Q: what form will the equivalence take?

— equ. of categories?

matching logic forms an institution

$$\mathbb{I} = (\mathrm{Sign}, \mathrm{Mod}, \mathrm{Sen}, \models)$$ satisfaction

- $\begin{cases} \mathrm{Mod} : \mathrm{Sign}^{op} \to \mathrm{Cat} & \Sigma \mapsto \text{models of } \Sigma \\ \mathrm{Sen} : \mathrm{Sign} \to \mathrm{Set} & \Sigma \mapsto \text{set of } \Sigma\text{-sentences} \end{cases}$

$\mathbb{I} \to \mathbb{I}'$ ::

[proof — in the technical report]

$$\begin{tikzcd} \mathrm{Sign} \arrow[rr, "\Phi"] \arrow[dr, "\mathrm{Mod}"'] & \arrow[d, phantom, "\overset{\beta}{\Rightarrow}"] & \mathrm{Sign}' \arrow[dl, "\mathrm{Mod}'"] \\ & \mathrm{Cat}^{op} & \end{tikzcd} \qquad \begin{tikzcd} \mathrm{Sign} \arrow[rr, "\Phi"] \arrow[dr, "\mathrm{Sen}"'] & \arrow[d, phantom, "\overset{\alpha}{\Leftarrow}"] & \mathrm{Sign}' \arrow[dl, "\mathrm{Sen}'"] \\ & \mathrm{Set} & \end{tikzcd}$$

$$m \models_{\Sigma} \alpha(f') \iff \beta(m) \models'_{\Phi(\Sigma)} f'$$

another formalization:

type refinement systems (monoidal closed bifibrations)

Functors are TRS — Melliès/Zeilberger

$$\begin{array}{ccc} E & & p^{-1}(b) \\ p\downarrow & & \\ B \ni b & \xrightarrow{f} & b' \end{array}$$

$\simeq$ $B^{op} \to Cat$

Sen: $Sign^{op} \to Cat$

Σ

$s \longmapsto Sub[\Sigma(-,s)]$

$\downarrow$

$\underline{Mod}$: $Sign^{op} \to Cat$

$$\begin{array}{ccc} \int Mod & = & (T, M) \\ \downarrow & & f\downarrow \\ Sign & & (T', M') \end{array} \qquad f(M) \xrightarrow{\varphi} M'$$

Sign signatures

Mod: $\mathrm{Sign}^{op} \to \mathrm{Cat}$ ~

Sen: $\mathrm{Sign} \to \mathrm{Cat}$ ~

$\int \mathrm{Mod} = (\Sigma, M)$
$\downarrow$ fibration
Sign

$\int \mathrm{Sen} = (\Sigma, \varphi)$
$\downarrow$ opfibration
Sign

- $\models_\Sigma \subseteq |\mathrm{Mod}(\Sigma)| \times \mathrm{Sen}(\Sigma)$

$$\begin{cases} M \mapsto \mathrm{Th}(M) = \{\varphi \mid M \models \varphi\} \\ \varphi \mapsto \mathrm{model}(\varphi) = \{M \mid M \models \varphi\} \end{cases}$$

$\mathrm{Th}: \mathrm{Mod} \to \mathrm{Set}$
$\mathrm{model}: \mathrm{Sen} \to \mathrm{Set}$

Galois connection!

— profunctors?

$\exists$ morph OSA $\to$ ML

"matching logic captures order-sorted algebra"