

## ADAPTIVE SLIDING MODE SPACECRAFT ATTITUDE CONTROL

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### ABSTRACT

*An adaptive sliding mode spacecraft attitude controller is derived in this paper. It has the advantage of not requiring knowledge of the inertia of the spacecraft, and rejecting unexpected external disturbances, with global asymptotic position and velocity tracking. The sliding manifold is designed using optimal control analysis of the quaternion kinematics. The sliding mode control law and the parameter adaptation law are designed using Lyapunov stability. Numerical simulations are performed to demonstrate both the nominal and the robust performance.*

### 1 INTRODUCTION

The attitude control problem has attracted much attention as it involves highly nonlinear characteristics of the governing motion equations. From the perspective of control, feedback control laws are sought for the purpose of asymptotic trajectory tracking, with the ability to reject unexpected external disturbances, and be insensitive to parameter variations. Previous efforts have been devoted to developing both open-loop and closed-loop control strategies. Although the open-loop formulation makes it easier to incorporate some optimal criterion, the resulting performance is inevitably sensitive to system uncertainties. Closed-loop control has been investigated to deal with both single-axis small angle rotations and three-axis large angle maneuvers. The latter problem is much more challenging as a larger region of operations makes the linear approximation of nonlinear dynamics invalid.

The simplest large-angle maneuver uses quaternion and

velocity feedback similar to a proportional derivative controller [1]. Model-based control techniques are also investigated such as sliding-mode control [2], and adaptive control [3]. Sliding-mode control was investigated for the purpose of robust attitude tracking for various attitude parameterizations (Rodrigues parameters [4, 5], Modified Rodrigues parameters [6], quaternions [7–9]). Adaptive attitude tracking control based on Lyapunov stability was studied for quaternions [10, 11] and rotation matrices [12].

In this paper, we develop an adaptive sliding-mode attitude tracking controller. A similar methodology has been applied to control of robot manipulators by Slotine *et. al.* [13]. Although the attitude dynamics cannot be directly transformed to the form of robot dynamics, the design of sliding manifold and the construction of a Lyapunov function in the present work achieve similar performance specifications. Compared with the existing sliding-mode attitude controller, our approach does not require any inertial information. The use of a sliding manifold reduces the design complexity and makes the controller have a simple form.

Unit quaternion is used to parameterize rotations since it is the minimal singularity-free rotation representation. Based on the quaternion kinematic relation, a sliding manifold is chosen according to the optimality criterion proposed in [14]. The dynamic equations of motion of the spacecraft actuated by either thrusters or momentum wheels are considered. Global asymptotic stability is shown using Lyapunov stability analysis.

The remainder of the paper is organized as follows. In Sec. 2, the quaternion kinematics and the spacecraft dynamics

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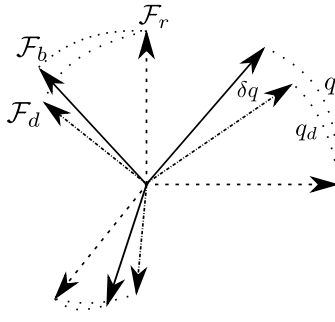


FIGURE 1. Definitions of the coordinate frames

are reviewed. In Sec.3, an optimal sliding manifold is presented along with its optimality proof and stability analysis. Then, a Lyapunov stability analysis is used to derive an asymptotically stabilizing sliding control law and a parameter adaptation law. A robust controller is designed to reject external disturbances. In Sec.4, numerical simulations are shown to demonstrate the closed-loop performance of the proposed controller.

## 2 PRELIMINARIES

### 2.1 Coordinate frames

We define three coordinate frames of interest. The inertial frame of reference, in which Newton's law is satisfied, is denoted as  $\mathcal{F}_r$ . We attach three mutually perpendicular axes to the spacecraft, and call this the body-fixed frame  $\mathcal{F}_b$ . The spacecraft is modeled as a rigid body actuated by either thrusters or momentum wheels in three orthogonal directions. The body axes are chosen to coincide the directions of actuations. The desired spacecraft attitude is described by a frame denoted  $\mathcal{F}_d$ . The frame definitions are depicted in Figure 1.

