# Problem Set 2

## Physical Layer

### 1. Noisy Channel Data Rates

The decibel is a measure of the ratio between two signal levels: $N_{db} = 10 \log_{10} (P_2/P_1)$, where $N_{db}$ = the number of decibels, $P_1$ = the input power level and $P_2$ = the output power level.

a. A telephone line is known to have a loss of 20db. The input signal power is measured as 0.25 watt and the output noise is measured as 10 µwatt. Using this information, calculate the output signal-to-noise ratio in dB.

$$N_{db} = 10log_{10}\frac{P_2}{P_1}$$
$$-20db = 10log_{10}\frac{P_2}{0.25watt}$$

Thus, the output power level is:

$$P_2 = 2.5 \times 10^{-3}\ watt$$

Then the output SNR is:

$$SNR_{db} = 10log_{10}\frac{P_2}{N_2}, where\ N_2\ is\ the\ output\ noise$$
$$SNR_{db} = 10log_{10}\frac{2.5 \times 10^{-3}\ watt}{10\ \mu watt} = 10log_{10}250 = 24\ db$$

**<u>The output signal-to-noise ratio in db is 24.</u>**

b. What is the capacity of this phone line with a frequency range of 300 Hz – 4800 Hz?
The bandwidth of this phone line is $B = 4800\text{ Hz} - 300\text{ Hz} = 4500\text{ Hz}$
According to the calculation from a, the output signal to nose ratio is

$$\frac{S}{N} = 250$$

Then the capacity is:

$$C = Blog_2\left(1 + \frac{S}{N}\right) = 4500 \times log_2(251) = 35872\ bps = 35.9\ kbps$$

**<u>Thus, the capacity of this phone line is 35.9 kbps.</u>**

c. If the attenuation rate of this phone line is 8 dB/km, and the minimum output signal is 0.005 watt, given the input signal from part a), how long can the phone line be before it requires a repeater?
The decimal loss of given input and output signal is:

$$N_{db} = 10log_{10}\frac{0.005}{0.25} = -17\ db$$

Then the length of the phone line is

$$L = \frac{17\ db}{8\ db/km} = 2.12\ km$$

**<u>Thus, the length of phone line should be smaller than 2.12 km before it requires a repeater.</u>**

## 2. Encoding

a. **Bit and baud rates.** Suppose, it is possible to send 128 different types of signals on a link, and that there is no noise. How many bits per second (bps) can such a link achieve at 4000 baud?
Each signal contains

$$log_2 128 = 7\ bits$$

**Then the data rate is:**

$$\underline{\mathbf{7 \times 4000 = 28000\ bps = 28\ kbps}}$$

b. **SNR**. What signal-to-noise ratio (in dB) is needed to put a 7 Gbps carrier on a 500-MHz line? (Note: for line speeds in networking, giga-, mega-, kilo- indicate powers of 1000, not 1024.)
According to the Clause Shannon equation:

$$C = Blog_2(1 + \frac{S}{N})$$

$$7000\ Mbps = 500MHz \times log_2(1 + \frac{S}{N})$$

$$\frac{S}{N} = 16383$$

Then

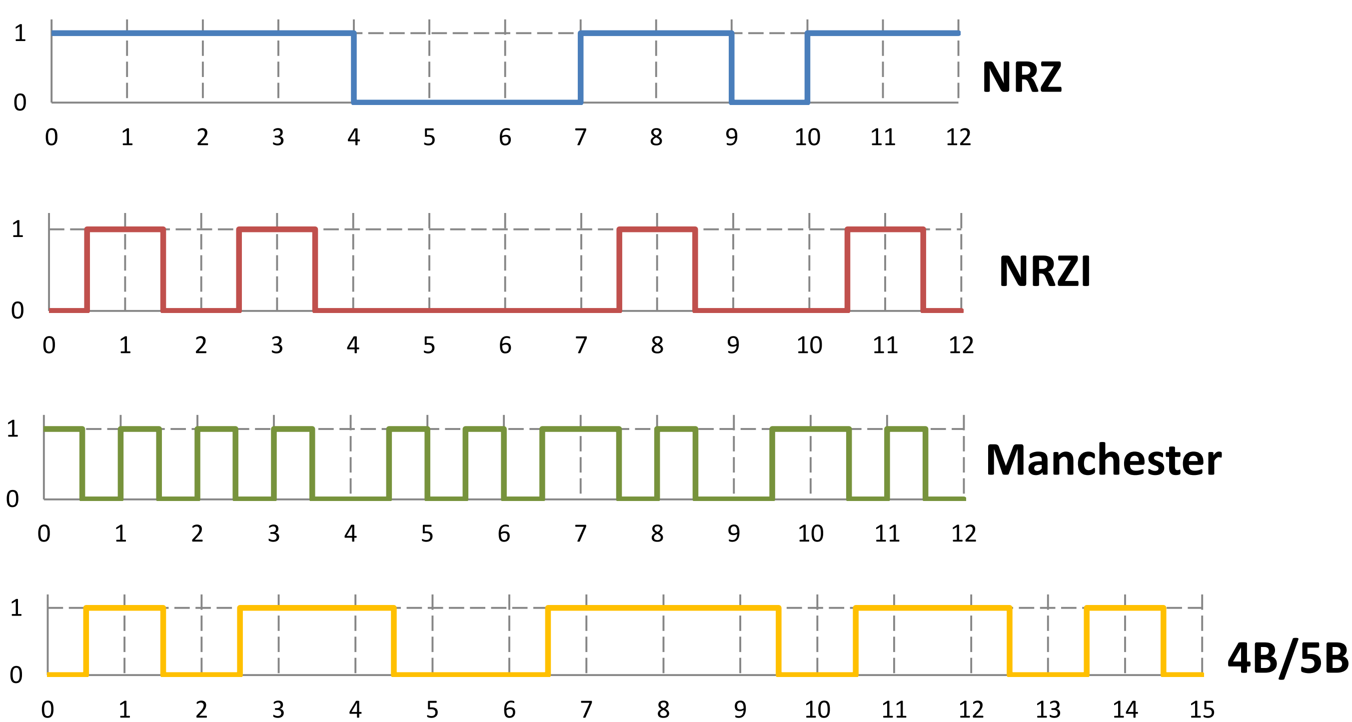
$$SNR(db) = 10log_{10}\left(\frac{S}{N}\right) = 10 \times log_{10}(16383) = 42\ db$$

