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1) Answer:

The Double Precision Floating point number is 64 bits. Its sign bit, exponent bits and mantissa bits are 1, 11 and 52 respectively. the minimum value of it is

$$2^{-1022} \approx 2.23 \times 10^{-308}$$

And the maximum value is

$$(1 + (1 - 2^{52})) \times 2^{1023} \approx 1.798 \times 10^{308}$$

2)Answer:

First, converting 2²4 into Single Precision Floating Format, we can get that

$$2^24 = 0.00011000.0000000...$$
(all the rest are zeros) = $(1.000...$ (total 23 zeros))₂ × 2^24

If we converting 1 into Single Precision Floating Format we can get that

$$1 = (1.000...(\text{total } 23 \text{ zeros}))_2 \times 2^0 = (0.00...01(\text{total } 23 \text{ zeros after decimal point}))_2 \times 2^2 + (0.00...01(\text{total } 23 \text{ zeros after decimal point}))_2 \times 2^2 + (0.00...01(\text{total } 23 \text{ zeros after decimal point}))_2 \times 2^2 + (0.00...01(\text{total } 23 \text{ zeros after decimal point}))_2 \times 2^2 + (0.00...01(\text{total } 23 \text{ zeros after decimal point}))_2 \times 2^2 + (0.00...01(\text{total } 23 \text{ zeros after decimal point}))_2 \times 2^2 + (0.00...01(\text{total } 23 \text{ zeros after decimal point}))_2 \times 2^2 + (0.00...01(\text{total } 23 \text{ zeros after decimal point}))_2 \times 2^2 + (0.00...01(\text{total } 23 \text{ zeros after decimal point}))_2 \times 2^2 + (0.00...01(\text{total } 23 \text{ zeros after decimal point}))_2 \times 2^2 + (0.00...01(\text{total } 23 \text{ zeros after decimal point}))_2 \times 2^2 + (0.00...01(\text{total } 23 \text{ zeros after decimal point}))_2 \times 2^2 + (0.00...01(\text{total } 23 \text{ zeros after decimal point}))_2 \times 2^2 + (0.00...01(\text{total } 23 \text{ zeros after decimal point}))_2 \times 2^2 + (0.00...01(\text{total } 23 \text{ zeros after decimal point}))_2 \times 2^2 + (0.00...01(\text{total } 23 \text{ zeros after decimal point}))_2 \times 2^2 + (0.00...01(\text{total } 23 \text{ zeros after decimal point}))_2 \times 2^2 + (0.00...01(\text{total } 23 \text{ zeros after decimal point}))_2 \times 2^2 + (0.00...01(\text{total } 23 \text{ zeros after decimal point}))_2 \times 2^2 + (0.00...01(\text{total } 23 \text{ zeros after decimal point}))_2 \times 2^2 + (0.00...01(\text{total } 23 \text{ zeros after decimal point}))_2 \times 2^2 + (0.00...01(\text{total } 23 \text{ zeros after decimal point}))_2 \times 2^2 + (0.00...01(\text{total } 23 \text{ zeros after decimal point}))_2 \times 2^2 + (0.00...01(\text{total } 23 \text{ zeros after decimal point}))_2 \times 2^2 + (0.00...01(\text{total } 23 \text{ zeros after decimal point}))_2 \times 2^2 + (0.00...01(\text{total } 23 \text{ zeros after decimal point}))_2 \times 2^2 + (0.00...01(\text{total } 23 \text{ zeros after decimal point}))_2 \times 2^2 + (0.00...01(\text{total } 23 \text{ zeros after decimal point}))_2 \times 2^2 + (0.00...01(\text{total } 23 \text{ zeros after decimal point}))_2 \times 2^2 + (0.00...01(\text{total } 23 \text{ zeros after decimal point}))_2 \times 2^2 + (0.00...01(\text{to$$

Then if we add these two numbers, it will be

$$(1.00...01(total\ 23\ zeros\ after\ decimal\ point))_2 \times 2^24$$

which has a 24 bits Mantissa. It exceeds the precision limit of the Single Precision Floating Format since there are only 23 bits of Mantissa.

Therefore 2²⁴ + 1 is still 2²⁴ in Single Precision Floating Format.

3)Answer:

$$111 = (11011111)_2$$

$$0.875 = (0.111)_2$$

$$111.875 = (1101111.111)_2 = (1.101111111)_2 \times 2^6$$

Since the offset is 129 and exponent is 10 bits, the exponent bits should be

$$129+6 = 135 = (0010000111)_2$$

Then the result should be

0 0010000111 1011111111000...(all the rest are zeros)

format sign+exponent+mantissa

4)Answer:

Since for single precision, the fraction is 23 bits, then the epsilon should be $2^-24 = 5.96e-08$. Since for double precision, the fraction is 52 bits, then the epsilon should be $2^-53 = 1.11e-16$.

9)Answer:

Based on the code for Gaussian Elimination in Q10, there is a 'while loop' outside the codes which cost $O(n^2)$. In the wile loop, there are several O(n) 'for loops' to do swapping and subtracting/dividing. Therefore the approximate is $O(n^3)$.

From another view, as the Gaussian Elimination just picking one pivot and doing elimination on the other rows, and there is n pivots to pick, therefore the approximate is $n(n-1)+(n-1)(n-2)+...2*1 = O(n^3)$.