

# A Project For Police Patrol District Design

## Group

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## Introduction

My project will use three different mathematical models to implement the optimization problem for police patrol district design.

Due to limited police resource (given a determined graph set of police service platforms), it is necessary to establish a mathematical model based on the actual conditions and needs of the city to allocate the jurisdiction of each police service platform and distribute the police resources. In this optimization, I generate the position of police service platforms(patrol centers) by random data. By default, I set the number of patrol centers be 1/4 of total traffic roads. The basic principle of optimization is the variance and the minimum cases that each patrol center is required to handle.

We need to find math models to find out:

- (1) Determine the area each patrol center should take control
- (2) The way to distribute patrol group to each traffic road depending on how many cases in the traffic roads.

By default, the number of patrol center would be 1/4 of total traffic roads, we will randomly generate those patrol centers (data generation).

In this project I will use matching algorithm, network flow model and linear programming to solve 3 particular problems that police officer will encounter in their work.

Problem 1: For a city given the coordinates of every traffic road and the coordinates of every patrol center, how to find out the distance between any two patrol centers and traffic roads, as well as which traffic roads will be covered in each patrol center? Could we get a solution on how to distribute the police force to each traffic road?

Problem 2: If 7 people are being murdered at the same time and there are only 10 police cars left in each patrol center, how to assign task for each patrol center so that without bias all 7 people will be saved as soon as possible?

Problem 3: Could we find out the minimum time to take down the criminal if he is reported escaping at some traffic road? Does it depend on the police's speed?

I will solve the 3 problems above using the following models:

1. Network flow model: each traffic road can be treated as a node in a network, and they will flow through each patrol center, what we're trying to do is to balance the flows to patrol center by minimizing the variance of each patrol center.

2. Linear programming model: We will use integer optimization model and minimax problem model to solve the decision variables that will provide us a solution that will guarantee that each murder prevention will have the same priority.

3. Matching algorithm: When we know the traffic roads that needed to be at each time interval we can solve for the matching between these traffic roads so that we can find out if it is possible to lock the criminal in the circle at time  $t$ .

## Data Generation

## Data Form

First an array expressing the locations of 40 traffic roads will be generated, then the adjacency matrix will be calculated from this array.

The city we will generate will be expressed as a 40x40 matrix which means it contains 40 traffic roads. Then the patrol center, the location where the criminal start to escape and the 7 traffic roads that the murder occurs will be generated as vectors.

## Math Model

Some constraints on the random function is consider as followed:

### The scales

Since the mean of random function seems to be  $1/2$  (uniformly distributed), Multiplying it by a fator (here is 5 as the lenght of the whole square city) makes it look more normal. To make these road concentarte around the  $2.5*2.5$  sqaure of the city we force 20 traffic roads to be in that range.

## Implementation

In [1]:

```

import Pkg; Pkg.add("GraphRecipes")
import Pkg; Pkg.add("FixedPointNumbers")
using Random
using Graphs, NamedArrays, Clp, Combinatorics, Gurobi, JuMP, PyPlot
using GraphRecipes
using Plots
using FixedPointNumbers
Random.seed!(12345);
rng = MersenneTwister(12345);

lmap=rand(Float64, (40, 2))

avg=0
rawmap=zeros(40, 40)
for i in 1:40
    for j in 1:2
        if(i<20)
            lmap[i, j]=lmap[i, j]*5/2+5/4
        else
            lmap[i, j]*=5
        end
    end
end
cmap=zeros(40, 40)
for i in 1:40
    avg+=lmap[i, 1]
end

for i in 1:40
    for j in 1:40
        if(rand(Float64)<4/40)
            cmap[i, j]=1
            cmap[j, i]=1
        end
    end
end

for i in 1:40
    for j in 1:40
        if(cmap[i, j]==1)
            rawmap[i, j]=sqrt((lmap[i, 1]-lmap[j, 1])^2+(lmap[i, 2]-lmap[j, 2])^2)

        end
    end
end

xs=zeros(40)
ys=zeros(40)
for i in 1:40
    xs[i]=lmap[i, 1]
    ys[i]=lmap[i, 2]
end
ps=zeros(10)

```

```
temp=shuffle(rng, Vector{1:40})
for i in 1:10
    ps[i]=temp[i]
end
colors= Array{RGB{FixedPointNumbers.Normed{UInt8, 8}}}(undef, 0)
po=0
for i in 1:40
    po=0
    for j in 1:10
        if (ps[j]==i)
            #println("pushing ", i)
            # println(colors)

            push!(colors, colorant"#389826")
            po=1
            continue
        end
    end

    if (po==0)
        push!(colors, colorant"#CB3C33")
    end
    # println(colors)

end
println("The patrol centers are on the node:", ps)

finalCitymap=zeros(40, 40)
for i in 1:40
    for j in 1:40
        finalCitymap[i, j]=rawmap[i, j]
    end
end
println("fianl city map is generated")
println(finalCitymap)

graphplot(rawmap,

    nodeshape=:circle,
    nodesize=0.1,

    curves=false,
    names = 1:40,
    fontsize = 10,
    nodecolor=colors,
    linecolor = :darkgrey
)
```

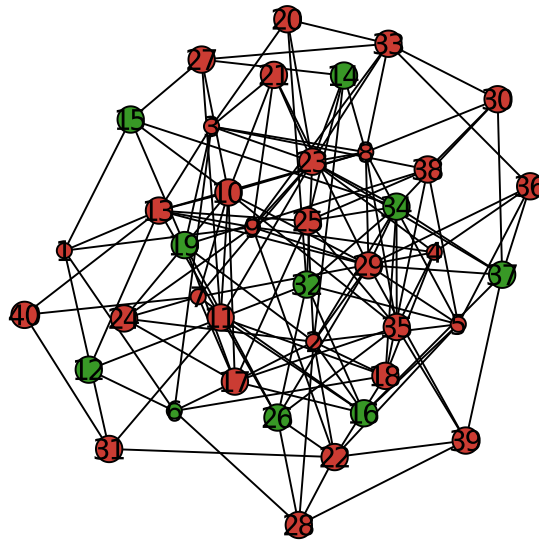
```
Updating registry at `C:\Users\10096\.julia\registries\General`
Updating git-repo `https://github.com/JuliaRegistries/General.git`
Resolving package versions...=====>] 100.0 % 0 %
Updating `C:\Users\10096\.julia\environments\v1.3\Project.toml`
[no changes]
Updating `C:\Users\10096\.julia\environments\v1.3\Manifest.toml`
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Resolving package versions...
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[no changes]
Updating `C:\Users\10096\.julia\environments\v1.3\Manifest.toml`
[no changes]
The patrol centers are on the node:[16.0, 37.0, 32.0, 26.0, 6.0, 12.0, 15.0, 34.0,
14.0, 19.0]
fianl city map is generated
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0.9147265457034718 0.0 0.0 0.0 0.0 2.3789009621823842 0.0; 0.0 0.0 0.0 2.53104807  
91965486 1.580577079296287 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0  
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 2.732261134514851 0.0 0.0 0.0 3.93868367706627  
3 0.0 0.0 0.0 2.5761927268572697 0.0 0.0 0.0; 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0  
0.0 0.0 0.0 0.0 0.0 0.0 1.9501025778009635 0.0 0.0 0.0 0.0 0.0 0.0 3.358105360022  
2823 0.0 0.0 0.0 0.0 0.0 4.199679633452012 2.1014807386917056 0.0 0.0 0.0 2.585227  
7489263042 0.0 2.5761927268572697 0.0 0.0 1.6780513530829027 0.0; 0.0 0.0 2.937593  
559732527 0.0 2.0416924919611175 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0  
1.7321586984694666 3.0531996119248523 0.0 0.0 0.0 0.0 0.0 2.6637477043374505 0.0  
0.0 0.0 0.0 1.64388213349605 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0; 0.0 0.0 0.0  
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 1.77021567  
35759558 0.0 0.0 0.0 0.0 0.0 3.0983129250124644 3.586342075124794 0.0 0.0 0.0 0.0  
0.0 2.3789009621823842 0.0 1.6780513530829027 0.0 0.0 0.0; 0.0 0.0 0.0 0.0 0.0 0.  
0 2.812897407812699 0.0 0.0 0.0 0.0 0.0 2.0829009303565447 0.0 0.0 0.0 0.0 0.0 0.0  
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 3.396198588856508 0.0 0.0 0.0 0.0 0.0  
0.0 0.0 0.0 0.0]



Out[1]:



## Problem1

### Analysis on the problem

We can think of every traffic road as vertex  $v_i, i \in \{\text{the set of patrol center index}\}$ .

The police need to arrive at the traffic road where the incident happened within 3 mins with a 60mph police car implies that the each patrol center should lie in the 3 miles circle of each traffic road so that it is possible for them to arrive on time.

So we can first figure out the shortest distance between each patrol center and each traffic road and represent this distance as a matrix  $D_{i,j}$

Then we find out each traffic road is covered by what patrol centers by iterating through matrix  $D_{i,j}$ , and represent it as a matrix  $C_{i,j}$  in which  $C_{i,j} = 1$  means that  $i$ th traffic road is covered by  $j$ th patrol center.

Then we check if Any traffic road can't be reached by any patrol center within 5 minutes, then connect them to the closest patrol center.

Then we will get the incident array  $In_j$  and size array  $Si_j$  which represent how many cases is in each traffic road and the number of cases that can be handled by each patrol center. Then we will try to lower the variance and overload of each patrol center by using networkflow optimizer.

## Math Model

In order to determine the shortest path between any two traffic roads in the city from the adjacency matrix we need to use warshall algorithm .

### 1.1 warshall algorithm.

let  $D_{i,j}$  be the shortest path from traffic road  $i$  to  $j$ , the idea is:

1)Get adjacency matrix  $A_{i,j}$  from the dataset , then initialize a new matrix that set the 0 entries to infinity.

2) for each pair of vertices  $u$  and  $v$ , we will check if there is another vertex  $w$  such that  $D_{u,w} + D_{w,v} > D_{u,v}$ .

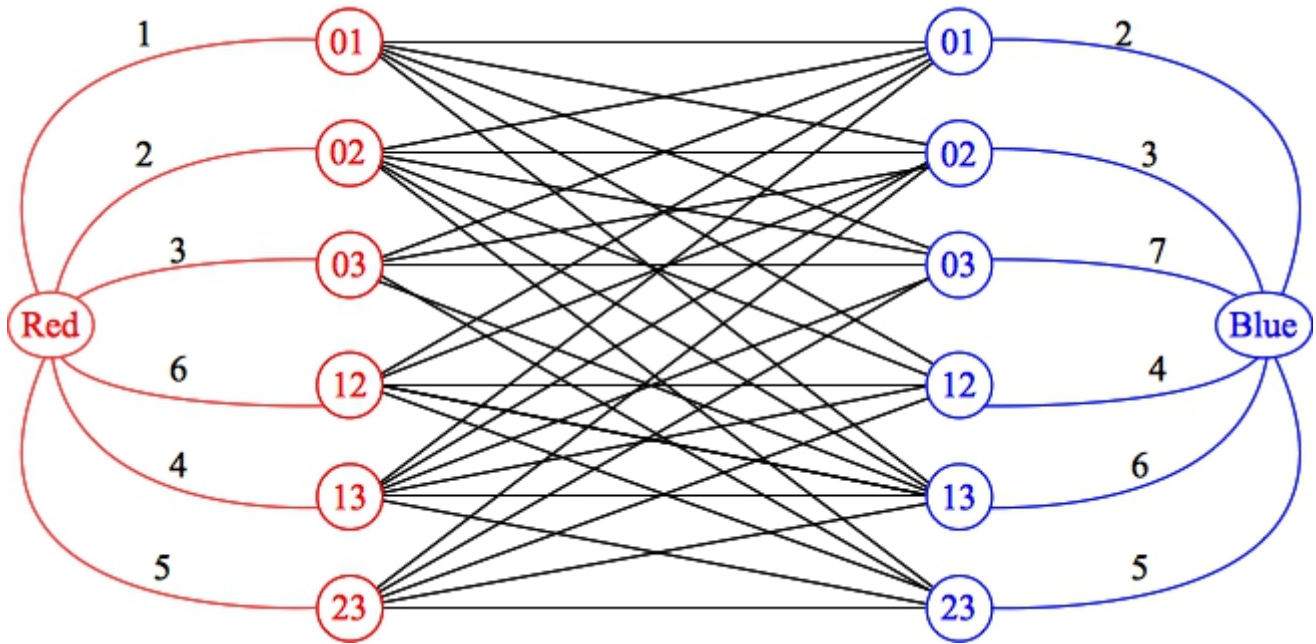
\*pusedo code of this algorithmn:

```
for k: 1 to n
  for i:=1 to n
    for j:=1 to n
      If  $D_{i,j} > D_{i,k} + D_{k,j}$ :
         $D_{i,j} = D_{i,k} + D_{k,j}$ 
```

### 1.2 create coverage matrix $C_{i,j}$ from $D_{i,j}$ and balance the overload of each police station

We first iterate through each entry of  $D_{i,j}$ , if  $D_{i,j} < 3miles$  set  $C_{i,j} = 1$ .

The idea is use networkflow problem as a way to set the constraint



Let  $n$  be the total number of patrol centers, and let  $m$  be the total number of traffic roads,  $sp_j$  be the working speed of each patrol center when it is at its full capacity,  $cf$  be the full capacity of each patrol center.

The variable for optimizer solver would be  $X_{i,j}$  constraint would be

$$X_{i,j} = C_{i,j} * X_{i,j}$$

$$\sum_{j=1}^n X_{i,j} = flowin_i$$

$$\sum_{i=1}^m X_{i,j} \leq maxload_j,$$

The incident array and patrol center maxload would be  $flowin_i$  and  $maxload_j$ .

The variance objective function is computed as follow  $z1 = \min \sum_{j=1}^n \frac{[\sum_{i=1}^m X_{i,j} - E(x)]^2}{n}$

Provided the comfortable load, the overload objective function would be

$$z2 = \min \left[ \frac{\sum_{i=1}^m X_{i,j}}{cf[j]} \right] * \left( \frac{\sum_{i=1}^m X_{i,j} - cf[j]}{sp[j]} \right)$$

## Implementation

In [2]:

```
import Pkg; Pkg.add("GraphRecipes")
import Pkg; Pkg.add("FixedPointNumbers")
import Pkg; Pkg.add("Ipopt")
import Pkg; Pkg.add("NLOpt")
using Random
using Graphs, NamedArrays, Clp, Combinatorics, Gurobi, JuMP, PyPlot
```

```
using GraphRecipes
using Plots
using FixedPointNumbers

Random.seed!(1234);
rng = MersenneTwister(1234);

lmap=rand(Float64, (40, 2))
#println(lmap)
avg=0
rawmap=zeros(40, 40)
for i in 1:40
    for j in 1:2
        if(i<20)
            lmap[i, j]=lmap[i, j]*5/2+5/4
        else
            lmap[i, j]*=5
        end
    end
end
cmap=zeros(40, 40)
for i in 1:40
    avg+=lmap[i, 1]
end
#println(avg)
for i in 1:40
    for j in 1:40
        if(rand(Float64)<4/40)
            cmap[i, j]=1
            cmap[j, i]=1
        end
    end
end
#println(cmap)

for i in 1:40
    for j in 1:40
        if(cmap[i, j]==1)
            rawmap[i, j]=sqrt((lmap[i, 1]-lmap[j, 1])^2+(lmap[i, 2]-lmap[j, 2])^2)
        end
    end
end

xs=zeros(40)
ys=zeros(40)
for i in 1:40
    xs[i]=lmap[i, 1]
    ys[i]=lmap[i, 2]
end

ps=zeros(10)
temp=shuffle(rng, Vector{1:40})
for i in 1:10
    ps[i]=temp[i]
end
colors= Array{RGB{FixedPointNumbers.Normed{UInt8, 8}}}(undef, 0)
po=0
for i in 1:40
```

