

# Mathematics for Scientific Oxbridge Interviews

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## 1 Introduction

Hi. If you're reading this then you're probably thinking of applying for a physical sciences course at Oxbridge, such as Natural Sciences at Cambridge, or some other more terrible course. My goal here is to give you more confidence in dealing with the mathematical questions you're likely to encounter at interview, focusing on a few skills that they seem to like.

Some of this will draw from my own experience in the interviews (which is hopefully at least a little valuable since I got in) and also includes some of the questions I was asked in the interviews. Most people (yourself probably included) are asked to sign some sort of NDA before the interview, saying you won't tell anyone what questions you are asked. For some reason I was never asked to sign anything. And since I'm not some private organisation selling interview questions to rich people (such organisations do unfortunately exist), I don't feel bad about telling you.

## 2 General Practicalities

Obviously, plan to arrive a couple of hours before your interview is due to start. Have a wander round the town, get a bite to eat, do whatever helps you feel relaxed and ready.

When you've signed in at the college, there will probably be some sort of space where you'll be allowed to wait before your interview – if your college is as nice as St Catharine's Cambridge you might even get the college library. Something I found quite effective at this time was reading some kind of popular science book/magazine during this time. It gets your brain into “science mode” without unnecessarily stressing you out – which might occur if you spend the time doing very difficult practice questions, for example.

You almost certainly will not need to bring your own stationary for the interview. Probably worth bringing a pen or two just in case, but don't expect to use them.

## 3 Question 1

My first interview was with the legendary Chemistry educator, Dr Peter Wothers, who was subsequently my 1st year Chemistry supervisor. The first question I was asked, by this giant of Chemistry, was the following:

“Could you please sketch the bonding in elemental nitrogen.”

Those of you doing A-level Chemistry will recognise that as an easy-ass question. :N≡N:

It is in the best interests of interviewer and interviewee alike that you are able to articulate your thoughts. Being able to talk with them about the questions they ask helps them do *their* job as well, and so to help calm your nerves, they probably won't ask anything too demanding straight away. That will come later.

That is not to say that your first question will *definitely* be super easy, though my first interview started with a Year 9 Chemistry question and my second started with something I said on my personal statement that I knew a lot about [in general though, don't expect Oxbridge to focus on your PS; they usually prefer other bits of evidence – like the interview itself – in deciding whether to admit you].

## 4 Graph Sketching

You will almost definitely be asked to sketch a graph in one, if not both, of your interviews. They seem to really like getting people to sketch graphs: it tests a lot of skills that are important for Science, many of which are not emphasised at A-level. So if you are asked to “Sketch the following function” and the following function is something like

$$f(x) = \left(2 - \frac{x}{a_0}\right) e^{-\frac{x}{2a_0}} \quad (1)$$

what do you do? Here are a few useful things to be thinking about, which can be remembered using a very spicy acronym.

### 4.1 Stationary Points

**Learn. Your. Calculus.** Calculus is a stupidly important skill for any Physical Scientist: probably half of my work in my first year involved some calculus. Integration will be discussed in a separate section, but in terms of differentiation, I'm afraid you're probably gonna have to learn, or be able to derive in seconds:

- Product, Quotient, and Chain Rules
- How to differentiate:
  - Polynomials
  - Exponentials
  - Trig functions. Yes, even the dumb ones like  $\cot x$
  - *Inverse* trig functions

Personally, I don't know what the derivative of  $\cot x$  is off the top of my head, nor do I know the derivative of  $\csc^{-1} x$  or something silly like that. But I know *exactly* what I would do to find them, and am confident it wouldn't take much more than a minute. This is unlikely to be enormously important at interview, but it's certainly something they might be justified in expecting from you, particularly if you take Further Maths (which you should do if your school offers it).

### 4.1.1 Maclaurin Series

Maclaurin series (AKA *power series*, or *Taylor series about  $x = 0$* ), are really useful ways of approximating almost any function  $f(x)$ , close to  $x = 0$ . They probably won't come up directly in the interview, but this section is here because so many of the tricks I want to show you rely on Maclaurin series; also they're gonna be really useful as you go further in your studies in Physical Science.