### 2.2 Kinematics

The unit quaternion is used to describe the spacecraft attitude,

$$q = \begin{bmatrix} \rho \\ q_4 \end{bmatrix} = \begin{bmatrix} \hat{e} \sin(\theta/2) \\ \cos(\theta/2) \end{bmatrix} \quad (1)$$

where  $\hat{e}$  is a unit vector representing the axis of rotation,  $\theta$  is the angle of rotation from  $\mathcal{F}_r$  to  $\mathcal{F}_b$ . It has to satisfy the following unity norm constraint,

$$\|q\|_2^2 = \rho^T \rho + q_4^2 = 1 \quad (2)$$

If the angular velocity of the spacecraft with respect to  $\mathcal{F}_r$ , expressed in  $\mathcal{F}_b$ , is denoted as  $\omega$ , the quaternion kinematic equa-

tion is given by,

$$\dot{q} = \frac{1}{2} \Xi(q) \omega \quad (3)$$

where

$$\Xi(q) = \begin{bmatrix} q_4 I_{3 \times 3} + [\rho \times] \\ -\rho^T \end{bmatrix}$$

$$[a \times] = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}, \quad a \in \mathbb{R}^3$$

The matrix  $\Xi(\bullet)$  obeys the following properties,

$$\begin{aligned} \Xi^T(q) \Xi(q) &= I_{3 \times 3} \\ \Xi^T(a) a &= 0_{3 \times 1}, \quad \forall a \in \mathbb{R}^4 \\ \frac{d}{dt} \{ \Xi^T(q) \dot{q} \} &= \Xi^T(q) \ddot{q} \end{aligned} \quad (4)$$

From these properties, one can show that if the desired attitude trajectory is specified by  $q_d = [\rho_d^T, q_{d4}]^T$ , the desired angular velocity  $\omega_d$  must obey,

$$\begin{aligned} \omega_d &= 2 \Xi^T(q_d) \dot{q}_d \\ \dot{\omega}_d &= 2 \Xi^T(q_d) \ddot{q}_d \end{aligned} \quad (5)$$

From the quaternion definition, one can see that  $q$  and  $-q$  represent the same physical rotation. Hence compared with algebraic subtraction, the error calculated from quaternion multiplication provides a better way because it resolves the sign ambiguity. The quaternion error and multiplication are defined as,

$$\begin{aligned} \delta q &= q \otimes q_d^{-1} = \begin{bmatrix} \Xi(q_d^{-1}), & q_d^{-1} \end{bmatrix} q = \begin{bmatrix} \delta \rho \\ \delta q_4 \end{bmatrix} = \begin{bmatrix} \Xi^T(q_d) q \\ q^T q_d \end{bmatrix} \\ q_d^{-1} &= \begin{bmatrix} -\rho_d \\ q_{d4} \end{bmatrix} \end{aligned} \quad (6)$$

$\delta q$  represents the rotation from  $\mathcal{F}_d$  to  $\mathcal{F}_b$ .

### 2.3 Dynamics

For a spacecraft having its three thrusters aligned with the body axes, the dynamic equations of motion are given by

$$J \dot{\omega} = -[\omega \times] J \omega + u \quad (7)$$

where  $J$  is the positive definite inertial matrix of the spacecraft,  $u$  is the torque generated by the thrusters.

If the spacecraft has three orthogonal momentum wheels instead, the equations of motion become,

$$\begin{aligned} (J - J_w)\dot{\omega} &= -[\omega \times](J\omega + J_w v) - u \\ J_w(\dot{\omega} + \dot{v}) &= u \end{aligned} \quad (8)$$

where  $J_w$  is the diagonal inertial matrix of the wheels,  $v$  is the wheel angular velocity, and  $u$  now is the torque applied to the wheel.

Eqn. (3) and one of (7) or (8) complete the dynamic model of the plant. The control objective is to make  $q \rightarrow q_d$  and  $\omega \rightarrow \omega_d$  as  $t \rightarrow \infty$ .

### 3 METHODOLOGY

We apply sliding mode control for controller synthesis. The idea of sliding mode control is to allow the transformation of a controller design problem for a general  $n$ -th order system to a simple stabilization problem with reduced order, i.e., stabilizing the dynamics associated with the switching function. Then for the equivalent reduced-order system, intuitive feedback control strategies can be applied.

