

A Model of Retail Banking and the Deposits Channel of Monetary Policy*

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Abstract

We develop a dynamic, search-theoretic model of bank deposit markets where relationships are bilateral, the demand for liquid assets is microfounded, and consumers are privately informed about their liquidity needs. As the policy rate rises, the deposit spread widens, and aggregate deposits shrink, in accordance with the deposits channel documented in Drechsler et al. (2017). We show that the deposit outflow originates from consumers with low liquidity needs. As banks become more informed about consumers' types (e.g., through big data), their market power increases but transmission weakens. As search frictions are reduced (e.g., through online banking), market power shrinks and transmission weakens.

JEL Classification: D82, D83, E40, E50

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1 Introduction

The role of banks in the transmission mechanism of monetary policy is a debated question.¹ A recent study by Drechsler et al. (2017) argues that bank market power in the deposit market is pivotal and provides evidence that monetary policy affects the real economy through the supply of deposits — a so-called *deposits channel*. The evidence, which we review in Section 1.1, includes the following observations. First, the deposit spread, defined as the difference between the federal funds rate and interest rates on deposits, is positive and increases with the policy rate. Second, the growth rate of bank deposits and the change in the policy rate are negatively correlated. Moreover, as the federal funds rate rises, the flow of deposits out of the banking system is larger in concentrated markets. Last, following an increase in the policy rate, banks with higher market power in deposit markets reduce their lending by more relative to other banks.

The purpose of this paper is to provide theoretical foundations for bank market power and the deposits channel of monetary policy from first principles. We will open the blackbox of the transmission mechanism and study how informational frictions and market structure matter for each component of the deposits channel (i.e., deposit rate and spread, individual and aggregate deposits, profitability of deposit contracts). In addition to showing that the deposits channel emerges naturally when there are informational asymmetries between banks and consumers, we use our model to address the following questions. Is bank market power necessary and/or sufficient for the transmission mechanism to operate? Does the origin of bank market power (e.g., consumer search and switching costs, banks' ability to price discriminate) matter for transmission? Relatedly, how do FinTech advances, such as mobile banking and crypto-payments, affect the transmission mechanism of monetary policy? What are the distributional effects of monetary policy across consumers with heterogeneous liquidity needs?

The main components of our theory include a microfounded demand for liquid assets, a role for banks in the provision of such assets, and an explicit description of the creation of contractual relations in the deposit market with game-theoretic foundations. The demand for liquid assets comes from households who lack commitment and are subject to idiosyncratic spending shocks, in the spirit of the New Monetarist literature surveyed in Lagos et al. (2017).² Banks provide means of payments that have a lower user cost than cash and finance firms' investment opportunities. All relationships between consumers and banks are bilateral. The terms of the deposit contracts are determined through negotiations or are posted unilaterally by banks depending on the version of the model. The existence of two-sided market power is microfounded with search and informational frictions.

We start with a complete-information economy where banks and nonfinancial agents have a common knowledge of each other's characteristics. A first contribution of our model is to provide a theory of the

¹The literature has identified different channels through which monetary policy can affect the real economy, a subset of them involving banks. For an overview of this literature, see, e.g., Ireland (2010).

²Symmetrically, in Appendix B we describe a decentralized market for bank loans where firms have financing needs due to idiosyncratic investment opportunities, as in Kiyotaki and Moore (2019).

deposit spread as an intermediation premium in over-the-counter banking markets. The determinants of the deposit spread include the policy rate, banks' bargaining power, and market concentration (or dilution) as captured by the number of banks per consumer. As the policy rate increases, the deposit spread widens, provided that banks have some bargaining power. The passthrough is positive because the outside option of the banked consumer, which is to hold her liquid wealth in the form of non-interest-bearing cash while searching for an alternative bank, becomes less valuable as the opportunity cost of cash increases.

The existence of a deposit spread passthrough, however, is inconsequential for deposits. When banks have complete information about their consumers, the deposit size, which maximizes the joint surplus of the consumer and her bank, is unaffected by changes in monetary policy. In contradiction with the empirical evidence, aggregate deposits increase with the policy rate because deposit contracts are more profitable to banks when interest rates are high. This finding shows that market power is not sufficient for the deposits channel to operate and that the information structure is critical.

In the second part of the paper, we assume consumers are heterogeneous in their liquidity needs and their preferences are private information. Banks engage in second-degree price discrimination by posting a menu of incentive-compatible deposit contracts. The optimal menu has a two-tier structure. Contracts for low-liquidity-needs consumers offer a zero deposit rate, which allows banks to appropriate the whole surplus. Consumers with larger liquidity needs enjoy a positive deposit rate and hold more liquidity than they would without a bank. Consistent with the evidence in Drechsler et al. (2017), average deposits shrink as the policy rate increases.

In addition to showing that our model can explain the deposits channel of monetary policy, we obtain three new insights. First, a new implication from our theory is that the size of the deposit spread passthrough varies across contracts: it is maximum for low deposits and it weakens as the deposit size increases. Second, the deposit outflow triggered by an increase in the policy rate is not distributed evenly across consumers: the outflow is concentrated on consumers with low liquidity needs. Third, if bank deposits are imperfectly liquid, the relationship between bank deposits and the policy rate is nonmonotone, i.e., bank deposits increase at low interest rates and decrease when the policy rate is above a threshold.

In order to capture the connection between market concentration and the strength of the transmission mechanism, we generalize our model by combining private information and two-sided bargaining powers. The terms of the deposit contracts depend on how easy it is for a consumer to find alternative offers, which we link to market concentration by assuming free entry in the deposit market. Deposit rates are lower in markets that are more concentrated. Moreover, the deposit spread passthrough and the strength of the deposits channel are higher when concentration is higher. We calibrate our model and show that bank market power has to be large to be consistent with the size of the passthrough and the elasticity of aggregate deposits with respect to the policy rate observed in the data.

Finally, we use our model to study the effects of technological advances in banking for the transmission

mechanism. The development of online banking improves consumers' outside options and reduces banks' market power, which promotes the accumulation of deposits by households but weakens the transmission mechanism. The introduction of digital currencies by the central bank (CBDC) also has the potential to raise the value of consumers' outside options by providing interest-bearing means of payments that can compete with bank deposits. Finally, the ability to collect data about consumers' liquidity needs allows banks to price discriminate, which shrinks consumers' informational rents but reduces the effectiveness of the deposits channel. These results emphasize the importance of identifying the source of variation of banks' market power to assess its effects on the transmission of monetary policy. Market power due to increased search frictions strengthens the deposits channel whereas market power due to a better knowledge of consumers' liquidity needs weakens the deposits channel.

1.1 Empirical evidence

We now review the main evidence on the deposits channel of monetary policy and banks' market power provided by Drechsler et al. (2017).³ We organize this evidence as a list of observations that will guide our modeling choices in the rest of the paper.

Observation #1a: The deposit spread passthrough is positive. There is a positive passthrough from the federal funds rate, to the deposit spread defined as the difference between the policy rate and the interest rate on bank deposits. A 100 bps increase in the Federal funds rate leads to an increase in the deposit spread by 54 bps according to Drechsler et al. (2017).

Observation #1b: The deposit spread passthrough is higher for more liquid deposits. Drechsler et al. (2017) distinguish three categories of deposits ranked by their liquidity: checking accounts are the most liquid; savings accounts; and, time deposits are the least liquid. As illustrated in Figure 1, the passthrough increases from 0.238 for small time deposits, to 0.415 for savings deposit, and 0.875 for checkable deposits.⁴

Observation #1c: The deposit spread passthrough is state-dependent. Wang (2018) documents that the deposit rate passthrough (one minus the deposit spread passthrough) is lower when the interest rate is lower. For checking and savings deposits, a 100 bps increase of the policy rate raises the deposit rate passthrough by about 0.3% percentage point at a 8-month time horizon.

Observation #2a: The growth rate of aggregate deposits is strongly negatively correlated with changes in the federal funds rate. Drechsler et al. (2017) find that the correlation between the growth rate of aggregate deposits and the year-over-year change in the federal funds rate is -0.49 . These correlations

³Additional evidence on banks' market power in the US deposit markets is provided by Hannan and Berger (1991), Neumark and Sharpe (1992), Degryse and Ongena (2008).

⁴These findings are similar to that of Figure 1 in Drechsler et al. (2017). Our data has been formatted to match various vintages of Call reports using the standard procedure in Kashyap and Stein (2000). We thank Russell Wong for providing us with this data.

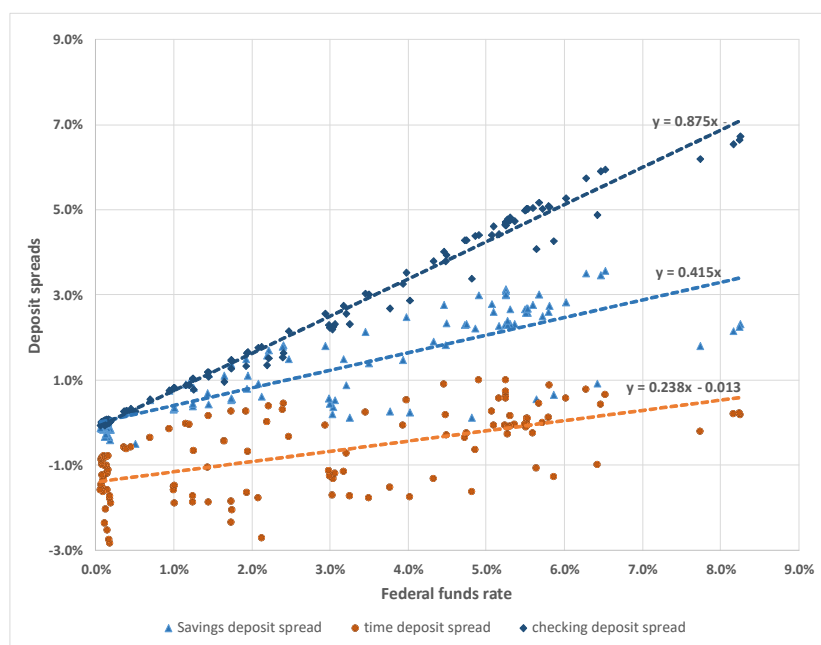


Figure 1: Passthrough from federal funds rate to deposit spreads. The data source is Call Reports from 1990 Q1 to 2020Q4.

are -0.28 and -0.55 for checkable and saving deposits, respectively. See the top and middle panels of Figure 2. Drechsler et al. (2017) also estimate the semi-elasticity of deposits with respect to deposit spreads and conclude that a 100 bps increase in the federal funds rate generates a 323 bps contraction in deposits.

Observation #2b. The growth rate of less-liquid deposits is positively correlated with the change in the federal funds rate. The correlation between the growth rate of deposits and the change in the federal funds rate is negative for checkable and savings deposits (top and middle panels of Figure 2), but positive, equal to 0.30, for small time deposits (bottom panel).

Observation #3a: Deposit rates and market concentration are negatively correlated. Berger and Hannan (1989) are the first to establish a relationship between local market concentration and the interest rates offered by banks for retail deposits. They found that banks in the most concentrated local markets pay deposit rates that are 25 to 100 basis points less than those paid in the least concentrated markets.

Observation #3b: The deposit spread passthrough increases with market power. Drechsler et al. (2017, Section 4) show that the deposit spread passthrough increases with market concentration, measured according to the Herfindahl–Hirschman Index (HHI), by about 12 percent from low to high concentration counties. They also consider an alternative measure of bank market power, namely the lack of financial sophistication of consumers proxied by age, income, and education. Following an increase in the policy

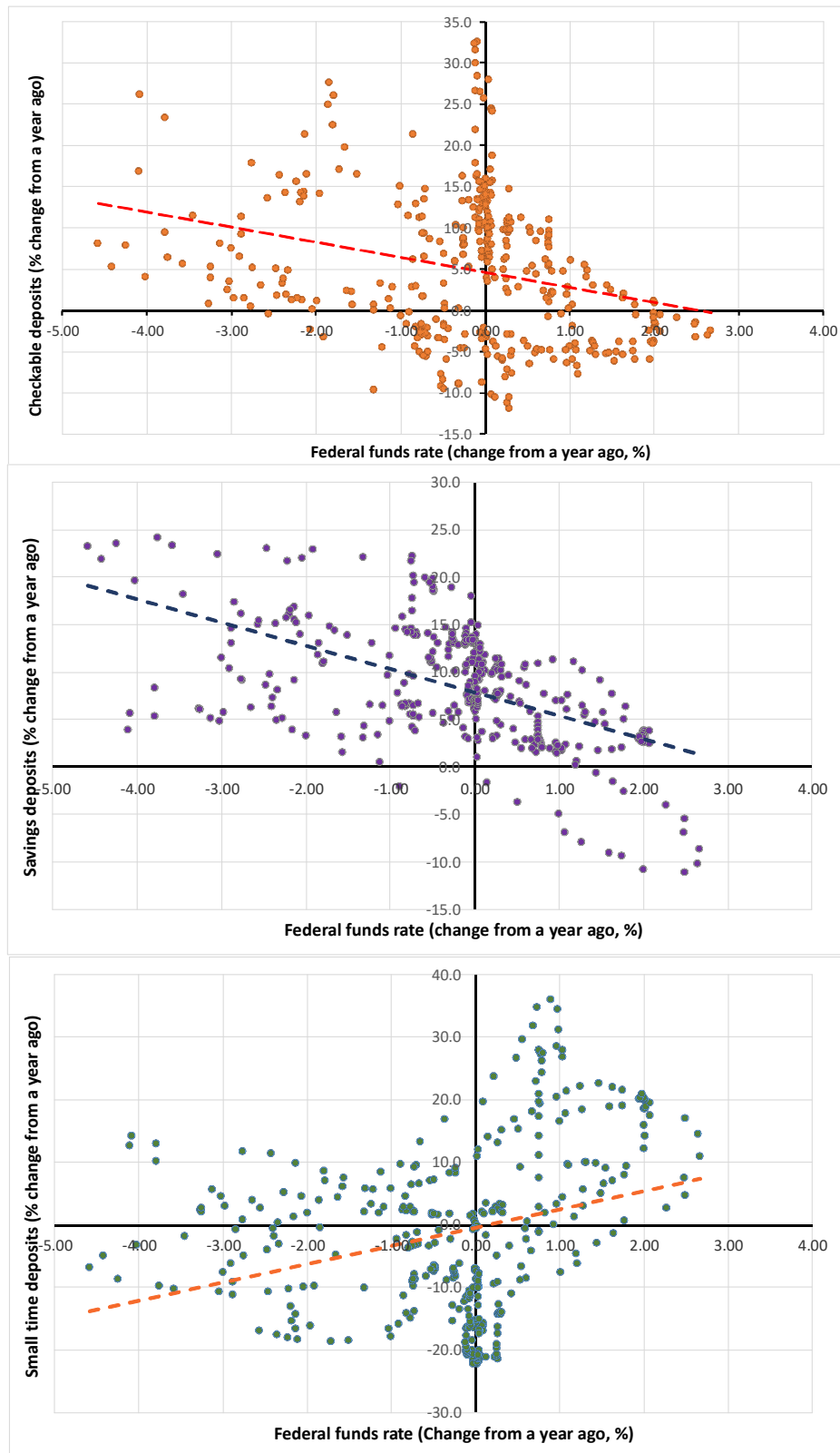


Figure 2: Relation between deposits and federal funds rate. Top: checkable deposits; Middle: savings deposits; Bottom: time deposits. The data on deposits is at monthly frequency from 1990 to 2019, is seasonally adjusted, and is expressed in percentage change from a year ago. It is obtained from the Federal Reserve Economic Data (Link: <https://fred.stlouisfed.org>).

rate, banks in counties with an older population, lower median household income, and less college education increase deposit spreads by more than banks in other counties.

Observation #4: The correlation between deposit growth and changes in the federal funds rate is more negative in more concentrated markets. Drechsler et al. (2017, Section 4) show that deposit growth is more sensitive to changes in the federal funds rate in more concentrated counties. Following a 100 bps increase in the Fed funds rate, deposits flow out by 38 bps more in high-concentration counties than low-concentration counties.⁵ Other proxies for market power (age, income, and education) have a similar effect as market concentration.

Table 1 recapitulates the empirical observations related to the deposits channel of monetary policy and gives a preview of the predictions of three versions of our model (bargaining under complete information, posting under private information, bargaining under private information).⁶ In order to account for all the observations, we will need private information and bargaining powers by agents on both sides of the market.

Observations	Models' predictions		
	<i>Complete information</i>	<i>Private Information</i>	
		Posting	Bargaining
1a. deposit spread passthrough	✓	✓	✓
1b. passthrough across deposits	✓	✓	✓
1c. state-dependent passthrough	✓	✓	✓
2a. aggregate deposits and policy rate	×	✓	✓
2b. disaggregated deposits and policy rate	✓	✓	✓
3a. deposit rate and market power	✓	×	✓
3b. passthrough and market power	✓	×	✓
4. deposits and market power	×	×	✓

Notes: ✓ means the model's prediction matches with data. × means the opposite.

Table 1: Summary of data and model predictions.

1.2 Literature

In addition to providing the evidence of a deposits channel that motivates this paper, Drechsler et al. (2017) construct a static model of monopolistic competition among banks where liquidity services enter the utility function through a CES aggregator of cash and bank deposits.⁷ While the model in Drechsler et al. (2017) provides clear intuitions and predictions that accord with their empirical results, our paper aims at deconstructing the transmission mechanism by providing joint microfoundations for the demand for liquidity

⁵Li et al. (2019) elaborate on the findings of Drechsler et al. (2017) and show that market power in the deposit market matters not only for prices and quantities (interest rate and supply of deposits) but for other terms of bank contracts such the maturity of the loans that banks offer.

⁶While we focus on the evidence regarding the deposits market, Drechsler et al. (2017) establishes a link between the contraction in deposits and the contraction in lending. They show that following an increase in the policy rate, banks that collect deposits in more concentrated markets reduce their lending more relative to other banks. Schaffer and Segev (2021) provide a critical reappraisal of these results. We extend our model in Appendix B to account for the lending part of the transmission mechanism.

⁷Drechsler et al. (2021) use a similar model to show it is optimal to use maturity transformation to hedge interest-rate risk. Related contributions on the role of liquid deposits for the transmission mechanism of monetary policy include Wang (2018) and Di Tella and Kurlat (2021).

and bank market power in a dynamic general equilibrium New Monetarist model. The game-theoretic approach to the terms of the deposit contract allows us to be explicit about the pricing of deposits, and the restrictions imposed upon it (e.g., the degree of price discrimination), and the nature of bank market power (e.g., information versus search costs). In terms of novel insights, we will show that bank market power is not a sufficient condition for the deposits channel to operate, i.e., banks can have market power and still deposits can be invariant to changes in monetary policy. It is because consumers' private information restricts banks' ability to price discriminate that the deposit channel emerges. A new testable implication from this finding is that the outflow of deposits following an increase in the policy rate should be concentrated on the agents at the bottom of the distribution of liquidity needs. Another insight is that the origin of bank market power matters for transmission. For instance, bank's market power due to search frictions strengthens the deposit channel while bank's market power due to a better knowledge of consumers' liquidity needs weakens it. Finally, by formalizing explicitly the liquidity of deposits we can show that transmission can differ at low and high interest rates.

Within the New Monetarist literature, different assumptions have been made regarding bank competition.⁸ In Williamson (2012), perfectly competitive banks provide insurance contracts to households where the idiosyncratic risk arises from the imperfect acceptability of bonds and private assets.⁹ Andolfatto et al. (2020) also describe competitive banks as providing insurance contracts but idiosyncratic shocks take the form of preference shocks for early or late consumption, as in Diamond and Dybvig (1983). The version of our model with multiple deposit categories shares some similarities with these approaches. Keister and Sanches (2020) construct a model where competitive banks issue liabilities backed by entrepreneurs' investment projects to study the competition between central bank digital currencies and banks' demand deposits.

New-Monetarist models where banks have market power include Rocheteau et al. (2018) and Bethune et al. (2021).¹⁰ The market structure is similar to our model in that contracts between entrepreneurs and banks are bilateral and relationships take time to form.¹¹ Similarly, Lagos and Zhang (2020) formalize banks as securities dealers in the market for consumer credit and emphasize the role of sellers' option to settle transactions with money as a mechanism to restrain banks' market power. These models focus on lending channels (for businesses or consumers) whereas our main focus is on the deposit market where agents with liquidity needs enter long-term deposit contracts with banks endowed with a superior investment technology. Our foundations for bank market power also differ in that consumers' outside options include the possibility to keep searching for alternative banks, which provides the connection between the strength of the transmission mechanism and market concentration. As a result of this description, our model delivers perfect competition at the limit when the speed of search goes to infinity. Finally, while the literature

⁸Vives (2016) provides a comprehensive discussion about the trade-offs between competition and stability in banking.

⁹The model has been applied to address a variety of monetary policy questions in Williamson (2016, 2018, 2019).

¹⁰Applications and extensions of this model include Silva (2019), Jackson and Madison (2021) and Liang (2021).

¹¹Alternative formulations of the banking sector with search and bargaining frictions include Wasmer and Weil (2004) and Petrosky-Nadeau and Wasmer (2017). This approach, which focuses on frictions in labor and credit markets, does not formalize the deposit market.

above assumes that banks have complete knowledge of their consumers' characteristics, we make consumers' liquidity needs private information. This assumption which is arguably natural (complete information is used for tractability, not for being realistic) is also necessary to obtain a negative relationship between aggregate deposits and the policy rate.

Alternative industrial organization approaches to imperfect competition in the deposit market are reviewed in Chapter 3 of Freixas and Rochet (2008) and have been recently applied to the study of central bank digital currencies. Andolfatto et al. (2020) adopts the model of a monopoly bank, as in Klein (1971) and Monti et al. (1972) while Chiu et al. (2019) formalizes bank market power as the outcome of Cournot competition.¹² These approaches raise the thorny issue of the choice of the appropriate strategic variables to describe competition among banks.¹³ Under Cournot competition, banks commit to a deposit capacity while the deposit rate is determined by a fictitious auctioneer to clear the market. Our approach to markets follows Osborne and Rubinstein (1990) where the determination of allocations and prices is entirely based on game theoretic foundations. Competition between banks is in terms of menus of deposit contracts that specify both prices and quantities or, alternatively, the utility that these contracts provide to consumers. Relative to Bertrand competition, perfect competition is only obtained at the limit when search costs vanish. The determination of deposit spreads in our model is analogous to the determination of bid-ask spreads in the model of over-the-counter markets of Duffie et al. (2005). While trade sizes are exogenous in Duffie et al. (2005), in our model deposit sizes are endogenous, which makes it closer to Lagos and Rocheteau (2009).

Gu et al. (2013) depart from the equilibrium approach and adopt mechanism design to explain the emergence of banks in an economy with limited commitment. Relatedly, Cavalcanti and Wallace (1999) developed a model with indivisible monies where banks are monitored agents and use an implementation approach to characterize incentive-feasible allocations with inside and outside monies. Various approaches to banking and financial intermediation are studied in Gu et al. (2020) with a focus on endogenous instability and multiplicity of equilibria.

The heterogeneous needs for liquid assets among consumers is formalized as in Lagos and Rocheteau (2005). The banks' mechanism design problem is set up according to the methodology in Mussa and Rosen (1978), Maskin and Riley (1984) and Jullien (2000). Faig and Jerez (2005), Ennis (2008), and Bajaj and Mangin (2020) introduced liquidity constraints into a similar mechanism design problem with directed search, undirected search, and consumer search under multilateral matching, respectively. Williamson (1987) also studies asymmetric information in banking contracts but using a costly monitoring approach. Finally, Bethune et al.

¹²Chiu et al. (2019) develop in their appendix a version of their model in the spirit of Burdett and Judd (1983) where competition is in terms of the deposit rate.

¹³According to Freixas and Rochet (2008):

“The (generalized) Monti-Klein model (...) suffers from the same criticisms as the Cournot model from which it is adapted. In particular, as emphasized originally by Bertrand, prices (here rates) may be more appropriate strategic variables for describing firms' (banks') behavior. As is well known, however, price competition à la Bertrand may go too far, since (1) existence of an equilibrium is not guaranteed, and (2) as soon as two firms are present, perfect competition is obtained.”

(2021) introduce private information over liquidity needs in an OTC asset market setting.

Our last extension with private information and bargaining is related to Inderst (2001). We adopt a different bargaining protocol and we consider the case with a continuum of consumer types. In Appendix C, we study an alternative version where consumers can keep searching for a deposit contract while banked. We formalize bank-to-bank transitions as in Mortensen (1998), which is a variant of Burdett and Judd (1983) and Burdett and Mortensen (1998). Yankov (2014) studies the dispersion of deposit rates using the Burdett-Judd model with complete information. The Burdett-Judd model with adverse selection has been studied by Garrett et al. (2019) and Lester et al. (2019).

2 Environment

Time, agents, and goods Time is continuous and indexed by $t \in \mathbb{R}_+$.¹⁴ The economy is composed of two types of agents: a unit measure of consumers-producers (thereafter simply called consumers) and a large measure of bankers. Bankers are infinitely lived while consumers die at rate $\delta > 0$ and are replaced by new consumers upon death. There are two perishable goods, $y \in \mathbb{R}_+$ and $c \in \mathbb{R}$. Good c is taken as the numéraire.

Preferences and technologies Consumers' preferences over good c and y are given by:

$$\mathbb{E} \left[\int_0^T e^{-\rho t} dC(t) + \sum_{n=1}^{+\infty} e^{-\rho t_n} u[y(t_n)] \mathbb{I}_{\{t_n \leq T\}} \right], \quad (1)$$

where $\rho > 0$ is the rate of time preference and T is the time horizon of the consumer, which is exponentially distributed with mean $1/\delta$. The function $C(t)$ is the cumulative net consumption of the numéraire good. Negative consumption is interpreted as production, i.e., consumers have the technology to produce the numéraire at unit cost. Consumption and production can take place in flows, $dC(t) = c(t)dt$, or in discrete quantities, $C(t^+) - C(t^-) \neq 0$.

The second term in (1) represents the preferences over good y . At random times, $\{t_n\}_{n=1}^{+\infty}$, the agent has the desire to consume good y , where $\{t_n\}_{n=1}^{+\infty}$ is a Poisson process with arrival rate, $\sigma > 0$. The utility of consumption, $u(y)$, satisfies $u' > 0$, $u'' < 0$, $u(0) = 0$, $u'(0) = +\infty$. The technology to produce good y is linear with the numéraire as the sole input, i.e., one unit of numéraire can be turned into one unit of good y . We denote y^* such that $u'(y^*) = 1$.

Bankers only value the numéraire and are risk neutral. Their preferences are given by

$$\mathbb{E} \left[\int_0^{+\infty} e^{-\rho t} dC(t) \right].$$

¹⁴Our environment of a monetary economy in continuous time is closely related to that in Choi and Rocheteau (2021) where we provide additional details regarding the methodology.

