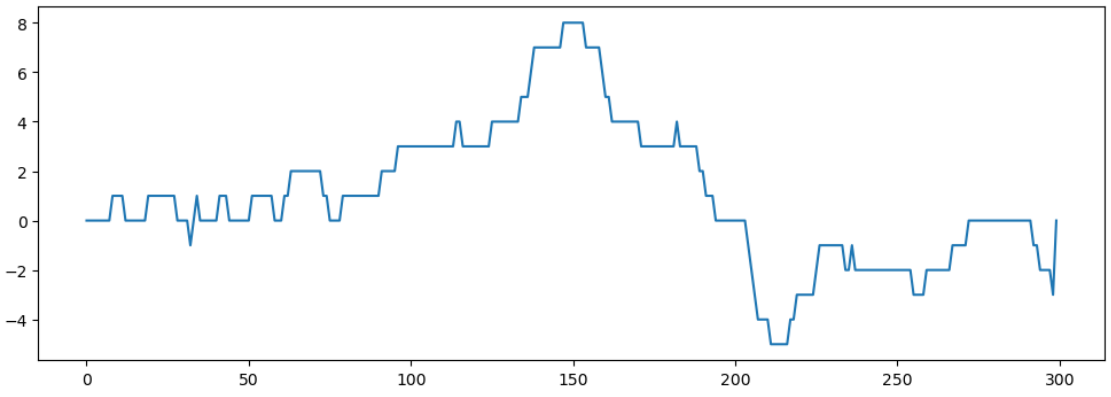
hw4

July 15, 2020

1 problem 1

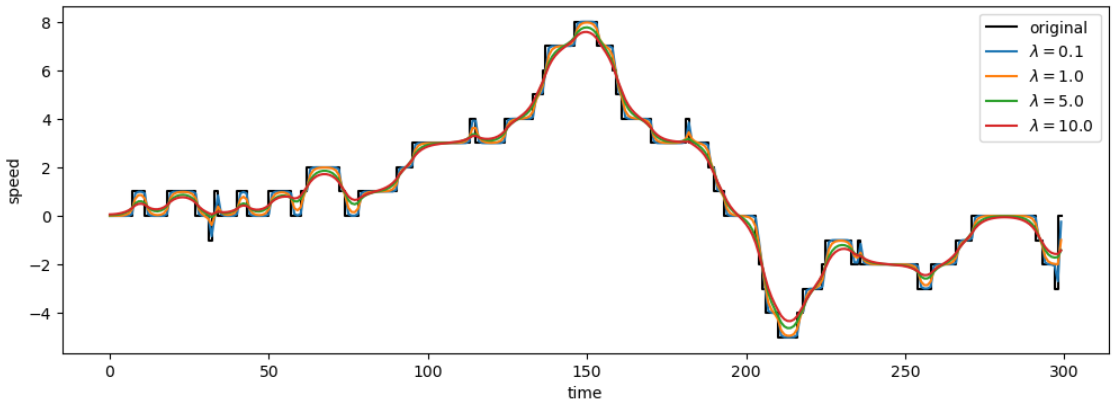
[1]:
```
using Random
# set a seed so we get the same output every time
seed = 393845
Random.seed!(seed)
# initialize the vector of speeds
val = 0; u = zeros(300); u[1] = val
# set a density that determines how often the speed changes # low density␣
 ↪corresponds to infrequent speed changes
dens = 0.1
# build speed vector for all times between now and time 299
for i in 2:299
    # if a uniform(0,1) variable is < density
    if rand() < dens
        # increase the speed by 1 mph
        val = val + 1
        u[i] = val
        # if a uniform(0,1) variable is >= 1 - density
    elseif rand() >= 1.0-dens
        # decrease the speed by 1 mph
        val = val - 1
        u[i] = val
    else# otherwise the speed stays the same
        u[i] = val
    end
end
# the final speed must be 0
u[300] = 0
# T = 300
T = length(u)
# plot the speeds (your figure should match the one in the assignment!)

using PyPlot
figure(figsize=(12,4))
plot(u,"-");
```



[2]:
```
import Pkg
Pkg.add("Gurobi")
T = 300
# try a variety of different regularization weights
lambdavals = [0.1, 1, 5, 10]
k = length(lambdavals)
y = zeros(T,k)


using JuMP, Gurobi
for (i,lambda) = enumerate(lambdavals)
    m = Model(Gurobi.Optimizer)
    set_optimizer_attribute(m,"OutputFlag",false)

    @variable(m, optv[1:T])

    @expression(m, LS, sum((u[i] - optv[i])^2 for i = 1:T))
    @expression(m, Rsmooth, sum((optv[i+1] - optv[i])^2 for i = 1:T-1))

    @objective(m, Min, LS + lambda*Rsmooth)

    optimize!(m)
    y[:,i] = value.(optv)
end

using PyPlot
figure(figsize=(12,4))
step(u,"k-",label="original")
for (i,lambda) = enumerate(lambdavals)
    plot(y[:,i],label=string(L"\lambda = ", lambda))
end
legend(loc ="best")
ylabel("speed")
```

```
xlabel("time");
```

```
  Updating registry at `~/.julia/registries/General`
  Updating git-repo
`https://github.com/JuliaRegistries/General.git`
[1mFetching: [========================================>]
100.0 %.0 % Resolving package versions…
```




```
  Updating `~/.julia/environments/v1.3/Project.toml`
 [no changes]
  Updating `~/.julia/environments/v1.3/Manifest.toml`
 [no changes]
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```

1.0.1 Plot the Pareto Curve.

[3]:
```
# create a function that takes a value for  then solves and reports solution␣
 ↪of tradeoff problem
function solveOpt(x)
    m = Model(Gurobi.Optimizer)
    @variable(m, optv[1:T])
    @variable(m, dv[1:T-1])
    @constraint(m, optv[1]==0)
    @constraint(m, optv[T]==0)
    for i = 1:T-1
        @constraint(m, dv[i] == optv[i+1] - optv[i])
```

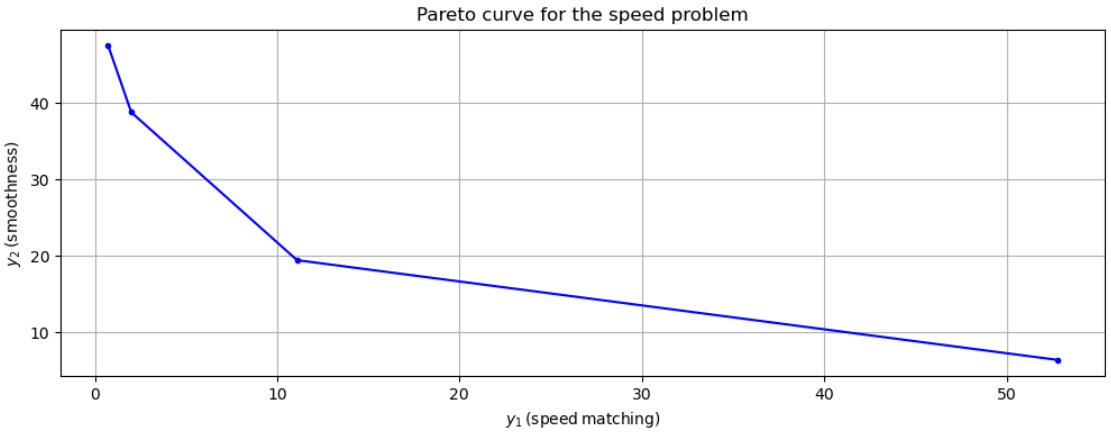
```
    end
    @objective(m, Min, sum(dv.^2) + x*sum((optv - u).^2))

    optimize!(m)
    y2 = value(sum(dv.^2))
    y1 = value(sum((optv - u).^2))
    opt = value.(optv)

    return (y1, y2, opt)
end

Npts = 4
y1 = zeros(Npts)
y2 = zeros(Npts)
for i in 1:4
    (y1[i],y2[i],x) = solveOpt(lambdavals[i])
end