```
po=0
for j in 1:10
    if (ps[j]==i)

        push!(colors, colorant"#389826")
        po=1
        break
    end
end
if(po==0)
    push!(colors, colorant"#CB3C33")
end

end
println("The patrol centers are on the node:", ps)

finalCitymap=zeros(40,40)
for i in 1:40
    for j in 1:40
        finalCitymap[i, j]=rawmap[i, j]
    end
end
#println("fianl city map is generated")
#println(finalCitymap)

plt=graphplot(rawmap,

    nodeshape=:circle,
    nodesize=0.1,

    curves=false,
    names = 1:40,
    fontsize = 10,
    nodecolor=colors,
    linecolor = :darkgrey
)

n=40
for i in 1:n
    for j in 1:n
        if (i==j)
            rawmap[i, i]=0

            elseif ( rawmap[i, j]==0)
                rawmap[i, j]=10000
            end
        end
    end
end
#println(rawmap)

for k in 1:n
    for i in 1:n
        for j in 1:n
```

```
        if(rawmap[i, j]>rawmap[i, k]+rawmap[k, j])
            rawmap[i, j]=rawmap[i, k]+rawmap[k, j]
        end
    end
end

end

end

#println(rawmap)
covermap=zeros(10, 40)
bcovermap=zeros(10, 40)
for i in 1:10
    for j in 1:40
        if(rawmap[Int(ps[i]), j]<3)
            covermap[i, j]= rawmap[Int(ps[i]), j]
            bcovermap[i, j]=1
        end
    end
end

end
#println(covermap)
#println(bcovermap)
for i in 1:10
    for j in 1:40
        if(j in ps)
            bcovermap[i, j]=0
        end
    end
end
for i in 1:10
    println("patrol center", i , "covers:")
    for j in 1:40
        if(bcovermap[i, j]==1)
            println(j, ", ")
        end
    end
end
end

printcovermap=zeros(40, 40)

for i in 1:10
    for j in 1:40
        if(covermap[i, j]>0)
            printcovermap[i, j]=covermap[i, j]
        end
    end
end

psl=8*rand(Float64, 10)
inflow=2*rand(Float64, 40)
expx=0
tdeg=zeros(40)
pl=zeros(10)
```

```

println("The crime cases in each traffic road is:",inflow)
println("The capacity of each patrol center is:",psl)
#println(pl)
#println(expx)
#println(bcovermap)

#println("Before optimization the variance is:",sum((pl[i]-expx)^2/10 for i in 1:10
))

m = Model(Gurobi.Optimizer)
@variable(m, x[1:10,1:40])

@constraint(m, con1[ i in 1:10, j in 1:40],x[i,j]==bcovermap[i,j]*x[i,j])
@constraint(m, con4[ i in 1:10, j in 1:40],x[i,j]>=0)
@constraint(m, con3[ i in 1:10],sum(x[i,j] for j in 1:40 )/inflow[i]>=0.8)
@constraint(m, con2[ j in 1:40],sum(x[i,j] for i in 1:10 ) <= inflow[j])
println("1")

@constraint(m, con5[ i in 1:10],sum(x[i,j] for j in 1:40) <= psl[i])
@objective(m, Min, sum((sum(x[i,j] for j in 1:40 )-sum(x[n,m]/10 for n in 1:10 for m
in 1:40))^2 for i in 1:10 )/10)

optimize!(m)
println("The how much cases should be handled by each patrol center:")
println(value.(x))
resultmap=zeros(40,40)
for i in 1:40
    for j in 1:40
        for k in 1:10
            if(i == ps[k])
                resultmap[i,j]=value.(x)[k,j]
                resultmap[j,i]=value.(x)[k,j]
            end
        end
    end
end
end

graphplot(resultmap,

    nodeshape=:circle,
    nodesize=0.1,
    linesize=0.001,
    curves=false,
    names = 1:40,
    fontsize = 10,
    nodecolor=colors,
    linecolor = :darkgrey
)

```