Sliding mode control design consists of two steps: (i) design a stable sliding manifold on which the control objective is achieved, and (ii) design a reaching law and the corresponding control input so that the switching function is attracted to 0.

Crassidis *et. al.* proposed an optimal sliding manifold [14]. The optimality is evaluated when we only consider the quaternion kinematic equation and treat  $\omega$  as the input. The following functional is minimized,

$$J^*(q(t), t) = \min_{\omega} \int_t^{\infty} \frac{1}{2} \left\{ r^2 \delta \rho^T \delta \rho + (\omega - \omega_d)^T (\omega - \omega_d) \right\} d\tau \quad (9)$$

subject to the kinematic constraint,

$$\dot{q} = \frac{1}{2} \Xi(q) \omega$$

and the endpoint constraint,

$$\delta q(\infty) = [0, 0, 0, 1]^T \quad (10)$$

where  $r > 0$  is the weighting factor,  $q_d$  and  $\omega_d$  satisfy Eqn.(5). Without loss of generality, we only consider  $\delta q_4(t) \geq 0$ .

There exists two main approaches to optimal control [15], via the calculus of variations (the maximum principle) or

Lagrangian $L(q, \omega, t)$	$\frac{1}{2} r^2 \delta \rho^T \delta \rho + \frac{1}{2} (\omega - \omega_d)^T (\omega - \omega_d)$
Lagrange multipliers	$\lambda \in \mathbb{R}^4$
Hamiltonian $H(q, \lambda, \omega, t)$	$L + \frac{1}{2} \lambda^T \Xi(q) \omega$
State equation	$\dot{q} = \frac{1}{2} \Xi(q) \omega$
Costate equation	$\dot{\lambda} = -\frac{\partial H}{\partial q}$
Stationary condition	$\frac{\partial H}{\partial \omega} = 0$
Boundary condition	$\delta q(\infty) = [0, 0, 0, 1]^T$

**TABLE 1.** The necessary conditions for optimality from calculus of variations

dynamic programming (the principle of optimality).

#### 3.1 Calculus of variations

In [16], the necessary conditions for optimality are derived from calculus of variations. For the functional minimization problem in Eqn.(9), the necessary conditions are summarized in Table 1. It can be shown by direct substitution that the following optimal angular velocity  $\omega^*$ ,

$$\omega^* = \omega_d - r \Xi^T(q_d) q \quad (11)$$

with  $\lambda^* = -2rq_d$ , satisfies all the conditions except the boundary condition. To prove the satisfaction of the boundary condition, we use the kinematic equation for  $\delta q$ ,

$$\begin{aligned} \delta \dot{\rho} &= \frac{1}{2} \delta q_4 (\omega - \omega_d) + \frac{1}{2} [\delta \rho \times] (\omega + \omega_d) \\ \delta \dot{q}_4 &= -\frac{1}{2} (\omega - \omega_d)^T \Xi^T(q_d) q \end{aligned} \quad (12)$$

and a Lyapunov function candidate,

$$V = \frac{1}{2} \delta \rho^T \delta \rho \quad (13)$$

The Lie derivative taken with respect to the kinematic relation is,

$$\dot{V} = -\frac{1}{2} r \delta q_4 \delta \rho^T \delta \rho \leq 0 \quad (14)$$

The Lyapunov function value will keep decreasing until  $\delta \rho = 0_{3 \times 1}$  and  $\delta q_4 = \pm 1$ . From Eqn.(12) and the minimizer  $\omega^*$ ,  $\delta q_4$  can converge only to 1 since,

$$\delta \dot{q}_4 = \frac{1}{2} r (1 - \delta q_4^2) \geq 0, \quad = 0 \text{ only if } \delta q_4 = 1 \quad (15)$$

Therefore, all the necessary conditions are satisfied. The optimal value  $J^*(q(t), t)$  can be derived to be,

$$\begin{aligned} J^*(q(t), t) &= \int_t^\infty r^2(1 - \delta q_4^2) d\tau \\ &= 2r \int_t^\infty \frac{r}{2}(1 - \delta q_4^2) d\tau \\ &= 2r \int_t^\infty \delta \dot{q}_4 \tau \\ &= 2r(1 - \delta q_4(t)) \end{aligned} \quad (16)$$

### 3.2 Dynamic programming

We can also prove optimality by showing that  $\omega^*$  satisfies the following Hamilton-Jacobi-Bellman partial differential equation,

$$\frac{\partial J^*}{\partial t}(q, t) = -H(q, \frac{\partial J^*}{\partial q}(q, t), \omega^*, t) \quad (17)$$

*Proof:* We expand  $\frac{\partial J^*}{\partial t}(q, t)$  by the chain rule,

$$\begin{aligned} LHS &= \frac{\partial J^*(q, t)}{\partial q_d} \frac{dq_d}{dt} \\ &= -rq^T \Xi(q_d) \omega_d \end{aligned} \quad (18)$$

On the other hand, by substituting  $\frac{\partial J^*}{\partial q} = -2rq_d$  into the Hamiltonian, the right-hand side becomes,

$$\begin{aligned} H(q, -2rq_d, \omega^*, t) &= r^2 \delta \rho^T \delta \rho - rq_d^T \Xi(q)(\omega_d - r\Xi^T(q_d)q) \\ &= -rq^T \Xi(q_d) \omega_d \end{aligned} \quad (19)$$

### 3.3 Optimal sliding surface

For optimal tracking performance, it is natural to select the following sliding manifold  $s(q, \omega, t) = 0_{3 \times 1}$ ,

$$s(q, \omega, t) = (\omega - \omega_d) + r \text{sgn}[\delta q_4] \Xi^T(q_d)q = 0_{3 \times 1} \quad (20)$$

Note that  $\text{sgn}[\delta q_4(t)]$  is added for generality. The stability of this sliding manifold has already been seen from the boundary condition, i.e.  $q \rightarrow q_d$  as  $t \rightarrow \infty$ . In view of the sliding condition, we can further show the velocity tracking,

$$\omega = \omega_d - r \underbrace{\Xi^T(q_d)q}_{\delta \rho \rightarrow 0} \rightarrow \omega_d, \quad t \rightarrow \infty \quad (21)$$

Therefore, both of the control objectives are satisfied as long as  $q$  and  $\omega$  are confined in the sliding manifold.

### 3.4 Linearity in system parameters

In Eqns (7) and (8), the inertia parameters  $J_{ij}, J_{wij}$ , where  $i, j = 1, 2, 3$ , appear linearly. To make this more explicit, we follow [10] to use the following linear operator  $\mathcal{L} : \mathbb{R}^3 \mapsto \mathbb{R}^{3 \times 6}$  acting on any three-dimension vector  $a = [a_1, a_2, a_3]^T$  by,

$$\mathcal{L}(a) = \begin{bmatrix} a_1 & 0 & 0 & 0 & a_3 & a_2 \\ 0 & a_2 & 0 & a_3 & 0 & a_1 \\ 0 & 0 & a_3 & a_2 & a_1 & 0 \end{bmatrix} \quad (22)$$

For  $J = J^T$ , it follows easily that,

$$\underbrace{\begin{bmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{bmatrix}}_J a = \mathcal{L}(a) \underbrace{\begin{bmatrix} J_{11} \\ J_{22} \\ J_{33} \\ J_{23} \\ J_{13} \\ J_{12} \end{bmatrix}}_{\mathfrak{J}} \quad (23)$$

We denote the column vector of the inertia parameters as  $\mathfrak{J}$ .

### 3.5 Sliding mode controller and parameter adaptation law

After finding a stable sliding manifold, we need to design a sliding control law and a parameter adaptation law to make the sliding manifold attractive. We propose the following laws for the thruster model,

$$\begin{aligned} u &= -F\hat{\mathfrak{J}} - Ks \\ \dot{\hat{\mathfrak{J}}} &= \Gamma^{-1} F^T s \end{aligned} \quad (24)$$

where

$$F = -[\omega \times] \mathcal{L}(\omega) - \mathcal{L}(\dot{\omega}_d) + \mathcal{L}(r \text{sgn}[\delta q_4] \delta \dot{\rho}) \quad (25)$$

$\Gamma, K$  are constant positive-definite matrices of compatible dimensions,  $\hat{\mathfrak{J}}$  is the on-line estimate of the spacecraft inertia.

