Reproduce conditional approach

XC

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Setting up

Model 1 (independent Matérns):	$b_o(h) \equiv 0$,
Model 2 (pointwise dependence):	$b_o(h) \equiv A\delta(h)$,
Model 3 (diffused dependence):	Model 4 with $\Delta = 0$
Model 4 (asymmetric dependence):	$b_o(h) \equiv \begin{cases} A\{1-(\|h-\Delta\|/r)^2\}^2, & \|h-\Delta\| \le r \\ 0, & \text{otherwise,} \end{cases}$

where $\Delta = (\Delta_1, \Delta_2)^{\mathrm{T}}$ is a shift-parameter vector that captures asymmetry, r is the aperture parameter, and A is a scaling parameter.

In Models 3 and 4, $b_o(h)$ is a shifted bisquare function defined on $\mathbb{R}^2$.

The covariance functions $C_{11}(\cdot)$ and $C_{2|1}(\cdot)$ are Matérn covariance functions.

For each model we also consider a *reversed* dependence, where we switch Y_2 and Y_1. This gives us a total of eight models to fit and compare.

```
### Model choice

model_names <- c("independent","pointwise","moving_average_delta0","moving_average")
image_path <- "../paper/art"
show_figs <- 1              ## show figs in document
print_figs <- 0             ## Print figures to file (leave =0)
LK_analysis <- 0            ## log-likelihood analysis
LOO_analysis <- 0           ## LOO analysis
Shifted_Pars_estimation <- 0  ## Fit shifted parimonious Matern
RF_estimation <- 0            ## Carry out LOO with RFields
useMPI <- 0                                ## MPI backend available?
```

The data

The data were made available through the package `RandomFields`. We first load the data

```
data(weather, package = "RandomFields")
weather <- weather %>% data.frame()
weather %>% head(4) %>% print()
```

```
##   pressure temperature    lon  lat
## 1 200.4844  0.60537720 -131.0 46.0
## 2 384.8516 -0.02233887 -124.4 41.9
## 3 156.8984 -0.26644897 -124.5 46.1
## 4 248.4297 -1.30670166 -124.7 47.3
```

The `weather` table contains four fields, with latitude, longitude, pressure forecasting errors, and temperature forecasting errors for December 13, 2003 at 4 p.m. in the North American Pacific Northwest.

Since pressure and temperature have different units, we find a scaling factor by taking the ratio of the sample variances of the two variates, and computing its square root.

We will use this factor to scale the pressure variable.

```
p_scale <- var(weather$pressure) / var(weather$temperature) %>%
  sqrt() %>%
  as.numeric()
```

From this data frame we extract Z_1 and Z_2 and concatenate them into one long vector Z through a function `form_Z`.

The vectors Z_1 and Z_2 are inverted if the model being analysed is greater than 4 (reversed model).

We also define `m1` as the number of observations of Y_1, `m2` as the number of observations of Y_2 and `m` as the total number of observations.

```
form_z <- function(model_num, scale = T){
  Z1 <- matrix(weather$temperature)
  Z2 <- matrix(weather$pressure)

  if(scale) Z2 <- Z2 / p_scale   # scale pressure

  if(model_num > 4) {
    temp <- Z1   # move temperature values out of Z1 name into temp
    Z1 <- Z2     # pressure values go into name Z1
    Z2 <- temp   # move original temperature values into Z2
  }

  Z <- rbind(Z1, Z2)  # concatenate
}


## Number of observations
m1 <- nrow(weather)
m2 <- nrow(weather)
m = m1 + m2
I_m1 <- Diagonal(m1)
```

Process Discretisation

We approximate the processes as a sum of elemental basis functions (tent functions) constructed on a triangulation.

The triangulation is formed using the mesher in the `INLA` package, while we provide a tailored function in the package `bicon`, `initFEbasis`

?initFEbasis: initialise a finite element basis which initialises an object of class FEBasis which defines a set of elemental 'tent' basis functions over a pre-specified triangulation in 2-D

which takes information from the `INLA` mesher and casts it into a `Mesh` object

We provide several methods associated with the `Mesh` class which will be useful for plotting later on.

Importantly, the `Mesh` object also contains information on the areas of the elements in the Voronoi tesselation, which will be used to approximate the integrations.

```r
## discretizing process Y1,Y2 using triangular grid

## constructing mesh
mesh <- inla.mesh.2d(loc = weather[c("lon", "lat")],
             cutoff = 0,        # minimum allowed dist between pts, two pts further apart at most of th
             max.edge = 0.75,   # the largest allowed tri edge length
             offset = 4)        # automatic extension dist

mesh_locs <- mesh$loc[, 1:2]  # [1:2071, 1:2]


## compute distances as in Gneiting(2010) -- greate-circle distance

### Greate circle distance: shortest distance between two points on the surface of a sphere, measured a

d <- RFearth2dist(as.matrix(mesh_locs)) # transform coods from earth (ellipsoid) to cartesian
```

```
## Angle mode switches to 'degree'.
```

```r
D <- as.matrix(d)
Dvec <- as.double(c(D))
```

