Lecture 12: Preview Control

Big picture
Problem formulation
Relationship to LQ
Solution

Review: optimal tracking

We consider controlling the system

$$x(k+1) = Ax(k) + Bu(k) \quad (1)$$
$$y(k) = Cx(k)$$

where

$$x \in \mathbb{R}^n,\ u \in \mathbb{R}^m,\ y \in \mathbb{R}^r$$

Optimal tracking with full reference information (homework 1):

$$\min_{U_0} J := \frac{1}{2}[y_d(N) - y(N)]^T S [y_d(N) - y(N)] + \frac{1}{2}\sum_{k=0}^{N-1}\Big([y_d(k) - y(k)]^T Q_y [y_d(k) - y(k)] + u(k)^T R u(k)\Big) \quad (2)$$

$$u^o(k) = -\left[R + B^T P(k+1) B\right]^{-1} B^T \left[P(k+1)Ax(k) + b^T(k+1)\right] \quad (3)$$

$$J_k^o(x(k)) = \frac{1}{2}x^T(k)P(k)x(k) + b(k)x(k) + c(k) \quad (4)$$

Overview of preview control

Preview control considers the same cost-function structure, with:

- **a N_p-step preview window**: the desired output signals in this window are known
- **post preview window**: after the preview window we assume we no longer know the desired output (due to, e.g., limited vision in the example of vehicle driving), but we assume the reference is generated from some models.
- e.g. (deterministic model)

$$y_d(k+N_p+l) = y_d(k+N_p),\ l>0 \tag{5}$$

- or (stochastic model):

$$\begin{aligned} x_d(k+1) &= A_d x_d(k) + B_d w_d(k) \\ y_d(k) &= C_d x_d(k) \end{aligned} \tag{6}$$

where $w_d(k)$ is white and Gaussian distributed. Note: if $A_d = I$, $B_d = 0$, $C_d = I$, $x_d(k+N_p) = y_d(k+N_p)$, then (6)$\Leftrightarrow$(5).

Structuring the future knowledge

Knowledge of the future trajectory can be built into

$$\underbrace{\left[\begin{array}{c} y_d(k+1) \\ y_d(k+2) \\ \vdots \\ y_d(k+N_p) \\ \hline x_d(k+N_p+1) \end{array}\right]}_{Y_d(k+1)} = \underbrace{\left[\begin{array}{cccc|c} 0 & I & 0 & \dots & 0 \\ 0 & 0 & I & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & 0 & C_d \\ \hline 0 & \dots & 0 & 0 & A_d \end{array}\right]}_{A_{Y_d}} \underbrace{\left[\begin{array}{c} y_d(k) \\ y_d(k+1) \\ \vdots \\ y_d(k+N_p-1) \\ \hline x_d(k+N_p) \end{array}\right]}_{Y_d(k)} + \underbrace{\left[\begin{array}{c} 0 \\ 0 \\ \vdots \\ 0 \\ \hline B_d \end{array}\right]}_{B_{Y_d}} \underbrace{w_d(k+N_p)}_{\bar{w}_d(k)} \tag{7}$$

The cost function

At time k

$$J_k = \frac{1}{1+N} \mathrm{E} \Big\{ (y(N+k) - y_d(N+k))^T S_y (y(N+k) - y_d(N+k)) + \sum_{j=0}^{N-1} \Big[(y(j+k) - y_d(j+k))^T Q_y (y(j+k) - y_d(j+k)) + u(j+k)^T R u(j+k) \Big] \Big\} \quad (8)$$

- a moving horizon cost
- only $u(k)$ is applied to the plant after we find a solution to minimize J_k.
- in deterministic formulation, we remove the expectation sign. In stochastic formulation, expectation is taken with respect to

$$\{ w_d(k+N_p), w_d(k+N_p+1), \ldots, w_d(k+N-1) \}$$

for the minimization of J_k.