The idea is to generate a polynomial,

$$P(x) = a + bx + cx^2 + \dots$$

called the Maclaurin series, where we choose the values of  $a, b, c, \dots$  to ensure that all the derivatives match the derivatives of the known function of interest,  $f(x)$ . If we can match *all* the derivatives, then the Maclaurin series is actually equal to the function<sup>1</sup>. If only the first few derivatives match, then the Maclaurin series will still be a pretty good approximation to  $f(x)$  near to  $x = 0$ . The more derivatives you match up, the better the approximation will be, but the more effort it is.

In our mission to generate a polynomial that looks like  $f(x)$  around  $x = 0$ , the first thing we should do is make sure that the actual *value* of the polynomial ("the 0th derivative", if you will) at  $x = 0$  is equal to the value of  $f(x)$  at  $x = 0$ . In other words,  $P(0) = f(0)$ . Putting  $x = 0$  into the definition of  $P(x)$ , we see that  $P(0) = a$ , so we set  $a = f(0)$ .

We then ensure that the first derivative of  $P(x)$  at  $x = 0$ , is equal to  $f'(0)$ . Differentiating  $P(x)$  and then setting  $x = 0$  just gives  $P'(0) = b$  (the constant  $a$  disappears,  $bx$  becomes  $b$ , and all the *higher-order* terms will have a factor of  $x$ , or  $x^2$ , etc. and thus die when we plug in  $x = 0$ ). Hence  $b = f'(0)$ .

You might think you have spotted the pattern, but be careful. Let's do the second derivative. The second derivative of  $P(x)$  at  $x = 0$  is  $P''(0) = 2c$ , **not** just  $c$ , so we set  $c = \frac{1}{2}P''(0)$  to compensate. Similarly, you will find that if we continue the series further, the  $x^3$  coefficient of  $P(x)$  will actually be  $d = \frac{1}{6}P'''(0)$ , because when you differentiate 3 times, factors of 3 *and* 2 will then come down from the cubic term.

You can calculate as many terms as you like, but if you are interested in the behaviour of  $f(x)$  near  $x = 0$ , the "higher-order" terms will not contribute much. If you are appending  $5x^8$  to a series you're only interested in for  $0 < x < 0.01$ , the extra 0.0000000000000005 will probably not make much of a difference.

To summarise, the Maclaurin series of  $f(x)$  is:

$$f(x) \approx f(0) + f'(0)x + \frac{1}{2!}f''(0)x^2 + \frac{1}{3!}f'''(0)x^3 + \dots + \frac{1}{n!}f^{(n)}(0)x^n + \dots$$

A few things we can find from this:

- The Maclaurin series of a polynomial is simply the polynomial itself (try working out the Maclaurin series for  $f(x) = 2x^2 - 5x + 6$ )
- $\sin x \approx x - \frac{1}{6}x^3$ , showing that for small values of  $x$ ,  $\sin x \approx x$ . Similarly  $\cos x \approx 1 - \frac{1}{2}x^2$
- Every derivative of  $e^x$  at  $x = 0$  is 1, so  $e^x \approx 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \dots$

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<sup>1</sup>At least, as far as a scientist is concerned. If you tell this to a mathematician they will probably attack you in some way.

### 4.1.2 Stationary Points

To get back on topic (which was graph sketching or something) you may be asked to locate the stationary points on a graph. They might be helpful anyway for making the general shape of the graph more clear – if you find your function has no stationary points, you ought not to draw it flat anywhere.

To locate a stationary point, differentiate the function and find the roots of the resulting function: each root is a stationary point and should be drawn flat on the graph. If you like, you can find the  $y$ -value of the function at the stationary point, and draw a short horizontal line at that point, to give your graph some shape as you fill it in.

Lastly, it might be important whether a stationary point is a minimum, maximum, or point of inflection. For example, if your function represents the potential energy of something, the stationary points are points at which it is in equilibrium (force is the minus derivative of potential energy, so force is 0 at a stationary point): minima are *stable* equilibrium points (a ball placed at these points will come back if you nudge it away) maxima and points of inflexion are *unstable* (a ball at these points will run away if you nudge it).

## 4.2 Axes

As you go about figuring out the shape of the graph, plotting any points on the axes provides some useful pivot points, and depending on what the function represents, these might be physically important values.

The intersection of the  $y$ -axis is usually pretty simple – just plug in  $x = 0$ . [There are cases like the function in figure 1 where this might be problematic – see later].

