Date :	Oct 7 2024	**Board** :	CBSE
Class :	12	**Session # :**	4
Subject :	Mathematics	**Assignment # :**	VaA1
Topic :	Vector Algebra	**Subtopic(s) :**	Scalar Quantities (e.g. Speed), Vector Quantities (e.g. Velocity, Acceleration, Force), Position Vector, Direction cosines, Types of Vectors
Lecture #:	1		

Definition 1 A quantity that has magnitude as well as direction is called a vector.

Position Vectors:

Position vectors are used in mathematics and physics to describe the location of a point in space relative to a reference origin. In a three-dimensional Cartesian coordinate system, a position vector $\mathbf{r}$ can be expressed as:

$$\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$$

where:

- x, y, and z are the coordinates of the point,
- $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ are the unit vectors in the directions of the x, y, and z axes, respectively.

Direction Cosines

Consider the position vector $\overline{\mathrm{OP}}$ (or $\vec{r}$) of a point P(x, y, z) as in Fig 10.3. The angles α, β, γ made by the vector $\vec{r}$ with the positive directions of x, y and z-axes respectively, are called its ***direction angles***. The cosine values of these angles, i.e., $\cos\alpha$, $\cos\beta$ and $\cos\gamma$ are called ***direction cosines*** of the vector $\vec{r}$, and usually denoted by ***l***, ***m*** and ***n***, respectively.

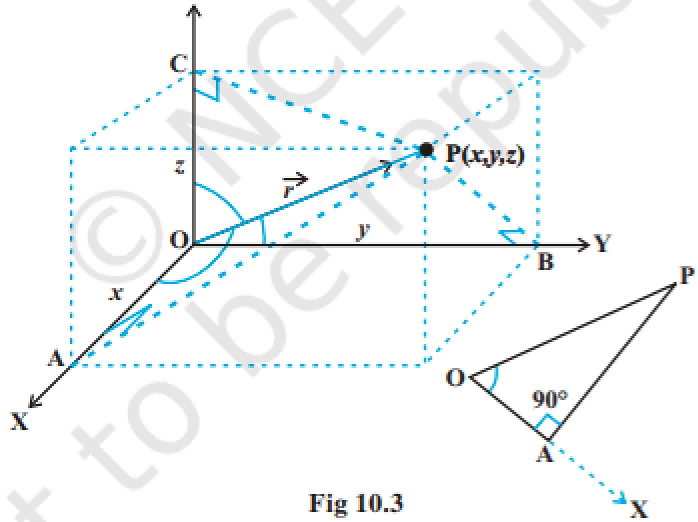


Fig 10.3

From Fig 10.3, one may note that the triangle OAP is right angled, and in it, we have $\cos\alpha = \frac{x}{r}$ (r stands for $|\vec{r}|$). Similarly, from the right angled triangles OBP and OCP, we may write $\cos\beta = \frac{y}{r}$ and $\cos\gamma = \frac{z}{r}$. Thus, the coordinates of the point P may also be expressed as (***lr***, ***mr***,***nr***). The numbers ***lr***, ***mr*** and ***nr***, proportional to the direction cosines are called as ***direction ratios*** of vector $\vec{r}$, and denoted as ***a***, ***b*** and ***c***, respectively.

Direction cosines are the cosines of the angles that a vector makes with the coordinate axes in a Cartesian coordinate system. They provide a way to describe the orientation of a vector in three-dimensional space.

Definition

For a vector $\mathbf{v}$ with components (v_x, v_y, v_z), the direction cosines are defined as:

- $l = \frac{v_x}{|\mathbf{v}|}$
- $m = \frac{v_y}{|\mathbf{v}|}$
- $n = \frac{v_z}{|\mathbf{v}|}$

where $|\mathbf{v}|$ is the magnitude of the vector, calculated as:

$$|\mathbf{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

Properties

1. **Range**: Each direction cosine l, m, and n can range from -1 to 1, depending on the orientation of the vector in relation to the coordinate axes.
2. **Normalization**: The sum of the squares of the direction cosines equals 1:
$$l^2 + m^2 + n^2 = 1$$
3. **Angles**: The angles α, β, and γ that the vector makes with the x, y, and z axes, respectively, are given by:
    - $\cos(\alpha) = l$
    - $\cos(\beta) = m$
    - $\cos(\gamma) = n$

