Harvard University
Computer Science 20

Midterm 1

Monday, February 22, 2016

PROBLEM 1

Prove that if you pick 5 distinct integers from $\{1, \ldots, 100\}$, some two differ by at most 24.

Solution.

PROBLEM 2

Let $A = \neg p \vee q$ and $B = p \oplus \neg q$. Show by writing out the two truth tables that these two formulas are not equivalent.

Solution.

PROBLEM 3

Perform the following operations in binary. Show your work.

(A) $1111_2 + 1111_2$
(B) $1010_2 - 100_2$

Solution.

PROBLEM 4

Let the domain of discourse be all Harvard CS courses. The predicate $P(c, d)$ means that course c is a prerequisite for course d. Assume that it is not possible for a course to be a prerequisite of itself. Write the following English sentences using quantificational formulas.

(A) There is a course that is a prerequisite for every other course.
(B) At least one course is a prerequisite for exactly one other course.

Solution.

PROBLEM 5

Let $a_1 = 1$ and for every $k \geq 1$, $a_{k+1} = 3a_k + 1$. Prove that for every $k \geq 1$, a_k is odd or even depending on whether k is odd or event, respectively. You may assume that the product of an even number and another number is even, and the product of two odd numbers is odd.

Solution.

PROBLEM 6

Prove that in any group of six people, at least two of them know the same number of people. (You don't know yourself, and if you know someone, then that person knows you too.)

Solution.