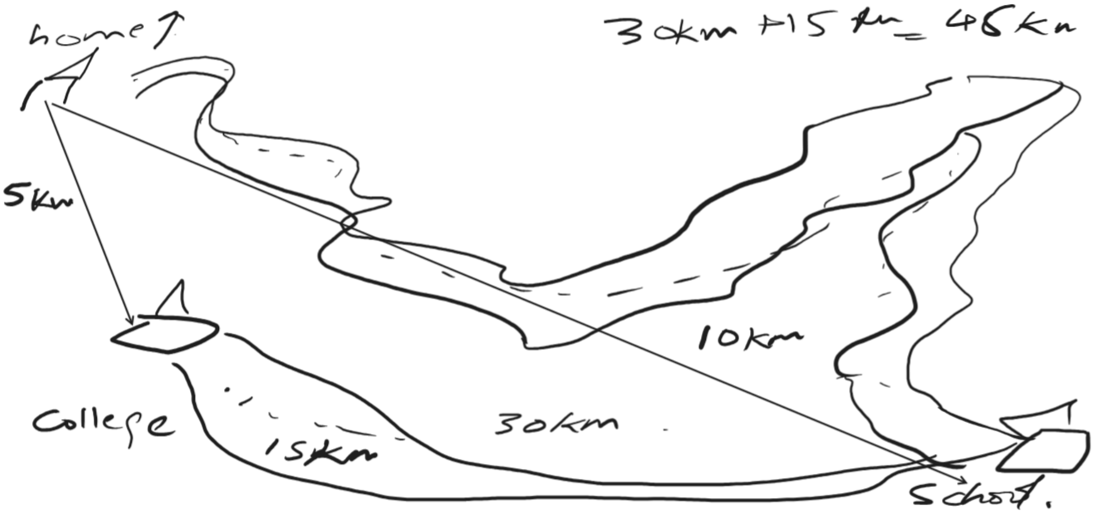
Date :	Oct 9 2024	**Board** :	CBSE
Class :	12	**Session # :**	5
Subject :	Mathematics	**Assignment # :**	VaA2
Topic :	Vector Algebra	**Subtopic(s) :**	VaA1 discussion, Vector joining two points, Vector Addition, Multiplication of vector by a scalar, Components of a vector, Vector joining two points
Lecture #:	2		

VaA1 discussion:

Distance & Displacement:



Distance - Scalar Quantity
Displacement (shortest distance) - Vector Quantity

Magnitude of a vector:

$$\vec{r} = 5i - 2j + 3k$$

magnitude

$$|\vec{r}| = \sqrt{(5)^2 + (-2)^2 + 3^2}$$

$$= \sqrt{25 + 4 + 9}$$

Angles:

$$\cos\alpha = \frac{1}{\sqrt{3}}$$

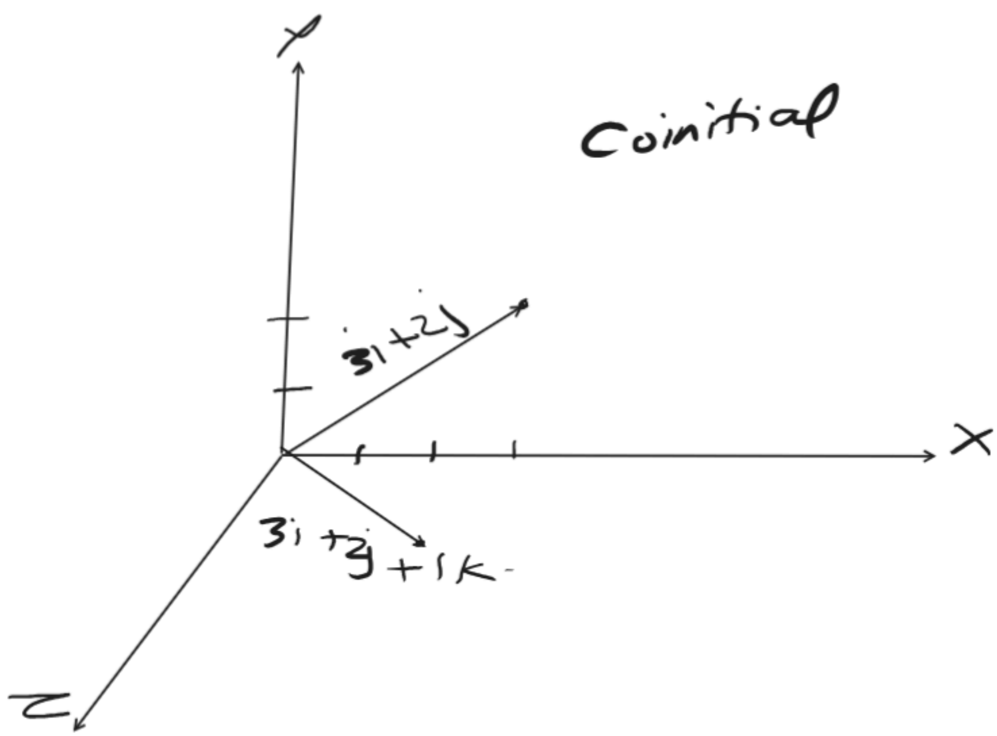
$$\alpha = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

