Throughout the course, we will use **EK** to refer to the course textbook "Erwin Kreyszig: *Advanced Engineering Mathematics*, 10th edition."

Problems to be turned in:

1. [Functions of a matrix] Suppose $A \in \mathbb{C}^{n\times n}$ has all its eigenvalues in the right half complex plane. Show that $I + A$ is invertible.
2. A square matrix $P \in \mathbb{C}^{n\times n}$ is called a projection matrix if $P = P^*$ and $P^2 = P$.
   (a) Find the eigenvalues of P
   (b) Show that $I \succeq P \succeq 0$
3. Find the square root of the matrix
$$A = \begin{bmatrix} 4 & 3 \\ 3 & 4 \end{bmatrix}$$
4. If $P \succ 0$, show that P^{-1} exists and is positive definite.
5. Compute the singular values of the following matrices
$$(a) \begin{bmatrix} 3 & \\ & -2 \end{bmatrix}, \ (b) \begin{bmatrix} 2 & \\ & 3 \end{bmatrix}, \ (c) \begin{bmatrix} 0 & 2 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$
6. Suppose that the maximum singular value of A is less than 1. Show that
   (a) $I - A^*A$ is invertible
   (b) $I - A^*A \succ 0$
7. Let $A \in \mathbb{C}^{n\times n}$ have eigenvalues $\{\lambda_1, \ldots, \lambda_n\}$
   (a) If $A = A^*$, show that the singular values of A are $\{|\lambda_1|, \ldots, |\lambda_n|\}$
   (b) If A is a unitary matrix, show that the singular values of A are all equal to 1
   (c) If $A = vw^*$ where $v, w \in \mathbb{C}^n$, find the singular values of A
8. True or false (provide your reasoning)
   (a) Suppse all the eigenvalues of a matrix A are zero. Is $A = 0$?
   (b) Suppose all the singular values of a matrix A are zero. Is $A = 0$?
9. **EK** P482: 9
10. **EK** P482: 13