```
Resolving package versions...
Updating `C:\Users\10096\.julia\environments\v1.3\Project.toml`
[no changes]
Updating `C:\Users\10096\.julia\environments\v1.3\Manifest.toml`
[no changes]
Resolving package versions...
Updating `C:\Users\10096\.julia\environments\v1.3\Project.toml`
[no changes]
Updating `C:\Users\10096\.julia\environments\v1.3\Manifest.toml`
[no changes]
Resolving package versions...
Updating `C:\Users\10096\.julia\environments\v1.3\Project.toml`
[no changes]
Updating `C:\Users\10096\.julia\environments\v1.3\Manifest.toml`
[no changes]
Resolving package versions...
Updating `C:\Users\10096\.julia\environments\v1.3\Project.toml`
[no changes]
Updating `C:\Users\10096\.julia\environments\v1.3\Manifest.toml`
[no changes]
The patrol centers are on the node:[28.0, 38.0, 31.0, 32.0, 33.0, 18.0,
37.0, 9.0, 27.0, 14.0]
patrol center1covers:
4,
5,
6,
10,
11,
13,
15,
24,
25,
35,
patrol center2covers:
3,
7,
17,
40,
patrol center3covers:
3,
6,
8,
11,
15,
20,
36,
39,
patrol center4covers:
2,
15,
40,
patrol center5covers:
2,
7,
8,
10,
11,
12,
13,
30,
35,
```

patrol center6covers:

1,  
2,  
3,  
4,  
5,  
6,  
7,  
8,  
10,  
11,  
12,  
13,  
15,  
16,  
17,  
19,  
21,  
23,  
24,  
25,  
29,  
30,  
40,

patrol center7covers:

1,  
4,  
5,  
6,  
7,  
8,  
10,  
11,  
15,  
22,  
25,  
30,  
40,

patrol center8covers:

1,  
2,  
4,  
5,  
6,  
7,  
8,  
10,  
11,  
13,  
15,  
19,  
22,  
24,  
25,  
29,

patrol center9covers:

3,  
4,  
8,  
12,  
35,

36,  
39,  
patrol center10covers:

1,  
2,  
4,  
5,  
6,  
7,  
8,  
10,  
11,  
13,  
15,  
16,  
19,  
20,  
21,  
24,  
25,  
30,  
40,

The crime cases in each traffic road is:[0.5718276446950155, 1.3845174738173975, 0.8503734992636205, 1.3824965879279967, 0.3184056555037603, 0.598524309133202, 1.7202321631674384, 0.41512443018434775, 0.5989031112857242, 0.5331078354561005, 1.4931418723189425, 0.13137181604552417, 0.4368628637867964, 1.236669749591039, 0.6753083494709444, 0.9357123475757518, 1.30356827784095, 0.4885013663970357, 0.3325931135571425, 1.7986910214784118, 1.9923329033975747, 1.3971925744414704, 0.4026703405981906, 0.12945863613795705, 1.0468071410053366, 1.2982245691005438, 0.5445022942154774, 0.2707600007581368, 0.5285204638442078, 0.830476415193218, 0.6620590788831353, 0.9881328537009058, 0.859528360179111, 0.535492496810166, 0.6291222809468517, 1.5607530921367947, 0.8761364795223496, 1.2727651714838593, 1.8708374093866027, 0.6233598069259565]

The capacity of each patrol center is:[4.0270087667610195, 4.466375568682446, 3.9194505340182513, 6.734265788704446, 5.780587148554769, 1.0864552901487912, 4.030355130748989, 3.019735970169796, 3.7698788113168007, 2.4436935880757478]

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Gurobi Optimizer version 9.0.2 build v9.0.2rc0 (win64)

Optimize a model with 860 rows, 400 columns and 1888 nonzeros

Model fingerprint: 0x7ea5d655

Model has 76300 quadratic objective terms

Coefficient statistics:

Matrix range [6e-01, 3e+00]  
Objective range [0e+00, 0e+00]  
QObjective range [2e-02, 2e-01]  
Bounds range [0e+00, 0e+00]  
RHS range [1e-01, 7e+00]

Presolve removed 823 rows and 288 columns

Presolve time: 0.09s

Presolved: 37 rows, 121 columns, 232 nonzeros

Presolved model has 5793 quadratic objective terms

Ordering time: 0.00s

Barrier statistics:

Free vars : 5  
AA' NZ : 1.710e+02  
Factor NZ : 7.030e+02

Factor Ops : 1.519e+04 (less than 1 second per iteration)

Threads : 1

Iter	Objective		Residual		Compl	Tim
	Primal	Dual	Primal	Dual		
0	3.34177392e+05	-3.58767858e+05	2.67e+04	4.28e-06	1.00e+06	0
1	1.33097958e+03	-5.18470985e+04	9.00e+02	1.44e-07	3.52e+04	0
2	3.43629988e+00	-4.87967714e+04	2.38e+00	3.80e-10	3.97e+02	0
3	3.15580630e+00	-5.01093327e+03	5.23e-02	8.37e-12	3.21e+01	0
4	3.13652729e+00	-1.83202866e+01	1.60e-04	2.53e-14	1.35e-01	0
5	1.85386452e+00	-4.93526712e+00	3.48e-05	5.55e-15	4.27e-02	0
6	1.30405298e+00	-3.49130221e+00	1.85e-05	3.00e-15	3.02e-02	0
7	5.99417399e-01	-3.20087402e-01	1.99e-06	4.44e-16	5.78e-03	0
8	3.95517964e-01	5.42350884e-02	2.06e-07	5.55e-17	2.15e-03	0
9	3.87906517e-01	3.02294484e-01	9.85e-09	1.86e-17	5.38e-04	0
10	3.74293083e-01	3.59125744e-01	9.10e-15	5.55e-17	9.54e-05	0
11	3.73004159e-01	3.72905736e-01	2.22e-16	1.37e-17	6.19e-07	0
12	3.72996753e-01	3.72996654e-01	8.88e-16	3.00e-17	6.19e-10	0
13	3.72996745e-01	3.72996745e-01	4.44e-16	1.96e-17	6.20e-13	0

Barrier solved model in 13 iterations and 0.10 seconds

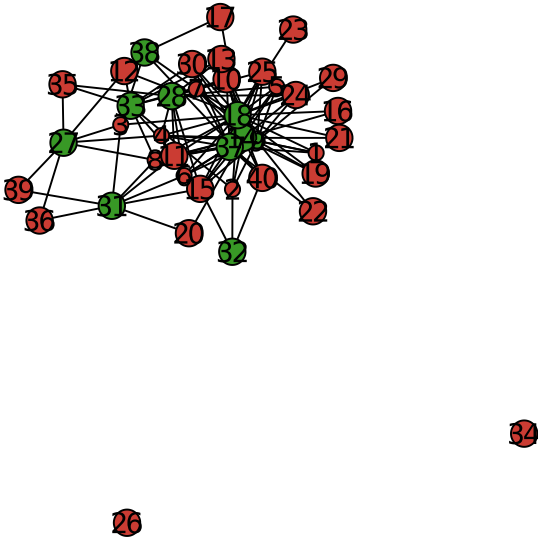
Optimal objective 3.72996745e-01

The how much cases should be handled by each patrol center:

[0.0 0.0 0.0 0.04995982177453549 0.04267315843344367 0.05173887941759819  
 4 0.0 0.0 0.0 0.048497143138645 0.050314067584685734 0.0 0.0487213153766  
 15996 0.0 0.03406229431360458 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.03155941  
 9514259786 0.051568651742917054 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.04  
 8367364464713285 0.0 0.0 0.0 0.0 0.0; 0.0 0.0 0.2903549509558251 0.0 0.0  
 0.0 0.3898619574512102 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.30272510915  
 92426 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0  
 0 0.0 0.0 0.0 0.0 0.0 0.1246719614919219; 0.0 0.0 0.07295210025834385 0.  
 0 0.0 0.07397948431137073 0.0 0.054566918951322846 0.0 0.0 0.09365083832  
 117115 0.0 0.0 0.0 0.05115433151716762 0.0 0.0 0.0 0.0 0.121119536562572  
 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.1048306942  
 9049104 0.0 0.0 0.10804489520298799 0.0; 0.0 0.6285029625408969 0.0 0.0  
 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.35282234630806203 0.0 0.0 0.0  
 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0  
 0.0 0.0 0.0 0.12467196149766638; 0.0 0.022066153050975052 0.0 0.0 0.0  
 0.0 0.030180228279870644 0.027174126726649723 0.0 0.030146511831760422  
 0.030997507078621735 0.021715863734876724 0.03021172286003626 0.0 0.0  
 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0319573536966  
 4022 0.0 0.0 0.0 0.0 0.030275057187088394 0.0 0.0 0.0 0.0 0.0; 0.0156003  
 69837172659 0.013917640031387138 0.015320374330892462 0.0170751292431059  
 2 0.01348080972228629 0.014916958485558345 0.016631080285450708 0.013472  
 378974237989 0.0 0.014490419237223495 0.016698320860869195 0.01204791321

```
330856 0.014397782073547333 0.0 0.013436195212943466 0.02164189493542492
0.018819844221892357 0.0 0.014974172280670056 0.0 0.02784974366935396 0.
0 0.017506896305207673 0.011844951966559287 0.016705293122827803 0.0 0.0
0.0 0.017118793745660825 0.016200524324639646 0.0 0.0 0.0 0.0 0.0 0.0 0.
0 0.0 0.0 0.12467196123471622; 0.11314487182800842 0.0 0.0 0.12808959175
921372 0.07312534782050178 0.10098726427270852 0.12412320241443348 0.071
75511743814768 0.0 0.09312833044268784 0.1287194025181583 0.0 0.0 0.0 0.
06211548695338626 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.10839479620193397 0.0 0.0 0.
12638423552544495 0.0 0.0 0.0 0.0 0.0 0.12154612188509235 0.0 0.0 0.0 0.0 0.
0 0.0 0.0 0.0 0.0 0.12467196147869823; 0.022309768705775553 0.0183438792
1648661 0.0 0.022893691098079705 0.018381763214187816 0.0206293306690669
2 0.022441473261306827 0.018334155595750714 0.0 0.019893745170375333 0.0
22597626445127347 0.0 0.01980200812230314 0.0 0.017789148713879852 0.0
0.0 0.0 0.021737070999648334 0.0 0.0 0.02537870669919373 0.0 0.01551297
3696365772 0.022905510892345674 0.0 0.0 0.0 0.023148691656758764 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0; 0.0 0.0 0.06947377598188598 0.08241
499421751816 0.0 0.0 0.0 0.05542324109747911 0.0 0.0 0.0 0.0348606785331
2425 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.07262728183077453 0.08236011912984949 0.0 0.0 0.08
196239824255315 0.0; 0.016134853163132245 0.01409633342024666 0.0 0.0168
34298028327352 0.014276393353390726 0.015676727966681913 0.0164271257689
4296 0.01427372555033854 0.0 0.015280092299759372 0.01650787599388145 0.
0 0.015197369257451873 0.0 0.013819642219858505 0.020460747941905087 0.0
0.0 0.01579816047612364 0.024348560407608365 0.02707256500396468 0.0 0.0
0.012512778949511716 0.016734630951399567 0.0 0.0 0.0 0.0 0.016362426399
948456 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.12467196121889916]
```

Out[2]:



## Result and Interpretation

1. The row of matrix represent how much patrol force from  $i^{th}$  row should be sent to  $j^{th}$  traffic road.
2. They should be integer in the real world, so more investigation will be need. Here a better optimization solution is to further use the integer programming.
3. The company owner should build their store more evenly so some store won't sold out their trading cards and no store should be over stocked.

## Problem 2

### Math Model

We can think of this problem as an integer optimization problem with decision variables.

$$X_{i,j} = \begin{cases} 1, & \text{if } i^{th} \text{ traffic road should be handled by } j^{th} \text{ patrol center} \\ 0, & \text{if } i^{th} \text{ traffic road should not be handled by } j^{th} \text{ patrol center} \end{cases}$$

And since one patrol center can only block one traffic road, and each traffic road only needs to be blocked by exactly one patrol center. These are two constraints we should consider in modeling.

What we need to do here is to extract the submatrix of problem 1  $C_{i,j}$  to represent the distance between  $n$  patrol centers and  $m$  traffic roads that need to be blocked.

### Solution

In [3]:

