# An Indirect Adaptive Approach to Reject Multiple Narrow-Band Disturbances in Hard Disk Drives ⋆

**Xu Chen** * **Masayoshi Tomizuka** *

* *Department of Mechanical Engineering, University of California, Berkeley, CA, 94720, USA (e-mails: {maxchen,tomizuka}@me.berkeley.edu).*

**Abstract:** This paper presents an indirect adaptive control scheme that rejects unknown multiple narrow-band disturbances in hard disk drive systems. The proposed algorithm first finds the model of the disturbance (the internal model) and then adaptively estimates its parameters. The design of a band-pass filter with multiple narrow pass-bands is then presented and used to construct a disturbance observer (DOB) for disturbance rejection. The proposed algorithm estimates the minimal amount of parameters, and is computationally simple. Evaluation of the proposed algorithm is performed on a benchmark problem for HDD track following.

*Keywords:* Adaptive control, disturbance rejection, multiple narrow-band disturbances, HDD

## 1. INTRODUCTION

In track following control of hard disk drives (HDDs), both the repeatable runout (RRO) and the non-repeatable runout (NRRO) contribute to Track Mis-Registration (TMR). RRO is synchronous with the HDD spindle rotation, and can be compensated by customized control algorithms such as adaptive feed-forward cancellation or repetitive control (Sacks et al. (1995)). NRRO, however, differs from track to track, and can appear at frequencies higher than the servo bandwidth (Ehrlich and Curran (1999)). Among the various components in NRRO, disk motion, such as disk fluttering due to turbulent air flow in the hard disk assembly, is the major contributor, and arises as multiple narrow-band disturbances [1] (Guo and Chen (2000); Ehrlich and Curran (1999); McAllister (1996)). With the rapid growth in HDD's storage density, the adverse influence of disk motion on the servo performance is becoming more and more an important issue. Rejection of multiple narrow-band disturbances is thus the key to achieve low TMR in track following.

Investigations of this important problem have been popular in the field of control theory. The existing solutions have mainly been rooted in rejecting disturbance with one narrow-band component. For example, Zheng and Tomizuka (2007, 2008) suggested direct and indirect adaptive disturbance observer (DOB) schemes to estimate and cancel the disturbance; Kim et al. (2005) proposed a parallel add-on peak filter to shape the open loop frequency response; Landau et al. (2005) achieved adaptive narrow-band disturbance rejection on an active suspension, based on Youla parametrization. Yet, the problem of *multiple* narrow-band disturbance rejection was seldom examined before. Landau et al. (2005)'s algorithm can be extended to reject $n$ narrow bands, but requires the estimation of $2n$ parameters.

This paper focuses on developing an adaptive control algorithm that rejects arbitrary number of unknown narrow-band disturbances in NRRO. The model of the disturbance, i.e., its internal model, is firstly derived. A new adaptive frequency identification method is then proposed to estimate the parameters of this model, which are then applied to construct a band-pass Q-filter with multiple narrow pass-bands. Finally, expanding the DOB structure in Zheng and Tomizuka (2008) to multiple narrow-band disturbance rejection, we form a disturbance observer with the newly designed Q-filter. Advantages of the proposed compensation scheme are: (1) it estimates the minimal number of parameters, which is equal to $n$, the number of narrow-band components; (2) it is stable over a wide range of frequencies, disturbances outside the servo bandwidth can also be compensated; (3) it has fast convergence rate, and is easy to implement.

The remainder of this paper is organized as follows. Section 2 formally defines the problem and introduces the proposed solution. Section 3 presents the proposed adaptive frequency identification scheme. The design of DOB with a multiple narrow band-pass Q-filter is shown in Section 4. An example of rejecting two narrow-band disturbances is provided in Section 5. Section 6 concludes the paper.

## 2. THE PROBLEM AND THE PROPOSED SOLUTION

Figure 1 shows the proposed block diagram for HDD track following. It reduces to the baseline feedback control loop if we remove the add-on compensator inside the dash-dotted box. Throughout the paper we use the well formulated open-source HDD benchmark simulation package (Hirata

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[1] Disturbances whose energy is concentrated at several frequencies.

(2007)) as a demonstration tool. The full-order plant model $G_p\left(z^{-1}\right)$ contains the dynamics of the HDD servo system including the power amplifier, the voice-coil motor, and the actuator mechanics. The dashed line in Fig. 2 shows the frequency response of $G_p\left(z^{-1}\right)$, which is a fourteenth-order transfer function with several high frequency resonances. The baseline feedback controller $C_{FB}\left(z^{-1}\right)$ is a third order PID controller cascaded with three notch filters. The baseline open loop system has a gain margin of 5.45 dB, a phase margin of 38.2 deg, and an open loop servo bandwidth of 1.19 kHz.