understandings 1

```r
str(mesh)
```

```
## List of 8
##  $ meta    :List of 5
##   ..$ call        : language inla.mesh.create(loc = loc, boundary = boundary1, interior = interior1,
##   ..$ fmesher.args: chr "--input=input.s --cutoff=0  --boundary=input.segm.bnd.idx  --boundarygrp=inp
##   ..$ time        : num [1:5, 1:5] 0 0 0 0 0 0 0 0 0 0 ...
##   .. ..- attr(*, "dimnames")=List of 2
##   .. .. ..$ : chr [1:5] "pre" "fmesher" "post" "object" ...
##   .. .. ..$ : NULL
##   ..$ prefix      : chr "/var/folders/wq/8wbjn9_s7jv6hzxn1m43d95w0000gn/T//RtmpzOW5Au/fmesher2caa1b8
##   ..$ is.refined  : logi TRUE
##  $ manifold: chr "R2"
##  $ n       : int 2071
##  $ loc     : num [1:2071, 1:3] -130 -114 -111 -111 -114 ...
##  $ graph   :List of 5
##   ..$ tv : int [1:3982, 1:3] 289 157 162 658 172 1275 336 1613 9 165 ...
##   ..$ vt : int [1:2071, 1] 2619 1734 3559 3375 3629 1738 3413 2356 2165 202 ...
##   ..$ tt : int [1:3982, 1:3] 2967 3869 461 3041 69 2399 3314 208 2705 22 ...
##   ..$ tti: int [1:3982, 1:3] 1 1 1 1 2 2 1 1 1 2 ...
##   ..$ vv :Formal class 'dgTMatrix' [package "Matrix"] with 6 slots
##   .. .. ..@ i       : int [1:12104] 1208 1238 1378 911 1046 1117 847 848 1943 1073 ...
##   .. .. ..@ j       : int [1:12104] 0 0 0 1 1 1 2 2 2 3 ...
##   .. .. ..@ Dim     : int [1:2] 2071 2071
##   .. .. ..@ Dimnames:List of 2
##   .. .. .. ..$ : NULL
##   .. .. .. ..$ : NULL
##   .. .. ..@ x       : num [1:12104] 1 1 1 1 1 1 1 1 1 1 ...
##   .. .. ..@ factors : list()
```

```
##  $ segm    :List of 2
##   ..$ bnd:List of 5
##   .. ..$ loc   : NULL
##   .. ..$ idx   : int [1:158, 1:2] 1 1209 580 1357 439 852 584 1354 219 1352 ...
##   .. ..$ grp   : int [1:158, 1] 0 0 0 0 0 0 0 0 0 0 ...
##   .. ..$ is.bnd: logi TRUE
##   .. ..$ crs   : NULL
##   .. ..- attr(*, "class")= chr "inla.mesh.segment"
##   ..$ int:List of 5
##   .. ..$ loc   : NULL
##   .. ..$ idx   : int[0 , 1:2]
##   .. ..$ grp   : int[0 , 1]
##   .. ..$ is.bnd: logi FALSE
##   .. ..$ crs   : NULL
##   .. ..- attr(*, "class")= chr "inla.mesh.segment"
##  $ idx     :List of 3
##   ..$ loc    : int [1:157] 9 10 11 12 13 14 15 16 17 18 ...
##   ..$ lattice: NULL
##   ..$ segm   : int [1:8] 1 2 3 4 5 6 7 8
##  $ crs     : NULL
##  - attr(*, "class")= chr "inla.mesh"
```

```
head(mesh$loc, 3)
```

```
##           [,1]     [,2] [,3]
## [1,] -129.5832 36.82639    0
## [2,] -114.1306 36.82639    0
## [3,] -110.8833 40.07370    0
```

```
tail(mesh$loc, 3)
```

```
##              [,1]     [,2] [,3]
## [2069,] -134.0416 45.11904    0
## [2070,] -124.3347 50.20053    0
## [2071,] -123.0588 50.49888    0
```

```
d <- RFearth2dist(as.matrix(mesh_locs))
```

```
## Angle mode switches to 'degree'.
```

```
d_matrx <- as.matrix(d)
rm(d_matrx)
rm(d)

dim(D) #2071 2071
```

```
## [1] 2071 2071
```

```
length(Dvec) # 2071 * 2071 = 4289041
```

```
## [1] 4289041
```

```
## obseration locs distance in cartesian
Dobs <- as.matrix(RFearth2dist(as.matrix(weather[c("lon", "lat")])))
```

```
## Angle mode switches to 'degree'.
```

```
Dobs_vec <- c(Dobs)
```