The intersection(s) of the  $x$ -axis (“roots”) require more thought. If the function is the product of several “sub-functions”, then the overall function has a root wherever any sub-function has a roots, because anything times 0 is 0. For instance for function (1) considered right at the start of this section, there are roots where  $2 - \frac{x}{a_0} = 0$  and where  $\exp\left(-\frac{2x}{a_0}\right) = 0$ . In this way you may be able to “break down” your function into several smaller functions when trying to find roots.

Polynomials, such as  $2 - \frac{x}{a_0}$ , always have a number of roots equal to the highest power of  $x$  they contain – the first half of this function therefore has one root, at  $x = 2a_0$ . Be aware though that in general some of these roots might be complex (and so won’t appear on your graph, like a quadratic which floats above the  $x$ -axis) or repeated (e.g.  $x^2$  has only one root despite being 2nd order).

The exponential function has **NO** roots (not even complex ones), only *tending* towards zero as  $x$  goes to infinity. The root at  $x = 2a_0$  is therefore the only root of the function in question.

Other functions (e.g.  $\sin x$ ) have a more complicated relationship with the  $x$ -axis, but this is something A-level does to death so I won’t bore you with more.

## 4.3 Limits

Try to break your function. What happens if you plug in  $\pm\infty$ ? What if you plug in  $x = 4$  to a function that has  $(x - 4)$  on the denominator? The behaviour of a graph in such silly cases is often scientifically important and they might well ask you about something like this.

### 4.3.1 $\infty$

Most functions increase or decrease without end as  $x$  gets larger and larger. These can sometimes be detected by mentally plugging in like a thousand and seeing how ridiculous the answer gets. If you are less sure, there are some more sophisticated techniques.

Let's consider three functions:

$$\begin{aligned}f_1(x) &= \frac{x+1}{x^2+x+1} \\f_2(x) &= \frac{2x^2-3x+1}{x^2+x+1} \\f_3(x) &= \frac{2x^2-3x+1}{x+1}\end{aligned}$$

How do they behave as  $x$  goes to infinity? Consider plugging in  $x = 1,000$ . This means  $x^2 = 1,000,000$  and  $x^2 + x + 1 = 1,001,001$ . This is pretty darn close to  $1,000,000$ , which is just  $x^2$ . In other words,  $x^2$  is the *dominant term* for large  $x$ . So if you are supposed to add  $x$  to  $x^2$ , then for large  $x$  you might as well not bother. Likewise, when you add 1 to  $x$  you can ignore the 1 – in other words, ignore all but the highest power in a polynomial: the rest will be negligible in comparison. Using fancy notation, we can write:

$$\begin{aligned}\lim_{x \rightarrow \infty} \frac{x+1}{x^2+x+1} &= \lim_{x \rightarrow \infty} \frac{x}{x^2} = \lim_{x \rightarrow \infty} \frac{1}{x} = 0 \\ \lim_{x \rightarrow \infty} \frac{2x^2-3x+1}{x^2+x+1} &= \lim_{x \rightarrow \infty} \frac{2x^2}{x^2} = \lim_{x \rightarrow \infty} 2 = 2 \\ \lim_{x \rightarrow \infty} \frac{2x^2-3x+1}{x+1} &= \lim_{x \rightarrow \infty} \frac{2x^2}{x} = \lim_{x \rightarrow \infty} 2x = \infty\end{aligned}$$

So even though the input to the functions was infinite, two of them gave finite limits.

What about something like:

$$\lim_{x \rightarrow \infty} \frac{e^x}{x}$$

Both the numerator and the denominator tend to  $\infty$ , but what about when you divide them? The correct answer is that it tends to infinity, as may be easily verified by expressing  $e^x$  as its Maclaurin series before dividing by  $x$ . If a fraction like this tends to  $\infty$  rather than 0 we say that the numerator *grows faster* than the denominator, and vice versa if the fraction tends to 0 (if the fraction tends to a finite limit that isn't zero, like  $f_2$  above, we say that the numerator and denominator grow at the same rate).