The reference $r$ is zero in track following control. The signals $d\left(k\right)$, $u\left(k\right)$, $n(k)$, and $PES$, are respectively the input disturbance, the control input, the output disturbance, and the position error signal. It is assumed that the multiple narrow-band disturbance of interest is contained in $d\left(k\right)$, and lies between 300 Hz and 2000 Hz (Guo and Chen (2000); Ehrlich and Curran (1999)).

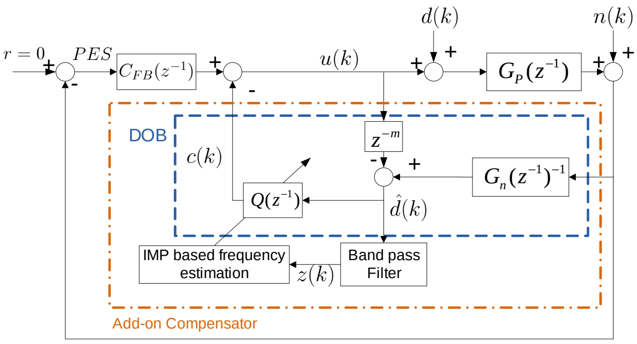


Fig. 1. Structure of the proposed control scheme

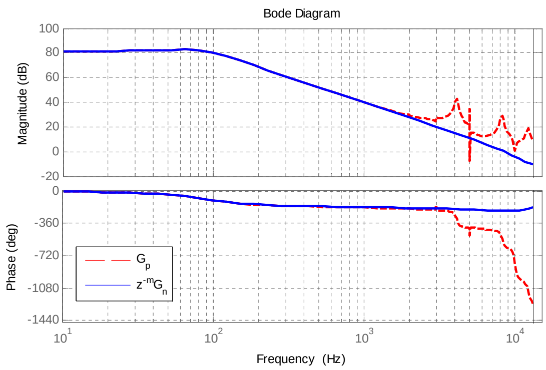


Fig. 2. Frequency response of $G_p\left(z^{-1}\right)$ and $z^{-m}G_n\left(z^{-1}\right)$

Figure 3 shows the spectrum of the position error signal on one track when the baseline controller is applied. It is observed that several sharp spikes are present due to the multiple narrow-band disturbances, which we aim to reject. The proposed solution is to add a compensator as shown in the dash-dotted box in Fig. 1. Within the compensator, the low-order nominal plant model $z^{-m}G_n\left(z^{-1}\right)$ matches the low-frequency dynamics of $G_p\left(z^{-1}\right)$ in the frequency response, as shown in Fig. 2. A stable inverse model $G_n\left(z^{-1}\right)^{-1}$ is needed in the design of our proposed compensator. If $G_n\left(z^{-1}\right)$ has minimum phase, its inverse can directly be assigned, if not, stable inversion techniques such as the ZPET method (Tomizuka (1987)) should be applied.

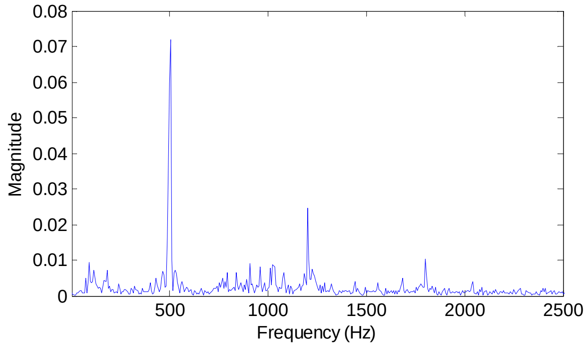


Fig. 3. PES spectrum with baseline controller

The compensation signal $c\left(k\right)$ is designed, by constructing the DOB, to approximate and cancel the multiple narrow-band disturbances. To see this point, notice first that the signal $\hat{d}\left(k\right)$ is expressed by, in the operator notation,

$$\hat{d}\left(k\right)=G_n\left(z^{-1}\right)^{-1}\left[G_p\left(z^{-1}\right)\left(u\left(k\right)+d\left(k\right)\right)+n\left(k\right)\right]-z^{-m}u\left(k\right). \quad (1)$$

Since below 2000 Hz, $G_p\left(z^{-1}\right)\approx z^{-m}G_n\left(z^{-1}\right)$, i.e., $G_n\left(z^{-1}\right)^{-1}G_p\left(z^{-1}\right)\approx z^{-m}$, Eq. (1) becomes

$$\hat{d}\left(k\right)\approx z^{-m}d\left(k\right)+G_n\left(z^{-1}\right)^{-1}n\left(k\right). \quad (2)$$

If in addition the output disturbance $n\left(k\right)$ is small, then the above equation is further simplified to

$$\hat{d}\left(k\right)\approx z^{-m}d\left(k\right)=d\left(k-m\right), \quad (3)$$

which implies that $\hat{d}\left(k\right)$ is a good estimate of the disturbance $d\left(k\right)$. Therefore, the multiple narrow-band disturbance is contained in $\hat{d}\left(k\right)$.

In reality, the influence of $n\left(k\right)$ can not be ignored. A band-pass filter $BP\left(z^{-1}\right)$ is constructed to filter out the signals in $\hat{d}\left(k\right)$ that are not of our interest. This is practical since the frequency region of the narrow-band disturbances is usually roughly known. The filtered signal $z\left(k\right)$ is finally a multiple narrow-band signal [2] with small noise-to-signal ratio, and can be applied for the parameter estimation scheme to be presented in Section 3.

With the estimated knowledge of the multiple narrow-band disturbance, a multiple band-pass filter $Q\left(z^{-1}\right)$ can then be constructed. The compensation signal $c\left(k\right)$ formed by filtering $\hat{d}\left(k\right)$ through $Q\left(z^{-1}\right)$, therefore contains only the multiple narrow-band disturbance. Adding the negative of $c\left(k\right)$ to the control input, we achieve the compensation.

## 3. ADAPTIVE DISTURBANCE IDENTIFICATION

### 3.1 *The Internal Model and the Adaptation Algorithm*

The multiple narrow-band disturbance in NRRO can be modeled as the sum of several sinusoidal signals (Ehrlich and Curran (1999); Guo and Chen (2000)). It is well known that any sinusoidal signal $x\left(k\right)$ satisfies

[2] More precisely, $z\left(k\right)$ and the multiple narrow-band disturbance in $\hat{d}\left(k\right)$ have the same amplitude but different phases.

$\left(1-2\cos(\omega)z^{-1}+z^{-2}\right)x(k)=0$, where $\omega=2\pi\Omega T_s$ is the frequency of $x(k)$ in radians [3]. The equality can either be verified by direct expansion or by noting that the zeros of the FIR filter $1-2\cos(\omega)z^{-1}+z^{-2}$ lie exactly at $e^{\pm j\omega}$ on the unit circle. The term $1/\left(1-2\cos(\omega)z^{-1}+z^{-2}\right)$ is named as the internal model of $x(k)$.

Extending the idea in the last paragraph, we can now develop the internal model of multiple narrow-band disturbances. Assume that the signal $z(k)$ contains $n$ narrow-band components. $z(k+1)$ will then satisfy

$$\prod_{i=1}^{n}\left(1-2\cos(\omega_i)z^{-1}+z^{-2}\right)z(k+1)=0, \tag{4}$$

where $\omega_i$ $(i=1,\ldots,n)$ is the frequency of the $i^{th}$ narrow-band component in $z(k)$.

The polynomial on the left hand side of Eq. (4) is

$$\begin{aligned}A\left(z^{-1}\right)=&\prod_{i=1}^{n}\left(1-2\cos(\omega_i)z^{-1}+z^{-2}\right)\\=&1+a_1z^{-1}+\cdots+a_nz^{-n}+\\&\cdots+a_1z^{-2n+1}+z^{-2n}\\=&1+\sum_{i=1}^{n-1}a_i\left(z^{-i}+z^{-2n+i}\right)+a_nz^{-n}+z^{-2n}.\end{aligned} \tag{5}$$

The values of $\omega_i's$ are unknown, $a_i's$ are thus unknown, and need to be estimated for constructing $A\left(z^{-1}\right)$. Choosing to directly estimate $a_i's$ makes the adaptation simple in computation, since $A\left(z^{-1}\right)$ is linear in $a_i's$. Notice that the coefficients of $A\left(z^{-1}\right)$ have a mirror symmetric form. Therefore only $n$ parameters need to be identified, which is the minimal possible number for $n$ narrow-band signals.

