

Q: what form will the equivalence take?

— equ. of categories?

matching logic forms an institution

$\mathbb{I} = (\text{Sign}, \text{Mod}, \text{Sen}, \models)$ satisfaction

- $\begin{cases} \text{Mod}: \text{Sign}^{\text{op}} \rightarrow \text{Cat} \\ \text{Sen}: \text{Sign} \rightarrow \text{Set} \end{cases} \quad \begin{aligned} \Sigma &\mapsto \text{models of } \Sigma \\ \Sigma &\mapsto \text{set of } \Sigma\text{-sentences} \end{aligned}$

$\mathbb{I} \rightarrow \mathbb{I}' ::$

[proof —
in the technical
report]

$\text{Sign} \xrightarrow{\mathbb{I}} \text{Sign}'$

$\text{Mod} \searrow \xRightarrow{\beta} \swarrow \text{Mod}'$

Cat^{op}

$\text{Sign} \xrightarrow{\mathbb{I}} \text{Sign}'$

$\text{Sen} \searrow \xleftarrow{\alpha} \swarrow \text{Sen}'$

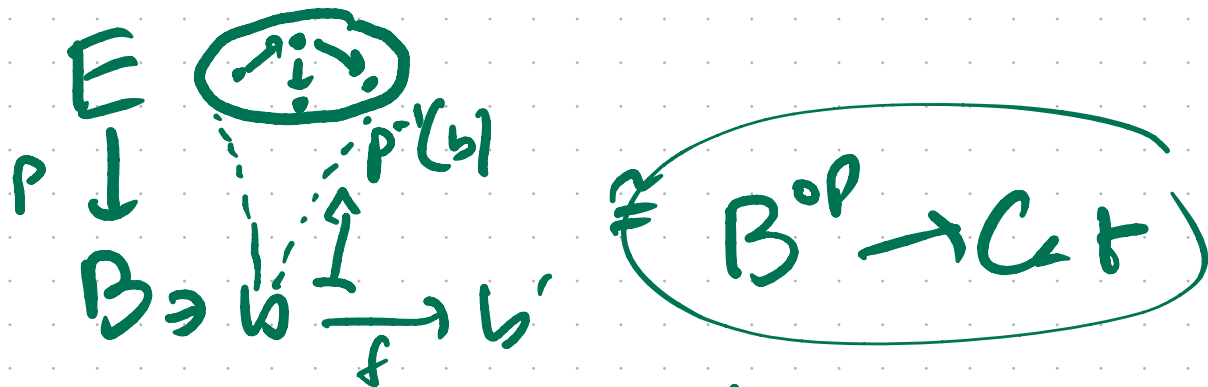
Set

$m \models_{\Sigma} \alpha(f') \Leftrightarrow \beta(m) \models'_{\beta(\Sigma)} f'$

another formalization:

type refinement systems (monoidal closed bifibrations)

Functors are TRS - Mellies/Zeilberger



Sen: $\text{Sign}^{op} \rightarrow \text{Cat}$

Σ

$s \mapsto \text{sub}(\Sigma(-, s))$

Mod: $\text{Sign}^{op} \rightarrow \text{Cat}$

$\int \text{Mod} = (T, M)$

\downarrow
Sign

$f \downarrow$
 (T', M')

$f(m) \mapsto m'$

Sign ^{signature}

Mod: $\text{Sign}^{\text{op}} \rightarrow \text{Cat} \sim$

$\mathcal{M}od = (\Sigma, \mathcal{M})$
 \downarrow fibration

Sen: $\text{Sign} \rightarrow \text{Cat} \sim$

Sign

$\mathcal{S}en = (\Sigma, \mathcal{C})$
 \downarrow opfibration
 Sign

• $F_{\Sigma} \subseteq |\text{Mod}(\Sigma)| \times \text{Sen}(\Sigma)$

$\{ M \mapsto \text{Th}(M) = \{ \varphi \mid M \models \varphi \} \}$

$\{ \varphi \mapsto \text{model}(\varphi) = \{ M \mid M \models \varphi \} \}$

Galois connection!

$\text{Th}: \text{Mod} \rightarrow \text{Set}$
 $\text{model}: \text{Sen} \rightarrow \text{Set}$

profunctors?

\exists morph

OSA \rightarrow ML

"matching logic captures order-sorted algebra"