Sampling Paths from a GP

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```
tinytex::install_tinytex()
```

General set-up

- a zero-centered GP(m(.), K(.,.))
- f is a function drawn from this GP
- a vector (x1, ... xn), the function values (f(x1), ... f(xn)) or (Y)
- must have a multivariate GP with
  - mean: (m(x1),..., m(xn))
  - covariance matrix Sigma with Sigma_{ij} = K(xi, xj)
- so we could make use of this property (with 2 moments determin the whole distribution), draw this function from GP:
  1. select a fine grid of x-coords
  2. use mvrnorm() from MASS to draw function values at these points
  3. then connect them with straight lines

Generate Covariance matrix

Generate Covariance matrix from a known kernel function at points x

```
cov_matrix <- function(x, kernel_fn, ...) {
  outer(x, x, function(a, b) kernel_fn(a, b, ...))
}
```

Sample

Given x coords, take N draws from the GP with K evaluated using kernel_fn at x

```
draw_samples <- function(x, N, kernel_fn, ...) {
  set.seed(03-07-2021)

  Y <- matrix(NA, nrow = length(x), ncol = N)
  for(i in 1:N) {
    K <- cov_matrix(x, kernel_fn)
    Y[, n]<- mvrnorm(1, mu = rep(0, length(x)), Sigma = K)
  }
  Y
}
```

Parameters

Use the following parameters for the rest code

```
x <- seq(0, 2, length.out = 201)           # x-coords
N <- 3                                     # no. of draws
col_list <- list("red", "blue", "black")   # col for lines of different draws
```

Squared exponential(SE) kernel

SE also known as *radial basis furntion (RBF) kernel* or the *Gaussian kernel* has the form: $K(x, x') = \sigma^2 exp(-\frac{||x-x'||^2}{2l^2})$,

where $\sigma^2 > 0$ and $l > 0$ are hyperparameters. $\sigma^2 > 0$ tells how variable the function is overall, and set to 1 for simplicity.

It's the most commonly used kernel as its computational tractability.

Now generates 3 draws from the SE kernel with $l = 0.2$

```
se_kernel <- function(x, y, Sigma = 1, l = 1) {
  Sigma^2 * exp(- (x - y)^2 / (2 * l^2))
}
```