```
using Random
using Graphs, NamedArrays, Clp, Combinatorics, Gurobi, JuMP, PyPlot
using GraphRecipes
using Plots
using FixedPointNumbers
using Ipopt
using Nlopt

Random.seed!(1234);
rng = MersenneTwister(1234);
#####
#####
#      function to find the shortest path between 2 traffic road
#####
#####
function Dijkstra(src, mat)
    visited=zeros(40)
    visited[src]=1
    dist=zeros(40)
    pre=zeros(40)
    minci=0
    for i in 1:40
        dist[i]=mat[src, i]
```

```

        pre[i]=src
    end

    for i in 1:40
        minc=10000000
        for j in 1:40
            if(visited[j]==0&&dist[j]<minc)
                minci=j
                minc=dist[j]
            end
        end
        visited[minci]=1
        for j in 1:40
            if(visited[j]==0&&mat[minci,j]!=10000000&&minc+mat[minci,j]<dist[j])
                dist[j]=minc+mat[minci,j]
                pre[j]=minci
            end
        end
    end

    return pre
end

function routea2b(a,b,mat)
    pre= Dijkstra(a,mat)
    i=Int(b)
    routeorder=[]
    print("route from ",a," to ",b," :")
    while(i!=a)
        push!(routeorder,i)
        i=Int(pre[i])
    end
    print(a,"->")

    for i in Iterators.reverse(1:length(routeorder)-1)
        print(routeorder[i],"->")
    end

end

#####
#####
#####
#generating data for the problem
#####
#####

lmap=rand(Float64, (40, 2))
#println(lmap)
avg=0
rawmap=zeros(40,40)
for i in 1:40
    for j in 1:2
        if(i<20)
            lmap[i,j]=lmap[i,j]*5/2+5/4
        else
            lmap[i,j]*=5
        end
    end
end
end
end

```



```
cmap=zeros(40,40)
for i in 1:40
    avg+=lmap[i,1]
end
println(avg/40)
for i in 1:40
    for j in 1:40
        if(rand(Float64)<4/40)
            cmap[i,j]=1
            cmap[j,i]=1
        end
    end
end
#println(cmap)

for i in 1:40
    for j in 1:40
        if(cmap[i,j]==1)
            rawmap[i,j]=sqrt((lmap[i,1]-lmap[j,1])^2+(lmap[i,2]-lmap[j,2])^2)
        end
    end
end

xs=zeros(40)
ys=zeros(40)
for i in 1:40
    xs[i]=lmap[i,1]
    ys[i]=lmap[i,2]
end
ps=zeros(10)
temp=shuffle(rng, Vector{Int}(1:40))
for i in 1:10
    ps[i]=temp[i]
end

lockRoad=zeros(7)
for i in 1:7
    lockRoad[i]=temp[10+i]
end

colors= Array{RGB{FixedPointNumbers.Normed{UInt8,8}}}(undef,0)
po=0
for i in 1:40
    po=0
    for j in 1:10
        if (ps[j]==i)
            push!(colors,colorant"#389826")
            po=1
            break
        end
    end
    if(po==0)
        push!(colors,colorant"#CB3C33")
    end
end
```

```
end
println("The patrol centers are on the node:", ps)

adj=zeros(40, 40)

for i in 1:n
    for j in 1:n
        if (i==j)
            rawmap[i, i]=0

            elseif ( rawmap[i, j]==0)
                rawmap[i, j]=10000
            end
            adj[i, j]=rawmap[i, j]
        end
    end
end
#println(rawmap)

for k in 1:n
    for i in 1:n
        for j in 1:n
            if(rawmap[i, j]>rawmap[i, k]+rawmap[k, j])
                rawmap[i, j]=rawmap[i, k]+rawmap[k, j]
            end
        end
    end
end

#println(rawmap)
covermap=zeros(10, 40)
bcovermap=zeros(10, 40)
for i in 1:10
    for j in 1:n
        if(rawmap[Int(ps[i]), j]<3)
            covermap[i, j]= rawmap[Int(ps[i]), j]
            bcovermap[i, j]=1
        end
    end
end
#println("covermap", covermap)
#println(bcovermap)
for i in 1:10

    for j in 1:40
        if(j in ps)
            bcovermap[i, j]=0
        end
    end
end

lockMap=zeros(10, 7)
for i in 1:10
```

```

    for j in 1:7
        lockMap[i, j]=rawmap[Int(ps[i]), Int(lockRoad[j])]
    end
end
#println(lockRoad)
println("murder locations", lockRoad)
#####
#####
#solution of the problem
#####
#####
m = Model(Gurobi.Optimizer)
@variable(m, x[1:10, 1:7], Bin)

@variable(m, r)
@constraint(m, con2[ i in 1:10 , j in 1:7], r>=lockMap[i, j]*x[i, j])
@constraint(m, con1[ i in 1:10], sum(x[i, j] for j in 1:7 )<=1)
@constraint(m, con4[ j in 1:7], sum(x[i, j] for i in 1:10)=1)

@objective(m, Min, r)
@time(optimize!(m))
println(value.(x))

for i in 1:10
    for j in 1:7
        if(value.(x)[i, j]==1)

            routea2b(Int(ps[i]), Int(lockRoad[j]), adj)