Scalar and vector quantities are fundamental concepts in physics and mathematics that describe different types of measurements. Here's a breakdown of each:

Scalar Quantities

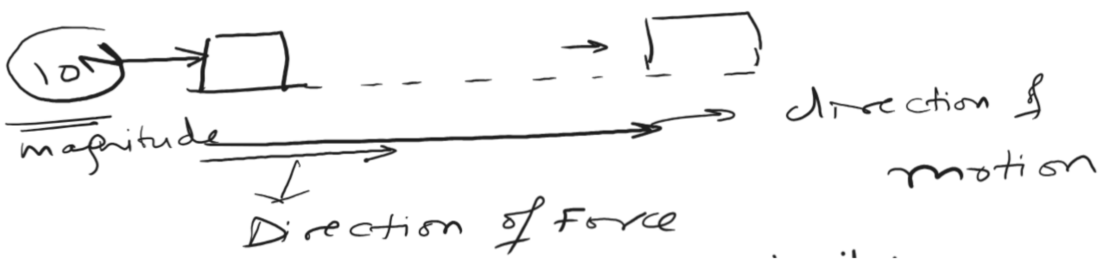
- **Definition**: Scalar quantities are measurements that are fully described by a magnitude (numerical value) alone. They do not have a direction.
- **Examples**:
  - **Temperature**: 25 degrees Celsius
  - **Mass**: 10 kilograms
  - **Speed**: 60 kilometers per hour
  - **Distance**: 100 meters
  - **Energy**: 50 joules

Vector Quantities

- **Definition**: Vector quantities are measurements that are described by both a magnitude and a direction. This means that to fully define a vector, you need to specify how much (magnitude) and in which way (direction).
- **Examples**:
  - **Velocity**: 60 kilometers per hour east
  - **Force**: 10 newtons downward
  - **Acceleration**: 9.8 meters per second squared downward
  - **Displacement**: 100 meters north

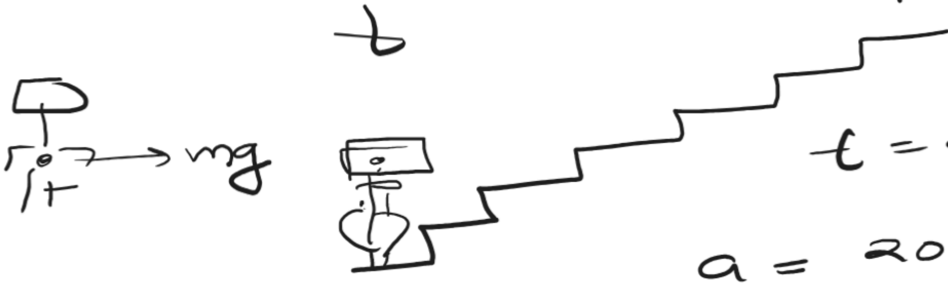
Key Differences

1. **Magnitude and Direction:**
    - Scalars have only magnitude.
    - Vectors have both magnitude and direction.
2. **Representation:**
    - Scalars are represented by numbers (e.g., 5 kg).
    - Vectors are often represented by arrows, where the length indicates magnitude and the arrowhead indicates direction (e.g., $\mathbf{v} = 30\,\mathrm{m/s\,north}$).
3. **Mathematical Operations:**
    - Scalars can be added, subtracted, multiplied, or divided using regular arithmetic.
    - Vectors require vector-specific operations, such as vector addition (using the head-to-tail method) and scalar multiplication.



