## Harvard University
## Computer Science 20

### Midterm 1

Monday, February 22, 2016

## PROBLEM 1

Prove that if you pick 5 integers from $\{1, \ldots, 100\}$, some two differ by at most 24.

**Solution.**

## PROBLEM 2

Let $A = \neg(\neg p \vee q) \to r$ and $B = \neg r \oplus (\neg p \wedge q)$. For which value(s) of $p, q$, and $r$ do $A$ and $B$ differ? Use a truth table.

**Solution.**

## PROBLEM 3

Perform the following operations in binary.

(A) $1111_2 + 1111_2$
(B) $1010_2 - 100_2$

**Solution.**

## PROBLEM 4

Let the domain of discourse be all Harvard CS courses. The predicate $P(c, d)$ means that course $c$ is a prerequisite for course $d$. Assume that it is not possible for a course to be a prerequesite of itself. Write the following English sentences using quantificational formulas.

(A) There is a course that is a prerequisite for every other course.
(B) At least one course is a prerequisite for exactly one other course.

**Solution.**

## PROBLEM 5

The Tribonacci numbers are defined by $T_0 = 1, T_1 = 1, T_2 = 2$, and $T_n = T_{n_1} + T_{n-2} + T_{n-3}$ for all $n \geq 3$. The beginning of the Tribonacci sequence is $1, 1, 2, 4, 7, 13, ....$ Use strong induction to prove that $T_n \leq 3^n$ for all natural numbers $n$.

**Solution.**




## PROBLEM 6


Prove that in any group of six people, at least two of them know the same number of people. Note that you don't know yourself, and that if A knows B then B knows A (thus "knows" is a symmetric relation).

**Solution.**