understandings 2

```
obs_locs <- weather[c("lon", "lat")] ## df 157 obs, 2 vars
str(obs_locs)
```

```
## 'data.frame':    157 obs. of  2 variables:
##  $ lon: num  -131 -124 -124 -125 -124 ...
##  $ lat: num  46 41.9 46.1 47.3 44.6 ...
```

```
obs_locs <- as.matrix(weather[c("lon", "lat")]) # [1:157, 1:2]

length(Dobs_vec)  # 24649
```

```
## [1] 24649
```

```
## Cast into custom Mesh object
## define a set of tent basis functions over a prespecified triangulation in 2D
Mesh <- initFEbasis(p = mesh_locs,
            t = mesh$graph$tv,
            K = mesh$graph$vv)

head(str(Mesh))
```

```
## Formal class 'FEBasis' [package "bicon"] with 3 slots
##   ..@ pars:List of 5
##   .. ..$ p   : num [1:2071, 1:2] -130 -114 -111 -111 -114 ...
##   .. ..$ t   : int [1:3982, 1:3] 289 157 162 658 172 1275 336 1613 9 165 ...
##   .. ..$ K   :Formal class 'dgTMatrix' [package "Matrix"] with 6 slots
##   .. .. .. ..@ i       : int [1:12104] 1208 1238 1378 911 1046 1117 847 848 1943 1073 ...
##   .. .. .. ..@ j       : int [1:12104] 0 0 0 1 1 1 2 2 2 3 ...
##   .. .. .. ..@ Dim     : int [1:2] 2071 2071
##   .. .. .. ..@ Dimnames:List of 2
##   .. .. .. .. ..$ : NULL
##   .. .. .. .. ..$ : NULL
##   .. .. .. ..@ x       : num [1:12104] 1 1 1 1 1 1 1 1 1 1 ...
##   .. .. .. ..@ factors : list()
##   .. ..$ vars:'data.frame':  2071 obs. of  4 variables:
##   .. .. ..$ x        : num [1:2071] -130 -114 -111 -111 -114 ...
##   .. .. ..$ y        : num [1:2071] 36.8 36.8 40.1 52.6 55.7 ...
##   .. .. ..$ n        : int [1:2071] 1 2 3 4 5 6 7 8 9 10 ...
##   .. .. ..$ area_tess: num [1:2071] 0.112 0.137 0.146 0.128 0.111 ...
##   .. ..$ pol :List of 2071
##   .. .. ..$ :Formal class 'gpc.poly' [package "gpclib"] with 1 slot
##   .. .. .. .. ..@ pts:List of 1
##   .. .. .. .. .. ..$ :List of 3
##   .. .. .. .. .. .. ..$ x   : num [1:4] -129 -130 -130 -129
##   .. .. .. .. .. .. ..$ y   : num [1:4] 36.8 36.8 37.1 37
##   .. .. .. .. .. .. ..$ hole: logi FALSE
##   .. .. ..$ :Formal class 'gpc.poly' [package "gpclib"] with 1 slot
##   .. .. .. .. ..@ pts:List of 1
##   .. .. .. .. .. ..$ :List of 3
##   .. .. .. .. .. .. ..$ x   : num [1:4] -114 -114 -114 -114
##   .. .. .. .. .. .. ..$ y   : num [1:4] 36.8 36.8 37.1 37.1
##   .. .. .. .. .. .. ..$ hole: logi FALSE
##   .. .. ..$ :Formal class 'gpc.poly' [package "gpclib"] with 1 slot
##   .. .. .. .. ..@ pts:List of 1
```

```
##   .. .. .. .. .. ..$ :List of 3
##   .. .. .. .. .. .. ..$ x   : num [1:4] -111 -111 -111 -111
##   .. .. .. .. .. .. ..$ y   : num [1:4] 39.7 40 40.3 40.3
##   .. .. .. .. .. .. ..$ hole: logi FALSE
##   .. .. ..$ :Formal class 'gpc.poly' [package "gpclib"] with 1 slot
##   .. .. .. .. ..@ pts:List of 1
##   .. .. .. .. .. ..$ :List of 3
##   .. .. .. .. .. .. ..$ x   : num [1:4] -111 -111 -111 -111
##   .. .. .. .. .. .. ..$ y   : num [1:4] 52.4 52.4 52.6 52.9
##   .. .. .. .. .. .. ..$ hole: logi FALSE
##   .. .. ..$ :Formal class 'gpc.poly' [package "gpclib"] with 1 slot
##   .. .. .. .. ..@ pts:List of 1
##   .. .. .. .. .. ..$ :List of 3
##   .. .. .. .. .. .. ..$ x   : num [1:4] -114 -114 -114 -114
##   .. .. .. .. .. .. ..$ y   : num [1:4] 55.4 55.5 55.7 55.7
##   .. .. .. .. .. .. ..$ hole: logi FALSE
##   .. .. ..