高顿财经
GOLDEN FINANCE

CFA Pre-Course
Time Value of Money

Lecturer: Angel

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1. Interest Rate

2. Present Value and Future Value（1）

3. Present Value and Future Value（2）

4. Evaluation of Cash Flow Stream

5. Return & Yield Measurements

Interest Rate

Time value of money

- Money available at the present time is worth more than the same amount in the future due to its potential earning capacity.
  - Provided money can earn interest, any amount of money is worth more the sooner it is received.
  - It concerns equivalence relationships between cash flows occurring on different dates.

Interest Rate

Cash flow additivity principle

- The amounts of money can only be added on if they are indexed at the same point in time.

Interest Rate

Interpretations of Interest Rate

- **Required rate of return:** minimum rate of return an investor must receive in order to accept the investment.
- **Discounted rate**: the rate at which we discount the future amounts to find their value today.
- **Opportunity cost:** the value that investors forgo by choosing a particular course of action.

Interest Rate

Components of interest rate

- Real risk-free interest rate
  - Single-period interest rate for risk-free security without inflation expected.
- Inflation premium
  - Compensating investors for expected inflation risk.

Interest Rate

Components of interest rate

- Risk premium
  - **Default risk premium**: compensating investors for the possibility that the borrower will fail to make the promised payments in time and in full amount.
  - **Liquidity premium**: compensating investors for the risk of loss relative to an investment's fair value if the investment needs to be converted to cash quickly.
  - **Maturity premium:** compensating investors for the increased sensitivity of the market value of debt to a change in market interest rates as maturity is extended.

Interest Rate

Components of interest rate

- Nominal interest rate = Real risk-free interest rate
  + Inflation premium
  + Default risk premium
  + Liquidity premium
  + Maturity premium
- Nominal risk-free interest rate = Real risk-free interest rate
  + Inflation premium

Interest Rate

Simple interest

- The annual interest rate times the principal.

Compounding interest

- The interest earned on interest is count in.

Interest Rate

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Example

- If the annual interest rate is 10% and the principal is $1000, what is the interest earned in 2 years under simple interest and compounding interest?

Answer:

- Under simple interest:

  Interest earned = $1000 × 10% × 2 = $200

- Under compounding interest:

  Interest earned = $1000 × (1+10%) ×(1+10%) - $1000 = $210

Interest Rate

Stated annual interest rate/Quoted interest rate (r_s)

- The annual interest rate that does not account for compounding within the year.

Compounding frequency (m)

- The number of compounding periods per year.
  - **Continuous compounding**: the number of compounding periods per year becomes infinite.

Periodic interest rate (r_s/m)

- Stated annual rate divided by the compounding frequency.

Interest Rate

Effective annual rate

- The rate by which a unit of currency will grow in a year with interest on interest included.

$$EAR = \left(1+\text{Periodic interest rate}\right)^{m} -1=\left(1+\frac{r_s}{m}\right)^{m} -1$$

  - For continuous compounding:

$$EAR=e^{r_s} -1$$

Interest Rate

Example

- If the stated annual rate is 8%, compute the effective annual rate with quarterly compounding.

- Answer:

  EAR = $(1 + 8\%/4)^4 - 1 = 1.0824 - 1 =$ **8.24%**

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Present Value and Future Value

Relationship between PV and FV

- **Present value (PV)**: the value of an initial investment.
- **Future value (FV)**: the value of an initial investment would be worth n periods from today.
  - Present value and future value are equivalent measures separated in time.

$$FV = PV \times (1+r)^n \quad \text{or} \quad PV = \frac{FV}{(1+r)^n}$$

where: r = periodic rate, n = number of periods

Present Value and Future Value

Relationships between PV and FV (Cont.)

- For a given interest rate, the FV increases with the number of periods.
- For a given number of periods, the FV increases with the interest rate.
- For a given interest rate, the farther in the future the amount to be received, the smaller that amount's PV.
- Holding time constant, the larger the interest rate, the smaller the PV of a future amount.

Present Value and Future Value

Example

- Suppose a $10,000 investment and a stated annual interest rate of 8%, compute the future value with monthly compounding and continuous compounding.

- **Answer:**
  - For monthly compounding:

$$FV_N = PV \times \left(1+\frac{r_s}{m}\right)^m = 10,000 \times (1+\frac{0.08}{12})^{12} = \$10,829.99$$

  - For continuous compounding:

$$FV = PV \times e^{r_s} = 10,000 \times e^{0.08} = \$10,832.87$$

Present Value and Future Value

Future value of a single cash flow

- **Example**: what is the future value of $200 invested today

  in two years when the interest rate is 10%?