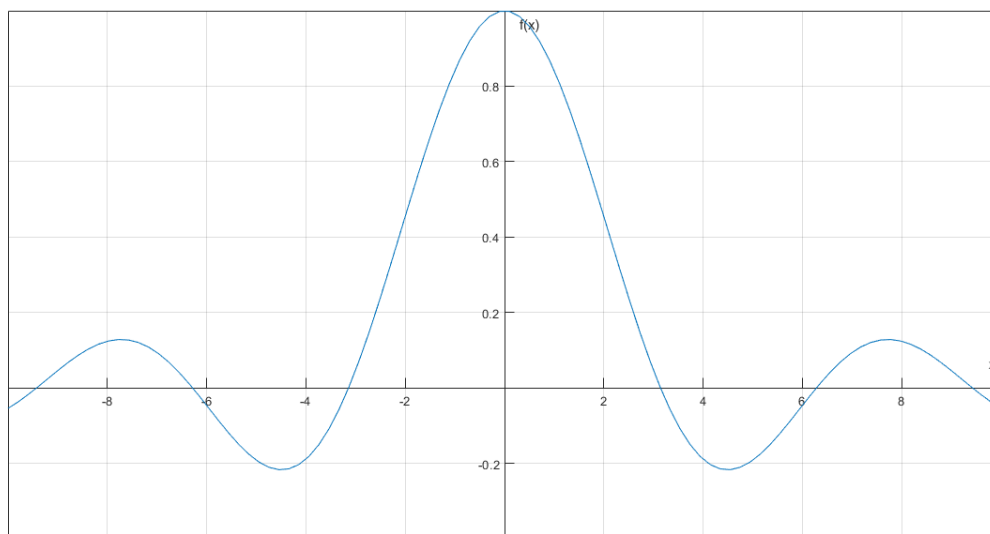
We can extend this analysis by bringing in other things into question like  $\ln x$  and  $x!$ . Eventually we get a hierarchy of functions. As  $x \rightarrow \infty$ :

$$c < \ln x < x < x^2 < \dots < e^x < x!$$

where  $c$  is a constant and the last one isn't just excited. Interestingly, according to Desmos  $x^x$  grows even faster than  $x!$  (!). Anyway if you have a feel for the ordering in the common hierarchy above you shouldn't come across any problems in this area.

### 4.3.2 How to divide by 0

Take the function  $f(x) = \sin(x)/x$ , plotted below.



**Figure 1** |  $f(x) = \sin(x)/x$

You may be puzzled by the fact that this graph appears to intersect the  $x$ -axis at  $y = 1$ , when the function clearly gives  $0/0$  if you try to plug in  $x = 0$ . Mathematically, the value of  $0/0$  could be anything, because  $0 \times \text{anything} = 0$ . If you get an answer of  $0/0$ , it doesn't necessarily mean you've done anything wrong, it just means you need to find a different way of going about the question.

If plugging in a certain value of  $x$  gives you  $0/0$ , or indeed  $\infty/\infty$ , there is a procedure called *L'Hôpital's Rule* which tells you what the function tends to as one gets closer and closer to that value of  $x$ . It's common knowledge in other countries but not in the UK for some reason.

Before discussing this rule, I need to make two technical disclaimers. Firstly the results of L'Hôpital's Rule don't show what the function is *equal to* at the problematic value of  $x$ :  $\frac{\sin 0}{0}$  is most certainly *not* equal to 1. It is undefined. The rule simply shows what the function *tends to*. Secondly, if you don't have  $0/0$  or  $\infty/\infty$ , but rather something like  $\infty/0$  or  $0/\infty$ , you don't need L'Hôpital's rule to tell you that these obviously tend to  $\infty$  and  $0$  respectively.

L'Hôpital's Rule is the following: if  $f(x_0) = g(x_0) = 0$ , or both  $\infty$ , then