# plot the points
using PyPlot
figure(figsize=(12,4))
plot( y1, y2, "b.-" )
xlabel(L"y_1\, (\sf speed\, matching)")
ylabel(L"y_2\, (\sf smoothness)");
title("Pareto curve for the speed problem")
grid()
```




```
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Gurobi Optimizer version 9.0.2 build v9.0.2rc0 (mac64)
Optimize a model with 301 rows, 599 columns and 899 nonzeros
```

```
Model fingerprint: 0x6ef1afe0
Model has 599 quadratic objective terms
Coefficient statistics:
  Matrix range     [1e+00, 1e+00]
  Objective range  [2e-01, 2e+00]
  QObjective range [2e-01, 2e+00]
  Bounds range     [0e+00, 0e+00]
  RHS range        [0e+00, 0e+00]
Presolve removed 2 rows and 2 columns
Presolve time: 0.00s
Presolved: 299 rows, 597 columns, 895 nonzeros
Presolved model has 597 quadratic objective terms
Ordering time: 0.00s

Barrier statistics:
 Free vars  : 597
 AA' NZ     : 2.980e+02
 Factor NZ  : 4.890e+03
 Factor Ops : 1.023e+05 (less than 1 second per iteration)
 Threads    : 1

                 Objective                Residual
Iter       Primal          Dual         Primal    Dual     Compl     Time
   0   2.68200000e+02  2.68200000e+02  0.00e+00 3.20e+00  0.00e+00     0s
   1   1.16930192e+01  1.16935322e+01  1.17e-15 3.20e-06  0.00e+00     0s
   2   1.16930192e+01  1.16930192e+01  4.02e-16 3.20e-12  0.00e+00     0s

Barrier solved model in 2 iterations and 0.00 seconds
Optimal objective 1.16930192e+01

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Gurobi Optimizer version 9.0.2 build v9.0.2rc0 (mac64)
Optimize a model with 301 rows, 599 columns and 899 nonzeros
Model fingerprint: 0x4fcf4f25
Model has 599 quadratic objective terms
Coefficient statistics:
  Matrix range     [1e+00, 1e+00]
  Objective range  [2e+00, 2e+01]
  QObjective range [2e+00, 2e+00]
  Bounds range     [0e+00, 0e+00]
  RHS range        [0e+00, 0e+00]
Presolve removed 2 rows and 2 columns
Presolve time: 0.00s
Presolved: 299 rows, 597 columns, 895 nonzeros
Presolved model has 597 quadratic objective terms
Ordering time: 0.00s
```

```
Barrier statistics:
 Free vars  : 597
 AA' NZ     : 2.980e+02
 Factor NZ  : 4.890e+03
 Factor Ops : 1.023e+05 (less than 1 second per iteration)
 Threads    : 1

                 Objective                Residual
Iter       Primal          Dual         Primal    Dual     Compl     Time
   0   2.68200000e+03  2.68200000e+03  0.00e+00 8.00e+00  0.00e+00     0s
   1   3.05244755e+01  3.05297784e+01  2.11e-15 8.00e-06  0.00e+00     0s
   2   3.05244755e+01  3.05244755e+01  1.61e-15 8.00e-12  0.00e+00     0s

Barrier solved model in 2 iterations and 0.00 seconds
Optimal objective 3.05244755e+01

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Gurobi Optimizer version 9.0.2 build v9.0.2rc0 (mac64)
Optimize a model with 301 rows, 599 columns and 899 nonzeros
Model fingerprint: 0x317aa5e7
Model has 599 quadratic objective terms
Coefficient statistics:
  Matrix range     [1e+00, 1e+00]
  Objective range  [1e+01, 8e+01]
  QObjective range [2e+00, 1e+01]
  Bounds range     [0e+00, 0e+00]
  RHS range        [0e+00, 0e+00]
Presolve removed 2 rows and 2 columns
Presolve time: 0.00s
Presolved: 299 rows, 597 columns, 895 nonzeros
Presolved model has 597 quadratic objective terms
Ordering time: 0.00s

Barrier statistics:
 Free vars  : 597
 AA' NZ     : 2.980e+02
 Factor NZ  : 4.890e+03
 Factor Ops : 1.023e+05 (less than 1 second per iteration)
 Threads    : 1

                 Objective                Residual
Iter       Primal          Dual         Primal    Dual     Compl     Time
   0   1.34100000e+04  1.34100000e+04  0.00e+00 2.00e+01  0.00e+00     0s
   1   4.87347025e+01  4.87614250e+01  3.19e-15 2.00e-05  0.00e+00     0s
   2   4.87347025e+01  4.87347025e+01  1.33e-15 2.00e-11  0.00e+00     0s

Barrier solved model in 2 iterations and 0.00 seconds
```

```
Optimal objective 4.87347025e+01

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Gurobi Optimizer version 9.0.2 build v9.0.2rc0 (mac64)
Optimize a model with 301 rows, 599 columns and 899 nonzeros
Model fingerprint: 0xde4e6344
Model has 599 quadratic objective terms
Coefficient statistics:
  Matrix range     [1e+00, 1e+00]
  Objective range  [2e+01, 2e+02]
  QObjective range [2e+00, 2e+01]
  Bounds range     [0e+00, 0e+00]
  RHS range        [0e+00, 0e+00]
Presolve removed 2 rows and 2 columns
Presolve time: 0.00s
Presolved: 299 rows, 597 columns, 895 nonzeros
Presolved model has 597 quadratic objective terms
Ordering time: 0.00s

Barrier statistics:
 Free vars  : 597
 AA' NZ     : 2.980e+02
 Factor NZ  : 4.890e+03
 Factor Ops : 1.023e+05 (less than 1 second per iteration)
 Threads    : 1

                 Objective                Residual
Iter       Primal          Dual         Primal    Dual     Compl     Time
   0   2.68200000e+04  2.68200000e+04  0.00e+00 2.00e+01  0.00e+00     0s
   1   5.47446843e+01  5.47982147e+01  2.94e-15 2.00e-05  0.00e+00     0s
   2   5.47446842e+01  5.47446843e+01  1.24e-15 2.00e-11  0.00e+00     0s

Barrier solved model in 2 iterations and 0.00 seconds
Optimal objective 5.47446842e+01
```
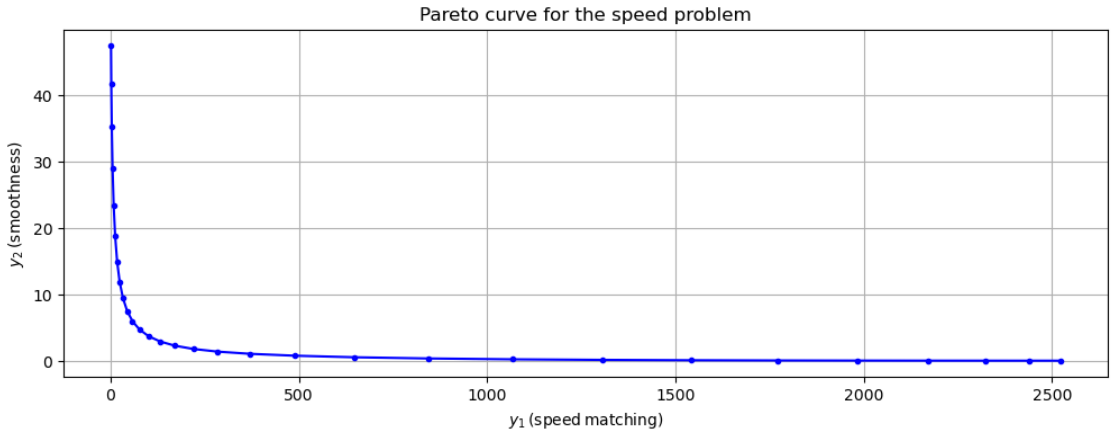

[4]:
```
Npts = 30
y1 = zeros(Npts)
y2 = zeros(Npts)

println(10 .^(range(-5,stop=1,length=Npts)))
for (i, ) in enumerate(10 .^(range(-5,stop=1,length=Npts)))
    (y1[i],y2[i],uu) = solveOpt( )
end;