Markets and money Goods are traded in competitive spot markets opened around the clock. Consumers who are hungry for good y cannot produce to finance their consumption. Moreover, consumers who lack commitment are not trusted to repay their debts. These frictions create a need for a means of payment for good y .

There is a quantity M_t of fiat money – a perfectly divisible and durable object that is intrinsically useless – growing at a constant rate $\pi \geq -\rho$. The revenue from money creation finances unproductive government consumption. Throughout our analysis, we identify the policy rate with the opportunity cost of holding a non-interest-bearing asset, such as fiat money or reserves, i.e., $i = \rho + \pi$.¹⁵

Bank deposits Alternative means of payment are provided by bankers in the form of deposits. Bankers have the technology to invest the funds they receive from consumers at some real interest rate r_b and can commit to return these funds on demand. In Appendix B, we endogenize r_b by formalizing the lending market where entrepreneurs with investment opportunities search for bank loans, as in Rocheteau et al. (2018) and Bethune et al. (2021), and the interbank market where banks trade funds competitively. In Section 4.3 and in Appendix E, we introduce bank deposits that are imperfectly liquid.

Deposit market Consumers form long-term relationships with bankers. A banker can only manage the account of a single consumer.¹⁶ The overall supply of deposits is determined by the free entry of bankers in the deposit market where the flow entry cost is $\kappa > 0$ (in utils or numéraire). In Drechsler et al. (2021), this cost is interpreted as the cost for the bank to operate a deposit franchise. It includes salaries, advertising, and the cost to attract and serve depositors. Each unbanked consumer meets an unmatched banker at Poisson rate $\alpha(\tau)$ where τ is the measure of unmatched bankers per unbanked consumer, $\alpha' > 0$, and $\alpha'' < 0$.¹⁷ A banker meets a potential consumer at rate $\alpha(\tau)/\tau$. Search frictions provide a tractable way to formalize imperfect competition in the market for deposit contracts. They capture the limited awareness of consumers of the banks in their area and the time to gather information about retail banking products and offers.¹⁸ Importantly, the frictions can be made arbitrarily small. The frictionless limit is obtained when $\alpha(\tau) \rightarrow +\infty$ for all $\tau > 0$, in which case we will show that banks have no market power and the outcome corresponds to the one from Bertrand competition. We assume that only unmatched consumers can search for a bank.

¹⁵It can also be interpreted as the interest rate on a risk-free bond that cannot be used to finance consumption of good y , e.g., because these bonds would take a small amount of time to be sold. The choice of i as our policy rate is consistent with Drechsler et al. (2017) who argue that “*while both Fed funds loans and T-bills are extremely safe short term securities, T-bills provide a higher level of liquidity services to a broader range of investors, and therefore command a liquidity premium*”. Moreover, Rocheteau, Wright, and Zhang (2018, Section 7) formalizes the interbank market and a demand for reserves and show the interest rate on interbank loans in equilibrium is i .

¹⁶This assumption is similar to the one-firm-one-job assumption in the frictional labor market of Pissarides (2000).

¹⁷We assume newborn consumers are initially unbanked and join a bank at Poisson rate α . Alternatively, we could assume that consumers are born with one bank offer that they can reject, in which case they have to search for alternative offers.

¹⁸For instance, Abrams (2019) argues that bank market power is exacerbated by consumers’ limited consideration of banks. Honka et al. (2017) report that the average consumer considers 6.8 banks among the 24 banks that populate the average metropolitan statistical area. According to the authors: “*A consumer searches among the banks he is aware of. Searching for information is costly for the consumer since it takes time and effort to contact financial institutions and is not viewed as pleasant by most consumers.*”

The absence of search by banked consumers is with no loss in generality given that banks are homogenous and provided that optimal contracts with upfront fees are allowed, e.g., as in Stevens (2004). In Appendix C, we introduce bank-to-bank transitions in a version of the model with private information and stationary deposit contracts.

As illustrated in Figure 3, the life cycle of a consumer is as follows. At the start of their lives, consumers are unbanked and search for a long-term relationship with a bank. Once such a relationship is formed, at rate α , consumers remain with their bank for the rest of their lives. (One can also interpret the δ -shock as a separation for exogenous reasons such as, e.g., a change in location.)

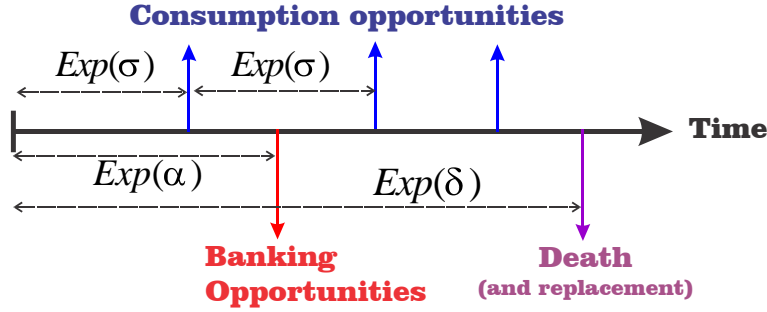


Figure 3: Life cycle of a consumer

3 A simple theory of the deposit spread passthrough

In this section, we present a simple theory of the determination of the interest rate on deposits and its relation to monetary policy. We focus on steady-state monetary equilibria where the rate-of-return of money is constant and equal to $-\pi$.

3.1 Consumers and bankers

We denote $V^u(m)$ the value function of an unbanked consumer with m real balances. From the linearity of preferences with respect to c , $V^u(m) = m + V^u$ (see Choi and Rocheteau, 2021), where V^u solves the HJB equation:

$$\rho V^u = \max_{m \geq 0} \{ -im + \sigma v(m) + \alpha(\tau)(V^b - V^u) - \delta V^u \}, \quad (2)$$

where $i \equiv \rho + \pi$ and $v(m)$ is a consumer's surplus from an opportunity to consume good y at Poisson rate σ ,

$$v(m) \equiv \max_{y \leq m} \{ u(y) - y \}. \quad (3)$$

According to (2) the unbanked consumer chooses his real balances, m , in order to maximize her expected surplus from trade, $\sigma v(m)$, net of the cost of holding real balances, im . At Poisson rate, $\alpha(\tau)$, the consumer finds a banker with whom to enter into a demand deposit contract. At Poisson rate, δ , the consumer consumes her wealth and dies, which generates a capital loss equal to V^u . Since good y is produced from the

numéraire at unit cost and is traded competitively, its price is equal to one. Hence, the producer's surplus is zero and is omitted from the HJB equations. According to (3), the consumer chooses y to maximize her utility of consumption net of the payment. She is subject to the feasibility constraint according to which the payment cannot be larger than her real balances.

From (2) the optimal choice of real balances of an unbanked consumer is such that

$$u'(y) = 1 + \frac{i}{\sigma}. \quad (4)$$

It equalizes the consumer's marginal utility to the producer's marginal cost augmented by a wedge equal to the expected cost of holding money, i/σ .

We now turn to a consumer under a demand deposit contract. The contract specifies a pair, (d, ϕ) , where d is the amount deposited at the bank expressed in terms of the numéraire and ϕ is the flow banking fee also expressed in the numéraire. In this formulation, the consumer pays ϕ to the banker in order to access its investment technology. The value function, V^b , solves:

$$\rho V^b = -\phi - s_b d + \sigma v(d) - \delta V^b, \quad (5)$$

where $s_b \equiv \rho - r_b$ is the opportunity cost of investing into banks' assets and $v(d)$ is the consumer surplus from a trade if the consumer has deposited d at the bank.

The expected discounted sum of the banker's profits from a deposit contract is Π solution to:

$$\rho \Pi = \phi - \delta \Pi. \quad (6)$$

So $\Pi = \phi/(\rho + \delta)$. The banker's profits are simply the fees charged to the customers discounted at rate $\rho + \delta$.

3.2 Demand deposit contract

The terms of the deposit contract, (d, ϕ) , are determined through negotiation. We adopt the generalized Nash solution where the banker's bargaining power is θ , i.e.,

$$(d, \phi) \in \arg \max_{d, \phi} [V^b(d, \phi) - V^u]^{1-\theta} [\Pi(d, \phi)]^\theta, \quad (7)$$

where the consumer's disagreement point, V^u , is taken as given by the two parties.¹⁹ In accordance with Drechsler et al. (2017), one can think of the consumer's bargaining power, $1 - \theta$, as related to their degree of financial sophistication. In the extensive-form game of Section 5, $1 - \theta$ is the probability that the consumer can counter the bank's initial offer.

¹⁹In Section 5, we provide an extensive-form game that generates the same outcome. At the start of the negotiation, the bank makes an offer. If the consumer accepts the offer, the negotiation ends. Otherwise, with probability $1 - \theta$, there is a second round where the consumer makes a take-it-or-leave-it offer to the bank. Alternatively, the generalized Nash solution in (7) corresponds to an extensive-form game with offers and counter-offers where the consumer can keep searching for an alternative bank, and the bank can look for an alternative consumer, while bargaining.

Lemma 1 (Terms of the deposit contract.) *The solution to the bargaining problem, (7), maximizes the joint surplus, $S \equiv [-s_b d + \sigma v(d) - (\rho + \delta) V^u] / (\rho + \delta)$, which gives,*

$$u'(d) = 1 + \frac{s_b}{\sigma}, \quad (8)$$

and splits that surplus according to the players' bargaining power, which gives

$$\phi = \frac{\theta (\rho + \delta)}{\rho + \delta + \alpha (1 - \theta)} [U(s_b) - U(i)], \quad (9)$$

where

$$U(s) \equiv \max_{d \geq 0} \{-s d + \sigma v(d)\}.$$

From (8) the individual demand for deposits increases with r_b (or decreases with s_b) but it is independent of the frictions in the deposit market, such as α and θ , and monetary policy as represented by i . From (9) banking fees are equal to a fraction of the gains enjoyed by the consumer from accessing the investment technology of the banker, $U(s_b) - U(i)$. This fraction increases with the banker's bargaining power (θ) but decreases with the speed at which the consumer can find another banker (α).²⁰

We illustrate the outcome of the negotiation in Figure 4 by plotting the indifference curves of the consumer with utility level $U = -(s + \sigma)d + \sigma u(d) - \phi$. The orange indifference curve represents the utility of the unbanked consumer, i.e., $s = i$ and $\phi = 0$. The dark-blue indifference curve is the utility of the banked consumer who has all the bargaining power, i.e., $s = s_b$ and $\phi = 0$. Finally, the light-blue indifference curve is the utility of the banked consumer who has no bargaining power, i.e., $s = s_b$ and $\phi = U(s_b) - U(i)$. For all θ , the outcome of the negotiation under complete information is such that $d = d^*$ as defined in (8).

3.3 Deposit spread

In order to obtain a theory of the deposit rate, we reinterpret the deposit contract as specifying a deposit level and a nominal interest rate per unit deposited. The nominal deposit rate is

$$\hat{i}_d = r_b + \pi - \frac{\phi}{d} = i - s_b - \frac{\phi(i)}{d}. \quad (10)$$

It is the nominal interest rate on banks' assets, $i_b = r_b + \pi$, reduced by the banker's profits per unit deposited, ϕ/d . The deposit spread is the difference between i and \hat{i}_d , i.e.,

$$\hat{s}_d \equiv i - \hat{i}_d = s_b + \frac{\phi(i)}{d}. \quad (11)$$

The deposit spread is equal to the bank spread, $s_b \equiv i - i_b$, augmented by an intermediation premium, ϕ/d .

We substitute $\phi(i)$ by its expression given by (9) and d by its expression from (8), i.e., $d = u'^{-1}(1 + s_b/\sigma)$, into (11) to obtain the following closed-form expression for the deposit spread:

$$\hat{s}_d = s_b + \frac{\theta (\rho + \delta) [U(s_b) - U(i)]}{[\rho + \delta + \alpha (1 - \theta)] u'^{-1}(1 + \frac{s_b}{\sigma})}. \quad (12)$$

²⁰If we allowed banked consumers to keep searching for a bank, the optimal contract would be payoff equivalent to the one in Lemma 1 and would specify an upfront fee equal to $\Phi = \phi/(\rho + \delta)$ and no additional fee afterwards. Under such a contract a consumer would have no strict incentive to switch to a different bank with the same investment technology.

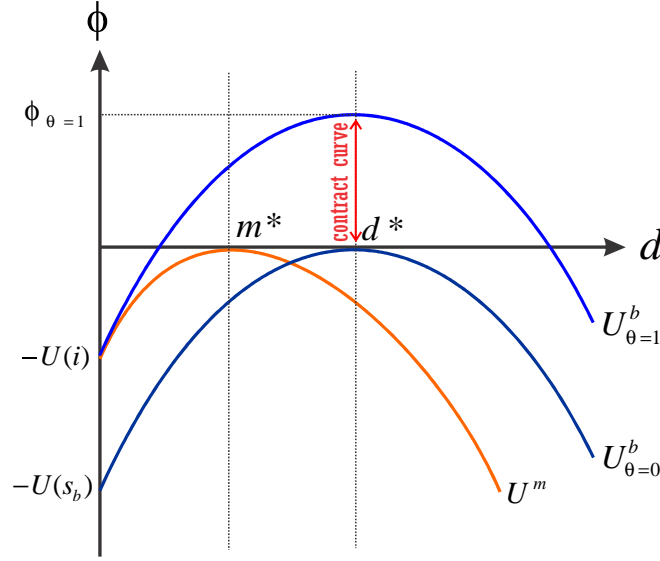


Figure 4: Contract curve under complete information

The deposit spread increases with bankers' bargaining power (or decreases with consumers' financial sophistication) and the policy rate but it decreases with the rate at which consumers find an alternative banker.

3.4 Entry in the deposit market

The condition that banks make zero expected discounted profits by entering the deposit market can be written as

$$\kappa = \frac{\alpha(\tau)}{\tau} \Pi. \quad (13)$$

The banker incurs a flow cost κ to search for a customer. At Poisson rate $\alpha(\tau)/\tau$ the banker finds a customer and enjoys the discounted sum of profits from a deposit contract, $\Pi = \phi/(\rho + \delta)$. We now relate ϕ to market tightness, τ . By generalized Nash bargaining, $V^b - V^u = (1 - \theta)\Pi/\theta$. We substitute $V^b - V^u$ by its expression into (2) to express the reservation utility of a consumer as

$$(\rho + \delta) V^u = U(i) + \alpha(\tau) \left(\frac{1 - \theta}{\theta} \right) \Pi = U(i) + \left(\frac{1 - \theta}{\theta} \right) \kappa \tau. \quad (14)$$

Next, we substitute (14) into $\phi = \theta [U(s_b) - (\rho + \delta) V^u]$ to obtain:

$$\phi = \theta [U(s_b) - U(i)] - (1 - \theta) \kappa \tau. \quad (15)$$

The first term on the right side is the product of the banker's bargaining power and the consumer gain from accessing the investment technology of the bank. The second term reflects market conditions and consumers' outside options.

Finally, we substitute ϕ given by (15) into the bank's profits given by (6), and from (13) we rewrite the free-entry condition as

$$(\rho + \delta) \kappa = \frac{\alpha(\tau)}{\tau} \theta [U(s_b) - U(i)] - \alpha(\tau) (1 - \theta) \kappa. \quad (16)$$

Equation (16) determines τ as a function of monetary policy, i , the rate of return of bank investments, r_b , and bankers' bargaining power, θ .

The operational notion of market power in Drechsler et al. (2017) is in terms of market concentration as measured by the Herfindahl-Hirschman Index. In the case of homogeneous banks, the Herfindahl-Hirschman Index is equal to market share (Philippon (2019) Box 2.1). In our model, the market share of a bank is simply $d/(n^b d) = 1/n^b$, i.e., it is the deposits at an individual bank relative to aggregate deposits. Hence, bank market share is equal to the inverse of the measure of banked consumers which, we will show later, is inversely related to τ .

3.5 Equilibrium of the deposit market

We now provide a graphical and analytical characterization of the equilibrium of the deposit market. We first simplify the expression for the deposit spread by substituting ϕ given by (15) into (11) to obtain:

$$\hat{s}_d = s_b + \frac{\theta [U(s_b) - U(i)] - (1 - \theta) \kappa \tau}{d}. \quad (17)$$

As the market becomes more concentrated, i.e., τ decreases, the deposit spread increases. If we use, from the free entry condition, that $\kappa = [\alpha(\tau)/\tau] \phi/(\rho + \delta)$, we can obtain an alternative expression of the deposit spread that does not depend directly on θ and i , i.e.,

$$\hat{s}_d = s_b + (\rho + \delta) \frac{\kappa \tau}{\alpha(\tau) d}. \quad (18)$$

According to this formulation, \hat{s}_d and τ are positively correlated. Since, from (16), τ is an increasing function of i , it follows immediately that the deposit spread increases with the policy rate.

In the left panel of Figure 5 we represent the joint determination of market tightness and deposit spread. The bank entry curve represents (13) where $\Pi = (\hat{s}_d - s_b)d/(\rho + \delta)$. The curve is upward sloping because as the deposit spread increases, deposit contracts become more profitable, and more bankers enter. The deposit spread curve represents (17). It is downward sloping because as market tightness increases, consumers' outside options improve, which drives the deposit spread down. There is a unique intersection of the two curves. An increase in the policy rate shifts the deposit spread curve upward, which leads to both a higher spread and higher market tightness.

In the right panel of Figure 5, we represent the relationship between aggregate deposits and market tightness. Aggregate deposits are equal to $D = n^b d$ where n^b denotes the steady-state measure of banked consumers and $n^u = 1 - n^b$.²¹ At a steady state, the flow of consumers who acquire a demand deposit contract must be equal to the flow of banked consumers who exit the market, i.e., $\alpha(\tau)n^u = \delta n^b$. Solving for n^b we obtain:

$$n^b = \frac{\alpha(\tau)}{\delta + \alpha(\tau)}. \quad (19)$$

²¹Our model generates a measure of unbanked consumers due to search frictions in the deposit market. This feature of the model, however, is not critical for our main results. Alternatively, we could assume that every newborn consumer receives one bank offer instantly. Subsequent offers arrive at Poisson rate α . The consumer's outside options and the bargaining problem between the consumer and the bank would be unchanged.

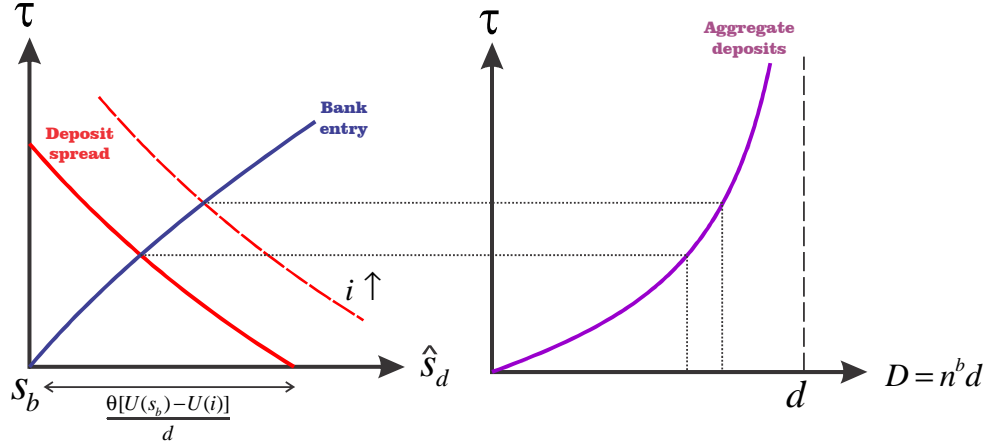


Figure 5: Equilibrium with entry: joint determination of deposit spread and market concentration.

Proposition 1 (Banking equilibrium.) Assume there is free entry of bankers in the deposit market. If $\pi + r_b > 0$ and $\theta > 0$, then there exists a unique equilibrium with $\tau > 0$. In any active equilibrium, $i_d < \pi + r_b$.

1. **Monetary policy and deposit spread.** The deposit pass through is given by

$$\frac{\partial \hat{s}_d}{\partial i} = \theta \frac{(\rho + \delta) [1 - \eta(\tau)]}{(\rho + \delta) [1 - \eta(\tau)] + (1 - \theta) \alpha(\tau)} \frac{u'^{-1} \left(1 + \frac{i}{\sigma}\right)}{u'^{-1} \left(1 + \frac{s_b}{\sigma}\right)} > 0. \quad (20)$$

Moreover, $\partial \hat{s}_d / \partial \theta > 0$ and $\partial \hat{s}_d / \partial \kappa > 0$.

2. **Deposit spreads and market concentration.** Suppose deposit markets differ in bank entry costs. Then there is a positive correlation between deposit spreads, \hat{s}_d , and market concentration, $1/\tau$. Moreover, if $\alpha(\tau) = \alpha_0 \tau^\eta$ with $\eta \in (0, 1)$, then the deposit spread passthrough, $\partial \hat{s}_d / \partial i$, is higher in a more concentrated market.

3. **Transmission to deposits.** If $i > s_b$, then individual deposits (d) are independent of i but aggregate deposits ($n^b d$) increase with i .

4. **Frictionless limit of the deposit market.** As κ goes to 0, τ approaches $+\infty$, n_b goes to 1, \hat{s}_d tends to s_b , and the spread passthrough, $\partial \hat{s}_d / \partial i$, goes to 0.

Proposition 1 allows us to compare the predictions of our model to the evidence on the deposits channel reviewed in Section 1.1. First, our model generates a positive passthrough from the policy rate to the deposit spread whenever $\theta > 0$. From (20) the size of the passthrough depends on market structure (e.g., matching technology and bargaining powers), and policy. These predictions are consistent with observation 1 in Table 1. Second, a change in i has no effect on d . Individual deposits are at their efficient level, which only depends on s_b . Aggregate deposits, however, increase as i increases because banks have incentives to spend additional resources to attract depositors whose outside option worsens. This prediction contradicts

observation 2a. Third, our model predicts that if two markets differ by their entry costs, then the market with the highest entry costs will have a higher concentration of bankers ($1/\tau$) and a larger deposit spread. This prediction is consistent with observation 3. In summary, our complete information model explains the deposit spread passthrough and its relation to monetary policy and market power, but it fails to account for the transmission to deposits.

The last part of Proposition 1 considers the frictionless limit where κ goes to 0. (Alternatively, we could take the limit as the efficiency of the matching process goes to infinity.) As the entry cost in the deposit market goes to zero, tightness in the deposit market goes to infinity and all consumers are banked. The rate of return on deposits approaches the rate of return on banks' assets and the passthrough from the policy rate to the deposit spread goes to zero.

Our model's prediction that the average deposit spread increases with i is robust to the introduction of different categories of bank deposits as shown in Appendix E. We show that the most liquid deposits decrease as i increases while the less liquid deposits increase. However, total deposits per banked consumer are unaffected. At the extensive margin, the measure of banked consumers increases.

4 Deposit contracts under private information

The result in Section 3 according to which individual deposits are unaffected by policy while aggregate deposits rise as i increases is at odds with the deposits channel documented in Drechsler et al. (2017). This result hinges on the assumption that banks have complete information about consumers' preferences so that deposit contracts are Pareto-efficient. In the following, we introduce consumer heterogeneity and relax the complete information assumption.

4.1 Second-degree price discrimination in the deposits market

We now assume that consumers are heterogeneous in terms of their liquidity needs: some consumers spend a lot, and hence wish to hold large quantities of liquid assets, while other consumers spend little. We formalize this idea by rewriting the utility of the consumer as $\varepsilon u(y)$ where $\varepsilon \in \mathbb{R}_+$ is a permanent idiosyncratic component. The cumulative distribution of ε across consumers is $\Upsilon(\varepsilon)$ with density $\gamma(\varepsilon)$ and support $[0, \bar{\varepsilon}]$. We assume that $1 - \Upsilon(\varepsilon)$ is log-concave, which implies that $\gamma(\varepsilon)/[1 - \Upsilon(\varepsilon)]$ is increasing with ε .²² Importantly, the preferences of a consumer are private information.

We follow Maskin and Riley (1984) and assume that bankers make take-it-or-leave-it offers to consumers, $\theta = 1$. A banker offers a menu of contracts (or a direct revelation mechanism), $\{[\phi(\varepsilon), d(\varepsilon)]\}$, where each contract specifies banking fees and deposit size as a function of ε . Upon meeting a banker, the consumer selects the contract in the menu corresponding to her type. We maintain the assumption of free entry of bankers so that the expected value of an unmatched banker is zero.

²²This mild assumption is slightly more restrictive than the standard mechanism design assumption that $\varepsilon - [1 - \Upsilon(\varepsilon)]/\gamma(\varepsilon)$ rises in ε .

Consumers' participation constraints. The value functions of the consumers solve the following two HJB equations:

$$(\rho + \delta) V^u(\varepsilon) = U(\varepsilon; i) + \alpha(\tau) [V^b(\varepsilon) - V^u(\varepsilon)] \quad (21)$$

$$(\rho + \delta) V^b(\varepsilon) = \max_{m \geq 0} \{-\phi(\varepsilon) - im - s_b d(\varepsilon) + \sigma \{\varepsilon u[d(\varepsilon) + m] - d(\varepsilon) - m\}\}, \quad (22)$$

where

$$U(\varepsilon; i) \equiv \max_{m \geq 0} \{-im + \sigma [\varepsilon u(m) - m]\}. \quad (23)$$

In (22), assuming the menu of contracts is incentive compatible, the consumer of type ε selects the contract, $[\phi(\varepsilon), d(\varepsilon)]$, in the menu offered by the banker. In (22), the banked consumer can supplement her bank deposits with cash, m . We denote $m(\varepsilon)$ the maximizer of the right side of (22).

Lemma 2 (*Participation to the deposit mechanism.*) *In any symmetric equilibrium, the consumers' participation constraints, $V^b(\varepsilon) \geq V^u(\varepsilon)$, hold if and only if*

$$\max_{m \geq 0} \{-\phi(\varepsilon) - im - s_b d(\varepsilon) + \sigma \{\varepsilon u[d(\varepsilon) + m] - d(\varepsilon) - m\}\} \geq U(\varepsilon; i) \quad \text{for all } \varepsilon \in [0, \bar{\varepsilon}]. \quad (24)$$

The constraint, (24), specifies that the flow expected utility of the consumer must be greater than the flow utility from being unbanked.²³ Note that this participation constraint omits the value from searching, $\alpha(V^b - V^u)$, as this term is zero when the constraint binds and is irrelevant otherwise.