To construct an adaptive estimation scheme, we substitute and expand Eq. (5) to Eq. (4), then move the terms containing $z(k)$, $z(k-1)$, $\ldots$, $z(k+1-2n)$ from the left side to the right side, to get the adaptation model:

$$\begin{aligned}z(k+1)=&-\sum_{i=1}^{n-1}a_i\left[z(k+1-i)+z(k+1-2n+i)\right]\\&-a_nz(k+1-n)-z(k+1-2n).\end{aligned} \tag{6}$$

Introduce the parameter vector to be estimated:

$$\theta=[a_1,a_2,\ldots,a_n]^T. \tag{7}$$

Introduce also the regressor vector at time $k$:

$$\phi(k)=[\phi_1(k),\phi_2(k),\ldots,\phi_n(k)]^T, \tag{8}$$

where

$$\phi_j(k)=-z(k+1-j)-z(k+1-2n+j) \quad j=1,...,n-1 \tag{9}$$

$$\phi_n(k)=-z(k+1-n) \tag{10}$$

Eq. (6) can then be simply represented by

$$z(k+1)=\phi(k)^T\theta-z(k+1-2n). \tag{11}$$

We can now define the *a priori* prediction of $z(k+1)$:

$$\hat{z}^o(k+1)=\phi(k)^T\hat{\theta}(k)-z(k+1-2n), \tag{12}$$

where $\hat{\theta}(k)$ is the predicted parameter vector at time $k$.

[3] $\Omega$ is the frequency in Hz, $T_s$ is the sampling time in seconds.

The *a priori* prediction error is then given by

$$e^o(k+1)=z(k+1)-\hat{z}^o(k+1)=-\phi(k)^T\tilde{\theta}(k), \tag{13}$$

where $\tilde{\theta}(k)=\hat{\theta}(k)-\theta$ is the parameter estimation error.

Correspondingly, we define the following *a posteriori* signals for later use in the stability analysis:

the *a posteriori* prediction of $z(k+1)$:

$$\hat{z}(k+1)=\phi(k)^T\hat{\theta}(k+1)-z(k+1-2n). \tag{14}$$

the *a posteriori* prediction error:

$$e(k+1)=-\phi(k)^T\tilde{\theta}(k+1). \tag{15}$$

With the above information, the following recursive least squares (RLS) parameter adaptation algorithm (PAA) can be constructed (Landau et al. (1998)).

$$\hat{\theta}(k+1)=\hat{\theta}(k)+\frac{F(k)\phi(k)e^o(k+1)}{1+\phi(k)^TF(k)\phi(k)} \tag{16}$$

$$e^o(k+1)=z(k+1)-\hat{z}^o(k+1) \tag{17}$$

$$\hat{z}^o(k+1)=\phi(k)^T\hat{\theta}(k)-z(k+1-2n) \tag{18}$$

$$F(k+1)=\frac{1}{\lambda(k)}\left[F(k)-\frac{F(k)\phi(k)\phi(k)^TF(k)}{\lambda(k)+\phi(k)^TF(k)\phi(k)}\right] \tag{19}$$

To improve the convergence rate, the forgetting factor $\lambda(k)$ is designed to increase from 0.95 to 1 (Ljung (1999)), obeying the rule $\lambda(k)=1-0.05\times0.995^k$.

As an example of the adaptation algorithm, when $n=2$,

$$\begin{aligned}A\left(z^{-1}\right)=&\left(1-2\cos(\omega_1)z^{-1}+z^{-2}\right)\times\\&\left(1-2\cos(\omega_2)z^{-1}+z^{-2}\right).\end{aligned} \tag{20}$$

Expanding Eq. (20) and introducing $a_1=-2\cos(\omega_1)-2\cos(\omega_2)$; $a_2=2+2\cos(\omega_1)\cdot2\cos(\omega_2)$, we obtain

$$A\left(z^{-1}\right)=1+a_1z^{-1}+a_2z^{-2}+a_1z^{-3}+z^{-4}. \tag{21}$$

The unknown parameter vector is thus $\theta=[a_1,a_2]^T$, and

$$z(k+1)=\phi(k)^T\theta-z(k-3) \tag{22}$$

$$\hat{z}^o(k+1)=\phi(k)^T\hat{\theta}(k)-z(k-3) \tag{23}$$

$$e^o(k+1)=z(k+1)-\hat{z}^o(k+1) \tag{24}$$

where

$$\phi(k)=\begin{bmatrix}-z(k)-z(k-2)\\-z(k-1)\end{bmatrix}. \tag{25}$$

$\theta$ can then be estimated according to Eqs. (16-19).

### *3.2 Stability and Convergence*

For stability analysis, we first transform the PAA to the *a posteriori* form. Pre-multipling $\phi^T(k)$ to Eq. (16) yields

$$\begin{aligned}\phi^T(k)\hat{\theta}(k+1)=&\phi^T(k)\hat{\theta}(k)\\&+\frac{\phi^T(k)F(k)\phi(k)}{1+\phi^T(k)F(k)\phi(k)}e^o(k+1).\end{aligned} \tag{26}$$

Subtracting $\phi^T(k)\theta$ from each side in Eq (26), and substituting in Eqs. (15) and (17), we have

$$e(k+1)=\frac{e^o(k+1)}{1+\phi^T(k)F(k)\phi(k)}. \tag{27}$$

Substituting Eq. (27) back to Eq. (16), we arrive at the PAA in the *a posteriori* form:

$$\hat{\theta}(k+1) = \hat{\theta}(k) + F(k)\,\phi(k)\,e(k+1) \tag{28}$$

$$e(k+1) = -\phi(k)^T\,\tilde{\theta}(k+1) \tag{29}$$

Subtracting $\theta$ from each side in Eq. (28) yields

$$\tilde{\theta}(k+1) = \tilde{\theta}(k) + F(k)\,\phi(k)\,e(k+1)\,. \tag{30}$$

Combining Eqs. (27) and (30), we can construct the equivalent feedback loop for the adaptive system as shown in Fig. 4.

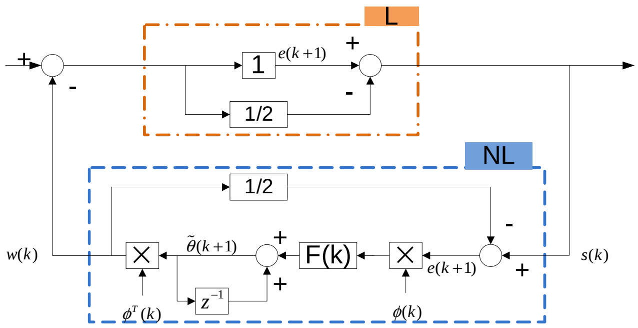


Fig. 4. Equivalent feedback loop of the adaptive system

The nonlinear block $NL$ in Fig. 4 is shown to be passive and satisfies the Popov Inequality (section 3.3.4 of Landau et al. (1998)). The linear block $L = 1 - 1/2$ is strictly positive real. Therefore, the parameter adaptation algorithm is asymptotically hyperstable. Applying further theorem 3.3.2 from Landau et al. (1998), we have

$$\lim_{k\to 0} e(k) = 0. \tag{31}$$

Substituting Eq. (29) to the above gives

$$\begin{aligned}
&\phi(k-1)^T\,\tilde{\theta}(k)\\
&= \sum_{i=1}^{n-1} \left(z(k-i) + z(k-2n+i)\right)\tilde{a}_i(k) + z(k-n)\,\tilde{a}_n(k)\\
&= \left(\sum_{i=1}^{n-1}\left(z^{-i} + z^{-2n+i}\right)\tilde{a}_i(k) + z^{-n}\tilde{a}_n(k)\right) z(k)\\
&\to 0 \qquad \text{as } k\to\infty.
\end{aligned} \tag{32}$$

Based on the assumption that $z(k)$ has $n$ independent frequency components, the Frequency Richness Condition for Parameter Convergence holds. Therefore, the only solution to the above equation is $\lim_{k\to\infty}\tilde{a}_i(k) = 0$, i.e., the parameters converge to their true values.

## 4. MULTIPLE BAND-PASS Q-FILTER DESIGN

With the estimated parameters $a_i's$, we are ready to design the Q-filter and turn on the adaptive DOB for the disturbance compensation. The Q-filter used in single narrow-band disturbance rejection (Zheng and Tomizuka (2008)) is given by

$$Q\left(z^{-1}\right) = \frac{(1-\alpha)\left(1-\alpha z^{-2}\right)}{1-\alpha\cdot 2\cos(\omega)\,z^{-1}+\alpha^2 z^{-2}}, \tag{33}$$

where the shaping coefficient $\alpha$ is a real number close to but smaller than 1. The above Q-filter has two poles close to $e^{\pm j\omega}$ but slightly shifted towards the origin. The magnitude response of $Q\left(z^{-1}\right)$ has a narrow pass-band centered at $\omega$. The closer $\alpha$ is to 1, the narrower the pass-band of $Q\left(z^{-1}\right)$.

For multiple narrow-band disturbance rejection, we extend Eq. (33) to

$$Q\left(z^{-1}\right) = \sum_{i=1}^{n} \frac{(1-\alpha_i)\left(1-\alpha_i z^{-2}\right)}{1-\alpha_i\cdot 2\cos(\omega_i)\,z^{-1}+\alpha_i^2 z^{-2}}. \tag{34}$$

For simplicity, we let $\alpha_i = \alpha = 0.998$. Recall the definition of $A(z^{-1})$:

$$\begin{aligned}
A\left(z^{-1}\right) &= \prod_{i=1}^{n}\left(1-2\cos(\omega_i)\,z^{-1}+z^{-2}\right) = 1 + a_1 z^{-1} + \\
&\quad \cdots + a_n z^{-n} + \cdots + a_1 z^{-2n+1} + z^{-2n}.
\end{aligned} \tag{35}$$