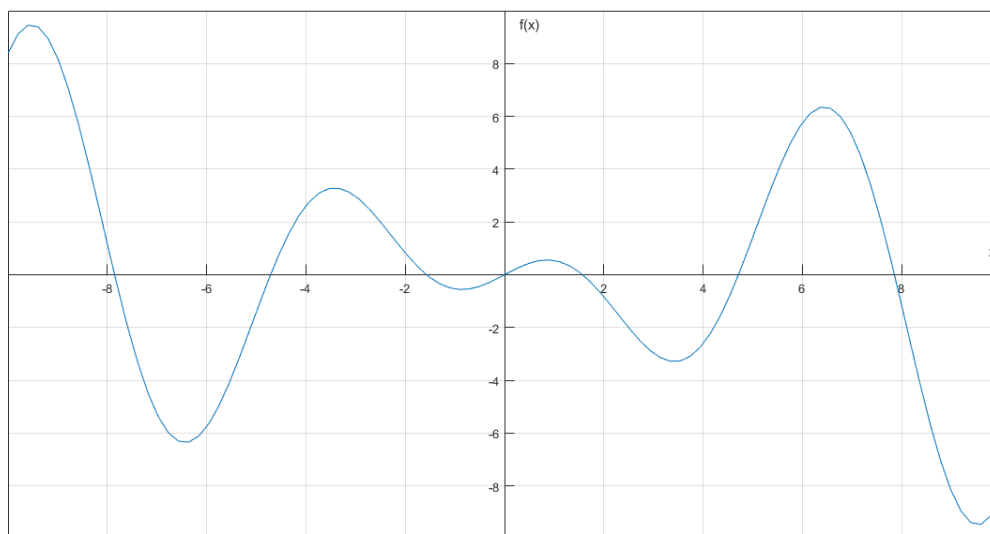
$$\lim_{x \rightarrow x_0} \frac{f(x)}{g(x)} = \lim_{x \rightarrow x_0} \frac{f'(x)}{g'(x)}$$

In other words, if you have the aforementioned problems with a function which is a fraction, you can differentiate the numerator and the denominator (*separately*, no quotient-rule-ing) and the new fraction will give you the right limit (if you still get  $0/0$  even after applying the rule, simply apply the rule again, finding the second derivatives). For example:

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = \lim_{x \rightarrow 0} \frac{\cos x}{1} = \frac{\cos 0}{1} = 1$$

Hence when the graph is about to cross the  $y$ -axis, it is at  $y = 1$ . You could also just use the Maclaurin series for  $\sin x$ :

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = \lim_{x \rightarrow 0} \frac{x - \frac{1}{6}x^3 + \dots}{x} = \lim_{x \rightarrow 0} (1 - \frac{1}{6}x^2 + \dots) = 1$$



**Figure 2** |  $f(x) = x \cos(x)$

Using the series expansions like this is more general, but usually slower, than L'Hôpital's Rule. Best bet would be to try L'Hôpital first, and if that doesn't work or is hard then try expanding the functions into their series.

## 4.4 Symmetry

Recognising some form of symmetry in a function can be a huge time-saver. For example, if you know that the function you are asked to sketch is symmetrical in the y-axis, you only have to bother thinking about  $x > 0$ , and the other half can be filled in automatically; Figure 1 is an example. Such functions are known as *even* functions. In symbols,  $f(-x) = f(x)$ . This family includes the functions  $x^2$ ,  $x^4$ ,  $x^6$ , ...,  $x^{-2}$ ,  $x^{-4}$ , ... and any constant, as you can see by substituting  $-x$  in for  $x$  in any of these functions. It should also be clear that, for example,  $2x^2 - x^4$  is also even. Likewise,  $\cos x$ , is also even – but you should really already know this, from the identity  $\cos(-x) = \cos(x)$ .

*Functions of even functions* are themselves even, for instance  $e^{-x^2}$ . If in doubt, just substitute in  $-x$  for every  $x$  and see what happens when you try to pull out all the minus signs.

A second important family of functions are the *odd* functions. In contrast to the even functions, their graphs show *rotational* symmetry,  $180^\circ$  about the origin; such as the graph shown in figure 2. They have the property that  $f(-x) = -f(x)$ , and so this family includes  $x$ ,  $x^3$ , ...,  $x^{-1}$ , ... as well as  $\sin x$ ,  $x^{71851} - 2x^{-135767}$ , but not  $x^{71851} - 2x^{-135768}$ .

Note also that for *any* odd function,  $f(0) = 0$ , and also  $\int_{-a}^a f(x) dx = 0$ . tHe pRoOf iS LeFt aS aN ExErCiSe tO ThE rEaDer.

### 4.4.1 Combinations of odd and even functions

Combining odd and even functions in various ways does **NOT** work like combining odd and even numbers. The following properties may be obvious; if not then try to prove them or consider some examples.