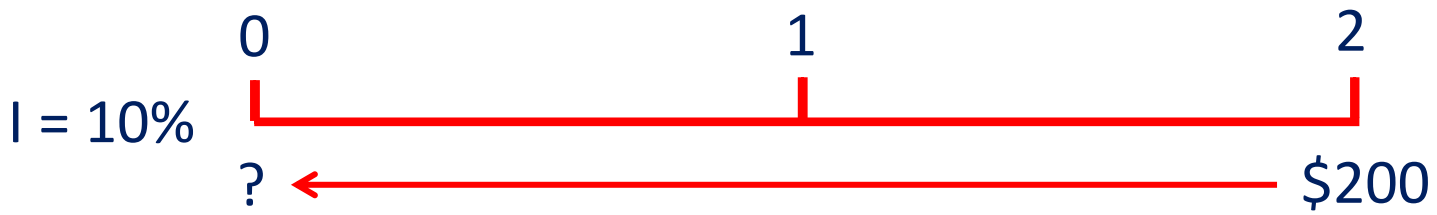
- **Answer:** $FV=200\times(1+10\%)\times(1+10\%)=200\times(1.1^2)=242$
  - Using financial calculator:

    N=2; I/Y=10; PV=200; PMT=0; CPT: FV= -242

Present Value and Future Value

Present value of a single cash flow

- **Example:** what is the present value of $200 to be received in two years when the interest rate is 10%?



- **Answer:** PV=200÷(1.1²)=165.29
  - Using financial calculator:

    N=2; I/Y=10; FV=200; PMT=0; CPT: PV= -165.29

Present Value and Future Value

Annuity

- A finite set of constant sequential cash flows.
  - **Ordinary annuity**: all constant cash flows occurring at the end of each period;
  - **Annuity due**: all constant cash flows occurring at the beginning of each period.

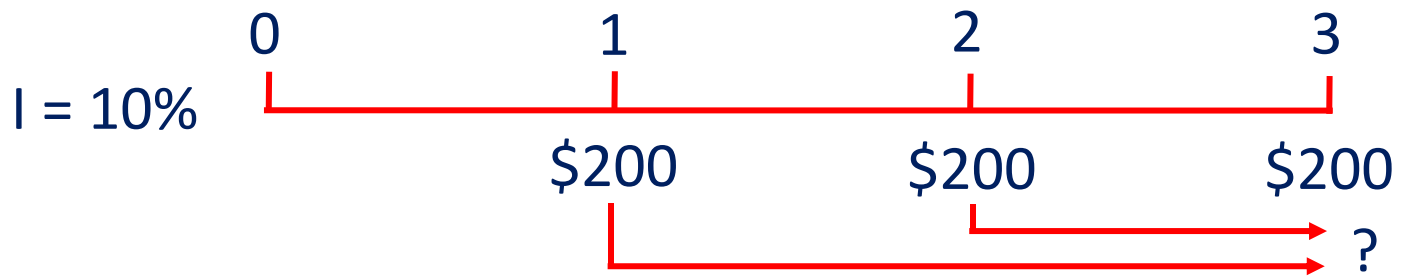
Perpetuity

- A set of constant never-ending sequential cash flows occurring at the end of each period.

Present Value and Future Value

Future value of an ordinary annuity

- **Example:** what is the value in 3 years time of $200 to be received at the end of each year for three years when the interest rate is 10%?



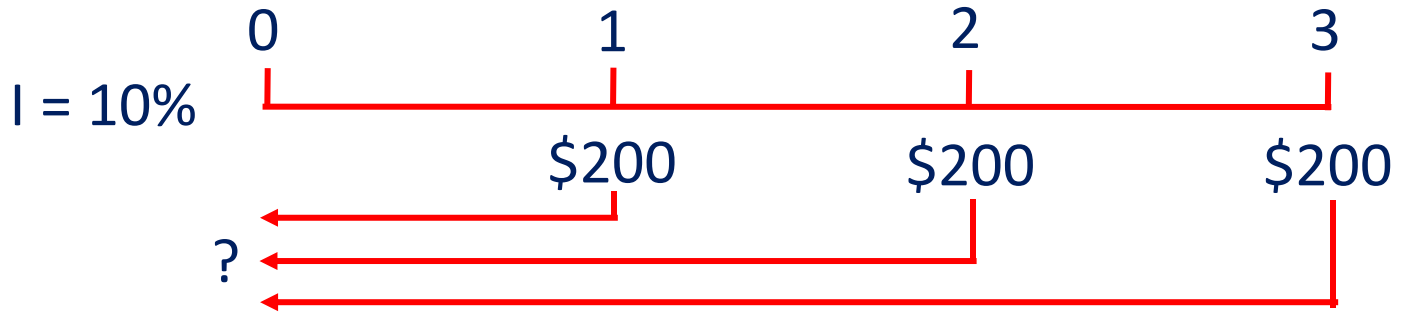
- **Answer:** $FV=200\times(1.1^2)+200\times(1.1)+200=662$
  - Using financial calculator:

