University of California, Berkeley
Department of Mechanical Engineering
ME 132 Dynamic Systems and Feedback

Fall 2013 **Midterm II**

Closed book and closed lecture notes;
Two 8.5 × 11 handwritten summary sheets allowed;
Scientific calculator with no graphics allowed;
Keep your phones in your bag, and set them off or in silent mode.

NAME: ________________________________

STUDENT ID #: ______________________

#1	#2	#3	#4	Total
10	10	10	20	50

Instructions:

- Write down your name and student ID.
- Read the problems carefully.
- Write your solutions clearly.
- Turn in your exam ON TIME. If we don't have your exam to grade, we will have to put a zero on the score sheet.
- This exam has 9 pages. The last page is empty.
- Do not turn to the next page until you are instructed to do so.

1. [10 points] True or false. Circle your choice in the provided table below. For the false statements, provide a one-sentence reasoning (remark: correct judgments with incorrect or no reasoning get zero point).

   (a) A standard PID controller is causal

   (b) In PID design, the integral action removes the steady-state error for step inputs

   (c) Modern control theory is based on state-space analysis

   (d) In PID design, the integral action perfectly rejects constant disturbances at the steady state

   (e) In PID design, high-gain feedback at low frequency comes from the "D term"

   (f) With a well-designed feedback controller, we can achieve good rejection for both the disturbance and the sensor noise at a particular frequency

   (g) When modeling a moving mass using state-space equations, it is reasonable to select the position and the negative of the position as the state variables

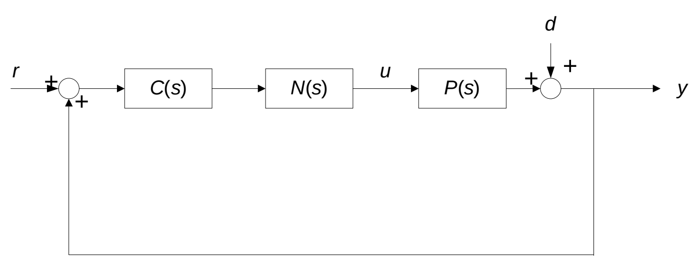
   (h) First-order LTI systems can have resonant behavior

   (i) The sensitivity function directly explains the disturbance rejection property of a feedback system

   (j) For the block diagram below, the sensitivity and complementary sensitivity functions are respectively

$$S(s) = \frac{1}{1 - P(s)N(s)C(s)}$$

$$T(s) = \frac{P(s)N(s)C(s)}{1 - P(s)N(s)C(s)}$$



Circle your choice and provide the reasoning for the false statements:

