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AN ALL STABILIZING CONTROL STRUCTURE AND ITS APPLICATIONS TO NARROW BAND DISTURBANCE REJECTION IN HARD DISK DRIVES

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Introduction

In hard disk drive (HDD) systems, the compensation of non-repeatable narrow-band disturbances is an important problem for high precision track following [1]. In this paper, we present first a new control structure that generates all the stabilizing controllers for HDD systems. Such a controller parameterization is then applied to provide a compensation scheme for narrow band disturbance rejection.

The Proposed Controller Structure

Figure 1 presents the proposed control structure for improved track following. In the block diagram, $G_p(z^{-1})$ is the plant that includes the dynamics of the HDD servo system; the reference r is zero in track following control; the interested narrow-band disturbances are lumped in $d(k)$ and modeled to enter at the output side of the plant; measurement noise and other runout disturbances are considered as additive errors in $d(k)$. The control aim is to regulate the position of the read/write head such that it follows the desired data track as closely as possible[1]. The plant is stabilized by an existing feedback controller $C(z^{-1})$, which provides the baseline robust stability and performance for the closed loop system. The proposed add-on compensator aims at designing the signal processing in the dash dotted box in **Fig. 1**, so as to minimize the influence of narrow-band disturbances in $d(k)$. Throughout our discussions, we assume all transfer functions are causal.

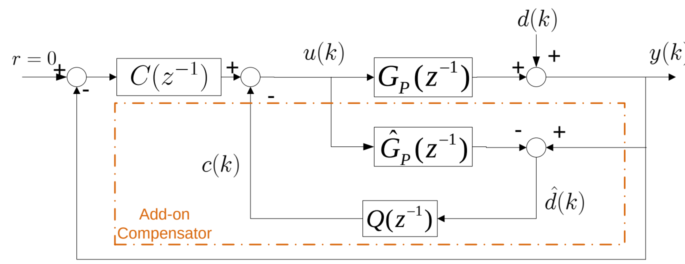

Fig. 1 The proposed control structure.

The proposed compensation scheme contains two main parts: $\hat{G}_p(z^{-1})$, the model of the plant, and a specially designed filter $Q(z^{-1})$. Intuitively, through the parallel configuration of $G_p(z^{-1})$ and $\hat{G}_p(z^{-1})$, an estimate of the disturbance, denoted as $\hat{d}(k)$, is firstly obtained. Designing $Q(z^{-1})$ to selectively inverse $G_p(z^{-1})$ at the interested narrow band disturbance frequencies, one can achieve the desired disturbance cancelation in $d(k)$.

Standard block diagram modification can transform **Fig. 1** to **Fig. 2**, from which the equivalent overall feedback controller is easily seen to be given by

$$C_{eq}(z^{-1}) = \frac{C(z^{-1}) + Q(z^{-1})}{1 - \hat{G}_p(z^{-1})Q(z^{-1})}. \qquad (1)$$

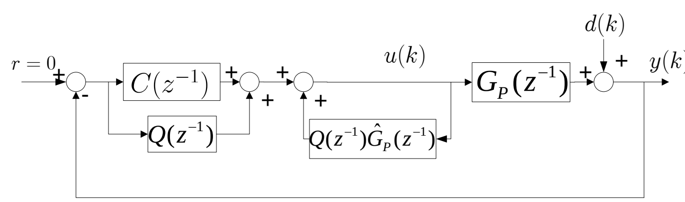

Fig. 2 Equivalent form of the block diagram in **Fig. 1**.

[1] Strictly speaking, the position error signal (PES), i.e., the input to $C(z^{-1})$ is measured instead of $y(k)$. Yet since $y(k)$ = -*PES* in track following, the knowledge of *PES* directly infers that of $y(k)$ in the design process.

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Proposition 1: When $G_p(z^{-1})$ and $C(z^{-1})$ are stable, if $\hat{G}_p(z^{-1}) = G_p(z^{-1})$, then the closed loop system is stable as long as $Q(z^{-1})$ in **Fig. 1** is stable. Moreover, any stabilizing controller (i.e., a controller that stabilizes the closed loop system) can be presented in the form of (1), by choosing a stable $Q(z^{-1})$.