Change in speed → velocity

5 m/sec → 20 m/sec



$t = 2\ \mathrm{sec}$

$$a = \frac{20 - 5}{2} \frac{\mathrm{m/sec}}{\mathrm{sec}}$$

$$= \frac{15}{2} = 7.5\ \mathrm{m/s^2}$$

Velocity → speed with direction

↓

vector quantity

↓

both magnitude and direction

speed

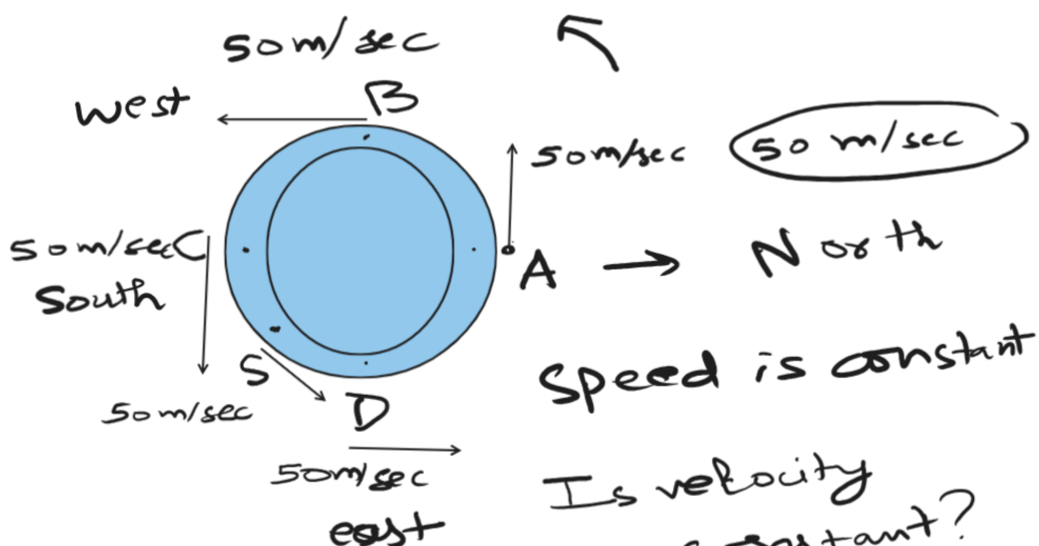
↓

Scalar quantity

↓

only magnitude

Is velocity constant?



constant velocity

linear motion

Magnitude of vectors:

To find the magnitude (or length) of a vector, you can use the following formula, depending on the vector's dimension.

In 2D

For a vector $\mathbf{v}$ with components (v_x, v_y), the magnitude is calculated as:

$$|\mathbf{v}| = \sqrt{v_x^2 + v_y^2}$$

In 3D

For a vector $\mathbf{v}$ with components (v_x, v_y, v_z), the magnitude is calculated as:

$$|\mathbf{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

Example Calculation

Example 1 (2D Vector):

If $\mathbf{v} = (3, 4)$:

$$|\mathbf{v}| = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$$

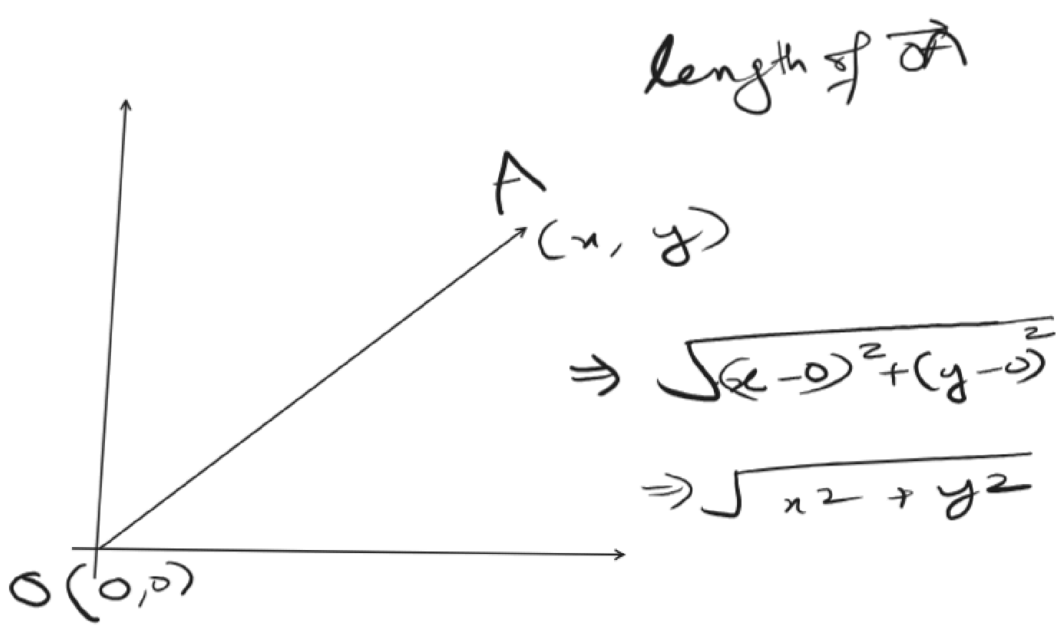
Example 2 (3D Vector):

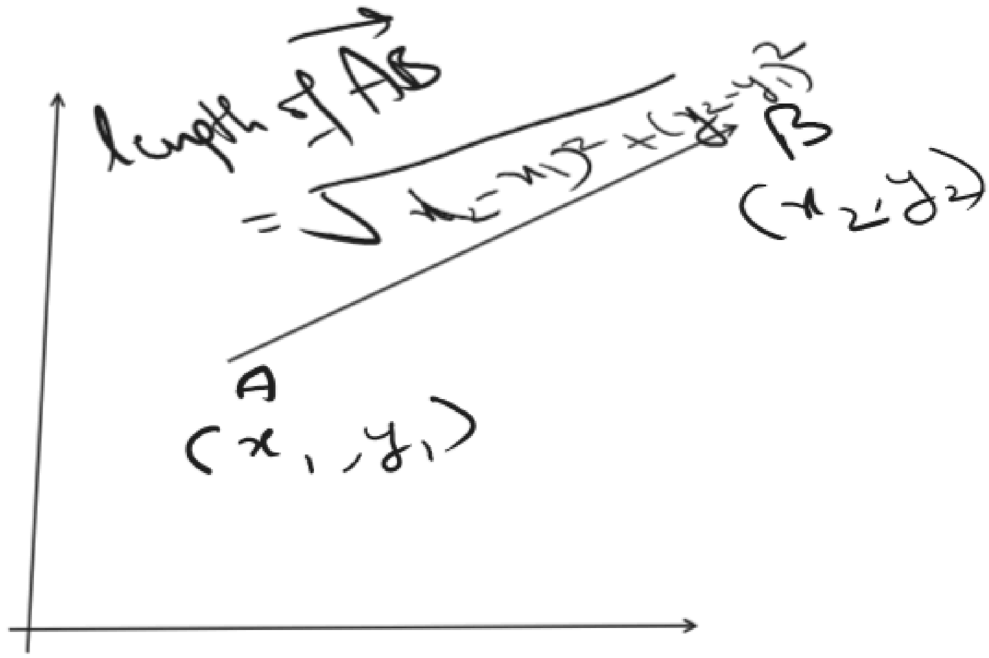
If $\mathbf{v} = (1, 2, 2)$:

$$|\mathbf{v}| = \sqrt{1^2 + 2^2 + 2^2} = \sqrt{1 + 4 + 4} = \sqrt{9} = 3$$

Key Points

- The magnitude of a vector is always a non-negative value.
- It represents the length of the vector in space.
- You can use the Pythagorean theorem for the calculation in both 2D and 3D.





Constructing Vectors from Coordinates in 2D and 3D:

To write a vector from coordinates, you simply express the vector in terms of its components along each axis. Here's how you can do it:

In 2D

If you have a point with coordinates (x, y), the corresponding vector $\mathbf{v}$ from the origin $(0, 0)$ to the point (x, y) can be written as:

$$\mathbf{v} = x\mathbf{i} + y\mathbf{j}$$

where:

- $\mathbf{i}$ is the unit vector in the x-direction.
- $\mathbf{j}$ is the unit vector in the y-direction.

Example (2D)

For the point $(3, 4)$:

$$\mathbf{v} = 3\mathbf{i} + 4\mathbf{j}$$

In 3D

If you have a point with coordinates (x, y, z), the corresponding vector $\mathbf{v}$ from the origin $(0, 0, 0)$ to the point (x, y, z) can be written as:

$$\mathbf{v} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$$

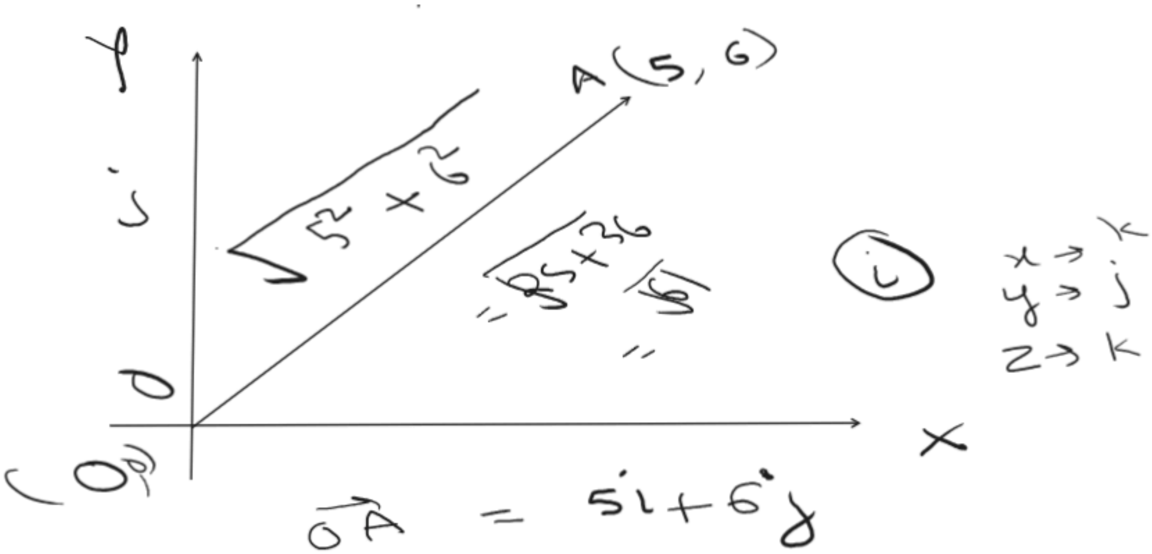
where:

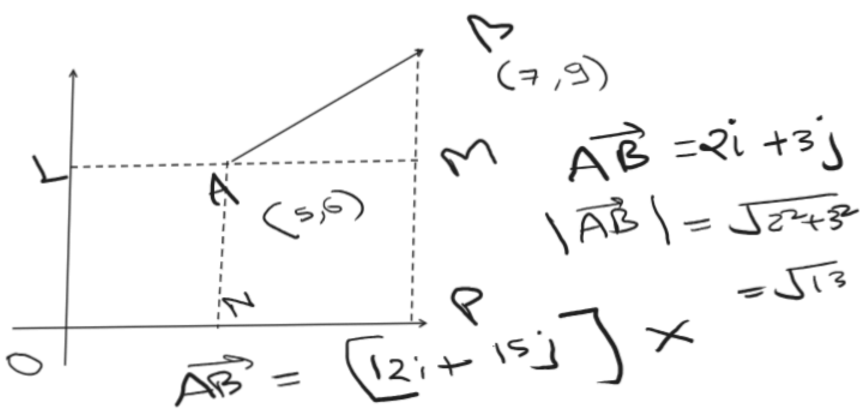
- $\mathbf{i}$ is the unit vector in the x-direction.
- $\mathbf{j}$ is the unit vector in the y-direction.
- $\mathbf{k}$ is the unit vector in the z-direction.

Example (3D)

For the point $(1, 2, 3)$:

$$\mathbf{v} = 1\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$$





Types of Vectors:

1. Zero Vector

- **Definition**: A vector with a magnitude of zero and no specific direction.
- **Notation**: Usually represented as $\mathbf{0}$.
- **Example**: $\mathbf{0} = 0\mathbf{i} + 0\mathbf{j} + 0\mathbf{k}$ in 3D.

2. Unit Vector

- **Definition**: A vector with a magnitude of one, used to indicate direction.
- **Notation**: Often represented with a hat, e.g., $\hat{\mathbf{u}}$.
- **Example**: To convert a vector $\mathbf{v}$ into a unit vector, divide it by its magnitude:

$$\hat{\mathbf{u}} = \frac{\mathbf{v}}{|\mathbf{v}|}$$

$$\vec{AB} = \boxed{a}\,i + \boxed{b}\,j$$

$$|\vec{AB}| = \sqrt{a^2 + b^2} =$$

$$\frac{2i}{\sqrt{13}} + \frac{3j}{\sqrt{13}}$$

$$4 + 9$$

$$\frac{\sqrt{13}}{\sqrt{13}}$$

3. Coinitial Vectors

- **Definition**: Vectors that originate from the same point (or the same initial point).
- **Example**: If $\mathbf{v_1} = 2\mathbf{i} + 3\mathbf{j}$ and $\mathbf{v_2} = 4\mathbf{i} + 5\mathbf{j}$, both vectors can be represented as starting from the same origin.

4. Collinear Vectors

- **Definition**: Vectors that lie along the same line, regardless of their magnitude or direction.
- **Example**: Vectors $\mathbf{a} = 2\mathbf{i} + 4\mathbf{j}$ and $\mathbf{b} = 4\mathbf{i} + 8\mathbf{j}$ are collinear because $\mathbf{b} = 2\mathbf{a}$.

5. Equal Vectors

- **Definition**: Vectors that have the same magnitude and direction.
- **Example**: If $\mathbf{v_1} = 3\mathbf{i} + 2\mathbf{j}$ and $\mathbf{v_2} = 3\mathbf{i} + 2\mathbf{j}$, then $\mathbf{v_1} = \mathbf{v_2}$.

6. Negative of a Vector

- **Definition:** A vector that has the same magnitude as the original vector but points in the opposite direction.
- **Notation**: If $\mathbf{v}$ is a vector, its negative is denoted as $-\mathbf{v}$.
- **Example**: If $\mathbf{v} = 5\mathbf{i} + 3\mathbf{j}$, then $-\mathbf{v} = -5\mathbf{i} - 3\mathbf{j}$.

Now complete this assignment #VaA1:

#VaA1:

1. Represent graphically a displacement of 40 km, 30° east of north.
2. Classify the following measures as scalars and vectors.
   (i) 10 kg (ii) 2 meters north-west (iii) 40°
   (iv) 40 watt (v) 10^{-19} coulomb (vi) 20 m/s^2
3. Classify the following as scalar and vector quantities.
   (i) time period (ii) distance
   (iii) force
   (iv) velocity (v) work done
4. In Fig 10.6 (a square), identify the following vectors.
   (i) Coinitial (ii) Equal
   (iii) Collinear but not equal

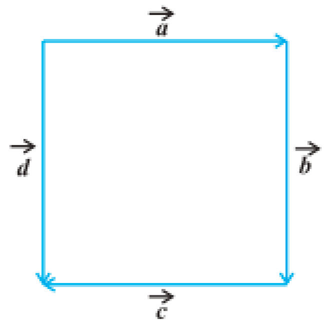


Fig 10.6

5. Answer the following as true or false.
   (i) $\vec{a}$ and $-\vec{a}$ are collinear.
   (ii) Two collinear vectors are always equal in magnitude.
   (iii) Two vectors having same magnitude are collinear.
   (iv) Two collinear vectors having the same magnitude are equal.

Scalar and Vector Quantities

1. **Scalar and Vector Identification:**
   - Classify the following quantities as either scalar or vector: temperature, displacement, mass, and force.
2. **Speed vs. Velocity:**
   - A car travels 100 meters east in 5 seconds. What is its speed and what is its velocity?
3. **Acceleration:**
   - A train increases its speed from 20 m/s to 60 m/s in 10 seconds. Calculate its average acceleration.
4. **Force Calculation:**
   - If a force of 10 N is applied to an object and it causes an acceleration of $2\,\mathrm{m/s^2}$, what is the mass of the object?

Position Vectors

5. **Position Vector:**

   - Write the position vector for a point located at coordinates (4, -3, 2) in 3D space.

6. **Magnitude of Position Vector:**

   - Calculate the magnitude of the position vector $\mathbf{r} = 3\mathbf{i} - 4\mathbf{j} + 5\mathbf{k}$.

Direction Cosines

7. **Finding Direction Cosines:**

   - Given a vector $\mathbf{v} = 2\mathbf{i} + 3\mathbf{j} + 6\mathbf{k}$, calculate its direction cosines.

8. **Angle with Axes:**

   - Determine the angles that the vector $\mathbf{v} = 1\mathbf{i} + 1\mathbf{j} + 1\mathbf{k}$ makes with the x, y, and z axes.

Types of Vectors

9. **Equal Vectors:**

   - Are the vectors $\mathbf{a} = 4\mathbf{i} + 2\mathbf{j}$ and $\mathbf{b} = 4\mathbf{i} + 2\mathbf{j}$ equal? Justify your answer.

10. **Negative of a Vector:**

    - If $\mathbf{p} = -5\mathbf{i} + 3\mathbf{j}$, what is $-\mathbf{p}$?

11. **Coinitial Vectors:**

    - Given the vectors $\mathbf{u} = 3\mathbf{i} + 2\mathbf{j}$ and $\mathbf{v} = 3\mathbf{i} + 2\mathbf{j} + 1\mathbf{k}$, are they coinitial? Explain.

12. **Collinear Vectors:**

    - Determine if the vectors $\mathbf{x} = 2\mathbf{i} + 4\mathbf{j}$ and $\mathbf{y} = k\mathbf{i} + 2k\mathbf{j}$ for some scalar k are collinear.

1. **Zero Vector:**
   - What is the magnitude of the zero vector?
   - Is the zero vector a scalar or a vector?
2. **Unit Vector:**
   - Find the unit vector in the direction of $\mathbf{v} = 3\mathbf{i} + 4\mathbf{j}$.
3. **Coinitial Vectors:**
   - If $\mathbf{a} = 2\mathbf{i} + \mathbf{j}$ and $\mathbf{b} = 2\mathbf{i} + 3\mathbf{j}$, are $\mathbf{a}$ and $\mathbf{b}$ coinitial? Explain.
4. **Collinear Vectors:**
   - Are the vectors $\mathbf{u} = 4\mathbf{i} + 2\mathbf{j}$ and $\mathbf{v} = 2\mathbf{i} + 1\mathbf{j}$ collinear? Justify your answer.
5. **Equal Vectors:**
   - Determine if $\mathbf{p} = 5\mathbf{i} + 3\mathbf{j}$ and $\mathbf{q} = 5\mathbf{i} + 3\mathbf{j}$ are equal vectors.

6. **Negative of a Vector:**
   - If $\mathbf{v} = -2\mathbf{i} + 4\mathbf{j}$, what is $-\mathbf{v}$?
7. **Magnitude of Vectors:**
   - Calculate the magnitude of the vector $\mathbf{v} = 6\mathbf{i} - 8\mathbf{j}$.
8. **Unit Vector:**
   - Convert the vector $\mathbf{w} = -1\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$ into a unit vector.
9. **Coinitial and Collinear:**
   - Given vectors $\mathbf{a} = 3\mathbf{i} + 4\mathbf{j}$ and $\mathbf{b} = k\mathbf{i} + \frac{4}{3}k\mathbf{j}$ for some scalar k, determine if they are coinitial and collinear.
10. **Equal and Negative Vectors:**
    - Let $\mathbf{c} = 2\mathbf{i} - 5\mathbf{j}$. Find the negative of $\mathbf{c}$ and check if $-\mathbf{c}$ equals $\mathbf{c}$.

End