*Proposition:* The proposed control law achieves asymptotic trajectory tracking.

*Proof:* Assume that  $\delta q_4$  is non-zero for a finite time, we have the time derivative of the switching function,

$$\dot{s} = \dot{\omega} - \dot{\omega}_d + r \text{sgn}[\delta q_4] \delta \dot{\rho} \quad (26)$$

Define the parameter estimate error to be  $\tilde{\mathfrak{J}} = \mathfrak{J} - \hat{\mathfrak{J}}$ . We assume that the inertial matrix of the spacecraft  $J$  is constant, then

$$\dot{\tilde{\mathfrak{J}}} = -\dot{\hat{\mathfrak{J}}} \quad (27)$$

Consider the following Lyapunov function candidate, which is a positive-definite function of the switching function and the parameter error,

$$V = \frac{1}{2}s^T J s + \frac{1}{2}\tilde{\mathcal{J}}^T \Gamma \tilde{\mathcal{J}} \quad (28)$$

Its Lie derivative can be written as,

$$\begin{aligned} \dot{V} &= s^T \left\{ J \left[ \dot{\omega} - \dot{\omega}_d + r \operatorname{sgn}[\delta q_4] \delta \dot{\rho} \right] \right\} + \tilde{\mathcal{J}}^T \Gamma \dot{\tilde{\mathcal{J}}} \\ &= s^T \left[ F \tilde{\mathcal{J}} + u \right] - \tilde{\mathcal{J}}^T \Gamma \dot{\tilde{\mathcal{J}}} \\ &= s^T \left[ F \tilde{\mathcal{J}} - K s \right] - \tilde{\mathcal{J}}^T \Gamma \Gamma^{-1} F^T s \\ &= -s^T K s \end{aligned} \quad (29)$$

which shows that  $\dot{V}$  is negative semi-finite. Hence  $s$  and  $\tilde{\mathcal{J}}$  are bounded. Invoke Barbalat's lemma,

$$\dot{V} = -2s^T K s \quad (30)$$

The boundedness of  $\dot{s}$  can be seen by combining Eqns.(7), (24) and (26). This implies the uniform continuity of  $\dot{V}$ , hence we conclude that  $\dot{V} \rightarrow 0$ . Equivalently  $s \rightarrow 0$  as  $t \rightarrow \infty$ . Furthermore, to analyze the convergence of parameter estimation, we consider

$$J \dot{s} + K s = F \tilde{\mathcal{J}} \quad (31)$$

All the terms except  $\dot{s}$  are uniformly continuous. Thus  $\dot{s}$  is uniformly continuous. From Barbalat's lemma again,  $\dot{s} \rightarrow 0$ . Therefore,

$$F \tilde{\mathcal{J}} \rightarrow 0 \quad (32)$$

To enforce the asymptotic parameter estimation, i.e.  $\tilde{\mathcal{J}} \rightarrow 0$ , the following persistent excitation condition must be satisfied,

$$\int_t^{t+T} F^T(\delta q, \omega, \omega_d, \dot{\omega}_d) F(\delta q, \omega, \omega_d, \dot{\omega}_d) d\tau \geq \epsilon I_{6 \times 6}, \quad \forall t \geq t_o \quad (33)$$

where  $T, t_o, \epsilon$  are some positive scalars.

For the momentum wheel model, the sliding control law and parameter adaptation laws are,

$$\begin{aligned} u &= -F \hat{\mathcal{J}} - G \hat{\mathcal{J}}_w - K s \\ \dot{\hat{\mathcal{J}}} &= \Gamma^{-1} F^T s \\ \dot{\hat{\mathcal{J}}}_w &= \Gamma^{-1} G^T s \\ G &= -[\omega \times] \mathcal{L}(v) + \mathcal{L}(\dot{\omega}_d) - \mathcal{L}(r \operatorname{sgn}[\delta q_4] \delta \dot{\rho}) \end{aligned} \quad (34)$$

The same stability analysis can be performed by using the following slightly modified Lyapunov function candidate  $V'$ ,

$$V' = \frac{1}{2}s^T J s + \frac{1}{2}\tilde{\mathcal{J}}^T \Gamma \tilde{\mathcal{J}} + \frac{1}{2}\tilde{\mathcal{J}}_w^T \Gamma \tilde{\mathcal{J}}_w \quad (35)$$

where  $\tilde{\mathcal{J}}_w = \hat{\mathcal{J}}_w - \hat{\mathcal{J}}_w$  represents the estimate error of the wheel inertia.