The banker's problem The problem of the banker is written as:

$$\Phi = \max_{\{\phi(\varepsilon), d(\varepsilon)\}} \int \phi(\varepsilon) d\Upsilon(\varepsilon) \quad (25)$$

subject to the consumers' individual rationality constraints, (24), and the incentive-compatibility constraints,

$$[\varepsilon, m(\varepsilon)] \in \arg \max_{\varepsilon', m \geq 0} \{-\phi(\varepsilon') - im - s_b d(\varepsilon') + \sigma \{\varepsilon u[d(\varepsilon') + m] - d(\varepsilon') - m\}\} \quad \forall \varepsilon \in [0, \bar{\varepsilon}]. \quad (26)$$

The objective of the banker in (25) is the expected value of the banker's fees. The incentive-compatibility constraint, (26), requires that a type- ε consumer weakly prefers $[\phi(\varepsilon), d(\varepsilon)]$ to any other contract in the menu offered by the banker taking into account that she can supplement bank deposits with real money balances. Finally, we impose $\phi(\varepsilon) \geq 0$ so that bankers have no incentive to renege on their contracts ex post.

The following Lemma shows that consumers hold no money under any optimal deposit contract.

Lemma 3 (*Optimal money holdings of banked consumers.*) *Any optimal menu of deposits contracts must be such that $m(\varepsilon) = 0$ for all ε , i.e.,*

$$\sigma \varepsilon u'[d(\varepsilon)] \leq i + \sigma \quad \text{for all } \varepsilon \in [0, \bar{\varepsilon}]. \quad (27)$$

²³Formally, we are solving for the optimal menu of contracts offered by an uninformed principal to an informed agent when the agent's reservation utility is endogenous and depends on his type. See Jullien (2000) for a treatment of the general problem where the reservation utility is exogenous.

The logic of Lemma 3 is as follows. Suppose there are consumers who accumulate cash in addition to their deposits. A profitable deviation for the banker consists in raising the deposit size it offers without paying any additional interest to the consumer. The consumer is indifferent while banks' profits increase. In the following, we ignore the constraint (27) but show it holds once we impose (24).

Solution to the banker's problem We solve the banker's problem by transforming it into an optimal control problem where the state variable is the consumer's flow utility,

$$\nu(\varepsilon) \equiv \max_{\varepsilon' \in [0, \bar{\varepsilon}]} \{-\phi(\varepsilon') - (s_b + \sigma)d(\varepsilon') + \sigma\varepsilon u[d(\varepsilon')]\}, \text{ for all } \varepsilon \in [0, \bar{\varepsilon}], \quad (28)$$

and the control variable is the deposit size, $d(\varepsilon)$. In the objective (25), we replace $\phi(\varepsilon)$ with its expression coming from (28), i.e.,

$$\phi(\varepsilon) = -\nu(\varepsilon) - (s_b + \sigma)d(\varepsilon) + \sigma\varepsilon u[d(\varepsilon)].$$

A menu of contracts can then be reexpressed as a pair of functions, $[\nu(\varepsilon), d(\varepsilon)]$. Applying the Envelope Theorem to (26),

$$\nu'(\varepsilon) = \sigma u[d(\varepsilon)] \text{ for all } \varepsilon \in [0, \bar{\varepsilon}]. \quad (29)$$

The solution to this optimal control problem is given by the following proposition.

Proposition 2 (Optimal banking contract under private information.) Assume $1 - \Upsilon(\varepsilon)$ is log-concave. The solution to the banker's problem, (25) subject to the incentive constraints (24) and (26), is given by:

$$u'[d(\varepsilon)] = \frac{i + \sigma}{\varepsilon\sigma} \text{ for all } \varepsilon < \tilde{\varepsilon} \quad (30)$$

$$u'[d(\varepsilon)] = \left\{ \varepsilon - \left[\frac{1 - \Upsilon(\varepsilon)}{\gamma(\varepsilon)} \right] \right\}^{-1} \left(1 + \frac{s_b}{\sigma} \right) \text{ for all } \varepsilon \geq \tilde{\varepsilon} \quad (31)$$

and

$$\phi(\varepsilon) = (i - s_b)d(\varepsilon) \text{ for all } \varepsilon \in (0, \tilde{\varepsilon}) \quad (32)$$

$$\phi(\varepsilon) = -U(\tilde{\varepsilon}; i) - \sigma \int_{\tilde{\varepsilon}}^{\varepsilon} u[d(x)] dx - (s_b + \sigma)d(\varepsilon) + \sigma\varepsilon u[d(\varepsilon)] \text{ for all } \varepsilon \geq \tilde{\varepsilon}, \quad (33)$$

where $\tilde{\varepsilon} \in (0, \bar{\varepsilon})$ is the unique solution to

$$\frac{1 - \Upsilon(\tilde{\varepsilon})}{\gamma(\tilde{\varepsilon})} = \left(\frac{i - s_b}{i + \sigma} \right) \tilde{\varepsilon}. \quad (34)$$

The optimal menu of contracts has a two-tier structure. Consumers with low spending needs, $\varepsilon \leq \tilde{\varepsilon}$, deposit the real balances they hold when they are unbanked. Bankers do not pay interest on such deposits (see below) and hence consumers are just indifferent between being affiliated to a bank or not. Bankers, however, enjoy strictly positive profits since they invest deposits into interest-bearing assets.

Consumers with high spending needs, $\varepsilon > \tilde{\varepsilon}$, are offered a positive interest on their deposits. Hence, they deposit more than the cash they hold when unbanked. However, they hold less deposits than under complete information except for the highest type, $\varepsilon = \bar{\varepsilon}$.

We illustrate the optimal schedule of deposits in the left panel of Figure 6. The red dashed curve, $d^{ci}(\varepsilon)$, is the complete-information deposit schedule given by $\varepsilon u'[d(\varepsilon)] = 1 + s_b/\sigma$. The blue dashed curve, $d^m(\varepsilon)$, is the schedule for real balances of unbanked consumers given by $\varepsilon u'[d(\varepsilon)] = 1 + i/\sigma$. The private-information schedule, denoted $d^{pi}(\varepsilon)$ and represented by a plain purple curve, is located in-between these two curves. It coincides with d^m for low ε and it reaches d^{ci} at $\varepsilon = \bar{\varepsilon}$. An increase i shifts d^m downward (light blue dashed curve), and hence it shifts deposits downward for low ε but it does not affect deposits for large ε (light purple dashed curve). The deposit rate is $i_d(\varepsilon) = i_b - \phi(\varepsilon)/d(\varepsilon)$. As shown in the right panel of Figure 6, it is zero for all $\varepsilon < \tilde{\varepsilon}$ and it rises in ε for $\varepsilon \geq \tilde{\varepsilon}$ by Lemma 5 in Appendix A.

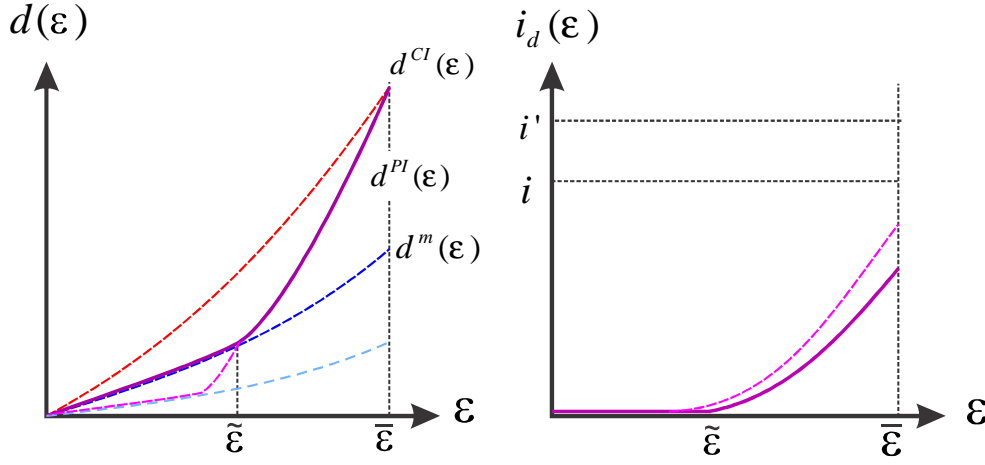


Figure 6: Banking contracts under private information

Free entry and definition of an equilibrium We close the model with the free-entry condition for bankers. We substitute $\Pi = \Phi/(\rho + \delta)$ into (13) to obtain:

$$\kappa = \frac{\alpha(\tau)}{\tau} \frac{\Phi}{\rho + \delta}. \quad (35)$$

An equilibrium is then a list, $\langle \phi(\varepsilon), d(\varepsilon), \Phi, \tau \rangle$, that solves (25), (30)-(31), (32)-(33), and (35). The equilibrium has a recursive structure. The menu of contracts is determined independently of tightness in the banking sector. Given the profits generated by the contract, (35) determines market tightness uniquely.

4.2 The bank deposits channel

The supply of deposits and the deposit spread are defined as follows:

$$D(i) \equiv \int_0^{\bar{\varepsilon}} d(\varepsilon) d\Upsilon(\varepsilon) \quad (36)$$

$$\hat{s}_d \equiv \int_0^{\bar{\varepsilon}} \hat{s}_d(\varepsilon) \frac{d(\varepsilon)}{D} \gamma(\varepsilon) d\varepsilon, \quad (37)$$

where D is the average deposit per banked consumer and \hat{s}_d averages the spreads of all individual contracts, $\hat{s}_d(\varepsilon) = s_b + \phi(\varepsilon)/d(\varepsilon)$.

Proposition 3 (*The deposits channel of monetary policy.*) *An increase in i leads to:*

1. *A decrease in individual deposits, $d(\varepsilon)$, for all $\varepsilon < \bar{\varepsilon}$, and a decreases in the average deposit per banked consumer, D , with*

$$D'(i) = \int_0^{\bar{\varepsilon}(i)} \frac{d\Upsilon(\varepsilon)}{\sigma \varepsilon u''[d(\varepsilon)]} < 0. \quad (38)$$

2. *An increase in the measure of banked consumers, $\partial n^b / \partial i > 0$;*

3. *A decrease in aggregate deposits, $n^b D$, if δ is small.*

4. *An increase in individual deposit spreads equal to:*

$$\frac{\partial \hat{s}_d(\varepsilon)}{\partial i} = 1 - \left(\frac{d(\varepsilon) - d(\bar{\varepsilon})}{d(\varepsilon)} \right) \mathbb{I}_{\{\varepsilon > \bar{\varepsilon}\}} \in (0, 1] \quad (39)$$

An increase in the average deposit spread, $\partial \hat{s}_d / \partial i > 0$.

Our model explains the following observations from Section 1.1. First, there is a positive passthrough from i to the deposit spread, $\hat{s}_d(\varepsilon) = i - \hat{i}_d(\varepsilon)$. The size of the passthrough, however, is not uniform across all deposits. For low ε , the deposit spread increases one-to-one with i , i.e., deposit rates stay at zero despite the increase in i . For high ε , the passthrough is less than one.

Second, an increase in i leads to a reduction in consumers' deposits. As i increases, the banker maximizes its profits by keeping the participation constraints of low- ε consumers binding, which requires reducing their deposit offers. The fraction of consumers for which the participation constraint binds shrinks, i.e., $\Upsilon(\bar{\varepsilon})$ decreases, as i increases. The right panel of Figure 7 illustrates the transmission of i to $d(\varepsilon)$ for different values for ε . If $i < s_b$, then there is no role for bank deposits and $d(\varepsilon) = 0$ for all ε . As i reaches s_b , all consumers matched with a bank deposit their money holdings, $m(s_b)$. As i increases above s_b , deposits start decreasing except for consumers in the neighborhood of $\bar{\varepsilon}$. If ε is not too low, above some value $\bar{\varepsilon}_\infty$, then $d(\varepsilon)$ remains constant once i passes a threshold. Otherwise, if $\varepsilon < \bar{\varepsilon}_\infty$, $d(\varepsilon)$ keeps decreasing. In the left panel, we plot $\bar{\varepsilon}$ as a function of i . This threshold is equal to $\bar{\varepsilon}$ when $i = s_b$, it decreases as i increases, and it approaches a lower bound, $\bar{\varepsilon}_\infty$, as i goes to $+\infty$.²⁴

²⁴The threshold, $\bar{\varepsilon}_\infty > 0$, solves $[1 - \Upsilon(\bar{\varepsilon}_\infty)] / \gamma(\bar{\varepsilon}_\infty) = \bar{\varepsilon}_\infty$. As i goes to $+\infty$, $d(\varepsilon)$ goes to zero for all $\varepsilon \leq \bar{\varepsilon}_\infty$. At the limit as cash becomes prohibitively costly to hold, unbanked consumers hold no cash and banks choose not to serve consumers below $\bar{\varepsilon}_\infty > 0$.

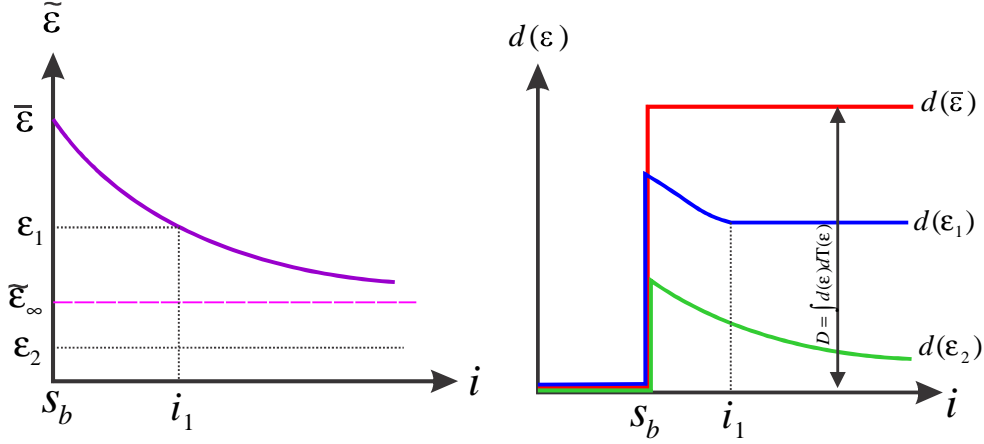


Figure 7: Left panel: threshold $\tilde{\varepsilon}$ as a function of i . Right panel: deposits $d(\varepsilon)$ as a function of i

The transmission of monetary policy to bank deposits has both an intensive and an extensive margin. On the intensive margin, an increase in i reduces $d(\varepsilon)$. On the extensive margin, an increase in i raises the measure of deposits contracts offered by bankers, i.e., τ increases, and the measure of banked consumers increases. If δ is sufficiently small so that most consumers are banked, a change in the measure of banked consumers induced by a change in i is small so that the effect on the intensive margin dominates and aggregate deposits fall with i . Formally, from (19), the effect of a change in the deposit creation rate, α , on the measure of banked consumers is $\partial n^b / \partial \alpha = (1 - n^b) / (\delta + \alpha)$, which tends to 0 as n^b approaches 1.

In summary, the main insight from Proposition 3 is that private information about consumers' liquidity needs subject banks to incentive-compatibility constraints, i.e., banks resort to the second-degree price discrimination. In order to satisfy these constraints, the optimal contract distorts individual deposit levels and generates binding participation constraints for consumers with low liquidity needs. It is precisely through those binding participation constraints that the deposits channel operates. Hence, a testable implication of our model is that the deposit outflow following an increase in the policy rate is not distributed evenly across all deposits but is concentrated on deposits at the bottom of the distribution.

4.3 Multiple bank deposit categories

We now assume that banks offer two types of deposits. Liquid (type-1) deposits, denoted d^1 , are invested in non-interest-bearing assets like cash or reserves and can be liquidated on demand. Hence, $s_b^1 = i$. Less-liquid (type-2) deposits, denoted d^2 , are invested at rate r_b^2 , with $0 < s_b^2 \equiv \rho - r_b^2 < i$, and can be liquidated when demanded with probability $\chi_2 < 1$. An alternative interpretation is that banks offer a single type of deposits (d^2) that are imperfectly liquid while consumers can hold both cash (d^1) and deposits (d^2).

Proposition 4 (Imperfectly liquid bank deposits.) *Suppose there are two types of deposits. Type-1 deposits are perfectly liquid but do not pay interest, $s_b^1 = i$. Type-2 deposits are such that $s_b^2 < i$ but are*

imperfectly liquid, $\chi_2 < 1$.

1. At $i = s_b^2/\chi_2$, $d^1(\varepsilon) = u'^{-1}[(i + \sigma)/(\varepsilon\sigma)]$ and $d^2(\varepsilon) = 0$ for all $\varepsilon \in [0, \bar{\varepsilon}]$.
2. As i increases from s_b^2/χ_2 to $+\infty$, $d^1(\varepsilon)$ decreases toward 0 and $\ell(\varepsilon) = d^1(\varepsilon) + d^2(\varepsilon)$ converges toward

$$\tilde{\ell}(\varepsilon) \equiv \left[\frac{\max\{\varepsilon - [1 - \Upsilon(\varepsilon)]/\gamma(\varepsilon), 0\} \sigma \chi_2}{s_b^2 + \sigma \chi_2} \right]^{\frac{1}{\alpha}}.$$

As i rises above s_b^2/χ_2 , consumers reduce their liquid deposits and their overall deposits. However, they increase their holdings of less-liquid deposits if i is not too large. This result is consistent with the observation 2b according to which the growth rate of less-liquid deposits is positively correlated with the change in the policy rate. We illustrate these findings in Figure 8. In the left panel, the deposit spread increases in i . In the right panel, aggregate deposits, $D \equiv \int_0^{\bar{\varepsilon}} d^2(\varepsilon) d\Upsilon(\varepsilon)$, can be non-monotone in i , which shows that the deposits channel can operate differently at different levels of the interest rate. This result shows the importance of formalizing liquidity differences across deposits in order to capture the nuances of the transmission mechanism.

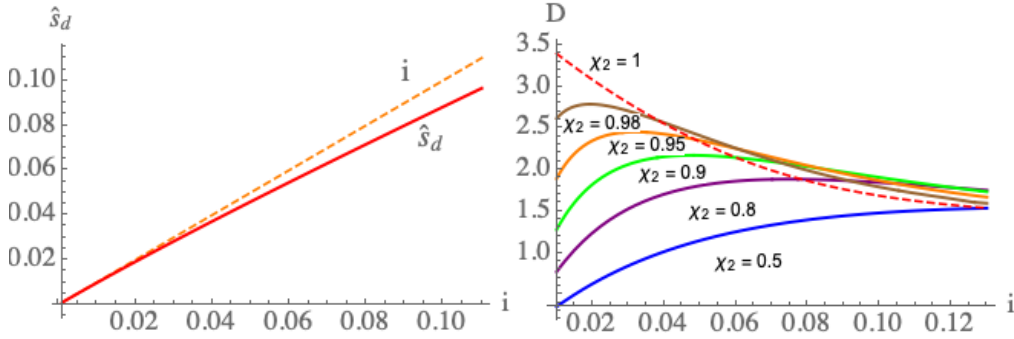


Figure 8: Outcome from second-degree price discrimination with $i = s_d^1$: $u(y) = y^{0.8}/0.8$, $\rho = 0.05$, $\sigma = 0.5$, $s_b^2 = 0$, $\varepsilon \sim \text{Exp}(2)$. In the left panel $\chi_2 = 0.8$.

5 Banks' market power and concentration

Our model with private information about consumers' liquidity needs of Section 4 explains the spread passthrough and the relationship between aggregate deposits and the policy rate. However, from Proposition 2, the terms of the deposit contracts are independent of α and the tightness in the deposit market. Hence, the model is not able to account for the relation between the strength of the transmission mechanism and market concentration. In order to explain this relationship, we now combine the approaches of Sections 3 and 4 by formalizing a one-sided-incomplete-information bargaining game with offers and counteroffers that gives bargaining powers to both banks and consumers.²⁵

²⁵In Appendix E we generalize the model with multiple deposit categories in Section 4.3 by assuming banks and consumers trade under the same bargaining game.

The extensive-form bargaining game is as follows. The bank makes an offer to the consumer that takes the form of a menu of contracts.²⁶ The consumer can either select a contract in the menu or reject the offer altogether. If the offer is rejected, the consumer has the possibility to make a take-it-or-leave-it counteroffer with probability $1 - \theta$. The game tree is represented in Figure 9. All initial moves by Nature, $\varepsilon \in [0, \bar{\varepsilon}]$, are part of the same information set of the bank since ε is consumer's private information. The concept of equilibrium is perfect Bayesian, i.e. strategies are sequentially rational and beliefs are updated according to Bayes' rule whenever possible. There is no belief updating unless the consumer rejects the bank's offer to make a counteroffer, but in this case the bank's belief about the consumer type is irrelevant since ε does not affect the bank's payoff directly. Moreover, under complete information, this game generates the same outcome as the generalized Nash solution used in Section 3.

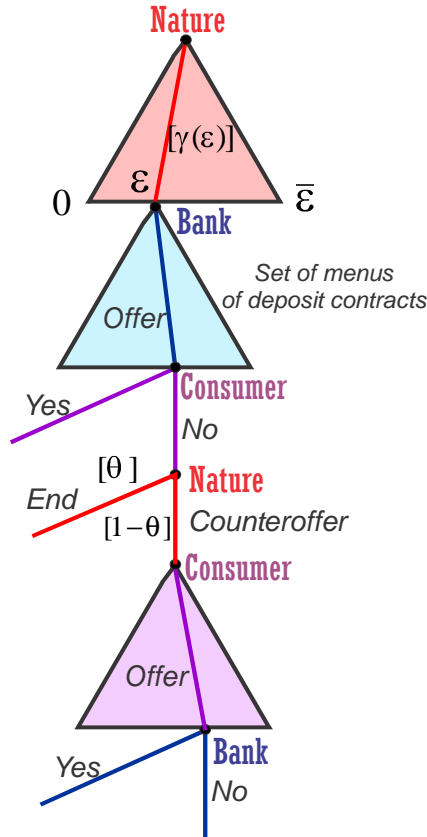


Figure 9: Tree of bargaining game under one-sided incomplete information

The value functions of unbanked and banked consumers, $V^u(\varepsilon)$ and $V^b(\varepsilon)$, respectively, solve (21) and (22). Off the equilibrium path, if the buyer rejects the bank's offer and has the opportunity to make a

²⁶In Appendix D we restrict the contract space by imposing that banks offer a single deposit rate to all consumers irrespective of the amount they deposit. In the context of our calibrated example where participation constraints bind for the vast majority of consumers, the two approaches have similar implications.

take-it-or-leave-it offer to the bank, with probability $1 - \theta$, then her expected utility is $\hat{V}^b(\varepsilon)$ solution to

$$(\rho + \delta) \hat{V}^b(\varepsilon) = U(\varepsilon; s_b) \quad (40)$$

and the bank earns no profit. According to (40), the consumer enjoys the payoff from investing directly in banks' assets at the spread s_b . The consumer accepts the bank's offer if

$$V^b(\varepsilon) \geq \theta V^u(\varepsilon) + (1 - \theta) \hat{V}^b(\varepsilon). \quad (41)$$

The value of accepting the offer, on the left side of (41), is larger than the expected value of rejecting it, on the right side of (41). If the bank's offer is rejected, then the consumer is either unbanked, with probability θ , or he has the opportunity to make a take-it-or-leave-it offer to the bank with probability $1 - \theta$. In the spirit of Lemma 2, in any symmetric equilibrium, we can rewrite (41) as a simpler condition.

Lemma 4 (*Incentive to accept banks' offers.*) *In any symmetric equilibrium where $V^b(\varepsilon)$ and $V^u(\varepsilon)$ solve (21)-(22), the consumers' participation constraints, (77), hold if and only if*

$$\nu(\varepsilon) \geq \underline{\nu}(\varepsilon) \equiv \frac{(1 - \theta)(\rho + \delta + \alpha)U(\varepsilon; s_b) + \theta(\rho + \delta)U(\varepsilon; i)}{(1 - \theta)(\rho + \delta + \alpha) + \theta(\rho + \delta)}. \quad (42)$$

The reservation value $\underline{\nu}(\varepsilon)$ is a weighted average of $U(\varepsilon; s_b)$ and $U(\varepsilon; i)$ where the weights depend on banks' bargaining power, θ , and search frictions, α .²⁷ By Lemma 4, the bank's optimal control problem is:

$$\Phi \equiv \max_{\{\nu(\varepsilon), d(\varepsilon)\}} \int_0^{\bar{\varepsilon}} \{-\nu(\varepsilon) - (s_b + \sigma)d(\varepsilon) + \sigma \varepsilon u[d(\varepsilon)]\} d\Upsilon(\varepsilon) \quad (43)$$

$$\text{s.t. } \nu'(\varepsilon) = \sigma u[d(\varepsilon)] \quad (44)$$

$$\nu(\varepsilon) \geq \underline{\nu}(\varepsilon). \quad (45)$$

The only difference relative to the screening model in Section 4 is the participation constraint, (45), that takes into account the possibility for the consumer to make a counter-offer to the bank.