# plot the points
```

```
using PyPlot
figure(figsize=(12,4))
plot( y1, y2, "b.-" )
xlabel(L"y_1\, (\sf speed\, matching)")
ylabel(L"y_2\, (\sf smoothness)");
title("Pareto curve for the speed problem")
grid()
```




```
[1.0e-5, 1.610262027560939e-5, 2.592943797404667e-5, 4.1753189365604006e-5,
6.723357536499335e-5, 0.00010826367338740541, 0.00017433288221999874,
0.000280721620394117 6, 0.00045203536563602454, 0.0007278953843983154,
0.001172102297533 4804, 0.0018873918221350976, 0.0030391953823131978,
0.004893900918477494, 0.007880462815669913, 0.01268961003167922,
0.020433597178569417, 0.03290344562312668, 0.05298316906283707,
0.0853167852417281, 0.1373823795883263, 0.2212216291070449, 0.3562247890262442,
0.5736152510448679, 0.9236708571873862, 1.4873521072935114, 2.3950266199874855,
3.856620421163471, 6.2101694189156165, 10.0]
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Gurobi Optimizer version 9.0.2 build v9.0.2rc0 (mac64)
Optimize a model with 301 rows, 599 columns and 899 nonzeros
Model fingerprint: 0xe05e8954
Model has 599 quadratic objective terms
Coefficient statistics:
  Matrix range     [1e+00, 1e+00]
  Objective range  [2e-05, 2e-04]
  QObjective range [2e-05, 2e+00]
  Bounds range     [0e+00, 0e+00]
  RHS range        [0e+00, 0e+00]
Presolve removed 2 rows and 2 columns
Presolve time: 0.00s
Presolved: 299 rows, 597 columns, 895 nonzeros
```

```
Presolved model has 597 quadratic objective terms
Ordering time: 0.00s

Barrier statistics:
 Free vars  : 597
 AA' NZ     : 2.980e+02
 Factor NZ  : 4.890e+03
 Factor Ops : 1.023e+05 (less than 1 second per iteration)
 Threads    : 1

                 Objective                Residual
Iter       Primal          Dual         Primal    Dual     Compl     Time
   0   2.68200000e-02  2.68200000e-02  0.00e+00 2.05e-02  0.00e+00     0s
   1   2.59905498e-02  2.59905515e-02  9.08e-16 2.05e-08  0.00e+00     0s

Barrier solved model in 1 iterations and 0.00 seconds
Optimal objective 2.59905498e-02

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Gurobi Optimizer version 9.0.2 build v9.0.2rc0 (mac64)
Optimize a model with 301 rows, 599 columns and 899 nonzeros
Model fingerprint: 0xe44814fa
Model has 599 quadratic objective terms
Coefficient statistics:
  Matrix range     [1e+00, 1e+00]
  Objective range  [3e-05, 3e-04]
  QObjective range [3e-05, 2e+00]
  Bounds range     [0e+00, 0e+00]
  RHS range        [0e+00, 0e+00]
Presolve removed 2 rows and 2 columns
Presolve time: 0.00s
Presolved: 299 rows, 597 columns, 895 nonzeros
Presolved model has 597 quadratic objective terms
Ordering time: 0.00s

Barrier statistics:
 Free vars  : 597
 AA' NZ     : 2.980e+02
 Factor NZ  : 4.890e+03
 Factor Ops : 1.023e+05 (less than 1 second per iteration)
 Threads    : 1

                 Objective                Residual
Iter       Primal          Dual         Primal    Dual     Compl     Time
   0   4.31872276e-02  4.31872276e-02  0.00e+00 3.30e-02  0.00e+00     0s
   1   4.11207765e-02  4.11207807e-02  5.73e-16 3.30e-08  0.00e+00     0s
```

```
Barrier solved model in 1 iterations and 0.00 seconds
Optimal objective 4.11207765e-02

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Gurobi Optimizer version 9.0.2 build v9.0.2rc0 (mac64)
Optimize a model with 301 rows, 599 columns and 899 nonzeros
Model fingerprint: 0xb6286d30
Model has 599 quadratic objective terms
Coefficient statistics:
  Matrix range     [1e+00, 1e+00]
  Objective range  [5e-05, 4e-04]
  QObjective range [5e-05, 2e+00]
  Bounds range     [0e+00, 0e+00]
  RHS range        [0e+00, 0e+00]
Presolve removed 2 rows and 2 columns
Presolve time: 0.00s
Presolved: 299 rows, 597 columns, 895 nonzeros
Presolved model has 597 quadratic objective terms
Ordering time: 0.00s

Barrier statistics:
 Free vars  : 597
 AA' NZ     : 2.980e+02
 Factor NZ  : 4.890e+03
 Factor Ops : 1.023e+05 (less than 1 second per iteration)
 Threads    : 1

                 Objective                Residual
Iter       Primal          Dual         Primal    Dual     Compl     Time
   0   6.95427526e-02  6.95427526e-02  0.00e+00 5.31e-02  0.00e+00     0s
   1   6.44969777e-02  6.44969878e-02  4.87e-16 5.31e-08  0.00e+00     0s

Barrier solved model in 1 iterations and 0.00 seconds
Optimal objective 6.44969777e-02

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Gurobi Optimizer version 9.0.2 build v9.0.2rc0 (mac64)
Optimize a model with 301 rows, 599 columns and 899 nonzeros
Model fingerprint: 0x29ccd1e1
Model has 599 quadratic objective terms
Coefficient statistics:
  Matrix range     [1e+00, 1e+00]
  Objective range  [8e-05, 7e-04]
  QObjective range [8e-05, 2e+00]
  Bounds range     [0e+00, 0e+00]
  RHS range        [0e+00, 0e+00]
```

```
Presolve removed 2 rows and 2 columns
Presolve time: 0.00s
Presolved: 299 rows, 597 columns, 895 nonzeros
Presolved model has 597 quadratic objective terms
Ordering time: 0.00s

Barrier statistics:
 Free vars  : 597
 AA' NZ     : 2.980e+02
 Factor NZ  : 4.890e+03
 Factor Ops : 1.023e+05 (less than 1 second per iteration)
 Threads    : 1

                 Objective                Residual
Iter       Primal          Dual         Primal    Dual     Compl     Time
   0   1.11982054e-01  1.11982054e-01  0.00e+00 4.28e-02  0.00e+00     0s
   1   9.99928095e-02  9.99928335e-02  1.14e-15 4.28e-08  0.00e+00     0s

Barrier solved model in 1 iterations and 0.00 seconds
Optimal objective 9.99928095e-02

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Gurobi Optimizer version 9.0.2 build v9.0.2rc0 (mac64)
Optimize a model with 301 rows, 599 columns and 899 nonzeros
Model fingerprint: 0x1fb94c79
Model has 599 quadratic objective terms
Coefficient statistics:
  Matrix range     [1e+00, 1e+00]
  Objective range  [1e-04, 1e-03]
  QObjective range [1e-04, 2e+00]
  Bounds range     [0e+00, 0e+00]
  RHS range        [0e+00, 0e+00]
Presolve removed 2 rows and 2 columns
Presolve time: 0.00s
Presolved: 299 rows, 597 columns, 895 nonzeros
Presolved model has 597 quadratic objective terms
Ordering time: 0.00s

Barrier statistics:
 Free vars  : 597
 AA' NZ     : 2.980e+02
 Factor NZ  : 4.890e+03
 Factor Ops : 1.023e+05 (less than 1 second per iteration)
 Threads    : 1

                 Objective                Residual
Iter       Primal          Dual         Primal    Dual     Compl     Time
```

```
   0   1.80320449e-01  1.80320449e-01  0.00e+00 6.88e-02  0.00e+00     0s
   1   1.52790310e-01  1.52790365e-01  1.92e-15 6.88e-08  0.00e+00     0s