    N=3; I/Y=10; PV=0; PMT=200; CPT: FV= -662

Present Value and Future Value

Present value of an ordinary annuity

- **Example:** what is the present value of $200 to be received at the end of each year for three years when the interest rate is 10%?



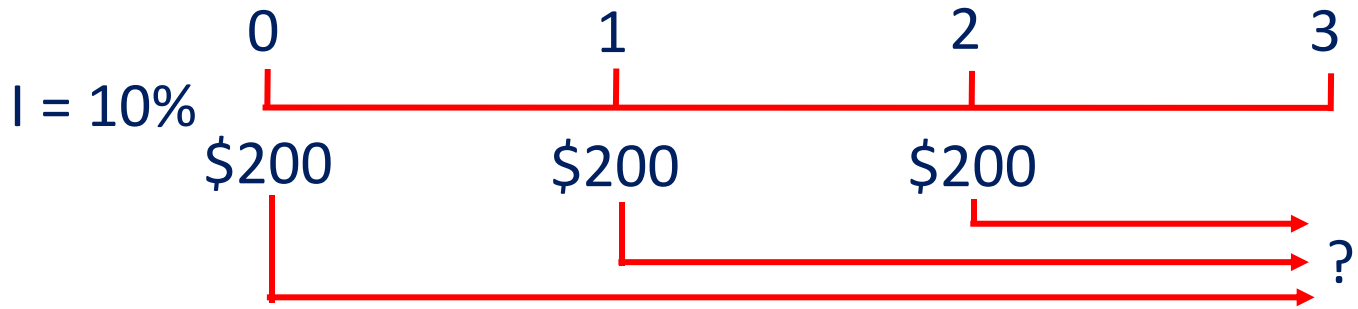
- **Answer:** PV=200÷(1.1)+200÷(1.1²)+200÷ (1.1³)=497.37
  - Using financial calculator:

    N=3; I/Y=10; FV=0; PMT=200; CPT: PV= -497.37

Present Value and Future Value

Future value of an annuity due

- **Example:** what is the value in 3 years time of $200 to be received at the beginning of each year for three years when the interest rate is 10%?



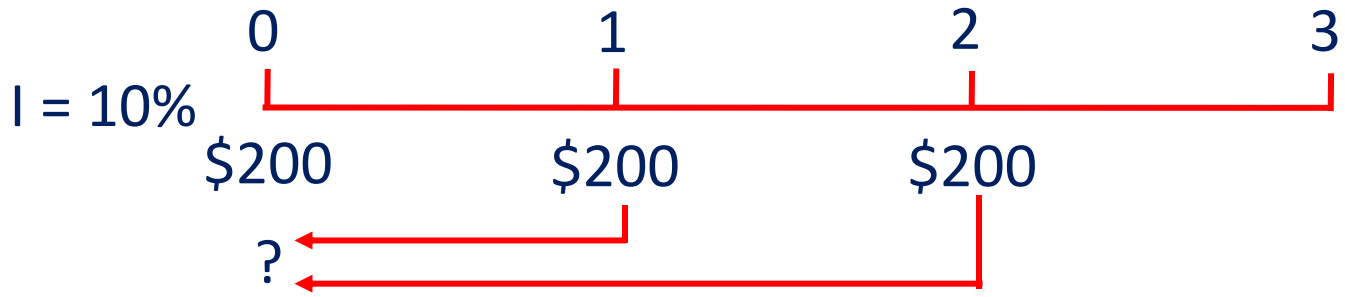
- **Answer:** FV = $200\times(1.1)+200\times(1.1^2)+200\times(1.1^3) = 728.2$
  - Using financial calculator (BGN Mode):

    N=3; I/Y=10; PV=0; PMT=200; CPT: FV= -728.2

Present Value and Future Value

Present value of an annuity due

- **Example:** what is the present value of $200 to be received at the start of each year for three years when the interest rate is 10%?