Proposition 5 (*Optimal banking contract under private information.*) *Assume $1 - \Upsilon(\varepsilon)$ is log-concave and $u(y) = y^{1-a}/(1-a)$ with $a \in (0, 1)$. The solution to the banker's problem, (43)-(45), is given by:*

$$d(\varepsilon) = \left[\frac{\varepsilon \sigma}{\underline{i}(i, s_b, \theta, \alpha) + \sigma} \right]^{1/a} \text{ for } \varepsilon < \tilde{\varepsilon} \quad (46)$$

$$d(\varepsilon) = \left\{ \varepsilon - \left[\frac{1 - \Upsilon(\varepsilon)}{\gamma(\varepsilon)} \right] \right\}^{1/a} \left(1 + \frac{s_b}{\sigma} \right)^{-1/a} \text{ for } \varepsilon \geq \tilde{\varepsilon} \quad (47)$$

where

$$\underline{i}(i, s_b, \theta, \alpha) \equiv \sigma \left\{ \left[\frac{(1 - \theta)(\rho + \delta + \alpha) \left(\frac{\sigma}{s_b + \sigma} \right)^{\frac{(1-a)}{a}} + \theta(\rho + \delta) \left(\frac{\sigma}{i + \sigma} \right)^{\frac{(1-a)}{a}}}{(1 - \theta)(\rho + \delta + \alpha) + \theta(\rho + \delta)} \right]^{-\frac{a}{1-a}} - 1 \right\} \quad (48)$$

²⁷Suppose we consider the same bargaining game under complete information. The bank chooses an offer that makes the consumer indifferent between accepting or rejecting. For all ε , $\nu(\varepsilon) = \underline{\nu}(\varepsilon)$ where $\nu(\varepsilon) = U(\varepsilon; s_b) - \phi^{ci}(\varepsilon)$. Substituting this expression into the left side of (42) and solving for the banking fee we obtain the expression (9) when terms of trade are determined according to the generalized Nash solution. This result confirms that the only change we made is in terms of the information structure.

and $\tilde{\varepsilon} \in (0, \bar{\varepsilon})$ is the unique solution to

$$\frac{1 - \Upsilon(\tilde{\varepsilon})}{\gamma(\tilde{\varepsilon})} = \tilde{\varepsilon} \left(\frac{\underline{i} - s}{\underline{i} + \sigma} \right). \quad (49)$$

The bank's flow profits are

$$\phi(\varepsilon) = d(\varepsilon) (\underline{i} - s_b) \quad \text{for } \varepsilon \in (0, \tilde{\varepsilon}) \quad (50)$$

$$\phi(\varepsilon) = -\underline{\nu}(\tilde{\varepsilon}) - \sigma \int_{\tilde{\varepsilon}}^{\varepsilon} u[d(x)] dx - (s_b + \sigma)d(\varepsilon) + \sigma \varepsilon u[d(\varepsilon)] \quad \text{for } \varepsilon \geq \tilde{\varepsilon}. \quad (51)$$

The menu of contracts offered by the bank is qualitatively similar to the one of the pure screening model described in Proposition 2, but with i replaced by the endogenous interest rate \underline{i} .²⁸ The contract is divided into two tiers. Above a threshold, $\tilde{\varepsilon}$, the consumer participation constraint, (45), is slack whereas below $\tilde{\varepsilon}$ it is binding. There are important differences. First, the terms of the deposit contracts depend on both consumers' ability to search for alternative banks, α , and banks' bargaining power, θ . In particular, the deposits of consumers with type $\varepsilon < \tilde{\varepsilon}$ given by (46) are in-between the real balances of unbanked consumers, $m(\varepsilon) = [\varepsilon\sigma/(\underline{i} + \sigma)]^{1/a}$, and the complete-information deposits, $d^{ci}(\varepsilon) = [\varepsilon\sigma/(s_b + \sigma)]^{1/a}$. Differentiating both sides of $\nu(\varepsilon) = \underline{\nu}(\varepsilon)$ with respect to ε where $\underline{\nu}(\varepsilon)$ is given by (42),

$$u[d(\varepsilon)] = \frac{(1 - \theta)(\rho + \delta + \alpha)}{(1 - \theta)\alpha + \rho + \delta} u[d^{ci}(\varepsilon)] + \frac{\theta(\rho + \delta)}{(1 - \theta)\alpha + \rho + \delta} u[m(\varepsilon)].$$

So the weight assigned to d^{ci} increases with α but decreases with θ . The second novelty is that the threshold, $\tilde{\varepsilon}$, is now a function of α . As α increases, $\tilde{\varepsilon}$ increases, which means that a larger fraction of the deposit contracts are determined by consumers' outside options.

We close the model with the free-entry condition for bankers which is given by (35). It is easy to check from (77) that an increase in i relaxes consumers' participation constraints, which allows banks to raise their expected revenue. As a result, τ increases with i . We now turn to the effects of monetary policy on equilibrium outcomes. In the following proposition, we adopt a linear matching function in the measure of consumers, i.e., $\alpha(\tau) \equiv \alpha$.

Proposition 6 (Monetary policy under incomplete information and two-sided bargaining powers.) Suppose $\theta > 0$ and $\alpha(\tau) \equiv \alpha$.

1. **Deposit spread passthrough and deposits channel.** For all ε , $\partial \hat{s}_d(\varepsilon)/\partial i > 0$. For all $\varepsilon < \tilde{\varepsilon}$, $\partial d(\varepsilon)/\partial i < 0$ and $\partial d(\varepsilon)/\partial i = 0$ otherwise. As i increases, $\tilde{\varepsilon}$ decreases.
2. **Bank market power and the transmission mechanism.** As α increases, $\partial \hat{s}_d(\varepsilon)/\partial i$ decreases (weakly) for all ε . If

$$a \leq 1 - \theta + \theta \left(\frac{s_b + \sigma}{\underline{i} + \sigma} \right)^{\frac{(1-a)}{a}}, \quad (52)$$

²⁸In Appendix D we restrict the contract space by assuming that banks can only offer a single deposit rate and the consumers choose the size of deposit. We show that if the bargaining power θ is sufficiently small, then banks' optimal choice is to offer a spread equals to \underline{i} . In this case, the contract under linear pricing is equivalent to the lower-tie of the contract under non-linear pricing.

then $|\partial d(\varepsilon)/\partial i|$ (weakly) decreases in α for all ε . Otherwise it is (weakly) hump-shaped as α increases from 0 to $+\infty$.

3. **Frictionless limit.** As $\alpha \rightarrow +\infty$, $\tilde{\varepsilon} \rightarrow \bar{\varepsilon}$, $d(\varepsilon) \rightarrow d^{ci}(\varepsilon)$, and $\phi(\varepsilon) \rightarrow 0$ for all $\varepsilon \in [0, \bar{\varepsilon}]$.

Our model generates the main observations of the deposits channel reviewed in Section 1.1. According to Part 1, there is a positive deposit spread, a positive spread passthrough and individual deposits for all consumers below $\tilde{\varepsilon}$ shrink as the policy rate increases. These results are consistent with observation 1 and 2 in Table 1. According to Part 2, as it becomes less frequent for consumers to receive bank offers, i.e., α decreases, the deposit spread passthrough increases and the strength of the transmission to deposits increases if (52) holds. These findings are consistent with observation 3 and 4 in Table 1. The last part considers the frictionless limit where consumers can generate bank offers almost instantly. The allocation and prices converge to that of a complete-information environment where consumers have all the bargaining power.²⁹

5.1 Calibrated example

We calibrate our model in order to quantify the market power of banks that is consistent with the observed strength of the deposits channel. The unit of time is a year and the rate of time preference is $\rho = 0.04$ as in Lagos and Wright (2005).³⁰ The utility function is $u(y) = y^{1-a}/(1-a)$ with $a \in (0, 1)$. The distribution of consumer types is given by an exponential distribution with mean 1.³¹ We interpret δ as the rate at which a consumer gets separated from her bank and set $\delta = 0.05$. This number is consistent with the J.D. Power 2019 U.S. Retail Banking Satisfaction Study according to which 4% of customers switched banks in the past year and a survey conducted by Bankrate reporting that the average U.S. adult has used the same primary checking account for about 16 years. We set the spread on banks' assets to $s_b = 0$. The average Federal fund rate in the sample is $i = 0.05$. The matching technology in the deposit markets is linear in the measure of consumers, so $\alpha(\tau) = \bar{\alpha}$, i.e., only banks generate congestion in the deposit market.

The remaining parameters to be calibrated are $(\theta, \bar{\alpha}, \sigma, a)$. We use the measure of unbanked households to calibrate $\bar{\alpha}$. According to the FDIC Survey of Household Use of Banking and Financial Services in 2019, 5.4% of U.S. households were unbanked, which implies $\bar{\alpha} = 0.88$.³² We choose θ to match the size of the deposit spread passthrough, $\partial \hat{s}_d / \partial i$. Drechsler et al. (2017) documents that a 100 bps increase in the policy rate leads to an average increase in the deposit spread by 54 bps, i.e., $\partial \hat{s}_d / \partial i = 0.54$. The pair, (σ, a) , targets the change of aggregate deposits with respect to i . Drechsler et al. (2017) report that a 100 bps increase

²⁹This result is consistent with the findings by Inderst (2001), who studied a matching model with private information where agents make take-it-leave-it offers and the proposer is chosen at random. He shows that if there is a finite set of consumer types and the matching frictions are sufficiently low, then the set of equilibria is independent of whether the consumers' type is private information or not. When there is a continuum of types, distortion vanish at the limit as matching frictions disappear.

³⁰In our continuous time model, the choice of the unit of time imposes a constraint on arrival rates of matching or consumption opportunities.

³¹Drăgulescu and Yakovenko (2001) argue that the distribution of individual income in the USA is well approximated by an exponential distribution.

³²Alternatively, we could choose $(\theta, \bar{\alpha})$ to target the spread and passthrough. But θ and $\bar{\alpha}$ cannot be separately identified because, by Proposition 5, $(\theta, \bar{\alpha})$ affect the banking contract only via $\underline{z}(i, s_b, \theta, \bar{\alpha})$ and, by (48), the relationship between \underline{z} and i remains unchanged as long as $(\rho + \delta + \bar{\alpha})(1 - \theta)/\theta$ is unchanged.

in the policy rate induces on average a 323 bps contraction in deposits. i.e., $(\partial D/\partial i)/D = -3.23$.³³ We normalize the data and the model such that the aggregate deposits $D = 1$ at $i = 0.05$.

We report the calibrated parameters in Table 2 and illustrate the model fit in Figure 10.

Parameter	Description	Targets	Value
ρ	Rate of time preference	Lagos-Wright (2005)	0.04
δ	Consumer's death rate	surveys of bank customers	0.05
s_b	Spread on banks' assets	abundant investment opportunities	0
$\bar{\alpha}$	Matching rate of consumers	fraction of unbanked households	0.88
θ	Bank's bargaining power	deposit spread passthrough	0.92
σ	Poisson rate of consumption shocks	semi-elasticity of deposits	0.5
a	Relative risk aversion	semi-elasticity of deposits	0.11

Table 2: Parameter values of the calibrated model

Our calibration results suggest that banks must have substantial market power to generate the transmission mechanism observed in the data. The bank's bargaining power is $\theta = 0.92$. The other two parameters, (σ, a) , are consistent with the calibration of money demands in New Monetarist models, e.g., Lagos and Wright (2005).³⁴

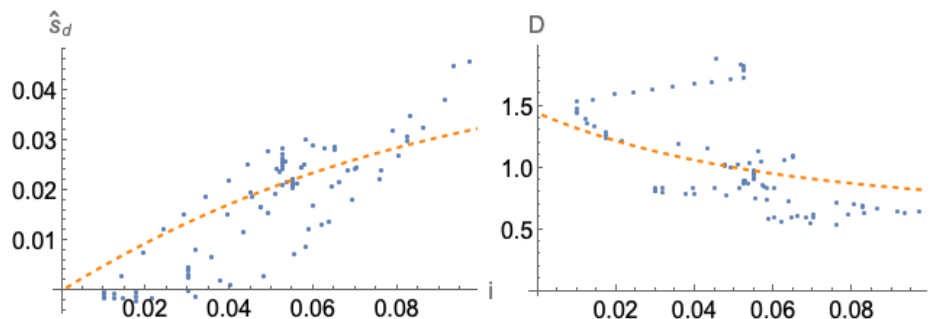


Figure 10: Fit of the calibrated model (Left) Average deposit spread (Right) Aggregate deposits

We illustrate the empirical observations from Section 1.1 through the lens of our model in Figure 11. In the top row, we plot the average deposit rate, $\hat{i}_d \equiv i - \int_0^{\bar{\varepsilon}} \phi(\varepsilon) d\Upsilon(\varepsilon)/D$, and average deposit spread, $\hat{s}_d \equiv s_b + \int_0^{\bar{\varepsilon}} \phi(\varepsilon) d\Upsilon(\varepsilon)/D$. Given the calibrated parameter values, the cutoff $\bar{\varepsilon}$ is large and most of the consumers fall into the lower tier of the contract. Therefore, \hat{i}_d is closely approximated by \underline{i} in (48). Both \hat{i}_d and \hat{s}_d increase in i , which reflects the incomplete passthrough from the policy to the deposit rate. As $\bar{\alpha}$ increases, banks' market power falls, and the curve representing the deposit rate shifts upward while the deposit spread shifts downward.

³³The data on core deposits and deposit rate are taken from Drechsler et al. (2017) and range from 1986 to 2007 at quarterly frequency. Core deposits are the sum of checking, savings, and small time deposits and amount to 9.3 trillion or 79% of bank liabilities in 2014. Figure 1 and 2 in Drechsler et al. (2017) use deposit and spread data from 1986 to 2013, we drop the data after 2008 because the spread became negative due to a financial crisis.

³⁴The curvature of the utility function, $a = 0.11$, is in the ballpark of the values that have been estimated in the literature (see, e.g., Craig and Rocheteau, 2008), and it is consistent with estimates from models with competitive goods markets, as in, e.g., Rocheteau and Wright (2009). The frequency of consumption opportunities, $\sigma = 0.5$, can be interpreted as the product of two parameters: the actual arrival rate of idiosyncratic consumption opportunities and a scaling parameter of the consumer surplus. So, consumption opportunities can arrive frequently, but the surplus they generate must be small.

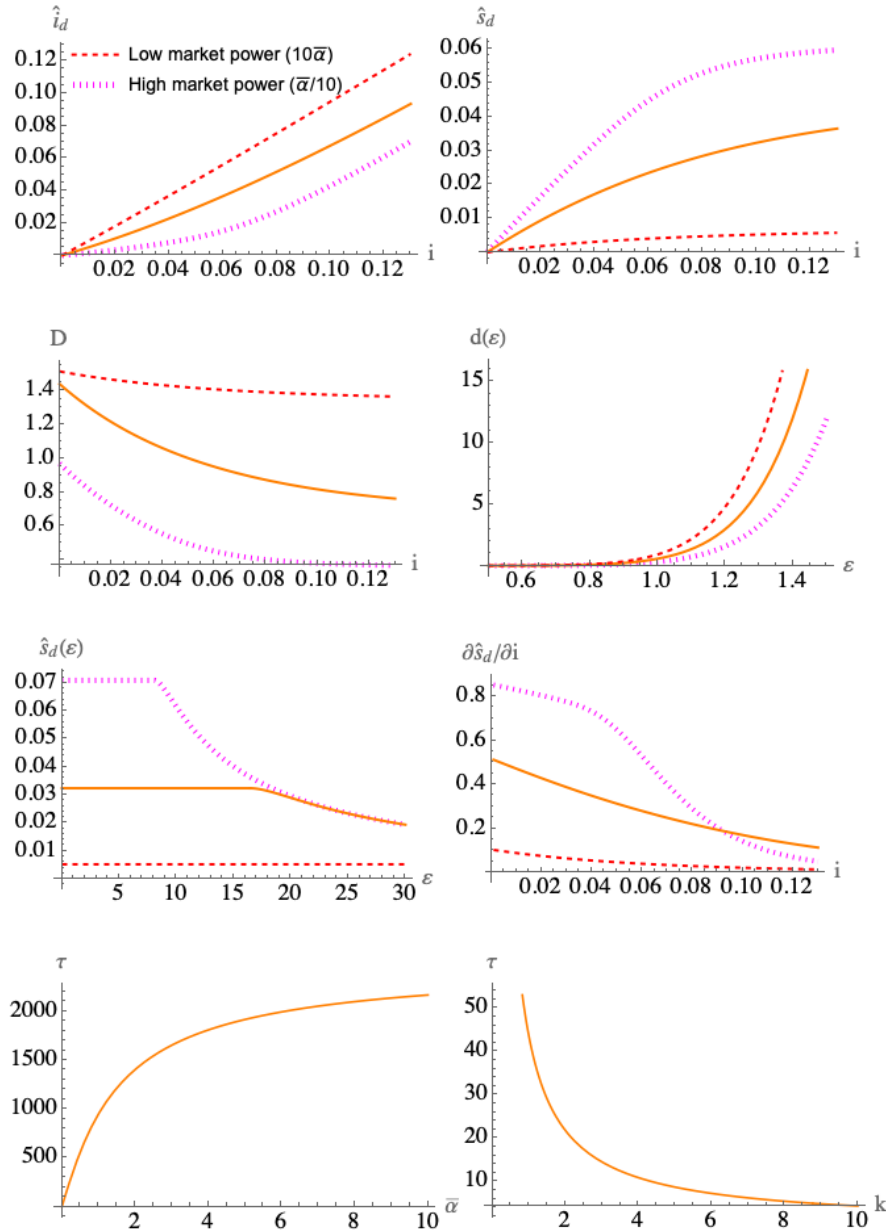


Figure 11: (Top-left) Average interest rate (Top-right) Average spread (Upper-middle-left) Aggregate deposits (Upper-middle-right) Deposits of various agent types (Lower-middle-left) Spread of various agent types (Lower-middle-right) Passthrough to the average spread. (Bottom) Measure of banks against matching efficiency (left) and entry cost (right). Orange lines represent the calibrated model. We increase the matching rate by 10 times in the red dashed lines and we reduce that by 10 times in the purple dotted lines.

In the second row, we plot average bank deposits, $D \equiv \int_0^{\bar{\varepsilon}} d(\varepsilon) d\Upsilon(\varepsilon)$, in the left panel and the deposit levels across consumer types, $d(\varepsilon)$, in the right panel. The relationship between D and i is negative but it flattens out as $\bar{\alpha}$ rises and the deposit market becomes more competitive. The effect on aggregate deposits can be sizable. For instance, suppose $i = 10\%$ and the deposit market becomes frictionless, $\bar{\alpha} \rightarrow +\infty$. The average deposits per consumer increase by about 50%. As shown by the right panel, this positive effect applies to all deposits irrespective of ε .

In the third row, we plot the deposit spread, $\hat{s}_d(\varepsilon)$, across consumer types in the left panel and the deposit spread passthrough, $\partial \hat{s}_d / \partial i$, as functions of the policy rate for different values of $\bar{\alpha}$ in the right panel. For $\varepsilon < \tilde{\varepsilon}$, $\hat{s}_d(\varepsilon)$ is constant, i.e., it is optimal for banks to use linear pricing. A higher $\bar{\alpha}$ reduces the deposit spread for all consumers. The size of the deposit spread passthrough falls in i and $\bar{\alpha}$, which illustrates its state dependence.

In the bottom row, we plot the market tightness, τ , against the matching efficiency, $\bar{\alpha}$, and the entry cost, k . As $\bar{\alpha}$ rises, banks have more incentive to enter the market because they can meet with consumers more quickly. But at the same time consumers' outside option improves and hence banks get a lower profit in each trade. In our calibrated example, the first effect dominates and the measure of banks rises in $\bar{\alpha}$, or equivalently the market concentration falls in $\bar{\alpha}$. As the entry cost k rises, banks have less incentive to participate and hence τ drops.

5.2 FinTech and the transmission mechanism of monetary policy

We now use our model to describe the impact of recent technological advances in banking and payments for the transmission mechanism of monetary policy. These advances include the development of online banking, the introduction of cryptocurrencies, and the ability of banks to collect extensive information about consumers' preferences and payment habits.³⁵

Online and mobile banking One aspect of the FinTech transformation is the development of online banking allowing consumers to open bank accounts online at lower costs and dispensing banks from a physical location. In the context of our model, these innovations lower the entry costs of banks and raise the efficiency of the matching process between banks and consumers, i.e., κ decreases and $\bar{\alpha}$ increases. As shown in Proposition 6 and Figure 11, as $\bar{\alpha}$ increases, both \hat{s}_d and $\partial \hat{s}_d / \partial i$ decrease. The average deposit per banked consumer increases and gets closer to its complete-information level but the transmission mechanism of monetary policy weakens, i.e., $|\partial D / \partial i|$ decreases. At the limit where $\bar{\alpha} \rightarrow +\infty$, deposit spreads are driven to zero and monetary policy loses its influence over the supply of deposits. If banks deposits are imperfectly liquid, i.e., can be liquidated on demand with probability $\chi_2 < 1$, then the equilibrium at the frictionless limit will be characterized by Proposition 12 in Appendix E, i.e., as i increases, d^2 increases and the deposits channel changes sign.

³⁵For a review of the FinTech revolution in the banking industry and its impact on competition, see OECD (2020).

Cryptomonies and digital payments Among the major recent FinTech advances, the blockchain technology has allowed the creation of crypto-monies outside of the traditional banking sector. The digital currencies that can have the most significant impact for the deposit market are those issued by the Central Bank (CBDC) as they can serve as means of payment and pay interest, thereby competing directly with bank deposits. In the context of our model, suppose there is a digital currency that pays interest i_{cbdc} and has the same acceptability as bank deposits. The interest rate spread on the digital currency is $s_{cbdc} \equiv i - i_{cbdc}$. If $i_{cbdc} > 0$ then the CBDC replaces cash. Provided $s_{cbdc} > s_b$, there is still a demand for bank deposits. The consumers' outside option is now $U(\varepsilon; s_{cbdc})$ where $U(\varepsilon; s)$ is given by (23). From Proposition 5, as i_{cbdc} increases (i.e., s_{cbdc} decreases), consumers' outside options improve, deposit spreads shrink, and deposits increase. So, the introduction of digital currencies can raise aggregate deposits if banks have market power.³⁶ The transmission mechanism is not affected qualitatively as s_{cbdc} becomes the new policy spread. The strength of the transmission, however, is state dependent and as s_{cbdc} decreases the passthrough increases.

Suppose next that bank deposits are imperfect substitutes for cash and CBDC. Formally, cash and CBDC are universally accepted while bank deposits are imperfectly liquid and can only serve as means of payments with probability χ_2 . In this case, we can use Corollary 4 to show that if s_{cbdc} decreases, then bank deposits will increase if s_{cbdc} is above a threshold and will decrease otherwise.

Big data and price discrimination A third aspect of the Fintech revolution is the ability of companies to collect data about consumers to better assess their liquidity and financial needs. For instance, a recent report from OECD (2020) argues that “[FinTech] allows much more targeted price discrimination.” In the context of our model, these innovations allow banks to recognize consumers' types, ε . Suppose there are two categories of consumers. A fraction ω have their preference type, ε , observable by banks, as in Section 3, while the remaining fraction, $1 - \omega$, have private information about their type, as in Sections 4 and 5. The former enjoy deposit contracts with terms given by Lemma 1 (where u is multiplied by ε) while the latter benefit from contracts with terms satisfying Proposition 5. As ω increases, more deposit contracts achieve efficiency and aggregate deposits increase. But aggregate deposits become less sensitive to monetary policy, i.e., the transmission weakens.

In Figure 12, we compare the deposits channel under complete ($\omega = 1$), private ($\omega = 0$), and mixed ($\omega = 0.5$) information. The deposit spread decreases with ω while deposits, D , increase with ω . The overall profit of each banking contract rises in ω as shown in the right panel, which captures the fact that banks have more market power when they are more informed about their consumers. As i rises, the spread increases regardless of the information structure, but the passthrough is larger when ω is smaller. The transmission of i to deposits in the right panel decreases as ω increases. This comparison suggests that as banks know more

³⁶This result is consistent with the findings in Andolfatto et al. (2020) when the bank is a monopoly and Chiu et al. (2019) when there is Cournot competition among banks.

about consumers' preferences, their profit increases but the deposits channel becomes weaker and monetary policy less effective.

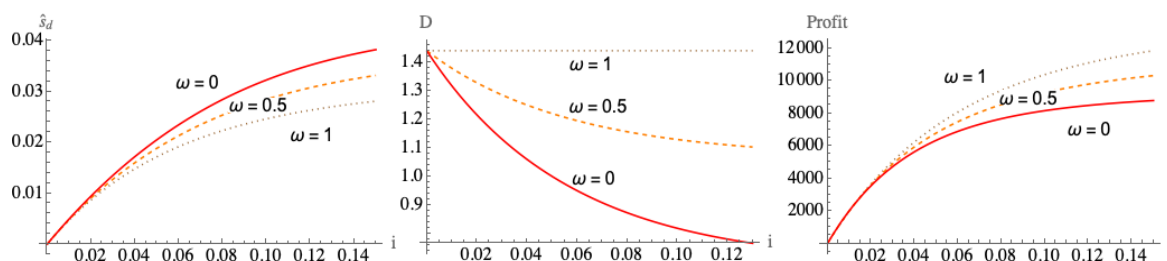


Figure 12: The deposits channel under various information structures.

6 Conclusion

We constructed a model of retail banking in which banks have market power in deposit (and loan) markets. We investigated the transmission mechanism of monetary policy and showed that when consumers have private information about their liquidity needs, a deposits channel emerges according to which an increase in the policy rate widens the deposit spread, and generates a contraction of aggregate deposits (and loans). This channel is not uniform across consumers and operates through those at the bottom of the distribution of deposit holdings – the same consumers from whom banks extract all the surplus. Moreover, by allowing for both private information and two-sided bargaining powers, we showed that the spread passthrough and the strength of the deposits channel are higher in more concentrated markets, in accordance with the evidence provided by Drechsler et al. (2017).

We used our model to study FinTech innovations in the banking industry that can reduce (e.g., on-line banking and crypto payments) or exacerbate (e.g., better information about consumers) bank market power. Innovations that reduce bank market power by improving consumers' outside options weaken the transmission mechanism of monetary policy. However, changes that reduce bank market power by limiting their information about consumers, thereby constraining banks' ability to price discriminate, strengthen the transmission mechanism. These results showcase the need to go deeper into our understanding of market power in banking.

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A Proofs of propositions and lemmas

Proof of Lemma 1. From (5) the surpluses of the consumer is:

$$V^b - V^u = \frac{-s_b d + \sigma v(d) - \phi - (\rho + \delta) V^u}{\rho + \delta}, \quad (53)$$

where $(\rho + \delta)V^u$ is interpreted as the consumer's reservation value of a deposit contract. The surplus of the banker is simply $\Pi = \phi/(\rho + \delta)$. The overall surplus from a deposit contract is

$$S \equiv V^b - V^u + \Pi = \frac{-s_b d + \sigma v(d) - (\rho + \delta) V^u}{\rho + \delta} \quad (54)$$

The solution to (7) maximizes the joint surplus with respect to d and ϕ splits the overall surplus according to agents' bargaining powers. Hence,

$$d \in \arg \max \{-s_b d + \sigma v(d)\} \quad (55)$$

$$\phi = \theta [U(s_b) - (\rho + \delta) V^u] \quad (56)$$

where $U(s_b) \equiv \max_d \{-s_b d + \sigma v(d)\}$. From (2), $(\rho + \delta) V^u = U(i) + \alpha(V^b - V^u)$ and, from the negotiation, $V^b - V^u = (\frac{1-\theta}{\theta})\phi/(\rho + \delta)$. Substituting into (56) we obtain (9). ■

Proof of Proposition 1. Existence and uniqueness of equilibrium. Define the following function:

$$\Gamma(\tau) \equiv \alpha(\tau) (1 - \theta) \kappa - \frac{\alpha(\tau)}{\tau} \theta [U(\rho - r_b) - U(i)] + (\rho + \delta) \kappa.$$

A solution to (16) is such that $\Gamma(\tau) = 0$. By the properties of $\alpha(\tau)$, the function $\Gamma(\tau)$ is strictly increasing with $\Gamma(0) = -\infty$ if $U(\rho - r_b) > U(i)$ and $\Gamma(0) > 0$ if $U(\rho - r_b) \leq U(i)$. Moreover, $\Gamma(+\infty) = +\infty$. So, there exists a positive solution to $\Gamma(\tau) = 0$ if and only if $U(\rho - r_b) > U(i)$, i.e., $\rho - r_b < i$. Using that $i = \rho + \pi$, the condition for existence can be reexpressed as $\pi + r_b > 0$. From the monotonicity of Γ , equilibrium is unique. From (10) and the fact that $\phi > 0$ in any active equilibrium it follows that $i_d < \pi + r_b$.