In summary, we have used the Lyapunov stability theory to show that the sliding manifold is always attractive under the proposed sliding control law and the parameter adaptation law. The control objective is achieved. Note that we do not require the knowledge of the inertial matrix. Also we note that although the parameter adaptation law is converging to a constant, that estimate does not necessarily converge to the true inertia of the spacecraft. The system should be subject to persistent excitation.

### 3.6 Robust controller

To take into account unexpected external disturbances in practice, we slightly modify Eqn. (7) by adding a combined disturbance input  $d$  that can be from air drag, solar pressure, gravity gradient, magnetic field, spherical harmonics,

$$J \dot{\omega} = -[\omega \times] J \omega + u + d \quad (36)$$

Although  $d$  is unknown, but its magnitude has known bounds  $D \in \mathbb{R}^3$ ,

$$|d_i(t)| \leq D_i, \quad \forall t > 0, i = \{1, 2, 3\} \quad (37)$$

The robust sliding mode controller is given by,

$$u = -F \hat{\mathcal{J}} - K s - k \operatorname{sgn}(s) \quad (38)$$

where  $k_i = D_i + \eta_i$  for  $i = 1, 2, 3$  and  $\eta_i$ 's are non-negative constants. With the same Lyapunov function used before, we can show that the Lie derivative is now,

$$\dot{V} \leq -\sum_{i=1}^3 \eta_i |s_i| - s^T K s \quad (39)$$

Again, the state variables are guaranteed to reach the sliding manifold regardless of unknown disturbances. To avoid control chattering after reaching the sliding manifold, saturation functions can replace sign functions [2].

## 4 Numerical Simulations

In this section, we show the proposed controller performance through numerical simulations. The proposed controller

is used to control the attitude of the Microwave Anisotropy Probe (MAP) spacecraft [6]. We assume that quaternions and angular velocities are available for full-state feedback. The desired attitude profile is specified in 3-1-3 Euler angles  $\{\phi, \theta, \psi\}$ ,

$$\begin{aligned}\dot{\phi} &= 0.001745 \text{ rad/sec} \\ \theta &= 0.3927 \text{ rad} \\ \dot{\psi} &= 0.04859 \text{ rad/sec}\end{aligned}\quad (40)$$

Then  $\phi, \psi$  can be obtained by integration. The desired quaternion trajectory can be computed by converting Euler angle parameterization to unit quaternions,

$$q_d = \begin{bmatrix} \sin(\frac{\theta}{2}) \cos(\frac{\phi-\psi}{2}) \\ \sin(\frac{\theta}{2}) \sin(\frac{\phi-\psi}{2}) \\ \cos(\frac{\theta}{2}) \sin(\frac{\phi+\psi}{2}) \\ \cos(\frac{\theta}{2}) \cos(\frac{\phi+\psi}{2}) \end{bmatrix} \quad (41)$$

By numerically differentiating  $q_d$ , we can compute  $\dot{q}_d, \ddot{q}_d$  and  $\omega_d, \dot{\omega}_d$  by Eqn. (5). We let the actual inertia matrix  $J$  be,

$$J = \begin{bmatrix} 20 & 5 & 1 \\ 5 & 17 & 3 \\ 1 & 3 & 15 \end{bmatrix} \quad (42)$$

Our proposed controller does not require knowledge of the inertia of the spacecraft, so we use an initial estimate  $\hat{J}(0)$  of the inertia with 30% error,

$$\hat{J}(0) = \begin{bmatrix} 26 & 1.6 & 1.4 \\ 1.6 & 13 & 1.2 \\ 1.4 & 1.2 & 8.5 \end{bmatrix} \quad (43)$$

