What makes a good prediction interval or probabilistic forecast?

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by

Beinan Xu

26401746

Department of Econometrics and Business Statistics

Monash University

Australia

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Contents

Abstract

This report is about introducing scoring rules and using it to evaluate the results of probabilistic forecasts. In the past few decades, probabilistic forecasts have a very important development and are attracting more and more attention. More and more organizations and individuals begin to use probability prediction instead of point prediction to carry out the future. However, the traditional evaluation methods of point prediction cannot effectively evaluate the results of probabilistic prediction. Because if we want to evaluate the probability prediction effectively, we should not only evaluate the sharpness of the prediction distribution but also evaluate its calibration. For evaluating the result of probabilistic forecasts, scoring rules is a very effective method. It can evaluate the sharpness of the prediction of distribution while assessing calibration. In this article, we have used different scoring rules to evaluate the different forecasting result base on different models at the index of ASX 200 and M3 datasets.

Chapter 1

Introduction

Probabilistic prediction is a method to forecast future uncertain events and development by generating probability prediction distribution. Base on the available information set, to maximize the sharpness of prediction distribution and subject to calibrate. (Gneiting & Katzfuss, 2014) Comparing the point forecasts can produce a single point result, such as predicted a stock price in the next day, probabilistic prediction can supply more information to the forecaster by assigning a probability distribution to each future possible outcome as supplying the probabilistic distribution on different prices on the second day. Obviously, probabilistic forecasting has more obvious advantages than point forecasting, so people begin to use probabilistic forecasts to predict activities rather than using point forecasts in many fields, such as finance, weather, medicine etc. For evaluating the results of probabilistic forecasting, the methods used to evaluate the results of point prediction cannot be effectively applied. Therefore, the scoring rules are used.

In this report, we will explain probability forecasts, sharpness and calibration at section 2. About the proper scoring rules and their formulas will be introduced in section 3. And in the next section, we will learn how to use the scoring rules to evaluate the results of probability prediction by using two case studies. In these two case studies, the forecasts all based on the Gaussian prediction distribution.

Chapter 2

Assessing probabilistic forecasts using scoring rules

The traditional prediction method is mainly based on point forecasts, which can provide forecasters with future development trend information under given significant level. But the future is extremely uncertain. It's hard to predict an accurate future through the past information. For example, when watching a football match, if the level of the two teams is very different, we can easily judge that the team is more likely to win, but how many goals is hard to know. At this point, the limitations of point forecasts are reflected. But the probabilistic forecasts can be given a probability distribution for all possible future results so that more information can be obtained to predict the uncertain future. If we can assign a different probability to different results in the game, the fans will be able to judge the result of the match.

There are two important factors to evaluate the results of probabilistic forecasts: calibration and sharpness. The meaning of sharpness refers to the centralization of the predicted distribution and the calibration refers to the statistical consistency between the predicted distribution and the observed value. Gneiting, Balabdaoui, and Raftery (2007) They affect the quality of probabilistic forecast. Therefore, to evaluate the calibration and sharpness of probability prediction is an important means to evaluate probability prediction results.

2.1 Distribution scoring rules

Scoring rules supply the summary measures to evaluate probabilistic forecasts, it assigns a numerical score under the predictive distribution and the events that needs to be predicted. Gneiting, Balabdaoui, and Raftery (2007) The function of scoring rules is to evaluate the calibration and the sharpness of the forecast distribution results at the same time, then evaluating the quality of probabilistic forecasts. For the results of produced scores, forecasters wish it can be minimized.

2.1.1 Property of scoring rules

Assume the result of probabilistic forecasts is F, $F \in \mathcal{F}$ where $\mathcal{F}$ is a suitable class of CDFs, and $G : \mathcal{F} \times \cdots \times \mathcal{F} \to \mathcal{F}$. Then the scoring rule will be $S(F, y)$, where $y \in R$ is the realized outcome.

The scoring rule S is proper relative to the class $\mathcal{F}$ if

$$S(F, G) \geq S(G, G)$$

for all $F, G \in \mathcal{F}$. Also when $F = G$, the two sides of equation are equal, then it meanings the scoring rules is strictly proper.

For variables on a continuous sample space, the most commonly used scoring rules are the logarithmic score (LogS), continuous ranked probability score (CRPS) and Dawid-Sebastiani score (DDS). They can be applied effectively for density forecasts.

2.1.2 Logarithmic score

For the scoring rules for evaluating probabilistic forecasts, the of the most commonly used rules is Logarithmic score (logS). It was first proposed at 1952 by Good. It is a modified version of relative entropy and can be calculated for real forecasts and realizations.Roulston and Smith (2002) It is a strictly proper scoring rules. But if the prediction is continuous, using ignorance is troublesome Peirolo (2010). Despite its shortcomings, it can directly evaluate the results through the forecast model. Therefore, logarithmic scoring rule can be used in many scenarios and is not limited to specific models.

The formula is:

$$LogS(F,y) = logF(y)$$

For this report, we use the scoring rules to evaluation the probabilistic forecasts under Gaussian predictive distributions. Then the formula of the logarithmic score can be rewritten as below.

$$LogS(N(\mu,\sigma^2),y) = \frac{(y-\mu)^2}{2\sigma^2} + log\sigma + \frac{1}{2}log2\pi$$

2.1.3 Continuous Ranked Probability Score

It is generally considered that it is unrealistic to limit the density forecasts. In the absence of restriction on density forecasts, the CRPS can define scoring rules directly in terms of predictive cumulative distribution functions. It focuses on observing the whole of forecast distributions rather than the special points in these distribution. It can use deterministic valus to evaluate the results of probabilistic forecasts. ALso, comparing with the CRPS, logarithmic score is a local strictly proper scoring rule. Therefore, there are not many restrictions on its use.

The formula of continuous ranked probability Score:

$$CRPS(F,y) = \int_{-\infty}^{\infty} (F(x) - 1\{y \leq x\})^2 dx$$

$$= E_F|Y-y| - \frac{1}{2}E_F|Y-Y'|$$

where Y and Y′ are independent random variables with CDF F and finite first moment Gneiting and Raftery (2007). The CPRS can compare the probabilistic forecasts and point forecasts because when the CRPS drop to the absolute error, the probabilistic forecast is a point forecast. Gneiting and Katzfuss (2014)

Also, when evaluating probabilistic forecasts under Gaussian predictive distribution the form will re-write:

$$CRPS(N(\mu,\sigma^2),y) = \sigma\left(\frac{y-\mu}{\sigma}\left(2\Phi\left(\frac{y-\mu}{\sigma}\right)-1\right)+2\varphi\left(\frac{y-\mu}{\sigma}\right)-\frac{1}{\sqrt{\pi}}\right)$$

2.1.4 Dawid-Sebastianti score

The CRPS can be easy to understand and convenient to use, but it has a limitation. It can be hard to compute for complex forecast distributions. Gneiting and Katzfuss (2014). Therefore, Therefore, when we need to evaluate the probabilistc forecasts under the complex distribution, choosing Dawid-Sebastiani score is a viable alternative.

The formula of DSS

$$DSS(F,y) = \frac{(y-\mu_F)^2}{\sigma_F^2} + 2log\sigma_F$$

Chapter 3

Probabilistic forecasts for the ASX 200 index

The ASX 200 is an index on the Australian Securities Exchange officially released on 31st March 2000. It uses market-weighted average calculations based on the 200 largest listed stocks in Australia. These stocks currently account for the Australian stock market value of 82%. It is considered to be the most important index to measure the operation of the Australian stock market.

We use the daily data over 10 years period until the beginning of 2018 to fit the models. In order to use financial data more efficiently for probabilistic forecasting, and for subsequent evaluation of result by scoring rules, we have processed the data and obtained a simple return for ASX 200 daily price. About the models, we choose to use ARIMA model and ARIMA-GARCH model to fit model. Since the simple return time series can be stationary, some models are not suitable to use as ETS model. Also, using these two models can be very intuitively and clearly to compare the results of forecasting and scoring.

In order to choose the appropriate model, we use auto.arima code from forecast package to automatically search for suitable ARIMA model, and setting the data before 2017 as train data, the data for 2017 as test data. Then the MA(3) model was selected. Then use the already found MA(3) model to select the Garch model.

Table 3.1: *Garch model select*

	AIC	BIC	SIC	HQIC
garch11	10.608	10.623	10.608	10.614
garch12	10.609	10.626	10.609	10.615
garch21	10.609	10.626	10.609	10.616
garch22	10.610	10.629	10.610	10.617
arch1	10.779	10.791	10.779	10.783
arch2	10.729	10.744	10.729	10.734

Table 3.2: *Scoring Rules for MA model and GARCH model*

	CRPS	LogS	DSS
GARCH	20.70	5.10	8.36
ARIMA	21.13	5.14	8.45

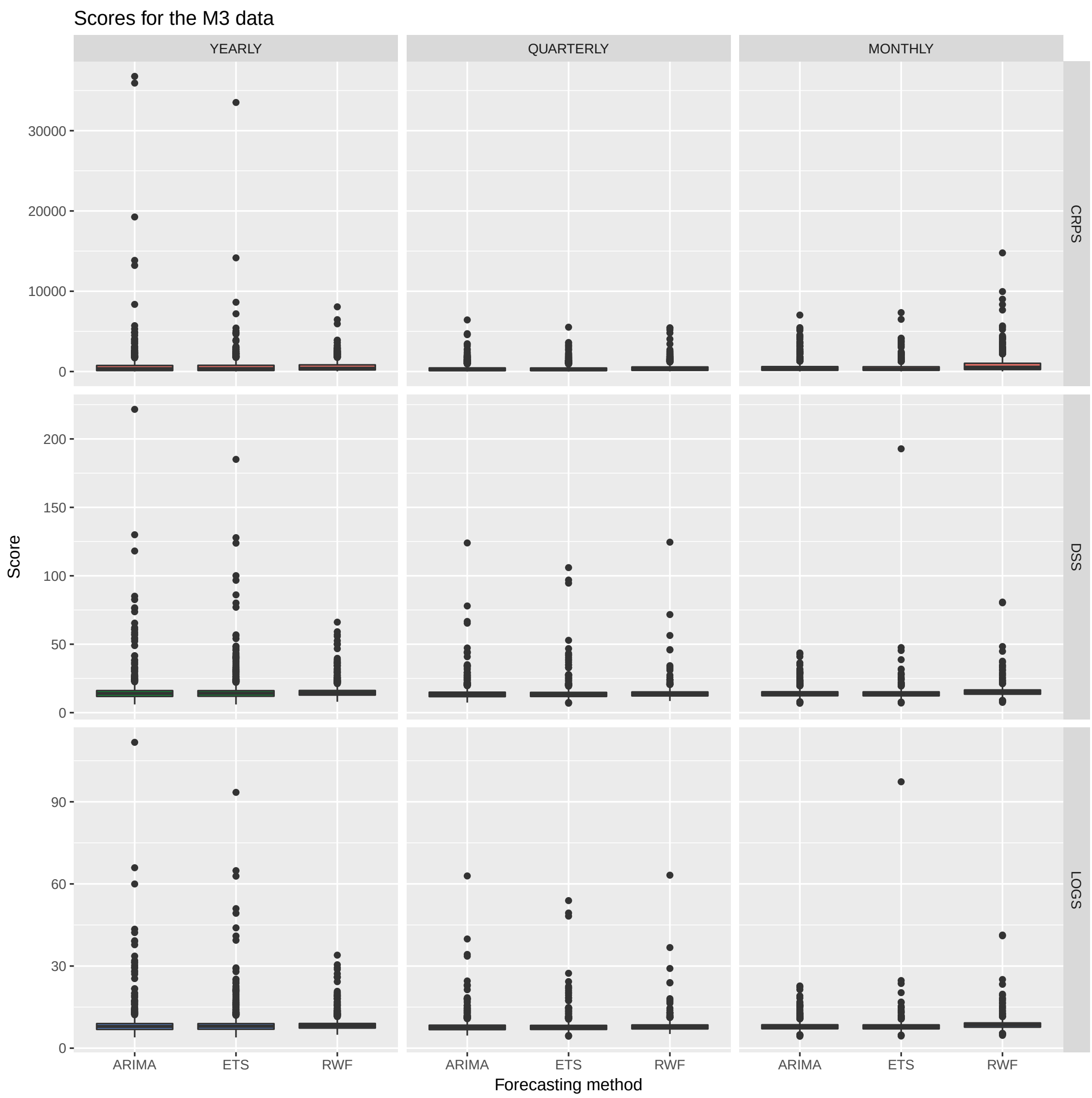
After comparing the AIC of each Garch model, the AIC of the MA(3)-Garch(1,1) is 10.60842, it is the smaller than other models. So, we consider using it to fit data. Then use these two models to predict the result at the year of 2017, and then evaluate the results of the two models by scoring rules. The results are displayed in the following table.

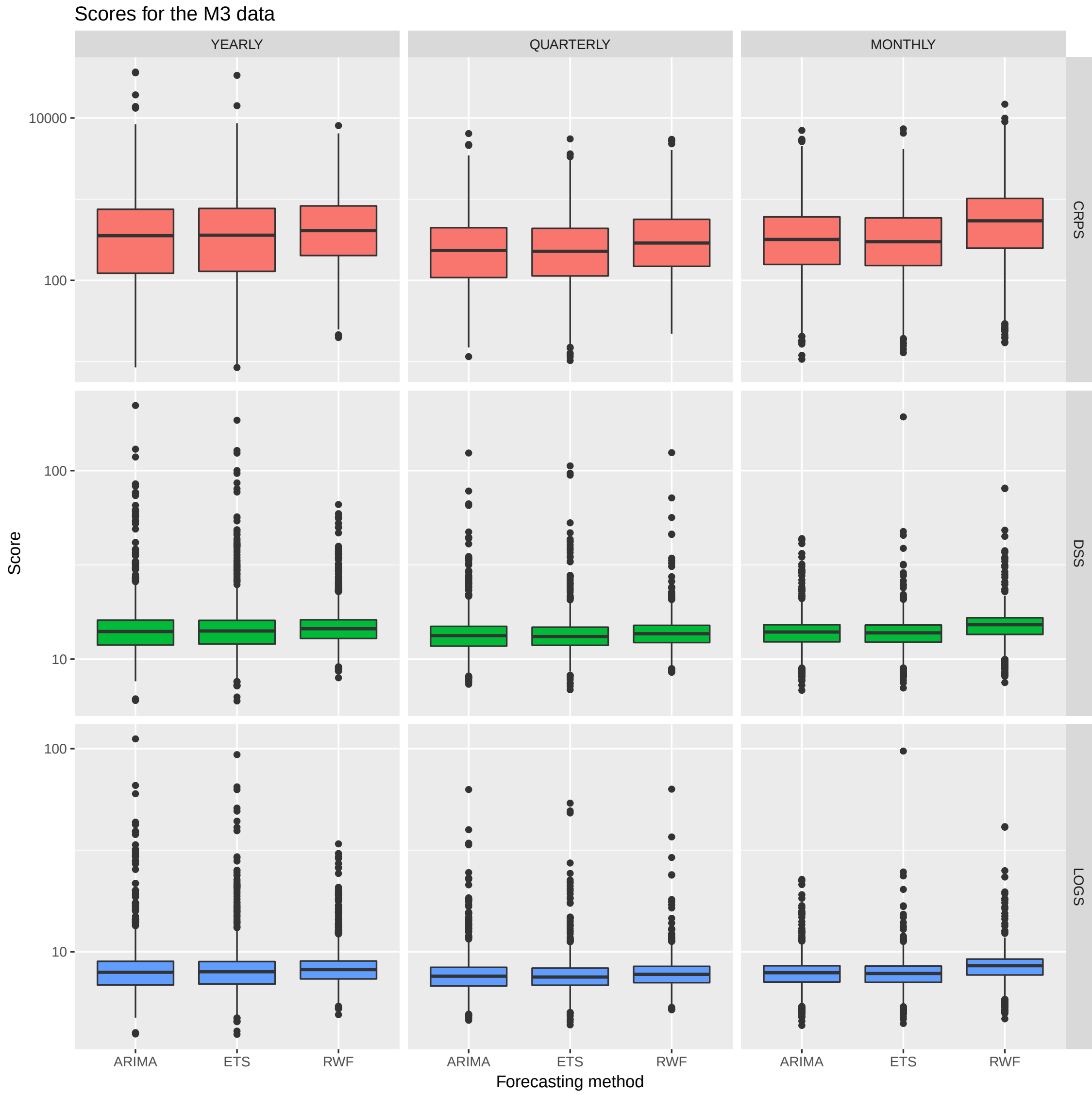
According to the table above, the results of three type scoring rules of MA(3)-garch(1,1) model are all smaller than the result of MA(3) Model. Therefore, it can be shown here that the garch model has a better prediction performance compared to MA(3).

Chapter 4

Probabilistic forecasts for the M3 competition data

The M3 dataset includes 3003 different type time series, it is from R packages Mcomp, so it can provide more information for evaluating probabilistic forecast by using scoring rule. Different from previous financial data, M3 datasets can use different models for predictive analysis at the same time. In this part, three prediction models are used, ARIMA model, ETS model, and Random walk model. After separately predicting these 3003 different time series, we reached 9009 forecast sets. Then each of these forecasting sets is evaluated by three different scoring rules separately. And to average the evaluation results for each different time series. Use these evaluation results to generate three boxplots, they represent the performance of different models to predict under different scoring rules.


Scores for the M3 data
YEARLY
QUARTERLY
MONTHLY
CRPS
DSS
LOGS
Score
30000
20000
10000
0
200
150
100
50
0
90
60
30
0
ARIMA
ETS
RWF
ARIMA
ETS
RWF
ARIMA
ETS
RWF
Forecasting method



Although it cannot be known from the above figure how many outliers are generated base on the different forecast model by different scoring rules, boxplots can show 5th, 25th, 50th, 75th and 95th percentiles of central prediction interval width. The width of the obvious random walk model is much narrower than that of other models, which means that its sharpness is sharpest, and the calibration is more accurate, although its mean value is not the lowest. Therefore, in the case of using M3 data sets, the quality of probability predictions derived from random walk model is even higher. This also proves that using scoring rules can simultaneously evaluate the sharpness and calibration of probabilistic results.

Chapter 5

Conclusion

In this report, we introduced what is the probabilistic forecasts, and calibration and sharpness. It also introduced scoring rules. For the three commonly used scoring rules, we show their original formulas and form under Gaussian predictive distribution. In the section of the case study, we first used two models to do probabilistic forecasts for the ASX200 index, to evaluate the forecasts results by using scoring rules. Then we learned how to use scoring rules to evaluate the outcome of forecasts. In the second case study, we used the M3 datasets and used multiple models to do probabilistic forecasts. We learned how the scoring rules evaluated both the probabilities and the results of car calibration and sharpness.

Chapter 6

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