Barrier solved model in 1 iterations and 0.00 seconds
Optimal objective 1.52790310e-01

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Gurobi Optimizer version 9.0.2 build v9.0.2rc0 (mac64)
Optimize a model with 301 rows, 599 columns and 899 nonzeros
Model fingerprint: 0xb6ca5cea
Model has 599 quadratic objective terms
Coefficient statistics:
  Matrix range     [1e+00, 1e+00]
  Objective range  [2e-04, 2e-03]
  QObjective range [2e-04, 2e+00]
  Bounds range     [0e+00, 0e+00]
  RHS range        [0e+00, 0e+00]
Presolve removed 2 rows and 2 columns
Presolve time: 0.00s
Presolved: 299 rows, 597 columns, 895 nonzeros
Presolved model has 597 quadratic objective terms
Ordering time: 0.00s

Barrier statistics:
 Free vars  : 597
 AA' NZ     : 2.980e+02
 Factor NZ  : 4.890e+03
 Factor Ops : 1.023e+05 (less than 1 second per iteration)
 Threads    : 1

                 Objective                Residual
Iter       Primal          Dual         Primal    Dual     Compl     Time
   0   2.90363172e-01  2.90363172e-01  0.00e+00 1.11e-01  0.00e+00     0s
   1   2.29560196e-01  2.29560317e-01  1.24e-15 1.11e-07  0.00e+00     0s
   2   2.29560196e-01  2.29560196e-01  3.09e-18 1.11e-13  0.00e+00     0s

Barrier solved model in 2 iterations and 0.00 seconds
Optimal objective 2.29560196e-01

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Gurobi Optimizer version 9.0.2 build v9.0.2rc0 (mac64)
Optimize a model with 301 rows, 599 columns and 899 nonzeros
Model fingerprint: 0xf2dd4d31
Model has 599 quadratic objective terms
Coefficient statistics:
  Matrix range     [1e+00, 1e+00]
```

```
  Objective range  [3e-04, 3e-03]
  QObjective range [3e-04, 2e+00]
  Bounds range     [0e+00, 0e+00]
  RHS range        [0e+00, 0e+00]
Presolve removed 2 rows and 2 columns
Presolve time: 0.00s
Presolved: 299 rows, 597 columns, 895 nonzeros
Presolved model has 597 quadratic objective terms
Ordering time: 0.00s

Barrier statistics:
 Free vars  : 597
 AA' NZ     : 2.980e+02
 Factor NZ  : 4.890e+03
 Factor Ops : 1.023e+05 (less than 1 second per iteration)
 Threads    : 1

                 Objective                Residual
Iter       Primal          Dual         Primal    Dual     Compl     Time
   0   4.67560790e-01  4.67560790e-01  0.00e+00 8.93e-02  0.00e+00     0s
   1   3.38518024e-01  3.38518282e-01  1.34e-15 8.93e-08  0.00e+00     0s

Barrier solved model in 1 iterations and 0.00 seconds
Optimal objective 3.38518024e-01

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Gurobi Optimizer version 9.0.2 build v9.0.2rc0 (mac64)
Optimize a model with 301 rows, 599 columns and 899 nonzeros
Model fingerprint: 0x97252e93
Model has 599 quadratic objective terms
Coefficient statistics:
  Matrix range     [1e+00, 1e+00]
  Objective range  [6e-04, 4e-03]
  QObjective range [6e-04, 2e+00]
  Bounds range     [0e+00, 0e+00]
  RHS range        [0e+00, 0e+00]
Presolve removed 2 rows and 2 columns
Presolve time: 0.00s
Presolved: 299 rows, 597 columns, 895 nonzeros
Presolved model has 597 quadratic objective terms
Ordering time: 0.00s

Barrier statistics:
 Free vars  : 597
 AA' NZ     : 2.980e+02
 Factor NZ  : 4.890e+03
 Factor Ops : 1.023e+05 (less than 1 second per iteration)
```

```
 Threads    : 1

                  Objective                Residual
Iter       Primal          Dual         Primal    Dual     Compl     Time
   0   7.52895386e-01  7.52895386e-01  0.00e+00 1.44e-01  0.00e+00     0s
   1   4.89041644e-01  4.89042171e-01  1.58e-15 1.44e-07  0.00e+00     0s
   2   4.89041644e-01  4.89041644e-01  1.02e-17 1.44e-13  0.00e+00     0s

Barrier solved model in 2 iterations and 0.00 seconds
Optimal objective 4.89041644e-01

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Gurobi Optimizer version 9.0.2 build v9.0.2rc0 (mac64)
Optimize a model with 301 rows, 599 columns and 899 nonzeros
Model fingerprint: 0x26ca8abb
Model has 599 quadratic objective terms
Coefficient statistics:
  Matrix range     [1e+00, 1e+00]
  Objective range  [9e-04, 7e-03]
  QObjective range [9e-04, 2e+00]
  Bounds range     [0e+00, 0e+00]
  RHS range        [0e+00, 0e+00]
Presolve removed 2 rows and 2 columns
Presolve time: 0.00s
Presolved: 299 rows, 597 columns, 895 nonzeros
Presolved model has 597 quadratic objective terms
Ordering time: 0.00s

Barrier statistics:
 Free vars  : 597
 AA' NZ     : 2.980e+02
 Factor NZ  : 4.890e+03
 Factor Ops : 1.023e+05 (less than 1 second per iteration)
 Threads    : 1

                  Objective                Residual
Iter       Primal          Dual         Primal    Dual     Compl     Time
   0   1.21235885e+00  1.21235885e+00  0.00e+00 2.31e-01  0.00e+00     0s
   1   6.90532312e-01  6.90533356e-01  2.04e-15 2.31e-07  0.00e+00     0s
   2   6.90532312e-01  6.90532312e-01  1.38e-17 2.32e-13  0.00e+00     0s

Barrier solved model in 2 iterations and 0.00 seconds
Optimal objective 6.90532312e-01

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Gurobi Optimizer version 9.0.2 build v9.0.2rc0 (mac64)
```

```
Optimize a model with 301 rows, 599 columns and 899 nonzeros
Model fingerprint: 0xb7d3cb89
Model has 599 quadratic objective terms
Coefficient statistics:
  Matrix range     [1e+00, 1e+00]
  Objective range  [1e-03, 1e-02]
  QObjective range [1e-03, 2e+00]
  Bounds range     [0e+00, 0e+00]
  RHS range        [0e+00, 0e+00]
Presolve removed 2 rows and 2 columns
Presolve time: 0.00s
Presolved: 299 rows, 597 columns, 895 nonzeros
Presolved model has 597 quadratic objective terms
Ordering time: 0.00s

Barrier statistics:
 Free vars  : 597
 AA' NZ     : 2.980e+02
 Factor NZ  : 4.890e+03
 Factor Ops : 1.023e+05 (less than 1 second per iteration)
 Threads    : 1

                 Objective                Residual
Iter       Primal          Dual         Primal    Dual     Compl     Time
   0   1.95221542e+00  1.95221542e+00  0.00e+00 1.86e-01  0.00e+00     0s
   1   9.50987336e-01  9.50989339e-01  2.14e-15 1.86e-07  0.00e+00     0s
   2   9.50987336e-01  9.50987336e-01  2.47e-17 1.86e-13  0.00e+00     0s

Barrier solved model in 2 iterations and 0.00 seconds
Optimal objective 9.50987336e-01

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Gurobi Optimizer version 9.0.2 build v9.0.2rc0 (mac64)
Optimize a model with 301 rows, 599 columns and 899 nonzeros
Model fingerprint: 0x22f297ca
Model has 599 quadratic objective terms
Coefficient statistics:
  Matrix range     [1e+00, 1e+00]
  Objective range  [2e-03, 2e-02]
  QObjective range [2e-03, 2e+00]
  Bounds range     [0e+00, 0e+00]
  RHS range        [0e+00, 0e+00]
Presolve removed 2 rows and 2 columns
Presolve time: 0.00s
Presolved: 299 rows, 597 columns, 895 nonzeros
Presolved model has 597 quadratic objective terms
Ordering time: 0.00s
```