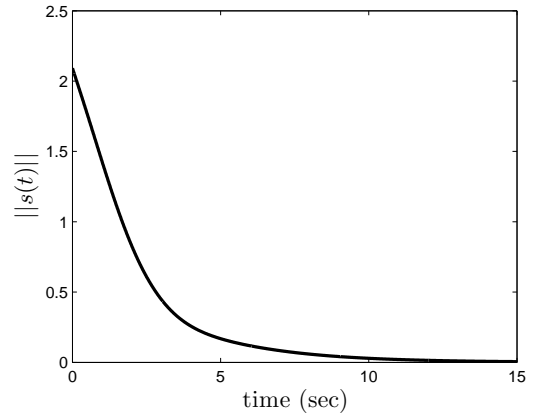
Furthermore,  $90^\circ$  error angle is used along  $[1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3}]^T$  as the initial condition,

$$q(0) = \begin{bmatrix} \frac{1}{\sqrt{3}} \sin(\pi/4) \\ \frac{1}{\sqrt{3}} \sin(\pi/4) \\ \frac{1}{\sqrt{3}} \sin(\pi/4) \\ \cos(\pi/4) \end{bmatrix} \otimes q_d(0) \quad (44)$$

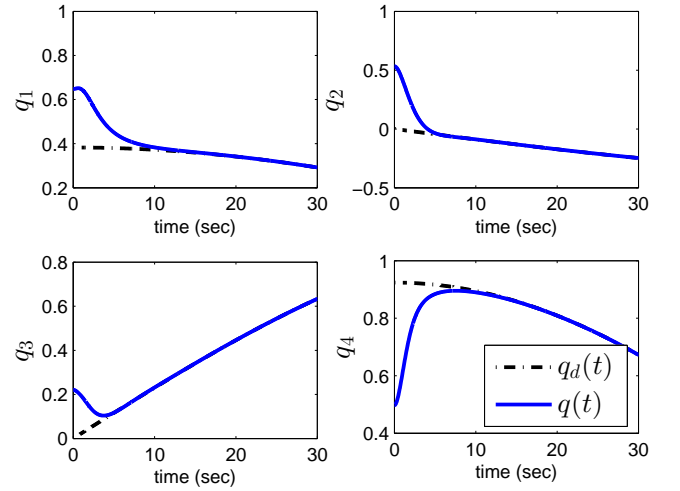
and the spacecraft is at rest initially  $\omega(0) = [0, 0, 0]^T$ . The controller parameters are set to be,  $r = 3, K = 10 \cdot I_{3 \times 3}, Q = I_{6 \times 6}$ .

#### 4.1 Nominal performance

Without external disturbance, we have shown that the state will be driven to the sliding manifold using Barbalat's lemma,



**FIGURE 2.** Plot of the norm of switching function  $s(t)$  which converges to 0



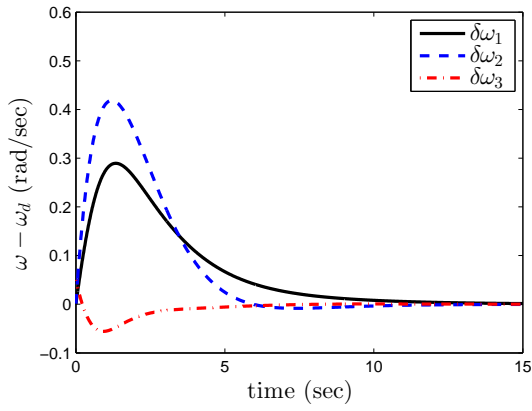
**FIGURE 3.** Plot of  $q(t)$  and  $q_d(t)$  showing asymptotic quaternion tracking

and consequently the control objectives are achieved. The convergence of the switching function is shown in Figure 2. The asymptotic quaternion and velocity tracking performance are plotted in Figures 3 and 4 respectively. The controlled thruster torque is shown in Figure 5.

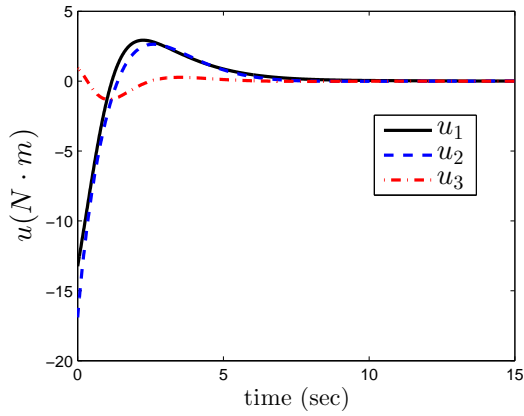
#### 4.2 Robust performance

If the spacecraft is subject to disturbances, the robust sliding mode controller in Eqn. (38) should be used. In our simulation, the following disturbance is used,

$$d(t) = [\sin(t), -1, \cos(t)]^T \quad (45)$$



**FIGURE 4.** Plot of the error velocity  $\delta\omega = \omega - \omega_d$  showing asymptotic velocity tracking



**FIGURE 5.** Plot of the controlled thruster torque

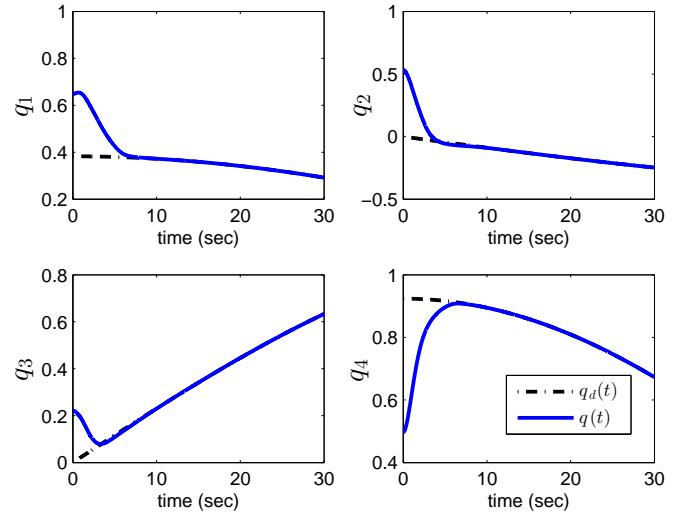
Therefore, in the controller design  $D_i = 1, i = \{1, 2, 3\}$ .  $\eta$  is chosen to be  $0_{3 \times 1}$  for simplicity. The robust quaternion and velocity tracking are shown in Figures 6 and 7. The robust control input is shown in Figure 8.

## 5 CONCLUSION

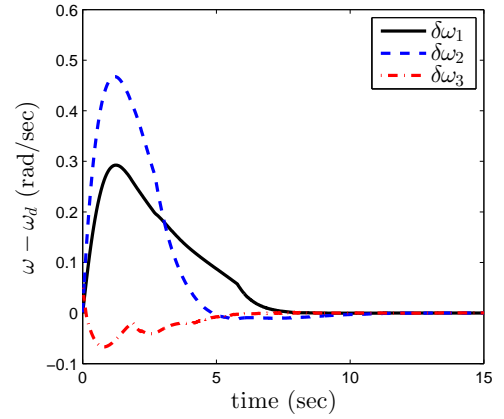
In this paper, an adaptive sliding mode attitude controller is designed for asymptotic quaternion and velocity tracking, which assumes no inertial information and can reject unknown external disturbances. The stability was shown through a Lyapunov analysis. Both the nominal performance and the robust performance are demonstrated in numerical simulations.

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**FIGURE 6.** Plot of  $q(t)$  and  $q_d(t)$  with the presence of disturbances

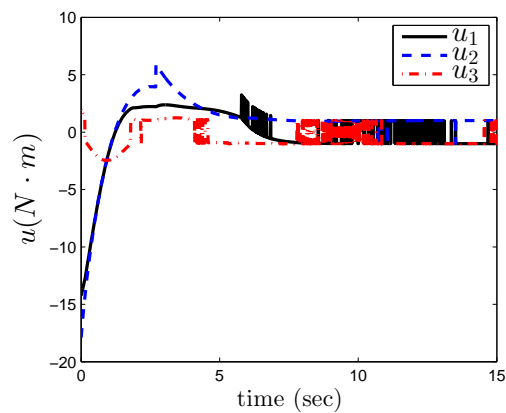


**FIGURE 7.** Plot of the error velocity  $\delta\omega$  with the presence of disturbances

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**FIGURE 8.** Plot of the robust control input

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