$ :Formal class 'gpc.poly' [package "gpclib"] with 1 slot
##   .. .. .. .. ..@ pts:List of 1
##   .. .. .. .. .. ..$ :List of 3
##   .. .. .. .. .. .. ..$ x   : num [1:5] -131 -131 -131 -131 -131
##   .. .. .. .. .. .. ..$ y   : num [1:5] 55.5 55.3 55.3 55.7 55.7
##   .. .. .. .. .. .. ..$ hole: logi FALSE
##   .. .. ..$ :Formal class 'gpc.poly' [package "gpclib"] with 1 slot
##   .. .. .. .. ..@ pts:List of 1
##   .. .. .. .. .. ..$ :List of 3
##   .. .. .. .. .. .. ..$ x   : num [1:4] -135 -135 -135 -135
##   .. .. .. .. .. .. ..$ y   : num [1:4] 51.2 51.2 51.7 51.4
##   .. .. .. .. .. .. ..$ hole: logi FALSE
##   .. .. ..$ :Formal class 'gpc.poly' [package "gpclib"] with 1 slot
##   .. .. .. .. ..@ pts:List of 1
##   .. .. .. .. .. ..$ :List of 3
##   .. .. .. .. .. .. ..$ x   : num [1:4] -135 -135 -135 -135
##   .. .. .. .. .. .. ..$ y   : num [1:4] 42.2 41.9 42.5 42.5
##   .. .. .. .. .. .. ..$ hole: logi FALSE
##   .. .. ..$ :Formal class 'gpc.poly' [package "gpclib"] with 1 slot
##   .. .. .. .. ..@ pts:List of 1
##   .. .. .. .. .. ..$ :List of 3
##   .. .. .. .. .. .. ..$ x   : num [1:6] -131 -131 -131 -131 -131 ...
##   .. .. .. .. .. .. ..$ y   : num [1:6] 45.9 45.7 45.7 46.2 46.2 ...
##   .. .. .. .. .. .. ..$ hole: logi FALSE
##   .. .. ..$ :Formal class 'gpc.poly' [package "gpclib"] with 1 slot
##   .. .. .. .. ..@ pts:List of 1
##   .. .. .. .. .. ..$ :List of 3
##   .. .. .. .. .. .. ..$ x   : num [1:6] -124 -124 -125 -125 -125 ...
##   .. .. .. .. .. .. ..$ y   : num [1:6] 41.9 41.7 41.8 42.1 42.1 ...
##   .. .. .. .. .. .. ..$ hole: logi FALSE
##   .. .. ..$ :Formal class 'gpc.poly' [package "gpclib"] with 1 slot
##   .. .. .. .. ..@ pts:List of 1
##   .. .. .. .. .. ..$ :List of 3
##   .. .. .. .. .. .. ..$ x   : num [1:6] -124 -124 -125 -125 -125 ...
##   .. .. .. .. .. .. ..$ y   : num [1:6] 46.1 45.8 45.9 46.4 46.5 ...
##   .. .. .. .. .. .. ..$ hole: logi FALSE
##   .. .. ..$ :Formal class 'gpc.poly' [package "gpclib"] with 1 slot
##   .. .. .. .. ..@ pts:List of 1
```

```
##   .. .. .. .. .. ..$ :List of 3
##   .. .. .. .. .. .. ..$ x   : num [1:6] -124 -125 -125 -125 -125 ...
##   .. .. .. .. .. .. ..$ y   : num [1:6] 47.2 47 47 47.4 47.6 ...
##   .. .. .. .. .. .. ..$ hole: logi FALSE
##   .. .. ..$ :Formal class 'gpc.poly' [package "gpclib"] with 1 slot
##   .. .. .. .. ..@ pts:List of 1
##   .. .. .. .. .. ..$ :List of 3
##   .. .. .. .. .. .. ..$ x   : num [1:6] -124 -125 -125 -125 -124 ...
##   .. .. .. .. .. .. ..$ y   : num [1:6] 44.4 44.3 44.5 44.7 44.9 ...
##   .. .. .. .. .. .. ..$ hole: logi FALSE
##   .. .. ..$ :Formal class 'gpc.poly' [package "gpclib"] with 1 slot
##   .. .. .. .. ..@ pts:List of 1
##   .. .. .. .. .. ..$ :List of 3
##   .. .. .. .. .. .. ..$ x   : num [1:5] -125 -125 -125 -125 -125
##   .. .. .. .. .. .. ..$ y   : num [1:5] 49.9 49.9 49.9 49.9 49.9
##   .. .. .. .. .. .. ..$ hole: logi FALSE
##   .. .. ..$ :Formal class 'gpc.poly' [package "gpclib"] with 1 slot
##   .. .. .. .. ..@ pts:List of 1
##   .. .. .. .. .. ..$ :List of 3
##   .. .. .. .. .. .. ..$ x   : num [1:4] -125 -125 -125 -125
##   .. .. .. .. .. .. ..$ y   : num [1:4] 49.9 49.8 49.9 49.9
##   .. .. .. .. .. .. ..$ hole: logi FALSE
##   .. .. ..$ :Formal class 'gpc.poly' [package "gpclib"] with 1 slot
##   .. .. .. .. ..