Proof: If $G_p(z^{-1})$ is stable, from the Youla parameterization theory, the structure of all stabilizing controllers is given by (see, e.g., [2]):

$$C_{all}(z^{-1}) = \frac{Q_0(z^{-1})}{1 - G_p(z^{-1})Q_0(z^{-1})}, \quad (2)$$

where $Q_0(z^{-1})$ is a stable causal transfer function.
Letting $C_{eq}(z^{-1})$ in (1) equal to $C_{all}(z^{-1})$ in (2), and using the assumption that $\hat{G}_p(z^{-1}) = G_p(z^{-1})$, we get

$$Q(z^{-1}) = \left(1 + G_p(z^{-1})C(z^{-1})\right)Q_0(z^{-1}) - C(z^{-1}), \quad (3)$$

$$Q_0(z^{-1}) = \frac{Q(z^{-1}) + C(z^{-1})}{1 + G_p(z^{-1})C(z^{-1})}. \quad (4)$$

For any stabilizing controller $C_{all}(z^{-1})$ given by (2), a unique and stable $Q_0(z^{-1})$ is defined. Note that $G_p(z^{-1})$ and $C(z^{-1})$ are both stable. Equation (3) thus provides a stable (and unique) $Q(z^{-1})$ that yields $C_{eq}(z^{-1}) = C_{all}(z^{-1})$ in (1). Therefore, $C_{all}(z^{-1})$ can be realized by the proposed controller structure. Similarly, $\forall$ a stable $Q(z^{-1})$, (4) provides a stable $Q_0(z^{-1})$, yielding a stabilizing controller $C_{all}(z^{-1})$. Using now $C_{eq}(z^{-1}) = C_{all}(z^{-1})$, we conclude that $C_{eq}(z^{-1})$ is also stabilizing.

■

In practical HDD systems, $G_p(z^{-1})$ has the characteristics of a damped second order system plus several high frequency resonance modes, and is often designed to be open loop stable [3]. In addition, stable nominal controllers are usually favored for $C(z^{-1})$. Therefore, the stability assumption on $G_p(z^{-1})$ and $C(z^{-1})$ are not difficult to satisfy in practice. Proposition 1 then infers that any controller that stabilizes the closed loop can be generated by choosing an appropriate $Q(z^{-1})$. Note that in **Fig. 1**, when $G_p(z^{-1}) = \hat{G}_p(z^{-1})$, the transfer function from $u(k)$ to $\hat{d}(k)$ is null, and $Q(z^{-1})$ plays the role of loop shaping.

Proposition 2: Given the same assumptions as those in Proposition 1, switching between different stable $Q(z^{-1})$'s does not lead to instability to the closed loop system. More generally, the closed loop system is stable if $Q(z^{-1})$ is time-varying but stable.

Proof: Omitted due to space limit.
Remark: Proposition 2 is particularly useful in adaptive configurations, where the loop shape has to be adjusted online for compensating uncertain or time-varying disturbances.

Perfect plant modeling has been assumed in the proceeding analysis, and the nominal stability of the proposed control structure has been shown by Proposition 1. To discuss the closed loop robust stability, one can derive from **Fig. 2**, the output sensitivity function:

$$G_{d2y}(z^{-1}) = \frac{1 - \hat{G}_p(z^{-1})Q(z^{-1})}{1 + G_p(z^{-1})C(z^{-1}) + Q(z^{-1})\left(G_p(z^{-1}) - \hat{G}_p(z^{-1})\right)}. \quad (5)$$

It is now seen that poles of the sensitivity function are composed of two parts: the (invariant) poles of the newly introduced $\hat{G}_p(z^{-1})Q(z^{-1})$, and the roots of the characteristic equation: $1 + G_p(z^{-1})\ C(z^{-1}) + Q(z^{-1})(G_p(z^{-1}) - \hat{G}_p(z^{-1})) = 0$. When there exist multiplicative model uncertainty $\Delta(z^{-1})$ and $G_p(z^{-1}) = \hat{G}_p(z^{-1})(1+\Delta(z^{-1}))$, Applying the small gain theory yields that the closed loop system is robust stable if

$$\left\|Q(z^{-1})\right\|_\infty < \left\|\frac{1 + G_p(z^{-1})C(z^{-1})}{\Delta(z^{-1})\hat{G}_p(z^{-1})}\right\|_\infty, \quad (6)$$

where $\|\cdot\|_\infty$ denotes taking the infinity norm of a transfer function. Equation (6) indicates that the magnitude of $Q(z^{-1})$ should be small at the frequencies where the plant is largely uncertain. For the specific HDD narrow-band rejection problem, the interested disturbances occur usually below 1.5 kHz [1], while large plant uncertainties appear above