```
Barrier statistics:
 Free vars  : 597
 AA' NZ     : 2.980e+02
 Factor NZ  : 4.890e+03
 Factor Ops : 1.023e+05 (less than 1 second per iteration)
 Threads    : 1

                 Objective                Residual
Iter       Primal          Dual         Primal    Dual     Compl     Time
   0   3.14357836e+00  3.14357836e+00  0.00e+00 3.00e-01  0.00e+00     0s
   1   1.27688589e+00  1.27688962e+00  1.48e-15 3.00e-07  0.00e+00     0s
   2   1.27688589e+00  1.27688589e+00  2.52e-17 3.00e-13  0.00e+00     0s

Barrier solved model in 2 iterations and 0.00 seconds
Optimal objective 1.27688589e+00

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Gurobi Optimizer version 9.0.2 build v9.0.2rc0 (mac64)
Optimize a model with 301 rows, 599 columns and 899 nonzeros
Model fingerprint: 0x5541de3e
Model has 599 quadratic objective terms
Coefficient statistics:
  Matrix range     [1e+00, 1e+00]
  Objective range  [4e-03, 3e-02]
  QObjective range [4e-03, 2e+00]
  Bounds range     [0e+00, 0e+00]
  RHS range        [0e+00, 0e+00]
Presolve removed 2 rows and 2 columns
Presolve time: 0.00s
Presolved: 299 rows, 597 columns, 895 nonzeros
Presolved model has 597 quadratic objective terms
Ordering time: 0.00s

Barrier statistics:
 Free vars  : 597
 AA' NZ     : 2.980e+02
 Factor NZ  : 4.890e+03
 Factor Ops : 1.023e+05 (less than 1 second per iteration)
 Threads    : 1

                 Objective                Residual
Iter       Primal          Dual         Primal    Dual     Compl     Time
   0   5.06198487e+00  5.06198487e+00  0.00e+00 4.83e-01  0.00e+00     0s
   1   1.67583920e+00  1.67584598e+00  8.26e-16 4.83e-07  0.00e+00     0s
   2   1.67583920e+00  1.67583920e+00  4.27e-17 4.83e-13  0.00e+00     0s
```

```
Barrier solved model in 2 iterations and 0.00 seconds
Optimal objective 1.67583920e+00

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Gurobi Optimizer version 9.0.2 build v9.0.2rc0 (mac64)
Optimize a model with 301 rows, 599 columns and 899 nonzeros
Model fingerprint: 0xeab8d028
Model has 599 quadratic objective terms
Coefficient statistics:
  Matrix range     [1e+00, 1e+00]
  Objective range  [6e-03, 5e-02]
  QObjective range [6e-03, 2e+00]
  Bounds range     [0e+00, 0e+00]
  RHS range        [0e+00, 0e+00]
Presolve removed 2 rows and 2 columns
Presolve time: 0.00s
Presolved: 299 rows, 597 columns, 895 nonzeros
Presolved model has 597 quadratic objective terms
Ordering time: 0.00s

Barrier statistics:
 Free vars  : 597
 AA' NZ     : 2.980e+02
 Factor NZ  : 4.890e+03
 Factor Ops : 1.023e+05 (less than 1 second per iteration)
 Threads    : 1

                 Objective                Residual
Iter       Primal          Dual         Primal    Dual     Compl     Time
   0   8.15112202e+00  8.15112202e+00  0.00e+00 3.89e-01  0.00e+00     0s
   1   2.16134789e+00  2.16135987e+00  2.50e-15 3.89e-07  0.00e+00     0s
   2   2.16134789e+00  2.16134789e+00  1.05e-16 3.89e-13  0.00e+00     0s

Barrier solved model in 2 iterations and 0.00 seconds
Optimal objective 2.16134789e+00

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Gurobi Optimizer version 9.0.2 build v9.0.2rc0 (mac64)
Optimize a model with 301 rows, 599 columns and 899 nonzeros
Model fingerprint: 0xb13edfe1
Model has 599 quadratic objective terms
Coefficient statistics:
  Matrix range     [1e+00, 1e+00]
  Objective range  [1e-02, 8e-02]
  QObjective range [1e-02, 2e+00]
  Bounds range     [0e+00, 0e+00]
```

```
  RHS range        [0e+00, 0e+00]
Presolve removed 2 rows and 2 columns
Presolve time: 0.00s
Presolved: 299 rows, 597 columns, 895 nonzeros
Presolved model has 597 quadratic objective terms
Ordering time: 0.00s

Barrier statistics:
 Free vars  : 597
 AA' NZ     : 2.980e+02
 Factor NZ  : 4.890e+03
 Factor Ops : 1.023e+05 (less than 1 second per iteration)
 Threads    : 1

                 Objective                Residual
Iter       Primal          Dual         Primal    Dual     Compl     Time
   0   1.31254423e+01  1.31254423e+01  0.00e+00 6.26e-01  0.00e+00     0s
   1   2.75690550e+00  2.75692624e+00  1.62e-15 6.26e-07  0.00e+00     0s
   2   2.75690550e+00  2.75690550e+00  9.54e-17 6.26e-13  0.00e+00     0s

Barrier solved model in 2 iterations and 0.00 seconds
Optimal objective 2.75690550e+00

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Gurobi Optimizer version 9.0.2 build v9.0.2rc0 (mac64)
Optimize a model with 301 rows, 599 columns and 899 nonzeros
Model fingerprint: 0x9616441b
Model has 599 quadratic objective terms
Coefficient statistics:
  Matrix range     [1e+00, 1e+00]
  Objective range  [2e-02, 1e-01]
  QObjective range [2e-02, 2e+00]
  Bounds range     [0e+00, 0e+00]
  RHS range        [0e+00, 0e+00]
Presolve removed 2 rows and 2 columns
Presolve time: 0.00s
Presolved: 299 rows, 597 columns, 895 nonzeros
Presolved model has 597 quadratic objective terms
Ordering time: 0.00s

Barrier statistics:
 Free vars  : 597
 AA' NZ     : 2.980e+02
 Factor NZ  : 4.890e+03
 Factor Ops : 1.023e+05 (less than 1 second per iteration)
 Threads    : 1
```

```
                  Objective                Residual
Iter       Primal          Dual         Primal    Dual     Compl     Time
   0   2.11354013e+01  2.11354013e+01  0.00e+00 5.04e-01  0.00e+00     0s
   1   3.49658349e+00  3.49661876e+00  2.55e-15 5.04e-07  0.00e+00     0s
   2   3.49658349e+00  3.49658349e+00  2.12e-16 5.04e-13  0.00e+00     0s

Barrier solved model in 2 iterations and 0.00 seconds
Optimal objective 3.49658349e+00

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Gurobi Optimizer version 9.0.2 build v9.0.2rc0 (mac64)
Optimize a model with 301 rows, 599 columns and 899 nonzeros
Model fingerprint: 0x047547fe
Model has 599 quadratic objective terms
Coefficient statistics:
  Matrix range     [1e+00, 1e+00]
  Objective range  [3e-02, 2e-01]
  QObjective range [3e-02, 2e+00]
  Bounds range     [0e+00, 0e+00]
  RHS range        [0e+00, 0e+00]
Presolve removed 2 rows and 2 columns
Presolve time: 0.00s
Presolved: 299 rows, 597 columns, 895 nonzeros
Presolved model has 597 quadratic objective terms
Ordering time: 0.00s

Barrier statistics:
 Free vars  : 597
 AA' NZ     : 2.980e+02
 Factor NZ  : 4.890e+03
 Factor Ops : 1.023e+05 (less than 1 second per iteration)
 Threads    : 1

