## Problems

**Section 1.1 and some fundamentals**

1. How many electron volts (eV) make one Joule?
2. If an electron moves from the positive to negative terminal of a (AAA) battery, how much does the potential energy of the electron change?
3. Consider a photon with a wavelength of . What is its energy in Joules and eV?
4. You have a source (say a light emitter) that emits photons of wavelength . What is the minimum energy that can be received by us in a time interval of 1 second? (Exclude the trivial answer of zero.)
5. How fast should a tennis ball travel so that its wavelength is 1 Angstrom? (Assume Newtonian mechanics. That is, neglect that particles cannot travel with a speed larger than the speed of light). What quantity is this wave in?
6. This problem introduces the student to the concept of *excitons*, a bound electron-hole pair. Consider a semiconductor where an electron has been excited from the valence to conduction band. The excitation leaves behind a hole in the valence band. As the electron and hole have opposite charges, they form a bound system like the hydrogen atom. That is, the electron goes around the hole due to the attractive force offered by Coulomb's law. This bound system is called an exciton. For simplicity, assume that the hole is infinitely heavy, and the electron has a mass of . Note that in reality, holes have a finite mass just like the electron, and the hole cannot be assumed to be a rest. But we will make this approximation anyway.

Using an analysis like Bohr's model for an atom, estimate the lowest energy level of the electron by assuming technologically important semiconductors (a) Silicon, (b) GaAs, and (c) Germanium. Assume the correct dielectric constant of the material but for simplicity, assume the rest mass of the electron.

**Section 0**

1. Consider a time independent Hamiltonian. If is the wave function at time , show that is a valid solution of the Schrödinger’s equation, .

**Sections 1.4 and 1.5**

1. is the eigenfunction of the one-dimensional Schrödinger equation. What is the (a) unit of the eigenfunction, (b) the meaning of and (c) meaning of |?
2. The effective mass of an electron in GaAs (Gallium Arsenide) is . Design a one-dimensional GaAs particle in a box such that when you put an electron in the energy level, it will emit green light of wavelength , and transitions to the energy level.
3. Consider a potential:
4. Derive expressions for the eigenfunction and energy eigenvalues. You do not have to normalize the wave function. (Hint: Determine the boundary condition that you will use at .)
5. Plot one example of the eigenfunction.
6. (Project-1-Group-1)Consider a potential (take ):
7. By solving Schrödinger’s equation, find the lowest eigenvalue (energy level) and the corresponding eigenfunction.
8. Plot the eigenfunction using a plotting package. Ensure that you present a zoomed-in plot of the eigenfunction for and .
9. Show that the eigenvalues form a discrete spectrum for energies less than and a continuous spectrum for higher energies. That is, for this Hamiltonian, there are both discrete quantum numbers with eigenenergies smaller than and a continuum of quantum numbers with eigenenergies larger than .

Find explicit from of the wave function by considering left and right incident waves incident with energy . Plot the transmission probability as a function of energy. Typewritten solution is required.

1. Consider a one-dimensional problem with the potential:

The eigenfunction corresponding to quantum number is, , where is the normalization constant. Find the expectation value of the position operator for an electron in the state:

2. an arbitrary .
3. Consider an electron constrained to lie on a purely one-dimensional ring with a circumference of . The potential energy on the one-dimensional ring is zero. (This is similar to the Particle in a Box (PiB), but the electron is in a ring).
   1. Derive an expression for the energy eigenvalues and eigenfunctions. Discuss the quantum numbers by including both a discussion of their sign and magnitude. Do these quantum number mean anything physical?
   2. What are the energy levels and eigenfunctions for ?
   3. Compare the energy levels and eigenfunctions of this problem with a PiB, where the box length () is .

[Hints: Pay attention to finding the correct boundary conditions. Using circular coordinates might make it easier. A particle in a ring classically will travel either clockwise or anticlockwise, so the eigenfunctions could potentially be clockwise or anticlockwise propagating waves in a ring.]

1. Consider a three-dimensional problem with the potential energy:

For the three-dimensional problem, the Hamiltonian is

Show that the eigenfunction

satisfies the Schrödinger equation. What are the energy eigenvalues?

1. (Project-1-Group-2:) Given a potential barrier [assume that is a number and is the Dirac delta function],
   1. Is the eigenfunction continuous across ? What is the physical reason for your answer?
   2. Is the derivative of the eigenfunction continuous across ? If the answer is negative, derive a relationship between the derivative of the eigenfunction at and .

Find explicit from of the wave function by considering left and right incident waves incident with energy . Plot the transmission probability as a function of energy. Solve for both positive and negative values of energy. Are there any bound states, and if yes, what are their energy levels. Typewritten solution is required.

1. (Project-1-Group-3 and Xichen:) Solve this problem numerically by discretizing the Schrödinger equation. Consider a potential:

Take . In solving this problem make sure to include at least 12 nm to the left of and to the right of .

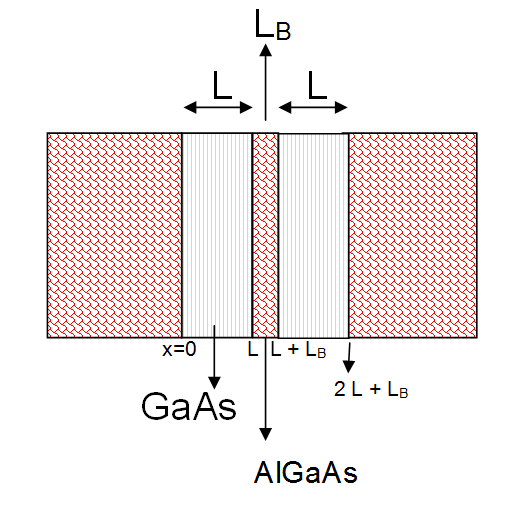
1. Find the lowest two eigenvalues by discretizing Schrödinger equation. Plot the corresponding eigenfunctions. Use a grid spacing of 0.08 nm.
2. Plot the lowest energy level as a function of grid spacing. Use grid spacings of 0.0.02 nm, 0.04, 0.08, 0.12, 0.2, 0.4, 0.8, 1.6 nm. Make observations and rationalize your results.

Repeat the above for the second lowest energy level.

1. Plot the lowest energy as a function of well width. Vary the well width from 1 to 12 nm in units of 1 nm. How do your results compare to that for a particle in a box? Fit the lowest energy level by an algebraic function.

**Section 1.6**

1. ([EE521-Project-1 - group 4](https://canvas.uw.edu/courses/1612336/groups)) Consider a **double quantum well**, where the wells are made of GaAs, and the barriers are made of AlGaAs, as shown in the figure below. (In solving the problem, you will only need the potential profile and mass of an electron in GaAs and AlGaAs. These values are supplied below.)



The potential energy is defined by (units meV):

Take and LB = 20 . Assume that the mass of an electron in both GaAs and AlGaAs is .

1. What are the lowest two eigenvalues? Plot the eigenfunctions corresponding to these eigenvalues. Are these eigenfunctions symmetric or antisymmetric about the center of the structure?
2. Plot the lowest energy level as a function of the quantum well width . Make sure to simultaneously change both GaAs well widths. Keep LB = 20 .
3. Plot the energy difference between the lowest two energy levels as a function of the quantum well width . Keep LB = 20 .Make sure to simultaneously change both GaAs well widths.
4. Plot the energy difference between the lowest two energy levels as a function of the AlGaAs barrier width (Vary from 5 to 30 in units of 5). Keep L = 120 .

Typewritten solution is required.

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**References**

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