chapter6

计63 肖朝军 2016011302

第3题

- **解题思路**：
  - **第(1)问**：该问主要是需要拟合一个多项式函数，这里我用了法方程方法来拟合这个最小二乘问题，解法相对常规。
  - **第(2)问**：该题需要拟合一个指数函数，先将该函数转化成如下的多项式函数，再同样利用法方程方法来解决即可。

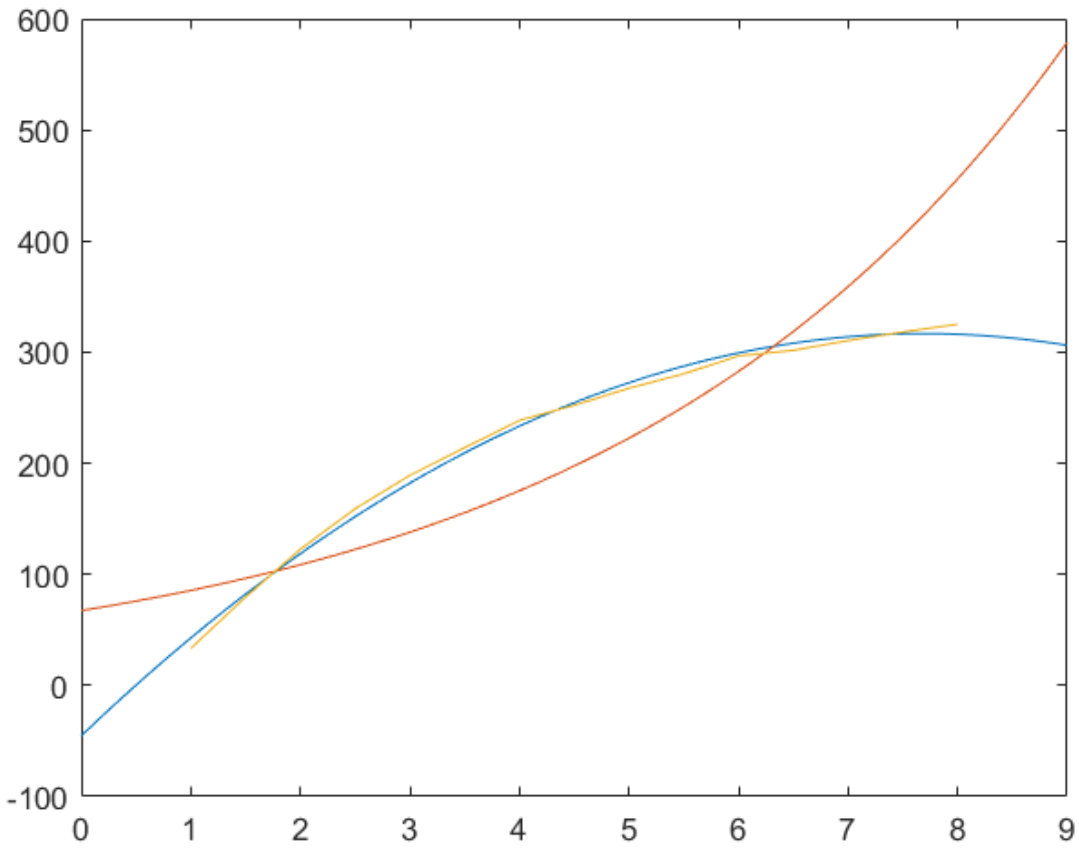
$$y = ae^{bt} \Rightarrow log(y) = log(a) + b \times t$$

- **代码**：

```
% 数据
t = (2:16) * 0.5;
y = [33.4; 79.5; 122.65; 159.05; 189.15; 214.15; 238.65; 252.2; 267.55; 280.50;
296.65; 301.65; 310.40; 318.15; 325.15];

% 用多项式进行拟合，利用法方程方法
A = zeros(length(t), 3);
for i = 1:length(t)
    A(i, 1) = 1;
    A(i, 2) = t(i);
    A(i, 3) = t(i)^2;
end
G = A.' * A;
b = A.' * y;
polyx = G\b;

disp(polyx);

% 用指数函数拟合
% 先对y取对数
y1 = log(y);
A1 = zeros(length(t), 2);
for i = 1:length(t)
    A1(i, 1) = 1;
    A1(i, 2) = t(i);
end

G = A1.' * A1;
b = A1.' * y1;
expx = G\b;
expx(1) = exp(expx(1));
disp(expx);
```

```
% 画图
xrange = 0:0.1:9;
polyy = polyx(1) + polyx(2) * xrange + polyx(3) * xrange.^2;
expy = expx(1) * exp(expx(2) * xrange);
plot(xrange, polyy);
hold on;
plot(xrange, expy);
hold on;
plot(t, y);
```

- **函数图像**：三条曲线函数图像如下，其中黄线代表原始数据点的连线，蓝线是多项式拟合的曲线，红线是指数函数拟合的曲线。从曲线可以看出，多项式曲线拟合效果要好一些。



- **计算结果**：

```
多项式参数:
    a = -45.2942;
    b = 94.1943;
    c = -6.1268;
指数参数:
    a = 67.3938;
    b = 0.2390;
```

