Harvard University
Computer Science 20

Problem Set 5

Due Wednesday, March 9, 2016 at 9:59am. All students should submit an electronic copy.

Problem set by **FILL IN YOUR NAME HERE**

Collaboration Statement: **FILL IN YOUR COLLABORATION STATEMENT HERE (See the syllabus for information)**

PART A (Graded by Ben)

PROBLEM 1 (4 points, suggested length of 1/3 page)

Show, by giving an example for each case, that the intersection of two uncountable sets can be: empty, finite, countably infinite, or uncountably infinite.

PROBLEM 2 (4+2 points, suggested length of 1 page)

(A) The Schröder-Bernstein Theorem states that for sets S and T, if there exist injective functions $f : S \to T$ and $g : T \to S$, then S and T have the same cardinality. Using the Schröder-Bernstein Theorem, show that the cardinality of the set of all real numbers between 0 and 100, inclusive, is the same as the cardinality of the set of all real numbers between, but not including, 0 and 100.

(B) Prove the finite case for the Schröder-Bernstein Theorem. That is, prove that for finite sets S and T, if there exist injective functions $f : S \to T$ and $g : T \to S$, then sets S and T have the same cardinality. (HINT: Check out the warm-up proof from the Relations and Functions lesson for some inspiration!)

PROBLEM 3 (2+2 points, suggested length of 1/2 page)

A robot wanders around a two-dimensional grid. The robot starts at (0,0) and is allowed to take four different types of steps:

1. (-2, +2)
2. (-4, +4)
3. (+1, -1)
4. (+3, -3)

For example, the robot might take the following stroll. The types of steps are denoted by each arrow's subscript:

$$(0,0) \rightarrow_1 (-2,2) \rightarrow_3 (-1,1) \rightarrow_2 (-5,5) \rightarrow_4 (-2,2) \rightarrow \ldots$$

(A) Describe a state machine model of this problem.
(B) Will the robot ever reach (1, 2)? Either find an appropriate path for the robot or use the Invariant Principle to prove that no such path exists.

PART B (Graded by Crystal)

PROBLEM 4 (3+1 points, suggested length of 1/2 page)

(A) Using string concatenation as the sole constructor, give a recursive definition of the set S of bit strings with no more than a single 1 in them
(e.g. 00010, 010, or 000)
(B) Is $0010 \in S$? How can you derive it from your base case?

PROBLEM 5 (2+2 points, suggested length of 1/2 page)

Let $A = \{5n : n \in \mathbb{N}\}$ and let S be the set defined as follows:

- Base Case: $5 \in S$
- Constructor Rule: If $x \in S$ and $y \in S$, $x + y \in S$

(A) Use induction to prove that $A \subseteq S$.
(B) Use structural induction to prove that $S \subseteq A$.