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Date :	Oct 2 2024	**Board** :	CBSE
Class :	12	**Session # :**	2
Subject :	Mathematics	**Assignment # :**	IA3
Topic :	Integration	**Subtopic(s) :**	IA2 Doubts, Substitution Method, Trigonometric identities

IA2 Doubts:

(1) $\int \sin 2x \, dx$

$$\frac{d(-\cos x)}{dx} = \sin x$$

$$\frac{1}{2} \frac{d}{dx}(-\cos 2x) = \sin 2x$$

$$\frac{d(-\cos 2x)}{dx}$$

$$= 2\sin(2x)$$

$$f(x) = e^{\sin(\cos(2x^2+3x))}$$

$$f'(x) = e^{\sin(\cos(2x^2+3x))} \times \cos(\cos(2x^2+3x))$$

$$\times -\sin(2x^2+3x) \times (4x+3)$$

Integration by substitution:

$$\int e^{\sin(x)} \times \cos x \, dx \quad \text{substitution}$$

Put $\sin(x) = t$

$\cos x \, dx = dt$

$$\int e^t \, dt = e^t + c$$

$$= e^{\sin x} + c$$

$$\int e^{2x} \, dx \qquad e^{mx+c}$$

$$\int e^{mx+c} \, dx = \frac{e^{mx \mp c}}{m}$$

$$\int e^{2x}\,dx$$

$$2x = t$$

$$2\,dx = dt \rightarrow$$

$$\frac{1}{2}\int e^{t}\,dt = \frac{1}{2}e^{t} + C$$

$$= \frac{1}{2}e^{2x} + C$$

$$\int \tan^{4}x \times \sec^{2}x\,dx$$

$$\tan x = t$$

$$\sec^{2}x\,dx = dt$$

$$\int t^{4}\,dt$$

$$\frac{t^{5}}{5} \Rightarrow \frac{\tan^{5}x}{5} + C$$

Method 1:

$$\int \frac{\tan^4\sqrt{x} \times \sec^2\sqrt{x}}{\sqrt{x}}\, dx$$

$$\int x^{-\frac{1}{2}} \boxed{\tan^4 x^{\frac{1}{2}}} \times \sec^2 x^{\frac{1}{2}}\; dx$$

$$\tan x^{\frac{1}{2}} = t$$

$$\frac{1}{2} \boxed{x^{-\frac{1}{2}} \times \sec^2 x^{\frac{1}{2}}}\, dx = dt$$

$$\int 2t^4\, dt = \frac{2t^5}{5} = \boxed{\frac{2\tan^5 x^{\frac{1}{2}}}{5} + C}$$

Method 2:

$$\int \frac{\tan^4\sqrt{x}\ \sec^2\sqrt{x}}{\sqrt{x}}\; dx$$

$$\sqrt{x} = a \qquad \frac{d(\sqrt{x})}{dx} = \frac{x^{-\frac{1}{2}}}{2} = \frac{1}{2\sqrt{x}}$$

$$\frac{1}{2\sqrt{x}}\, dx = da$$

$$\frac{1}{\sqrt{x}}\, dx = 2da$$

$$2\int \tan^4(a)\sec^2(a)\; da$$

Put $\tan(a) = b$

$$\sec^2 a \, da = db$$

$$2\int b^4 \, db \Rightarrow \frac{2b^5}{5} + C$$

$$\Rightarrow \frac{2\tan^5 a}{5} + C$$

$$\Rightarrow \frac{2\tan^5 \sqrt{x}}{5} + C$$

$$\int \frac{e^{\tan^{-1}x}}{1+x^2} \, dx = e^{\tan^{-1}x} + C$$

$$\tan^{-1}x = t$$

$$\frac{dx}{1+x^2} = dt$$

$$\int e^t \, dt = e^t + C$$

$$= e^{\tan^{-1}x} + C$$

Method 1:

$$\int x^2\sqrt{x^3}\,dx \qquad a^m \times a^n$$

$$a^{m+n}$$

$$x^2 \times x^{\frac{3}{2}}$$

$$x^{2+\frac{3}{2}} = \int x^{\frac{7}{2}}\,dx = \frac{x^{\frac{7}{2}+1}}{\frac{7}{2}+1}$$

Method 2:

$$\int x^2 \sqrt{x^3}\, dx$$

$$x^3 = t$$

$$3x^2\, dx = dt$$

$$x^2\, dx = \frac{dt}{3}$$

$$\int \frac{1}{3}\sqrt{t}\, dt$$

$$\frac{1}{3}\,\frac{t^{\frac{1}{2}+1}}{\frac{1}{2}+1} = \frac{1\times 2\, t^{\frac{3}{2}}}{3 \cdot 3}$$

$$= \frac{2}{9}\, t^{\frac{3}{2}} = \frac{2}{9}\, x^{\frac{9}{2}}$$

$$\int x^2 \sqrt{x^3}\, dx \qquad \frac{2\left(\sqrt{x^3}\right)^3}{9}$$

Method 3:

$$x^3 = t^2 \Rightarrow t = \sqrt{x^3}$$

$$3x^2\, dx = 2t\, dt$$

$$\int t \times \frac{2t\, dt}{3} \qquad \int \frac{2t^2\, dt}{3} = \frac{2t^{3}}{9} \qquad \sqrt{x^3} \qquad \sqrt{t^2} = t$$

