

Homework #2

Due July 1 @ 11:59pm

Submission requirements

Upload a **single PDF file** of your IJulia notebook for this entire assignment. Clearly denote which question each section of your PDF corresponds to.

Problem 1 -- Life on Mars?

It has been determined that the most cost-effective way to feed our colony on Mars is to farm genetically modified bacteria that contain all the nutrients required for a well-balanced diet (yum!). The food production farm anticipates needing to produce the following numbers of tons of bacteria each of the next six months:

Month	Jul	Aug	Sep	Oct	Nov	Dec
Demand	10	5	8	20	15	3

The current rate of production is 10 tons/month. The production can be increased by introducing a carefully constructed enzyme cocktail that costs \$240 per ton of increased production. Similarly, the production can be slowed with a different set of enzymes that cost \$210 per ton of decreased production.

The bacteria can be stored in a cryogenic chamber for use at a later time. It costs \$60 per ton to hold up to 3 tons of bacteria in storage. If more than 3 tons are kept in storage, an extra cooling unit must be turned on, so the cost for any additional inventory is \$75 per ton. There is 1 ton currently in storage.

If demand is not met in a month, it must be met by the end of the year and costs \$80 per ton of missed demand resulting from the costs of feeding the colonists in other, less nutrient-rich ways. We also require that there are at least 2 tons in storage at the end of December, since production typically goes down in January.

How should we produce bacteria in the next six months to meet demand while minimizing costs?

Would you save money or lose money if you instead met demand every month? Why, and how much money do you save/lose?

Problem 2 -- Martian Transports

Now that the colony has been established on Mars, we must figure out how to efficiently move our 25 colonists to different areas, using our complex, fully enclosed tunnel system.

There are five different locations in the colony that need to be accessible. The options are to build a tunnel between a pair of locations, to provide air-based transport between a pair of locations, or to use our newly discovered teleportation technology between a pair of locations.

The locations must be visited in a specific order to ensure safe air-cycling and decontamination procedures:

1. Airlock
2. Medbay
3. Mess hall
4. Science labs
5. Dormitories

On each leg of this sequence of locations, the colonists must use one of the three types of transportation (tunnel, aircraft, teleportation). We'll only install the infrastructure once, so all colonists must use the same mode of transportation between the same pair of locations (e.g., only a tunnel exists between locations 2 and 3).

The following table lists the cost per colonist of installing each of the three modes along each leg:

Pairs of locations	1-2	2-3	3-4	4-5
Tunnel	30	25	40	60
Aircraft	25	40	45	50
Teleportation	40	20	50	45

Because of the infrastructure required to build things on Mars, it also costs money to change from one type of transportation to another. For example, if you start with a tunnel on route 1-2, it costs \$0 to build a tunnel from 2-3, but it costs \$500 to then build an aircraft that goes from 2-3 or \$1200 to build a teleportation system from 2-3.

From/To	Tunnel	Aircraft	Teleportation
Tunnel	0	500	1200
Aircraft	800	0	1000
Teleportation	1500	1000	0

How should the transportation be installed to minimize the cost? What is the minimum cost for transporting the 25 colonists from location 1 to location 5?

Problem 3 -- Mission to Mars

We are trying to select flight crews for 4 different missions to establish our first colony on Mars. Each flight crew consists of two people, and there are four skillsets that are considered "mission critical" that our 8 astronauts have in varying degrees. Each astronaut has reported her or his skill on these four areas, with "0" meaning they are an amateur and "20" meaning they are an expert. The table below shows each astronaut's skillsets:

Skill	Astronaut 1	2	3	4	5	6	7	8
Computing/IT	20	14	0	13	0	0	8	8
Piloting	12	0	0	10	15	20	8	9
Mechanical engineering	0	20	12	0	8	11	14	12
Health/safety	0	0	0	0	17	0	0	16

To further complicate matters, each astronaut also has varying levels of skill (again ranked 0-20) in 5 different areas related to establishing the colony. The table below summarizes these data:

Skill	Astronaut 1	2	3	4	5	6	7	8
Agriculture	18	12	15	0	0	0	8	0
Electronics	10	0	9	14	15	8	12	13
Astrobiology	0	17	0	11	13	10	0	0
Air \& water	0	0	14	0	0	12	16	0
Geology	0	0	0	0	12	18	0	18

To create a valid crew, we must ensure that every pair of astronauts has at least 10/20 in at least two different skill areas, both for the flight portion and for the colony establishment.

Answer the following questions regarding this mission plan. For the first 3 questions, do not build an optimization model to answer them.

(a) Is it possible to construct a plan where each of the 8 astronauts gets to fly?

(b) If yes, find at least two valid assignments of the 8 astronauts to the 4 flights. If no, soften the requirement to the skills being nonzero in at least two different areas for each set of skills and find two valid crews.

(c) For the valid assignments you found in part (b), list all the skills in which at least one astronaut has a score of $\geq 10/20$ on each of the 4 flights.

Now build an optimization problem to answer the following question:

(d) What is the assignment of pilots to flights that gives the maximum total score across all skillsets (ignoring the requirement of skills being at least 10/20)?

OPTIONAL ALTERATION TO THIS MODEL: If you have already built and solved your model, you do not need to modify it at this point. As written, this problem has a trivial structure so is only interesting from a modeling standpoint. If you would like to make your solution slightly more interesting, you have the option of adding the following piece to it:

Suppose further that each of the 8 astronauts has stated their preferences (10 being most, 0 least) for each of the 4 possible flights. Change your model to maximize the total score across all skillsets PLUS the total preference of the astronauts to the flights.

Flight	Astronaut	1	2	3	4	5	6	7	8
1		1	5	4	3	6	4	6	4
2		3	0	0	5	1	0	0	10
3		2	1	3	0	7	8	4	1
4		8	1	4	5	10	1	0	0

Problem 4 -- Keeping Mars Safe

Our Martian colony -- Terra Nova -- is split into three districts: Financial, Residential, and Commercial. To travel around Terra Nova, everyone shares a set of oxygen tanks and pressure suits (collectively referred to as "safety equipment") that are distributed all over the districts. The numbers of oxygen tanks and pressure suits currently in each district are given below:

District	Oxygen	Pressure suits
Financial	45	210
Residential	150	250
Commercial	120	190

There are two major transportation hubs in the colony. The safety equipment gets returned to the transportation hubs each night for re-use the following day. The previous strategic planner determined that the ratio of oxygen tanks to pressure suits (can be thought of as percent of the total amount of safety equipment) must be no more than ± 5 percentage points different from the whole colony (i.e., if 30% of the colony's safety equipment is oxygen tanks, Hub 1 must keep oxygen tanks between 25 and 35% of the safety equipment). Assume for simplicity that the current percent of oxygen tanks in the colony is 49%.

The distances (in kilometers) between the districts and the transportation hubs is as follows:

District	Hub 1	Hub 2
Financial	1.0	2.3
Residential	2.1	0.9
Commercial	1.5	1.1

Each transportation hub must have between 350 and 500 units of safety equipment each night. Determine how to transport safety equipment to the transportation hubs tonight to meet requirements while minimizing the total distance the equipment must be moved. Note that you may not need to move every single unit of safety equipment to meet these requirements.

In []: