

**Harvard University
Computer Science 20**

Problem Set 5

Due Wednesday, March 9, 2016 at 9:59am. All students should submit an electronic copy.

Problem set by ****FILL IN YOUR NAME HERE****

Collaboration Statement: ****FILL IN YOUR COLLABORATION STATEMENT HERE
(See the syllabus for information)****

PART A (Graded by Ben)

PROBLEM 1 (4 points, suggested length of 1/3 page)

Show, by giving an example for each case, that the intersection of two uncountable sets can be: empty, finite, countably infinite, or uncountably infinite.

PROBLEM 2 (4+2 points, suggested length of 1 page)

(A) The Schröder-Bernstein Theorem states that for sets S and T , if there exist injective functions $f : S \rightarrow T$ and $g : T \rightarrow S$, then S and T have the same cardinality. Using the Schröder-Bernstein Theorem, show that the cardinality of the set of all real numbers between 0 and 100, inclusive, is the same as the cardinality of the set of all real numbers between, but not including, 0 and 100.

(B) Prove the finite case for the Schröder-Bernstein Theorem. That is, prove that for finite sets S and T , if there exist injective functions $f : S \rightarrow T$ and $g : T \rightarrow S$, then sets S and T have the same cardinality. (HINT: Check out the warm-up proof from the Relations and Functions lesson for some inspiration!)

PROBLEM 3 (2+2 points, suggested length of 1/2 page)

A robot wanders around a two-dimensional grid. He starts at (0,0) and is allowed to take four different types of steps:

1. (-2, +2)
2. (-4, +4)
3. (+1, -1)
4. (+3, -3)

For example, the robot might take the following stroll. The types of his steps are denoted by each arrow's subscript:

$$(0, 0) \rightarrow_1 (-2, 2) \rightarrow_3 (-1, 1) \rightarrow_2 (-5, 5) \rightarrow_4 (-2, 2) \rightarrow \dots$$

- (A) Describe a state machine model of this problem.
- (B) Will the robot ever reach $(1, 2)$? Either find an appropriate path for the robot or use the Invariant Principle to prove that no such path exists.

PART B (Graded by Crystal)

PROBLEM 4 (3+1 points, suggested length of 1/2 page)

- (A) Using string concatenation as the sole constructor, give a recursive definition of the set S of bit strings with no more than a single 1 in them
(e.g. 00010, 010, or 000)
- (B) Is $0010 \in S$? How can you derive it from your base case?

PROBLEM 5 (2+2 points, suggested length of 1/2 page)

Let $A = \{5n : n \in \mathbb{N}\}$ and let S be the set defined as follows:

- Base Case: $5 \in S$
- Constructor Rule: If $x \in S$ and $y \in S$, $x + y \in S$

- (A) Use induction to prove that $A \subseteq S$.
- (B) Use structural induction to prove that $S \subseteq A$.