                  Objective                Residual
Iter       Primal          Dual         Primal    Dual     Compl     Time
   0   3.40335341e+01  3.40335341e+01  0.00e+00 8.12e-01  0.00e+00     0s
   1   4.42120444e+00  4.42126366e+00  1.26e-15 8.12e-07  0.00e+00     0s
   2   4.42120444e+00  4.42120444e+00  1.84e-16 8.12e-13  0.00e+00     0s

Barrier solved model in 2 iterations and 0.00 seconds
Optimal objective 4.42120444e+00

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Gurobi Optimizer version 9.0.2 build v9.0.2rc0 (mac64)
Optimize a model with 301 rows, 599 columns and 899 nonzeros
Model fingerprint: 0x053addbf
```

```
Model has 599 quadratic objective terms
Coefficient statistics:
  Matrix range     [1e+00, 1e+00]
  Objective range  [4e-02, 3e-01]
  QObjective range [4e-02, 2e+00]
  Bounds range     [0e+00, 0e+00]
  RHS range        [0e+00, 0e+00]
Presolve removed 2 rows and 2 columns
Presolve time: 0.00s
Presolved: 299 rows, 597 columns, 895 nonzeros
Presolved model has 597 quadratic objective terms
Ordering time: 0.00s

Barrier statistics:
 Free vars  : 597
 AA' NZ     : 2.980e+02
 Factor NZ  : 4.890e+03
 Factor Ops : 1.023e+05 (less than 1 second per iteration)
 Threads    : 1

                 Objective                Residual
Iter       Primal          Dual         Primal    Dual     Compl     Time
   0   5.48029076e+01  5.48029076e+01  0.00e+00 1.31e+00  0.00e+00     0s
   1   5.57330713e+00  5.57340559e+00  1.22e-15 1.31e-06  0.00e+00     0s
   2   5.57330713e+00  5.57330713e+00  2.01e-16 1.31e-12  0.00e+00     0s

Barrier solved model in 2 iterations and 0.00 seconds
Optimal objective 5.57330713e+00

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Gurobi Optimizer version 9.0.2 build v9.0.2rc0 (mac64)
Optimize a model with 301 rows, 599 columns and 899 nonzeros
Model fingerprint: 0x0164b8e5
Model has 599 quadratic objective terms
Coefficient statistics:
  Matrix range     [1e+00, 1e+00]
  Objective range  [7e-02, 5e-01]
  QObjective range [7e-02, 2e+00]
  Bounds range     [0e+00, 0e+00]
  RHS range        [0e+00, 0e+00]
Presolve removed 2 rows and 2 columns
Presolve time: 0.00s
Presolved: 299 rows, 597 columns, 895 nonzeros
Presolved model has 597 quadratic objective terms
Ordering time: 0.00s

Barrier statistics:
```

```
 Free vars  : 597
 AA' NZ     : 2.980e+02
 Factor NZ  : 4.890e+03
 Factor Ops : 1.023e+05 (less than 1 second per iteration)
 Threads    : 1

                 Objective                Residual
Iter       Primal          Dual         Primal    Dual     Compl     Time
   0   8.82470412e+01  8.82470412e+01  0.00e+00 1.05e+00  0.00e+00     0s
   1   6.99708441e+00  6.99724691e+00  1.14e-15 1.05e-06  0.00e+00     0s
   2   6.99708441e+00  6.99708441e+00  4.23e-16 1.05e-12  0.00e+00     0s

Barrier solved model in 2 iterations and 0.00 seconds
Optimal objective 6.99708441e+00

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Gurobi Optimizer version 9.0.2 build v9.0.2rc0 (mac64)
Optimize a model with 301 rows, 599 columns and 899 nonzeros
Model fingerprint: 0xabe34abd
Model has 599 quadratic objective terms
Coefficient statistics:
  Matrix range     [1e+00, 1e+00]
  Objective range  [1e-01, 8e-01]
  QObjective range [1e-01, 2e+00]
  Bounds range     [0e+00, 0e+00]
  RHS range        [0e+00, 0e+00]
Presolve removed 2 rows and 2 columns
Presolve time: 0.00s
Presolved: 299 rows, 597 columns, 895 nonzeros
Presolved model has 597 quadratic objective terms
Ordering time: 0.00s

Barrier statistics:
 Free vars  : 597
 AA' NZ     : 2.980e+02
 Factor NZ  : 4.890e+03
 Factor Ops : 1.023e+05 (less than 1 second per iteration)
 Threads    : 1

                 Objective                Residual
Iter       Primal          Dual         Primal    Dual     Compl     Time
   0   1.42100859e+02  1.42100859e+02  0.00e+00 1.70e+00  0.00e+00     0s
   1   8.74473062e+00  8.74499733e+00  1.58e-15 1.70e-06  0.00e+00     0s
   2   8.74473062e+00  8.74473062e+00  3.89e-16 1.70e-12  0.00e+00     0s

Barrier solved model in 2 iterations and 0.00 seconds
Optimal objective 8.74473062e+00
```

```
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Gurobi Optimizer version 9.0.2 build v9.0.2rc0 (mac64)
Optimize a model with 301 rows, 599 columns and 899 nonzeros
Model fingerprint: 0xccc5f4f0
Model has 599 quadratic objective terms
Coefficient statistics:
  Matrix range     [1e+00, 1e+00]
  Objective range  [2e-01, 1e+00]
  QObjective range [2e-01, 2e+00]
  Bounds range     [0e+00, 0e+00]
  RHS range        [0e+00, 0e+00]
Presolve removed 2 rows and 2 columns
Presolve time: 0.00s
Presolved: 299 rows, 597 columns, 895 nonzeros
Presolved model has 597 quadratic objective terms
Ordering time: 0.00s

Barrier statistics:
 Free vars  : 597
 AA' NZ     : 2.980e+02
 Factor NZ  : 4.890e+03
 Factor Ops : 1.023e+05 (less than 1 second per iteration)
 Threads    : 1

                 Objective                Residual
Iter       Primal          Dual         Primal    Dual     Compl     Time
   0   2.28819618e+02  2.28819618e+02  0.00e+00 2.73e+00  0.00e+00     0s
   1   1.08815012e+01  1.08819371e+01  1.04e-15 2.73e-06  0.00e+00     0s
   2   1.08815012e+01  1.08815012e+01  3.85e-16 2.73e-12  0.00e+00     0s

Barrier solved model in 2 iterations and 0.00 seconds
Optimal objective 1.08815012e+01

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Gurobi Optimizer version 9.0.2 build v9.0.2rc0 (mac64)
Optimize a model with 301 rows, 599 columns and 899 nonzeros
Model fingerprint: 0x3ba7f00b
Model has 599 quadratic objective terms
Coefficient statistics:
  Matrix range     [1e+00, 1e+00]
  Objective range  [3e-01, 2e+00]
  QObjective range [3e-01, 2e+00]
  Bounds range     [0e+00, 0e+00]
  RHS range        [0e+00, 0e+00]
Presolve removed 2 rows and 2 columns
```

```
Presolve time: 0.00s
Presolved: 299 rows, 597 columns, 895 nonzeros
Presolved model has 597 quadratic objective terms
Ordering time: 0.00s

Barrier statistics:
 Free vars  : 597
 AA' NZ     : 2.980e+02
 Factor NZ  : 4.890e+03
 Factor Ops : 1.023e+05 (less than 1 second per iteration)
 Threads    : 1

                 Objective                Residual
Iter       Primal          Dual         Primal    Dual     Compl     Time
   0   3.68459542e+02  3.68459542e+02  0.00e+00 2.20e+00  0.00e+00     0s
   1   1.34817572e+01  1.34824672e+01  2.28e-15 2.20e-06  0.00e+00     0s
   2   1.34817572e+01  1.34817572e+01  6.11e-16 2.20e-12  0.00e+00     0s

Barrier solved model in 2 iterations and 0.00 seconds
Optimal objective 1.34817572e+01

