

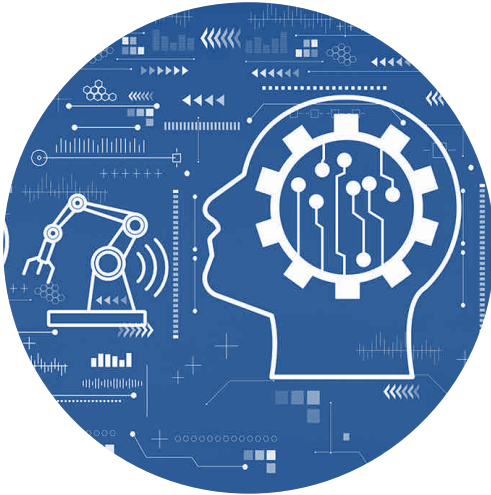
Model Predictive Control of Lettuce Greenhouse

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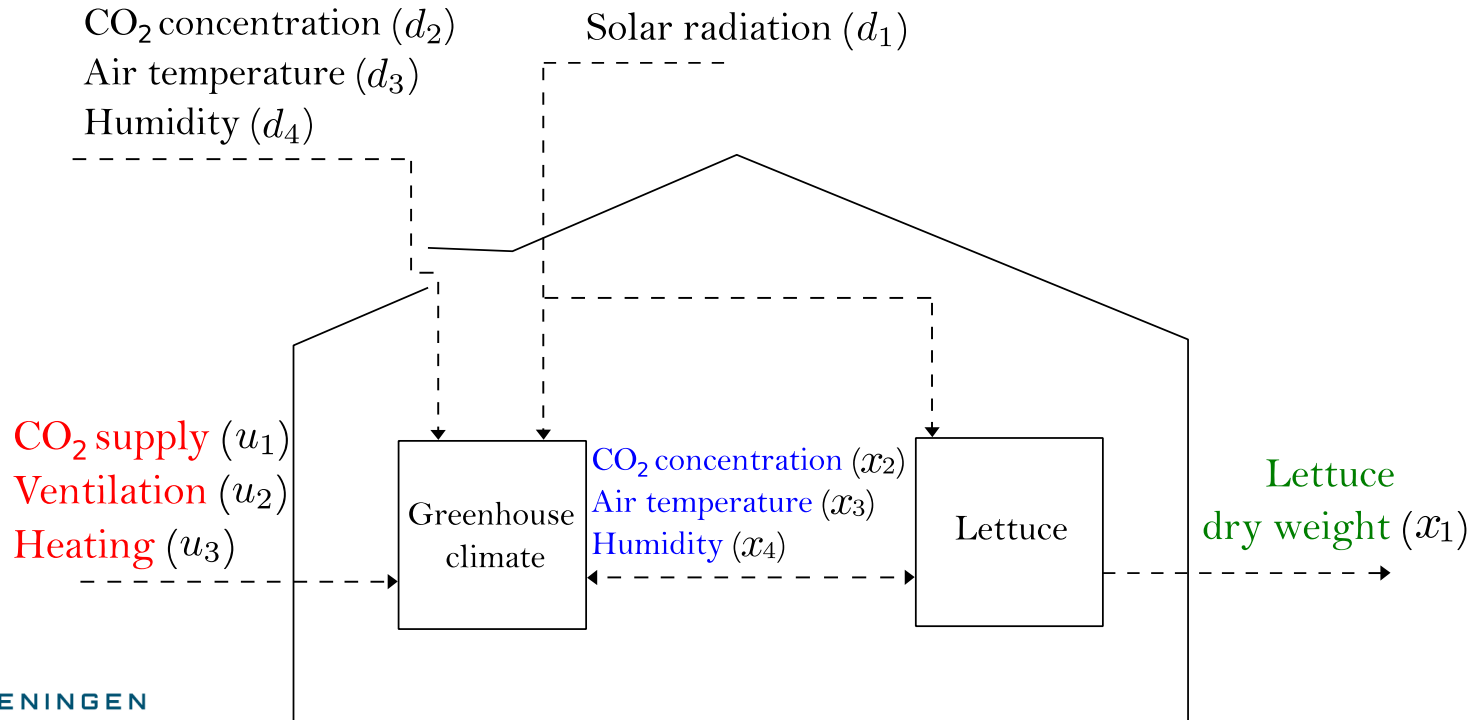
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Course materials



states	disturb.	control
x_1 : dry-weight	d_1 : radiation	u_1 : CO2 injection
x_2 : indoor CO2	d_2 : outdoor CO2	u_2 : ventilation
x_3 : indoor temp.	d_3 : outdoor temp.	u_3 : heating
x_4 : indoor humidity	d_4 : outdoor humidity	