- Odd  $\times$  Odd = Even (hence Figure 1)
- Even  $\times$  Even = Even
- Even  $\times$  Odd = Odd (hence Figure 2)
- Odd + Odd = Odd
- Even + Even = Even
- Even + Odd = Neither odd nor even

Note the last point - unlike integers, not all functions are simply one or the other. Consider  $x + 1$  for example, which if you sketch it is clearly neither; likewise  $x + x^2$ , and  $e^x$ . Further, there is exactly one (single-valued) function which has the weird property of being both odd *and* even – what is it?

## 4.5 Asymptotes

Asymptotes provide a sense of “structure” to a graph if they have them, and the interviewer will be impressed if you can draw a dotted line well. If the limit of your function is a line, dash it on.

Asymptotes may be vertical, horizontal, or oblique. Vertical asymptotes occur when the function suddenly goes to infinity, such as at  $x = 4$  if your function has  $x - 4$  on a denominator. Horizontal asymptotes occur when the function tends to a finite value at  $\pm\infty$ .

The reason for a separate section for asymptotes is mostly so that the acronym would work. But I also wanted to mention oblique asymptotes because your interviewers will probably be old and this is something they had to do a lot of in school. These can sometimes be detected using the methods in the limit section, for example  $y = 2x - 1/x^2$ . For large  $x$ , the second term will all but disappear, and the graph will tend to the graph of  $y = 2x$ .

Another place that oblique asymptotes might pop up is if you are asked to sketch a graph which is not really of a function, but something more like:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

which is the general equation for a hyperbola (see Figure 3).

As  $x$  and  $y$  get very large, the 1 begins to become irrelevant, and we are left simply with

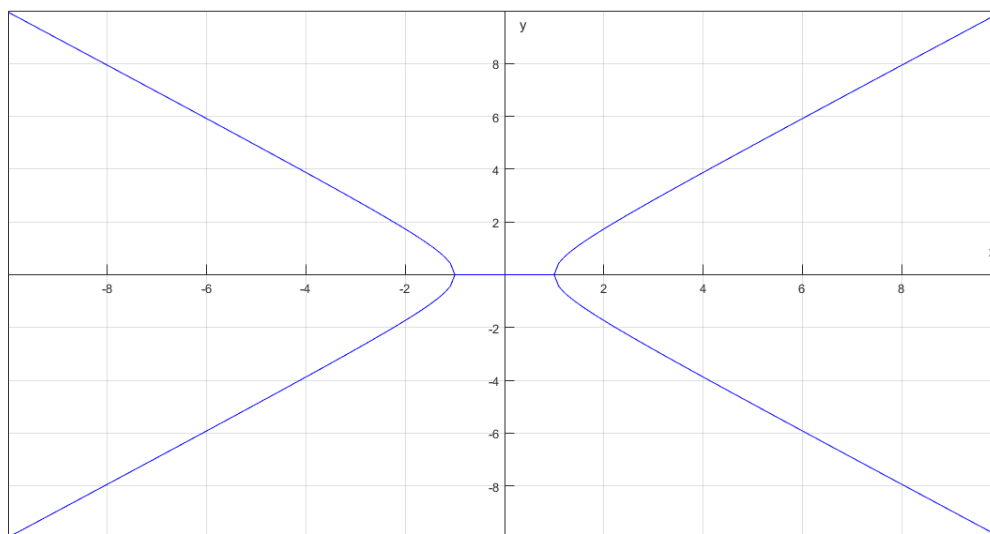
$$\frac{x^2}{a^2} = \frac{y^2}{b^2}$$

which if you rearrange it describes two straight lines through the origin.

## 5 General skills

Graph sketching will almost certainly be important in your interview, but if they *only* ask you to draw some graphs they will be wasting their time. There are lots of other maths things you will probably be asked to do, and they don't fit in to the acronym and aren't just for graphs; here are some more general pointers.





**Figure 3** |  $x^2 - y^2 = 1$

## 5.1 Never do nothing

This is probably the most important section, and it applies to all subjects, even non-scientific ones, so tell your friends.