(a): true, false

(b): true, false

(c): true, false

(d): true, false

(e): true, false

(f): true, false

(g): true, false

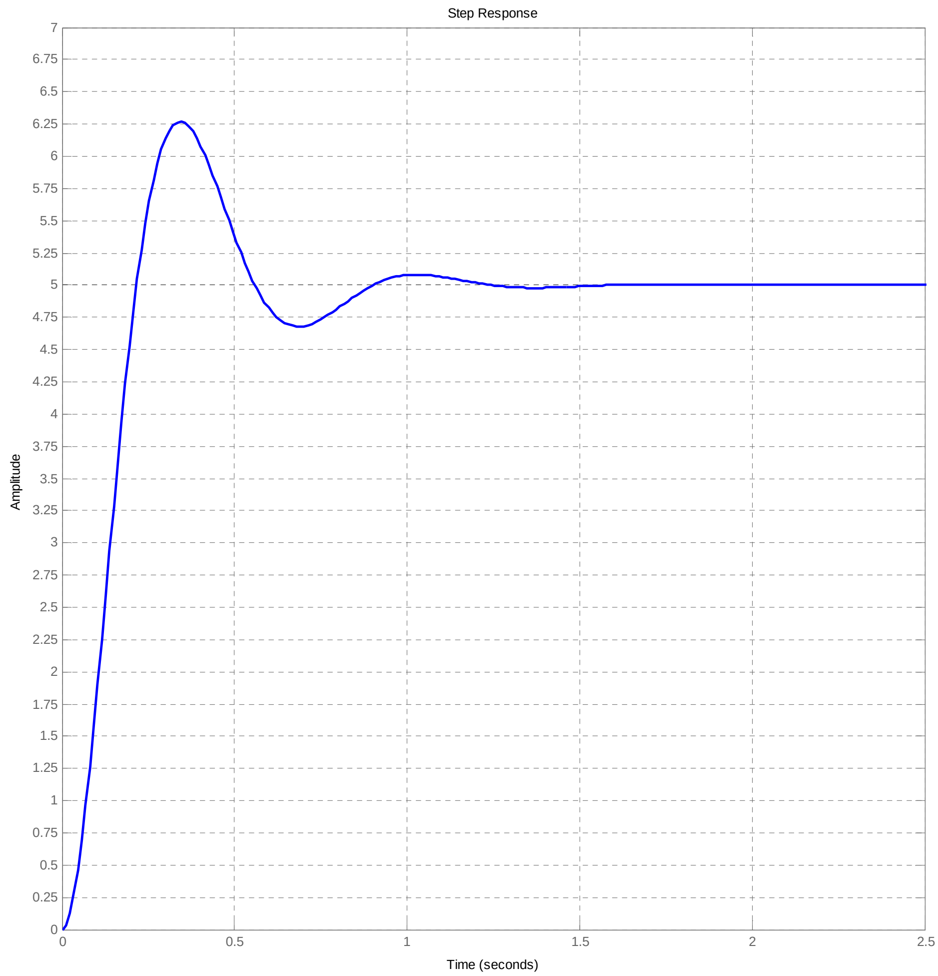
(h): true, false

(i): true, false

(j): true, false

2. [10 points] System analysis.

(a) [4 points] Shown below is a time-domain step response of a linear time invariant system.



i. Mark the 5% settling time and peak time on the figure. Compute the maximum overshoot M_p. $M_p =$ ______

ii. From the following four transfer functions,

$$\frac{500}{s^2+8s+100}, \quad \frac{500}{s^2+0.8s+100}, \quad \frac{100-400s}{s^2+8s+100}, \quad \frac{100000}{(s^2+8s+100)(s+200)}$$

circle the one(s) that may have generated the step response. Provide your reasoning.

(b) [6 points] A set of unit step responses $\{G[1,1]\ G[1,2]\ G[2,1]\ G[2,2]\ G[3,1]\ G[3,2]\}$ and six transfer functions (A to F) are provided below. In the provided box, match each step response with its corresponding transfer function. Assume zero initial condition in all systems from A to F.

