Homework 1

Due: Friday, Sept 4, 11:59 PM PST

1. Consider a *stable roommate problem* involving four students (A, B, C, D): Each student ranks the other three in a strict order of preference. A matching involves forming two pairs of the students, and it is considered stable if no two *separated* (not matched) students would prefer each other over their current roommates.
   Prove or disprove: A stable matching always exists in this scenario.

2. Consider an instance of the Stable Matching Problem in which there exists a man m and a woman w such that m is ranked first on the preference list of w and w is ranked first on the preference list of m.
   Prove or disprove: In every stable matching for this instance, m is matched with w.

3. Consider the stable matching problem with n women and n men (where $n \geq 2$).
   Prove or disprove: It is not possible for any preference lists, that the Gale-Shapley algorithm with men proposing can match every woman to her least preferred man.

4. There are six students in a class: Harry, Ron, Hermione, Luna, Neville, and Ginny. This class requires them to pair up and work on pair programming. Each has preferences over who they want to be paired with. The preferences are:

   Harry: Hermione > Neville > Ron > Ginny > Luna
   Ron: Ginny > Neville > Hermione > Harry > Luna
   Hermione: Ron > Neville > Ginny > Harry > Luna
   Luna : Harry > Ron > Ginny > Hermione > Neville
   Neville: Harry > Ron > Hermione > Ginny > Luna
   Ginny: Neville > Harry > Hermione > Ron > Luna

   A given matching (of students into pairs) is not stable if there are two potential partners who are not currently paired but prefer each other over their respective current partners.
   Prove or Disprove: No matching is stable for the preferences given above.

Ungraded Problems:

1. Prove or Disprove: It is possible to have an instance of the Stable Matching problem in which two women have the same best valid partner.

2. Prove or Disprove: In every instance of the Stable Matching Problem, there is a stable matching containing a pair (m, w) such that m is ranked first on the preference list of w and w is ranked first on the preference list of m.

3. Consider the stable matching problem with n women and n men (where $n \geq 2$). Suppose the preference lists are such that for any woman w, her most preferred man m, does not have w as his first preference
   Prove or disprove: It is possible that the Gale-Shapley algorithm with men proposing can match every woman to her most preferred man.

4. Suppose the Gale-Shapley algorithm with men proposing is run once. Then, a particular woman w falsely reports her preferences by swapping two men's positions in her actual preference list. With this alteration, the Gale-Shapley algorithm with men proposing is run once again.
   Prove or disprove: It is possible for the woman w to get a more preferred partner (as per her actual preference list), by doing the swapping alteration in her preferences.