It may be that if you are trying to sketch a graph, evaluate a sum, or suggest what Dostoevsky really meant in this passage, and no matter how hard you look you can't seem to find any easy shortcuts. This is fine. There may in fact not be any, or they may be deliberately disguised. The best thing to do at this point (in the case of the graph) is to simply try plotting in some points you think would be easy to plot; there is little else you can really do. If there is a trigonometric function, try plugging in  $\pi$ . If there is an exponential, try plugging in  $\ln 2$  (whose value is useful to know as about 0.7).

Crucially, the very worst thing you can do during an interview is nothing. The interviewers are selecting the people they want to do supervisions with next year, and they don't want to do a supervision with a lemon. They want to be engaged and conversed with – perhaps even challenged. **Never do nothing**, and **always think out loud**. If you have absolutely no idea what to do or don't understand, that's not necessarily a bad thing in itself, but for heaven's sake ask for a hint/clue/prompt (good interviewers will notice when you're getting stuck and give you one anyway). If you have a few ideas but aren't super-confident in any of them, talk about them, about why they might or might not work – the interviewers will still be impressed if you say why an approach would *not* work, maybe all the other candidates tried it!

In any case, if you don't tell them what you're thinking, they'll never know if you're on the right track (and probably nor will you). In my second interview, a complicated integral had  $1 + x^2$  on the denominator, so I suggested (*aloud*) maybe substituting in  $x = \tan \theta$ . Although being sternly told in a gruff Russian accent that “It wouldn't help” is a little disheartening, it at least prompts you to think of something else instead of wasting any more time thinking about  $\tan \theta$ .

Here's another question I was asked in my second interview which illustrates a similar point.

I was asked to evaluate:

$$\sum_{n=0}^{\infty} 2^{-\frac{1}{\pi} \left[ \arctan(1+n^2) + \arctan\left(\frac{1}{1+n^2}\right) \right]} 2n$$

(Incidentally, I showed this to Albert Chang in my year and he randomly guessed “Uh, is it 2?” which (spoilers) is the correct answer.)

You’re probably about as freaked out by this thing as I was. I reckon that was the point: to see what you do when faced with something so stupidly complicated that there’s no clever way to immediately proceed. There is a massive shortcut here, but they probably weren’t expecting anyone to see it right away. I certainly didn’t.

The point is, in this situation, there is little to do except to start evaluating terms in the sum. You should not intend to evaluate every single term in the series individually, as this would require an infinite amount of time and you only have about 20 minutes. Rather, you should intend to set off along the path and expect to uncover the shortcut somewhere along the way. Have a go and see if it leaps out at you (you’ll know when it happens. If you’re struggling, read the next section first then have a go).

## 5.2 Trigonometry

This is an important subject that could come up pretty much across the board, and they’ll be expecting a good understanding.

Side note: you may also be familiar with hyperbolic trigonometric functions: you’d be unlucky to be asked a question on them but they might be a good flex if they come up in conversation. In general I can’t think of anything wrong with flexing in an interview if you’re confident, as long as it’s not completely out of the blue and unrelated to what you’re talking about.

### 5.2.1 Identities

A few useful things you should probably know:

- Derivatives and integrals
- Sum angle formulae (e.g.  $\sin(A + B)$ ) and hence double angle formulae
- $\sin^2 x + \cos^2 x = 1$ , and its children  $1 + \cot^2 x = \csc^2 x$ ;  $\tan^2 x + 1 = \sec^2 x$
- Other identities like  $\sin^2 x = \frac{1}{2} (1 - \cos 2x)$  which are really just rearrangements of the cosine double angle identity. These are especially useful for integrals with  $\sin^2 x$  or  $\cos^2 x$  in them.

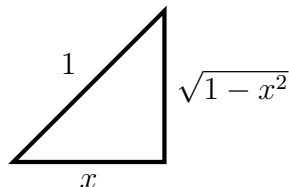
On the last point, there is often a shortcut to such integrals. Over a whole number of cycles (or even a whole number of quarter cycles) you can easily show that the *average* value of  $\sin^2$  and  $\cos^2$  is  $1/2$ . This makes certain integrals massively easier:

$$\int_0^{3\pi/2} \sin^2 x \, dx = \frac{1}{2} \times \frac{3\pi}{2} = \frac{3\pi}{4}$$

No substitutions or identities needed. I showed this trick to my 3rd year relativity supervisor, who had studied at Stanford and ETH Zürich, and he said he had never noticed that before and would definitely use it in future.

