Reduced-Complexity and Robust Youla Parameterization for Discrete-time Dual-input-single-output Systems

Xu Chen and Masayoshi Tomizuka

Abstract— **A loop-shaping idea for dual-input-single-output (DISO) systems is presented in this paper. We explore special forms of Youla parameterization for obtaining stabilizing controllers for DISO systems, and discuss concepts to achieve reduced-complexity formulations with clearer design intuitions. The algorithm is verified by a benchmark control problem of dual-stage hard disk drive systems, using real-system models and actual vibration data.**

Index Terms— **precision control, dual-stage systems, loop shaping, linear feedback control, vibration rejection**

I. Introduction

Practical mechanical systems are commonly subjected to multiple control objectives. Take a modern hard disk drive system as an example, the servo controller needs to position the actuator such that the read/write head can rapidly move between different tracks on the spinning disk, and stay on track afterward as precisely as possible. Besides accurate command following, the feedback control system has to also address disturbances generated from windage, vibrations from audio speakers, and non-circular tracks, etc [1]. These combined performance requirements are common for general precision control, where special task patterns (e.g., repeatable trajectory [2]) or cumulative imperfections in mechanical parts (gears, motors, etc) generates various disturbances for servo design. As a result, standard feedback control techniques alone (such as PID and H_∞ design) are usually not sufficient to achieve the mixed performance requirements.[1]

A hardware solution to such requirements is dual-stage actuation, which combines two actuators for enhanced positioning: a coarse actuator with a long movement range but a limited dynamic bandwidth, and a fine actuator with a short stroke but a large bandwidth. In this paper, we discuss add-on loop-shaping ideas via Youla parameterization for dual-stage systems under different control objectives. Youla parameterization, also known as the parameterization for all stabilizing controllers, is broadly investigated in the H_∞- and adaptive-control fields [3], [4]. It provides that any stabilizing controllers can be parameterized using a fixed structure, if a perfect model of the plant is available. The design of the central Q filter in this scheme however does not have a commonly agreed rule, particularly for multi-input-multi-output systems [3], [4]. One main reason for the variance in designing the Q filter is that it heavily depends on the parameterization of the plant model. In this paper, we focus on the special class of dual-input-single-output (DISO) systems and discuss new investigations about Youla parameterization for add-on feedback design. We show that for DISO (and more generally multiple-input-single-output systems), we can formulate a reduced-complexity Youla parameterization and obtain simpler realizations from the loop-shaping perspective. Discrete-time Youla design is known to have subtle differences compared to the continuous-time version of the problem. We provide, in the digital-control framework, a special coprime parameterization scheme such that the Q-filter design can be approximately separated from the plant characteristics, and analyze the robustness issue in the presence of model mismatch. This investigation is useful, for example, in various applications of dual-stage actuation in precision control (see, e.g., [5]), where the control of dual-input-single-output systems has substantial importance in the servo performance.

II. Standard Youla Parameterization

Define the set

$$\mathcal{S} := \{\text{stable, proper, and rational transfer functions}\}.$$

For a single-input-single-output (SISO) discrete-time system $P(z^{-1})$, Youla parameterization starts with a coprime factorization of the plant: $P(z^{-1}) = N(z^{-1})/D(z^{-1})$, where $N(z^{-1})$ and $D(z^{-1})$ are coprime over $\mathcal{S}$, i.e., there exists $U(z^{-1})$ and $V(z^{-1})$ in $\mathcal{S}$ such that $U(z^{-1})N(z^{-1}) + V(z^{-1})D(z^{-1}) = 1$. If $P(z^{-1})$ can be stabilized by a negative-feedback controller $C(z^{-1}) = X(z^{-1})/Y(z^{-1})$, with $X(z^{-1})$ and $Y(z^{-1})$ coprime over $\mathcal{S}$, then Youla parameterization [6] provides that any stabilizing feedback controller can be parameterized as

$$C_{all}(z^{-1}) = \frac{X(z^{-1}) + D(z^{-1})Q(z^{-1})}{Y(z^{-1}) - N(z^{-1})Q(z^{-1})} : \; Q(z^{-1}) \in \mathcal{S} \quad (1)$$

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[1] see, for example, the control structure for hard disk drives in [1].

Here it is required to have $Y(\infty) - N(\infty)Q(\infty) \neq 0$ for the closed loop to be well-posed, which is easily satisfied for practical problems. Two important concepts are implied by the above theorem. First, a controller parameterized as (1) is guaranteed to generate a stable closed loop, as long as $Q(z^{-1})$ is stable, proper, and rational. Second, any stabilizing controllers can be realized in the form of (1).