- **Answer:** PV = 200+200÷(1.1)+200÷(1.1^2) = 547.11
  - Using financial calculator (BGN Mode):

    N=3; I/Y=10; FV=0; PMT=200; CPT: FV= -547.11

Present Value and Future Value

Present value of perpetuity

- $PV = \frac{A}{r}$
  - A = the periodic payment to be received forever
- **Example:** a preferred stock will pay $8 per year forever and the rate of return is 10%. What is its value?
- **Answer:** PV = 8 ÷ 0.1 = 80

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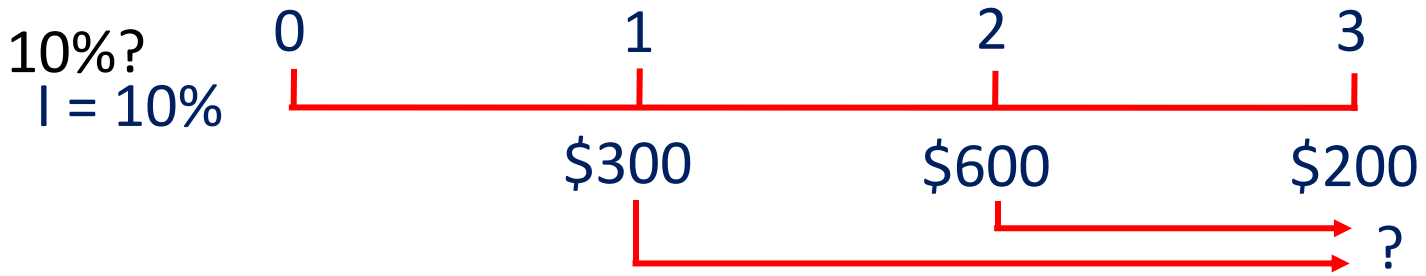
Present Value and Future Value

Future value of a series of unequal cash flow

- **Example:** what is the total value in 3 years time of $300 received at the end of 1st year, $600 at the end of 2nd year, and $200 at the end of 3rd year when the interest rate is 10%?

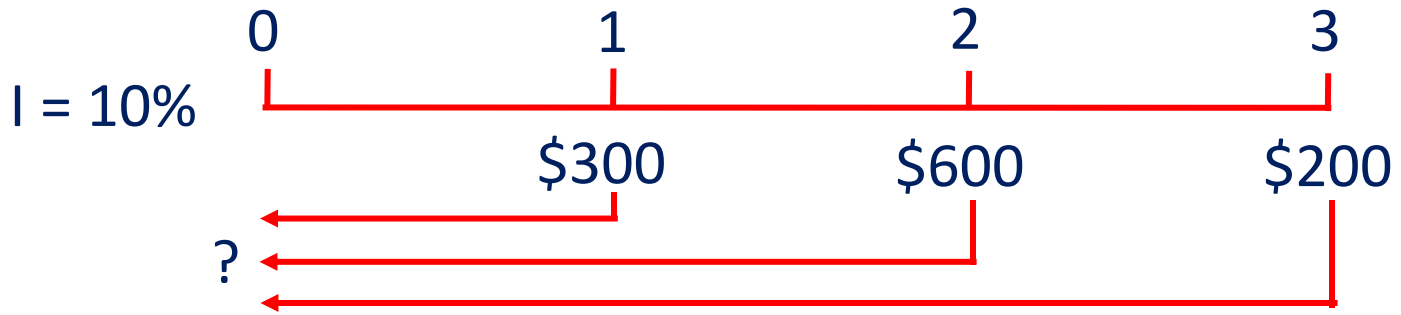


- **Answer:** $FV = 200+600\times(1.1)+300\times(1.1^2) = 1233$

Present Value and Future Value

Present value of a series of unequal cash flow

- **Example:** what is the total present value of $300 received at the end of 1st year, $600 at the end of 2nd year, and $200 at the end of 3rd year when the interest rate is 10%?



- **Answer:** PV = $300 \div (1.1) + 600 \div (1.1^2) + 200 \div (1.1^3) = 918.86$

Present Value and Future Value

Discount rate or growth rate

- **Example:** Elmer has won his $4 million state lottery and has been offered 20 annual payments of $200,000 each beginning today or a single payment of $2,267,000. What is the annual discount rate used to calculate the lump-sum pay-out amount?