Augmenting the system

Augmenting the plant with the reference model yields

$$\boxed{ \begin{bmatrix} x(k+1) \\ Y_d(k+1) \end{bmatrix} = \underbrace{\begin{bmatrix} A & 0 \\ 0 & A_{Y_d} \end{bmatrix}}_{A_e} \underbrace{\begin{bmatrix} x(k) \\ Y_d(k) \end{bmatrix}}_{x_e(k)} + \underbrace{\begin{bmatrix} B \\ 0 \end{bmatrix}}_{B_e} u(k) + \underbrace{\begin{bmatrix} 0 \\ B_{Y_d} \end{bmatrix}}_{B_{w,e}} \bar{w}_d(k) } \quad (9)$$

and

$$\begin{aligned} y(j+k) - y_d(j+k) &= Cx(k+j) - [I, 0, \ldots, 0] Y_d(k+j) \\ &= \underbrace{[C, -I, 0, \ldots, 0]}_{C_e} x_e(k+j) \end{aligned}$$

Translation to a standard LQ

$$y(j+k) - y_d(j+k) = \underbrace{[C, -I, 0, \ldots, 0]}_{C_e} x_e(k+j)$$

Hence

$$J_k = \frac{1}{1+N} \mathrm{E}\left\{ (y(N+k) - y_d(N+k))^T S_y (y(N+k) - y_d(N+k)) \right.$$
$$\left. + \sum_{j=0}^{N-1} \left[(y(j+k) - y_d(j+k))^T Q_y (y(j+k) - y_d(j+k)) + u(j+k)^T R u(j+k) \right] \right\}$$

is nothing but

$$J_k = \frac{1}{1+N} \mathrm{E}\left\{ x_e(N+k)^T C_e^T S_y C_e x_e(N+k) \right.$$
$$\left. + \sum_{j=0}^{N-1} \left[x_e(j+k)^T C_e^T Q_y C_e x_e(j+k) + u(j+k)^T R u(j+k) \right] \right\} \quad (10)$$

Solution of the preview control problem

The equivalent formulation

$$x_e(k+1) = A_e x_e(k) + B_e u(k) + B_{w,e} \bar{w}_d(k)$$

$$J_k = \frac{1}{1+N} \mathrm{E}\left\{ x_e(N+k)^T C_e^T S_y C_e x_e(N+k) \right.$$
$$\left. + \sum_{j=0}^{N-1} \left[x_e(j+k)^T C_e^T Q_y C_e x_e(j+k) + u(j+k)^T R u(j+k) \right] \right\}$$

is a standard LQ (deterministic formulation) or a standard LQG problem with exactly known state (stochastic formulation). Hence

$$u^o(k) = -\left[B_e^T P(k+1) B_e + R \right]^{-1} B_e^T P(k+1) A_e x_e(k)$$

$$P(k) = -A_e^T P(k+1) B_e \left[B_e^T P(k+1) B_e + R \right]^{-1} B_e^T P(k+1) A_e$$
$$+ A_e^T P(k+1) A_e + C_e^T Q_y C_e$$

where $P(k+N) = C_e^T S_y C_e$

Remark

Let $u^o(k) = K_e x_e(k) = \begin{bmatrix} K_{e1}(k) & K_{e2}(k) \end{bmatrix} x_e(k)$, the closed-loop matrix is

$$\begin{aligned} A_e - B_e K_e(k) &= \begin{bmatrix} A & 0 \\ 0 & A_{Y_d} \end{bmatrix} - \begin{bmatrix} B \\ 0 \end{bmatrix} \begin{bmatrix} K_{e1}(k) & K_{e2}(k) \end{bmatrix} \\ &= \begin{bmatrix} A - BK_{e1}(k) & -BK_{e2}(k) \\ 0 & A_{Y_d} \end{bmatrix} \end{aligned}$$

- the closed-loop eigenvalue from A_{Y_d} will not be changed.
- The Riccati equation may look ill conditioned if A_{Y_d} contains marginally stable eigenvalues. This, however, does not cause a problem. For additional details, see the course reader or come to the instructor's office hour .

Summary

1. Big picture
2. Formulation of the optimal control problem
3. Translation to a standard LQ