Greenhouse Model

$$\begin{pmatrix} \frac{dx_1(t)}{dt} \\ \frac{dx_2(t)}{dt} \\ \frac{dx_3(t)}{dt} \\ \frac{dx_4(t)}{dt} \end{pmatrix} = \begin{pmatrix} \frac{p_{1,1}\phi_{\text{phot,c}}(t) - p_{1,2}x_1(t)2^{x_3(t)/10-5/2}}{p_{2,1}} \\ \frac{1}{p_{2,1}} \left(-\phi_{\text{phot,c}}(t) + p_{2,2}x_1(t)2^{x_3(t)/10-5/2} + u_1(t)10^{-6} - \phi_{\text{vent,c}}(t) \right) \\ \frac{1}{p_{3,1}} \left(u_3(t) - (p_{3,2}u_2(t)10^{-3} + p_{3,3})(x_3(t) - d_3(t)) + p_{3,4}d_1(t) \right) \\ \frac{1}{p_{4,1}} \left(\phi_{\text{transp,h}}(t) - \phi_{\text{vent,h}}(t) \right) \end{pmatrix}$$

Change of dry weight:

Weight gain from photosynthesis – weight loss due to respiration

$$\phi_{\text{phot,c}}(t) = \left(1 - \exp(-p_{1,3}x_1(t)) \right) \left(p_{1,4}d_1(t)(-p_{1,5}x_3(t)^2 + \dots \right. \\ \left. p_{1,6}x_3(t) - p_{1,7})(x_2(t) - p_{1,8}) \right) / \varphi(t),$$

$$\varphi(t) = p_{1,4}d_1(t) + (-p_{1,5}x_3(t)^2 + p_{1,6}x_3(t) - p_{1,7})(x_2(t) - p_{1,8}),$$

Greenhouse Model

$$\begin{pmatrix} \frac{dx_1(t)}{dt} \\ \frac{dx_2(t)}{dt} \\ \frac{dx_3(t)}{dt} \\ \frac{dx_4(t)}{dt} \end{pmatrix} = \begin{pmatrix} p_{1,1}\phi_{\text{phot,c}}(t) - p_{1,2}x_1(t)2^{x_3(t)/10-5/2} \\ \frac{1}{p_{2,1}} \left(\underbrace{-\phi_{\text{phot,c}}(t)}_{\text{green}} + \underbrace{p_{2,2}x_1(t)2^{x_3(t)/10-5/2}}_{\text{orange}} + \underbrace{u_1(t)10^{-6}}_{\text{red}} - \underbrace{\phi_{\text{vent,c}}(t)}_{\text{blue}} \right) \\ \frac{1}{p_{3,1}} \left(u_3(t) - (p_{3,2}u_2(t)10^{-3} + p_{3,3})(x_3(t) - d_3(t)) + p_{3,4}d_1(t) \right) \\ \frac{1}{p_{4,1}} (\phi_{\text{transp,h}}(t) - \phi_{\text{vent,h}}(t)) \end{pmatrix}$$

Change of indoor CO₂ concentration:

- Consumption of photosynthesis + increase due to respiration
- + CO₂ injection – the mass exchange of CO₂ through the vents

$$\phi_{\text{vent,c}}(t) = (u_2(t)10^{-3} + p_{2,3})(x_2(t) - d_2(t)),$$

Ventilation

difference in CO₂ concentration between indoor and outdoor

Greenhouse Model

$$\begin{pmatrix} \frac{dx_1(t)}{dt} \\ \frac{dx_2(t)}{dt} \\ \frac{dx_3(t)}{dt} \\ \frac{dx_4(t)}{dt} \end{pmatrix} = \begin{pmatrix} p_{1,1}\phi_{\text{phot,c}}(t) - p_{1,2}x_1(t)2^{x_3(t)/10-5/2} \\ \frac{1}{p_{2,1}} \left(-\phi_{\text{phot,c}}(t) + p_{2,2}x_1(t)2^{x_3(t)/10-5/2} + u_1(t)10^{-6} - \phi_{\text{vent,c}}(t) \right) \\ \frac{1}{p_{3,1}} \left(\underline{u_3(t)} - \underline{(p_{3,2}u_2(t)10^{-3} + p_{3,3})(x_3(t) - d_3(t))} + \underline{p_{3,4}d_1(t)} \right) \\ \frac{1}{p_{4,1}} \left(\phi_{\text{transp,h}}(t) - \phi_{\text{vent,h}}(t) \right) \end{pmatrix}$$

Change of indoor temperature:

– heating + heat loss through the vents + heat gain from radiation

Greenhouse Model

$$\begin{pmatrix} \frac{dx_1(t)}{dt} \\ \frac{dx_2(t)}{dt} \\ \frac{dx_3(t)}{dt} \\ \frac{dx_4(t)}{dt} \end{pmatrix} = \begin{pmatrix} p_{1,1}\phi_{\text{phot},c}(t) - p_{1,2}x_1(t)2^{x_3(t)/10-5/2} \\ \frac{1}{p_{2,1}} \left(-\phi_{\text{phot},c}(t) + p_{2,2}x_1(t)2^{x_3(t)/10-5/2} + u_1(t)10^{-6} - \phi_{\text{vent},c}(t) \right) \\ \frac{1}{p_{3,1}} \left(u_3(t) - (p_{3,2}u_2(t)10^{-3} + p_{3,3})(x_3(t) - d_3(t)) + p_{3,4}d_1(t) \right) \\ \frac{1}{p_{4,1}} \left(\phi_{\text{transp},h}(t) - \phi_{\text{vent},h}(t) \right) \end{pmatrix}$$

Change of indoor humidity:

+ the canopy transpiration – exchange of H₂O through the vents

$$\phi_{\text{vent},h}(t) = (u_2(t)10^{-3} + p_{2,3})(x_4(t) - d_4(t)),$$

Determined by
ventilation rate and
humidity difference

$$\phi_{\text{transp},h}(t) = p_{4,2} \left(1 - \exp(-p_{1,3}x_1(t)) \right)$$

$$\left(\frac{p_{4,3}}{p_{4,4}(x_3(t) + p_{4,5})} \exp\left(\frac{p_{4,6}x_3(t)}{x_3(t) + p_{4,7}}\right) - x_4(t) \right)$$

affected by biomass,
indoor temperature
and humidity

Greenhouse Model

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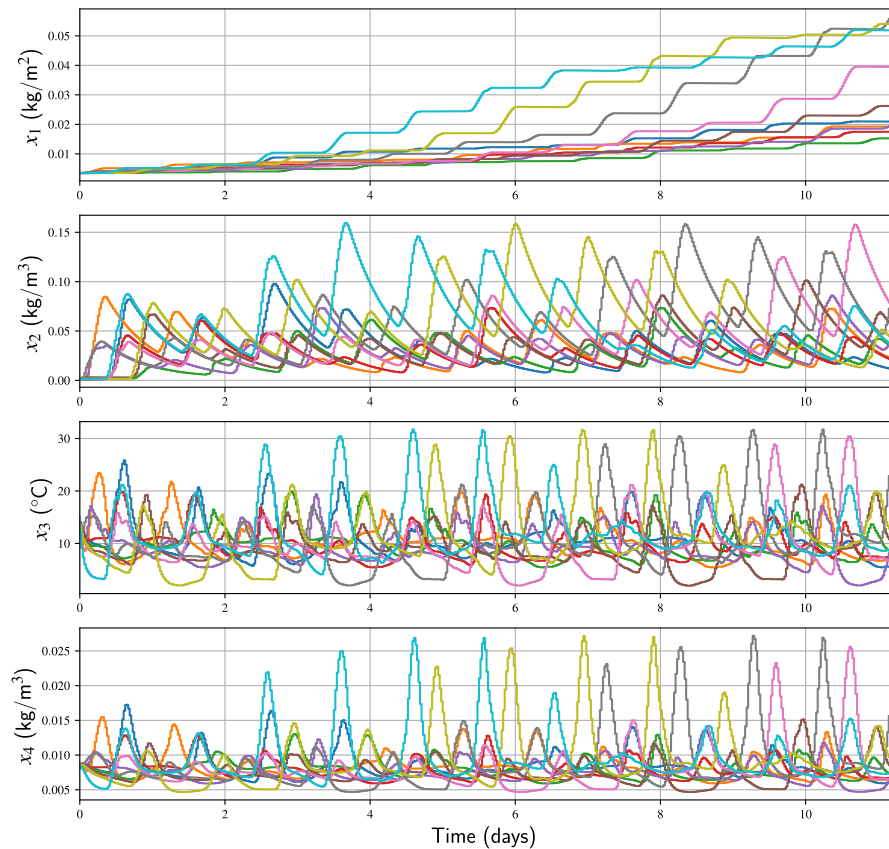
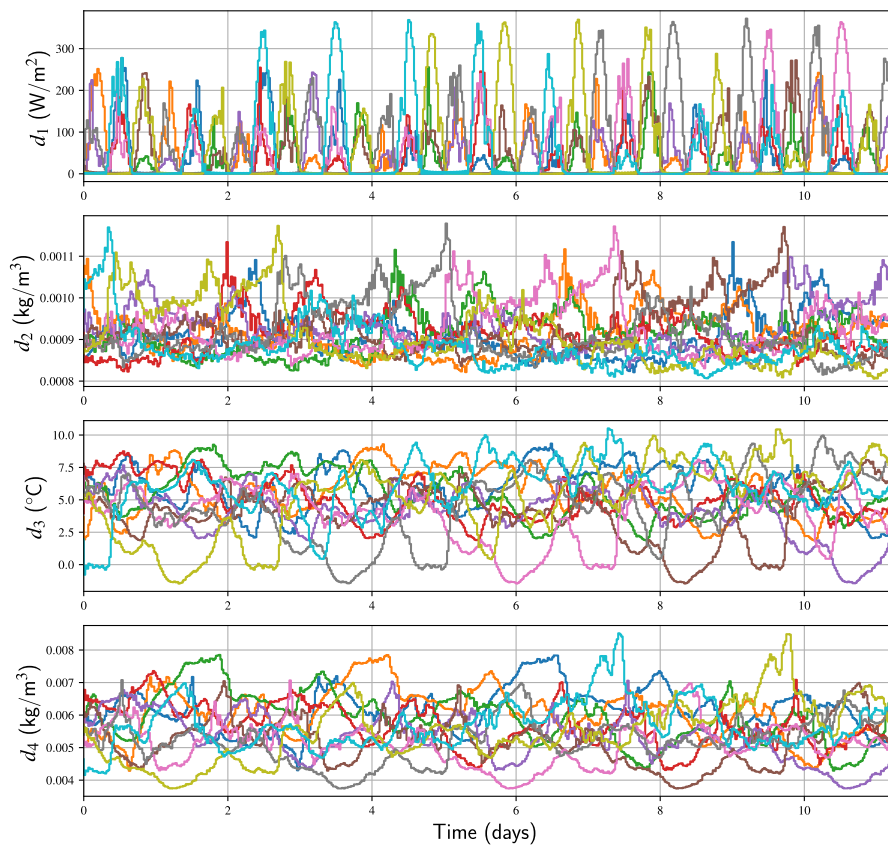
$$\begin{pmatrix} \frac{dx_1(t)}{dt} \\ \frac{dx_2(t)}{dt} \\ \frac{dx_3(t)}{dt} \\ \frac{dx_4(t)}{dt} \end{pmatrix} = \underbrace{\begin{pmatrix} p_{1,1}\phi_{\text{phot},c}(t) - p_{1,2}x_1(t)2^{x_3(t)/10-5/2} \\ \frac{1}{p_{2,1}}(-\phi_{\text{phot},c}(t) + p_{2,2}x_1(t)2^{x_3(t)/10-5/2} + u_1(t)10^{-6} - \phi_{\text{vent},c}(t)) \\ \frac{1}{p_{3,1}}(u_3(t) - (p_{3,2}u_2(t)10^{-3} + p_{3,3})(x_3(t) - d_3(t)) + p_{3,4}d_1(t)) \\ \frac{1}{p_{4,1}}(\phi_{\text{transp},h}(t) - \phi_{\text{vent},h}(t)) \end{pmatrix}}_{f_c(x(t), u(t), d(t), p)}$$

with

$$\begin{aligned} \phi_{\text{phot},c}(t) &= \left(1 - \exp(-p_{1,3}x_1(t))\right) \left(p_{1,4}d_1(t) \left(-p_{1,5}x_3(t)^2 + \dots\right. \right. \\ &\quad \left. \left. p_{1,6}x_3(t) - p_{1,7}\right)(x_2(t) - p_{1,8})\right) / \varphi(t), \\ \varphi(t) &= p_{1,4}d_1(t) + (-p_{1,5}x_3(t)^2 + p_{1,6}x_3(t) - p_{1,7})(x_2(t) - p_{1,8}), \\ \phi_{\text{vent},c}(t) &= (u_2(t)10^{-3} + p_{2,3})(x_2(t) - d_2(t)), \\ \phi_{\text{vent},h}(t) &= (u_2(t)10^{-3} + p_{2,3})(x_4(t) - d_4(t)), \\ \phi_{\text{transp},h}(t) &= p_{4,2} \left(1 - \exp(-p_{1,3}x_1(t))\right) \\ &\quad \left(\frac{p_{4,3}}{p_{4,4}(x_3(t) + p_{4,5})} \exp\left(\frac{p_{4,6}x_3(t)}{x_3(t) + p_{4,7}}\right) - x_4(t)\right). \end{aligned}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = f\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}, \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \end{bmatrix}\right)$$

$$f: (\mathbb{R}^4, \mathbb{R}^3, \mathbb{R}^4) \rightarrow \mathbb{R}^4$$



Preparation

- Go to: https://github.com/xcheng20/Greenhouse_Control_Exercise
- Download all the files in the repository
- Open 'exercise_student_version.ipynb' in python notebook
- Install **casadi**

`pip install casadi` (needs pip -V>=8.1)

Scan for the exercise



Task 1: Heuristic Feedforward Controller

The temperature setpoint switches depending on a daylight proxy $d_k^{(0)}$ (the first disturbance channel):

$$\text{sp_temp}_k = \begin{cases} 18, & d_k^{(0)} > 10W/m^2, \\ 12, & \text{otherwise.} \end{cases}$$

During the simulation loop, the heuristic controller compares the current temperature with the setpoint using a deadband rule:

- If the temperature is too low (below setpoint - deadband), maximum heating and minimum ventilation are applied
- If the temperature is too high (above setpoint + deadband), maximum ventilation and minimum heating are applied
- CO₂ injection is kept constant throughout.

Task 2: MPC Controller

Now, implement the NMPC to control the climate of the greenhouse using **CasADi** and its embedded **IPOPT** solver. Set **Np** as the *prediction horizon*, and the temperature setpoints are used the same ones as before. Then the the objective is to minimize the tracking error for temperature over the prediction horizon.

Dynamics and outputs

The system is represented by the discrete-time model:

$$X_{i+1} = f_d(X_i, U_i, d_i, p, h), \quad Y_i = g(X_i, U_i, d_i, p, h), \quad i = 0, \dots, N_p - 1.$$

Objective function

At each stage i , the cost penalizes: (1) Deviation of the temperature output $Y_i^{(2)}$ from its setpoint. (2) Input magnitude (regularization)

$$\ell_i = (Y_i^{(2)} - \text{sp}_i)^2 - \lambda_1 (Y_i^{(0)})^2 + \lambda_2 \|U_i\|_2^2, \quad \lambda_1 = 0.1, \lambda_2 = 10^{-3},$$

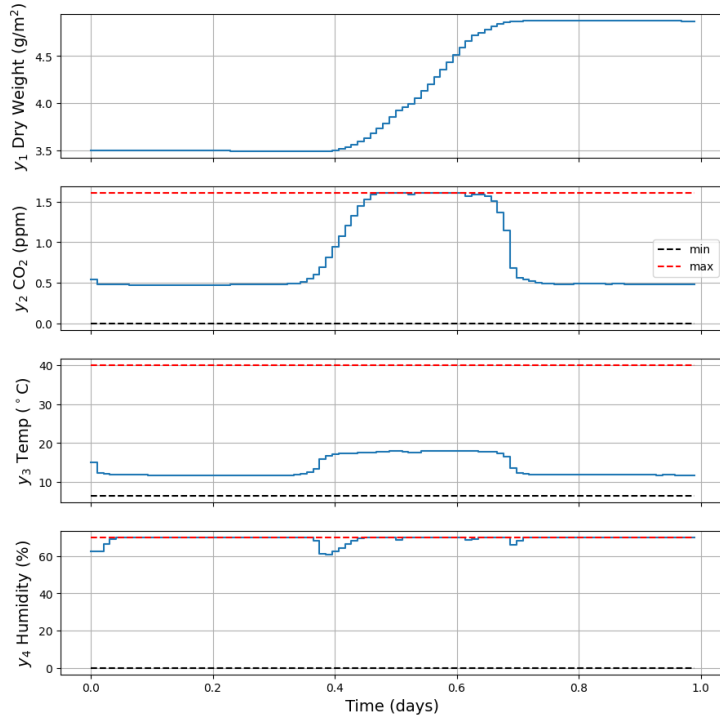
where the second term is used to regulate the CO₂ injection by considering the yield, and the third term is to penalize input energy. This leads to the total cost: $J = \sum_{i=0}^{N_p-1} \ell_i$.

Constraints

- Initial state: $X_0 = x_0$
- Input bounds: $u_{\min} \leq U_i \leq u_{\max}$.
- Output bounds (selected indices): $y_{\min}^{(1)} \leq Y_i^{(1)} \leq y_{\max}^{(1)}, y_{\min}^{(2)} \leq Y_i^{(2)} \leq y_{\max}^{(2)}, Y_i^{(3)} \leq y_{\max}^{(3)}$.

Results of NMPC

Greenhouse State Outputs



Control Inputs

