

# Project4

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A) Xichen Li: I worked on project-4 all independently.

(1) Look at equation (9-50) and discuss in detail the range on the Bloch sphere that can be traversed using pulses of various width. Assume that the initial state is  $|0\rangle$ .

*Discussion of (9-50)*

$$\psi = e^{\frac{i\Delta t}{2}} [\cos(\Omega' t) - \frac{i\Delta}{2\Omega'} \sin(\Omega' t)] e^{-\frac{iE_{10}t}{\hbar}} \phi_{10} - \frac{iu_{21}}{\hbar} \frac{1}{\Omega'} e^{-\frac{i\Delta t}{2}} [\sin(\Omega' t)] e^{-\frac{iE_{20}t}{\hbar}} \phi_{20}$$

where

$$\begin{aligned}\Omega' &= \sqrt{\Omega^2 + \frac{\Delta^2}{4}} \\ \Omega &= \frac{|u_{21}|}{\hbar} \\ \Delta &= \omega - \omega_{21}\end{aligned}$$

In the Bloch sphere, a qubit state can be expressed as:

$$\psi = \cos\left(\frac{\theta}{2}\right)|0\rangle + e^{i\phi} \sin\left(\frac{\theta}{2}\right)|1\rangle$$

The phase factor  $\phi$  in the equation above doesn't change the probability of  $|0\rangle$  and  $|1\rangle$  and thus it will be neglected in the following analysis.

The analysis will assume two difference cases: 1) the detuning is zero and the ac driving frequency is equal to the natural or resonant frequency. 2) the detuning is not zero and the ac driving frequency is different from the natural or resonant frequency.

**1) Detuning is zero ( $\Delta = 0$ ):**

$$\begin{aligned}P_1(t) &= \cos^2(\Omega t) \\ P_2(t) &= \sin^2(\Omega t)\end{aligned}$$

The possibility of both  $P_1(t)$  and  $P_2(t)$  changes between 0 and 1 periodically with a frequency of  $2\Omega$ . This also means by applying a pulse signal with a frequency equal to  $\Omega$ , a qubit with the initial state  $|0\rangle$  can be rotated between 0 and 180 degree on the Bloch sphere. For example, if we apply a pulse width equal to  $0.5\pi/\Omega$  to the initial state  $|0\rangle$ , the qubit will be rotated by 180 degree on the Bloch sphere and then the probability of getting a state  $|1\rangle$  will be equal to 1. To better understand this, I will use following numbers as an example ( $\Delta = 0, \Omega = 2\pi * 5\text{e}6$ ):

```

In [53]: import numpy as np
          #import numpy.pi as pi
          %matplotlib notebook
          import matplotlib.pyplot as plt
          import scipy.integrate as spi

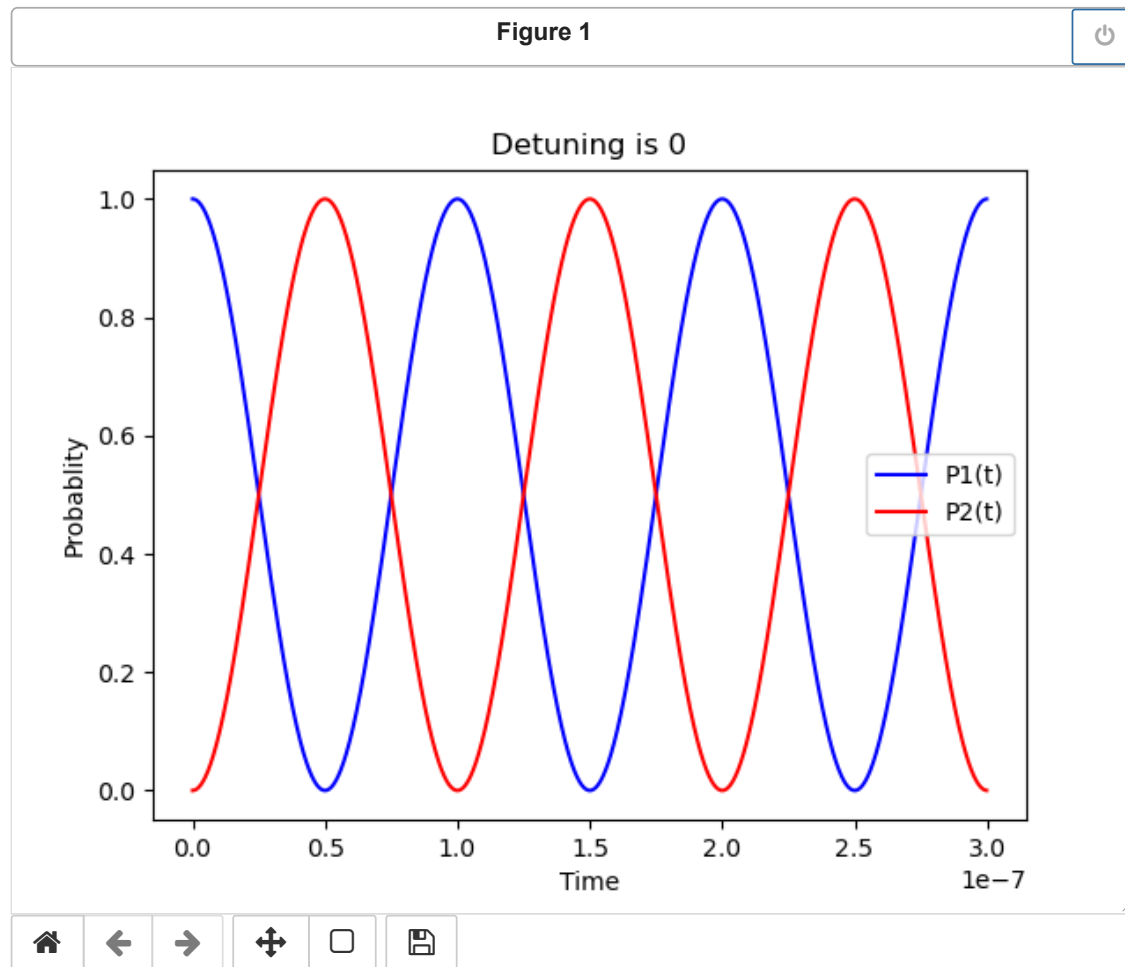
          #eta = 6.63e-34/2/np.pi #Reduced Plank constant in eV.s
          #q = 1.6e-19
          #u21=6.5e-6*1e-3*q
          #w21=2*q/eta
          #w=2.05*q/eta
          #omega=abs(u21)/eta
          delta=0
          omega=2*np.pi*5e6
          omegap=np.sqrt(omega**2+delta**2/4)

          Tend=1e-6
          dt=10e-9
          t = np.arange(0,Tend,dt)
          P1=(np.cos(omegap*t))**2 + delta**2/4/omegap**2*(np.sin(omegap*t))**2
          P2=omega**2/omegap**2*(np.sin(omegap*t))**2

          Tend=3e-7
          dt=2e-10
          t = np.arange(0,Tend,dt)
          P1=(np.cos(omegap*t))**2 + delta**2/4/omegap**2*(np.sin(omegap*t))**2
          P2=omega**2/omegap**2*(np.sin(omegap*t))**2

          plt.figure()
          plt.plot(t, P1, color='b', ls='-')
          plt.plot(t, P2, color='r', ls='-')
          #plt.plot(t, P1+P2, color='g', ls='-')
          plt.legend(['P1(t)', 'P2(t)', 'P1(t)+P2(t)'])
          plt.title('Detuning is 0')
          plt.xlabel('Time')
          plt.ylabel('Probablility')

```



Out[53]: Text(0, 0.5, 'Probability')

In the example above,  $\Omega/2\pi$  is equal to  $5\text{ MHz}$ . By applying an pulse signal with a frequency of  $5\text{ MHz}$  and width of  $0.05\mu\text{s}$ ,  $P_1(t)$  changes from 1 to 0 while  $P_2(t)$  changes from 0 to 1. As that said, the initial state on the Bloch sphere is rotated by 180 degree which is the maximum range. If we apply a pulse signal with a width less than  $0.05\mu\text{s}$ , the phase rotated on the Bloch sphere will be between 0 and 180 degree and  $P_1(t)$  will be between 0 and 1.