**Thus, 42db signal-to-noise ratio is needed.**

## 3. Encoding and Channel Capacity

a. Show the NRZ, Manchester, NRZI and 4B/5B encoding signals (the resulting NRZI signal for 4B/5B), using a diagram similar to that in the class slides, for the data bit sequence 1111 0001 1011. To be definite, suppose the NRZI signals begin at low voltage.

**The 4B/5B encoding is: 11101 01001 10111**
**The following diagrams show encoding signal of the given bit sequence:**



b. In 1962, Bell Labs introduced the first version of their Transmission System 1 (T-1). Subsequent specifications carried multiples of the basic T1 data rates. What signal-to-noise ratio is needed to put a T3, 672channel, carrier on a 75-MHz line?

The T3 has data rate of 44.736 Mbps (https://en.wikipedia.org/wiki/T-carrier).

$$44.736\ Mbps = 75\ MHz \times log_2(1+\frac{S}{N})$$

Then, the signal to noise ratio is:

$$\frac{S}{N} = 0.512$$

We need 0.512 signal-to-noise ratio to put T2, 672 channel, carrier on a 75 MHz line.

## 4. Modulation

a. A modem constellation diagram has data points at the following coordinates: (1, 1), (1, -1), (-1, 1), (-1, -1), (2, 2), (2, -2), (-2, 2), (-2, -2), (3, 3), (3, -3), (-3, 3), (-3, -3), (4, 4), (4, -4), (-4, 4), and (-4, -4). How many bps can a modem with these parameters achieve at 1600 baud?
Since there are 16 data points in the constellation diagram, each symbol contains

$$log_2 16 = 4\ bits$$

The data rate is:

$$4 \times 1600 = 6400\ bsp$$

**Thus, 6400 bps can be achieved at 1600 baud.**

b. A modem constellation diagram has data points at (-12, 4) and (-48, -16). Does the modem use phase modulation and/or amplitude modulation? Explain your answer.

Since these two points have different distance from the origin, this modem might use combination of amplitude and phase modulation.

## 5. Framing

Consider the data bit sequence 0000 1010 1111 1111 0101 0000 0110 0000 1111 1111 0000 1110. In this problem, you will frame these bits in three ways.

a. First, frame the bits with byte stuffing as used in the BISYNC protocol. You need show only the body (including stuffed bytes) and the sentinel bits. DLE is ASCII character 16 (decimal), STX is 2, and ETX is 3.

STX: 0000 0010
**Body:**
**0000 1010 1111 1111 0101 0000 0110 0000 1111 1111 0000 1110**
ETX: 0000 0011

b. Second, frame the bits using bit stuffing as defined by the HDLC protocol. Again, you need show only the (stuffed) data bits and the sentinel bits.

Frame Marker: 0111 1110
Body:
**0000 1010 1111 10111 0101 0000 0110 0000 1111 10111 0000 1110**
Frame Marker: 0111 1110

c. Third, frame the bits into 8-bit RS-232 characters. Use “0” to represent start bits and “1” to represent stop bits.
The frame is:

**0**0000 101**1**
**0**0 1111 111**1**
**0**11 0101 01**1**
**0**000 0110 **1**
**0**0000 111**1**
**0**1 1111 00**1**
**0**00 1110**1**

d. Now, counting only the bits that you wrote, calculate the efficiency (as a percentage of real data per bit sent) of your answers to (a), (b), and (c).

