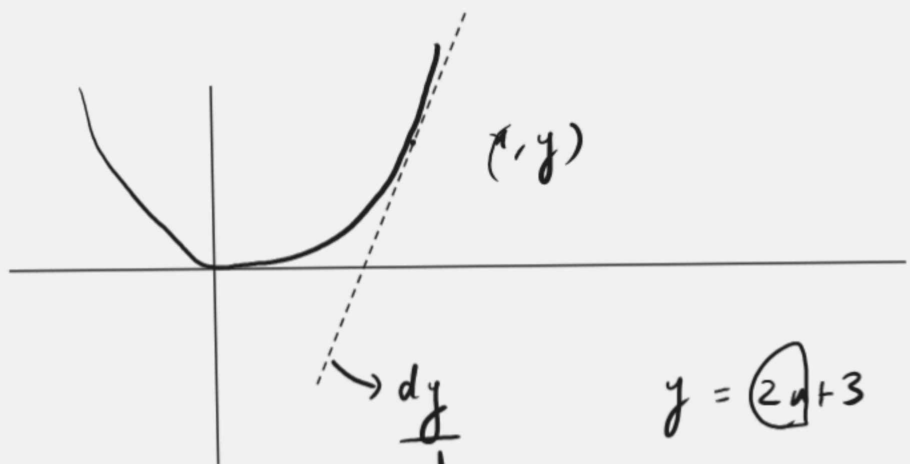
$$\boxed{\frac{e^x - c_1}{e^x}}$$

$$\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$$

$$\int \frac{dy}{1+y^2} = \int \frac{dx}{1+x^2}$$

$$\boxed{\tan^{-1} y = \tan^{-1} x + C}$$

(variable separable)



$\frac{dy}{dx}$

$y = 2x + 3$

$(2,3)$: $\frac{dy}{dx} = 2 =$

$$\frac{y^3}{3} = x^2 + c$$

$$(-2, 3)$$

$$\frac{3^3}{3} = (-2)^2 + C$$

$$9 = 4 + C$$

$$\boxed{5 = C}$$

$$\boxed{\frac{y^3}{3} = x^2 + 5}$$

$$y = (3x^2 + 15)^{\frac{1}{3}}$$

$$20 \log(P) = t + C$$

$$P = ?$$

$$P = e^{\frac{t}{20}} \times e^{C_1} \rightarrow C$$

$$P = Ce^{\frac{t}{20}}$$

initial value of

$$P_1 = \text{Rs } 1000$$

$$t_1 = 0$$

$$1000 = C e^{\frac{0}{20}}$$

$$\boxed{1000 = C \times 1}$$

$$P = 1000\, e^{t/20}$$

$$t_2 = ?\ ,\quad P_2 = 2 \times 1000 = 2000$$

$$2000 = 1000\, e^{\frac{t_2}{20}}$$

$$2 = e^{t_2/20}$$

Take log.

$$\log(2) = \log_e e^{t_2/20}$$

$$\log(2) = \frac{t_2}{20} \log_e e$$

$$\log_e(2) = \frac{t_2}{20} \times 1$$

$$\boxed{20 \log_e 2 = t_2}$$

Now complete this assignment #DeA1:

#DeA1:

Example 4 Find the general solution of the differential equation $\frac{dy}{dx} = \frac{x+1}{2-y}, (y \neq 2)$

Example 5 Find the general solution of the differential equation $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$.

Example 6 Find the particular solution of the differential equation $\frac{dy}{dx} = -4xy^2$ given that $y = 1$, when $x = 0$.

Example 7 Find the equation of the curve passing through the point (1, 1) whose differential equation is $x\,dy = (2x^2 + 1)\,dx$ $(x \neq 0)$.

Example 8 Find the equation of a curve passing through the point (–2, 3), given that the slope of the tangent to the curve at any point (x, y) is $\frac{2x}{y^2}$.

Example 9 In a bank, principal increases continuously at the rate of 5% per year. In how many years Rs 1000 double itself?

For each of the differential equations in Exercises 1 to 10, find the general solution:

1. $\frac{dy}{dx} = \frac{1-\cos x}{1+\cos x}$

2. $\frac{dy}{dx} = \sqrt{4-y^2}$ $(-2 < y < 2)$

3. $\frac{dy}{dx} + y = 1\,(y \neq 1)$

4. $\sec^2 x \tan y\, dx + \sec^2 y \tan x\, dy = 0$

5. $(e^x + e^{-x})\, dy - (e^x - e^{-x})\, dx = 0$

6. $\frac{dy}{dx} = (1+x^2)(1+y^2)$

7. $y \log y\, dx - x\, dy = 0$

8. $x^5 \frac{dy}{dx} = -y^5$

9. $\frac{dy}{dx} = \sin^{-1} x$

10. $e^x \tan y\, dx + (1 - e^x) \sec^2 y\, dy = 0$

For each of the differential equations in Exercises 11 to 14, find a particular solution satisfying the given condition:

11. $(x^3 + x^2 + x + 1)\dfrac{dy}{dx} = 2x^2 + x$; $y = 1$ when $x = 0$

12. $x(x^2 - 1)\dfrac{dy}{dx} = 1$; $y = 0$ when $x = 2$

13. $\cos\left(\dfrac{dy}{dx}\right) = a$ $(a \in \mathbf{R})$; $y = 1$ when $x = 0$

14. $\dfrac{dy}{dx} = y\tan x$; $y = 1$ when $x = 0$

15. Find the equation of a curve passing through the point (0, 0) and whose differential equation is $y' = e^x \sin x$.

16. For the differential equation $xy\frac{dy}{dx}=(x+2)(y+2)$, find the solution curve passing through the point (1, –1).

17. Find the equation of a curve passing through the point (0, –2) given that at any point (x, y) on the curve, the product of the slope of its tangent and y coordinate of the point is equal to the x coordinate of the point.

18. At any point (x, y) of a curve, the slope of the tangent is twice the slope of the line segment joining the point of contact to the point (– 4, –3). Find the equation of the curve given that it passes through (–2, 1).

19. The volume of spherical balloon being inflated changes at a constant rate. If initially its radius is 3 units and after 3 seconds it is 6 units. Find the radius of balloon after t seconds.

20. In a bank, principal increases continuously at the rate of r% per year. Find the value of r if Rs 100 double itself in 10 years ($\log_e 2 = 0.6931$).

21. In a bank, principal increases continuously at the rate of 5% per year. An amount of Rs 1000 is deposited with this bank, how much will it worth after 10 years ($e^{0.5} = 1.648$).

22. In a culture, the bacteria count is 1,00,000. The number is increased by 10% in 2 hours. In how many hours will the count reach 2,00,000, if the rate of growth of bacteria is proportional to the number present?

23. The general solution of the differential equation $\frac{dy}{dx}=e^{x+y}$ is

(A) $e^x + e^{-y} = C$ (B) $e^x + e^y = C$

(C) $e^{-x} + e^y = C$ (D) $e^{-x} + e^{-y} = C$

End