@ pts:List of 1
##   .. .. .. .. .. ..$ :List of 3
##   .. .. .. .. .. .. ..$ x   : num [1:5] -128 -128 -128 -128 -128
##   .. .. .. .. .. .. ..$ y   : num [1:5] 49.7 49.7 49.7 49.8 49.8
##   .. .. .. .. .. .. ..$ hole: logi FALSE
##   .. .. ..$ :Formal class 'gpc.poly' [package "gpclib"] with 1 slot
##   .. .. .. .. ..@ pts:List of 1
##   .. .. .. .. .. ..$ :List of 3
##   .. .. .. .. .. .. ..$ x   : num [1:5] -128 -128 -128 -128 -128
##   .. .. .. .. .. .. ..$ y   : num [1:5] 49.6 49.6 49.7 49.7 49.7
##   .. .. .. .. .. .. ..$ hole: logi FALSE
##   .. .. ..$ :Formal class 'gpc.poly' [package "gpclib"] with 1 slot
##   .. .. .. .. ..@ pts:List of 1
##   .. .. .. .. .. ..$ :List of 3
##   .. .. .. .. .. .. ..$ x   : num [1:5] -123 -124 -124 -124 -124
##   .. .. .. .. .. .. ..$ y   : num [1:5] 49.3 49.1 49.4 49.6 49.5
##   .. .. .. .. .. .. ..$ hole: logi FALSE
##   .. .. ..$ :Formal class 'gpc.poly' [package "gpclib"] with 1 slot
##   .. .. .. .. ..@ pts:List of 1
##   .. .. .. .. .. ..$ :List of 3
##   .. .. .. .. .. .. ..$ x   : num [1:6] -129 -129 -129 -129 -129 ...
##   .. .. .. .. .. .. ..$ y   : num [1:6] 51.3 51.1 51.1 51.5 51.7 ...
##   .. .. .. .. .. .. ..$ hole: logi FALSE
##   .. .. ..$ :Formal class 'gpc.poly' [package "gpclib"] with 1 slot
##   .. .. .. .. ..@ pts:List of 1
##   .. .. .. .. .. ..$ :List of 3
##   .. .. .. .. .. .. ..$ x   : num [1:5] -126 -126 -126 -126 -126
##   .. .. .. .. .. .. ..$ y   : num [1:5] 48.8 48.8 48.9 49 48.9
##   .. .. .. .. .. .. ..$ hole: logi FALSE
##   .. .. ..$ :Formal class 'gpc.poly' [package "gpclib"] with 1 slot
##   .. .. .. .. ..@ pts:List of 1
```

```
##   .. .. .. .. .. ..$ :List of 3
##   .. .. .. .. .. .. ..$ x   : num [1:5] -126 -126 -126 -126 -126
##   .. .. .. .. .. .. ..$ y   : num [1:5] 48.7 48.7 48.8 48.8 48.8
##   .. .. .. .. .. .. ..$ hole: logi FALSE
##   .. .. ..$ :Formal class 'gpc.poly' [package "gpclib"] with 1 slot
##   .. .. .. .. ..@ pts:List of 1
##   .. .. .. .. .. ..$ :List of 3
##   .. .. .. .. .. .. ..$ x   : num [1:5] -130 -130 -130 -130 -130
##   .. .. .. .. .. .. ..$ y   : num [1:5] 50.8 50.7 50.7 50.9 50.9
##   .. .. .. .. .. .. ..$ hole: logi FALSE
##   .. .. ..$ :Formal class 'gpc.poly' [package "gpclib"] with 1 slot
##   .. .. .. .. ..@ pts:List of 1
##   .. .. .. .. .. ..$ :List of 3
##   .. .. .. .. .. .. ..$ x   : num [1:5] -130 -130 -130 -130 -130
##   .. .. .. .. .. .. ..$ y   : num [1:5] 50.9 50.9 51.1 51.1 51
##   .. .. .. .. .. .. ..$ hole: logi FALSE
##   .. .. ..$ :Formal class 'gpc.poly' [package "gpclib"] with 1 slot
##   .. .. .. .. ..@ pts:List of 1
##   .. .. .. .. .. ..$ :List of 3
##   .. .. .. .. .. .. ..$ x   : num [1:5] -124 -124 -125 -125 -124
##   .. .. .. .. .. .. ..$ y   : num [1:5] 43.2 43.1 43.1 43.4 43.5
##   .. .. .. .. .. .. ..$ hole: logi FALSE
##   .. .. ..$ :Formal class 'gpc.poly' [package "gpclib"] with 1 slot
##   .. .. .. .. ..@ pts:List of 1
##   .. .. .. .. .. ..$ :List of 3
##   .. .. .. .. .. .. ..$ x   : num [1:6] -123 -123 -124 -124 -124 ...
##   .. .. .. .. .. .. ..$ y   : num [1:6] 48.3 48.2 48.2 48.4 48.4 ...
##   .. .. .. .. .. .. ..$ hole: logi FALSE
##   .. .. ..$ :Formal class 'gpc.poly' [package "gpclib"] with 1 slot
##   .. .. .. .. ..@ pts:List of 1
##   .. .. .. .. .. ..