PROBLEM 1

Prove that if you pick 5 integers from $\{1, \ldots, 100\}$, some two differ by at most 24.

Solution.

PROBLEM 2

Let $A = \neg(\neg p \vee q) \to r$ and $B = \neg r \oplus (\neg p \wedge q)$. For which value(s) of p, q, and r do A and B differ? Use a truth table.

Solution.

PROBLEM 3

Let the domain of discourse be all Harvard CS courses. The predicate $P(c, d)$ means that course c is a prerequisite for course d. Assume that it is not possible for a course to be a prerequesite of itself. Write the following English sentences using quantificational formulas.

(A) There is a course that is a prerequisite for every other course.
(B) At least one course is a prerequisite for exactly one other course.

Solution.

PROBLEM 4

The Tribonacci numbers are defined by $T_0 = 1, T_1 = 1, T_2 = 2$, and $T_n = T_{n_1} + T_{n-2} + T_{n-3}$ for all $n \geq 3$. The beginning of the Tribonacci sequence is $1, 1, 2, 4, 7, 13, \ldots$. Use strong induction to prove that $T_n \leq 3^n$ for all natural numbers n.

Solution.

PROBLEM 5

While wandering the halls of Hogwarts, Harry stumbles upon a note that reads: "Danger lies before you, while safety lies behind. Drink some of us to help you, the combination you must find". Harry sees four bottles of mysterious potion, one colored red, one blue, one yellow, and one green. A sign next to the door reads:

1. Drink two of these potions, one must be colored blue.

2. If blue you do not have, the other three will tide you through.

3. Pair not blue with red

4. Or face your fate with dread

Luckily, Harry sees all the colored bottles. Define r, y, b, g:

1. r: Harry drinks the red potion

2. y: Harry drinks the yellow potion

3. b: Harry drinks the blue potion

4. g: Harry drinks the green potion

(A) Write a proposition that evaluates to True if Harry selects a valid combination of potions.
(B) Draw a logic circuit that implements the proposition in (a) with as few gates as possible from this list: NOT, OR, AND, XOR, NAND, XNOR.

Solution.