**2) Detuning is not zero ( $\Delta \neq 0$ ):**

$$P_1(t) = \cos^2(\Omega' t) + \frac{\Delta^2}{4\Omega'^2} \sin^2(\Omega' t)$$

$$P_2(t) = \frac{\Omega^2}{\Omega'^2} \sin^2(\Omega' t)$$

The possibility  $P_1(t)$  changes between  $\frac{\Delta^2}{4\Omega'^2}$  and 1, while the possibility  $P_2(t)$  changes between  $\frac{\Omega'^2}{\Omega'^2}$ . Both of their oscillation frequency is equal to  $2\Omega'$ .

Assuming the initial state is  $|0\rangle$  and  $P_1(t) = 1$ , we can find  $P_1(t)$  will never reach 0, which means the maximum range on the Bloch sphere is smaller than 180 degree. The expression of the maximum phase can be calculated from:

$$\begin{aligned}\frac{\Delta^2}{4\Omega'^2} &= \cos^2\left(\frac{\theta}{2}\right) \\ \frac{\Delta}{2\Omega'} &= \cos\left(\frac{\theta}{2}\right) \\ \theta &= 2\arccos\left(\frac{\Delta}{2\Omega'}\right)\end{aligned}$$

Here,  $\theta$  is the maximum phase range the state can be rotated on the Bloch sphere.

To better understand this, I will use following numbers as an example ( $\Delta = 2\pi * 20e6$ ,  $\Omega = 2\pi * 5e6$ ):

```

In [54]: delta=2*np.pi*20e6
omega=2*np.pi*5e6
omegap=np.sqrt(omega**2+delta**2/4)

Tend=1e-6
dt=10e-9
t = np.arange(0,Tend,dt)
P1=(np.cos(omegap*t))**2 + delta**2/4/omegap**2*(np.sin(omegap*t))**2
P2=omega**2/omegap**2*(np.sin(omegap*t))**2

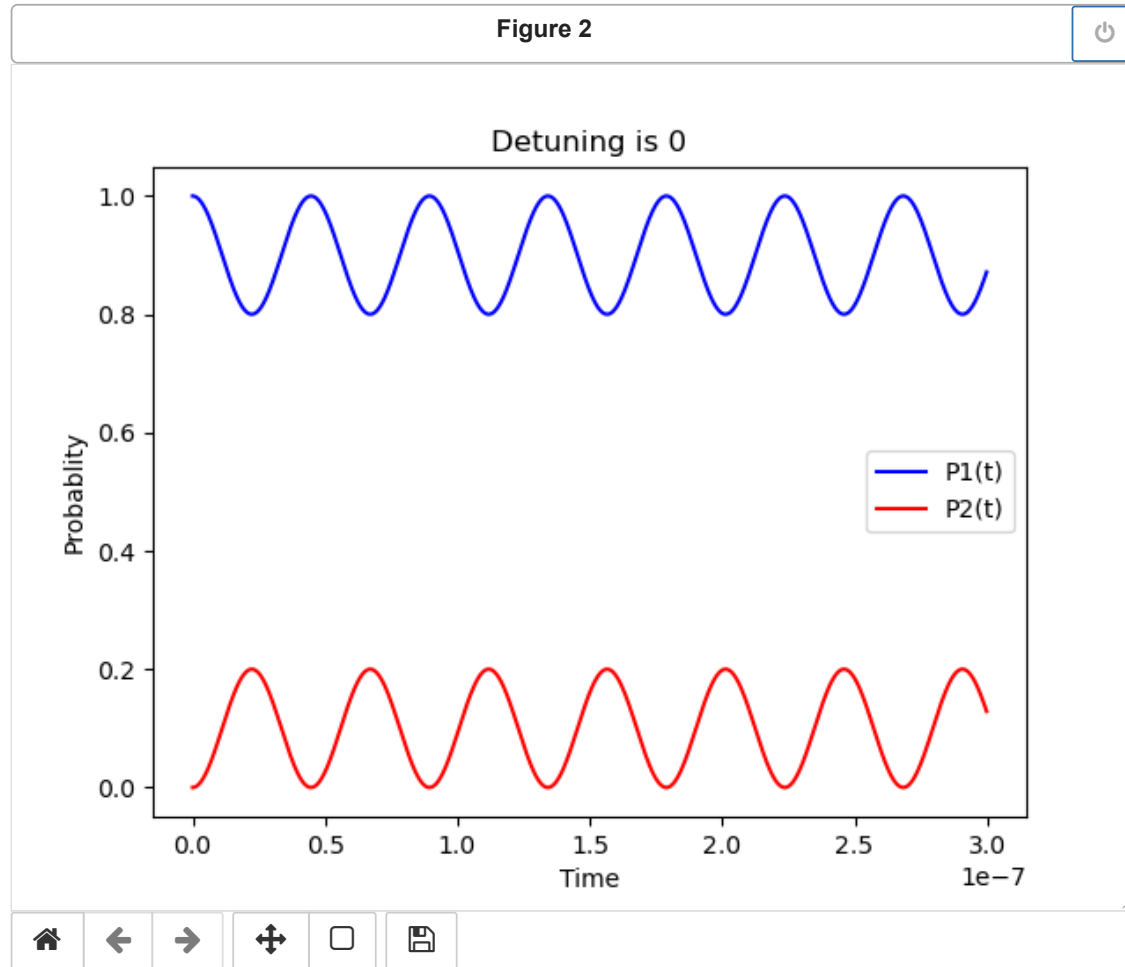
print(delta**2/4/omegap**2)
print(omegap/np.pi/2)
print(2*np.arccos(delta/2/omegap)/np.pi*180)

Tend=3e-7
dt=2e-10
t = np.arange(0,Tend,dt)
P1=(np.cos(omegap*t))**2 + delta**2/4/omegap**2*(np.sin(omegap*t))**2
P2=omega**2/omegap**2*(np.sin(omegap*t))**2

plt.figure()
plt.plot(t, P1, color='b', ls='-')
plt.plot(t, P2, color='r', ls='-')
#plt.plot(t, P1+P2, color='g', ls='-')
plt.legend(['P1(t)', 'P2(t)', 'P1(t)+P2(t)'])
plt.title('Detuning is 0')
plt.xlabel('Time')
plt.ylabel('Probablity')

0.7999999999999999
11180339.887498949
53.13010235415599