$A: \frac{1}{s+1}$

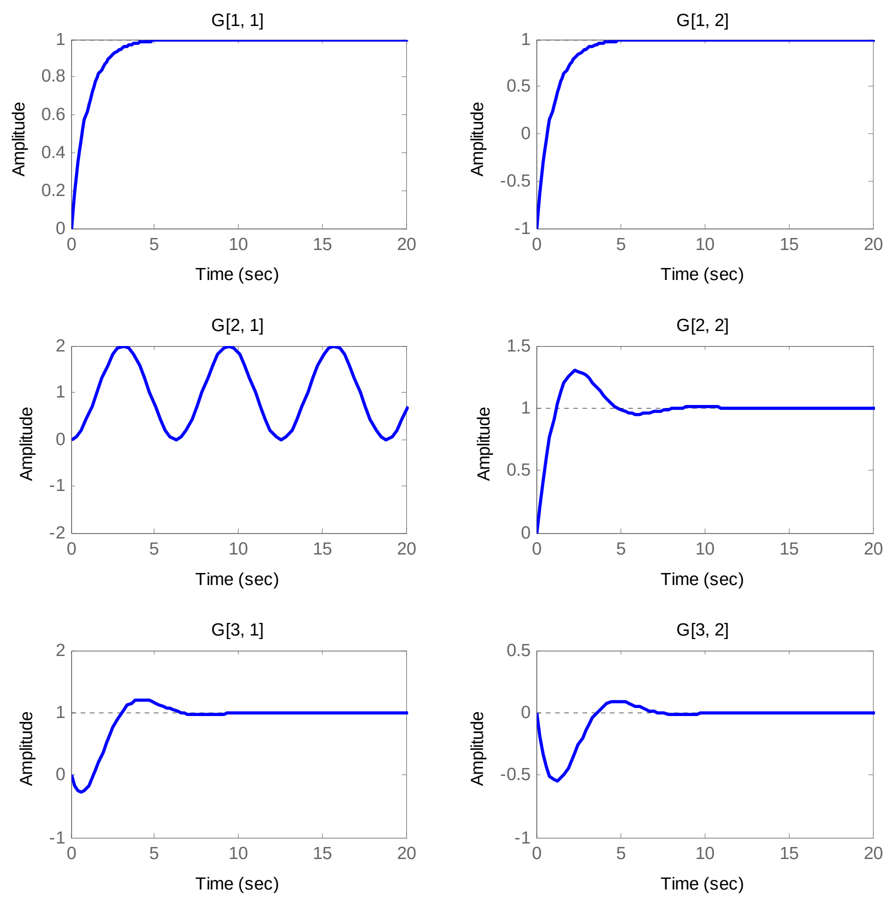
$B: \frac{-s}{s^2+2\times0.5s+1}$

$C: \frac{1}{s^2+1}$

$D: \frac{s+1}{s^2+2\times0.5s+1}$

$E: \frac{1-s}{s+1}$

$F: \frac{-s+1}{s^2+2\times0.5s+1}$



Answer (fill in the blanks):

A matches $G[\ \_\_\_,\_\_\_\ ]$; B matches $G[\ \_\_\_,\_\_\_\ ]$

C matches $G[\ \_\_\_,\_\_\_\ ]$; D matches $G[\ \_\_\_,\_\_\_\ ]$

E matches $G[\ \_\_\_,\_\_\_\ ]$; F matches $G[\ \_\_\_,\_\_\_\ ]$

3. [10 points] For each of the differential equations below, derive the **general form** of the solution. Each of you solutions in (a)-(c) should contain two free constants.

   (a) [2 points] $\ddot{y}(t) + 5\dot{y}(t) + 6y(t) = 0$

   Final result:

   (b) [2 points] $\ddot{y}(t) + 5\dot{y}(t) + 6y(t) = 18u(t)$ where $u(t)$ is a standard unit step signal.

   Final result:

   (c) [2 points] $\ddot{y}(t) + 5\dot{y}(t) + 6y(t) = 18u(t) - 18\dot{u}(t)$ where $u(t)$ is a standard unit step signal.

   Final result:

   (d) [4 points] Obtain the full solution of (c) if the initial conditions are $y(0) = 0$ and $\dot{y}(0) = 0$. The result should contain no undetermined coefficients.

   Final result:

4. [20 points] System design and analysis.

(a) [5 points] Consider the state-space description of a system

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -100 & -4 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r$$
$$y = \begin{bmatrix} 100 & 3 \end{bmatrix} x + 2r$$

i. Obtain the transfer function from r to y.

Final result:
$G_{r\to y}(s) =$

ii. Now consider another system

$$\left[\begin{array}{c} \dot{x}_1 \\ \dot{x}_2 \\ \hline y_1 \\ y_2 \end{array}\right] = \left[\begin{array}{cc|c} 0 & 1 & 0 \\ -100 & -4 & 1 \\ \hline 100 & 3 & 2 \\ 0 & 100 & 0 \end{array}\right] \left[\begin{array}{c} x_1 \\ x_2 \\ \hline r \end{array}\right]$$

Obtain the transfer function from r to y_1.

Final result:
$G_{r\to y_1}(s) =$

(b) [2 points] *Suppose* the derived transfer function in part (a)-i is

$$G_{r\to y}(s) = \frac{24s + 100}{s^2 + 25s + 100} \tag{1}$$

(warning: this is not the right answer for part (a)-i. Do NOT copy it to the answer box there). Equation (1) is the closed-loop transfer function from r to y in the feedback system in Fig. 1. Derive k_P and k_I, the coefficients of the PI controller.

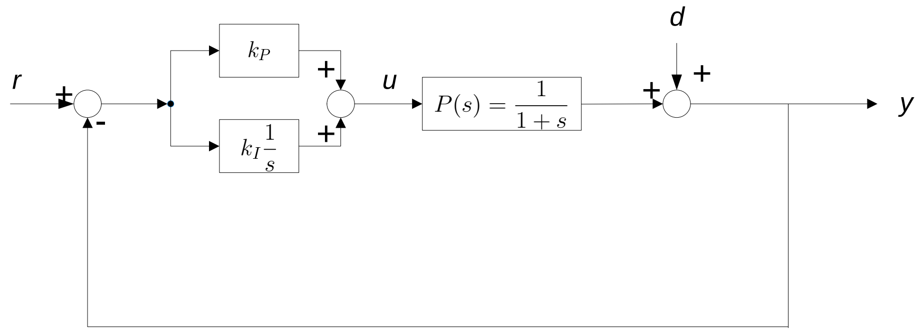


Figure 1: Block diagram for problem 4

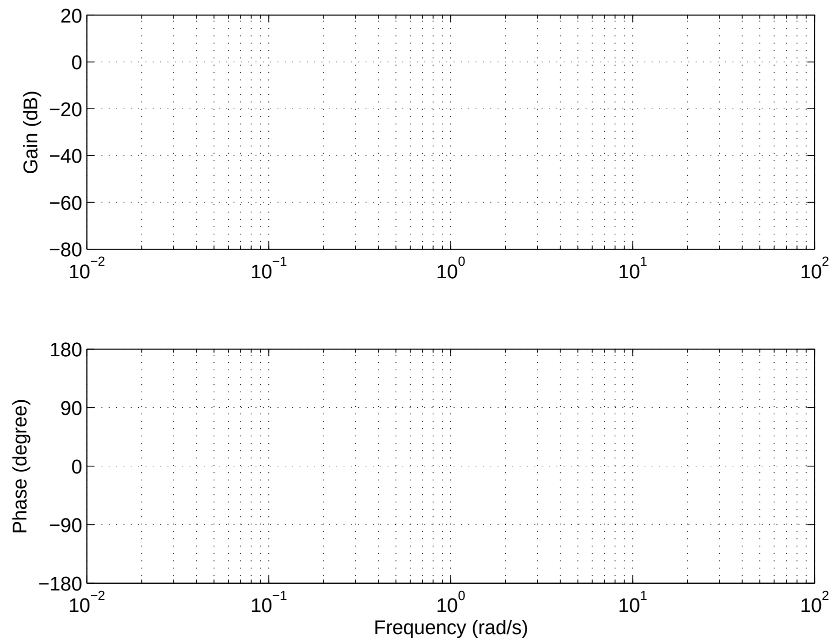
Final result:
$k_P =$
$k_I =$

(c) [5 points] Sketch, on the provided figure, the magnitude responses of

$$P(s) = \frac{1}{1+s}, \quad G_{r\to y}(s) = \frac{24s+100}{s^2+25s+100} \tag{2}$$

and the phase response of

$$P(s) = \frac{1}{1+s}$$



(d) [2 points] Consider the step responses of the two systems $G_{r\to y}(s)$ and $P(s)$ in Equation (2). Assume zero initial condition(s). Which one has shorter transient response? Provide your reasoning.

Conclusion (circle one):

$G_{r\to y}(s)$ has shorter transient

$P(s)$ has shorter transient

(e) [6 points] Consider the sinusoidal responses of the two systems $G_{r\to y}(s)$ and $P(s)$ in Equation (2), with respect to the input $u(t) = \sin\left(3t + \frac{\pi}{4}\right),\ t \geq 0$ ($u(t) = 0$ if $t < 0$). At steady state, which system provides better tracking to $u(t)$? Derive this steady-state output.
Remark: you don't have to compute the numerical values of the magnitude and phase.

Conclusion (circle one):

$G_{r\to y}(s)$ is better

$P(s)$ is better