Your Name:

# Harvard University
## Computer Science 20
## Midterm 2

Wednesday, March 23, 2016

### PROBLEM 1

For each of the following, state whether the set is finite, countably infinite, or uncountable. No justification required.

(A) The set of all total functions with domain $\{0, 1\}$ and codomain $\{0, 1\}$.
(B) The set of all total functions with domain $\mathbb{N}$ and codomain $\{1\}$.
(C) The set of all total functions with domain $\mathbb{N}$ and codomain $\{0, 1\}$.
(D) The set of all total functions with domain $\{0, 1\}$ and codomain $\mathbb{N}$.

**Solution.**

### PROBLEM 2

Draw state machines that only accept strings in the following set. Assume that the alphabet is $\Sigma = \{0, 1\}$; that is, for all possible input strings $s$ we have $s \in \Sigma^*$.

$$\{w : w \text{ starts with 0 and contains the substring 101, i.e. } w = 0x101y \text{ for some } x \text{ and } y\}$$

**Solution.**

### PROBLEM 3

Let $G$ be a directed graph with $n$ vertices. Show that if $G$ has a path of length greater than $n$, then $G$ has a cycle (a path has length $k$ if it contains $k$ edges).

**Solution.**

### PROBLEM 4

Let $G = (V, E)$ be a directed acyclic graph. Define a relation $R$ on $V$ by $(v_1, v_2)$ which is an element of $R$ iff there is a path from $v_1$ to $v_2$.

(A) Is $R$ reflexive? Prove your answer.
(B) Is $R$ symmetric? Prove your answer.
(C) Is $R$ transitive? Prove your answer.

**Solution.**

## PROBLEM 5

(A) Using set-builder notation, give a formal description of the union of two sets $A$ and $B$.
(B) Using set-builder notation, give a formal description of the complement of a set $A$.
(C) Let $|A| = n$ and $|B| = m$. If $A \subseteq B$, what is $|A \cap B|$?
(D) Let $|A| = n$ and $|B| = m$. If $A \subseteq B$, what is $|A - B|$?
(E) What is the power set of $\{h, a, i\}$?

**Solution.**

## PROBLEM 6

Let S be the set defined as follows:

- Base Case: $(1, 2) \in S$
- Constructor Rules: If $(x, y) \in S$, then C1: $(x + 2, y) \in S$ and C2: $(y, x) \in S$

Use structural induction to prove that for any pair (x,y) in S, x and y can not both be odd or both be even.

**Solution.**

## PROBLEM 7

Please provide a graph of binary relations over the set $\{1, 2, 3\}$ that fulfils the following properties, or explain why it would be impossible to construct one.
(A) Symmetric transitive
(B) Asymmetric, reflexive.
(C) Transitive, symmetric, irreflexive.

**Solution.**