Negative:

$$x = -5 + \sqrt{2}$$

$$-x = 5 - \sqrt{2}$$

Coinitial Vectors:



Collinear Vectors:

$$\vec{a} = 2i + 4j$$

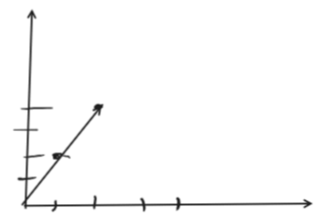
$$\vec{b} = ki + 2kj \qquad (k \text{ scalar})$$

$k = 1$

$\vec{b} = 1i + 2j$

$k = 2$

$\vec{b} = 2i + 4j$



$\vec{a} = m\vec{b}$ $\{m$ & n are scalar$\}$

or $\vec{b} = n\vec{a}$

$\vec{a}$ & $\vec{b}$ will be collinear vectors

If $\vec{a} = a_1 i + a_2 j$

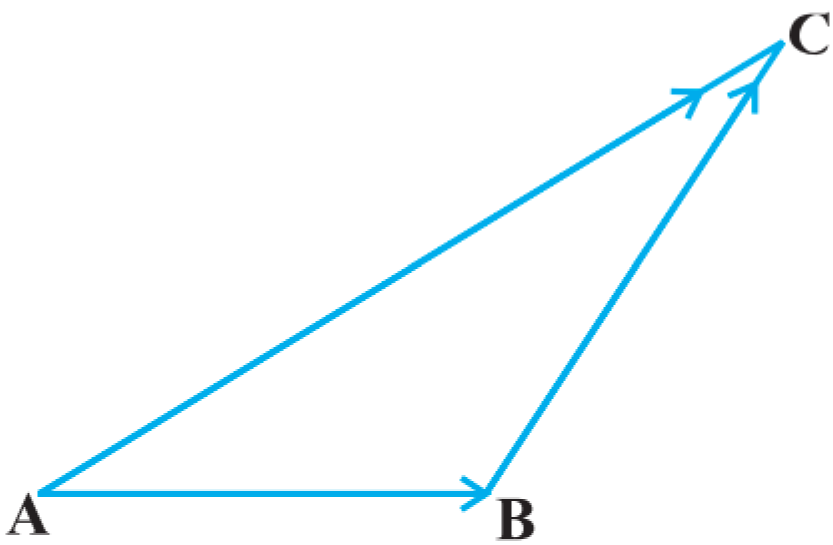
$\vec{b} = b_1 i + b_2 j$

$\longrightarrow$ If $\dfrac{a_1}{b_1} = \dfrac{a_2}{b_2}$

then $\vec{a}$ & $\vec{b}$ are collinear.