Parts 1-3. Comparative statics are summarized in the following table:

exogenous→ endogenous↓	r_b	i	θ	κ	δ
τ	+	+	+	-	-
n^b	+	+	+	-	-
m	0	-	0	0	0
d	+	0	0	0	0

From (16),

$$\frac{\partial \tau}{\partial i} = \left\{ (\rho + \delta) \kappa \frac{[1 - \eta(\tau)]}{\alpha(\tau)} + (1 - \theta) \kappa \right\}^{-1} \theta m,$$

where $\eta(\tau) \equiv \tau \alpha'(\tau)/\alpha(\tau)$. From (18),

$$\frac{\partial \hat{s}_d}{\partial i} = (\rho + \delta) \frac{\kappa}{d} \frac{[1 - \eta(\tau)]}{\alpha(\tau)} \frac{\partial \tau}{\partial i}.$$

Hence, substituting $\partial\tau/\partial i$ in the expression above we obtain (20). In order to establish $\partial\hat{s}_d/\partial\theta > 0$ and $\partial\hat{s}_d/\partial\kappa > 0$, we use (18) according to which

$$\hat{s}_d = s_b + (\rho + \delta) \frac{\kappa\tau}{\alpha(\tau)d}.$$

The result $\partial\hat{s}_d/\partial\theta > 0$ follows from the fact that \hat{s}_d increases with τ and $\partial\tau/\partial\theta > 0$. From (16),

$$\frac{\tau\kappa}{\alpha(\tau)} = \frac{\theta [U(s_b) - U(i)]}{(\rho + \delta) + \alpha(\tau) (1 - \theta)}.$$

Hence, the deposit spread can be rewritten as:

$$\hat{s}_d = s_b + \frac{(\rho + \delta)}{d} \frac{\theta [U(s_b) - U(i)]}{(\rho + \delta) + \alpha(\tau) (1 - \theta)}.$$

Using that $\partial\tau/\partial\kappa < 0$, it follows that $\partial\hat{s}_d/\partial\kappa > 0$.

From (20), if $\alpha(\tau) = \alpha_0\tau^\eta$, then

$$\frac{\partial\hat{s}_d}{\partial i} = \theta \frac{(\rho + \delta) (1 - \eta)}{(\rho + \delta) (1 - \eta) + (1 - \theta) \alpha_0\tau^\eta} \frac{u'^{-1} \left(1 + \frac{i}{\sigma}\right)}{u'^{-1} \left(1 + \frac{s_b}{\sigma}\right)},$$

which is decreasing in τ , i.e., the passthrough is larger in more concentrated markets.

Part 4. From (16),

$$(\rho + \delta)\kappa = \frac{\alpha(\tau)}{\tau} \{ \theta [U(s_b) - U(i)] - (1 - \theta) \tau \kappa \}.$$

As κ goes to 0, $\kappa\tau$ stays bounded away from 0. To see this, suppose $\kappa\tau \rightarrow 0$. Since $\theta [U(s_b) - U(i)] > 0$, $\alpha(\tau)/\tau \rightarrow 0$ and τ goes to $+\infty$. The equation can be rewritten as:

$$(\rho + \delta)\kappa\tau = \alpha(\tau) \{ \theta [U(s_b) - U(i)] - (1 - \theta) \tau \kappa \}$$

Using that $\theta [U(s_b) - U(i)] - (1 - \theta) \tau \kappa \rightarrow \theta [U(s_b) - U(i)]$ and $\alpha(\tau) \rightarrow +\infty$, the right side approaches $+\infty$ while the left side converges to 0. It is a contradiction. So $\lim_{\kappa \rightarrow 0} \kappa\tau > 0$. Suppose next that $\phi = \theta [U(s_b) - U(i)] - (1 - \theta) \tau \kappa$ stays bounded away from 0. Then, $\alpha(\tau)/\tau \rightarrow 0$ and τ goes to $+\infty$. But then

$$\alpha(\tau) \{ \theta [U(s_b) - U(i)] - (1 - \theta) \tau \kappa \} \rightarrow +\infty,$$

which is a contradiction with the left side being finite. It follows that $\phi \rightarrow 0$, $\hat{s}_d \rightarrow s_b$, and $\tau \kappa \rightarrow \theta [U(s_b) - U(i)] / (1 - \theta)$. Moreover,

$$\alpha(\tau) = \frac{(\rho + \delta)\kappa\tau}{\theta [U(s_b) - U(i)] - (1 - \theta) \tau \kappa} \rightarrow +\infty,$$

i.e., $\tau \rightarrow \infty$. From (19), as $\alpha(\tau)$ approaches infinity, n_b goes to 1. From (20), $\alpha(\tau) \rightarrow +\infty$ implies $\partial\hat{s}_d/\partial i \rightarrow 0$.

■

Proof of Lemma 2. The consumer is willing to participate in the mechanism offered by a banker if $V^b(\varepsilon) \geq V^u(\varepsilon)$, which from (21)-(22) can be reexpressed as

$$\max_{m \geq 0} \{ -\phi(\varepsilon) - im - s_b d(\varepsilon) + \sigma \{ \varepsilon u[d(\varepsilon) + m] - d(\varepsilon) - m \} \} \geq U(\varepsilon; i) + \alpha(\tau) [V^b(\varepsilon) - V^u(\varepsilon)], \quad (57)$$

where $V^b(\varepsilon)$ and $V^u(\varepsilon)$ are continuation values that depend on the distribution of mechanisms offered by other bankers. The reservation utility on the right side is composed of two terms. The first term is the flow utility of an unbanked consumer with the same preference type, ε . The second term captures the value of searching for an alternative banker.

“If” part. If (24) binds at some ε , then $V^b(\varepsilon) = V^u(\varepsilon)$ by (21) and (22), so (57) binds. Similarly, if (24) holds strictly, then so is (57).

“Only if” part. If (57) binds for some ε , then by (21), (22) and the symmetry in strategies, $V^b(\varepsilon) = V^u(\varepsilon)$. Therefore, (24) binds. If (57) is slack for some ε , then $V^b(\varepsilon) > V^u(\varepsilon)$ and (24) must hold strictly. Therefore, the individual rationality constraint (57) is equivalent to (24) in any symmetric equilibrium. ■

Proof of Lemma 3. The proof is by contradiction. Suppose there is a subset $\widehat{\mathcal{E}} \subset [0, \bar{\varepsilon}]$ with positive measure such that $m(\varepsilon) > 0$ for all $\varepsilon \in \widehat{\mathcal{E}}$. A deviation from the bank consists in offering alternative contracts, $[\phi'(\varepsilon), d'(\varepsilon)]$, for all $\varepsilon \in \widehat{\mathcal{E}}$ constructed as follows:

$$\begin{aligned} d'(\varepsilon) &= d(\varepsilon) + m(\varepsilon) \\ \phi'(\varepsilon) &= \phi(\varepsilon) + (i - s_b)m(\varepsilon). \end{aligned}$$

By construction consumers in $\widehat{\mathcal{E}}$ are indifferent between contracts $[\phi(\varepsilon), d(\varepsilon)]$ and $[\phi'(\varepsilon), d'(\varepsilon)]$, i.e.,

$$\begin{aligned} -\phi(\varepsilon) - im(\varepsilon) - s_b d(\varepsilon) + \sigma \{ \varepsilon u[d(\varepsilon) + m(\varepsilon)] - d(\varepsilon) - m(\varepsilon) \} = \\ -\phi'(\varepsilon) - s_b d'(\varepsilon) + \sigma \{ \varepsilon u[d'(\varepsilon)] - d'(\varepsilon) \}. \end{aligned}$$

So, if the IC and IR constraints, (24) and (26), hold for the original contracts, $[\phi(\varepsilon), d(\varepsilon)]$, then they also hold for the new contracts, $[\phi'(\varepsilon), d'(\varepsilon)]$. Finally, since $m(\varepsilon) > 0$ for all $\varepsilon \in \widehat{\mathcal{E}}$, $\phi'(\varepsilon) > \phi(\varepsilon)$ for all $\varepsilon \in \widehat{\mathcal{E}}$. Hence, the deviation is profitable and $m(\varepsilon)$ cannot be positive for a positive measure of consumers. ■

Proof of Proposition 2. Part 1. Reformulation as an optimal control problem. We first show how to rewrite the bank problem as an optimal control problem where the state variable is $\nu(\varepsilon)$ defined in (28). Assuming $\phi(\varepsilon)$ and $d(\varepsilon)$ are differentiable functions of ε , the first-order condition of (28) is:

$$\phi'(\varepsilon) + (s_b + \sigma)d'(\varepsilon) = \sigma \varepsilon u'[d(\varepsilon)] d'(\varepsilon). \quad (58)$$

From (58), we obtain the law of motion of the state variable as (29). Next we establish that incentive-compatibility implies that $d(\varepsilon)$ is nondecreasing in the consumer’s type, ε . From the IC constraints, (26),

$$\sigma \varepsilon' \{ u[d(\varepsilon')] - u[d(\varepsilon)] \} \geq \phi(\varepsilon') - \phi(\varepsilon) + (s_b + \sigma) [d(\varepsilon') - d(\varepsilon)] \geq \sigma \varepsilon \{ u[d(\varepsilon')] - u[d(\varepsilon)] \}. \quad (59)$$

If $\varepsilon' > \varepsilon$, then the inequality

$$\varepsilon' \{ u[d(\varepsilon')] - u[d(\varepsilon)] \} \geq \varepsilon \{ u[d(\varepsilon')] - u[d(\varepsilon)] \}.$$

implies $u[d(\varepsilon')] \geq u[d(\varepsilon)]$, and hence $d(\varepsilon') \geq d(\varepsilon)$. So deposits are nondecreasing in ε . From Proposition 23.D.2 in Mas-Colell et al. (1995), the law of motion of $\nu(\varepsilon)$, (29), and the result that $d(\varepsilon)$ is nondecreasing provide necessary and sufficient conditions for incentive-compatibility.

By the definition of $\nu(\varepsilon)$ we can express $\phi(\varepsilon)$ as a function of $\nu(\varepsilon)$ and $d(\varepsilon)$, i.e.,

$$\phi(\varepsilon) = -\nu(\varepsilon) - (s_b + \sigma)d(\varepsilon) + \sigma\varepsilon u[d(\varepsilon)],$$

so as to rewrite the bank's problem, (25), as an optimal control problem with free endpoints and a pure state constraint:

$$\max_{\{\nu(\varepsilon), d(\varepsilon)\}} \int_0^{\bar{\varepsilon}} \{-\nu(\varepsilon) - (s_b + \sigma)d(\varepsilon) + \sigma\varepsilon u[d(\varepsilon)]\} d\Upsilon(\varepsilon) \quad (60)$$

$$\text{s.t. } \nu'(\varepsilon) = \sigma u[d(\varepsilon)] \quad (61)$$

$$\nu(\varepsilon) \geq U(\varepsilon; i). \quad (62)$$

The nonnegativity of $\phi(\varepsilon)$ is conjectured and checked later. The Hamiltonian is:

$$H(\nu, d, \mu, \xi, \varepsilon) \equiv [-\nu(\varepsilon) - (s_b + \sigma)d + \sigma\varepsilon u(d)] \gamma(\varepsilon) + \mu\sigma u(d) + \xi(\varepsilon) [\nu(\varepsilon) - U(\varepsilon; i)],$$

where $\mu(\varepsilon)$ is the costate variable, and $\xi(\varepsilon)$ is the Lagrange multiplier associated with the participation constraint. From the Maximum Principle, necessary conditions for an optimum are:

$$[\varepsilon\gamma(\varepsilon) + \mu(\varepsilon)] \sigma u'[d(\varepsilon)] = (s_b + \sigma) \gamma(\varepsilon) \quad (63)$$

$$\mu'(\varepsilon) = \gamma(\varepsilon) - \xi(\varepsilon), \quad (64)$$

with the complementary slackness conditions,

$$\mu(0)\nu(0) = \mu(\bar{\varepsilon})\nu(\bar{\varepsilon}) = 0.$$

Equation (63) is the first-order condition, $\partial H / \partial d = 0$. Equation (64) is the law of motion of the costate variable, $\mu'(\varepsilon) = -\partial H / \partial \nu$.

We construct a candidate solution by conjecturing that there is $\tilde{\varepsilon} > 0$ such that $\nu(\varepsilon) \geq U(\varepsilon; i)$ binds for all $\varepsilon < \tilde{\varepsilon}$. We will show that such an $\tilde{\varepsilon} \in (0, \bar{\varepsilon})$ exists and is unique. After that we show that the sufficiency conditions for a maximum are satisfied. At the end we argue our solution is unique.

Part 2. IR constraints bind for $\varepsilon < \tilde{\varepsilon}$. Under this conjecture, at $\varepsilon = 0$, $\nu(\varepsilon) = U(0; i) = 0$. Hence, the condition, $\mu(0)\nu(0) = 0$, holds. The condition $\nu(\varepsilon) = U(\varepsilon; i)$ requires $\nu'(\varepsilon) = \sigma u[d(\varepsilon)] = \partial U(\varepsilon; i) / \partial \varepsilon = \sigma u[m(\varepsilon)]$, i.e., $d(\varepsilon) = m(\varepsilon)$ where, from (23), $m(\varepsilon)$ solves

$$\sigma\varepsilon u'[m(\varepsilon)] = i + \sigma. \quad (65)$$

It follows from (63) that the closed-form solution for the costate variable is:

$$\mu(\varepsilon) = -\left(\frac{i - s_b}{i + \sigma}\right) \varepsilon \gamma(\varepsilon) < 0 \text{ for all } \varepsilon \in (0, \tilde{\varepsilon}). \quad (66)$$

By the definition of $\nu(\varepsilon)$, $\phi(\varepsilon) = -\nu(\varepsilon) - (s_b + \sigma)d(\varepsilon) + \sigma u[d(\varepsilon)]$. Therefore the banking fees are equal to

$$\phi(\varepsilon) = (i - s_b) d(\varepsilon) > 0 \quad \text{for all } \varepsilon \in (0, \tilde{\varepsilon}).$$

From (64) the expression for the Lagrange multiplier associated with the participation constraint is $\xi(\varepsilon) = \gamma(\varepsilon) - \mu'(\varepsilon)$. Substituting $\mu'(\varepsilon)$ by the derivative of $\mu(\varepsilon)$ defined in (66) we obtain

$$\xi(\varepsilon) = \gamma(\varepsilon) + \left(\frac{i - s_b}{i + \sigma} \right) [\gamma(\varepsilon) + \varepsilon \gamma'(\varepsilon)] \quad \text{for all } \varepsilon \in (0, \tilde{\varepsilon}). \quad (67)$$

We will check later that $\xi(\varepsilon) > 0$ for all $\varepsilon \in (0, \tilde{\varepsilon})$.

Part 3. IR constraints are slack for $\varepsilon \geq \tilde{\varepsilon}$. Hence, $\xi(\varepsilon) = 0$ for all $\varepsilon > \tilde{\varepsilon}$. It follows from (64) and (66):

$$\mu(\varepsilon) = - \left(\frac{i - s_b}{i + \sigma} \right) \tilde{\varepsilon} \gamma(\tilde{\varepsilon}) + \Upsilon(\varepsilon) - \Upsilon(\tilde{\varepsilon}), \quad \text{for all } \varepsilon \geq \tilde{\varepsilon}. \quad (68)$$

Provided $\nu(\tilde{\varepsilon}) > 0$, the optimality condition for a free end-point problem is $\mu(\tilde{\varepsilon}) = 0$, which, from (68), gives

$$\frac{1 - \Upsilon(\tilde{\varepsilon})}{\gamma(\tilde{\varepsilon})} = \left(\frac{i - s_b}{i + \sigma} \right) \tilde{\varepsilon}. \quad (69)$$

The right side of (69) is increasing in $\tilde{\varepsilon}$ while the left side is decreasing in $\tilde{\varepsilon}$ due to the log-concavity of $1 - \Upsilon$. Moreover, the left side is greater than the right side at $\tilde{\varepsilon} = 0$ and it is smaller at $\tilde{\varepsilon} = \bar{\varepsilon}$. Hence, there is a unique $\tilde{\varepsilon} \in (0, \bar{\varepsilon})$ solution to (69).

We now use the definition of $\tilde{\varepsilon}$ to show that the Lagrange multiplier in (67) is always positive. If $\gamma'(\varepsilon) \geq 0$ the condition $\xi(\varepsilon) > 0$ is satisfied, so we focus on $\gamma'(\varepsilon) < 0$. From (67), we rewrite the Lagrange multiplier as

$$\xi(\varepsilon) = \gamma(\varepsilon) \left[1 + \left(\frac{i - s_b}{i + \sigma} \right) \right] \gamma(\varepsilon) + \left(\frac{i - s_b}{i + \sigma} \right) \varepsilon \gamma'(\varepsilon).$$

From (69), for all $\varepsilon < \tilde{\varepsilon}$,

$$\frac{1 - \Upsilon(\varepsilon)}{\gamma(\varepsilon)\varepsilon} > \left(\frac{i - s_b}{i + \sigma} \right).$$

Hence, under the assumption $\gamma'(\varepsilon) < 0$,

$$\xi(\varepsilon) > \left[1 + \left(\frac{i - s_b}{i + \sigma} \right) \right] \gamma(\varepsilon) + \frac{1 - \Upsilon(\varepsilon)}{\gamma(\varepsilon)} \gamma'(\varepsilon).$$

We rearrange the terms to obtain:

$$\xi(\varepsilon) > \left(\frac{i - s_b}{i + \sigma} \right) \gamma(\varepsilon) + \frac{[1 - \Upsilon(\varepsilon)] \gamma'(\varepsilon) + [\gamma(\varepsilon)]^2}{\gamma(\varepsilon)}.$$

From the log-concavity of $1 - \Upsilon(\varepsilon)$, $[\gamma(\varepsilon)]^2 + [1 - \Upsilon(\varepsilon)] \gamma'(\varepsilon) \geq 0$. Hence, $\xi(\varepsilon) > 0$ for all $\varepsilon \in (0, \tilde{\varepsilon})$.³⁷

Using the definition of $\tilde{\varepsilon}$ in (69) the expression for $\mu(\varepsilon)$, (68), can be simplified to:

$$\mu(\varepsilon) = \Upsilon(\varepsilon) - 1 \leq 0 \quad \text{for all } \varepsilon \geq \tilde{\varepsilon}. \quad (70)$$

³⁷Note that $\xi(\varepsilon)$ is discontinuous at $\varepsilon = \tilde{\varepsilon}$. This discontinuity is consistent with standard results in optimal control because by Theorem 1 on page 317 of Seierstad and Sydsaeter (1987), the Lagrange multiplier on the state constraint is not necessarily continuous. In general, there is no guarantee that the Lagrange multiplier is continuous at points where the state constraint becomes active, see Chertovskih et al. (2019) for more details.

Substituting this expression into (63), the expression for $d(\varepsilon)$ is:

$$\left\{ \varepsilon - \left[\frac{1 - \Upsilon(\varepsilon)}{\gamma(\varepsilon)} \right] \right\} u' [d(\varepsilon)] = 1 + \frac{s_b}{\sigma} \quad \text{for all } \varepsilon \geq \tilde{\varepsilon}.$$

By the log-concavity of $1 - \Upsilon(\varepsilon)$, the first term in brackets is increasing in ε . Hence, $d(\varepsilon)$ is increasing in ε .

We also need to show $d(\varepsilon) > m(\varepsilon)$ for $\varepsilon > \tilde{\varepsilon}$. By (65) and the expression for $d(\varepsilon)$,

$$\left[1 + \frac{\mu(\varepsilon)}{\varepsilon \gamma(\varepsilon)} \right] \frac{u' [d(\varepsilon)]}{u' [m(\varepsilon)]} = \frac{1 + s_b/\sigma}{1 + i/\sigma}.$$

By (68)

$$1 + \frac{\mu(\varepsilon)}{\tilde{\varepsilon} \gamma(\tilde{\varepsilon})} = \frac{1 + s_b/\sigma}{1 + i/\sigma} + \frac{\Upsilon(\varepsilon) - \Upsilon(\tilde{\varepsilon})}{\tilde{\varepsilon} \gamma(\tilde{\varepsilon})} \geq \frac{1 + s_b/\sigma}{1 + i/\sigma}, \quad \text{for all } \varepsilon \geq \tilde{\varepsilon} \quad (71)$$

where the inequality is strict when $\varepsilon > \tilde{\varepsilon}$. Hence for $\varepsilon > \tilde{\varepsilon}$, $u' [d(\varepsilon)]/u' [m(\varepsilon)] < 1$ or equivalently $d(\varepsilon) > m(\varepsilon)$.

The value of the state variable is:

$$\nu(\varepsilon) = U(\tilde{\varepsilon}; i) + \sigma \int_{\tilde{\varepsilon}}^{\varepsilon} u [d(x)] dx \quad \text{for all } \varepsilon \geq \tilde{\varepsilon}.$$

The result that $\nu(\varepsilon) > U(\varepsilon; i)$ for all $\varepsilon > \tilde{\varepsilon}$ follows from the observation that:

$$\nu(\varepsilon) = \int_0^{\varepsilon} \sigma u [d(\varepsilon)] d\varepsilon > U(\varepsilon; i) = \int_0^{\varepsilon} \sigma u [m(\varepsilon)] d\varepsilon, \quad (72)$$

and $d(\varepsilon) \geq m(\varepsilon)$ with a strict inequality when $\varepsilon > \tilde{\varepsilon}$. Finally, the banking fees are:

$$\phi(\varepsilon) = -U(\tilde{\varepsilon}; i) - \sigma \int_{\tilde{\varepsilon}}^{\varepsilon} u [d(x)] dx - (s_b + \sigma)d(\varepsilon) + \sigma \varepsilon u [d(\varepsilon)] \quad \text{for all } \varepsilon \geq \tilde{\varepsilon}.$$

In order to establish that $\phi(\varepsilon)$ is increasing, we differentiate $\phi(\varepsilon)$ to obtain:

$$\phi'(\varepsilon) = \{-(s_b + \sigma) + \sigma \varepsilon u' [d(\varepsilon)]\} d'(\varepsilon) \quad \text{for all } \varepsilon \geq \tilde{\varepsilon}. \quad (73)$$

We use the first-order condition for $d(\varepsilon)$, (31), to rewrite this derivative as:

$$\phi'(\varepsilon) = (s_b + \sigma) \left\{ \frac{1 - \Upsilon(\varepsilon)}{\varepsilon \gamma(\varepsilon) - [1 - \Upsilon(\varepsilon)]} \right\} d'(\varepsilon) \quad \text{for all } \varepsilon \geq \tilde{\varepsilon}.$$

For $\varepsilon \geq \tilde{\varepsilon}$, the denominator is positive by (69) and the log-concavity of $1 - \Upsilon$. Hence, $\phi'(\varepsilon) > 0$ for $\varepsilon \in (\tilde{\varepsilon}, \bar{\varepsilon})$.

Part 4. Sufficiency. We now check the Mangasarian sufficiency conditions. Given our solution for μ , the Hamiltonian function, $H(\nu, d, \mu, \xi, \varepsilon)$, is jointly concave in (ν, d) . To see this, note that H is additively separable in ν and d . It is linear in ν (hence, concave); and, provided that $\mu(\varepsilon) + \varepsilon \gamma(\varepsilon) \geq 0$, which holds from (66) and (70), it is concave in d due to the strict concavity of $u(d)$. Finally, the terminal conditions, $\mu(0)\nu(0) = 0$ and $\mu(\bar{\varepsilon})\nu(\bar{\varepsilon}) = 0$, hold since $\nu(0) = 0$ and $\mu(\bar{\varepsilon}) = 0$.