A main benefit of Youla parameterization is that the closed-loop transfer functions become affine in $Q(z^{-1})$. Of particular interest is the sensitivity function:

$$\begin{aligned} S(z^{-1}) &= \frac{1}{1+P(z^{-1})C_{all}(z^{-1})} \\ &= \underbrace{\frac{1}{1+P(z^{-1})C(z^{-1})}}_{S_o(z^{-1})\text{: original sensitivity}} \left[1 - \frac{N(z^{-1})}{Y(z^{-1})}Q(z^{-1})\right] \end{aligned} \tag{2}$$

which defines the bandwidth for disturbance rejection.

Remark: the original Youla formulation requires additionally the Bezout identity $X(z^{-1})N(z^{-1}) + Y(z^{-1})D(z^{-1}) = 1$, which gives instead $S_o(z^{-1}) = Y(z^{-1})D(z^{-1})$ and $S(z^{-1}) = \left(Y(z^{-1}) - N(z^{-1})Q(z^{-1})\right)D(z^{-1})$. This assumption can be dropped without losing generality, and has been used in a group of literatures including [4] and [7].

For general multiple-input-multiple-output systems, dimensions of transfer functions play important roles. We have:

Theorem 1: consider a n_u-input-n_y-output plant P. let $P = ND^{-1}$, with $N \in \mathcal{S}^{n_y \times n_u}$ and $D \in \mathcal{S}^{n_u \times n_u}$ right coprime over $\mathcal{S}$, be stabilized by a controller $C = XY^{-1}$ (in a negative feedback loop), with $X \in \mathcal{S}^{n_u \times n_y}$ and $Y \in \mathcal{S}^{n_y \times n_y}$ coprime over $\mathcal{S}$. Then the set of all stabilizing controllers for P is given by

$$\left\{(X + DQ)(Y - NQ)^{-1} : \ Q \in \mathcal{S}^{n_u \times n_y}\right\} \tag{3}$$

or in the left-coprime format

$$\left\{(\bar{Y} - \bar{Q}\bar{N})^{-1}(\bar{X} + \bar{Q}\bar{D}) : \ \bar{Q} \in \mathcal{S}^{n_u \times n_y}\right\} \tag{4}$$

where $P = \bar{D}^{-1}\bar{N}$ is the left-coprime factorization of P with $\bar{N} \in \mathcal{S}^{n_y \times n_u}$, $\bar{D} \in \mathcal{S}^{n_y \times n_y}$; $C = \bar{Y}^{-1}\bar{X}$ is the left-coprime factorization of C with $\bar{X} \in \mathcal{S}^{n_u \times n_y}$, $\bar{Y} \in \mathcal{S}^{n_u \times n_u}$. For well-posedness, the mild requirements $\det(Y(\infty) - N(\infty)Q(\infty)) \neq 0$ and $\det(\bar{Y}(\infty) - \bar{Q}(\infty)\bar{N}(\infty)) \neq 0$ are assumed in the above formulation.

Here the pair $(N \in \mathcal{S}^{n_y \times n_u}, D \in \mathcal{S}^{n_u \times n_u})$ is called right coprime over $\mathcal{S}$ if there exists $U \in \mathcal{S}^{n_u \times n_y}$, $V \in \mathcal{S}^{n_u \times n_u}$ such that $UN + VD = I_{n_u}$. The pair $(\bar{N} \in \mathcal{S}^{n_y \times n_u}, \bar{D} \in \mathcal{S}^{n_y \times n_y})$ is called left coprime over $\mathcal{S}$ if there exists $\bar{U} \in \mathcal{S}^{n_u \times n_y}$, $\bar{V} \in \mathcal{S}^{n_y \times n_y}$ such that $\bar{N}\bar{U} + \bar{D}\bar{V} = I_{n_y}$. For simplicity, we have omitted the index (z^{-1}) in the above theorem, and will adopt this format for long equations in the remainder of the paper.

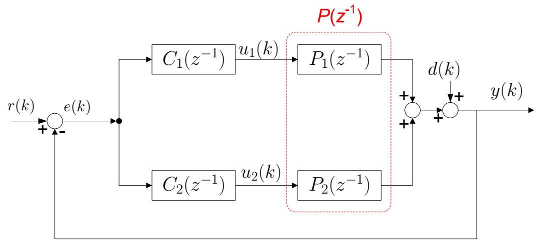

Fig. 1. General control structure for DISO systems

III. Proposed Youla Parameterization for DISO Systems

Consider now the parameterization for dual-input-single-output systems, with the general control structure shown in Fig. 1. Here the plant $P(z^{-1}) = \left[P_1(z^{-1}),\ P_2(z^{-1})\right]$, hence a stabilizing feedback controller should be single-output-dual-output (SIDO): $C(z^{-1}) = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix}$. Three sets of elements need to be constructed to achieve Youla parameterization: the plant parameterization, the design of the baseline controller $C(z^{-1})$, and the choice of $Q(z^{-1})$. The design of $C_1(z^{-1})$ and $C_2(z^{-1})$ usually considers the dynamic characteristics of $P_1(z^{-1})$ and $P_2(z^{-1})$ (see, e.g., [5]). The final step of Q-filter design depends heavily on the choice in the first two elements above, and on the desired servo enhancement. For this special class of MIMO system, the generalized (and more complicated) form of Youla parameterization in Theorem 1 certainly works, however at the expense of reduced tuning intuitions and increased computation [recall (3) and (4)]. To simplify such practical obstacles, we discuss a special Youla parameterization for DISO systems.

A. Reduced-complexity formulation for DISO systems

Notice that regardless of the fact that $P(z^{-1})$ is a dual-input system, the *open-loop* transfer function, i.e., the transfer function from $e(k)$ to $y(k)$ with the bottom feedback path cut open in Fig. 1, is always single-input-single-output:

$$L(z^{-1}) = P_1(z^{-1})C_1(z^{-1}) + P_2(z^{-1})C_2(z^{-1}).$$

If we treat $L(z^{-1})$ as a plant, then the baseline closed loop is composed of just two single-input-single-output elements: $L(z^{-1})$ and an identity feedback controller, as shown in Fig. 2. Among all transfer functions, the identity block is the simplest to perform coprime factorization. On another practical side, $L(z^{-1})$ is always a "single-rate" system while the plant input u_2 might in practice be updated faster than the output y in Fig. 1, which additionally complicates the parameterization.

We propose to perform Youla parameterization for the fictitious plant $L(z^{-1})$. Consider a coprime factorization

$$L(z^{-1}) = N(z^{-1})/D(z^{-1}). \tag{5}$$

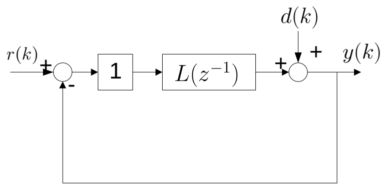

Fig. 2. SISO viewpoint of DISO feedback systems

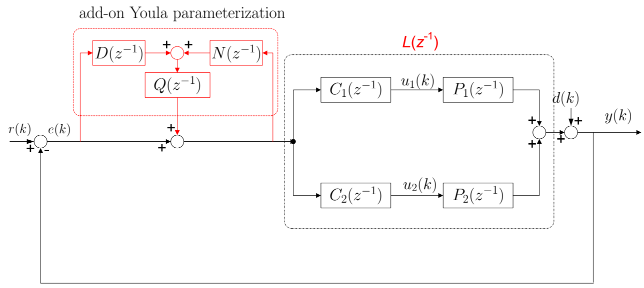

Fig. 3. Proposed Youla parameterization for DISO systems

For Fig. 2 we can choose $X(z^{-1}) = Y(z^{-1}) = 1$ for the fictitious-controller factorization and obtain a simplified form of (1) for the DISO system:

$$C_{all}(z^{-1}) = \frac{1+D(z^{-1})Q(z^{-1})}{1-N(z^{-1})Q(z^{-1})},\ Q(z^{-1}) \in \mathcal{S}. \tag{6}$$

With (6), stability of the closed-loop system is guaranteed when no model mismatch exists. The sensitivity function in (2) is reduced to

$$S(z^{-1}) = S_o(z^{-1})(1-N(z^{-1})Q(z^{-1})) \tag{7}$$

where $S_o(z^{-1}) = 1/(1 + N(z^{-1})D^{-1}(z^{-1})) = 1/(1 + P(z^{-1})C(z^{-1}))$ is again the baseline sensitivity function. Loop-shaping schemes for SISO Youla parameterization can now be applied for the DISO system. Compared to the general MIMO formulation where $Q(z^{-1})$ and $\bar{Q}(z^{-1})$ will be single-input-dual-output in (3) and (4), (7) has reduced complexity and depends only on one SISO Q filter. This simplifies the algorithm greatly, especially when adaptive control is required for finding a proper $Q(z^{-1})$.

One realization of (6) is provided in Fig. 3. Since we designed $C_1(z^{-1})$ and $C_2(z^{-1})$ to form the baseline loop transfer function $L(z^{-1})$ first, the parameterization of (6) is an add-on scheme, and can be switched on or off depending on the operation environments.