3 kHz, where the lightly damped (and possibly time-varying) resonance modes and other unstructed perturbations are difficult to fully model. Therefore the proposed system still has the required capacity for disturbance compensation below 1.5 kHz.

Application to Narrow-band Disturbance Rejection

We discuss now one specific $Q(z^{-1})$ design for narrow-band disturbance rejection in HDD systems. In this problem, the vibration disturbance can be modeled as the sum of several sinusoidal signals, whose energy is largely concentrated at multiple frequencies [1]. From (5), we see that, to achieve rejection of the vibration disturbance, it suffices if the frequency response $1-\hat{G}_p(e^{-j\omega})Q(e^{-j\omega})$ has small magnitude when $\omega = \omega_i$'s, where ω_i is the i_{th} interested disturbance frequency. Additionally, it is desired to bring as little influence as possible to the loop shape at other frequency regions, where the existing controller $C(z^{-1})$ is usually well tuned for good baseline system performance.

In HDD systems, $\hat{G}_p(z^{-1})$ usually approximates a second order damped system with a pair of stable poles below 200 Hz, in the high frequency region, $\hat{G}_p(z^{-1})$ has a double integrator type dynamics, with a real zero in the z-plane. If unstable, the high frequency zero can be shifted to the inside of the unit circle, creating a strictly stable inverse of $\hat{G}_p(z^{-1})$. This change of high frequency nominal modeling in $\hat{G}_p(z^{-1})$ has small influence to the middle and low frequency disturbance rejection[2]. Such a design step can also be supported from the robust stability requirement discussed in the last section, where we note that regardless of the possible high frequency unstable zero, the magnitude of $Q(z^{-1})$ has to be kept small above 3 kHz for robust stability.

It is thus feasible to design $Q(z^{-1})=\hat{G}_p^{-1}(z^{-1})Q_0(z^{-1})$, such that

$$1-G_p(z^{-1})Q(z^{-1})\approx 1-Q_0(z^{-1}), \quad (7)$$

[2] An optimization problem can be constructed to guide the unstable zero manipulation design, where aiming at achieving accurate model match in the interested middle and low frequency regions, and the mean time constraints the high frequency magnitude of the inverse transfer function.

where $Q_0(z^{-1})$ is a combinations of band-pass filters designed to pass only the interested narrow-band components in the disturbance estimate $\hat{d}(k)$.

The HDD benchmark problem in [3] was applied for the design demonstration. Recursive least squares was used to identify the vibration frequencies. **Figure 3** demonstrates the FFT of the position error signals with and without compensation. It can be observed that the proposed algorithm effectively attenuates the narrow-band disturbances around 800 Hz and 1100 Hz, with small influence to the error spectrum at other frequencies.

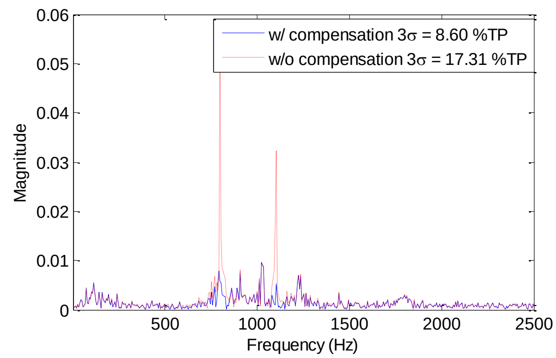

Fig. 3 FFT of the PES with and without compensation.

Conclusion

In this paper, a control structure is discussed for loop shaping design in control systems. It was shown that under mild conditions, such a control scheme generates all the stabilizing controllers, and in addition, can be readily adopted for adaptive control. The proposed algorithm was applied to narrow-band disturbance rejection, which was supported by simulation results on a HDD benchmark problem.

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