Eq. (34) can then be expressed as

$$\begin{aligned}
Q\left(z^{-1}\right) &= \frac{(1-\alpha)\left(1-\alpha z^{-2}\right)B_Q\left(z^{-1}\right)}{\prod_{i=1}^{n}\left(1-\alpha\cdot 2\cos(\omega_i)\,z^{-1}+\alpha^2 z^{-2}\right)}\\
&= \frac{(1-\alpha)\left(1-\alpha z^{-2}\right)B_Q\left(z^{-1}\right)}{A\left(\alpha z^{-1}\right)}.
\end{aligned} \tag{36}$$

where $A\left(\alpha z^{-1}\right)$ is obtained by replacing every $z^{-1}$ by $\alpha z^{-1}$ in Eq. (35), and $B_Q\left(z^{-1}\right)$ is a polynomial of $z^{-1}$.

### *4.1 The Case of two narrow-band disturbances*

When $n = 2$, direct expansion in Eq (34) gives

$$Q\left(z^{-1}\right) = \frac{(1-\alpha)\left(1-\alpha z^{-2}\right)\left(2+\alpha a_1 z^{-1}+2\alpha^2 z^{-2}\right)}{1+\alpha a_1 z^{-1}+\alpha^2 a_2 z^{-2}+\alpha^3 a_1 z^{-3}+\alpha^4 z^{-4}}, \tag{37}$$

where $a_1 = -2\cos(\omega_1) - 2\cos(\omega_2)$ and $a_2 = 2 + 2\cos(\omega_1)\times 2\cos(\omega_2)$.

Notice that $\alpha$, $a_1$ and $a_2$ completely determine $Q\left(z^{-1}\right)$. With the estimated $\hat{a}_1$ and $\hat{a}_2$ in section 3, the Q-filter can then be constructed according to Eq. (37), which has a frequency response as shown in Fig. 5. Notice that at the central frequencies, the magnitude and the phase of $Q\left(z^{-1}\right)$ are 1 (0 dB) and 0 deg, respectively. Therefore, passing a broad band disturbance $\hat{d}(k)$ through $Q\left(z^{-1}\right)$, one gets the exact multiple narrow-band signals at 500 Hz and 1200 Hz.

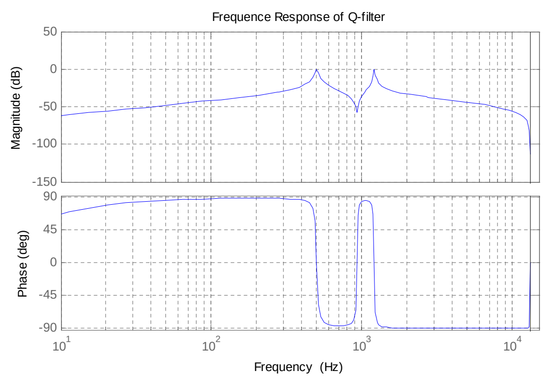


Fig. 5. Frequency response of the proposed Q-filter (central frequencies: 500 Hz and 1200 Hz)

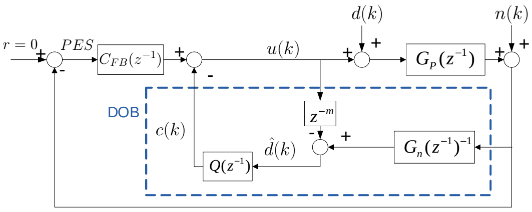


Fig. 6. Block diagram of the closed loop system with the proposed multiple narrow-band DOB

The error rejection function $S\left(z^{-1}\right)$ (a.k.a. the sensitivity function), is the transfer function from the output disturbance $n\left(k\right)$ to the position error signal $PES$ in Fig. 6. When the DOB is turned on, $S\left(z^{-1}\right)$ can be derived as

$$S\left(z^{-1}\right)=\frac{1}{1+C_{eq}\left(z^{-1}\right)G_p\left(z^{-1}\right)}, \tag{38}$$

where

$$C_{eq}\left(z^{-1}\right)=\frac{G_{FB}\left(z^{-1}\right)+Q\left(z^{-1}\right)G_n\left(z^{-1}\right)^{-1}}{1-z^{-m}Q\left(z^{-1}\right)} \tag{39}$$

is the equivalent feedback controller.

Figure 7 shows the frequency response of the sensitivity function for the closed loop system with the proposed DOB. With the add-on compensation scheme, PES at 500 Hz and 1200 Hz gets greatly attenuated due to the deep notches in the magnitude response at the corresponding frequencies, while the influence on the sensitivity at other frequencies is neglectable.