B. Coprime parameterization for the fictitious plant

The simplest form of (5) is that $N(z^{-1}) = 1$. This occurs if $L(z^{-1}) = 1/L^{-1}(z^{-1})$ is a valid coprime parameterization, and will provide a beneficial result of $S(z^{-1}) = S_o(z^{-1})(1-Q(z^{-1}))$. If $L^{-1}(z^{-1})$ is not causal itself, this ideal concept of factorization can be be approximated by $L(z^{-1}) = z^{-m}/L_m^{-1}(z^{-1}) \triangleq z^{-m}\left[z^{-m}L^{-1}(z^{-1})\right]^{-1}$. Namely, we added delays so that $N(z^{-1}) = z^{-m}$, and that $z^{-m}L^{-1}(z^{-1})$ is proper and realizable.

Remark 1: it is not uncommon for practical plants to have input delays. In a dual-input-single-output setting, the proposed fictitious plant actually has the advantage of reduced delay effect by its construction. Consider, for example, $L(z^{-1}) = P_1(z^{-1})C_1(z^{-1}) + P_2(z^{-1})C_2(z^{-1})$ where $P_1(z^{-1}) = z^{-m_1}P_{1m}(z^{-1})$, $P_2(z^{-1}) = z^{-m_2}P_{2m}(z^{-1})$ with $P_{1m}(z^{-1})$ and $P_{2m}(z^{-1})$ being minimum-phase, and $m_1 > m_2$. We assume for simplicity the controllers do not introduce additional delays. Then $L(z^{-1}) = z^{-m_2}[z^{m_2-m_1}P_{1m}(z^{-1})C_1(z^{-1}) + P_{2m}(z^{-1})C_2(z^{-1})]$. The intermediate delay $z^{m_2-m_1}$ will be absorbed in the transfer function in the bracketed term (consider a simple example of $P_1 = z^{-2}$, $P_2 = z^{-1}$, $C_1 = 0.56$, and $C_2 = 1.5$). The total delay for $L(z^{-1})$ is hence $m_2 = \min\{m_1, m_2\}$, i.e., the minimum of the delays among all actuators.

Remark 2: for continuous-time systems the above parameterization is not as trivial, since delays are formatted as exponential transfer functions.

We now apply the above concept to form an inverse-based coprime factorization of $L(z^{-1})$ for Fig. 3. If $L^{-1}(z^{-1})$ is stable, then $L(z^{-1}) = z^{-m}\left[z^{-m}L^{-1}(z^{-1})\right]^{-1}$ can be directly used. If not, we provide an approximation and form a robust Youla parameterization scheme. Notice that high-frequency uncertainties always exist for a practical mechanical system and an exact transfer function of $L(z^{-1})$ is intrinsically not possible. Practical feedback design has an effective servo bandwidth above which the control efforts are suggested to be kept small [8]. It is hence more reasonable to obtain an inverse design at selective frequency regions. Denote $\hat{L}^{-1}(z^{-1})$ as the *nominal* inverse for $L(z^{-1})$. At frequencies where no large model uncertainties are expected (usually low and middle frequencies), we enforce correct model matching between $\hat{L}(z^{-1})$ and $L(z^{-1})$; when there are large model uncertainties (commonly at high frequencies), we construct constrains such that $\hat{L}^{-1}(z^{-1})$ has a limited magnitude response.

The latter condition in the last paragraph allows us to also achieve the second requirement of coprime factorization, that $\hat{L}^{-1}(z^{-1})$ should be stable. This is detailed as follows: it is a well-established concept that non-minimum-phase zeros cause control limitations. Particularly, unstable zeros at low frequencies should be avoided in general feedback design (see, e.g., [9]) while high-frequency unstable zeros may naturally arise due to system uncertainties and discretization [10]. By sacrificing perfect model matching at very high frequencies (which is not practical anyway), we gain the opportunity to search for a *stable* $\hat{L}^{-1}(z^{-1})$ that retains the information of $L(z^{-1})$ at the desired servo frequencies. Recall that general feedback design commonly achieves a loop shape as shown in Fig. 4. For simple cases it is possible to construct $\hat{L}^{-1}(z^{-1})$ by