第8题

- **解题思路**：该题主要考察三次样条插值，给出的条件属于第一种边界条件的三次样条插值，按照公式计算即可求得样条函数值(即求得M)，再按照以下公式求 `S(x)`、`S'(x)`、`S''(x)`

$$S(x)=M_j\frac{(x_{j+1}-x)^3}{6h_j}+M_{j+1}\frac{(x-x_j)^3}{6h_j}+(f_j-\frac{M_jh_j^2}{6})(\frac{x_{j+1}-x}{h_j})+(f_{j+1}-\frac{M_{j+1}h_j^2}{6})(\frac{x-x_j}{h_j})$$

$$S^{'}(x)=-M_j\frac{(x_{j+1}-x)^2}{2h_j}+M_{j+1}\frac{(x-x_j)^2}{2h_j}+\frac{f_{j+1}-f_j}{h_j}-\frac{M_{j+1}-M_j}{6}h_j$$

$$S^{''}(x)=M_j\frac{x_{j+1}-x}{h_j}+M_{j+1}\frac{x-x_j}{h_j}$$

- **代码**：

```
% 数据
x = [0.520, 3.1, 8.0, 17.95, 28.65, 39.62, 50.65, 78, 104.6, 156.6, 208.6, 260.7,
312.50, 364.4, 416.3, 468, 494, 507,520].';
y = [5.288, 9.4, 13.84, 20.20, 24.90, 28.44, 31.10, 35, 36.9, 36.6, 34.6, 31.0, 26.34,
20.9, 14.8, 7.8, 3.7, 1.5, 0.2].';

dy0 = 1.86548;
dyn = -0.046115;

n = length(x);
h = zeros(n, 1);
u = zeros(n, 1);
lambda = zeros(n, 1);
d = zeros(n, 1);

for i = 1:n - 1
    h(i) = x(i + 1) - x(i);
end

% 计算 lambda、mui
for i = 2:n - 1
    u(i) = h(i - 1) / (h(i - 1) + h(i));
    lambda(i) = h(i) / (h(i - 1) + h(i));
    d(i) = y(i - 1) / (h(i - 1) * (h(i - 1) + h(i)));
    d(i) = d(i) + y(i + 1) / (h(i) * (h(i - 1) + h(i)));
    d(i) = d(i) - y(i) / (h(i - 1) * h(i));
    d(i) = d(i) * 6;
end

u(n) = 1;
lambda(1) = 1;
d(1)=(6 / h(1)) * ((y(2) - y(1)) / h(1) - dy0);
d(n) = (6 / h(n - 1)) * (dyn - (y(n) - y(n - 1)) / h(n - 1));
```

```
% 利用追赶法求解方程
M = zeros(n, 1);
b = zeros(n, 1) + 2;
f = d;
for i = 2:n
    m = u(i) / b(i - 1);
    b(i) = b(i) - m * lambda(i - 1);
    f(i) = f(i) - m * f(i - 1);
end
M(n) = f(n) / b(n);
for i = fliplr(1:n - 1)
    M(i) = (f(i) - lambda(i) * M(i + 1)) / b(i);
end
% 上面已经求到了解

%计算改点
testx = [2, 30, 130, 350, 515];

for p = testx
    fprintf('for x = %d\t', p);
    scope = 0;
    for j = 1:n - 1
        if (x(j) <= p && x(j + 1) >= p)
            scope = j;
            break;
        end
    end
    % 求S(x)
    sx = M(scope) * (x(scope + 1) - p)^3 / (6 * h(scope)) + M(scope + 1) * (p -
x(scope))^3 / (6 * h(scope)) + (y(scope) - M(scope) * h(scope)^2 / 6) * (x(scope + 1)
- p) / h(scope) + (y(scope + 1) - M(scope + 1) * h(j)^2 / 6) * (p - x(scope)) /
h(scope);
    fprintf('S(x) = %f\t\t', sx);

    % 求S'(x)
    sx_ = - M(scope) * (x(scope + 1) - p)^2 / (2 * h(scope)) + M(scope + 1) * (p -
x(scope))^2 / (2 * h(scope)) + (y(scope + 1) - y(scope)) / h(scope) - (M(scope + 1) -
M(scope)) * h(scope) / 6;
    fprintf('S''(x) = %f\t\t', sx_);

    % 求S''(x)
    sx__ = M(scope) * (x(scope + 1) - p) / h(scope) + M(scope + 1) * (p - x(scope)) /
h(scope);
    fprintf('S''''(x) = %f\n', sx__);

end
```

- **运行结果**:

```
for x = 2       S(x) = 7.825155     S'(x) = 1.556835        S''(x) = -0.221260
for x = 30      S(x) = 25.386235    S'(x) = 0.354874        S''(x) = -0.007843
for x = 130     S(x) = 37.213841    S'(x) = -0.010392       S''(x) = -0.001382
for x = 350     S(x) = 22.475111    S'(x) = -0.107784       S''(x) = -0.000230
for x = 515     S(x) = 0.542713     S'(x) = -0.089906       S''(x) = 0.008120
```