Method 1:

$$\int x\sqrt{1+2x^2}\,dx$$

$$1+2x^2 = t$$

$$4x\,dx = dt$$

$$\int \frac{1}{4}\sqrt{t}\,dt$$

Method 2:

$$\int x\sqrt{1+2x^2}\,dx$$

$$1+2x^2 = t^2 \Rightarrow t=\sqrt{1+2x^2}$$

$$4x\,dx = 2t\,dt$$

$$x\,dx = \frac{\cancel{2}t\,dt}{\cancel{4}\,2}$$

$$\int t\times\frac{t\,dt}{2} \quad \int\frac{t^2}{2}dt = \frac{t^3}{6}$$

$$= \frac{\left(\sqrt{1+2x^2}\right)^3}{6}$$

$$+ C$$

Now try these questions:

Example 5 Integrate the following functions w.r.t. x:

(i) $\sin mx$ (ii) $2x \sin (x^2 + 1)$

(iii) $\dfrac{\tan^4 \sqrt{x} \sec^2 \sqrt{x}}{\sqrt{x}}$ (iv) $\dfrac{\sin (\tan^{-1} x)}{1+x^2}$

Example 6 Find the following integrals:

(i) $\int \sin^3 x \cos^2 x \, dx$ (ii) $\int \dfrac{\sin x}{\sin (x+a)} dx$ (iii) $\int \dfrac{1}{1+\tan x} dx$

Integrate the functions in Exercises 1 to 37:

1. $\dfrac{2x}{1+x^2}$ **2.** $\dfrac{(\log x)^2}{x}$ **3.** $\dfrac{1}{x + x \log x}$

4. $\sin x \sin (\cos x)$ **5.** $\sin (ax+b) \cos (ax+b)$

6. $\sqrt{ax+b}$ **7.** $x\sqrt{x+2}$ **8.** $x\sqrt{1+2x^2}$

9. $(4x+2)\sqrt{x^2+x+1}$ **10.** $\dfrac{1}{x-\sqrt{x}}$ **11.** $\dfrac{x}{\sqrt{x+4}}, x>0$

12. $(x^3-1)^{\frac{1}{3}} x^5$ **13.** $\dfrac{x^2}{(2+3x^3)^3}$ **14.** $\dfrac{1}{x (\log x)^m}, x>0, m \neq 1$

15. $\dfrac{x}{9-4x^2}$ **16.** e^{2x+3} **17.** $\dfrac{x}{e^{x^2}}$

18. $\dfrac{e^{\tan^{-1} x}}{1+x^2}$

19. $\dfrac{e^{2x}-1}{e^{2x}+1}$

20. $\dfrac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}$

21. $\tan^2(2x-3)$

22. $\sec^2(7-4x)$

23. $\dfrac{\sin^{-1}x}{\sqrt{1-x^2}}$

24. $\dfrac{2\cos x - 3\sin x}{6\cos x + 4\sin x}$

25. $\dfrac{1}{\cos^2 x\,(1-\tan x)^2}$

26. $\dfrac{\cos\sqrt{x}}{\sqrt{x}}$

27. $\sqrt{\sin 2x}\cos 2x$

28. $\dfrac{\cos x}{\sqrt{1+\sin x}}$

29. $\cot x \log \sin x$

30. $\dfrac{\sin x}{1+\cos x}$

31. $\dfrac{\sin x}{(1+\cos x)^2}$

32. $\dfrac{1}{1+\cot x}$

33. $\dfrac{1}{1-\tan x}$

34. $\dfrac{\sqrt{\tan x}}{\sin x \cos x}$

35. $\dfrac{(1+\log x)^2}{x}$

36. $\dfrac{(x+1)(x+\log x)^2}{x}$

37. $\dfrac{x^3\sin\left(\tan^{-1}x^4\right)}{1+x^8}$

Choose the correct answer in Exercises 38 and 39.

38. $\displaystyle\int \frac{10x^9+10^x \log_{e^{10}} dx}{x^{10}+10^x}$ equals

(A) $10^x - x^{10} + C$

(B) $10^x + x^{10} + C$

(C) $(10^x - x^{10})^{-1} + C$

(D) $\log(10^x + x^{10}) + C$

39. $\displaystyle\int \frac{dx}{\sin^2 x\cos^2 x}$ equals

(A) $\tan x + \cot x + C$

(B) $\tan x - \cot x + C$

(C) $\tan x \cot x + C$

(D) $\tan x - \cot 2x + C$

Trigonometric Identities:

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

1. $sin(a) + sin(b) = 2sin(\frac{a+b}{2})cos(\frac{a-b}{2})$

2. $sin(a) - sin(b) = 2cos(\frac{a+b}{2})sin(\frac{a-b}{2})$

3. $cos(a) + cos(b) = 2cos(\frac{a+b}{2})cos(\frac{a-b}{2})$

4. $cos(a) - cos(b) = -2sin(\frac{a+b}{2})sin(\frac{a-b}{2})$

sin(a+b)+sin(a-b)= 2sin(a)cos(b)

sin(a+b)-sin(a-b) = 2 cos(a)sin(b)

cos(a+b)+cos(a-b)=2cos(a)cos(b)

cos(a+b)-cos(a-b)= -2sin(a)sin(b)

$$\sin A \cos B = \frac{1}{2} [\sin(A+B) + \sin(A-B)]$$

$$\cos A \sin B = \frac{1}{2} [\sin(A+B) - \sin(A-B)]$$

$$\cos A \cos B = \frac{1}{2} [\cos(A+B) + \cos(A-B)]$$

$$\sin A \sin B = \frac{1}{2} [\cos(A-B) - \cos(A+B)]$$

End