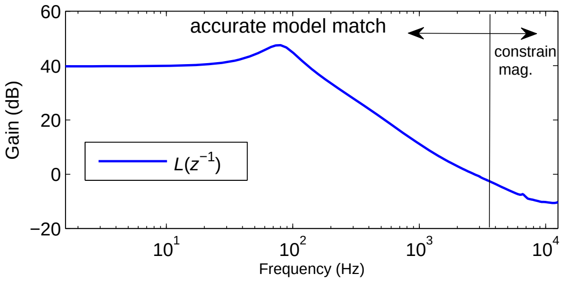

Fig. 4. General loop shape in servo design

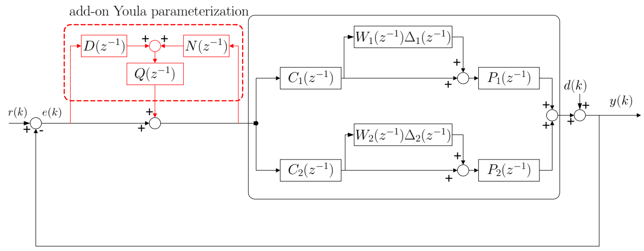

Fig. 5. Block diagram for the proposed robust Youla parameterization

principles of frequency response. For general cases, we discuss next an H_∞ formulation to optimally solve the problem.

Consider the following minimization problem

$$\min_{L_m^{-1}(z^{-1})\in\mathbf{S}} \left\| \begin{bmatrix} W_{o1}(z^{-1})\left(L_m^{-1}(z^{-1})L(z^{-1})-z^{-m}\right) \\ W_{o2}(z^{-1})L_m^{-1}(z^{-1})L(z^{-1}) \end{bmatrix} \right\|_\infty . \quad (8)$$

Here we search among the set of stable, proper, and rational transfer functions to find $L_m^{-1}(z^{-1})$ such that the weighted combination of $\|L_m^{-1}(z^{-1})L(z^{-1})-z^{-m}\|_\infty$ and $\|L_m^{-1}(z^{-1})L(z^{-1})\|_\infty$ are minimized. If it is feasible to achieve a zero cost for the first term, we have $L_m^{-1}(z^{-1}) = z^{-m}L^{-1}(z^{-1})$, hence perfect model matching at all frequencies. For the second term, $L_m^{-1}(z^{-1}) = 0$ clearly achieves a zero cost, namely, the magnitude of $L_m^{-1}(z^{-1})$ is kept small. After adding the weighting functions $W_{o1}(z^{-1})$ and $W_{o2}(z^{-1})$ we can select the frequency regions so that $\|L_m^{-1}(z^{-1})L(z^{-1})-z^{-m}\|_\infty$ is small at the interested (low and middle) frequencies and $\|L_m^{-1}(z^{-1})L(z^{-1})\|_\infty$ is small at high frequencies.

By the above formulation, (8) falls into the framework of H_∞ control, and can be efficiently solved using, for example, the robust control toolbox in MATLAB. If needed, standard model-reduction techniques can then be applied to obtain a lower-order $L_m^{-1}(z^{-1})$.

C. The final implementation form

With the discussions in the above subsection, an approximate coprime factorization for $L(z^{-1})$ is

$$L(z^{-1}) = \frac{N(z^{-1})}{D(z^{-1})} \approx \frac{z^{-m}}{L_m^{-1}(z^{-1})} = \frac{z^{-m}}{z^{-m}\hat{L}^{-1}(z^{-1})} \quad (9)$$

and (7) becomes

$$S(e^{-j\omega}) \approx S_o(e^{-j\omega})(1-e^{-mj\omega}Q(e^{-j\omega})) \quad (10)$$

in the frequency domain. We can see that if $e^{-mj\omega}Q(e^{j\omega})$ has unity gains at certain frequencies (e.g., when $Q(z^{-1})$ is a low-pass filter), then $S(e^{-j\omega})$ has small gains at the same frequencies; if $|Q(e^{-j\omega})| \approx 0$ then $S(e^{-j\omega}) \approx S_o(e^{-j\omega})$. Add-on loop shaping has become quite intuitive in this simplified affine form of (10).

IV. Robustness Against Model Mismatch

This section considers the robustness of the proposed control scheme. This is necessary due to the approximate coprime factorization for $L(z^{-1})$. One difference from the standard robust-control analysis is that $L(z^{-1})$ is a fictitious plant that is composed of not only the plants $P_1(z^{-1})$, $P_2(z^{-1})$ but also the controllers $C_1(z^{-1})$ and $C_2(z^{-1})$. Assume the plants are perturbed to $\tilde{P}_i(z^{-1}) = P_i(z^{-1})(1+W_i(z^{-1})\Delta_i(z^{-1}))$: $i = 1,2$, as shown in Fig. 5, with $\|\Delta_i(z^{-1})\|_\infty \le 1$ and $W_i(z^{-1})$ being the uncertainty weighting functions. In the proposed Youla scheme, the overall feedback controller is given by

$$\bar{C}(z^{-1}) = \begin{bmatrix} \bar{C}_1(z^{-1}) \\ \bar{C}_2(z^{-1}) \end{bmatrix} = \frac{1+D(z^{-1})Q(z^{-1})}{1-N(z^{-1})Q(z^{-1})} \begin{bmatrix} C_1(z^{-1}) \\ C_2(z^{-1}) \end{bmatrix} . \quad (11)$$

Robust stability analysis seeks to find the minimum perturbation such that

$$\det\left(I+\tilde{P}(z^{-1})\bar{C}(z^{-1})\right) = 0. \quad (12)$$

When nominal stability holds, the nominal characteristic polynomial $1+P(z^{-1})\bar{C}(z^{-1}) = 1+P_1(z^{-1})\bar{C}_1(z^{-1})+P_2(z^{-1})\bar{C}_2(z^{-1})$ is nonzero evaluated at any frequencies. Dividing this quantity on both sides of (12), and after some algebra, we obtain

$$1+\frac{P_1\bar{C}_1}{1+P\bar{C}}W_1\Delta_1+\frac{P_2\bar{C}_2}{1+P\bar{C}}W_2\Delta_2 = 0. \quad (13)$$

Theorem 2: For full perturbations where $\Delta_i(e^{j\omega})$ can take any complex value with $|\Delta_i(e^{j\omega})| \le 1$, the proposed scheme is robustly stable if and only if

$$\frac{\left|P_1(e^{-j\omega})W_1(e^{-j\omega})\bar{C}_1(e^{-j\omega})\right| + \left|P_2(e^{-j\omega})W_2(e^{-j\omega})\bar{C}_2(e^{-j\omega})\right|}{\left|1+P_1(e^{-j\omega})\bar{C}_1(e^{-j\omega})+P_2(e^{-j\omega})\bar{C}_2(e^{-j\omega})\right|} < 1 \quad (14)$$

where $\bar{C}_i$ is from (11).

Proof: (due to space limit, only the key concepts are provided here) The worst-case minimum perturbation happens when $|\Delta_1(e^{-j\omega})| = |\Delta_2(e^{-j\omega})| = |\Delta_o(e^{-j\omega})|$ and the perturbation directions are such that

$$1-\left|\frac{P_1(e^{-j\omega})\bar{C}_1(e^{-j\omega})W_1(e^{-j\omega})}{1+P(e^{-j\omega})\bar{C}(e^{-j\omega})}\right|\left|\Delta_o(e^{-j\omega})\right| - \left|\frac{P_2(e^{-j\omega})\bar{C}_2(e^{-j\omega})W_2(e^{-j\omega})}{1+P(e^{-j\omega})\bar{C}(e^{-j\omega})}\right|\left|\Delta_o(e^{-j\omega})\right| = 0$$

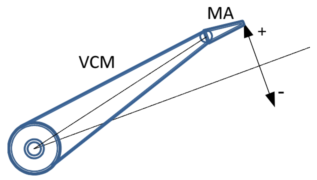

Fig. 6. Schematic structure of dual-stage hard disk drives

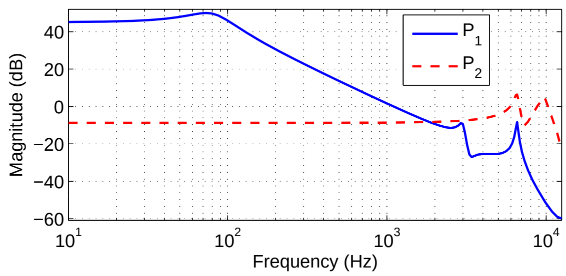

Fig. 7. Magnitude responses of the plant

in (13). Reordering terms and using $P\bar{C} = P_1\bar{C}_1 + P_2\bar{C}_2$ yield the boundary point for robust stability. ∎

Equation (14) is the general result that covers uncertainties in both actuators. A practical concern for piezo-actuator based control system is the influence of the uncertainty in the gain of the actuator. In this case we have a *scalar* perturbation $W_2\Delta_2$ and can assume $W_1 = 0$ in (13). The characteristic equation becomes

$$1 + \frac{P_2(z^{-1})\bar{C}_2(z^{-1})}{1 + P(z^{-1})\bar{C}(z^{-1})} W_2\Delta_2 = 0. \tag{15}$$

Notice that the perturbation $W_2\Delta_2 \in \mathbf{R}$ here. This way the maximum gain perturbations (note there are two of them: positive and negative perturbations) can be obtained by computing the gain margins of $\pm P_2(z^{-1})C_2(z^{-1})/[1 + P_1(z^{-1})C_1(z^{-1}) + P_2(z^{-1})C_2(z^{-1})]$.

V. Application Example

With $S(e^{-j\omega}) \approx S_o(e^{-j\omega})(1 - e^{-mj\omega}Q(e^{-j\omega}))$ in (10), loop-shaping design can simply concentrate on the add-on element $1 - z^{-m}Q(z^{-1})$.[2] We provide an application example about vibration rejection, which is an important problem for many practical mechanical systems, including the dual-stage hard disk drive systems to be discussed in this section. We use the benchmark system on Page 195 of the book [1] as a demonstration tool. It is a typical mechanical system that contains a voice coil motor (VCM) and a piezo microactuator (MA) as shown in Fig. 6. The plant models, whose magnitude responses are shown in Fig. 7, are obtained from an actual test drive system. The disturbance data we use are from real vibration tests.

[2] Of course, the robust stability condition discussed in Section IV should be respected (normally violations occur only when $Q(z^{-1})$ has large gains at high frequencies).

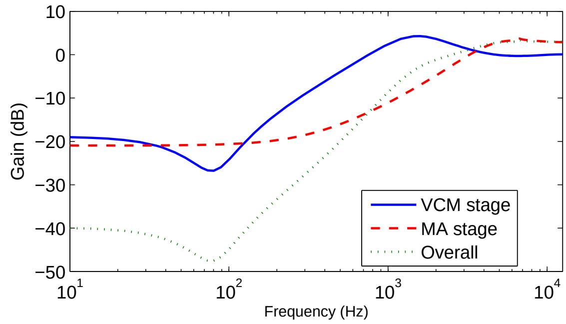

Fig. 8. Magnitude responses in the decoupled-sensitivity design

The baseline controllers use the decoupled-sensitivity scheme where $C_1(z^{-1}) = C_v(z^{-1})\left(1 + C_{ma}(z^{-1})P_2(z^{-1})\right)$ and $C_2(z^{-1}) = C_{ma}(z^{-1})$ (the subscripts v and ma denote VCM and MA respectively). In this way we have $1 + L(z^{-1}) = \left(1 + P_1(z^{-1})C_v(z^{-1})\right)\left(1 + P_2(z^{-1})C_{ma}(z^{-1})\right)$ so that $S_o(z^{-1}) = 1/(1 + L(z^{-1})) = S_{1o}(z^{-1})S_{2o}(z^{-1})$ where $S_{1o}(z^{-1}) = 1/(1 + P_1(z^{-1})C_v(z^{-1}))$ and $S_{2o}(z^{-1}) = 1/(1 + P_2(z^{-1})C_{ma}(z^{-1}))$ are respectively the decoupled sensitivities as if two independent feedback loops about $P_1(z^{-1})$ and $P_2(z^{-1})$ are formed. Fig. 8 shows the magnitude responses of the decoupled sensitivities. Fig. 4 presents the achieved baseline magnitude response for $L(z^{-1})$. There are multiple resonances which are compensated by notch filters in the system. Hence although $L(z^{-1})$ has an order of 22, the magnitude response in Fig. 4 is quite smooth. After the inverse design and model reduction, $L_m(z^{-1})$ is just a 4th-order transfer function, with $m = 1$ and

$$L_m(z^{-1}) = \frac{0.6301 - 1.698z^{-1} + 1.511z^{-2} - 0.4438z^{-3}}{1 - 3.895z^{-1} + 5.687z^{-2} - 3.69z^{-3} + 0.8977^{-4}}$$

which is a minimum-phase system.

Due to the form of $1 - z^{-m}Q(z^{-1})$, the design of the Q filter falls into the same class of problem as that in [11], [12]. The following band-pass type filter

$$Q(z^{-1}) = \frac{(\alpha^2 - 1 - a^2(\alpha - 1)) - (\alpha - 1)az^{-1}}{1 + a\alpha z^{-1} + \alpha^2 z^{-2}}, \tag{16}$$

or more generally $Q(z^{-1}) = B_Q(z^{-1})/A_Q(z^{-1})$ with

$$A_Q(z^{-1}) = 1 + \sum_{i=1}^{n-1} a_i(\alpha^i z^{-i} + \alpha^{2n-i}z^{-2n+i}) + a_n\alpha^n z^{-n} + \alpha^{2n}z^{-2n}$$

$$B_Q(z^{-1}) = \sum_{i=1}^{2n} (\alpha^i - 1)a_i z^{-i+1}, \; a_i = a_{2n-i} \tag{17}$$

can achieve sample loop shapes as shown in Figs. 9 and 10.

Fig. 9 suits for rejecting strong vibrations at several frequencies. Fig. 10 expands the range of disturbance attenuation and suits for mixed structural vibrations that come from a frequency-rich excitation source. In both cases $S(z^{-1})$ has very small gain (the depth of the

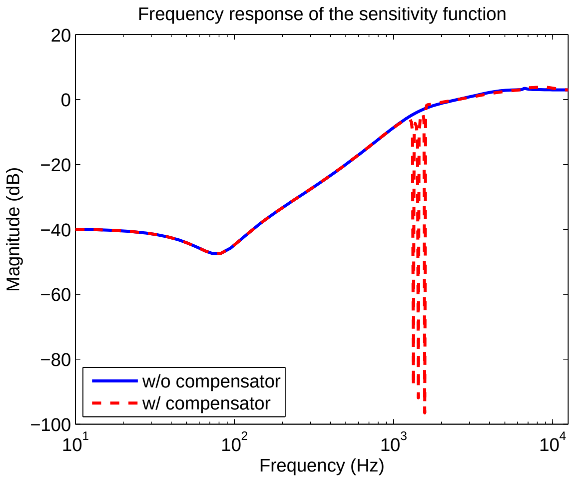

Fig. 9. Sensitivity function for narrow-band loop shaping

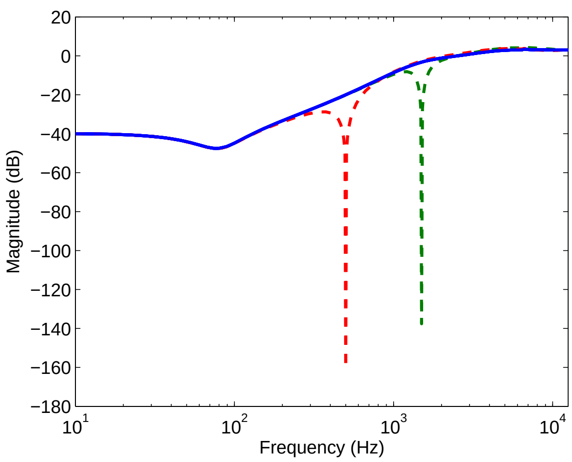

Fig. 10. Sensitivity function for wide-band vibration rejection

notch is tunable) at multiple bands of frequencies while amplifications at other frequencies are very small. The coefficients a_i's in (16) and (17) determine the notch frequencies in Figs. 9 and 10. For (16), we have $a = -2\cos(2\pi\Omega T_s)$, where Ω is the desired notch frequency in Hz. For (17), a_i and Ω_i are connected by $A_Q(z^{-1}) = \prod_{i=1}^{n}(1-2\cos(2\pi\Omega_i T_s)\alpha z^{-1}+\alpha^2 z^{-2})$ [11].

Fig. 11 shows the error spectra with and without the proposed Youla parameterization. (17) was used to construct a 2-band band-pass $Q(z^{-1})$, to form a loop shape in the class of Fig. 10. We observe that the strong frequency components at around 1000 and 2500 Hz have been successfully rejected without visual amplification of other error components. Notice that (17) is linear in the coefficients a_i's. An adaptive algorithm can also be constructed for updating the parameters of $Q(z^{-1})$ online to automatically find the vibration frequencies.

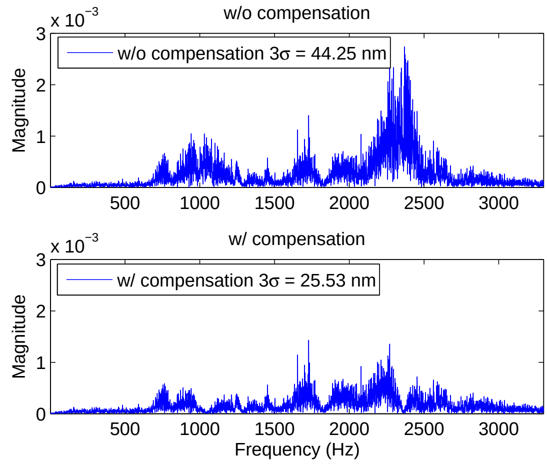

Fig. 11. Error spectra with and without compensation

VI. Conclusion

In this paper we have introduced a loop-shaping concept for dual-input-single-output systems. Several application examples have demonstrated the validity of the algorithm and explained its design intuitions.

Finally, we note that the algorithm can be readily extended to general multiple-input-single-output systems, where the loop transfer function $L(z^{-1})$ is single-input-single-output and can be treated as the fictitious plant for the proposed Youla parameterization.

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