```



Out[54]: Text(0, 0.5, 'Probability')

In the example above,  $\Omega/2\pi$  is equal to  $5\text{ MHz}$  and  $\Delta/2\pi$  is  $20\text{ MHz}$ . As we can see from the figure above, by applying an ac potential with a width of approximately  $0.0224\mu\text{s}$ ,  $P_1(t)$  has a minimum value that is equal to  $\frac{\Omega^2}{\Omega'^2} = 0.8$ . This corresponds to a phase shift of  $2\cos(\frac{\Delta}{2\Omega'})$  which is approximately  $53.1$  degree. If we apply a pulse signal with a width less than  $0.0224\mu\text{s}$ , the phase rotated on the Bloch sphere will be between  $0$  and  $53.1$  degree and  $P_1(t)$  will be between  $0.8$  and  $1$ .

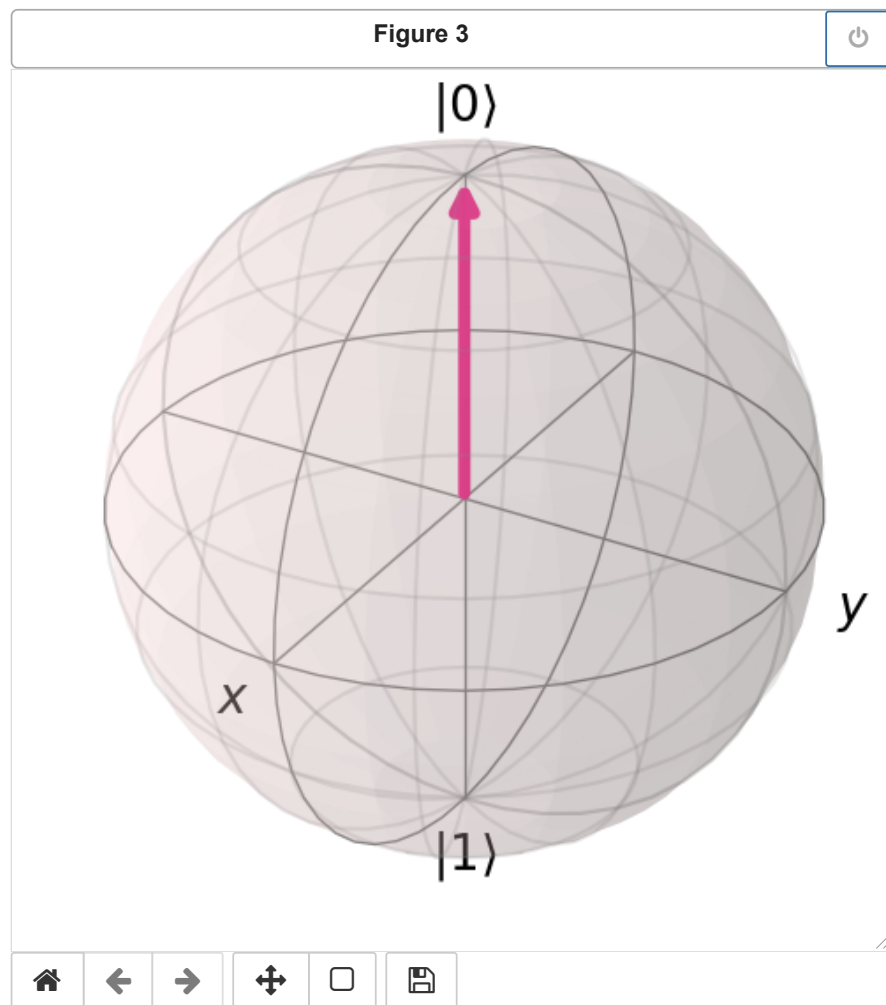
**(2) Draw the points reached on the Bloch sphere if the initial state is  $|0\rangle$ .**

**1) Detuning is zero ( $\Delta = 0$ ):**

The points on the bloch Sphere is shown below and the phase factor is neglected.

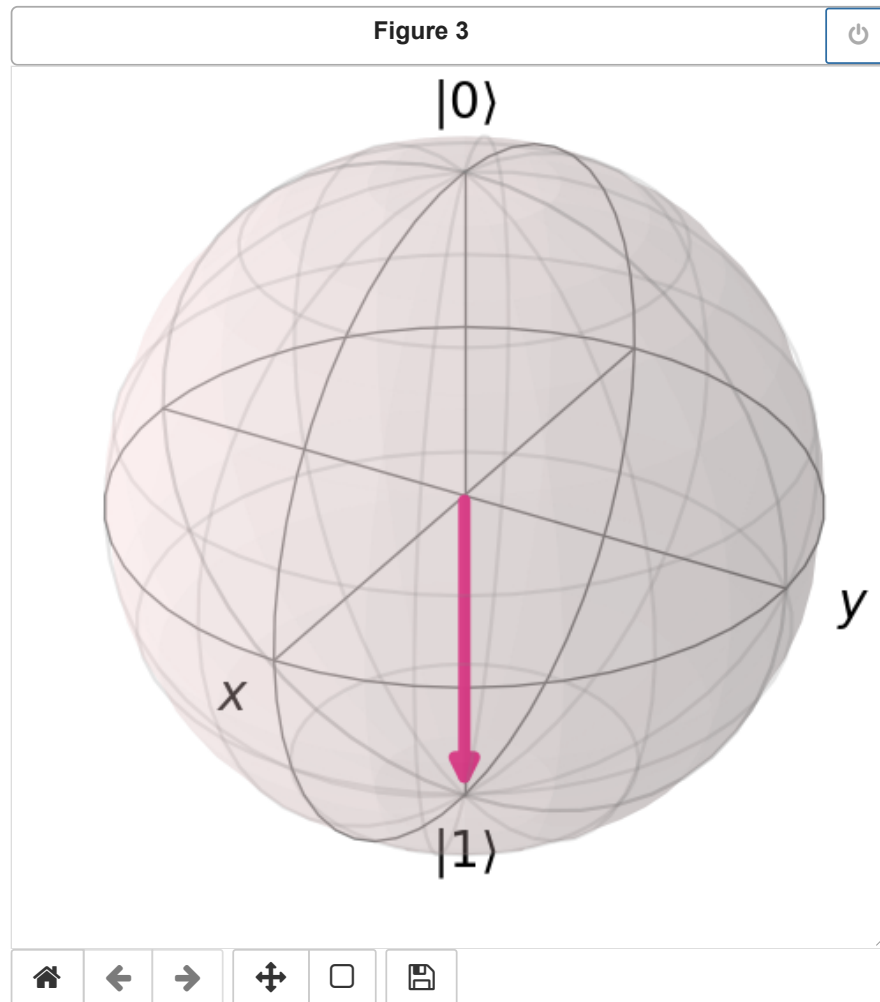
```
In [55]: from qiskit.visualization import plot_bloch_vector  
plot_bloch_vector([1, 0, np.pi/3], coord_type='spherical')
```





The figure above is the initial state  $|0\rangle$ .

```
In [56]: delta=0 #Detuning
omega=2*np.pi*5e6
omegap=np.sqrt(omega**2+delta**2/4)
phi=(2*np.arccos(delta/2/omegap))
#print(phi)
plot_bloch_vector([1, phi, 0], coord_type='spherical')
```

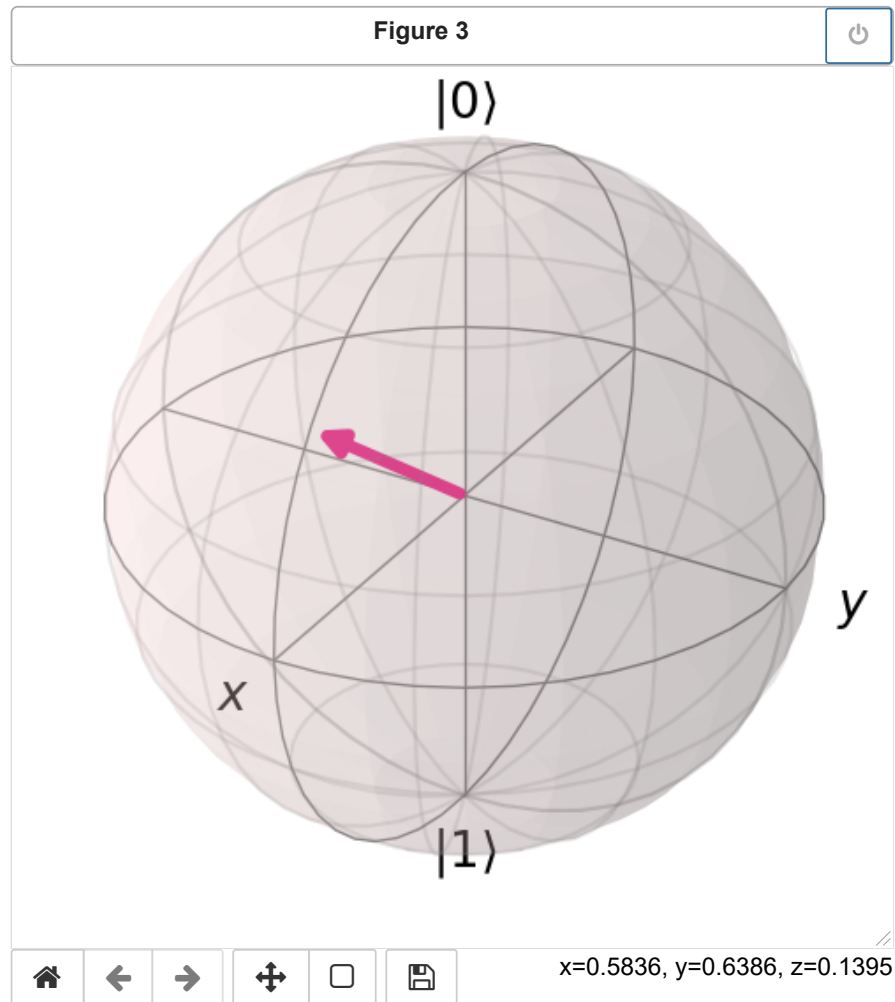


The figure above is the state after applying a pulse with a maximum phase rotation range. Note: any points on the Bloch sphere between the  $|0\rangle$  and  $|1\rangle$  should be able to be achieved by adjusting the pulse width.

**1) Detuning is zero ( $\Delta = 0$ ):**

The points on the Bloch Sphere are shown below and the phase factor is neglected. The initial state is still  $|0\rangle$  which is already plotted.

```
In [57]: delta=2*np.pi*20e6
omega=2*np.pi*5e6
omegap=np.sqrt(omega**2+delta**2/4)
phi=(2*np.arccos(delta/2/omegap))
#print(phi)
plot_bloch_vector([1, phi, 0], coord_type='spherical')
```



The figure above is the state after applying a pulse with a maximum phase rotation range. The maximum phase is approximately 53 degree. Note: any points on the Bloch sphere between the  $\theta = 0$  and  $\theta = 53$  should be able to be achieved by adjusting the pulse width.

In [ ]:

In [ ]:

