4 Jose list of important stuff

Chapter 1:

- Completeness theorem
- Model existence lemma
- Compactness
- Decidability
- Building deductive closure

Chapter 2:

- Induction
- Ordinal arithmetic
- Equivalent forms of ordinal arithmetic
- Harthog's (ordinal of next highest size)
- 2 types of question: Cantor Normal Form, or prove/disprove equalities and inequalities. Counterexamples list as above.
- Ordinals well-ordered

Chapter 3:

- State and prove Zorn's lemma - where did we use AOC? Upper bound of each chain, and x_α
- Knaster-Tarski
- Schroder-Bernstein

Chapter 4:

- Completeness, compactness
- Use compactness to show not axiomatisable (can add constants to language)
- Upwards/downwards L.S.

Chapter 5:

- Axioms for set theory
- Collapsing theorem
- $\in$ induction/foundation

- Von Neumann hierarchy/rank (5 things to remember)
    1. V_α transitive
    2. They form hierarchy
    3. They cover universe of sets
    4. Rank is computed recursively
    5. Rank of an ordinal is itself

Chapter 6:

- $\aleph_\alpha \aleph_\alpha = \aleph_\alpha$
- Assuming AOC, all sets cardinality some $\aleph$
- In ZF, $\aleph$s cardinalities of well-ordered sets
- 3 cardinal tools listed above

LST tricks

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1 Ordinals

- Counterexamples: try 1, 2, ω, $\omega+1$, ω^2, ω^2+1, ϵ_0, ω_1, ω_2 (latter two may work for cardinality reasons).
- If an identity is true its basically always possible by induction, sometimes synthetic is easier.
- Can use right-distributivity of ordinals to show certain equalities, e.g $\omega + \omega^2 = \omega(1+\omega) = \omega\omega = \omega^2$.
- To show the existence of a least ordinal with a given property, it suffices to show one ordinal has this property.
- The reason the maps $\beta \mapsto \alpha^\beta$, $\beta \mapsto \alpha\beta$ and $\beta \mapsto \alpha + \beta$ (important: the operation is on the right) are nice is because they are example of normal functions. For example, any normal function f has (unbounded) fixed points - see here for more.
- Can use Cantor Normal Form in questions. In particular, an important result from this is $\alpha + \beta = \beta + \alpha$ for when $\alpha < \beta * \omega$ or $\beta < \alpha * \omega$.
- Can show one side successor, other side a limit to show equality does not hold

2 Cardinals

- We only have three tools: $\lambda < 2^\lambda$ (Cantor's Theorem), $\gamma(\aleph_\alpha) = \aleph_{\alpha+1}$ and $\alpha \leq \beta$ if there is an injection $\alpha \to \beta$. Basically anything you do has to come from a combination of these.

3 Axiomatising

- If you need to have e.g uncountably many things in your model, add a constant for each element in w1 and have an axiom which says your thing exists for each constant e.g 2022 P3. Remember to make sure all these things you're adding are distinct.

- Anything which has arbitrarily large finite models but can't have an infinite model cannot be axiomatised because of compactness e.g finite amount of groups, theory of well orderings ¡- (2018 P3)
- Any theory where your model is upper bounded in cardinality dies to upper lowenheim skolem