        end
    end
    println()
end
end

```

2.491098786608659

WARNING: using NLOpt.optimize! in module Main conflicts with an existing identifier.

The patrol centers are on the node:[28.0, 38.0, 31.0, 32.0, 33.0, 18.0, 37.0, 9.0, 27.0, 14.0]

murder locations[39.0, 30.0, 34.0, 17.0, 5.0, 15.0, 6.0]

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Gurobi Optimizer version 9.0.2 build v9.0.2rc0 (win64)

Optimize a model with 87 rows, 71 columns and 280 nonzeros

Model fingerprint: 0xdbe669c7

Variable types: 1 continuous, 70 integer (70 binary)

Coefficient statistics:

Matrix range [9e-01, 7e+00]

Objective range [1e+00, 1e+00]

Bounds range [0e+00, 0e+00]

RHS range [1e+00, 1e+00]

Found heuristic solution: objective 4.5836477

Presolve removed 9 rows and 9 columns

Presolve time: 0.01s

Presolved: 78 rows, 62 columns, 244 nonzeros

Variable types: 1 continuous, 61 integer (61 binary)

Root relaxation: objective 1.062601e+00, 76 iterations, 0.01 seconds

Nodes		Current Node			Objective Bounds			Work
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node
Time								
0	0	1.38328	0	20	4.58365	1.38328	69.8%	-
0s								
H	0				4.0729796	1.38328	66.0%	-
0s								
H	0				3.9474676	1.38328	65.0%	-
0s								
0	0	1.38328	0	20	3.94747	1.38328	65.0%	-
0s								

Explored 1 nodes (123 simplex iterations) in 0.06 seconds

Thread count was 8 (of 8 available processors)

Solution count 3: 3.94747 4.07298 4.58365

Optimal solution found (tolerance 1.00e-04)

Best objective 3.947467604050e+00, best bound 3.947467604050e+00, gap 0.0000%

2.432449 seconds (2.16 M allocations: 108.432 MiB, 2.94% gc time)

[-0.0 -0.0 0.0 -0.0 0.0 -0.0 -0.0; -0.0 -0.0 -0.0 1.0 0.0 -0.0 -0.0; -0.0 -0.0 0.0 -0.0 -0.0 -0.0 -0.0; 0.0 -0.0 0.0 0.0 0.0 -0.0 -0.0; -0.0 1.0 -0.0 0.0 -0.0 -0.0 -0.0; -0.0 -0.0 0.0 -0.0 1.0 -0.0 -0.0; -0.0 -0.0 0.0 -0.0 -0.0 1.0 -0.0; -0.0 -0.0 1.0 -0.0 -0.0 -0.0 -0.0; 1.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0; -0.0 -0.0 0.0 -0.0 -0.0 1.0 -0.0]

route from 38 to 17 :38->

route from 33 to 30 :33->

route from 18 to 5 :18->6->5->

route from 37 to 6 :37->6->

route from 9 to 34 :9->34->

route from 27 to 39 :27->39->

route from 14 to 15 :14->15->

## Result and Interpretation

In this case, if there are 7 people are murdered, the minimum total distance is 3.947467604050e+00.

We can obtain 7 routes from the output are

route from 38 to 17 :38->

route from 33 to 30 :33->

route from 18 to 5 :18->6->5->

route from 37 to 6 :37->6->

route from 9 to 34 :9->34->

route from 27 to 39 :27->39->

route from 14 to 15 :14->15->

## Problem3

### Math model

Since the criminal is escaping we need to calculate the possible area the criminal could be by drawing the circle with  $r_c = v_c * t$  where  $v$  is the escaping speed of the criminal and  $t$  is the time the criminal has been escaping.

Then we need to iterate through the traffic roads in the row of the traffic road where the criminal start to escaping and add it to the blocking set  $B_i$ .

Then we need to calculate the police reaction circles with  $r_p = v_p * y$ , so that we can iterate through the patrol center to construct possible edges from set of patrol center  $P_i$  to  $B_i$  and represent this bipartite graph using a matrix  $U_{i,j}$ .

Then then we iterate through each entry of patrol center to traffic road matrix  $T_{i,j}$  which contains the distance between each patrol center and each traffic road. Set  $U_{i,j} = T_{i,j}$  if these 2 condition is satisfied:

- 1)  $T_{i,j} \leq r_p$
- 2)  $i \in B_i$

Then We need to find a matching  $M_i$  in the bipartite graph  $U_{i,j}$ .

### 3.1 matching model in bipartite graph

We will use hungry algorithm to find the matching in  $U_{i,j}$ . The idea of this algorithm is that :.

1) If you can find an augmented path in bipartite graph, there is a better matching that is closer to perfect match than the current path.

2) The subgraph don't have augmented graph implies that there is not augmented graph

used code

```
U[n][m]
used[m]
r2l[n]
main:
    for :i to n:
        initialize used
        if find(i) all+=1

find(x):
    for: i to m:
        if (U[x][i]==true&&used[i]==false):
            used[i]=1
            if (r2l[i]==0 | find(r2l)):
                r2l=x
                return true

    return false
```

## Solution



In [4]:

```

using Random
using Graphs, NamedArrays, Clp, Combinatorics, Gurobi, JuMP, PyPlot
using GraphRecipes
using Plots
using FixedPointNumbers

#####
#####
#####      #generating data for the problem
#####
#####
Random.seed!(1234);
rng = MersenneTwister(1234);

lmap=rand(Float64, (40, 2))

avg=0
rawmap=zeros(40, 40)
for i in 1:40
    for j in 1:2
        if(i<20)
            lmap[i, j]=lmap[i, j]*5/2+5/4
        else
            lmap[i, j]*=5
        end
    end
end
cmap=zeros(40, 40)
for i in 1:40
    avg+=lmap[i, 1]
end

for i in 1:40
    for j in 1:40
        if(rand(Float64)<4/40)
            cmap[i, j]=1
            cmap[j, i]=1
        end
    end
end

```

```
end
  #println(cmap)

for i in 1:40
  for j in 1:40
    if(cmap[i, j]==1)
      rawmap[i, j]=sqrt((lmap[i, 1]-lmap[j, 1])^2+(lmap[i, 2]-lmap[j, 2])^2)
    end
  end
end

xs=zeros(40)
ys=zeros(40)
for i in 1:40
  xs[i]=lmap[i, 1]
  ys[i]=lmap[i, 2]
end

ps=zeros(10)
temp=shuffle(rng, Vector{1:40})
for i in 1:10
  ps[i]=temp[i]
end
colors= Array{RGB{FixedPointNumbers.Normed{UInt8, 8}}}(undef, 0)
po=0
for i in 1:40
  po=0
  for j in 1:10
    if (ps[j]==i)
      push!(colors, colorant"#389826")
      po=1
      break
    end
  end
  if(po==0)
    push!(colors, colorant"#CB3C33")
  end
end

end
println("The patrol centers are on the node:", ps)

finalCitymap=zeros(40, 40)
for i in 1:40
  for j in 1:40
    finalCitymap[i, j]=rawmap[i, j]
  end
end

plt=graphplot(rawmap,

               nodeshape=:circle,
```

```
        nodesize=0.1,

        curves=false,
        names = 1:40,
        fontsize = 10,
        nodecolor=colors,
        linecolor = :darkgrey
    )

n=40

for i in 1:n
    for j in 1:n
        if (i==j)
            rawmap[i,i]=0

            elseif ( rawmap[i,j]==0)
                rawmap[i,j]=10000
            end
        end
    end
end
#println(rawmap)

for k in 1:n
    for i in 1:n
        for j in 1:n
            if(rawmap[i,j]>rawmap[i,k]+rawmap[k,j])
                rawmap[i,j]=rawmap[i,k]+rawmap[k,j]
            end
        end
    end
end

#println(rawmap)
covermap=zeros(10,40)
bcovermap=zeros(10,40)
for i in 1:10
    for j in 1:40
        if(rawmap[Int(ps[i]),j]<3)
            covermap[i,j]= rawmap[Int(ps[i]),j]
            bcovermap[i,j]=1
        end
    end
end
#println(covermap)
#println(bcovermap)
for i in 1:10

    for j in 1:40
        if(j in ps)
            bcovermap[i,j]=0
        end
    end
end
end
```

```

printcovermap=zeros(40,40)

for i in 1:10
    for j in 1:40
        if(covermap[i,j]>0)
            printcovermap[i,j]=covermap[i,j]
        end
    end
end

#####
#####
#####
#solution of the problem
#####
#####

function find(i,mat,usd,m,n,match) #function of hugarian algorithmn
    #println("i:", i, "j:", j)
    for j in 1:n
        # println("i:", i, "j:", j)
        if (mat[Int(i), j]==1&&usd[j]!=1)

            usd[j]=1
            if(match[j]==0 || (find(Int(match[j]), mat, usd, m, n, match)))
                match[j]=i
                return true
            end
        end
    end

    return false
end

cmt=temp[11]
vcar=0.1
vpcar=0.16
#there are t=100 limit
t=0
#####
#####
#####
#main loop to find min(t)
#####
#####
while(t<100)
    t+=1
    rc=0
    rc=t*vcar+0.3
    rp=t*vpcar
    setB=[]
    println("Iterating t=", t)
    # println("rc", rc)
    for i in 1:40
        if(rawmap[Int(cmt), i]<rc)
            push!(setB, i)
        end
    end
    maA=zeros(10, length(setB))
    start=zeros(2)

```

```

for i in 1:10
    for j in 1:length(setB)
        if(rawmap[Int(ps[i]),setB[j]]<rp)
            #println("d():",rawmap[Int(ps[i]),setB[j]], " < ",rp)
            maA[i,j]=1
            start[1]=i
            start[2]=j
        end
    end
end
println("At this time criminal's range: ",setB," starting from node :",temp[11
])
println("The roads that patrol center can block:",maA)
#println(start)
#if(length(setB)==0||start[1]==0)
    #continue
#end
flg=0
match=zeros(length(setB))
for i in 1:10
    used=zeros(length(setB))

    if( find(i,maA,used,10,length(setB),match)==true)
        perfect =1
        for i in 1:length(setB)
            if (match[i]==0)
                perfect=0
            end
        end
        if (perfect==1)
            println("Match found",match," at time t=",t)
            flg=1
            break
        end
    end
    println("current Match",match)
end

if(flg==1)

    break
end

end
end

```

```
The patrol centers are on the node:[28.0, 38.0, 31.0, 32.0, 33.0, 18.0,
37.0, 9.0, 27.0, 14.0]
Iterating t=1
At this time criminal's range: Any[39] starting from node :39
The roads that patrol center can block:[0.0; 0.0; 0.0; 0.0; 0.0; 0.0; 0.
0; 0.0; 0.0; 0.0]
current Match[0.0]
current Match[0.0]
current Match[0.0]
current Match[0.0]
current Match[0.0]
current Match[0.0]
current Match[0.0]
current Match[0.0]
current Match[0.0]
current Match[0.0]
Iterating t=2
At this time criminal's range: Any[39] starting from node :39
The roads that patrol center can block:[0.0; 0.0; 0.0; 0.0; 0.0; 0.0; 0.
0; 0.0; 0.0; 0.0]
current Match[0.0]
current Match[0.0]
current Match[0.0]
current Match[0.0]
current Match[0.0]
current Match[0.0]
current Match[0.0]
current Match[0.0]
current Match[0.0]
current Match[0.0]
Iterating t=3
At this time criminal's range: Any[36, 39] starting from node :39
The roads that patrol center can block:[0.0 0.0; 0.0 0.0; 0.0 0.0; 0.0
0.0; 0.0 0.0; 0.0 0.0; 0.0 0.0; 0.0 0.0; 0.0 0.0; 0.0 0.0]
current Match[0.0, 0.0]
current Match[0.0, 0.0]
current Match[0.0, 0.0]
current Match[0.0, 0.0]
current Match[0.0, 0.0]
current Match[0.0, 0.0]
current Match[0.0, 0.0]
current Match[0.0, 0.0]
current Match[0.0, 0.0]
current Match[0.0, 0.0]
Iterating t=4
At this time criminal's range: Any[36, 39] starting from node :39
The roads that patrol center can block:[0.0 0.0; 0.0 0.0; 0.0 0.0; 0.0
0.0; 0.0 0.0; 0.0 0.0; 0.0 0.0; 0.0 0.0; 0.0 0.0; 0.0 0.0]
current Match[0.0, 0.0]
current Match[0.0, 0.0]
current Match[0.0, 0.0]
current Match[0.0, 0.0]
current Match[0.0, 0.0]
current Match[0.0, 0.0]
current Match[0.0, 0.0]
current Match[0.0, 0.0]
current Match[0.0, 0.0]
current Match[0.0, 0.0]
Iterating t=5
At this time criminal's range: Any[36, 39] starting from node :39
The roads that patrol center can block:[0.0 0.0; 0.0 0.0; 0.0 0.0; 0.0
```

0.0; 0.0 0.0; 0.0 0.0; 0.0 0.0; 0.0 0.0; 0.0 0.0; 0.0 0.0]

current Match[0.0, 0.0]

current Match[0.0, 0.0]

current Match[0.0, 0.0]

current Match[0.0, 0.0]

current Match[0.0, 0.0]

current Match[0.0, 0.0]

current Match[0.0, 0.0]

current Match[0.0, 0.0]

current Match[0.0, 0.0]

current Match[0.0, 0.0]

Iterating t=6

At this time criminal's range: Any[36, 39] starting from node :39

The roads that patrol center can block:[0.0 0.0; 0.0 0.0; 0.0 0.0; 0.0 0.0; 0.0 0.0; 0.0 0.0; 0.0 0.0; 0.0 0.0]

current Match[0.0, 0.0]

current Match[0.0, 0.0]

current Match[0.0, 0.0]

current Match[0.0, 0.0]

current Match[0.0, 0.0]

current Match[0.0, 0.0]

current Match[0.0, 0.0]

current Match[0.0, 0.0]

current Match[0.0, 0.0]

current Match[0.0, 0.0]

Iterating t=7

At this time criminal's range: Any[36, 39] starting from node :39

The roads that patrol center can block:[0.0 0.0; 0.0 0.0; 0.0 0.0; 0.0 0.0; 0.0 0.0; 0.0 0.0; 0.0 0.0; 0.0 0.0]

current Match[0.0, 0.0]

current Match[0.0, 0.0]

current Match[0.0, 0.0]

current Match[0.0, 0.0]

current Match[0.0, 0.0]

current Match[0.0, 0.0]

current Match[0.0, 0.0]

current Match[0.0, 0.0]

current Match[0.0, 0.0]

current Match[0.0, 0.0]

Iterating t=8

At this time criminal's range: Any[36, 39] starting from node :39

The roads that patrol center can block:[0.0 0.0; 0.0 0.0; 0.0 0.0; 0.0 0.0; 0.0 0.0; 0.0 0.0; 0.0 0.0; 0.0 0.0]

current Match[0.0, 0.0]

current Match[0.0, 0.0]

current Match[0.0, 0.0]

current Match[0.0, 0.0]

current Match[0.0, 0.0]

current Match[0.0, 0.0]

current Match[0.0, 0.0]

current Match[0.0, 0.0]

current Match[0.0, 0.0]

current Match[0.0, 0.0]

Iterating t=9

At this time criminal's range: Any[36, 39] starting from node :39

The roads that patrol center can block:[0.0 0.0; 0.0 0.0; 0.0 0.0; 0.0 0.0; 0.0 0.0; 0.0 0.0; 0.0 0.0; 0.0 0.0]

current Match[0.0, 0.0]

current Match[0.0, 0.0]

current Match[0.0, 0.0]

current Match[0.0, 0.0]

```
current Match[0.0, 0.0]
current Match[0.0, 0.0]
current Match[0.0, 0.0]
current Match[0.0, 0.0]
current Match[0.0, 0.0]
current Match[0.0, 0.0]
```

Iterating t=10

At this time criminal's range: Any[1, 36, 39] starting from node :39

The roads that patrol center can block:[0.0 0.0 0.0; 0.0 0.0 0.0; 0.0 0.0 0.0; 0.0 0.0 0.0; 0.0 0.0 0.0; 0.0 0.0 0.0; 0.0 0.0 0.0; 0.0 0.0 0.0; 0.0 0.0 0.0; 0.0 0.0 0.0]

```
current Match[0.0, 0.0, 0.0]
current Match[0.0, 0.0, 0.0]
current Match[0.0, 0.0, 0.0]
current Match[0.0, 0.0, 0.0]
current Match[0.0, 0.0, 0.0]
current Match[0.0, 0.0, 0.0]
current Match[0.0, 0.0, 0.0]
current Match[0.0, 0.0, 0.0]
current Match[0.0, 0.0, 0.0]
current Match[0.0, 0.0, 0.0]
current Match[0.0, 0.0, 0.0]
```

Iterating t=11

At this time criminal's range: Any[1, 36, 39] starting from node :39

The roads that patrol center can block:[0.0 0.0 0.0; 0.0 0.0 0.0; 0.0 0.0 0.0; 0.0 0.0 0.0; 0.0 0.0 0.0; 0.0 0.0 0.0; 0.0 0.0 0.0; 0.0 0.0 0.0; 0.0 0.0 0.0; 0.0 0.0 0.0]