Part 5. Uniqueness. Finally, we argue that our conjecture about the two-tier structure of the menu of contracts is the only way to construct a solution. At $\varepsilon = 0$, the consumer can never get any surplus from monetary trades. Since $\phi_0 \geq 0$, $\nu(0) \leq 0$. But the IR constraint $\nu(0) \geq U(0; i) = 0$ must hold, hence $\nu(0) = U(0; i) = 0$. Suppose next that as ε rises above 0, the menu of contracts transitions into an nonempty

interval, $[\tilde{\varepsilon}, \tilde{\varepsilon}]$, where $\tilde{\varepsilon} < \bar{\varepsilon}$ and IR is slack in the interior of the interval but binds at the end points of the interval. By the proof logic of Part 3, $d(\varepsilon) > m(\varepsilon)$, for $\varepsilon \in (\tilde{\varepsilon}, \tilde{\varepsilon}]$. Hence by (72),

$$\nu(\varepsilon) = U(\tilde{\varepsilon}; i) + \sigma \int_{\tilde{\varepsilon}}^{\varepsilon} u[d(x)] dx > U(\varepsilon; i)$$

for $\varepsilon \in (\tilde{\varepsilon}, \tilde{\varepsilon}]$ and the IR constraint cannot bind at $\tilde{\varepsilon}$, leading to a contradiction. Therefore, once the IR becomes slack at $\tilde{\varepsilon}$, it must stay slack for all larger ε . ■

Lemma 5 *The deposit rate is $\hat{i}_d(\varepsilon) = 0$ for all $\varepsilon < \tilde{\varepsilon}$ and it rises in ε for $\varepsilon \geq \tilde{\varepsilon}$.*

Proof. To prove this claim, we will show the expression in the large parenthesis in (74) falls in ε for $\varepsilon \geq \tilde{\varepsilon}$. Since $d'(\varepsilon) > 0$ by Proposition 2, the derivative of this expression is proportional to

$$\begin{aligned} \frac{\partial}{\partial \varepsilon} \left(\frac{\{\varepsilon u[d(\varepsilon)] - d(\varepsilon)\} - \int_0^{\varepsilon} u[d(x)] dx}{d(\varepsilon)} \right) &\propto \varepsilon u'[d(\varepsilon)] d(\varepsilon) - \varepsilon u[d(\varepsilon)] + \int_0^{\varepsilon} u[d(x)] dx \\ &= \varepsilon u'[d(\varepsilon)] d(\varepsilon) - (1 + s_b/\sigma) d(\varepsilon) - \phi(\varepsilon)/\sigma \end{aligned}$$

where the second line uses (33). By (30) and (32) the last expression is 0 at $\varepsilon = \tilde{\varepsilon}$. We argue that this expression falls in ε :

$$\begin{aligned} \frac{\partial}{\partial \varepsilon} \left\{ [\varepsilon u'[d(\varepsilon)] - (1 + \frac{s_b}{\sigma})] d(\varepsilon) - \frac{\phi(\varepsilon)}{\sigma} \right\} &= d(\varepsilon) \frac{\partial [\varepsilon u'[d(\varepsilon)] - (1 + s_b/\sigma)]}{\partial \varepsilon} \\ &\quad + d'(\varepsilon) \left\{ \varepsilon u'[d(\varepsilon)] - \left(1 + \frac{s_b}{\sigma}\right) \right\} - \frac{\phi'(\varepsilon)}{\sigma} \\ &= d(\varepsilon) \frac{\partial}{\partial \varepsilon} \left[\frac{1 - \Upsilon(\tilde{\varepsilon})}{\gamma(\tilde{\varepsilon})} u'[d(\varepsilon)] \right] < 0 \end{aligned}$$

where the second line uses (31) and (73). The inequality is true by the log-concavity of $1 - \Upsilon$, $d'(\varepsilon) > 0$ and the concavity of u . ■

Proof of Proposition 3. Part 1. The derivative, (38), follows directly from

$$D(i) \equiv \int_0^{\tilde{\varepsilon}} d(\varepsilon) d\Upsilon(\varepsilon) = \int_0^{\tilde{\varepsilon}(i)} u'^{-1} \left(\frac{i + \sigma}{\varepsilon \sigma} \right) d\Upsilon(\varepsilon) + \int_{\tilde{\varepsilon}(i)}^{\tilde{\varepsilon}} u'^{-1} \left[\frac{\gamma(\varepsilon)}{\varepsilon \gamma(\varepsilon) - 1 + \Upsilon(\varepsilon)} \left(1 + \frac{s_b}{\sigma} \right) \right] d\Upsilon(\varepsilon).$$

Parts 2 and 3. We differentiate $n^b = \alpha(\tau)/[\delta + \alpha(\tau)]$ to obtain:

$$\frac{\partial n^b}{\partial i} = \frac{\alpha'(\tau) \delta}{[\delta + \alpha(\tau)]^2} \frac{\partial \tau}{\partial i},$$

where, from (35),

$$\frac{\partial \tau}{\partial i} = \frac{\alpha(\tau)}{[1 - \eta(\tau)](\rho + \delta) \kappa} \frac{\partial \Phi}{\partial i},$$

with $\eta(\tau) \equiv \tau \alpha'(\tau)/\alpha(\tau)$ and, from (32)-(33),

$$\frac{\partial \Phi}{\partial i} = \int_0^{\tilde{\varepsilon}} d(\varepsilon) d\Upsilon(\varepsilon) + d(\tilde{\varepsilon}) [1 - \Upsilon(\tilde{\varepsilon})].$$

So

$$\frac{\partial n^b/n^b}{\partial i} = \frac{\eta(\tau)\delta n^b}{[1 - \eta(\tau)]\alpha(\tau)\Phi} \left\{ \int_0^{\bar{\varepsilon}} d(\varepsilon)d\Upsilon(\varepsilon) + d(\bar{\varepsilon})[1 - \Upsilon(\bar{\varepsilon})] \right\} > 0.$$

As $\delta \rightarrow 0$, $\partial\Phi/\partial i$ is unaffected, $\partial\tau/\partial i < +\infty$, and hence $\partial n^b/\partial i \rightarrow 0$. Hence, from (38), there is a threshold for δ below which

$$\frac{\partial(n^b D)}{\partial i} = \frac{\partial n^b}{\partial i} D + n^b \frac{\partial D}{\partial i} < 0.$$

Part 3. The deposit spread is

$$\hat{s}_d(\varepsilon) = s_b + \frac{\phi(\varepsilon)}{d(\varepsilon)} = i\mathbb{I}_{\{\varepsilon \leq \bar{\varepsilon}\}} + \left(\sigma \frac{\{\varepsilon u[d(\varepsilon)] - d(\varepsilon)\} - \int_0^{\varepsilon} u[d(x)] dx}{d(\varepsilon)} \right) \mathbb{I}_{\{\varepsilon > \bar{\varepsilon}\}}. \quad (74)$$

To obtain (39), note that

$$\sigma \int_0^{\varepsilon} u[d(x)] dx = U(\bar{\varepsilon}; i) + \sigma \int_{\bar{\varepsilon}}^{\varepsilon} u[d(x)] dx.$$

The derivative of the right side with respect to i is $-d(\bar{\varepsilon})$. The average spread is

$$\hat{s}_d = s_b + \int_0^{\bar{\varepsilon}} \frac{\phi(\varepsilon)\gamma(\varepsilon)}{D(i)} d\varepsilon.$$

Using that $\phi(\varepsilon)$ is increasing in i and D is decreasing in i , it follows that $\partial\hat{s}_d/\partial i > 0$. ■

Proof of Proposition 4.

The proof uses Proposition 13, which characterizes the optimal contract with two general deposit categories under posting and private information. Proposition 4 concerns the limit when $s_b^1 \rightarrow i$.

From (148), $\bar{\varepsilon} \rightarrow \bar{\varepsilon}$ at the limit $s_b^1 \rightarrow i$. Then the solution of (d^1, ℓ) is given by (151)-(152):

$$\left\{ \varpi(\varepsilon)\chi_1\sigma d^1 - (i - s_b^2 + \sigma\chi_1) (d^1)^{1+a} \right\} \chi_2 = \left\{ \varpi(\varepsilon)\sigma\chi_2\ell - (s_b^2 + \sigma\chi_2) (\ell)^{1+a} \right\} \chi_1, \quad (75)$$

$$\sigma\chi_1\varepsilon (d^1)^{-a} + \sigma\chi_2\varepsilon (\ell)^{-a} = i + \sigma. \quad (76)$$

As i rises, the curves labelled FOCs in Figure 17, representing (75), shift up and the threshold \tilde{d}^1 falls by (161). The curve ICM, representing (76), shifts downward in i . Hence d^1 falls in i . As $i \uparrow +\infty$, the threshold $\tilde{d}^1 \rightarrow 0$ in the left panel. Moreover, the slope of FOCs explodes to $+\infty$ in both panels. By (76), the curve ICM becomes an L-shape curve that asymptote to $+\infty$ when $d^1 = 0$ and equals to 0 for all $d^1 > 0$. Hence, $d^1 \rightarrow 0$ as $i \uparrow +\infty$. If $\varpi(\varepsilon) > 0$, then (75) is represented by FOCs in the left panel and $\ell \rightarrow \tilde{\ell}$ where $\tilde{\ell}$ is given by (160); otherwise (75) corresponds to FOCs in the right panel and $\ell \rightarrow 0$. ■

Proof of Lemma 4. We multiply both sides of (41) by $\rho + \delta$ and we use (21), (22) and (40), to reexpress the consumer's participation constraint as:

$$\nu(\varepsilon) \geq \theta [U(\varepsilon; i) + \alpha\Delta V(\varepsilon)] + (1 - \theta)U(\varepsilon; s_b), \quad (77)$$

where $\Delta V(\varepsilon) \equiv V^b(\varepsilon) - V^u(\varepsilon)$. The utility of the contract must be larger than a weighted average of the reservation utility of the consumer, $U(\varepsilon; i) + \alpha\Delta V(\varepsilon)$, and the complete-information utility when the

consumer has all the bargaining power, $U(\varepsilon; s_b)$. From (21) and (22), the difference $\Delta V(\varepsilon) \equiv V^b(\varepsilon) - V^u(\varepsilon)$ can be written as

$$(\rho + \delta) \Delta V(\varepsilon) = \nu^*(\varepsilon) - U(\varepsilon; i) + \alpha \Delta V(\varepsilon),$$

where $\nu^*(\varepsilon)$ is the flow surplus of a type- ε banked consumer in a symmetric equilibrium. Solving for $\Delta V(\varepsilon)$,

$$\Delta V(\varepsilon) = \frac{\nu^*(\varepsilon) - U(\varepsilon; i)}{(\rho + \delta + \alpha)}.$$

Substituting this expression into the participation constraint (77),

$$\nu(\varepsilon) \geq \theta \left[U(\varepsilon; i) + \alpha \frac{\nu^*(\varepsilon) - U(\varepsilon; i)}{(\rho + \delta + \alpha)} \right] + (1 - \theta) U(\varepsilon; s_b).$$

In a symmetric equilibrium where $\nu(\varepsilon) = \nu^*(\varepsilon)$, the inequality is equivalent to (42). ■

Proof of Proposition 5. The proof is similar to the one of Proposition 2 and hence we omit the parts

that are repetitive. The Hamiltonian of the bank's problem is:

$$\begin{aligned} H(\nu, d, \mu, \xi, \varepsilon) \equiv & [-\nu(\varepsilon) - (s_b + \sigma)d + \sigma \varepsilon u(d)] \gamma(\varepsilon) + \mu \sigma u(d) \\ & + \xi(\varepsilon) [\nu(\varepsilon) - \underline{\nu}(\varepsilon)], \end{aligned}$$

where $\mu(\varepsilon)$ is the costate variable, and $\xi(\varepsilon)$ is the Lagrange multiplier associated with the participation constraint. From the Maximum Principle, necessary conditions for an optimum are:

$$[\varepsilon \gamma(\varepsilon) + \mu(\varepsilon)] \sigma u'(d(\varepsilon)) = (s_b + \sigma) \gamma(\varepsilon) \quad (78)$$

$$\mu'(\varepsilon) = \gamma(\varepsilon) - \xi(\varepsilon), \quad (79)$$

with the complementary slackness conditions,

$$\mu(0) \nu(0) = \mu(\bar{\varepsilon}) \nu(\bar{\varepsilon}) = 0.$$

As before, we conjecture that there is $\tilde{\varepsilon} > 0$ such that $\nu(\varepsilon) \geq \theta [U(\varepsilon; i) + \alpha \Delta V(\varepsilon)] + (1 - \theta) U(\varepsilon; s_b)$ binds for all $\varepsilon < \tilde{\varepsilon}$. We will show that such an $\tilde{\varepsilon} \in (0, \bar{\varepsilon})$ exists and is unique.

Part 1. IR constraints bind for $\varepsilon < \tilde{\varepsilon}$. If the participation constraint binds over some nonempty open interval, then $\nu(\varepsilon) = \underline{\nu}(\varepsilon)$ and from (42),

$$\nu'(\varepsilon) = \sigma u[d(\varepsilon)] = \frac{(1 - \theta)(\rho + \delta + \alpha)U'(\varepsilon; s_b) + \theta(\rho + \delta)U'(\varepsilon; i)}{(1 - \theta)(\rho + \delta + \alpha) + \theta(\rho + \delta)}.$$

Moreover, $\partial U(\varepsilon; i)/\partial \varepsilon = \sigma u[m(\varepsilon)]$ and $\partial U(\varepsilon; s_b)/\partial \varepsilon = \sigma u[d^s(\varepsilon)]$, where $m(\varepsilon)$ and $d^s(\varepsilon)$ solve

$$\begin{aligned} \sigma \varepsilon u'[m(\varepsilon)] &= i + \sigma \\ \sigma \varepsilon u'[d^s(\varepsilon)] &= s^b + \sigma. \end{aligned}$$

So $m(\varepsilon)$ are the real balances of an unbanked consumer while $d^s(\varepsilon)$ represents the complete-information level of deposits. Hence:

$$\omega\{u'[d(\varepsilon)]\} = \frac{(1-\theta)(\rho+\delta+\alpha)\omega\left(\frac{s_b+\sigma}{\sigma\varepsilon}\right) + \theta(\rho+\delta)\omega\left(\frac{i+\sigma}{\sigma\varepsilon}\right)}{(1-\theta)(\rho+\delta+\alpha) + \theta(\rho+\delta)},$$

where $\omega(x) \equiv u \circ u'^{-1}$. If the utility function is CRRA, $u(y) = y^{1-a}/(1-a)$ with $a \in (0, 1)$, then $\omega(x) = x^{\frac{-(1-a)}{a}}/(1-a)$ and

$$\varepsilon u'[d(\varepsilon)] = \varepsilon [d(\varepsilon)]^{-a} = \frac{[(1-\theta)(\rho+\delta+\alpha) + \theta(\rho+\delta)]^{\frac{a}{1-a}}}{\left[(1-\theta)(\rho+\delta+\alpha) \left(\frac{\sigma}{s_b+\sigma}\right)^{\frac{(1-a)}{a}} + \theta(\rho+\delta) \left(\frac{\sigma}{i+\sigma}\right)^{\frac{(1-a)}{a}} \right]^{\frac{a}{1-a}}}.$$

Hence, the deposit levels are:

$$d(\varepsilon) = \varepsilon^{\frac{1}{a}} \left[\frac{(1-\theta)(\rho+\delta+\alpha) \left(\frac{\sigma}{s_b+\sigma}\right)^{\frac{(1-a)}{a}} + \theta(\rho+\delta) \left(\frac{\sigma}{i+\sigma}\right)^{\frac{(1-a)}{a}}}{(1-\theta)(\rho+\delta+\alpha) + \theta(\rho+\delta)} \right]^{\frac{1}{1-a}}, \quad (80)$$

which corresponds to (46).

By the definition of \underline{i} in (48), we can rewrite $d(\varepsilon) = [\sigma\varepsilon/(\underline{i} + \sigma)]^{1/a}$ and $\varepsilon u'[d(\varepsilon)] = \underline{i}/\sigma + 1$. By the definition of $\phi(\varepsilon)$ and the IC condition,

$$\phi'(\varepsilon) = \{-(s_b + \sigma) + \sigma\varepsilon u'[d(\varepsilon)]\} d'(\varepsilon).$$

Since $\varepsilon u'[d(\varepsilon)]$ is independent of ε and $d(0) = 0$, the fee, $\phi(\varepsilon)$, is

$$\phi(\varepsilon) = d(\varepsilon) (\underline{i} - s_b), \quad \text{for all } \varepsilon < \tilde{\varepsilon}.$$

From (78) that the closed-form solution for the costate variable is:

$$\begin{aligned} \mu(\varepsilon) &= \varepsilon\gamma(\varepsilon) \left[\frac{s_b + \sigma}{\sigma\varepsilon u'[d(\varepsilon)]} - 1 \right] \\ &= \varepsilon\gamma(\varepsilon) \left\{ \left[\frac{(1-\theta)(\rho+\delta+\alpha) + \theta(\rho+\delta) \left(\frac{s_b+\sigma}{i+\sigma}\right)^{\frac{(1-a)}{a}}}{(1-\theta)(\rho+\delta+\alpha) + \theta(\rho+\delta)} \right]^{\frac{a}{1-a}} - 1 \right\} \\ &= -\varepsilon\gamma(\varepsilon) \left(\frac{\underline{i} - s_b}{\underline{i} + \sigma} \right) \end{aligned} \quad (81)$$

for all $\varepsilon \in (0, \tilde{\varepsilon})$.

Part 2. IR constraints are slack for $\varepsilon \geq \tilde{\varepsilon}$. Hence, $\xi(\varepsilon) = 0$ for all $\varepsilon > \tilde{\varepsilon}$. It follows from (79) and (81):

$$\mu(\varepsilon) = \mu(\tilde{\varepsilon}) + \Upsilon(\varepsilon) - \Upsilon(\tilde{\varepsilon}), \quad \text{for all } \varepsilon \geq \tilde{\varepsilon}. \quad (82)$$

Provided $\nu(\tilde{\varepsilon}) > 0$, the optimality condition for a free end-point problem is $\mu(\tilde{\varepsilon}) = 0$, which, from (81), gives

$$\frac{1 - \Upsilon(\tilde{\varepsilon})}{\gamma(\tilde{\varepsilon})} = \tilde{\varepsilon} \left(\frac{\underline{i} - s_b}{\underline{i} + \sigma} \right), \quad (83)$$

which corresponds to (49). The right side of (83) is increasing in $\tilde{\varepsilon}$ while the left side is decreasing in $\tilde{\varepsilon}$ due to the log-concavity of $1 - \Upsilon$. Moreover, the left side is greater than the right side at $\tilde{\varepsilon} = 0$ and it is smaller at $\tilde{\varepsilon} = \bar{\varepsilon}$. Hence, there is a unique $\tilde{\varepsilon} \in (0, \bar{\varepsilon})$ solution to (83).

By (78) and $\mu(\varepsilon) = -1 + \Upsilon(\varepsilon)$, the deposits $d(\varepsilon)$ solves

$$u'[d(\varepsilon)] = \left\{ \varepsilon - \left[\frac{1 - \Upsilon(\varepsilon)}{\gamma(\varepsilon)} \right] \right\}^{-1} \left(1 + \frac{s_b}{\sigma} \right) \quad \text{for all } \varepsilon \geq \tilde{\varepsilon}.$$

Using that u is CRRA,

$$d(\varepsilon) = \left\{ \varepsilon - \left[\frac{1 - \Upsilon(\varepsilon)}{\gamma(\varepsilon)} \right] \right\}^{1/a} \left(1 + \frac{s_b}{\sigma} \right)^{-1/a} \quad \text{for all } \varepsilon \geq \tilde{\varepsilon},$$

which corresponds to (47).

For $\varepsilon \leq \tilde{\varepsilon}$, the value of the state variable, $\nu(\varepsilon)$, is equal to $\underline{\nu}(\varepsilon)$ by Lemma 4. Therefore by integrating over the IC constraint (44),

$$\nu(\varepsilon) = \underline{\nu}(\tilde{\varepsilon}) + \sigma \int_{\tilde{\varepsilon}}^{\varepsilon} u[d(x)] dx \quad \text{for all } \varepsilon \geq \tilde{\varepsilon}.$$

Using this expression and the definition of the fees we can reexpress $\phi(\varepsilon)$ by (51) for $\varepsilon \geq \tilde{\varepsilon}$. ■

Proof of Proposition 6. Part 1. From (46)-(47) and (50)-(51), the spread passthrough is given by:

$$\begin{aligned} \frac{\partial \hat{s}_d(\varepsilon)}{\partial i} &= \frac{\left(\frac{i}{\sigma} + 1\right)^{\frac{1}{a}}}{(i + \sigma)} \frac{\theta(\rho + \delta)}{a[(1 - \theta)(\rho + \delta + \alpha) + \theta(\rho + \delta)]} \left(\frac{\sigma}{i + \sigma}\right)^{\frac{1+a}{a}} > 0 \quad \text{for all } \varepsilon < \tilde{\varepsilon} \\ &= \frac{\theta(\rho + \delta)m(\varepsilon)}{d(\varepsilon)[(1 - \theta)(\rho + \delta + \alpha) + \theta(\rho + \delta)]} > 0 \quad \text{for all } \varepsilon > \tilde{\varepsilon}. \end{aligned}$$

The effects of the policy rate on deposits are given by:

$$\begin{aligned} \frac{\partial d(\varepsilon)}{\partial i} &= \frac{-\varepsilon^{1/a} \left(\frac{\sigma}{i + \sigma}\right) \theta(\rho + \delta)}{a(i + \sigma)[(1 - \theta)(\rho + \delta + \alpha) + \theta(\rho + \delta)]} \left(\frac{\sigma}{i + \sigma}\right)^{\frac{1-a}{a}} < 0 \quad \text{for all } \varepsilon < \tilde{\varepsilon} \\ &= 0 \quad \text{otherwise.} \end{aligned}$$

Part 2. Using that i is decreasing in α , it follows immediately that $\partial \hat{s}_d(\varepsilon)/\partial i$ decreases in α . Next, we compute:

$$\begin{aligned} \frac{\partial \ln [|\partial d(\varepsilon)/\partial i|]}{\partial \alpha} &= \frac{a(1 - \theta)}{1 - a} \left[(1 - \theta)(\rho + \delta + \alpha) + \theta(\rho + \delta) \left(\frac{s^b + \sigma}{i + \sigma}\right)^{\frac{(1-a)}{a}} \right]^{-1} \\ &\quad - \frac{1 - \theta}{1 - a} [(1 - \theta)(\rho + \delta + \alpha) + \theta(\rho + \delta)]^{-1} \\ &\propto a - \frac{(1 - \theta)(\rho + \delta + \alpha) + \theta(\rho + \delta) \left(\frac{s^b + \sigma}{i + \sigma}\right)^{\frac{(1-a)}{a}}}{(1 - \theta)(\rho + \delta + \alpha) + \theta(\rho + \delta)}. \end{aligned}$$

The right side strictly falls in α and it is negative when $\alpha \rightarrow +\infty$. When $\alpha = 0$, the right side is negative if and only if (52) holds. Therefore if (52) holds, then $|\partial d(\varepsilon)/\partial i|$ falls in α and otherwise it is hump-shaped in α for $\alpha \in (0, +\infty)$. ■

B Two-sided banking markets: deposit and lending

In this appendix, we endogenize r_b by assuming that bank deposits serve to finance loans to entrepreneurs. The structure of the lending market is symmetric to the one of the deposit market, i.e., relations between banks and borrowers are bilateral and take time to form and the terms of the lending contracts are negotiated.³⁸ There is free entry of bankers in both the deposit and the lending markets. The two markets are connected through a competitive interbank market where banks can borrow/lend funds at rate r_b . An interpretation is that banks are large institutions composed of a multitude of agents, called bankers, that specialize in the different activities of the banks. One can think of r_b as a shadow price of funds within large banks.³⁹

B.1 The lending market

We introduce a unit measure of a new type of agents called *entrepreneurs*. These agents die at Poisson rate δ_e and are replaced by new entrepreneurs. They have linear preferences for the consumption of the numéraire good and discount the future at rate ρ . Each entrepreneur can operate a technology, $f(k)$, with $f(0) = 0$, $f' > 0$, $f'' < 0$, $f'(0) = +\infty$, and $f'(+\infty) = 0$, where k is capital. The numéraire good can be transformed into capital one to one, and vice versa. Capital does not depreciate. We denote k^* the solution to $f'(k^*) = \rho$. With no loss in generality, we describe loan contracts where entrepreneurs roll over the principal of the loan. At the time of their death, entrepreneurs' capital is used to repay their debt to the bank.

The matching technology in the lending market is denoted $\beta(\tau^\ell)$ where τ^ℓ is the measure of banks per entrepreneur. It satisfies $\beta(0) = 0$, $\beta' > 0$, $\beta'' < 0$, $\beta'(0) = +\infty$, and $\beta'(+\infty) = 0$. The matching rate of an entrepreneur is $\beta(\tau^\ell)$ while the matching rate of a bank is $\beta(\tau^\ell)/\tau^\ell$. The flow participation cost of a bank in the lending market is $\kappa^\ell > 0$.

B.2 Loan contracts

The lifetime expected discounted utility of an unbanked entrepreneur in a steady-state equilibrium, E^u , solves the following HJB equation:

$$\rho E^u = \beta(\tau^\ell) (E^b - E^u) - \delta_e E^u. \quad (84)$$

The unbanked entrepreneur meets a bank at rate $\beta(\tau^\ell)$, in which case it can finance its investment in k and enjoys the surplus $E^b - E^u$, and it dies at rate δ_e . The value function of a banked entrepreneur solves

$$\rho E^b = f(k) - r_b k - \phi^\ell - \delta_e E^b. \quad (85)$$

³⁸A similar description of the market for bank loans can be found in Rocheteau et al. (2018) and Bethune et al. (2021). Relative to these papers, here entrepreneurs cannot accumulate liquid assets ex-ante to finance their investment opportunity.

³⁹This interpretation is consistent with Drechsler et al. (2017, p.1844) who view "the decision of how many deposits to raise at a given branch [as] independent of the decision of how many loans to make at that branch" since deposits at one branch can be allocated to any lending opportunity received by the bank.

The banked entrepreneur produces the output flow $f(k)$ and repays the cost of the funds to the bank, $r_b k$, plus additional lending fees, ϕ^ℓ .

A lending contract is a pair, (k, ϕ^ℓ) , where k is the amount of capital financed by the bank and ϕ^ℓ are the flow profits of the bank. The surplus of the entrepreneur is

$$E^b - E^u = \frac{f(k) - r_b k - \phi^\ell - (\rho + \delta_e) E^u}{\rho + \delta_e}, \quad (86)$$

where $(\rho + \delta_e) E^u$ is the reservation utility of the entrepreneur. The surplus of the bank is $\Pi^\ell = \phi^\ell / (\rho + \delta_e)$. The terms of the lending contracts are determined by generalized Nash bargaining where the bargaining power of the bank is θ^ℓ , i.e.,

$$(k, \phi^\ell) \in \arg \max \left[E^b(k, \phi^\ell) - E^u \right]^{1-\theta^\ell} \left[\Pi^\ell(\phi^\ell) \right]^{\theta^\ell}. \quad (87)$$

The solution is:

$$f'(k) = r_b \quad (88)$$

$$\phi^\ell = \theta^\ell [f(k) - r_b k - (\rho + \delta_e) E^u]. \quad (89)$$

According to (88), the marginal product of capital is equal to the real interest rate in the interbank market. According to (89), the profits of the bank are equal to a fraction θ^ℓ of the match surplus.

Free entry in the lending market implies

$$\kappa^\ell = \frac{\beta(\tau^\ell)}{\tau^\ell} \Pi^\ell = \frac{\beta(\tau^\ell)}{\tau^\ell} \frac{\phi^\ell}{\rho + \delta_e}. \quad (90)$$

The bank entry cost, κ^ℓ , is equal to the expected discounted profits from a lending contract. By the same reasoning as in Section 3, the free entry condition and the bargaining solution can be used to rewrite banks' flow profits as:

$$\phi^\ell = \theta^\ell [f(k) - r_b k] - (1 - \theta^\ell) \tau^\ell \kappa^\ell. \quad (91)$$

The profits of the banks are equal to a fraction θ^ℓ of the entrepreneurs' profits net of a fraction $(1 - \theta^\ell)$ of banks' entry costs per entrepreneur. We interpret the lending rate as $r^\ell = r^b + \phi^\ell / k$, which is equal to:

$$r^\ell = r^b + \frac{\theta^\ell [f(k) - r_b k] - (1 - \theta^\ell) \tau^\ell \kappa^\ell}{k}. \quad (92)$$

The lending rate increases with banks' bargaining power.

B.3 Supply of loans

In order to determine the tightness of the lending market, we substitute ϕ^ℓ from (91) into (90):

$$(\rho + \delta_e) \kappa^\ell = \frac{\beta(\tau^\ell)}{\tau^\ell} \left\{ \theta^\ell [f(k) - r_b k] - (1 - \theta^\ell) \tau^\ell \kappa^\ell \right\}. \quad (93)$$

Market tightness, τ^ℓ , is a decreasing function of r_b . In a steady state, the measure of banked entrepreneurs is

$$n^e = \frac{\beta(\tau^\ell)}{\beta(\tau^\ell) + \delta_e}.$$

The aggregate demand for capital is defined as $K^d \equiv n^e k$, which can be reexpressed as:

$$K^d = \frac{\beta [\tau^\ell(r_b)]}{\beta [\tau^\ell(r_b)] + \delta_e} f'^{-1}(r_b). \quad (94)$$

It decreases with r_b from $+\infty$ when $r_b = 0$ to 0 when $r_b = +\infty$.