- **Answer:** using financial calculator (BGN Mode):

  N=20; FV=0; PV=2,267,000; PMT=-200,000; CPT: I/Y= 7%.

Present Value and Future Value

Number of periods

- **Example:** Elmer has won his $4 million state lottery and has been offered 20 annual payments of $200,000 each beginning today or a single payment of $2,267,000. If Elmer can choose the amount of his annual pay-out, based on a 7% discount rate, how many payments of $232,631 could Elmer receive if his first payment were today?
- **Answer:** using financial calculator (BGN Mode):

  FV=0; PV=2,267,000; PMT=-232,631; I/Y= 7%; CPT: N=15.

Present Value and Future Value

Size of payment

- **Example:** what is the monthly payment on a $100K, 30-year home loan with stated rate of 6%?

- **Answer:** using financial calculator:

  N=30×12=360; I/Y=6/12=0.5; PV=100,000; FV=0;

  CPT: PMT= -599.55.

Practice 1

A financial product offers to pay a sum of $2,500 per annum for an infinite period in return for an upfront investment of $38,462. What is the interest rate implicit within this product?

A. 6.5%

B. 7%

C. 7.5%

Answer: A

The present value of the perpetuity at 6.5% is:

$2,500/0.065 = $38,462

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Evaluation of Cash Flow Streams

Net Present Value (NPV)

- The present value of its cash inflows(benefits) minus the present value of its cash outflows(costs).
- Calculation of NPV:
  - Identify all cash flows;
  - Determine the discount rate or opportunity cost (r);
  - Find the present value of each cash flow;
  - Sum up all present value to get NPV.

$$NPV = CF_0 + \frac{CF_1}{(1+r)^1} + \frac{CF_2}{(1+r)^2} + \ldots + \frac{CF_n}{(1+r)^n}$$

Evaluation of Cash Flow Streams

Net Present Value (Cont.)

- Apply the NPV rules:
  - If NPV > 0, undertake the project;
  - If NPV ≤ 0, should not undertake the project;
  - For mutually exclusive projects (can only invest in one), choose the one with higher positive NPV.

Evaluation of Cash Flow Streams

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Example:

- A project requires an initial outlay of $2 million, cash flows at end of year 1, 2, 3 are $0.5 million, $0.75 million, $1.35 million, respectively. If the discount rate is 10% per year, calculate the net present value.

Answer:

$$NPV = -2 + 0.5/(1.10) + 0.75/(1.10)^2 + 1.35/(1.10)^3$$

$$= \$0.089 \text{ mil.}$$

Evaluation of Cash Flow Streams

Internal rate of return (IRR):

- The discount rate that makes net present value equal to zero.

$$NPV = 0 = CF_0 + \frac{CF_1}{(1+IRR)^1} + \frac{CF_2}{(1+IRR)^2} + \ldots + \frac{CF_n}{(1+IRR)^n}$$

- Apply the IRR rules:
  - IRR > opportunity cost of capital, undertake the project .
  - IRR ≤ opportunity cost of capital, should not undertake the project.

Evaluation of Cash Flow Streams

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Example

- A project requires an initial outlay of $2 million, cash flows at end of year 1, 2, 3 are $0.5 million, $0.75 million, $1.35 million, respectively. If the discount rate is 10% per year, calculate the IRR.

Answer:

$$0 = -2 + 0.5/(1+IRR) + 0.75/(1+IRR)^2 + 1.35/(1+IRR)^3$$

IRR = 12.13%

Evaluation of Cash Flow Streams

Problems with IRR rules

- NPV and IRR rules give the same accept or reject decision when projects are independent, but may rank projects differently if projects are mutually exclusive when:
  - The size or scale of the projects differs;
  - The timing of the projects' cash flows differs.
- Stick to the NPV rule when NPV's and IRR's suggestions are conflict.
- When the signs of cash flows change more than once, there can be more than one IRR.

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Portfolio Return Measurement

Holding period return

- The return that an investor earns over a specified holding period.

$$\text{HPR} = \frac{P_1 - P_0 + D_1}{P_0}$$

Example

- Stock purchased nine months ago for $29 just paid a dividend of $1.30 and is valued at $30.50. Calculate the nine-month holding period return.