$ :List of 3
##   .. .. .. .. .. .. ..$ x   : num [1:7] -123 -123 -123 -123 -123 ...
##   .. .. .. .. .. .. ..$ y   : num [1:7] 50 49.9 49.8 49.9 50 ...
##   .. .. .. .. .. .. ..$ hole: logi FALSE
##   .. .. ..$ :Formal class 'gpc.poly' [package "gpclib"] with 1 slot
##   .. .. .. .. ..@ pts:List of 1
##   .. .. .. .. .. ..$ :List of 3
##   .. .. .. .. .. .. ..$ x   : num [1:6] -123 -123 -123 -124 -123 ...
##   .. .. .. .. .. .. ..$ y   : num [1:6] 49.4 49.2 49.3 49.5 49.9 ...
##   .. .. .. .. .. .. ..$ hole: logi FALSE
##   .. .. ..$ :Formal class 'gpc.poly' [package "gpclib"] with 1 slot
##   .. .. .. .. ..@ pts:List of 1
##   .. .. .. .. .. ..$ :List of 3
##   .. .. .. .. .. .. ..$ x   : num [1:6] -121 -121 -122 -122 -122 ...
##   .. .. .. .. .. .. ..$ y   : num [1:6] 51 50.9 51 51.4 51.4 ...
##   .. .. .. .. .. .. ..$ hole: logi FALSE
##   .. .. ..$ :Formal class 'gpc.poly' [package "gpclib"] with 1 slot
##   .. .. .. .. ..@ pts:List of 1
##   .. .. .. .. .. ..$ :List of 3
##   .. .. .. .. .. .. ..$ x   : num [1:7] -126 -126 -126 -127 -127 ...
##   .. .. .. .. .. .. ..$ y   : num [1:7] 49.4 49.4 49.2 49.1 49.4 ...
##   .. .. .. .. .. .. ..$ hole: logi FALSE
##   .. .. ..$ :Formal class 'gpc.poly' [package "gpclib"] with 1 slot
##   .. .. .. .. ..@ pts:List of 1
```

```
##   .. .. .. .. .. ..$ :List of 3
##   .. .. .. .. .. .. ..$ x   : num [1:5] -124 -124 -124 -124 -124
##   .. .. .. .. .. .. ..$ y   : num [1:5] 49.1 49.1 49.2 49.3 49.4
##   .. .. .. .. .. .. ..$ hole: logi FALSE
##   .. .. ..$ :Formal class 'gpc.poly' [package "gpclib"] with 1 slot
##   .. .. .. .. ..@ pts:List of 1
##   .. .. .. .. .. ..$ :List of 3
##   .. .. .. .. .. .. ..$ x   : num [1:6] -123 -123 -123 -123 -123 ...
##   .. .. .. .. .. .. ..$ y   : num [1:6] 48.8 48.7 48.6 48.8 49 ...
##   .. .. .. .. .. .. ..$ hole: logi FALSE
##   .. .. ..$ :Formal class 'gpc.poly' [package "gpclib"] with 1 slot
##   .. .. .. .. ..@ pts:List of 1
##   .. .. .. .. .. ..$ :List of 3
##   .. .. .. .. .. .. ..$ x   : num [1:7] -129 -129 -129 -129 -129 ...
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##   .. .. .. .. .. .. ..$ hole: logi FALSE
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##   .. .. .. .. .. ..$ :List of 3
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##   .. .. .. .. .. .. ..$ hole: logi FALSE
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##   .. .. .. .. .. .. ..$ hole: logi FALSE
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##   .. .. .. .. .. .. ..$ y   : num [1:5] 46.7 46.7 46.7 46.8 46.8
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##   .. .. .. .. .. ..$ :List of 3
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##   .. .. .. .. .. ..$ :List of 3
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##   .. .. .. .. .. .. ..$ hole: logi FALSE