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Gurobi Optimizer version 9.0.2 build v9.0.2rc0 (mac64)
Optimize a model with 301 rows, 599 columns and 899 nonzeros
Model fingerprint: 0x2ac4dd60
Model has 599 quadratic objective terms
Coefficient statistics:
  Matrix range     [1e+00, 1e+00]
  Objective range  [4e-01, 4e+00]
  QObjective range [4e-01, 2e+00]
  Bounds range     [0e+00, 0e+00]
  RHS range        [0e+00, 0e+00]
Presolve removed 2 rows and 2 columns
Presolve time: 0.00s
Presolved: 299 rows, 597 columns, 895 nonzeros
Presolved model has 597 quadratic objective terms
Ordering time: 0.00s

Barrier statistics:
 Free vars  : 597
 AA' NZ     : 2.980e+02
 Factor NZ  : 4.890e+03
 Factor Ops : 1.023e+05 (less than 1 second per iteration)
 Threads    : 1

                 Objective                Residual
Iter       Primal          Dual         Primal    Dual     Compl     Time
```

```
   0   5.93316409e+02  5.93316409e+02  0.00e+00 3.54e+00  0.00e+00     0s
   1   1.66179501e+01  1.66191035e+01  1.44e-15 3.54e-06  0.00e+00     0s
   2   1.66179501e+01  1.66179501e+01  8.33e-16 3.54e-12  0.00e+00     0s

Barrier solved model in 2 iterations and 0.00 seconds
Optimal objective 1.66179501e+01

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Gurobi Optimizer version 9.0.2 build v9.0.2rc0 (mac64)
Optimize a model with 301 rows, 599 columns and 899 nonzeros
Model fingerprint: 0xcb5eb3d1
Model has 599 quadratic objective terms
Coefficient statistics:
  Matrix range     [1e+00, 1e+00]
  Objective range  [7e-01, 6e+00]
  QObjective range [7e-01, 2e+00]
  Bounds range     [0e+00, 0e+00]
  RHS range        [0e+00, 0e+00]
Presolve removed 2 rows and 2 columns
Presolve time: 0.00s
Presolved: 299 rows, 597 columns, 895 nonzeros
Presolved model has 597 quadratic objective terms
Ordering time: 0.00s

Barrier statistics:
 Free vars  : 597
 AA' NZ     : 2.980e+02
 Factor NZ  : 4.890e+03
 Factor Ops : 1.023e+05 (less than 1 second per iteration)
 Threads    : 1

                 Objective                Residual
Iter       Primal          Dual         Primal    Dual     Compl     Time
   0   9.55394884e+02  9.55394884e+02  0.00e+00 5.70e+00  0.00e+00     0s
   1   2.03485748e+01  2.03504449e+01  1.48e-15 5.70e-06  0.00e+00     0s
   2   2.03485748e+01  2.03485748e+01  7.70e-16 5.70e-12  0.00e+00     0s

Barrier solved model in 2 iterations and 0.00 seconds
Optimal objective 2.03485748e+01

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Gurobi Optimizer version 9.0.2 build v9.0.2rc0 (mac64)
Optimize a model with 301 rows, 599 columns and 899 nonzeros
Model fingerprint: 0xc5d6f77a
Model has 599 quadratic objective terms
Coefficient statistics:
```

```
  Matrix range     [1e+00, 1e+00]
  Objective range  [1e+00, 9e+00]
  QObjective range [1e+00, 2e+00]
  Bounds range     [0e+00, 0e+00]
  RHS range        [0e+00, 0e+00]
Presolve removed 2 rows and 2 columns
Presolve time: 0.00s
Presolved: 299 rows, 597 columns, 895 nonzeros
Presolved model has 597 quadratic objective terms
Ordering time: 0.00s

Barrier statistics:
 Free vars  : 597
 AA' NZ     : 2.980e+02
 Factor NZ  : 4.890e+03
 Factor Ops : 1.023e+05 (less than 1 second per iteration)
 Threads    : 1

                 Objective                Residual
Iter       Primal          Dual         Primal    Dual     Compl     Time
   0   1.53843610e+03  1.53843610e+03  0.00e+00 4.59e+00  0.00e+00     0s
   1   2.47026478e+01  2.47056753e+01  2.72e-15 4.59e-06  0.00e+00     0s
   2   2.47026478e+01  2.47026478e+01  1.55e-15 4.59e-12  0.00e+00     0s

Barrier solved model in 2 iterations and 0.00 seconds
Optimal objective 2.47026478e+01

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Gurobi Optimizer version 9.0.2 build v9.0.2rc0 (mac64)
Optimize a model with 301 rows, 599 columns and 899 nonzeros
Model fingerprint: 0x89136391
Model has 599 quadratic objective terms
Coefficient statistics:
  Matrix range     [1e+00, 1e+00]
  Objective range  [2e+00, 1e+01]
  QObjective range [2e+00, 2e+00]
  Bounds range     [0e+00, 0e+00]
  RHS range        [0e+00, 0e+00]
Presolve removed 2 rows and 2 columns
Presolve time: 0.00s
Presolved: 299 rows, 597 columns, 895 nonzeros
Presolved model has 597 quadratic objective terms
Ordering time: 0.00s

Barrier statistics:
 Free vars  : 597
 AA' NZ     : 2.980e+02
```

```
 Factor NZ  : 4.890e+03
 Factor Ops : 1.023e+05 (less than 1 second per iteration)
 Threads    : 1

                 Objective                Residual
Iter       Primal          Dual         Primal    Dual     Compl     Time
   0   2.47728524e+03  2.47728524e+03  0.00e+00 7.39e+00  0.00e+00     0s
   1   2.96498214e+01  2.96547166e+01  3.01e-15 7.39e-06  0.00e+00     0s
   2   2.96498214e+01  2.96498214e+01  1.67e-15 7.39e-12  0.00e+00     0s

Barrier solved model in 2 iterations and 0.00 seconds
Optimal objective 2.96498214e+01

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Gurobi Optimizer version 9.0.2 build v9.0.2rc0 (mac64)
Optimize a model with 301 rows, 599 columns and 899 nonzeros
Model fingerprint: 0xc4d5038e
Model has 599 quadratic objective terms
Coefficient statistics:
  Matrix range     [1e+00, 1e+00]
  Objective range  [3e+00, 2e+01]
  QObjective range [2e+00, 3e+00]
  Bounds range     [0e+00, 0e+00]
  RHS range        [0e+00, 0e+00]
Presolve removed 2 rows and 2 columns
Presolve time: 0.00s
Presolved: 299 rows, 597 columns, 895 nonzeros
Presolved model has 597 quadratic objective terms
Ordering time: 0.00s

Barrier statistics:
 Free vars  : 597
 AA' NZ     : 2.980e+02
 Factor NZ  : 4.890e+03
 Factor Ops : 1.023e+05 (less than 1 second per iteration)
 Threads    : 1

                 Objective                Residual
Iter       Primal          Dual         Primal    Dual     Compl     Time
   0   3.98907835e+03  3.98907835e+03  0.00e+00 1.19e+01  0.00e+00     0s
   1   3.50547858e+01  3.50626938e+01  3.68e-15 1.19e-05  0.00e+00     0s
   2   3.50547858e+01  3.50547858e+01  1.22e-15 1.19e-11  0.00e+00     0s

Barrier solved model in 2 iterations and 0.00 seconds
Optimal objective 3.50547858e+01