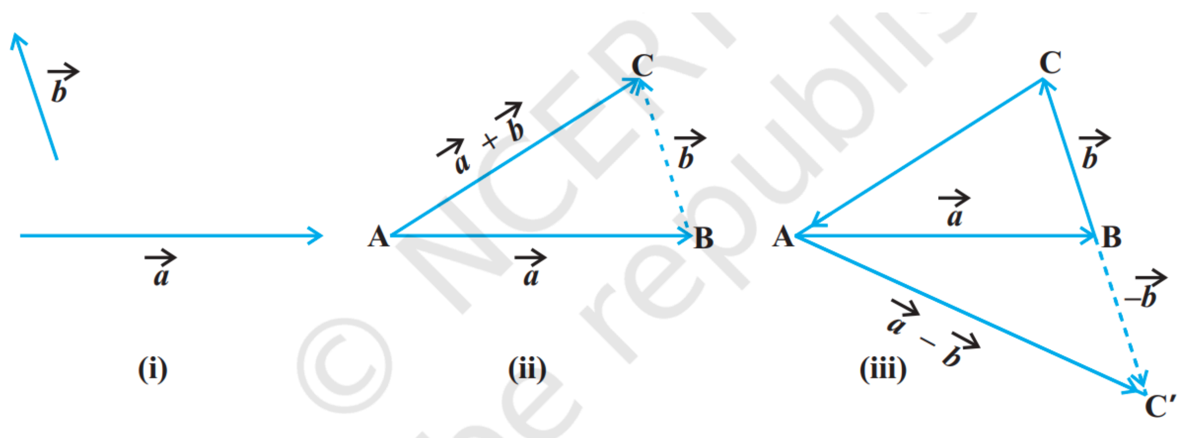
Vector Addition:

Triangle Law:

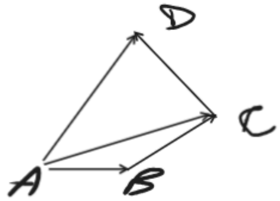


$$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$$

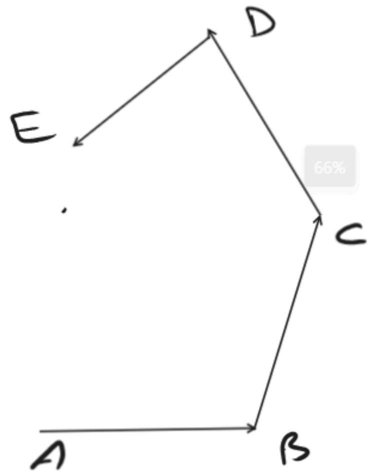
This is known as the *triangle law of vector addition.*



Examples:

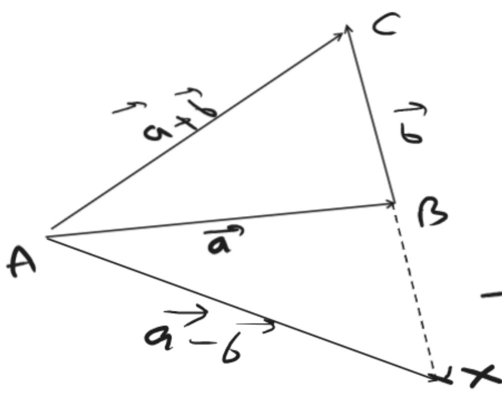


$$\vec{AB}+\vec{BC}+\vec{CD} = \vec{AD}$$



$$\vec{AB}+\vec{BC}+\vec{CD}+\vec{DE}$$

$$= \vec{AE}$$



$$\vec{AC} = \vec{AB}+\vec{BC}$$

$$\vec{AB}+\vec{BX} = \vec{AX}$$

$-\vec{b}$ $\quad \vec{a}+(-\vec{b}) = \vec{AX}$

$$\vec{a}-\vec{b}$$

Properties of vector addition

Property 1 For any two vectors $\vec{a}$ and $\vec{b}$,

$$\vec{a} + \vec{b} = \vec{b} + \vec{a} \qquad \text{(Commutative property)}$$

Property 2 For any three vectors a, b and c

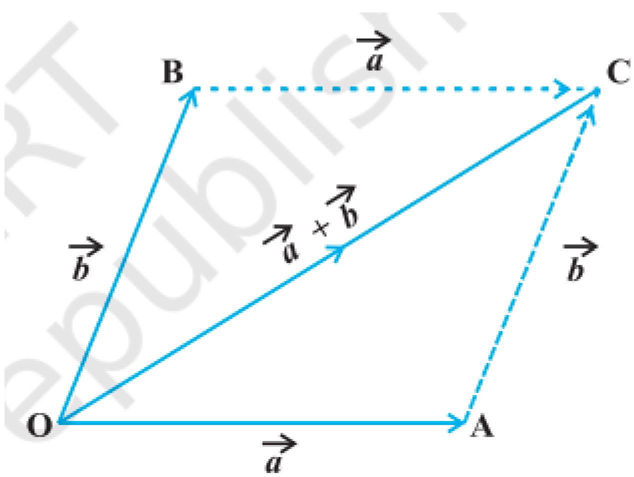
$$(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c}) \qquad \text{(Associative property)}$$

$$\vec{a} + \vec{0} = \vec{0} + \vec{a} = \vec{a}$$

Here, the zero vector $\vec{0}$ is called the *additive identity* for the vector addition.

Parallelogram law of vector addition:

The parallelogram law of vector addition is a geometric way to add two vectors. It states that if you have two vectors, $\mathbf{A}$ and $\mathbf{B}$, you can represent their sum, $\mathbf{A} + \mathbf{B}$, as the diagonal of a parallelogram formed by these two vectors.



Multiplication of vector by a scalar:

1. **Vector Definition**: A vector is represented as $\mathbf{v} = \langle v_1, v_2, v_3 \rangle$, where v_1, v_2, and v_3 are its components.
2. **Scalar Definition**: A scalar is just a single number, say k.
3. **Multiplication**: To multiply the vector by the scalar, you multiply each component of the vector by the scalar:

$$k \cdot \mathbf{v} = k \cdot \langle v_1, v_2, v_3 \rangle = \langle k \cdot v_1, k \cdot v_2, k \cdot v_3 \rangle$$

Example

If you have a vector $\mathbf{v} = \langle 2, 3 \rangle$ and a scalar $k = 4$:

$$4 \cdot \mathbf{v} = 4 \cdot \langle 2, 3 \rangle = \langle 4 \cdot 2, 4 \cdot 3 \rangle = \langle 8, 12 \rangle$$

Components of a vector:

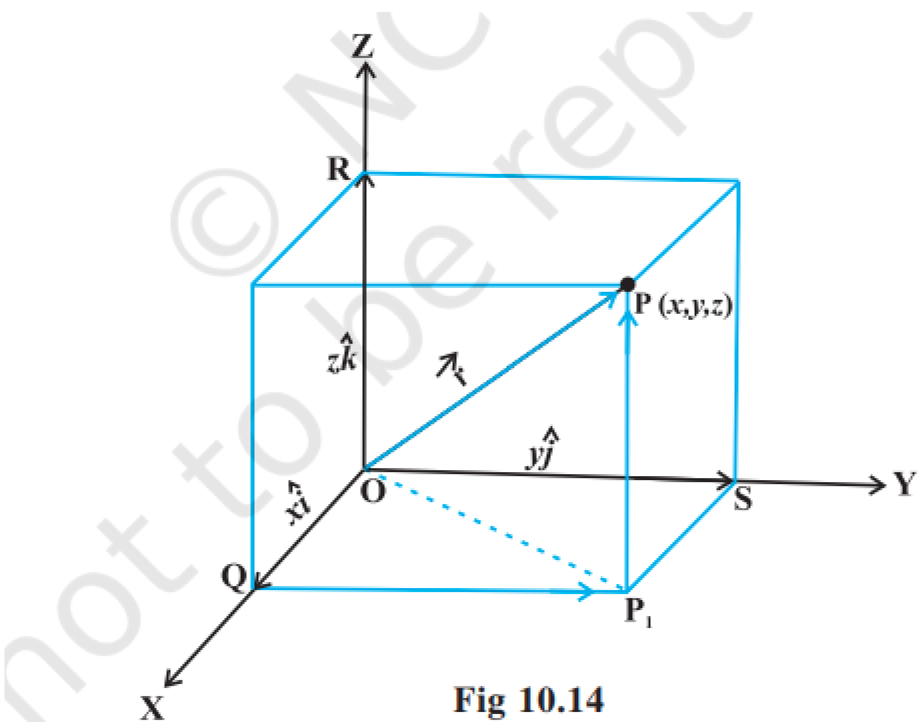


Fig 10.14

If $\vec{a}$ and $\vec{b}$ are any two vectors given in the component form $a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$, respectively, then

(i) the sum (or resultant) of the vectors $\vec{a}$ and $\vec{b}$ is given by

$$\vec{a} + \vec{b} = (a_1 + b_1)\hat{i} + (a_2 + b_2)\hat{j} + (a_3 + b_3)\hat{k}$$

(ii) the difference of the vector $\vec{a}$ and $\vec{b}$ is given by

$$\vec{a} - \vec{b} = (a_1 - b_1)\hat{i} + (a_2 - b_2)\hat{j} + (a_3 - b_3)\hat{k}$$