For (a):

If we only count the body, 48 bits were sent for 48 bits of data.
The efficiency is 48/48 = 100%

If include the STX and ETX:
The efficiency is 48/64 = 75%

For (b):

If we only count the body, 50 bits were sent for 48 bits of data.
The efficiency is 48/50 = 96%

If include the Frame Marker:
The efficiency is 48/66 = 73%

For (c):

62 bits were sent for 48 bits of data
The efficiency is 48/62 = 77%

## 6. Error Detection

a. A CRC is constructed to generate a 4-bit checksum for an 11-bit message. The generator polynomial is $x^4 + x^3 + 1$. Encode the data bit sequence **10000110100**. Now assume that bit 4 (counting from the most significant bit) in the code word is in error and show how the error is detected.

Since the generator is of degree 4, multiply the message with $x^4$, this gives T(x) = 10000110100<u>0000</u>. We divide the T(x) by $x^4 + x^3 + 1$, which is corresponds to 11001. The following process shows the detailed polynomial long division operation:

```
        -----------------
11001 ) 100001101000000
        11001
        -----
         10011
         11001
         -----
          10101
          11001
          -----
           11000
           11001
           -----
            00011000
               11001
               -----
                  0001000
```

The remainder is 1000, then, we know the message we send is **10000110100<u>1000</u>**, which would be exactly divisible by 11001.

When bit 4 is corrupted, the message becomes: **100101101001<u>000</u>**, divide it by 11001 gives a none zero remainder 1101. This concludes that there are errors in the massage.
<u>Detailed calculation:</u>

```
         ----------------
 11001 ) 100101101001000
         11001
         -----
          10111
          11001
          -----
           11101
           11001
           -----
            010001
             11001
             -----
              10000
              11001
              -----
               10010
               11001
               -----
                10111
                11001
                -----
                 11100
                 11001
                 -----
                  010100
                   11001
                   -----
                   01101
```

b. The bit sequence **10010110011** corresponds to the polynomial $x^{10} + x^7 + x^5 + x^4 + x + 1$. Divide this polynomial by the CRC generator polynomial $x^3 + 1$ and report the remainder as a polynomial. Is the bit sequence correctly encoded with the given generator (i.e., is the remainder 0)?

The generator polynomial $x^3 + 1$ corresponds to 1001.
<u>**Dividing the bit sequence by 1001 gives a nonzero remainder 101, which corresponds to the polynomial $x^2 + 1$. This means the bit sequence is NOT correctly encoded with the given generator.**</u>

Detailed calculation:

```
        ------------
 1001 ) 10010110011
        1001
        ----
         00001100
             1001
             ----
              1011
              1001
              ----
               0101
```

c. Suppose a 4 bit CRC is appended to an *n* bit message according to the CRC polynomial $x^4 + x + 1$. The encoded message thus has *n* + 4 bits. What is the largest value of *n* such that any double bit error can be detected? (Hint: any error sequence corresponds to a polynomial that is the product of C(x) and some other polynomial.)

Since $G(x) = x^4 + x + 1$ can check two bits error of length up to $2^4 - 1 = 15$, if G(x) is primitive.

To check if G(x) is primitive, divide $x^m + 1$ by G(x), where m < 15.
**The result shows that G(x) is a primitive polynomial and it can detect double bit error up to length 15, and the largest n value is 15.**

Proof that this CRC cannot detect double bit error separated by 16: E(x) = 1000000000000001.

```
        -----------------
10011 ) 1000000000000001
        10011
        -----
         0011000
           10011
           -----
            10110
            10011
            -----
             010100
              10011
              -----
               011100
                10011
                -----
                 11110
                 10011
                 -----
                  11010
                  10011
                  -----
                   10011
                   10011
                   -----
                   00000
```

## 7. Networking Utilities

a. The Unix utility ping can be used to find the round trip time (RTT) to various Internet hosts. See the man page for ping to see how to use it and the -s option with other options to see how you can control the time between ICMP packet transmissions, and to display the resulting round trip times. Upon interrupting execution of ping, the min, average and maximum RTT will also be displayed. Report the average (average over ten pings) round trip times for pings to the following domains:

   i. cs.illinois.edu
   rtt min/avg/max/mdev = 52.700/53.527/54.254/0.540 ms
   **Average(RTT) = 53.527 ms**

   ii. www.illinois.edu
   --- illinois.edu ping statistics ---
   10 packets transmitted, 0 received, 100% packet loss, time 9011ms

   iii. www.nps.gov
   rtt min/avg/max/mdev = 6.191/6.943/7.796/0.437 ms
   **Average(RTT) = 6.943 ms**