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```

```
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Gurobi Optimizer version 9.0.2 build v9.0.2rc0 (mac64)
Optimize a model with 301 rows, 599 columns and 899 nonzeros
Model fingerprint: 0x86734b0c
Model has 599 quadratic objective terms
Coefficient statistics:
  Matrix range     [1e+00, 1e+00]
  Objective range  [5e+00, 4e+01]
  QObjective range [2e+00, 5e+00]
  Bounds range     [0e+00, 0e+00]
  RHS range        [0e+00, 0e+00]
Presolve removed 2 rows and 2 columns
Presolve time: 0.00s
Presolved: 299 rows, 597 columns, 895 nonzeros
Presolved model has 597 quadratic objective terms
Ordering time: 0.00s

Barrier statistics:
 Free vars  : 597
 AA' NZ     : 2.980e+02
 Factor NZ  : 4.890e+03
 Factor Ops : 1.023e+05 (less than 1 second per iteration)
 Threads    : 1

                 Objective                Residual
Iter       Primal          Dual         Primal    Dual     Compl     Time
   0   6.42346139e+03  6.42346139e+03  0.00e+00 9.58e+00  0.00e+00     0s
   1   4.06434329e+01  4.06561985e+01  3.66e-15 9.58e-06  0.00e+00     0s
   2   4.06434328e+01  4.06434329e+01  1.33e-15 9.58e-12  0.00e+00     0s

Barrier solved model in 2 iterations and 0.00 seconds
Optimal objective 4.06434328e+01

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Gurobi Optimizer version 9.0.2 build v9.0.2rc0 (mac64)
Optimize a model with 301 rows, 599 columns and 899 nonzeros
Model fingerprint: 0x74555718
Model has 599 quadratic objective terms
Coefficient statistics:
  Matrix range     [1e+00, 1e+00]
  Objective range  [8e+00, 6e+01]
  QObjective range [2e+00, 8e+00]
  Bounds range     [0e+00, 0e+00]
  RHS range        [0e+00, 0e+00]
Presolve removed 2 rows and 2 columns
Presolve time: 0.00s
Presolved: 299 rows, 597 columns, 895 nonzeros
```

```
Presolved model has 597 quadratic objective terms
Ordering time: 0.00s

Barrier statistics:
 Free vars  : 597
 AA' NZ     : 2.980e+02
 Factor NZ  : 4.890e+03
 Factor Ops : 1.023e+05 (less than 1 second per iteration)
 Threads    : 1

                 Objective                Residual
Iter       Primal          Dual         Primal    Dual     Compl     Time
   0   1.03434560e+04  1.03434560e+04  0.00e+00 1.54e+01  0.00e+00     0s
   1   4.60290882e+01  4.60496831e+01  2.66e-15 1.54e-05  0.00e+00     0s
   2   4.60290882e+01  4.60290883e+01  1.60e-15 1.54e-11  0.00e+00     0s

Barrier solved model in 2 iterations and 0.00 seconds
Optimal objective 4.60290882e+01

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Gurobi Optimizer version 9.0.2 build v9.0.2rc0 (mac64)
Optimize a model with 301 rows, 599 columns and 899 nonzeros
Model fingerprint: 0xd5cf0845
Model has 599 quadratic objective terms
Coefficient statistics:
  Matrix range     [1e+00, 1e+00]
  Objective range  [1e+01, 1e+02]
  QObjective range [2e+00, 1e+01]
  Bounds range     [0e+00, 0e+00]
  RHS range        [0e+00, 0e+00]
Presolve removed 2 rows and 2 columns
Presolve time: 0.00s
Presolved: 299 rows, 597 columns, 895 nonzeros
Presolved model has 597 quadratic objective terms
Ordering time: 0.00s

Barrier statistics:
 Free vars  : 597
 AA' NZ     : 2.980e+02
 Factor NZ  : 4.890e+03
 Factor Ops : 1.023e+05 (less than 1 second per iteration)
 Threads    : 1

                 Objective                Residual
Iter       Primal          Dual         Primal    Dual     Compl     Time
   0   1.66556744e+04  1.66556744e+04  0.00e+00 2.48e+01  0.00e+00     0s
   1   5.08186507e+01  5.08518604e+01  3.77e-15 2.48e-05  0.00e+00     0s
```

```
   2   5.08186507e+01  5.08186507e+01  1.55e-15 2.48e-11  0.00e+00     0s

Barrier solved model in 2 iterations and 0.00 seconds
Optimal objective 5.08186507e+01

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Gurobi Optimizer version 9.0.2 build v9.0.2rc0 (mac64)
Optimize a model with 301 rows, 599 columns and 899 nonzeros
Model fingerprint: 0xde4e6344
Model has 599 quadratic objective terms
Coefficient statistics:
  Matrix range     [1e+00, 1e+00]
  Objective range  [2e+01, 2e+02]
  QObjective range [2e+00, 2e+01]
  Bounds range     [0e+00, 0e+00]
  RHS range        [0e+00, 0e+00]
Presolve removed 2 rows and 2 columns
Presolve time: 0.00s
Presolved: 299 rows, 597 columns, 895 nonzeros
Presolved model has 597 quadratic objective terms
Ordering time: 0.00s

Barrier statistics:
 Free vars  : 597
 AA' NZ     : 2.980e+02
 Factor NZ  : 4.890e+03
 Factor Ops : 1.023e+05 (less than 1 second per iteration)
 Threads    : 1

                 Objective                Residual
Iter       Primal          Dual         Primal    Dual     Compl     Time
   0   2.68200000e+04  2.68200000e+04  0.00e+00 2.00e+01  0.00e+00     0s
   1   5.47446843e+01  5.47982147e+01  2.94e-15 2.00e-05  0.00e+00     0s
   2   5.47446842e+01  5.47446843e+01  1.24e-15 2.00e-11  0.00e+00     0s

Barrier solved model in 2 iterations and 0.00 seconds
Optimal objective 5.47446842e+01
```

2 problem 2

2.1 (a)&(b)

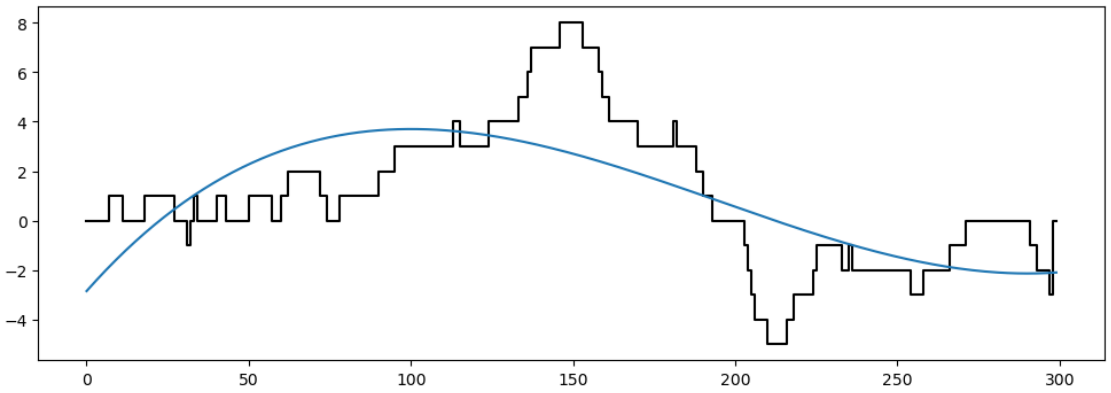
[5]:
```
# order of polynomial to use
k = 3

# fit using a function of the form f(x) = p x^k + q x^(k-1) + ... + rk x + s
# our matrix A will need to have m rows, where m is the number of data points
m = 300
# initialize A to be a matrix of zeros with m rows and k+1 columns
# (there will be coefficients on x^3, x^2, x^1, and x^0, so 4 columns)
A = zeros(m,k+1)


for i = 1:m
    for j = 1:k+1
        A[i,j] = i^(k+1-j)
    end
end

coef = [1.7e-6, -0.001, 0.15, -3]
y_poly= A*coef

using PyPlot
figure(figsize=(12,4))
step(u,"k-",label="original")
plot(y_poly,label=string(L"y_poly"))
```



[5]:
```
1-element Array{PyCall.PyObject,1}:
 PyObject <matplotlib.lines.Line2D object at 0x14e00d650>
```

The coefficient is $[1.7e-6, -0.001, 0.15, -3]$.

2.2 (c)

[6]:
```
using LinearAlgebra
# calculate the 2-norm error
println("2-normerror:", norm(u-y_poly))
```

```
2-normerror:36.88763139118808
```

In a broad sense, the polynomial technique fits the speed sequence worse than the regularization method, but its smoothness is better.