(iii) the vectors $\vec{a}$ and $\vec{b}$ are equal if and only if

$$a_1 = b_1,\ a_2 = b_2 \quad \text{and} \quad a_3 = b_3$$

(iv) the multiplication of vector $\vec{a}$ by any scalar λ is given by

$$\lambda\vec{a} = (\lambda a_1)\hat{i} + (\lambda a_2)\hat{j} + (\lambda a_3)\hat{k}$$

The addition of vectors and the multiplication of a vector by a scalar together give the following distributive laws:

Let $\vec{a}$ and $\vec{b}$ be any two vectors, and k and m be any scalars. Then

(i) $k\vec{a} + m\vec{a} = (k + m)\vec{a}$

(ii) $k(m\vec{a}) = (km)\vec{a}$

(iii) $k(\vec{a} + \vec{b}) = k\vec{a} + k\vec{b}$

Vector joining two points:

1. Define the Points

Let's say you have two points:

- Point A with coordinates (x_1, y_1, z_1)
- Point B with coordinates (x_2, y_2, z_2)

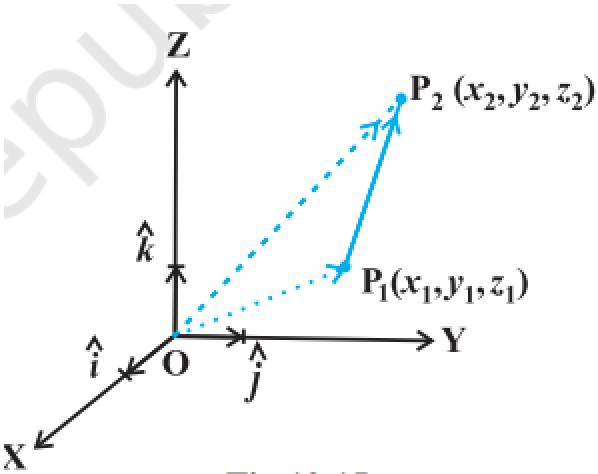
2. Vector Representation

The vector $\mathbf{AB}$ that joins points A and B can be calculated using the formula:

$$\mathbf{AB} = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle$$

3. Explanation

- The components of the vector $\mathbf{AB}$ represent the change in each coordinate from point A to point B:
  - $x_2 - x_1$: Change in the x-direction.
  - $y_2 - y_1$: Change in the y-direction.
  - $z_2 - z_1$: Change in the z-direction.



4. Example

For example, if you have:

- Point $A(1, 2, 3)$
- Point $B(4, 5, 6)$

The vector $\mathbf{AB}$ would be:

$$\mathbf{AB} = \langle 4 - 1, 5 - 2, 6 - 3 \rangle = \langle 3, 3, 3 \rangle$$

Direction Ratios:

Direction ratios are a set of three numbers that are proportional to the direction cosines of a line in three-dimensional space. They represent the direction of a vector, and are often denoted by l, m, and n.

1. Definition

For a vector $\mathbf{v} = \langle x, y, z \rangle$ in 3D space, the direction ratios are given as:

- $l = kx$
- $m = ky$
- $n = kz$

where k is a non-zero scalar. The direction ratios can thus be any scalar multiple of the vector's components.

2. **Relation to Direction Cosines**

The direction ratios are related to the direction cosines, which are the cosines of the angles that the vector makes with the positive x, y, and z axes. The direction cosines are denoted as:

- $\cos\alpha = \frac{l}{\sqrt{l^2+m^2+n^2}}$
- $\cos\beta = \frac{m}{\sqrt{l^2+m^2+n^2}}$
- $\cos\gamma = \frac{n}{\sqrt{l^2+m^2+n^2}}$

3. **Properties**

- Direction ratios are not unique. For example, the direction ratios $2, 3, 4$ and $1, \frac{3}{2}, 2$ both represent the same direction.
- If a line has direction ratios l, m, n, then any non-zero multiples of these ratios (like kl, km, kn for any non-zero k) will point in the same direction.

Example

For a vector $\mathbf{v} = \langle 3, 4, 5 \rangle$:

- Direction ratios could be $l = 3$, $m = 4$, $n = 5$.
- You could also have direction ratios $l = 6$, $m = 8$, $n = 10$ (which are proportional to the original values).

Understanding direction ratios is essential for working with vectors in 3D space and can help in many applications involving geometry and physics!