```
current Match[0.0, 0.0, 0.0]
current Match[0.0, 0.0, 0.0]
current Match[0.0, 0.0, 0.0]
current Match[0.0, 0.0, 0.0]
current Match[0.0, 0.0, 0.0]
current Match[0.0, 0.0, 0.0]
current Match[0.0, 0.0, 0.0]
current Match[0.0, 0.0, 0.0]
current Match[0.0, 0.0, 0.0]
current Match[0.0, 0.0, 0.0]
current Match[0.0, 0.0, 0.0]
```

Iterating t=12

At this time criminal's range: Any[1, 36, 39] starting from node :39

The roads that patrol center can block:[0.0 0.0 0.0; 0.0 0.0 0.0; 0.0 0.0 0.0; 0.0 0.0 0.0; 0.0 0.0 0.0; 0.0 0.0 0.0; 0.0 0.0 0.0; 0.0 0.0 0.0; 1.0 0.0 0.0]

```
current Match[0.0, 0.0, 0.0]
current Match[0.0, 0.0, 0.0]
current Match[0.0, 0.0, 0.0]
current Match[0.0, 0.0, 0.0]
current Match[0.0, 0.0, 0.0]
current Match[0.0, 0.0, 0.0]
current Match[0.0, 0.0, 0.0]
current Match[0.0, 0.0, 0.0]
current Match[0.0, 0.0, 0.0]
current Match[10.0, 0.0, 0.0]
```

Iterating t=13

At this time criminal's range: Any[1, 36, 39] starting from node :39

The roads that patrol center can block:[0.0 0.0 0.0; 0.0 0.0 0.0; 0.0 0.0 0.0; 0.0 0.0 0.0; 0.0 0.0 0.0; 0.0 0.0 0.0; 0.0 1.0 0.0; 1.0 0.0 0.0]

```
current Match[0.0, 0.0, 0.0]
current Match[0.0, 0.0, 0.0]
current Match[0.0, 0.0, 0.0]
current Match[0.0, 0.0, 0.0]
current Match[0.0, 0.0, 0.0]
```



```

current Match[0.0, 0.0, 0.0]
current Match[0.0, 0.0, 0.0]
current Match[0.0, 0.0, 0.0]
current Match[0.0, 9.0, 0.0]
current Match[10.0, 9.0, 0.0]
Iterating t=14
At this time criminal's range: Any[1, 30, 36, 39] starting from node :39
The roads that patrol center can block:[0.0 0.0 0.0 0.0; 0.0 0.0 0.0 0.
0; 0.0 0.0 1.0 0.0; 0.0 0.0 0.0 0.0; 0.0 0.0 0.0 0.0; 0.0 0.0 0.0 0.0;
0.0 0.0 0.0 0.0; 0.0 0.0 0.0 0.0; 0.0 0.0 1.0 0.0; 1.0 1.0 0.0 0.0]
current Match[0.0, 0.0, 0.0, 0.0]
current Match[0.0, 0.0, 0.0, 0.0]
current Match[0.0, 0.0, 3.0, 0.0]
current Match[0.0, 0.0, 3.0, 0.0]
current Match[0.0, 0.0, 3.0, 0.0]
current Match[0.0, 0.0, 3.0, 0.0]
current Match[0.0, 0.0, 3.0, 0.0]
current Match[0.0, 0.0, 3.0, 0.0]
current Match[0.0, 0.0, 3.0, 0.0]
current Match[10.0, 0.0, 3.0, 0.0]
Iterating t=15
At this time criminal's range: Any[1, 30, 36, 39] starting from node :39
The roads that patrol center can block:[0.0 0.0 0.0 0.0; 0.0 0.0 0.0 0.
0; 0.0 0.0 1.0 0.0; 0.0 0.0 0.0 0.0; 0.0 0.0 0.0 0.0; 1.0 1.0 0.0 0.0;
0.0 0.0 0.0 0.0; 0.0 0.0 0.0 0.0; 0.0 0.0 1.0 0.0; 1.0 1.0 0.0 0.0]
current Match[0.0, 0.0, 0.0, 0.0]
current Match[0.0, 0.0, 0.0, 0.0]
current Match[0.0, 0.0, 3.0, 0.0]
current Match[0.0, 0.0, 3.0, 0.0]
current Match[0.0, 0.0, 3.0, 0.0]
current Match[6.0, 0.0, 3.0, 0.0]
current Match[6.0, 0.0, 3.0, 0.0]
current Match[6.0, 0.0, 3.0, 0.0]
current Match[6.0, 0.0, 3.0, 0.0]
current Match[10.0, 6.0, 3.0, 0.0]
Iterating t=16
At this time criminal's range: Any[1, 29, 30, 36, 39] starting from node
:39
The roads that patrol center can block:[0.0 0.0 0.0 0.0 0.0; 0.0 0.0 0.0
0.0 0.0; 0.0 0.0 0.0 1.0 0.0; 0.0 0.0 0.0 0.0 0.0; 0.0 0.0 1.0 0.0 0.0;
1.0 0.0 1.0 0.0 0.0; 0.0 0.0 0.0 0.0 0.0; 0.0 1.0 0.0 0.0 0.0; 0.0 0.0
0.0 1.0 1.0; 1.0 0.0 1.0 0.0 0.0]
current Match[0.0, 0.0, 0.0, 0.0, 0.0]
current Match[0.0, 0.0, 0.0, 0.0, 0.0]
current Match[0.0, 0.0, 0.0, 3.0, 0.0]
current Match[0.0, 0.0, 0.0, 3.0, 0.0]
current Match[0.0, 0.0, 5.0, 3.0, 0.0]
current Match[6.0, 0.0, 5.0, 3.0, 0.0]
current Match[6.0, 0.0, 5.0, 3.0, 0.0]
current Match[6.0, 8.0, 5.0, 3.0, 0.0]
Match found[6.0, 8.0, 5.0, 3.0, 9.0] at time t=16

```

## Result and Interpretation

It seems like if the criminal is last seen in this random traffic road, police car needs to be 1.5 times faster than the car of criminal in order to catch him. Because the selection of patrol centers is random and the result won't be too good .

Obviously not every traffic road can be intercepted, because although the siren did work you can't drive 110 mph in the city. Other fun things to do would be pack the main of the program above and pass combinations of police station in to find a better maximum possible intercept node of the whole city.

## Conclusion

To summarize this project, the original three questions are almost solved, although there are some limitations to fit in the realistic situation. (e.g. Recall the Integer Program in Problem1) One of my findings is that for a city (given the coordinates of every traffic road and the coordinates of every patrol center), it is possible to use warshall alogrithmn to find out the distance between any two patrol centers and traffic roads, as well as which traffic roads will be covered in each patrol center. Finally we can get a optimized solution (valued by the evenness of police cases accepted by all centers) on how to distribute the police force to each traffic road. Another finding is that for problem3, we find the ratio of the speed of the police and the speed of the crimnal such that the criminal can be taken down needs to be 1.5 times.

There are many aspects we can investigate in the future, we can implement another obective function and plot the tradeoff curve between the the balance of the patrol distribution and the efficient of patrol distribution.

It would be also interesting to discuss if there is a correlation between the second objective in the first problem and their performance in second problem. Though first we would need to figure out how to evaluate their performance in problem 2. But this can be evaluated by suming all the objective by calculating the linear combinations (mutiply by weight).

In [ ]: