Lecture 9: LQG/Loop Transfer Recovery (LTR)

Big picture
Loop transfer recovery
Target feedback loop
Fictitious KF

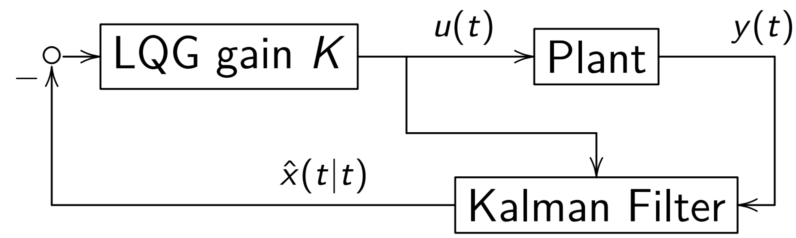
Big picture

Where are we now?

- LQ: optimal control, guaranteed robust stability under basic assumptions in stationary case
- KF: optimal state estimation, good properties from the duality between LQ and KF
- LQG: LQ+KF with separation theorem
- frequency-domain feedback design principles and implementations

Stability robustness of LQG was discussed in one of the homework problems: the nice robust stability in LQ (good gain and phase margins) is lost in LQG.
LQG/LTR is one combined scheme that uses many of the concepts learned so far.

Continuous-time stationary LQG solution

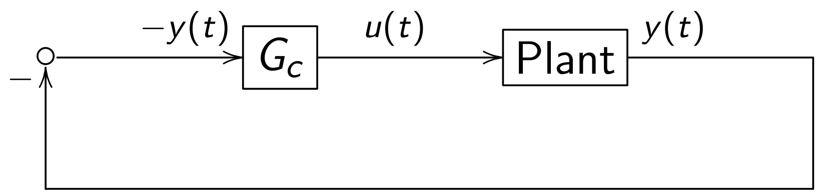


$$u(t) = -K\hat{x}(t|t)$$

$$\frac{d\hat{x}(t|t)}{dt} = A\hat{x}(t|t) + Bu(t) + F\left(y(t) - C\hat{x}(t|t)\right)$$

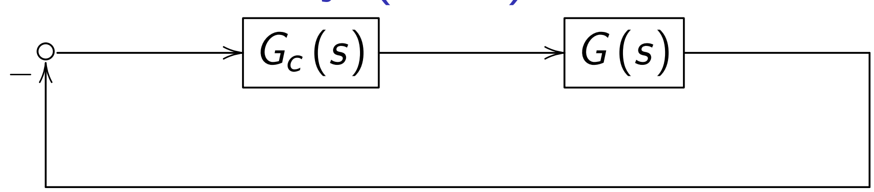
$$= (A - BK - FC)\hat{x}(t|t) + Fy(t)$$

$\iff$



$$G_c(s) = K(sI - A + BK + FC)^{-1}F \tag{1}$$

Loop transfer recovery (LTR)



Theorem (Loop Transfer Recovery (LTR))

If a $m \times m$ dimensional $G(s)$ has only minimum phase transmission zeros, then the open-loop transfer function

$$G(s)\,G_c(s) = \left[C(sI - A)^{-1}B\right]\left[K(sI - A + BK + FC)^{-1}F\right]$$

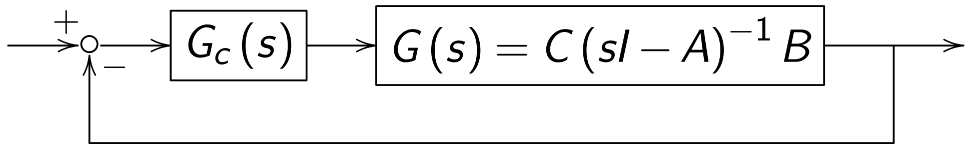
$$\xrightarrow{\rho \to 0} C(sI - A)^{-1}F \tag{2}$$

K and ρ are from the LQ [(A, B) controllable, (A, C) observable]

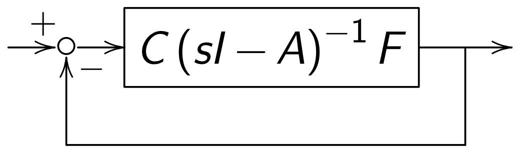
$$J = \int_0^\infty \left(x^T(t)C^TCx(t) + \rho\, u^T(t)Nu(t)\right) dt \tag{3}$$

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{4}$$

Loop transfer recovery (LTR)



converges, as $\rho \to 0$, to the *target feedback loop*



key concepts:

- regard LQG as an output feedback controller
- will design F such that $C(sI - A)^{-1}F$ has a good loop shape
- not a conventional optimal control problem
- not even a stochastic control design method

Selection of F for the target feedback loop

standard KF procedure: given noise properties (W, V, etc), KF gain F comes from RE

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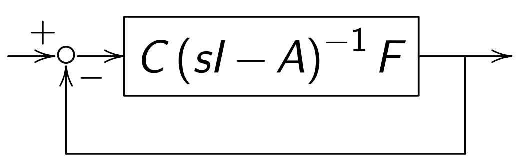
fictitious KF for target feedback loop design: want to have good behavior in



select W and V to get a desired F (hence a *fictitious* KF problem):

$$\dot{x}(t) = Ax(t) + Lw(t), \qquad E[w(t)w^T(t+\tau)] = I\delta(\tau)$$
$$y(t) = Cx(t) + v(t), \qquad E[v(t)v^T(t+\tau)] = \mu I\delta(\tau)$$

which gives

$$F = \frac{1}{\mu}MC^T, \quad AM + M^TA + LL^T - \frac{1}{\mu}MC^TCM = 0,\ M \succ 0 \qquad (5)$$

The target feedback loop from fictitious KF

$$\dot{x}(t) = Ax(t) + Lw(t), \qquad E[w(t)w^T(t+\tau)] = I\delta(\tau)$$
$$y(t) = Cx(t) + v(t), \qquad E[v(t)v^T(t+\tau)] = \mu I\delta(\tau)$$

Return difference equation for the fictitious KF is

$$[I_m + G_F(s)][I_m + G_F(-s)]^T = I_m + \frac{1}{\mu}[C\Phi(s)L][C\Phi(-s)L]^T$$

where $G_F(s) = C(sI - A)^{-1}F$ and $\Phi(s) = (sI - A)^{-1}$. Then

$$\sigma[I_m + G_F(j\omega)] = \sqrt{\lambda\left\{[I_m + G_F(j\omega)][I_m + G_F(-j\omega)]^T\right\}}$$
$$= \sqrt{1 + \frac{1}{\mu}\{\sigma[C\Phi(j\omega)L]\}^2} \geq 1$$

The (nice) target feedback loop from fictitious KF

$$\sigma[I_m + G_F(j\omega)] = \sqrt{\lambda\left\{[I_m + G_F(j\omega)][I_m + G_F(-j\omega)]^T\right\}}$$
$$= \sqrt{1 + \frac{1}{\mu}\{\sigma[C\Phi(j\omega)L]\}^2} \geq 1$$

gives:

- $\sigma_{\max} S(j\omega) = \sigma_{\max}[I + G_F(j\omega)]^{-1} \leq 1$, namely

  no disturbance amplification at any frequency

- $\sigma_{\max} T(j\omega) = \sigma_{\max}[I - S(j\omega)] \leq 2$, hence,

  guaranteed closed loop stable if $\sigma_{\max}\Delta(j\omega) < 1/2$