Now complete this assignment #VaA2:

#VaA2:

Example 4 Find the values of x, y and z so that the vectors $\vec{a} = x\hat{i} + 2\hat{j} + z\hat{k}$ and $\vec{b} = 2\hat{i} + y\hat{j} + \hat{k}$ are equal.

Example 5 Let $\vec{a} = \hat{i} + 2\hat{j}$ and $\vec{b} = 2\hat{i} + \hat{j}$. Is $|\vec{a}| = |\vec{b}|$? Are the vectors $\vec{a}$ and $\vec{b}$ equal?

Example 6 Find unit vector in the direction of vector $\vec{a} = 2\hat{i} + 3\hat{j} + \hat{k}$

Example 7 Find a vector in the direction of vector $\vec{a} = \hat{i} - 2\hat{j}$ that has magnitude 7 units.

Example 8 Find the unit vector in the direction of the sum of the vectors, $\vec{a} = 2\hat{i} + 2\hat{j} - 5\hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} + 3\hat{k}$.

Example 9 Write the direction ratio's of the vector $\vec{a} = \hat{i} + \hat{j} - 2\hat{k}$ and hence calculate its direction cosines.

Example 10 Find the vector joining the points P(2, 3, 0) and Q(– 1, – 2, – 4) directed from P to Q.

1. Compute the magnitude of the following vectors:

$$\vec{a} = \hat{i} + \hat{j} + k; \quad \vec{b} = 2\hat{i} - 7\hat{j} - 3\hat{k}; \quad \vec{c} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} - \frac{1}{\sqrt{3}}\hat{k}$$

2. Write two different vectors having same magnitude.
3. Write two different vectors having same direction.
4. Find the values of x and y so that the vectors $2\hat{i} + 3\hat{j}$ and $x\hat{i} + y\hat{j}$ are equal.
5. Find the scalar and vector components of the vector with initial point (2, 1) and terminal point (– 5, 7).
6. Find the sum of the vectors $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = -2\hat{i} + 4\hat{j} + 5\hat{k}$ and $\vec{c} = \hat{i} - 6\hat{j} - 7\hat{k}$.
7. Find the unit vector in the direction of the vector $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$.
8. Find the unit vector in the direction of vector $\overline{PQ}$, where P and Q are the points (1, 2, 3) and (4, 5, 6), respectively.
9. For given vectors, $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = -\hat{i} + \hat{j} - \hat{k}$, find the unit vector in the direction of the vector $\vec{a} + \vec{b}$.
10. Find a vector in the direction of vector $5\hat{i} - \hat{j} + 2\hat{k}$ which has magnitude 8 units.
11. Show that the vectors $2\hat{i} - 3\hat{j} + 4\hat{k}$ and $-4\hat{i} + 6\hat{j} - 8\hat{k}$ are collinear.
12. Find the direction cosines of the vector $\hat{i} + 2\hat{j} + 3\hat{k}$.
13. Find the direction cosines of the vector joining the points A (1, 2, –3) and B (–1, –2, 1), directed from A to B.
14. Show that the vector $\hat{i} + \hat{j} + \hat{k}$ is equally inclined to the axes OX, OY and OZ.

17. Show that the points A, B and C with position vectors, $\vec{a} = 3\hat{i} - 4\hat{j} - 4\hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - 3\hat{j} - 5\hat{k}$, respectively form the vertices of a right angled triangle.

18. In triangle ABC (Fig 10.18), which of the following is not true:

(A) $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{0}$

(B) $\overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{AC} = \vec{0}$

(C) $\overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{AC} = \vec{0}$

(D) $\overrightarrow{AB} - \overrightarrow{CB} + \overrightarrow{CA} = \vec{0}$

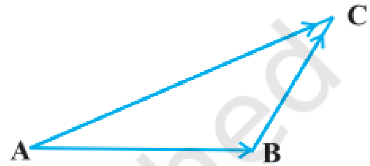


Fig 10.18

19. If $\vec{a}$ and $\vec{b}$ are two collinear vectors, then which of the following are incorrect:

(A) $\vec{b} = \lambda\vec{a}$, for some scalar λ

(B) $\vec{a} = \pm\vec{b}$

(C) the respective components of $\vec{a}$ and $\vec{b}$ are not proportional

(D) both the vectors $\vec{a}$ and $\vec{b}$ have same direction, but different magnitudes.

End