B.4 Interbank market and equilibrium

We can now close the model by using the market clearing condition in the interbank market to determine r_b . The supply of capital is equal to aggregate bank deposits:

$$K^s \equiv n^b d = \frac{\alpha [\tau^d(r_b; i)]}{\delta_c + \alpha [\tau^d(r_b; i)]} d(r_b), \quad (95)$$

where we now use the superscript “ d ” to refer to variables related to the deposit market, e.g., τ^d and κ^d , and the death rate of consumers is indexed by subscript “ c ”. From (8) and (16), both τ^d and d depend on r_b . As r_b increases, the measure of banks per consumer increases and a consumer’s demand for deposits increases as well. From (94) and (95), market clearing in the interbank market, $K^d = K^s$, can be rewritten as

$$\frac{\beta [\tau^\ell(r_b)]}{\beta [\tau^\ell(r_b)] + \delta_e} f'^{-1}(r_b) = \frac{\alpha [\tau^d(r_b; i)]}{\delta_c + \alpha [\tau^d(r_b; i)]} d(r_b). \quad (96)$$

An equilibrium of the banking market reduces to a $r_b \in (-\infty, \rho]$ that solves (96).

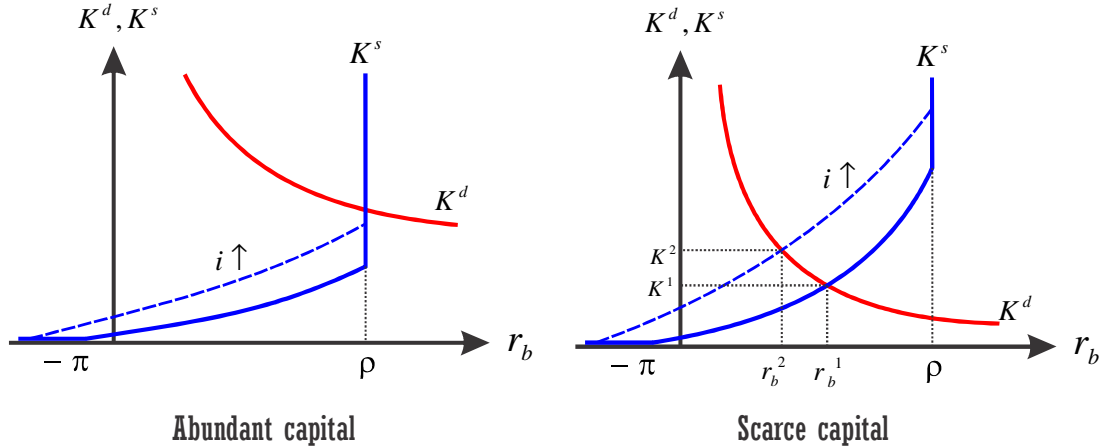


Figure 13: Equilibrium of the capital market

We define $\tau_0^d(i)$ as the solution to (16) when $r_b = \rho$, i.e., $s_b = 0$. Similarly, we define τ_0^ℓ as the unique solution to (93) if $r_b = \rho$.

Proposition 7 *There exists a unique general equilibrium of the deposit and lending markets. There are two regimes.*

1. Abundant capital. *If the following condition holds,*

$$\frac{\beta(\tau_0^\ell)}{\beta(\tau_0^\ell) + \delta_e} k^* \geq \frac{\alpha(\tau_0^d)}{\delta_c + \alpha(\tau_0^d)} y^*, \quad (97)$$

then $r_b = \rho$ and $k = k^$. If the inequality is strict, a small increase in i has no effect on aggregate capital, K , and r_b .*

2. Scarce capital. *If (97) does not hold, then $r_b < \rho$. An increase in i reduces r_b and raises K .*

3. Frictionless lending market. *Suppose $\kappa^\ell \rightarrow 0$. Then, $\tau^\ell \rightarrow +\infty$, $\beta(\tau^\ell) \rightarrow +\infty$, $n_e \rightarrow 1$, $K \rightarrow f'^{-1}(r_b)$, and $r_\ell - r_b \rightarrow 0$.*

Proof of Proposition 7.

Existence and uniqueness of the equilibrium. In order to establish the monotonicity of left side of (96), we use two observations: (i) $f'^{-1}(r_b)$ is decreasing in r_b from $+\infty$ to 0 (by the Inada conditions); from (93) τ^ℓ is decreasing in r^b . It follows that the left side of (96) is decreasing in r_b from $+\infty$ to 0 as r_b increases from 0 to $+\infty$. In order to establish the monotonicity of right side of (96), we observe that: $d(r_b) = u'^{-1}[1 + (\rho - r_b)/\sigma]$ is increasing in r_b from 0 when $r_b = -\infty$ to any $d \in [y^*, +\infty)$ as $r_b = \rho$ (d is set valued); from (16), τ^d is decreasing in s_b and hence increasing in r_b . It follows that the right side is increasing from 0 to $[y^*, +\infty)$ as r_b increases from $-\infty$ to ρ . From these characterizations of the left and right sides of (96) we conclude that there is a unique $r_b \in (0, \rho]$ solution to (96).

Regime with abundant capital. The solution to (96) is $r_b = \rho$ if and only if the left side of (96) intersects the right side in its vertical part, i.e.,

$$\frac{\beta(\tau_0^\ell)}{\beta(\tau_0^\ell) + \delta_e} k^* \in \left[\frac{\alpha(\tau_0^d)}{\delta + \alpha(\tau_0^d)} y^*, +\infty \right),$$

where we used that $f'^{-1}(\rho) = k^*$, and the notations according to which τ_0^ℓ and τ_0^d are the values of market tightness in the lending and deposit markets when $r_b = \rho$. If the inequality in (97) is strict, the equilibrium remains in the regime with abundant capital following a small change in i . It follows that a small change in i has no effect on $r_b = \rho$, and hence no effect on market tightness in the lending and deposit markets, and no effect on K .

Regime with scarce capital. If the inequality in (97) is not satisfied, i.e., the left side of (96) intersects the right side in its upward-sloping part, then $r_b < \rho$. An increase in i raises $\tau^d(r_b; i)$ and hence increases the right side of (96). For the interbank market to clear, r_b must decrease, which reduces K .

Finally, the proof of Part 3 is analogous to the proof of Part 4 of Proposition 1 and is therefore omitted.

■

There are two regimes.⁴⁰ In the first regime, k^* is large enough to back deposits when the real interest rate is ρ and the deposit spread is $s_b = 0$. In the second regime, k^* is too low relative to the demand for deposits. The real interest rate, r_b , falls below ρ .

The effects of monetary policy differ in the two regimes. In the first regime with abundant capital, an increase in i has no effect on r_b and K . Aggregate capital is unaffected because even though the measure of banked consumers increases, consumers hold deposits in excess of their spending needs. Hence, they can reduce their individual holdings to allow the interbank market to clear at the price $r_b = \rho$. Bank profits increase because the consumers' gains from being banked are larger.

In the second regime with scarce capital, an increase in i raises the aggregate capital stock and reduces the rate of return on deposits. It is the standard Tobin effect according to which an increase in the opportunity cost of holding money induces agents to substitute away from money into capital.

⁴⁰Proposition 7 is a generalization of the results in Lagos and Rocheteau (2008) who consider an economy where fiat money and capital compete as media of exchange. In our economy, capital is used to back bank deposits and banks have bargaining power.

C Search while banked

In the main text, consumers can search sequentially for the best contract, but once they accept a contract they keep it until there is exogenous separation. In this appendix, we introduce a connection between banks' market power and market concentration by allowing banked consumers to keep shopping for deposit contracts. We maintain the assumptions that consumers' preferences are private information.

In order to keep the model tractable, we define a household (or a collective of consumers) as a unit measure of heterogeneous consumers with preference types distributed according to $\Upsilon(\varepsilon)$.⁴¹ The role of the household is strictly limited to the search for a bank. Each consumer within the household is individualistic and unmonitored: she chooses her deposit contract in the menu offered by the bank and manages her own asset holdings, i.e., there is no pooling of asset holdings. Moreover, credit between members of a same household is not feasible as consumers are not trustworthy to repay their debt to anyone. The decision by the household to join a bank is based on a utilitarian criterion. We will show later that in equilibrium the decision to accept a bank's offer is unanimous within the household. The economy is composed of a unit measure of households and τ is now the measure of bankers per household. A banker can only manage the deposits of one household. The rate at which banked and unbanked households receive bank offers is $\alpha(\tau)$ and the matching rate of a banker is $\alpha(\tau)/\tau$. The death shock, δ , affects all members of a household. (Alternatively, we think of δ as an exogenous termination of the contract between the bank and the household.) In a steady state the measure of banked households is $n^b = \alpha/(\alpha + \delta)$.

A menu of contracts is a list, $\{\nu(\varepsilon), d(\varepsilon)\}$, composed of a contract for each consumer type.⁴² It is posted before matches are formed and cannot be made contingent on the banking status of the household, which is private information. The value of a menu for the household is $\boldsymbol{\nu} = \int \nu(\varepsilon) d\Upsilon(\varepsilon)$.

Distribution of deposit contracts We distinguish two equilibrium distributions for $\boldsymbol{\nu}$. The cumulative distribution of the values of menus posted by banks is $F(\boldsymbol{\nu})$. The cumulative distribution of $\boldsymbol{\nu}$ across banked households is $G(\boldsymbol{\nu})$. The two distributions are related as follows. The flow of unbanked households who are offered deposit contracts worth less than $\boldsymbol{\nu}$ is $\alpha n^u F(\boldsymbol{\nu})$ where n^u is the measure of unbanked households. The flow of banked households with a menu of contracts worth less than $\boldsymbol{\nu}$ who obtain a new menu of contracts with utility greater than $\boldsymbol{\nu}$ or who become unbanked is $n^b G(\boldsymbol{\nu}) \{\delta + \alpha [1 - F(\boldsymbol{\nu})]\}$. In a steady state these two flows are equal, which gives

$$G(\boldsymbol{\nu}) = \frac{\delta F(\boldsymbol{\nu})}{\delta + \alpha [1 - F(\boldsymbol{\nu})]}, \quad (98)$$

where we used that $n^u/n^b = \delta/\alpha$.

⁴¹This formulation simplifies the bank's problem as the bank only has to think about attracting and retaining a representative collective of consumers instead of considering the trade-off between flow profits and turnover for each individual consumer. Papers that tackle this problem with two states include Garrett et al. (2019) and Lester et al. (2019).

⁴²We restrict the contracts posted by banks to be stationary. For instance, banking fees cannot depend on how long the collective has been the client of a given bank. For a model where the terms of the contracts can depend on the duration of the match, see, e.g., Stevens (2004) in the context of the labor market.

The discounted sum of the profits of the bank are:

$$\Pi(\boldsymbol{\nu}) = \frac{\Phi(\boldsymbol{\nu})}{\rho + \delta + \alpha [1 - F(\boldsymbol{\nu})]}, \quad (99)$$

where $\Phi(\boldsymbol{\nu}) \equiv \int_0^{\bar{\varepsilon}} \phi(\varepsilon; \boldsymbol{\nu}) d\Upsilon(\varepsilon)$ represents the flow profits from a menu of contracts guaranteeing utility $\boldsymbol{\nu}$ to the household. The effective discount rate of the flow profits, the denominator of (99), is the sum of the rate of time preference, ρ , the death rate, δ , and the quit rate, $\alpha [1 - F(\boldsymbol{\nu})]$.

Optimal search strategy The value function of an unbanked household, $W^u = \int_0^{\bar{\varepsilon}} V^u(\varepsilon) d\Upsilon(\varepsilon)$, solves:

$$(\rho + \delta)W^u = U(i) + \alpha(\tau) \int \max\{W^b(\boldsymbol{\nu}) - W^u, 0\} dF(\boldsymbol{\nu}), \quad (100)$$

where $U(i) = \int_0^{\bar{\varepsilon}} U(\varepsilon; i) d\Upsilon(\varepsilon)$ is the sum of the flow utilities of the members of the household and $W^b(\boldsymbol{\nu}) = \int_0^{\bar{\varepsilon}} V^b(\varepsilon; \boldsymbol{\nu}) d\Upsilon(\varepsilon)$ is the average expected utility of a household under a menu of contracts worth $\boldsymbol{\nu}$. The unbanked household receives a bank offer at Poisson rate $\alpha(\tau)$ and it accepts the offer if $W^b(\boldsymbol{\nu}) \geq W^u$. The value function of a banked household solves:

$$(\rho + \delta)W^b(\boldsymbol{\nu}) = \boldsymbol{\nu} + \alpha(\tau) \int \max\{W^b(\boldsymbol{x}) - W^b(\boldsymbol{\nu}), 0\} dF(\boldsymbol{x}). \quad (101)$$

The household enjoys the aggregate utility flow, $\boldsymbol{\nu}$, and receives an alternative bank offer at Poisson rate $\alpha(\tau)$. Since $W^b(\boldsymbol{\nu})$ is increasing, a household switches to a new bank whenever it receives an offer of higher value than his current offer, $\boldsymbol{x} > \boldsymbol{\nu}$. The comparison of (100) and (101) shows that $W^u = W^b(\boldsymbol{\nu})$ if $\boldsymbol{\nu} = U(i)$. Hence, an unbanked household accepts any offer such that $\boldsymbol{\nu} \geq U(i)$.

Optimal deposit contracts We now turn to the characterization of $\Phi(\boldsymbol{\nu})$, the maximum profits of a bank that guarantees utility $\boldsymbol{\nu}$ to a household. The bank's optimal control problem is:

$$\Phi(\boldsymbol{\nu}) \equiv \max_{\{(\nu(\varepsilon), d(\varepsilon))\}} \int_0^{\bar{\varepsilon}} \{-\nu(\varepsilon) - (s_b + \sigma)d(\varepsilon) + \sigma \varepsilon u[d(\varepsilon)]\} d\Upsilon(\varepsilon) \quad (102)$$

$$\text{s.t. } \nu'(\varepsilon) = \sigma u[d(\varepsilon)] \quad (103)$$

$$\nu(\varepsilon) \geq U(\varepsilon; i) \quad (104)$$

$$\int_0^{\bar{\varepsilon}} \nu(\varepsilon) d\Upsilon(\varepsilon) \geq \boldsymbol{\nu} \quad (105)$$

The novelty relative to the monopoly problem in the previous section is the constraint, (105), which specifies that the menu of contracts must generate a flow utility for the household at least equal to $\boldsymbol{\nu}$. The participation constraint, (104), allows each member of a household to opt out of the banking contract and hold real money balances instead.

The utility that a banking contract can offer in any equilibrium is bounded below by the value of the pure monopoly offer, $\boldsymbol{\nu}^m$, characterized in Proposition 2. The upper bound, denoted $\boldsymbol{\nu}^c$, corresponds to the case where banks make no profits, $\phi(\varepsilon) = 0$ for all ε , and deposits correspond to their complete-information level, namely $d(\varepsilon) = u'^{-1}[(1 + s_b/\sigma)/\varepsilon]$ for all ε .

Proposition 8 (Optimal deposit contracts.) A menu of deposits contracts of value $\nu \in [\nu^m, \nu^c]$ is characterized as follows. The deposit levels are:

$$u' [d(\varepsilon)] = \frac{i + \sigma}{\varepsilon \sigma} \text{ for all } \varepsilon \leq \tilde{\varepsilon}, \quad (106)$$

$$u' [d(\varepsilon)] = \left\{ \varepsilon - (1 - \zeta) \left[\frac{1 - \Upsilon(\varepsilon)}{\gamma(\varepsilon)} \right] \right\}^{-1} \left(1 + \frac{s_b}{\sigma} \right) \text{ for all } \varepsilon \geq \tilde{\varepsilon}, \quad (107)$$

where $\tilde{\varepsilon}$ is the unique solution to

$$\left(\frac{i - s_b}{i + \sigma} \right) \tilde{\varepsilon} = (1 - \zeta) \left[\frac{1 - \Upsilon(\tilde{\varepsilon})}{\gamma(\tilde{\varepsilon})} \right], \quad (108)$$

and $\zeta \in [0, 1]$ is the unique solution to

$$\int_0^{\tilde{\varepsilon}} \varpi \left(\frac{i + \sigma}{x \sigma} \right) [1 - \Upsilon(x)] dx + \int_{\tilde{\varepsilon}}^{\bar{\varepsilon}} \varpi \left[\left[x - (1 - \zeta) \left[\frac{1 - \Upsilon(x)}{\gamma(x)} \right] \right]^{-1} \left(1 + \frac{s_b}{\sigma} \right) \right] [1 - \Upsilon(x)] dx = \frac{\nu}{\sigma}, \quad (109)$$

where $\varpi(x) \equiv u \circ u'^{-1}(x)$.

The menu of contracts offered by banks depend on the shadow price, ζ , which is the Lagrange multiplier associated with the constraint (105), i.e., $\int_0^{\bar{\varepsilon}} [\nu(\varepsilon) - \nu] d\Upsilon(\varepsilon) \geq 0$.⁴³ Equivalently, $\zeta = -\Phi'(\nu)$, which can be interpreted as the slope of the contract curve between banks and households of consumers. From (109) there is a positive relationship between ζ and ν . If $\zeta = 0$, the offer is the one of a monopoly bank. If $\zeta = 1$, then the offer is the one of a competitive bank that makes no profit under complete information.

All menus of deposit contracts have the same two-tier structure where, from (108), the threshold, $\tilde{\varepsilon}$, separating the two tiers decreases with ζ and approaches 0 as ζ tends to 1. Only deposit levels below $\tilde{\varepsilon}$ depend on the policy rate, i . From (107), $d(\varepsilon)$ increases in ζ for all $\varepsilon \geq \tilde{\varepsilon}$. Banks that offer higher utility levels to consumers distort deposits less relative to the complete information case.

The next Corollary shows that in equilibrium the decision to switch to a different bank is unanimous as all members of a household order banks' offers according to ν .⁴⁴

Corollary 1 (Ranking of menus of deposits contracts.) The flow utilities, $\nu(\varepsilon; \nu)$, are nondecreasing in ν for all $\varepsilon \in [0, \bar{\varepsilon}]$.

Equilibrium distribution of banks' offers From the free-entry condition, any contract offered in equilibrium must solve:

$$\kappa = \frac{\alpha(\tau)}{\tau} [n^u + n^b G(\nu)] \Pi(\nu) \quad \forall \nu \in \text{supp}(F). \quad (110)$$

The bank finds a household at rate α/τ . This household accepts the menu of contracts if it is unbanked or it is banked but its utility is less than what the bank offers. Substituting $G(\nu)$ by its expression given by

⁴³We added this constraint to the objective in (102) after multiplying it by a Lagrange multiplier, ζ . This integral constraint on the collective utility is formally called an *isoperimetric constraint*. The way we introduce the Lagrange multiplier ζ is consistent with the Isoperimetric Theorem in Chapter 3-8 in Pierre (1986).

⁴⁴This property is analogous to the rank-ordering of the menu of contracts in Garrett et al. (2019) and Lester et al. (2019).

(98) and substituting $\Pi(\boldsymbol{\nu})$ by the expression given by (99),

$$\kappa = \frac{\alpha(\tau)}{\tau} \frac{\delta\Phi(\boldsymbol{\nu})}{\{\delta + \alpha(\tau)[1 - F(\boldsymbol{\nu})]\} \{\rho + \delta + \alpha(\tau)[1 - F(\boldsymbol{\nu})]\}} \quad \forall \boldsymbol{\nu} \in \text{supp}(F). \quad (111)$$

We can now characterize the distribution of posted offers, F .

Lemma 6 (*Distributions of deposit contracts.*) *The equilibrium distribution of banks' offers is*

$$F(\boldsymbol{\nu}) = \frac{2(\alpha + \delta) + \rho - \sqrt{\rho^2 + 4\frac{\alpha}{\tau\kappa}\delta\Phi(\boldsymbol{\nu})}}{2\alpha} \quad \text{for all } \boldsymbol{\nu} \in [\boldsymbol{\nu}^m, \bar{\boldsymbol{\nu}}], \quad (112)$$

where τ solves

$$\kappa = \frac{\alpha(\tau)}{\tau} \frac{\delta\Phi(\boldsymbol{\nu}^m)}{[\delta + \alpha(\tau)][\rho + \delta + \alpha(\tau)]}, \quad (113)$$

and $\bar{\boldsymbol{\nu}}$ solves

$$\Phi(\bar{\boldsymbol{\nu}}) = \frac{\delta(\rho + \delta)\Phi(\boldsymbol{\nu}^m)}{[\delta + \alpha(\tau)][\rho + \delta + \alpha(\tau)]}. \quad (114)$$

The distribution of bank offers is continuous over the support $[\boldsymbol{\nu}^m, \bar{\boldsymbol{\nu}}]$. The least attractive offer is the monopoly offer while the most attractive offer, $\bar{\boldsymbol{\nu}}$, has a value less than the complete-information offer. The distribution admits no mass point by a similar logic as in Burdett and Judd (1983). If there was a mass point at some $\boldsymbol{\nu}$, some banks would deviate and offer a marginally better menu of contracts worth $\boldsymbol{\nu} + \epsilon$ so as to attract a strictly larger flow of consumers. The tightness of the deposits market is uniquely determined by (113) which describes the entry decision of a bank offering the monopoly menu of deposit contracts. The best offer is determined by (114) so that banks are indifferent between making this offer that retains all the consumers until there is exogenous separation and the monopoly offer that retains consumers only until they find any other offer.

We are now ready to define an equilibrium as a list $\langle F(\boldsymbol{\nu}), \Phi(\boldsymbol{\nu}), \tau \rangle$ solution to (102), and (112) and (113). The equilibrium has a simple recursive structure. The value of the contracts, $\Phi(\boldsymbol{\nu})$, can be solved independently. Given $\Phi(\boldsymbol{\nu}^m)$, one can determine market tightness from (113). Given τ , one can compute the meeting rate with banks, $\alpha(\tau)$, and the distribution of offers from (112). The next proposition establishes a relation between market concentration and bank offers.

Proposition 9 (*Market concentration and bank offers*) *Consider two markets h and ℓ that differ by their entry costs, $\kappa^h > \kappa^\ell$.*

1. *Bank concentration in market h is larger than that in market ℓ , i.e., $\tau^h < \tau^\ell$.*
2. *Market ℓ offers more competitive contracts than market h , i.e., $\bar{\boldsymbol{\nu}}^h < \bar{\boldsymbol{\nu}}^\ell$.*
3. *The rate of bank-to-bank transitions, $\alpha(\tau)[1 - F(\boldsymbol{\nu})]$, is larger in market ℓ than market h .*

If $\alpha(\tau) = \bar{\alpha}$, then F and G decrease in the first-order stochastic dominance sense as κ rises.

In Proposition 9 we generate differences in concentration across markets through differences in entry costs — markets with higher costs have fewer banks per consumer. Banks offer contracts of higher value in the less concentrated market. Moreover, consumers in that market transition more often between banks. The next proposition describes how the deposits channel operates in an economy where consumers transition from bank to bank.

Proposition 10 (*Monetary policy and bank market power.*)

1. Deposits channel and market power. Assume $\alpha(\tau) = \bar{\alpha}$, $u(y) = y^{1-a}/(1-a)$ where $a \in (0, 1)$. Suppose $i \approx s_b$ and consider an infinitesimal increase in i .

(a) Bank concentration, $1/\tau$, decreases.

(b) The distributions of bank offers, G and F , decrease in the first-order stochastic dominance sense.

(c) The distribution of deposits per bank falls in the first-order stochastic dominance sense.

2. Monetary policy at the frictionless limit. As $\bar{\alpha} \rightarrow +\infty$, $1/\tau \rightarrow +\infty$. The distribution, F , tends to

$$F(\boldsymbol{\nu}) = 1 - \sqrt{\frac{\Phi(\boldsymbol{\nu})}{\Phi(\boldsymbol{\nu}^m)}} \quad (115)$$

and G converges to a mass point at $\boldsymbol{\nu} = \boldsymbol{\nu}^c$, i.e., $\hat{i}_d(\varepsilon) = i_b$ and $\hat{s}_d(\varepsilon) = s_b$ for all ε . Aggregate deposits, D , are independent of i .

An increase in i worsens consumers' outside options and raises bank profits at all ζ . Hence, more banks enter and τ rises, i.e., the market becomes less concentrated. This lower concentration in the deposit market does not imply that banks have less market power since bank entry is triggered by higher rents. In fact, as i rises, the distributions of bank offers and bank deposits worsen in a first-order stochastic sense. In the top-left panel of Figure 14 we illustrate how the probability density of $\boldsymbol{\nu}$ among the banked consumers, g , changes with i .⁴⁵ As i rises the probability mass moves to the left and becomes more spread out.

At the frictionless limit $\bar{\alpha} \rightarrow \infty$, almost all consumers enjoy the competitive contract, $\boldsymbol{\nu} = \boldsymbol{\nu}^c$. Banks earn vanishingly small profits and market tightness approaches 0. The passthrough from the policy rate to deposit rates is one and the passthrough to the deposit spread is zero. The deposits channel vanishes in that aggregate deposits do not depend on i .

In the top-right panel of Figure 14, we present a numerical example to show how the spread of each menu varies with ε . For $\varepsilon < \tilde{\varepsilon}$, the spread equals to i and for $\varepsilon \geq \tilde{\varepsilon}$ it falls monotonically in ε . In the bottom-left panel, we illustrate how average bank deposits, D , vary with the policy rate. As $\bar{\alpha}$ rises, the

⁴⁵The parameters are $u(y) = y^{0.8}/0.8$, $\rho = 0.05$, $\sigma = 0.5$, $s_b^2 = 0$, $\varepsilon \sim \text{Exp}(2)$, $\chi_2 = 0.8$, $\bar{\alpha} = 0.1$ and $i = 0.1$.