##   .. .. ..$ :Formal class 'gpc.poly' [package "gpclib"] with 1 slot
##   .. .. .. .. ..@ pts:List of 1
##   .. .. .. .. .. ..$ :List of 3
##   .. .. .. .. .. .. ..$ x   : num [1:5] -120 -121 -121 -121 -120
##   .. .. .. .. .. .. ..$ y   : num [1:5] 41.4 41.2 41.4 41.8 41.7
##   .. .. .. .. .. .. ..$ hole: logi FALSE
##   .. .. ..$ :Formal class 'gpc.poly' [package "gpclib"] with 1 slot
##   .. .. .. .. ..@ pts:List of 1
##   .. .. .. .. .. ..$ :List of 3
##   .. .. .. .. .. .. ..$ x   : num [1:5] -124 -124 -124 -124 -124
##   .. .. .. .. .. .. ..$ y   : num [1:5] 40.7 40.8 41.2 41.2 41
##   .. .. .. .. .. .. ..$ hole: logi FALSE
##   .. .. ..$ :Formal class 'gpc.poly' [package "gpclib"] with 1 slot
##   .. .. .. .. ..@ pts:List of 1
##   .. .. .. .. .. ..$ :List of 3
##   .. .. .. .. .. .. ..$ x   : num [1:7] -118 -119 -119 -119 -119 ...
##   .. .. .. .. .. .. ..$ y   : num [1:7] 45.8 45.8 46 46.1 46.2 ...
##   .. .. .. .. .. .. ..$ hole: logi FALSE
##   .. .. ..$ :Formal class 'gpc.poly' [package "gpclib"] with 1 slot
##   .. .. .. .. ..@ pts:List of 1
##   .. .. .. .. .. ..$ :List of 3
##   .. .. .. .. .. .. ..$ x   : num [1:7] -124 -124 -124 -124 -124 ...
##   .. .. .. .. .. .. ..$ y   : num [1:7] 46.1 46 45.9 46.1 46.2 ...
##   .. .. .. .. .. .. ..$ hole: logi FALSE
##   .. .. ..$ :Formal class 'gpc.poly' [package "gpclib"] with 1 slot
##   .. .. .. .. ..@ pts:List of 1
```

```
##   .. .. .. .. .. ..$ :List of 3
##   .. .. .. .. .. .. ..$ x   : num [1:5] -122 -122 -122 -122 -122
##   .. .. .. .. .. .. ..$ y   : num [1:5] 47.5 47.5 47.6 47.6 47.6
##   .. .. .. .. .. .. ..$ hole: logi FALSE
##   .. .. ..$ :Formal class 'gpc.poly' [package "gpclib"] with 1 slot
##   .. .. .. .. ..@ pts:List of 1
##   .. .. .. .. .. ..$ :List of 3
##   .. .. .. .. .. .. ..$ x   : num [1:6] -118 -118 -118 -118 -118 ...
##   .. .. .. .. .. .. ..$ y   : num [1:6] 44.6 44.5 44.7 45.1 45.1 ...
##   .. .. .. .. .. .. ..$ hole: logi FALSE
##   .. .. ..$ :Formal class 'gpc.poly' [package "gpclib"] with 1 slot
##   .. .. .. .. ..@ pts:List of 1
##   .. .. .. .. .. ..$ :List of 3
##   .. .. .. .. .. .. ..$ x   : num [1:6] -122 -123 -123 -123 -123 ...
##   .. .. .. .. .. .. ..$ y   : num [1:6] 48.5 48.6 48.7 48.8 49 ...
##   .. .. .. .. .. .. ..$ hole: logi FALSE
##   .. .. ..$ :Formal class 'gpc.poly' [package "gpclib"] with 1 slot
##   .. .. .. .. ..@ pts:List of 1
##   .. .. .. .. .. ..$ :List of 3
##   .. .. .. .. .. .. ..$ x   : num [1:6] -119 -119 -119 -119 -119 ...
##   .. .. .. .. .. .. ..$ y   : num [1:6] 43.5 43.3 43.4 43.8 43.8 ...
##   .. .. .. .. .. .. ..$ hole: logi FALSE
##   .. .. ..$ :Formal class 'gpc.poly' [package "gpclib"] with 1 slot
##   .. .. .. .. ..@ pts:List of 1
##   .. .. .. .. .. ..$ :List of 3
##   .. .. .. .. .. .. ..$ x   : num [1:6] -116 -116 -116 -117 -116 ...
##   .. .. .. .. .. .. ..$ y   : num [1:6] 43.5 43.2 43.2 43.6 43.9 ...
##   .. .. .. .. .. .. ..$ hole: logi FALSE
##   .. .. ..$ :Formal class 'gpc.poly' [package "gpclib"] with 1 slot
##   .. .. .. .. ..@ pts:List of 1
##   .. .. .. .. .. ..$ :List of 3
##   .. .. .. .. .. .. ..$ x   : num [1:5] -124 -124 -124 -124 -124
##   .. .. .. .. .. .. ..$ y   : num [1:5] 41.6 41.6 41.7 41.9 41.9
##   .. .. .. .. .. .. ..$ hole: logi FALSE
##   .. .. ..$ :Formal class 'gpc.poly' [package "gpclib"] with 1 slot
##   .. .. .. .. ..@ pts:List of 1
##   .. .. .. .. .. ..$ :List of 3
##   .. .. .. .. .. .. ..$ x   : num [1:6] -123 -124 -124 -124 -123 ...
##   .. .. .. .. .. .. ..$ y   : num [1:6] 47.8 47.8 48.1 48.2 48.2 ...
##   .. .. .. .. .. .. ..$ hole: logi FALSE
##   .. .. .. [list output truncated]
##   ..@ n   : int 2071
##   ..@ fn  : list()
```

```
## NULL
```

Establish the dimension of our grids

Since we will be evaluating Y_1 and Y_2 on the same grid, `n1` = `n2`.

```
## Mesh size
n1 <- nrow(mesh_locs)   # 2071
n2 <- nrow(mesh_locs)
n <- n1 + n2            # 4142
```

As in the first document (simulation example in Section 3.2), we will approximate the integration using the rectangular rule.

When using finite elements, this reduces to using the areas of the Voronoi tessellation as integration weights.

We first compute the vector of displacements h which will be of length (`n2` $\times$ `n1`)

```
## mesh integration points
h <- matrix(0, n1 * n2, 2)
area <- rep(0, n1 * n2)
for (i in 1:n2) {
  h[((i - 1) * n1 + 1):(i * n1), ] <- t(t(mesh_locs) - mesh_locs[i, ])
  area[((i - 1) * n1 + 1):(i * n1)] <- Mesh["area_tess"]
}

h1_double <- as.double(h[, 1])
h2_double <- as.double(h[, 2])
```

The displacements (`h1`,`h2`) and the areas `areas` will be used to construct the matrix `B` using the function `bisquare_B`.

Organising the observations

In order to map the process to the observations we construct an incidence matrix, which contains a `1` wherever the observation coincides with a vertex on the triangulation and a `0` otherwise.

The dimension of this incidence matrix is (`m1` + `m2`) $\times$ (`n1` + `n2`), where `m1`, `m2`, are the number of observations in Z_1, Z_2.

Since in this problem we have co-located (shared) observations, we find the incidence matrix for one of the observations, Z_1,

and then form the whole incidence matrix by simply constructing a block diagonal matrix (using `bdiag`). We find the points with which the observation locations coincide by using the function `left_join`, which returns an `NA` if no observation coincides with the vertex.

```
mesh_locs <- data.frame(lon = mesh_locs[, 1], lat = mesh_locs[, 2])

indx <- which(!is.na(left_join(mesh_locs, weather)$temperature)) # index of coincidence
```

```
## Joining, by = c("lon", "lat")
```

```
length(indx)                    # 157
```

```
## [1] 157
```

```
C1 <- sparseMatrix(i = 1:m1, j = indx,
                   x = 1, dims = c(m1, n1))  # incidence matrix of Z1

C <- bdiag(C1, C1)    # incidence matrix of Z1, Z2
```

Maximum likelihood estimation

Since the optimisation algorithm requires a parameter vector of the same length (irrespective of the model number)

we first define a function `append_theta` that takes the parameter vector associated with the model in question and appends it so it is of the required size (in this case of length 12).

```
append_theta <- function(theta,model_num) {
  if(model_num %in% c(1,5)) {
    theta <- c(theta,rep(0,4))
    theta[10] <- 0.001
  } else if(model_num %in% c(2,6)) {
    theta <- c(theta,rep(0,3))
    theta[10] <- 0.001
  } else if(model_num %in% c(3,7)) {
    theta <- c(theta,rep(0,2))
  }
  theta
}
```

the <- c(0, 3, 2.3) c(the, rep(0, 4))

Next, we require a function that, given the parameter vector `theta` and the model number `model_num`, returns the required matrices and vectors used in fitting. These are the matrices

$$\texttt{SY} = \begin{bmatrix} \Sigma_{11} & \Sigma_{11}B^{\mathrm{T}} \\ B\Sigma_{11} & \Sigma_{2|1} + B\Sigma_{11}B^{\mathrm{T}} \end{bmatrix}, \qquad \texttt{So} = \begin{bmatrix} \tau_1^2 I_m & 0 \\ 0 & \tau_2^2 I_m \end{bmatrix}. \tag{1}$$

We then add these two together to obtain the matrix $\mathrm{cov}((Y_1^{\mathrm{T}}, Y_2^{\mathrm{T}})^{\mathrm{T}})$ which, recall that for this example is identical to $\mathrm{cov}((Z_1^{\mathrm{T}}, Z_2^{\mathrm{T}})^{\mathrm{T}})$ since the data is equal to the process at the observed locations.

If `whole_mesh` is `TRUE`, then the process covariance matrix is evaluated over the entire mesh (used for cokriging at unobserved locations).