### 5.2.2 Complicated expressions

Some trigonometric expressions are simpler than they look. Something like  $\tan(\arccos(x))$  looks a bit menacing, but it may actually be written as  $\sqrt{1-x^2}/x$ . This is most easily seen by drawing the following right-angled triangle:



In this triangle, the Adjacent and Hypotenuse are chosen to be  $x$  and  $1$ , such that the lower-left angle is  $\arccos(x)$ . The Opposite is then  $\sqrt{1-x^2}$  by Pythagoras, and so the tangent of the lower-left angle is  $\tan(\arccos(x)) = \sqrt{1-x^2}/x$ . The latter expression is generally nicer to deal with (e.g. to differentiate), and might make simplifying a larger expression significantly easier.

The key to this method is choosing two sides of your triangle to make the angle equal to what you want it to be. For example, if your expression involves the angle  $\arctan(7x/5)$ , then you should put the Opposite equal to  $7x$  and the Adjacent equal to  $5$ . (You could equally put the Opposite equal to  $7x/5$  and the Adjacent equal to  $1$  – this just corresponds to a larger, but *similar* triangle with all the angles and ratios the same. I just find it easier not to bother with fractions.)

This can also be used to solve similar problems with weird-looking trig expressions. Indeed, it's how I figured out the answer to that crazy summation on the last page – give it a go!

## 5.3 Integration

As with differentiation, this is a lot of what I spent my first year of uni doing. You should know what to do with:

- Polynomials
- Trig functions (and weird and wacky combinations thereof) (perhaps hyperbolic ones too)
- Logs and exponents
- Expressions where you can solve them with trigonometric substitutions (e.g. where there's a  $\sqrt{1-x^2}$ )
- Integration by substitution
- Integration by parts
- Integration by partial fractions

There isn't much to say here other than you should learn how to do the above. Practice makes perfect. In fact, even with knowledge of all the above techniques, it may still be very,

very difficult to know what to do. For example, it took me rather a lot of prompting in my interview to be able to work out the following integral:

$$\int_0^\infty \frac{\ln x}{1+x^2} dx$$

Hint: don't bother substituting in  $x = \tan \theta$ .

There is one method I would recommend learning about, if nothing else because even Cambridge don't teach us about it and it's a massive timesaver some of the time. I have no idea why this method isn't better known. It'd be a bit bulky to tell you how it works here, but there are some OK explanations online about how it works. say you have an integral where you just *know* you're gonna have to integrate by parts several times, e.g.

$$\int x^5 e^x dx$$

Rather than writing out all the lines of working and all the intermediate steps after each application of integration by parts, there exists an elegant method, known as **tabular integration**. Tabular integration gives an advantage over regular repeated integration by parts in a similar sense to the way that dividing polynomials by inspection gives an advantage over dividing them by long division. Regardless of whether it comes up at your interview, I just think it's a really neat trick, if you want you can learn more about it here, or it's demonstrated by Mr Escalante in the film *Stand and Deliver*<sup>2</sup>.

Finally, if you're dealing with an indefinite integral, don't, whatever you do, forget the +C.

## 6 Conclusion

Sorry this was a bit longer than I intended it to be. Not everything in here will come up, but hopefully some of what's in here will prove useful. If there's an inevitable mistake in here, or a technique you think should be added in here, feel free to email me at the address on the front page [you should probably cc a teacher in to the email for safeguarding].

Other than what I've said above, you should endeavour to do lots of practice questions. Of course, it's gonna be difficult to replicate the actual environment, but in terms of a good bank of questions, Isaac Physics ([isaacphysics.org](http://isaacphysics.org)) was pretty invaluable to me in preparing for my interview. Not only does it often have questions with several progressively-more-difficult steps (à la Oxbridge interview), but you can also sort by topic, giving you a chance to shore up any weak points you feel you have. Also, it was founded by admissions tutors at Cambridge, so it couldn't be much better targeted.

I'll probably update this for future years, so let me know if there's something you'd like to change/add (you'll also get a place in the following section!)

## 7 Acknowledgements

Many thanks to the following who have spotted errors/suggested improvements so far.

- Ibrahim Ezzeldin

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<sup>2</sup>For this performance, Edward James Olmos was nominated for an Oscar, though at the time of writing he has not yet been nominated for a Fields Medal