Harvard University
Computer Science 20

In-Class Problems 11

Friday, February 19, 2016

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Midterm Review This in-class midterm review is a puzzle. Each problem gives you one piece of the puzzle. Solve all the problems and put together the pieces to discover the keyword. Enjoy!

PROBLEM 1

How many base cases does the following proof by induction require?

The Tribonacci numbers are defined by $T_0 = 1, T_1 = 1, T_2 = 2$, and $T_n = T_{n-1} + T_{n-2} + T_{n-3}$ for all $n \geq 3$. The beginning of the Tribonacci sequence is $1, 1, 2, 4, 7, 13, ...$

Proof. Let $P(n)$ be the predicate $T_n \leq 3^n$. Base cases: [???]. Inductive step: assume $P(1), ..., P(n)$ holds for $n \geq$ [???]. Then $T_{n+1} = T_n + T_{n-1} + T_{n-2}$
$\leq 3^n + 3^{n-1} + 3^{n-2}$
$= 3^{n+1}(\frac{1}{3} + \frac{1}{9} + \frac{1}{27})$
$= 3^{n+1}\frac{13}{27}$
$\leq 3^{k+1}$

0	1	2	3	4	5	¿5
G	R	S	E	H	L	M

Problem 1 Clue

Solution.
3 base cases are required.

PROBLEM 2

Construct a truth table for $(p \leftrightarrow q) \rightarrow (q \leftrightarrow r)$. How many rows in the truth table are True?

0	1	2	3	4	5	6	7	8	> 8
E	N	A	V	B	R	I	Z	X	Q

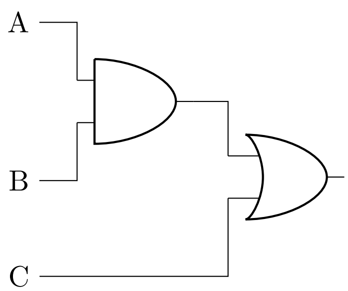
Problem 2 Clue

Solution.

6 rows are True:

p	q	r	$(p \leftrightarrow q) \rightarrow (q \leftrightarrow r)$
1	1	1	1
1	1	0	0
1	0	1	1
1	0	0	1
0	1	1	1
0	1	0	1
0	0	1	0
0	0	0	1

PROBLEM 3



Which of the following does the above logic circuit compute? (For the purpose of this problem, assume $1 + 1 = 1$.)

1. $A \cdot B + C$
2. $A + B + C$
3. $A + B \cdot C$
4. $A \cdot B \cdot C$
5. $(A + B) \cdot C$

1	2	3	4	5
S	E	O	T	R

Problem 3 Clue

Solution.
(1).

PROBLEM 4

Which of the following quantificational logic statements are true?

1. $\forall n \in \mathbb{N}, \exists m \in \mathbb{N}. n \cdot m = 1$

2. $\forall n \in \mathbb{N}, \forall m \in \mathbb{N}. n + m = n$
3. $\exists n \in \mathbb{N}, \forall m \in \mathbb{N}. n + m = n$
4. $\exists n \in \mathbb{N}, \forall m \in \mathbb{N}. n \cdot m = n + m$
5. $\exists n \in \mathbb{N}, \forall m \in \mathbb{N}. n \cdot m = m$

1	2	3	4	5
M	N	R	E	L

Problem 4 Clue

Solution.
(5) is true; $n = 1$.

PROBLEM 5

(BONUS) What is the least value of m for which the following is true? "In any set of m propositions, all involving only p, two of the propositions are logically equivalent."

2	5	9	12	17	26	50	> 50
A	W	B	K	U	O	N	T

Problem 5 Clue

Solution.
By the pigeonhole principle, 5, since there are only 4 possible truth tables.

PROBLEM 6

Final Answer:

Solution.
LEWIS