  Answer: HPR = (30.50 + 1.30 -29)/29 = 9.66%

Portfolio Return Measurement

Time-weighted return (TWR)

- The compound return that $1 initially invested in the portfolio over a stated measurement period.
- Calculation of TWR:
  - Break the overall evaluation period into sub-periods based on the dates of significant cash inflows and outflows;
  - Calculate the HPRs for each sub-periods;
  - Link or compound HPRs to obtain an annual rate of return.

$$TWR = \left[\left(\frac{\text{End Value}_1}{\text{Begin Value}_1} \right) \left(\frac{\text{End Value}_2}{\text{Begin Value}_2} \right) ..... \left(\frac{\text{End Value}_n}{\text{Begin Value}_n} \right) \right]^{\frac{1}{N}} - 1$$

Portfolio Return Measurement

Money-weighted return (MWR)

- MWR accounts for the timing and amount of all cash flows into and out of the portfolio.
  - If more funds to invest at an unfavorable time, MWR will tend to be depressed;
  - If more funds to invest at a favorable time, MWR will tend to be elevated.
- Calculation of MWR: similar to IRR.

$$CF_0 + \frac{CF_1}{1+MWR} + \ldots + \frac{CF_N}{(1+MWR)^N} = 0$$

Portfolio Return Measurement

TWR vs. MWR

- **Time weighted return:**
  - Not affected by cash withdrawals or additions;
  - Periods can be any length between significant cash flows.
- **Money weighted return:**
  - Assign more weights to the return of larger cash flows;
  - Affected by cash withdrawals or additions;
  - Periods must be equal length.
    - ✓ Use shortest period with no significant cash flows.

Portfolio Return Measurement

TWR vs. MWR (Cont.)

- **Example:** Eric invests $1,000 in an account. After one year, the value of his investment is $1,200 and Eric adds another $800 into the account. At the end of Year 2, the total value of the investment is $2,200. Calculate the annual TWR and MWR.

Answer:

TWR = $[(1.2)(1.1)]^{1/2} - 1 = 14.89\%$; MWR = 13.623%.

- Using your calculator to calculate MWR:

  $CF_0 = -1{,}000$; $CF_1 = -800$; $CF_2 = 2{,}200$; CPT: IRR = 13.623%.

Money Market Yields

Holding period yield (HPY)

- HPY = (Ending Value/Beginning Value) - 1

Bank discount yield (BDY)

- BDY = (Discount/Face Value) × (360/Days to maturity)
  - Discount rate, simple interest, 360-day annualized.

Money Market Yield (MMY)

- **MMY** = (Discount/Price) × (360/Days to maturity)
  - Add-on rate, simple interest, 360-day annualized.

Money Market Yields

Bond Equivalent Yield (BEY)

- BEY = (Discount/Price) × (365/Days to maturity)
  - Add-on rate, simple interest, 365-day annualized;
  - Only for money market, not available for capital market.

Effective annual yield (EAY)

- EAY = $(1+HPY)^{365/Days} - 1$
  - Add-on rate, compound interest, 365-day annualized.

Money Market Yields

Example

- A 90-day T-bill is purchased for $997.40. What are the bank discount yield, holding period yield, money market yield, and the effective yield?

Answer:

Bank discount yield: [(1,000 – 997.40)/1,000] × 4 = 1.04%;

90-day holding period return: 1,000/997.4 – 1 = 0.2607%;

Money market yield: 0.2607 × (360/90) = 1.0428%;

Effective annual yield: $(1{,}000/997.4)^{365/90} - 1 = 1.0614\%$.

Practice (1)

If a bank offers a stated annual interest rate of 3.98%, and compounds quarterly, what is the effective annual rate the bank is offering?

A. 13.33%

B. 16.90%

C. 4.040%

Answer: C

Effective annual rate = $(1 + 3.98\%/4)^4 - 1 = 4.040\%$

Practice (2)

Supersuds is planning to spend $8 million on advertising. The company expects this expenditure to result in annual incremental cash flows of $1.2 million in perpetuity. What is the net present value and IRR of this project if Supersuds' opportunity cost of capital is 11%?

	NPV	IRR
A.	$2.91 million	4%
B.	$10.91 million	4%
C.	$2.91 million	15%

Practice (2)

Answer: C

PV of incremental cash flow = 1.2/0.11 = 10.91 Mil.

NPV = 10.91 – 8 = 2.91 Mil.

IRR = 1.2/8 = 15%

You're a Champion!

Thanks for staying with us. You have finished this chapter.