   iv. www.cambridge.uk
   rtt min/avg/max/mdev = 23.885/24.481/25.472/0.443 ms
   **Average(RTT) = 24.481 ms**

   v. sydney.edu.au
   rtt min/avg/max/mdev = 163.136/164.086/164.667/0.685 ms

**Average(RTT) = 164.086 ms**

b. The Unix utility traceroute is like ping, but it sends packets that are limited to go one hop, then two hops, then three hops, and so on, towards a given destination, and the intermediate routers are reported. Read the man page for traceroute and experiment with it. Try traceroute on the servers that you just pinged. Report the number of routers that are encountered along the way. Answer these questions: what is the relation between this number and the RTT? Is the number of hops the only factor affecting the RTT?

i. cs.illinois.edu

1 192.168.1.1 1.206ms 0.467ms 0.306ms
2 50.35.64.1 4.106ms * *
3 74.40.70.61 5.901ms * *
4 74.40.1.101 7.010ms * *
5 74.40.5.126 5.829ms * *
6 206.81.80.77 7.010ms * *
7 * * *
8 * * *
9 * * *
10 64.57.20.252 50.789ms 48.761ms 50.920ms
11 64.57.21.86 49.637ms * *
12 72.36.126.101 51.756ms 52.689ms 52.686ms
13 72.36.126.65 51.804ms * *
14 72.36.127.2 51.626ms * *
15 130.126.0.142 52.018ms 52.784ms 53.311ms
16 130.126.0.133 52.848ms * *
17 130.126.0.133 53.629ms * *
18 130.126.0.241 54.227ms 52.908ms 52.676ms
19 * * *
20 * * *
21 192.17.90.136 53.069ms 53.160ms 53.198ms

**Number of routers = 21**

ii. www.illinois.edu

--- illinois.edu ping statistics ---
10 packets transmitted, 0 received, 100% packet loss, time 9011ms

iii. www.nps.gov

1 192.168.1.1 0.612ms 0.965ms 0.921ms
2 50.35.64.1 4.957ms * *
3 74.40.70.61 5.831ms * *
4 74.40.1.101 5.617ms * *
5 74.40.5.126 5.267ms * *
6 74.43.94.9 5.485ms * *
7 23.222.151.84 7.254ms 6.503ms 6.174ms

**Number of routers = 7**

iv. www.cambridge.uk

1 192.168.1.1 0.393ms 1.180ms 0.310ms
2 50.35.64.1 4.529ms * *
3 74.40.70.61 12.848ms * *
4 74.40.1.101 24.833ms * *
5 74.40.3.137 23.833ms * *

6 74.40.1.222 34.201ms * *
7 74.40.1.186 26.264ms * *
8 74.40.1.225 22.915ms * *
9 74.40.3.150 23.867ms * *
10 198.32.176.14 23.003ms 23.313ms 22.528ms
11 204.2.241.58 24.965ms 25.532ms 26.354ms
12 64.98.4.6 25.285ms 24.209ms 24.348ms
13 98.124.245.24 24.348ms 24.101ms 24.625ms
**Number of routers = 13**

v. sydney.edu.au
1 192.168.1.1 0.752ms 0.327ms 0.306ms
2 50.35.64.1 5.191ms * *
3 74.40.70.61 6.244ms * *
4 74.40.1.101 23.678ms * *
5 74.40.3.137 23.645ms * *
6 74.40.1.222 23.767ms * *
7 74.40.1.186 23.647ms * *
8 74.40.1.225 23.351ms * *
9 74.40.3.150 26.656ms * *
10 198.32.176.14 25.704ms 23.386ms 25.494ms
11 129.250.5.238 23.906ms 24.569ms 23.103ms
12 140.174.28.138 25.259ms * *
13 202.158.194.176 162.410ms 162.956ms 162.759ms
14 113.197.15.146 163.575ms * *
15 138.44.5.47 163.725ms 163.513ms 163.709ms
16 * * *
17 * * *
18 129.78.5.8 163.387ms 164.141ms 163.193ms
**Number of routers = 18**

What is the relation between this number and the RTT? Is the number of hops the only factor affecting the RTT?

There is no clear relation between the number of routers and the RTT.
The number of hops is not the only factor affecting the RTT. Since the SpeedOf Light the size of packet, the bandwidth, and switch queuing delays.
If two servers has the same distances from me, then the more hops it takes to get to the server, the longer the RTT is, for example, the [www.nps.gov](www.nps.gov) encounters less routers than cs.illinois.edu and it has less RTT than cs.illinois.edu.
However, if the distance varies a lot, the SpeedOfLight might dominate the RTT. For example, although sydney.edu.au encounters less routers than cs.illinois.edu, its RTT is much more than cs.illinois.edu. This is probably because of the SpeedOfLight factor.