Optimal Strategies for New Recruit using Linear Programming

Nataraj Dasgupta*

Abstract: Optimal combination of choices that maximise, minimise or equalise payoff for individual parties in the New Recruit problem can be estimated quantitatively using an Optimisation Model formulation. This can be especially useful for selecting options from distributive and integrative preference with incongruent payoffs. More concretely, (Mixed Integer) Linear Programming may be used to estimate payoffs with and without fixed constraints such as reservation values for co-operative and competitive scenarios with a objective function that maximises value creation.

The results from applying an optimisation model in the New Recruit problem indicate that the most co-operative (value-creation) scenario would result in the recruiter and candidate each receiving 6,600 points. On the other hand, minimum co-operation results in a 18,000 point difference with the candidate and recruiter receiving 12,000 and -6,000 points respectively.

Keywords: negotiation, optimisation, pareto-efficiency

Contents

* E-mail address: nataraj.dasgupta18@alumni.ic.ac.uk

1 Overview

Basic tenets of (mixed integer) linear programming can often be used to estimate optimal bipartite payoff in various day-to-day real-world problems. The New Recruit problem is one such use case that can be represented as a set of constraints with an objective function to maximise or minimise payoff for co-operative and competitive solution. The best payoff for any one party can be obtained simply by selecting options that have the highest value in each category. Finding co-operative and/or competitive payoffs, however, may be more challenging due to the non-congruent nature of payoff across categories.

An optimisation formulation also allows us to determine optimal payoff subject to boundary conditions. For instance, if the employer offers a max salary of $ 86,000 and a max bonus of 6% and if these limits are non-negotiable, what choices would result in an optimal co-operative payoff ? Or conversely, in a competitive scenario, what choices could result in maxmising the payoff for say the candidate, whilst minimising the payoff for the recruiter. A few such examples have been discussed to demonstrate the utility of an optimisation-based approach.

2 Introduction

The parties to the negotiation exercise are a recruiter and a candidate. There are 8 categories of "issues": Bonus, Job Assignment, Vacation Time, Start Date, Moving Expense Coverage, Insurance Coverage, Salary and Job Location. The payoff matrix corresponding to each party has been provided in Table 2.

Due to the competing interests of the two parties, points assigned across options may be congruent, distributive or integrative. For instance, whilst a candidate earns the highest points for negotiating a 25-day vacation allowance, the recruiter receives no points[1].

2.1 Data

Each option in the payoff matrix (Table 2) has been assigned an index number (i) and a binary decision variable, b_i that takes the value 1 if the corresponding option i, is chosen and 0 otherwise. In our problem formulation, the recruiter and candidate payoffs have been similarly denoted by the variables, r_i and c_i respectively.

3 Optimisation Model Formulation

In general, optimisation models have 3 main aspects: 1) Decision Variables, an n-dimensional vector of variables to be estimated; 2) Objective Function, i.e., the equation we seek to maximise or minimise; and 3) Constraints, i.e., boundary conditions for the problem.

3.1 Decision Variables

Let b_i be the binary decision variable associated with option i. No other decision variables are necessary for the given model.

3.2 Objective Function

The objective function maximises the combined payoff of both the recruiter and the candidate, i.e., the optimiser will attempt to find the best combination of binary decisions that will result in the maximum total payoff for the recruiter and the candidate. In matrix notation, the combined payoff can be represented in

[1] An exhaustive search to find all sets of possible choices would yield $5^8 = 390,625$ combinations. As such, this can be easily computed. However, real-world issues are more complex and often computationally intractable

terms of row and column vectors as follows,

$$\text{Total Payoff} = \begin{bmatrix} r_1 & r_2 & \dots & r_{40} \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_{40} \end{bmatrix} + \begin{bmatrix} c_1 & c_2 & \dots & c_{40} \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_{40} \end{bmatrix} \tag{1}$$

where r_i and c_i represent the i^{th} payoff for the recruiter and candidate respectively and b_i represent the binary decision variable for the i^{th} option as shown in Table 2.

Hence, our formal **objective function** is as follows,

$$\text{maximise} \sum_{i=1}^{40} (r_i \cdot b_i) + \sum_{i=1}^{40} (c_i \cdot b_i) \tag{2}$$

3.3 Optimisation Constraints

Constraint: Only 1 option may be chosen from each category

$$\begin{aligned}
\text{Only 1 Bonus:} \quad & b_1 + b_2 + b_3 + b_4 + b_5 && = 1 \\
\text{Only 1 Job Assignment:} \quad & b_6 + \cdots + b_{10} && = 1 \\
\text{Only 1 Vacation Time:} \quad & b_{11} + \cdots + b_{15} && = 1 \\
\text{Only 1 Starting Date:} \quad & b_{15} + \cdots + b_{20} && = 1 \\
\text{Only 1 Moving Expense Coverage:} \quad & b_{21} + \cdots + b_{25} && = 1 \\
\text{Only 1 Insurance Coverage:} \quad & b_{26} + \cdots + b_{30} && = 1 \\
\text{Only 1 Salary:} \quad & b_{31} + \cdots + b_{35} && = 1 \\
\text{Only 1 Location:} \quad & b_{36} + \cdots + b_{40} && = 1
\end{aligned} \tag{3}$$

3.3.1 Case 1: Maximise co-operation

In order for the payoffs to be co-operative, we add a constraint for the absolute value of the difference between the recruiter and candidate payoff to be 0, i.e.,

$$\left\| \sum_{i=1}^{40} (r_i \cdot b_i) - \sum_{i=1}^{40} (c_i \cdot b_i) \right\| = 0 \tag{4}$$

Generally, it is advisable to use ≤ or ≥ constraints because a feasible solution with equal payoff might not exist. In the case of the New Recruit problem, however, the solver was able to find a feasible solution where both payoffs were the same.

3.3.2 Case 2: Maximise co-operation under non-negotiable constraints

The second case explores a scenario where the maximum salary and bonus offered by the employer have an upper limit of \$ 86,000 and 6% respectively. We add these limits as additional constraints. Further we include an additional constraint that restricts the maximum difference between the recruiter - candidate payoff to no more than ± 1,000 points. Hence, in summary, we add 3 new constraints as follows:

$$\begin{aligned}
\text{Bonus should be} \leq 6\%: \quad & b_3 + b_4 + b_5 = 1 \\
\text{Salary should be} \leq \$\ 86{,}000: \quad & b_{33} + b_{34} + b_{35} = 1 \\
\text{Max abs. difference} \leq 1{,}000: \quad & \left\| \sum_{i=1}^{40} (r_i \cdot b_i) - \sum_{i=1}^{40} (c_i \cdot b_i) \right\| \leq 1{,}000
\end{aligned} \tag{5}$$

where b_3, b_4 & b_5 are the decision variables for Bonus = 6%, 4% and 2% and $b_{33} + b_{34} + b_{35}$ are the decision variables for Salary = \$ 86,000, \$ 84,000 & \$ 82,000 respectively. By adding these constraints we limit the choices only to the corresponding options from each category.

3.3.3 Case 3: Minimise co-operation

In the third & final example, we explore a scenario where the *difference* between the recruiter and candidate payoffs have been *maximised*, i.e., the least co-operative scenario where one party wishes to maximise individual payoff and minimise the counterparty's payoff.

We can maximise the absolute difference between the payoffs by simply replacing the original objective function in Eqn. 2 as follows:

New Objective Function:

$$\text{maximise} \left\| \sum_{i=1}^{40} (r_i \cdot b_i) - \sum_{i=1}^{40} (c_i \cdot b_i) \right\| \tag{6}$$

4 Model Results

The results indicated that in max co-operation, each party would receive 6,600 points. For max co-operation with limits on max salary and bonus and a max. difference of 1,000 points, the recruiter and candidate received 3,300 and 3,900 points respectively. Finally, min co-operation results in a solution with an 18,000 point difference.Note that there may also be other payoff combinations in the feasible solution space.

Case	Objective	Objective Fn.	Constraints	Recruiter	Candidate	Total	Diff.
1	Max co-op	Eqn. 2	Eqn. 3 & 4	6,600	6,600	13,200	0
2	Max co-op + conditions	Eqn. 2	Eqn. 3 & 5	3,300	3,900	7,200	600
3	Min co-op	Eqn. 6	Eqn. 3	-6,000	12,000	6,000	18,000

Tab. 1: Parameter Selection and Results

	Most Co-op, Fixed Choices			Most Co-op, Fixed Choices			Least Co-operative		
Topic	**Options**	**Recruiter**	**Candidate**	**Options**	**Recruiter**	**Candidate**	**Options**	**Recruiter**	**Candidate**
Bonus	10%	0	4,000	6%	800	2,000	10%	0	4,000
Division	Division A	0	0	Division A	0	0	Division A	0	0
Vacation	5 days	4,000	0	10 days	3,000	400	25 days	0	1,600
StartDate	July 1	1,200	1,200	July 1	1,200	1,200	June 1	0	2,400
Moving	100%	0	3,200	80%	400	1,600	100%	0	3,200
Insurance	Plan E	3,200	0	Plan A	0	800	Plan A	0	800
Salary	\$86,000	-3,000	-3,000	\$86,000	-3,000	-3,000	\$90,000	-6,000	0
Location	San Francisco	1,200	1,200	Altanta	900	900	New York City	0	0
		6,600	6,600		3,300	3,900		-6,000	12,000
		Difference	0			300			18000

5 Appendix

5.1 Payoff Matrix

	Options	Index (i)	Decision Variable	Recruiter (r)	Candidate (c)
Bonus	10%	1	b_1	0	4,000
	8%	2	b_2	400	3,000
	6%	3	b_3	800	2,000
	4%	4	b_4	1,200	1,000
	2%	5	b_5	1,600	0
Division	Division A	6	b_6	0	0
	Division B	7	b_7	-600	-600
	Division C	8	b_8	-1,200	-1,200
	Division D	9	b_9	-1,800	-1,800
	Division E	10	b_{10}	-2,400	-2,400
Vacation	25 days	11	b_{11}	0	1,600
	20 days	12	b_{12}	1,000	1,200
	15 days	13	b_{13}	2,000	800
	10 days	14	b_{14}	3,000	400
	5 days	15	b_{15}	4,000	0
Start Date	June 1st	16	b_{16}	0	2,400
	June 15th	17	b_{17}	600	1,800
	July 1st	18	b_{18}	1,200	1,200
	July 15th	19	b_{19}	1,800	600
	August 1st	20	b_{20}	2,400	0
Moving	100%	21	b_{21}	0	3,200
	90%	22	b_{22}	200	2,400
	80%	23	b_{23}	400	1,600
	70%	24	b_{24}	600	800
	60%	25	b_{25}	800	0
Insurance	Plan A	26	b_{26}	0	800
	Plan B	27	b_{27}	800	600
	Plan C	28	b_{28}	1,600	400
	Plan D	29	b_{29}	2,400	200
	Plan E	30	b_{30}	3,200	0
Salary	$ 90,000	31	b_{31}	-6,000	0
	$ 88,000	32	b_{32}	-4,500	-1,500
	$ 86,000	33	b_{33}	-3,000	-3,000
	$ 84,000	34	b_{34}	-1,500	-4,500
	$ 82,000	35	b_{35}	0	-6,000
Location	San Francisco	36	b_{36}	1,200	1,200
	Altanta	37	b_{37}	900	900
	Chicago	38	b_{38}	600	600
	Boston	39	b_{39}	300	300
	New York City	40	b_{40}	0	0

Tab. 2: Recruiter / Candidate Payoff