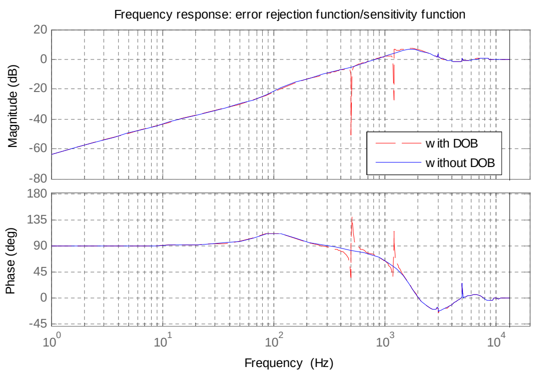


Fig. 7. Frequency response of the sensitivity function

Stability of DOB (see Kempf and Kobayashi (1999)) requires the nominal model $z^{-m}G_n\left(z^{-1}\right)$ to have no zeros outside the unit circle and that

$$\left|Q\left(e^{j\omega}\right)\right|<\frac{1}{\left|\Delta\left(e^{j\omega}\right)\right|}\quad\forall\omega, \tag{40}$$

where $\Delta(z^{-1})=\left[G_p(z^{-1})-z^{-m}G_n(z^{-1})\right]/z^{-m}G_n(z^{-1})$ represents the multiplicative model mismatch. Plotting the magnitude responses of $1/\Delta\left(z^{-1}\right)$ and $Q\left(z^{-1}\right)$ in Fig. 8, we see that the multiple narrow-band DOB is stable as long as the narrow-band disturbance arises below 3000 Hz.

### *4.2 The Case of $n$ narrow-band disturbances*

For the general case of $n$ narrow-band disturbances

$$Q\left(z^{-1}\right)=\left(1-\alpha\right)\left(1-\alpha z^{-2}\right)\frac{B_Q\left(z^{-1}\right)}{A\left(\alpha z^{-1}\right)} \tag{41}$$

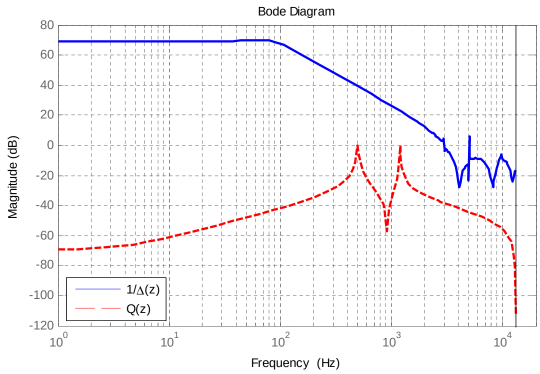


Fig. 8. Magnitude responses of $1/\Delta\left(z^{-1}\right)$ and $Q\left(z^{-1}\right)$

where $A\left(\alpha z^{-1}\right)=1+a_1\alpha z^{-1}+\cdots+a_n\alpha^n z^{-n}+\cdots+a_1\alpha^{2n-1}z^{-2n+1}+\alpha^{2n}z^{-2n}$.

Derivation of the $B_Q\left(z^{-1}\right)$ is best done by using a Computer Algebra System such as *Maple* or *Mathematica*. We have, for $n=3$,

$$\begin{aligned}B_Q\left(z^{-1}\right)=3&+2\alpha a_1 z^{-1}+\alpha^2\left(a_2+3\right)z^{-2}\\&+2\alpha^3 a_1 z^{-3}+3\alpha^4 z^{-4}.\end{aligned} \tag{42}$$

For $n=4$,

$$\begin{aligned}B_Q\left(z^{-1}\right)=4&+3\alpha a_1 z^{-1}+\alpha^2\left(2a_2+4\right)z^{-2}\\&+\alpha^3\left(a_3+3a_1\right)z^{-3}+\alpha^4\left(2a_2+4\right)z^{-4}\\&+3\alpha^5 a_1 z^{-5}+4\alpha^6 z^{-6}.\end{aligned} \tag{43}$$

By induction, we can get the general form of $B_Q\left(z^{-1}\right)$:

$$\begin{aligned}B_Q\left(z^{-1}\right)=&\sum_{i=0}^{n-2}b_i\left(\alpha^i z^{-i}+\alpha^{2(n-1)-i}z^{-2(n-1)+i}\right)\\&+b_{n-1}\alpha^{n-1}z^{-n+1},\end{aligned} \tag{44}$$

where $b_0=n$; $b_1=\left(n-1\right)a_1$; $b_i=\left(n-i\right)a_i+b_{i-2}$; $i=2,\ldots n-1$.

## 5. SIMULATION RESULT

The proposed adaptive compensator for multiple narrow-band disturbance rejection is implemented in the HDD benchmark simulation package (Hirata (2007)). The baseline control system is as shown in Section 2. The disturbances include the torque disturbance, the disk flutter disturbance, the RRO, and the measurement noise. The system has a sampling time of $3.788\times10^{-5}$ sec. Two narrow-band disturbances at 500 Hz and 1200 Hz were injected at the input to the plant.

In the simulated track following, the first five revolutions were run without compensation. It is seen in Fig. 9 that the peak values of PES exceeded the standard PES upper-bound of 15% Track Pitch (TP). The dotted line in Fig. 10 presents the spectrum of the PES without compensation. We can see that the PES had strong energy components at 500 Hz and 1200 Hz. Without compensation, the Track Mis-Registration (TMR), defined as 3 times the standard deviation of the PES, was 21.87% TP.

The proposed algorithm was applied to improve the HDD track following performance. The multiple band-pass filter

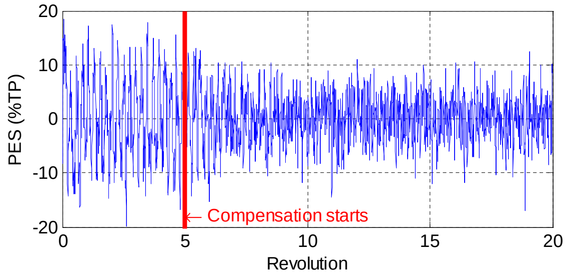


Fig. 9. PES time trace

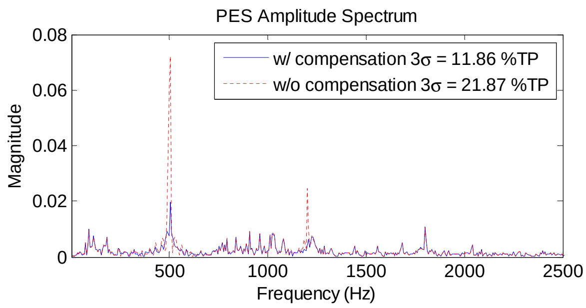


Fig. 10. PES spectrum with and without the proposed compensator

$BP\left(z^{-1}\right)$ was designed using the *Signal Processing Toolbox* in *MATLAB*. $BP\left(z^{-1}\right)$ has a magnitude response as shown in Fig. 11. The estimation of the parameters was turned on at the beginning of the simulation. The initial guess of the parameter vector was set to half of its true value. Figure 12 shows the estimated parameters $\hat{a}_1$ and $\hat{a}_2$ converged to their true values within half a revolution, i.e., 0.00415 sec.

With the estimated parameters $\hat{a}_1$ and $\hat{a}_2$, the Q-filter was constructed and turned on at the fifth revolution. Figure 9 shows the resulting PES time trace. It is seen that the PES was reduced now to less than 10% TP. In Fig. 10, we observe that the strong energy concentrations at 500 Hz and 1200 Hz were greatly attenuated, while the spectrum of the PES at other frequencies was almost identical to that without compensation. The TMR was reduced to 11.86% TP, implying a 45.8% improvement.

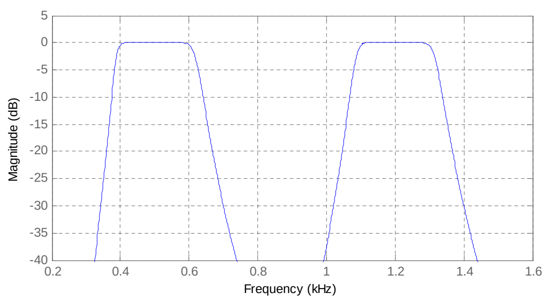


Fig. 11. Magnitude response of the multiple bandpass filter

## 6. CONCLUSION

In this paper, an indirect adaptive control scheme was proposed to reject multiple narrow-band disturbances in HDD track following. Simulation on a realistic open-source HDD

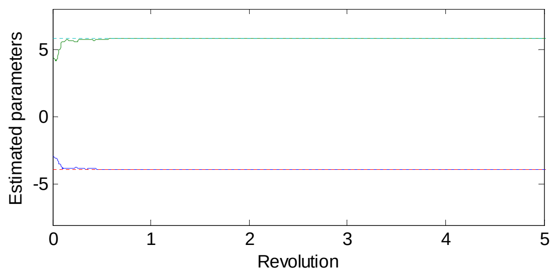


Fig. 12. Online parameter estimation of the internal model for two narrow-band signals

benchmark problem showed that the proposed algorithm significantly reduced PES and TMR. The proposd method is suitable for compensating disturbances within narrow frequency regions.

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