Airframe Design

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1 Nomenclature

- Coefficient of Drag (3D), C_D : $C_D = \frac{D}{q_\infty S_{ref}}$
- Coefficient of Lift (3D), C_L : $C_L = \frac{L}{q_\infty S_{ref}}$
- Coefficient of Moment (3D), C_M : this is made up of the roll (l), pitch (m), and yaw moments (n):
  - $C_l = \frac{l}{q_\infty S_{ref} b_{ref}}$
  - $C_m = \frac{m}{q_\infty S_{ref} b_{ref}}$
  - $C_m = \frac{n}{q_\infty S_{ref} b_{ref}}$
- Longitudinal Static Stability Derivative: $\frac{\partial C_{mcg}}{\partial \alpha} < 0$, but it is often short-handed, as $C_{m,\alpha} < 0$
- Lateral Static Stability Derivative: $C_{n,\beta}$
- Roll Stability Derivative: $C_{l,\beta}$
- Horizontal Tail Volume Coefficient: $V_h = \frac{S_h x_h}{S_w c_w}$ where x is the distance between the aircraft's center of gravity and the aerodynamic center of the horizontal tail
- Vertical Tail Volume Coefficient: $V_v = \frac{S_v x_v}{S_w b_w}$ where x is the distance between the aircraft's center of gravity and the aerodynamic center of the horizontal tail
- Wing Mean Geometric Chord, $\bar{c}$: $\frac{S_{ref}}{b}$, note that this is not used much
- Wing Mean Aerodynamic Chord, c_{mac} : $c_{mac} = \frac{2}{S}\int_0^{b/2} c^2\, dy$, where $S = 2\int_0^{b/2} c\, dy$

  It is a chord-weighted average chord.
- Wing Span, b : the lateral extent of the wing (both sides)

- Wing Dihedral, ϕ : angle at which the wing is rotated up from level (think boat hull)
- Wing Twist, θ : the angle at which a cross section of a wing is pointed (think angle of attack)
- Wing Sweep, Λ : angle at which the wing is perpendicular to flow
- Wing Aspect Ratio : $\frac{b^2}{S_{ref}}$
- Induced Drag, C_{Di}: Also called "vortex drag", it is the energy left behind in the wake, in the downwash. $\frac{C_L^2}{\pi AR}\sum_{n=1}^{N} n(\frac{A_n}{A_1})^2$
- Inviscid Span Efficiency, e : The coefficient terms in induced drag. $e = (\sum_{n=1}^{N} n(\frac{A_n}{A_1})^2)^{-1}$
- Airframe Stability Derivatives : A partial derivative that expresses how the forces on a vehicle change as its position changes. Think: how do the forces/moments react as the airframe moves? Some of the flight conditions are: α, β, p, q, and r.
- Wake Vortex: a linear vortex that trails behind a wing (vortex filaments)
- Downwash: downward moving air behind a wing

2 Introduction

The aim of this analysis is to use the vortex lattice method to model wings and an airframe to start building an understanding of wings and airframes. In order to do this, I analyzed three main questions. They are: how does the efficiency of a wing change as it becomes more elliptical, how do horizontal and vertical tail volume ratios effect the stability derivatives of an airframe, and how does the angle of attack affect the lift coefficient when using the vortex lattice method.

3 Methods

In preparation to answer these questions, I wrote a function that prepares a grid of points that approximate an elliptical wing. The root chord and the wing span define the shape of the ellipse. The function also accepts as arguments a wing angle, an x translation, a z translation, the number of chord-wise points used to approximate the wing, and the number of span-wise sections used to approximate the wing. This function returns a grid of points representing the wing, a reference area for the wing, and a reference chord. The code for this function can be found here.

To analyze the efficiency of elliptical wings, I used a loop to vary the number of sections used to represent the same elliptical wing. Because of the nature of an ellipse and the limitations of the vortex lattice method, the end of the wing had to be left off to avoid having overlapping points. Because this significantly changes the shape of the wing used, I chose to use a unique reference area and reference chord for each iteration, as well as to skip modeling wings with fewer than 10 sections so that no more than a tenth of the wing would be missing. The reference area and reference chord unique to each representation of the perfectly elliptical wing were used to calculate the coefficients of lift, drag, and moment for each representation of the wing.

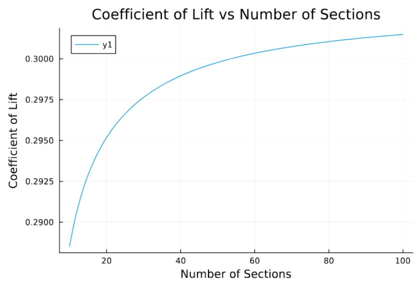
For the second question, which is about how horizontal and vertical tail volume ratios affect the stability derivatives of an aircraft, I modeled a simple airframe with a wing, a horizontal tail wing, and a vertical tail wing. All three wings were constructed using the function that I prepared, and due to the limitations of VortexLattice, I had to put each one at a small angle off of the x-y and y-z planes. In order to vary the vertical and horizontal volume ratios, I varied the placement of the tail wing portions by moving them both back together a small distance with each iteration. The stability derivatives of interest were $C_{m,\alpha}$, $C_{n,\beta}$, and $C_{roll,\beta}$ to judge the lateral, longitudinal, and roll stabilities, respectively.

The third question brings us back to a single wing to analyze how the Vortex Lattice method relates the angle of attack of a wing to the lift coefficient. The methods used were simple; an elliptical wing at one degree relative to the x-y plane was kept constant throughout the simulations, as the angle of attack varied from -10 to 25 degrees. The large range of angles of attack was chosen to show stall.

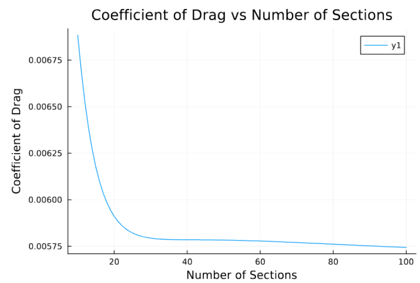
The code for the above analysis of all 3 questions can be found here.

4 Results

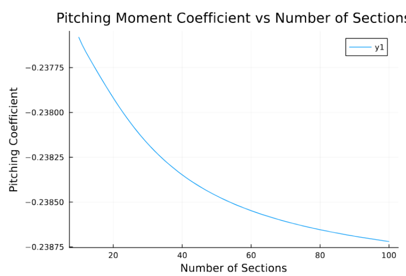
Running the VortexLattice Julia package on the elliptical wings, each with a different number of sections used for the approximation, yielded the results shown in Figure 1. It can be seen that the lift coefficient increased while the drag and moment coefficients decreased as the number of segments used to represent the wing increased.



(a) As the number of sections used to model the elliptical wing increased so did the coefficient of lift.



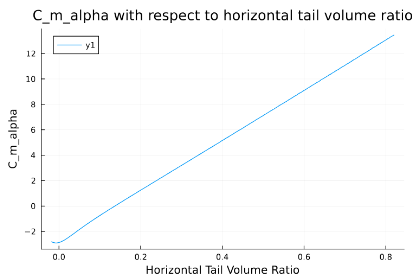
(b) As the number of sections used to model the elliptical wing increased the coefficient of lift decreased.



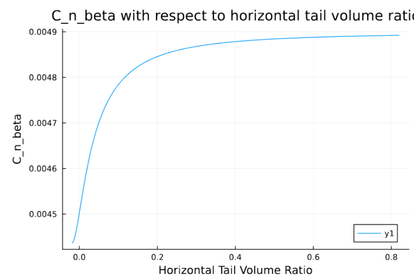
(c) As the number of sections used to model the elliptical wing increased the moment coefficient decreased.

Figure 1: As the number of sections used to model the elliptical wing increased, so did its efficiency.

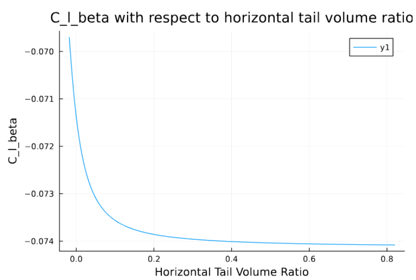
Running the analysis with VortexLattice on the airframe with a wing and a tail at varying distances back from the leading edge of the wing yielded both a set of horizontal and vertical tail volume ratios as well as the corresponding stability derivatives. These were plotted to show how the stability derivatives change with the volume ratios. Figure 2 shows the graphs corresponding to the horizontal volume ratios and Figure 3 shows the graphs corresponding to the vertical volume ratios. Notice that the curves are the same for both Figures 2 and 3. This is because the volume ratios were varied together by altering the variable they both shared, that is, the x position of the tail.



(a) The longitudinal stability derivative as the horizontal tail volume varies.

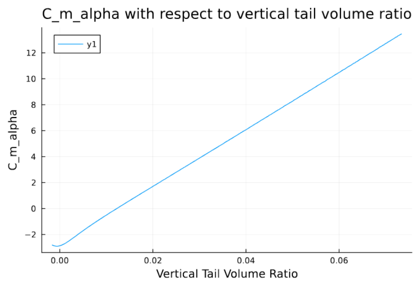


(b) The lateral stability derivative as the horizontal tail volume varies.

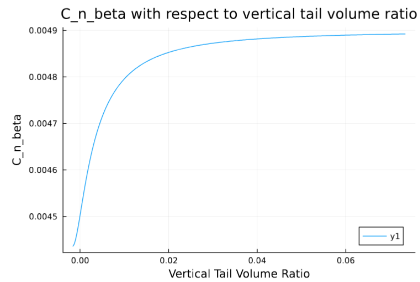


(c) The roll stability derivative as the horizontal tail volume varies.

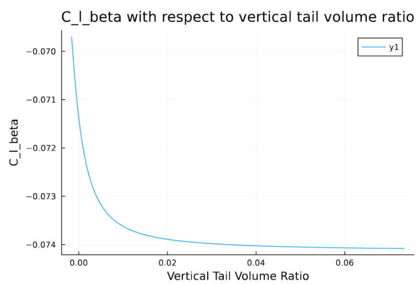
Figure 2: Selected stability derivatives as the horizontal tail volume ratio varies.



(a) The longitudinal stability derivative as the vertical tail volume varies.



(b) The lateral stability derivative as the vertical tail volume varies.



(c) The roll stability derivative as the vertical tail volume varies.

Figure 3: Selected stability derivatives as the vertical tail volume ratio varies.

As mentioned in the Methods section, VortexLattice was used to perform an analysis on the same wing while varying the angle of attack. This analysis gave us the lift coefficients plotted in Figure 4. As will be explored in the Discussion section, one can see that the coefficient of lift is quite nearly linear with respect to the angle of attack.

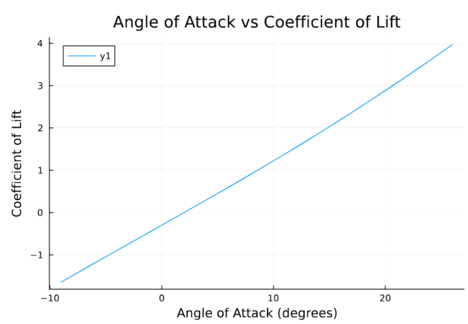


Figure 4: The Coefficient of Lift for a wing as predicted by VortexLattice.

5 Discussion

Looking at the lift, drag, and moment coefficients of an elliptical wing approximated using an increasing number of segments, it can be seen that the wing becomes more efficient as the shape of the wing becomes more elliptical. This is evident because the coefficient of lift increases while the coefficients of both drag and moment decrease. This means that as the wing becomes more elliptical, it provides more lift with less drag and a weaker pitching moment. This makes it so that the wing requires less thrust and a weaker corrective moment can be used to bring the wing into trim.

When designing an aircraft, it is important to take into consideration the stability derivatives of the aircraft. $C_{m,\alpha}$ indicates how the pitching moment (m) reacts to changes in the angle of attack (α). For the aircraft to stabilize itself longitudinally, $C_{m,\alpha}$ should be negative. Likewise, the important stability derivative for lateral stability is $C_{n,\beta}$ and it indicates how the yaw moment (n) reacts to changes in the sideslip angle (β). For stability, this derivative should be positive. And finally, for roll stability, the derivative used is $C_{l,\beta}$. This derivative should also be negative.

From the analysis performed for the second question, it can be seen that increasing the horizontal and vertical tail volume ratios by moving the tail further back from the wing almost linearly increases $C_{m,\alpha}$, increases $C_{n,\beta}$ along a roughly logarithmic curve, and decreases $C_{l,\beta}$ along a curve that appears to be the inverse of a logarithm. Using the data here, the airframe modeled for this question would have the correct signs for the mentioned stability derivatives if a horizontal tail volume ratio was selected that was less than roughly 0.15, and the vertical tail volume ratio was selected to be less than approximately 0.015.

Knowing that the wing was modeled as a 2-dimensional ellipse for VortexLattice, it makes sense that the coefficient of lift that VortexLattice would predict would be linear with the angle of attack, and that is almost exactly what Figure 4 shows. This means that the lift is almost exclusively from Newton's third law and that VortexLattice likely gives an underestimate of the lift produced by a wing with an airfoil cross section. Additionally, it does not predict any sort of stall.

A final note on using VortexLattice is that all of the wings had to be placed at an angle for VortexLattice to work properly. This is because of I used chord-wise segments as well as span-wise segments to create the grid representing the wings. When the chord-wise segments were perfectly in line with each other, they each produced a horseshoe vortex in VortexLattice, and then the vortex filaments from the leading edge would overlap with those further back, causing the quantities to diverge. An example of this can be seen plotted in Figure 5 where an analysis identical to that used for the third question (relating angle of attack and lift coefficient) was applied to that same wing, only with the wing being perfectly level.

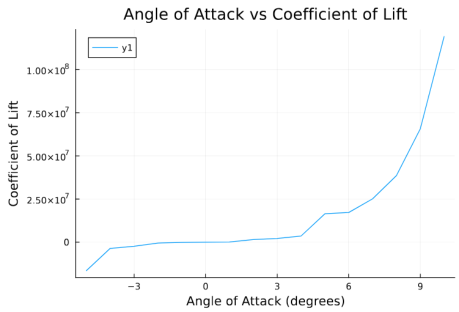


Figure 5: The predicted Coefficient of Lift for a perfectly horizontal wing as predicted by VortexLattice.

6 Resources

Ning, A. (2022). Computational Aerodynamics. (Original work published 2020)
Ning, A. (2024). Flight Vehicle Design [Review of Flight Vehicle Design]. (Original work published 2018)