$$\left\{\begin{array}{l}
\mathrm{Mod} : \boxed{\mathrm{Sign}} \longrightarrow \mathrm{Cat}^{op} \xrightarrow{\ 1\text{-}1\ } \mathrm{Set}^{op} \\
\mathrm{Sen} : \mathrm{Sign} \longrightarrow \mathrm{Set} \xrightarrow{\ P\ } \mathrm{Set}^{op} \\
\models_\Sigma
\end{array}\right.$$

$$\begin{tikzcd}
\Sigma \arrow[d, "\varphi"'] & {|\mathrm{Mod}(\Sigma)|} \arrow[r, "\models_{\Sigma}"] & \mathrm{Sen}(\Sigma) \arrow[d, "\mathrm{Sen}(\varphi)"] \\
\Sigma' & {|\mathrm{Mod}(\Sigma')|} \arrow[u, "{|\mathrm{Mod}(\varphi)|}"] \arrow[r, "\models_{\Sigma'}"'] \arrow[ur, phantom, "*"] & \mathrm{Sen}(\Sigma')
\end{tikzcd}$$

$$\models : \mathrm{Sign} \ \underset{P(\mathrm{Sen}(-))}{\overset{|\mathrm{Mod}(-)|}{\underset{\Downarrow}{\rightrightarrows}}} \ \mathrm{Set}^{op}$$

$$\begin{tikzcd}
{|\mathrm{Mod}(\Sigma)|} \arrow[r, "\models_\Sigma"] & P(\mathrm{Sen}(\Sigma)) \\
{|\mathrm{Mod}(\Sigma')|} \arrow[u, "{|\mathrm{Mod}(\varphi)|}"] \arrow[r, "\models_{\Sigma'}"'] \arrow[ur, phantom, "*"] & P(\mathrm{Sen}(\Sigma')) \arrow[u, "P(\mathrm{Sen}(\varphi))"']
\end{tikzcd}$$

$$\frac{m' \models_{\Sigma'} \mathrm{Sen}(\varphi)(s)}{\mathrm{Mod}(\varphi)(m') \models_\Sigma s}$$

$$\mathbb{I}\mathrm{ns} := \underline{\mathrm{obj}} \ \ (\mathrm{Sign}, \underbrace{\mathrm{Mod}, \mathrm{Sen}, \models})^{*}$$

$$\begin{array}{lll}
\mathbb{R}\mathrm{oom} := & \underline{\mathrm{obj}} & \left(M, S, R : |M| \to P(S)\right) \\
 & & \ \ f \downarrow \Rightarrow \downarrow g \qquad \qquad \downarrow \qquad \qquad \downarrow \\
 & \underline{\mathrm{nor}} & \left(M', S', R' : |M'| \to P(S')\right)
\end{array}$$

$$\oint [(-)^{op}, \mathbb{R}\mathrm{oom}] = \mathbb{I}\mathrm{ns}$$

<u>2-category</u>

$$\mathrm{Sign} \longmapsto (\mathrm{Mod}, \mathrm{Sen}, \models)$$

$$
\begin{tikzcd}
T \arrow[r, "y"] & {[T^{op}, Set]} \arrow[r, "\Omega"] & Hey \\
\oint \Omega y \arrow[u] & &
\end{tikzcd}
$$

$$
\begin{tikzcd}
& & \cdots \\
\omega : Thy \arrow[r] & CartClosedFibs \arrow[ur] &
\end{tikzcd}
$$

structural types

$$
\begin{tikzcd}
T \arrow[d, "F"'] \\
T'
\end{tikzcd}
\qquad
\begin{tikzcd}
\oint \Omega y_T \arrow[r] \arrow[d] & \oint \Omega y_{T'} \arrow[d] \\
T \arrow[r] & T'
\end{tikzcd}
$$

$$ML : Thy \longrightarrow (M, S, R)$$

power comes from axioms

LTL <u>linear temporal logic</u> (properties of systems)

automata generate languages ⇒ demand formulas, then model-check

FOL <u>first-order logic</u> (<u>specify</u> systems)

ex. arrays

* people don't appreciate <u>axioms</u>.