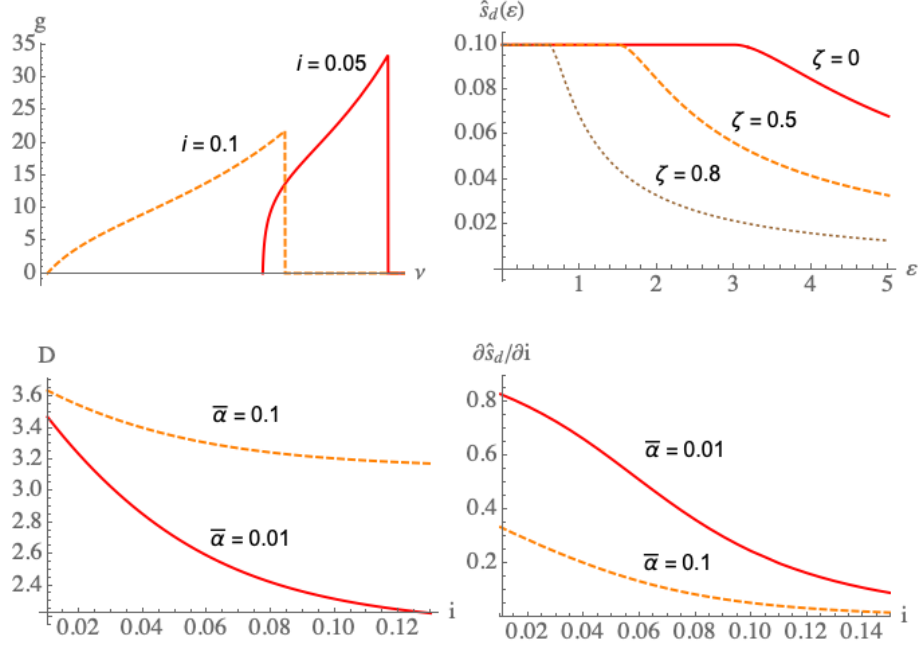


Figure 14: (Top-left) Distribution of ν (Top-right) Deposit spread of various menus (Bottom-left) Aggregate deposit (Bottom-right) Average deposit passthrough.

deposit market becomes more competitive and the aggregate deposit become less sensitive to a change in i . In the bottom-right panel we define the average deposit spread as

$$\hat{s}_d \equiv s_b + \frac{\int_{\nu^m}^{\bar{\nu}} \Phi(\nu) dG(\nu)}{\int_{\nu^m}^{\bar{\nu}} D(\nu) dG(\nu)}$$

and we plot the passthrough, $\partial \hat{s}_d / \partial i$. The average spread rises in i while the passthrough falls in i . As $\bar{\alpha}$ rises, the market is more competitive and the spread becomes less sensitive to changes in i .

Proofs of propositions and lemmas

Proof of Proposition 8. Setting up the Hamiltonian. We add the constraint (105) to the objective, (102), to form the following Lagrangian:

$$\mathcal{L}[\nu(\varepsilon), d(\varepsilon), \xi(\varepsilon), \zeta] = \int_0^{\bar{\varepsilon}} \{-\nu(\varepsilon) - (s_b + \sigma)d(\varepsilon) + \sigma \varepsilon u[d(\varepsilon)] + \xi(\varepsilon)[\nu(\varepsilon) - U(\varepsilon; i)] + \zeta[\nu(\varepsilon) - \nu]\} dY(\varepsilon), \quad (116)$$

where ζ is the Lagrange multiplier associated with the constraint (105) and $\xi(\varepsilon)$ is the Lagrange multiplier associated with the participation constraints, (104). The Lagrangian is maximized with respect to $\nu(\varepsilon)$ and $d(\varepsilon)$ given $\zeta \geq 0$ and $\xi(\varepsilon) \geq 0$ and subject to (103). The Lagrange multiplier, $\zeta \geq 0$, satisfies complementary slackness:

$$\zeta \left[\int_0^{\bar{\varepsilon}} \nu(\varepsilon) dY(\varepsilon) - \nu \right] = 0. \quad (117)$$

Similarly, the Lagrange multipliers, $\xi(\varepsilon) \geq 0$, satisfy the complementary slackness condition,

$$\xi(\varepsilon) [\nu(\varepsilon) - U(\varepsilon; i)] = 0. \quad (118)$$

From (116) we build the Hamiltonian function:

$$H(\nu, d, \mu, \xi, \zeta, \varepsilon) \equiv \{-\nu - (s_b + \sigma)d + \sigma \varepsilon u(d) + \zeta(\nu - \nu)\} \gamma(\varepsilon) + \mu \sigma u(d) + \xi[\nu - U(\varepsilon; i)],$$

where $\mu(\varepsilon)$ is the costate variable. From the Maximum Principle, the necessary conditions for an optimum are:

$$[\varepsilon \gamma(\varepsilon) + \mu(\varepsilon)] \sigma u'(d(\varepsilon)) = (s_b + \sigma) \gamma(\varepsilon) \quad (119)$$

$$\mu'(\varepsilon) = (1 - \zeta) \gamma(\varepsilon) - \xi(\varepsilon). \quad (120)$$

Equation (119) corresponds to the first-order condition, $\partial H / \partial d = 0$. Equation (120) is the law of motion of the costate variable, $\mu'(\varepsilon) = -\partial H / \partial \nu$. The key novelty relative to the problem of a pure monopoly is the term $1 - \zeta$ on the right side of (120) that scales down the co-state variable. The solution must also satisfy the terminal condition for a free-end-point problem,

$$\mu(0)\nu(0) = \mu(\tilde{\varepsilon})\nu(\tilde{\varepsilon}) = 0. \quad (121)$$

The characterization of the solution follows the same steps as in the proof of Proposition 2.

Lower tier of bank's mechanism. We conjecture a solution such that there is a threshold, $\tilde{\varepsilon}$, below which participation constraints bind, $\xi > 0$. At $\varepsilon = 0$, $\nu(0) = U(0; i) = 0$. Since $\nu(\varepsilon) = U(\varepsilon; i)$ for all $\varepsilon < \tilde{\varepsilon}$, then $\nu'(\varepsilon) = \partial U(\varepsilon; i) / \partial \varepsilon$ for all $\varepsilon < \tilde{\varepsilon}$, which implies $\sigma u[d(\varepsilon)] = \sigma u[m(\varepsilon)]$ where $m(\varepsilon)$ is the solution to $\varepsilon u'[m(\varepsilon)] = (i + \sigma) / \sigma$. Hence, for all $\varepsilon < \tilde{\varepsilon}$, $d(\varepsilon) = m(\varepsilon)$ solves (106). From (119) the value of the costate variable is

$$\mu(\varepsilon) = \left(\frac{s_b - i}{i + \sigma} \right) \varepsilon \gamma(\varepsilon) \text{ for all } \varepsilon < \tilde{\varepsilon}. \quad (122)$$

From (120) the expression for the Lagrange multiplier associated with the participation constraint is $\xi(\varepsilon) = (1 - \zeta) \gamma(\varepsilon) - \mu'(\varepsilon)$. Substituting $\mu'(\varepsilon)$ by the derivative of $\mu(\varepsilon)$ defined in (122) we obtain

$$\xi(\varepsilon) = (1 - \zeta) \gamma(\varepsilon) + \left(\frac{i - s_b}{i + \sigma} \right) [\gamma(\varepsilon) + \varepsilon \gamma'(\varepsilon)] \text{ for all } \varepsilon < \tilde{\varepsilon}.$$

By the same reasoning as in the proof of Proposition 2, we can check that $\xi(\varepsilon) > 0$ for all $\varepsilon \in (0, \tilde{\varepsilon})$.

Upper tier of bank's mechanism. Above the threshold, $\tilde{\varepsilon}$, consumers' participation constraints are slack, i.e., for all $\varepsilon > \tilde{\varepsilon}$, $\xi = 0$. Hence, from (120),

$$\begin{aligned} \mu(\varepsilon) &= \mu(\tilde{\varepsilon}) + (1 - \zeta) \int_{\tilde{\varepsilon}}^{\varepsilon} \gamma(\varepsilon) d\varepsilon \\ &= \left(\frac{s_b - i}{i + \sigma} \right) \tilde{\varepsilon} \gamma(\tilde{\varepsilon}) + (1 - \zeta) [\Upsilon(\varepsilon) - \Upsilon(\tilde{\varepsilon})], \end{aligned} \quad (123)$$

where the second equality is obtained by replacing $\mu(\tilde{\varepsilon})$ by its expression given by (122). The condition $\mu(\bar{\varepsilon}) = 0$ gives (108). Moreover, (123) simplifies to

$$\mu(\varepsilon) = -(1 - \zeta) [1 - \Upsilon(\varepsilon)] \quad \text{for all } \varepsilon > \tilde{\varepsilon}. \quad (124)$$

By substituting the expression for $\mu(\varepsilon)$ given by (124) into (119) we show that deposits solve (107).

Determination of the Lagrange multiplier, ζ . Integrating (103) and using that $\nu(0) = 0$, the utility of consumer ε is

$$\nu(\varepsilon) = \int_0^\varepsilon \sigma u[d(x; \zeta)] dx. \quad (125)$$

By definition, $\boldsymbol{\nu} \equiv \int_0^{\bar{\varepsilon}} \nu(\varepsilon) d\Upsilon(\varepsilon)$, which, from (125), can be reexpressed as

$$\int_0^{\bar{\varepsilon}} \int_0^{\bar{\varepsilon}} \mathbb{I}_{\{x < \varepsilon\}} u[d(x; \zeta)] dx d\Upsilon(\varepsilon) = \frac{\boldsymbol{\nu}}{\sigma}. \quad (126)$$

By changing the order of integration (126) can be rewritten as:

$$\int_0^{\bar{\varepsilon}} \int_0^{\bar{\varepsilon}} \mathbb{I}_{\{\varepsilon > x\}} d\Upsilon(\varepsilon) u[d(x; \zeta)] dx = \frac{\boldsymbol{\nu}}{\sigma}. \quad (127)$$

Using that $\int_0^{\bar{\varepsilon}} \mathbb{I}_{\{\varepsilon > x\}} d\Upsilon(\varepsilon) = 1 - \Upsilon(x)$, (127) becomes

$$\int_0^{\bar{\varepsilon}} u[d(x; \zeta)] [1 - \Upsilon(x)] dx = \frac{\boldsymbol{\nu}}{\sigma}. \quad (128)$$

From (106)-(107), using the notation $\varpi(x) \equiv u \circ u'^{-1}(x)$,

$$\begin{aligned} u[d(x; \zeta)] &= \varpi\left(\frac{i + \sigma}{x\sigma}\right) \quad \text{for all } x \leq \tilde{\varepsilon}, \\ u[d(x; \zeta)] &= \varpi\left\{\left\{x - (1 - \zeta) \left[\frac{1 - \Upsilon(x)}{\gamma(x)}\right]\right\}^{-1} \left(1 + \frac{s_b}{\sigma}\right)\right\} \quad \text{for all } x \geq \tilde{\varepsilon}. \end{aligned}$$

Substituting these expressions into (128) we obtain (109). Using that $u[d(x; \zeta)]$ is increasing in ζ for all $x \geq \tilde{\varepsilon}$, the second term on the left side of (109) is an increasing function of ζ . When $\zeta = 0$, then $u[d(x; \zeta)]$ and $\tilde{\varepsilon}$ are equal to their expressions in Proposition 2 that characterizes the menu of deposits contracts offered by a monopoly. Hence, $\boldsymbol{\nu} = \boldsymbol{\nu}^m$. When $\zeta = 1$, then, from (108), $\tilde{\varepsilon} = 0$, and $\varepsilon u'[d(\varepsilon)] = (1 + s_b/\sigma)$ for all ε , which corresponds to the deposit levels under complete information. Equation (109) reduces to

$$\begin{aligned} \boldsymbol{\nu} &= \int_0^{\bar{\varepsilon}} \sigma u[d^c(\varepsilon)] [1 - \Upsilon(\varepsilon)] d\varepsilon \\ &= \int_0^{\bar{\varepsilon}} \sigma u[d^c(\varepsilon)] \int_0^{\bar{\varepsilon}} \mathbb{I}_{\{x \geq \varepsilon\}} d\Upsilon(x) d\varepsilon \\ &= \int_0^{\bar{\varepsilon}} \int_0^x \sigma u[d^c(\varepsilon)] d\varepsilon d\Upsilon(x) \\ &= \int_0^{\bar{\varepsilon}} \nu^c(x) d\Upsilon(x) = \boldsymbol{\nu}^c. \end{aligned}$$

where the second equality is obtained by using the definition of $1 - \Upsilon(\varepsilon)$; the third equality follows from changing the order of integration; and the fourth equality follows from the definition of $\nu(x)$. We used the

superscript “c” to indicate complete-information levels. In summary, we showed that $\nu = \nu^c$ when $\zeta = 1$. So Equation (109) determines a unique $\zeta \in [0, 1]$ for all $\nu \in [\nu^m, \nu^c]$ and ζ is an increasing function of ν .

Sufficiency. By the same reasoning as in Proposition 2, the Hamiltonian function, $H(\nu, d, \mu, \xi, \zeta, \varepsilon)$, is jointly concave in (ν, d) given our solution to the necessary conditions for μ . Hence, by the Mangasarian sufficiency conditions, our proposed solution is an optimum. ■

Proof of Corollary 1. This result follows directly from (125),

$$\nu(\varepsilon) = \int_0^\varepsilon \sigma u[d(x; \zeta)] dx,$$

and the fact that, from (106)-(107), $d(x; \zeta)$ are nondecreasing in ζ while from (109) ζ is increasing in ν . ■

Proof of Lemma 6. By the standard argument from Burdett and Judd (1983), the distribution has no mass point. By contradiction, suppose there is a ν_0 such that $F(\nu_0^+) - F(\nu_0^-) > 0$. A bank offering ν_0 can deviate and offer $\nu_0 + \varepsilon$, where $\varepsilon \approx 0$, thereby raising its profits by $\alpha [F(\nu_0^+) - F(\nu_0^-)] \Phi(\nu_0 + \varepsilon) > 0$. Second, banks offering the lowest point on the support of F cannot retain any household, i.e., households move to a different bank as soon as they get an opportunity. Hence, such banks cannot do better than offering the monopoly menu of contracts, $\underline{\nu} = \nu^m$, in which case $F(\nu^m) = 0$. From (111) this gives (113), which pins down tightness in the banking market. The best contract offered in equilibrium is $\nu = \bar{\nu}$ such that $F(\bar{\nu}) = 1$, which from (111) gives (114). The fact that $\Phi(\bar{\nu}) > 0$ implies that $\bar{\nu} < \nu^c$. Finally, we can reexpress (111) as a quadratic equation in $1 - F(\nu)$,

$$\alpha^2 [1 - F(\nu)]^2 + [\alpha(\rho + \delta) + \alpha\delta] [1 - F(\nu)] - \left[\frac{\alpha}{\tau\kappa} \delta \Phi(\nu) - \delta(\rho + \delta) \right] = 0. \quad (129)$$

The product of the two roots is equal to

$$\frac{-\delta \left[\frac{\alpha}{\tau\kappa} \Phi(\nu) - (\rho + \delta) \right]}{\alpha^2} < 0,$$

where we used that, from (111), $\frac{\alpha(\tau)}{\tau\kappa} \Phi(\nu) - (\rho + \delta) > 0$. We solve (129) in closed form and retain the positive root to obtain (112). ■

Proof of Proposition 9. Claim 1. Market tightness is uniquely determined by (113). The right side of (113) is decreasing in τ . Hence, $\kappa^h > \kappa^\ell$ implies $\tau^h < \tau^\ell$.

Claim 2. From (114), $\Phi(\bar{\nu})$ is a decreasing function of τ . From (102), $\Phi(\nu)$ is a decreasing function of ν . Hence, $\tau^h < \tau^\ell$ implies $\bar{\nu}^h < \bar{\nu}^\ell$.

Claim3. Denote $z = \alpha [1 - F(\nu)]$ and rewrite (129)

$$z^2 + (\rho + 2\delta)z - \left[\frac{\alpha}{\tau\kappa} \delta \Phi(\nu) - \delta(\rho + \delta) \right] = 0. \quad (130)$$

From (113)

$$\frac{\alpha(\tau)}{\tau\kappa} = \frac{[\delta + \alpha(\tau)] [\rho + \delta + \alpha(\tau)]}{\delta \Phi(\nu^m)}$$

where the right side rises in τ . From Claim 1, $\tau^h < \tau^\ell$ and hence $\alpha(\tau^h)/(\tau^h \kappa^h) < \alpha(\tau^\ell)/(\tau^\ell \kappa^\ell)$. So the parabola representing the quadratic equation (130) when $\kappa = \kappa^\ell$ is located below the parabola when $\kappa = \kappa^h$. Hence, the positive root of (130) is such that $z^\ell > z^h$.

If $\alpha(\tau) = \bar{\alpha}$, then clearly $F(\boldsymbol{\nu})$ increases in κ . By (98) $G(\boldsymbol{\nu})$ also increases in the first-order stochastic dominance sense in κ . ■

Proof of Proposition 10. Claim 1. The profit $\Phi(\boldsymbol{\nu}^m)$ rises in i because an increase in i relaxes the participation constraints (104) of the bank's problem and hence raises its profits. Therefore, by (113), τ rises in i .

Claim 2. By equation (109), there is a one-to-one mapping between $\boldsymbol{\nu}$ and the Lagrange multiplier $\zeta = \Phi'(\boldsymbol{\nu})$ of (105). Since each contract is indexed by $\boldsymbol{\nu}$, we can also index each contract by ζ . Let \hat{F} be the distribution of ζ among menu of contracts posted by banks. We first show \hat{F} falls in i in the first-order stochastic dominance (FOSD) sense. Let Φ_ζ be the profit of a bank when the value of the Lagrange multiplier is ζ . By (112) and (113), \hat{F} is given by

$$\hat{F}(\zeta) = \frac{1}{2\bar{\alpha}} \left(2(\bar{\alpha} + \delta) + \rho - \sqrt{\rho^2 + 4(\delta + \bar{\alpha})(\rho + \delta + \bar{\alpha}) \frac{\Phi_\zeta}{\Phi_0}} \right).$$

If the ratio Φ_ζ/Φ_0 falls in i for all $\zeta \in (0, 1]$, then $\hat{F}(\zeta)$ rises and the distribution of posted ζ falls in i in the FOSD sense. To show Φ_ζ/Φ_0 falls in i , it suffices to show $\partial \ln(\Phi_\zeta)/\partial \zeta \partial i \leq 0$. The sign of this cross derivative is the same as

$$\frac{\partial \ln(\Phi_\zeta)}{\partial \zeta \partial i} \propto \frac{\partial^2 \Phi_\zeta / \partial \zeta \partial i}{\partial \Phi_\zeta / \partial i} - \frac{\partial \Phi_\zeta / \partial \zeta}{\Phi_\zeta}. \quad (131)$$

We argue the right side is strictly negative when $i \approx s_b$. By (102) we can calculate the derivative $\partial \Phi_\zeta / \partial i$ as

$$\begin{aligned} \frac{\partial \Phi_\zeta}{\partial i} &= \frac{\partial}{\partial i} \int_0^{\bar{\varepsilon}} \left\{ - \left(\int_0^{\varepsilon} \sigma u[d(z; \zeta)] dz \right) - (s_b + \sigma) d(\varepsilon; \zeta) + \sigma \varepsilon u[d(\varepsilon; \zeta)] \right\} d\Upsilon(\varepsilon) \\ &= \int_0^{\bar{\varepsilon}} \frac{d(\varepsilon; \zeta)}{a} \left(\frac{1 - \Upsilon(\varepsilon)}{\varepsilon \gamma(\varepsilon)} - \frac{i - s_b}{i + \sigma} \right) d\Upsilon(\varepsilon) > 0. \end{aligned} \quad (132)$$

The integrand in the second line is strictly positive by (106), (108) and the log-concavity of $1 - \Upsilon$. Next, by (132) the cross derivative is given by

$$\frac{\partial^2 \Phi_\zeta}{\partial i \partial \zeta} = - \frac{\zeta d(\bar{\varepsilon}; \zeta) [1 - \Upsilon(\bar{\varepsilon})]}{a(1 - \zeta) \left[1 + \bar{\varepsilon} \left(\frac{\gamma(\bar{\varepsilon})}{1 - \Upsilon(\bar{\varepsilon})} + \frac{\gamma'(\bar{\varepsilon})}{\gamma(\bar{\varepsilon})} \right) \right] \gamma(\bar{\varepsilon})} \leq 0 \quad (133)$$

where

$$d(\bar{\varepsilon}; \zeta) = \left(\frac{\sigma(1 - \zeta)[1 - \Upsilon(\bar{\varepsilon})]}{(i - s)\gamma(\bar{\varepsilon})} \right)^{1/a}.$$

By (102) the derivative with respect to ζ is

$$\frac{\partial \Phi_\zeta}{\partial \zeta} = \frac{\zeta(\sigma + s_b)}{a} \int_{\bar{\varepsilon}}^{\infty} \frac{d(\varepsilon; \zeta)}{[\varepsilon - (1 - \zeta) \frac{1 - \Upsilon(\varepsilon)}{\gamma(\varepsilon)}]^2} \left(\frac{1 - \Upsilon(\varepsilon)}{\gamma(\varepsilon)} \right) d\Upsilon(\varepsilon).$$

By (102) we can rewrite the profit Φ_ζ as

$$\begin{aligned}\Phi_\zeta = & (i - s_b) \int_0^{\tilde{\varepsilon}} d(\varepsilon; \zeta) d\Upsilon(\varepsilon) \\ & + \int_{\tilde{\varepsilon}}^{\bar{\varepsilon}} \left\{ - \left(\int_0^{\varepsilon} \sigma u[d(z; \zeta)] dz \right) - (s_b + \sigma) d(\varepsilon; \zeta) + \sigma \varepsilon u[d(\varepsilon; \zeta)] \right\} d\Upsilon(\varepsilon).\end{aligned}$$

As $i \rightarrow s_b$, $\tilde{\varepsilon} \rightarrow \bar{\varepsilon}$ by (108). By the expressions above, as $i \rightarrow s_b$, $[\partial^2 \Phi_\zeta / \partial i \partial \zeta](i - s_b) \rightarrow -\infty$, $\partial \Phi_\zeta / \partial \zeta \rightarrow 0$, $\partial \Phi_\zeta / \partial i > 0$ and $\Phi_\zeta / (i - s_b) \geq \int_0^{\tilde{\varepsilon}} d(\varepsilon; \zeta) d\Upsilon(\varepsilon) > 0$. By (132) and (106) at the limit $\partial \Phi_\zeta / \partial i$ is proportional to $\int_0^{\tilde{\varepsilon}} \varepsilon^{1/a} d\Upsilon(\varepsilon)$ which is finite as $\bar{\varepsilon}$ is finite. Therefore, the right side of (131) is strictly negative when $i \approx s_b$ for $\zeta \in (0, 1]$. Hence, Φ_ζ / Φ_0 falls in i and $\hat{F}(\zeta)$ falls in i in the FOSD sense when $i \approx s_b$.

Fixing ζ , ν falls in i by (109). Since the distribution of ζ falls in i in the FOSD sense, the distribution F also falls in i in the FOSD sense. By (98), the distribution G also falls in the FOSD sense in i .

Claim3. By Proposition 8, fixing ζ , $d(\varepsilon; \zeta)$ falls in i for $\varepsilon < \tilde{\varepsilon}$ and otherwise it is constant in i . It follows that the deposit held by a household with a contract indexed by ζ , $d_c(i, \zeta) \equiv \int_0^{\tilde{\varepsilon}} d(\varepsilon; \zeta) d\Upsilon(\varepsilon)$, falls in i . By (107), $d(\varepsilon; \zeta)$ rises in ζ for $\varepsilon \geq \tilde{\varepsilon}$ and otherwise is constant in ζ . It follows that $d_c(i, \zeta)$ rises in ζ and falls in i . Since \hat{F} falls in i in the FOSD sense, so does the distribution of $d_c(i, \zeta)$.

Frictionless limit. As $\bar{\alpha} \rightarrow \infty$, from (112)-(113), the distribution F is given by (115). The distribution G collapses to a single mass point at $\nu = \nu^c$ by (98). The tightness $\tau \rightarrow 0$ by (113). Since measure 1 of consumers are banked with competitive contracts, their deposits are given by $d(\varepsilon) = u'^{-1}[(1 + s_b/\sigma)/\varepsilon]$ for all ε . Hence the aggregate deposits are independent of i . ■

D Linear pricing

In the main text we placed no restriction on deposit contracts and only assumed they had to be incentive compatible and individually rational. Here we study a more restrictive version of banking contract, i.e. banks can only offer contracts using linear pricing. Consider the bargaining model in Section 5, but now assume banks can only offer a single interest rate spread \hat{s}_d to the consumers. The quantity of deposit, d , is chosen by the consumers. The bank's profits are $\phi = (\hat{s}_d - s_b)d$.

We focus on symmetric equilibria where all banks offer the same $\hat{s}_d \in [s_b, i]$ to the consumers. We first derive consumers' IR constraint by considering their outside option payoff during a meeting. In a symmetric equilibrium, the value functions of the unbanked and banked consumers solve:

$$(\rho + \delta) V^u(\varepsilon) = U(\varepsilon; i) + \alpha(\tau) [V^b(\varepsilon) - V^u(\varepsilon)] \quad (134)$$

$$(\rho + \delta) V^b(\varepsilon) = U(\varepsilon; \hat{s}_d). \quad (135)$$

If a consumer rejects the bank's offer and has the opportunity to make a take-it-or-leave-it offer to the bank, then her expected utility is

$$(\rho + \delta) \hat{V}^b(\varepsilon) = U(\varepsilon; s_b). \quad (136)$$

Given \hat{V}^b , the consumer accepts the bank's offer if the following constraint is satisfied

$$V^b(\varepsilon) \geq \theta V^u(\varepsilon) + (1 - \theta) \hat{V}^b(\varepsilon). \quad (137)$$

Using the expressions for V^u and V^b in (134) and (135), we can reexpress (137) as

$$U(\varepsilon; \hat{s}_d) \geq \frac{(1 - \theta)(\rho + \delta + \alpha)U(\varepsilon; s_b) + \theta(\rho + \delta)U(\varepsilon; i)}{(1 - \theta)(\rho + \delta + \alpha) + \theta(\rho + \delta)}, \quad (138)$$

which holds when \hat{s}_d is sufficiently small. By the definition of $U(\varepsilon; i)$ and assume $u(y) = y^{1-a}/(1-a)$,

$$U(\varepsilon; i) = (\sigma\varepsilon)^{\frac{1}{a}} \frac{a}{1-a} \left(\frac{1}{i + \sigma} \right)^{\frac{1-a}{a}}.$$

Hence the constraint (138) is independent of ε . As a result, all consumers make the same participation decision. Using the expression for $U(\varepsilon; i)$, one can show that (138) binds when $\hat{s}_d = \underline{i}(i, s_b, \theta, \alpha, \sigma)$, where $\underline{i}(i, s_b, \theta, \alpha, \sigma)$ is defined in (48). Hence, in a symmetric equilibrium, the constraint is satisfied if and only if

$$\hat{s}_d \leq \underline{i}(i, s_b, \theta, \alpha, \sigma).$$

Next, we examine the bank's optimal choice of \hat{s}_d . The bank's expected profit is given by

$$\Phi = \max_{\hat{s}_d \leq \underline{i}} \left\{ \int (\hat{s}_d - s_b) d(\varepsilon) dF(\varepsilon) \right\}.$$

Given \hat{s}_d , if a consumer chooses to participate, then the optimal choice of deposit is

$$d(\varepsilon) = \left(\frac{\varepsilon}{\hat{s}_d/\sigma + 1} \right)^{1/a}.$$

Therefore, the bank's profits can be reexpressed as

$$\Phi = \max_{\hat{s}_d \leq \underline{i}} \left\{ \frac{\hat{s}_d - s_b}{(\hat{s}_d/\sigma + 1)^{1/a}} \right\} \int \varepsilon^{1/a} dF(\varepsilon). \quad (139)$$

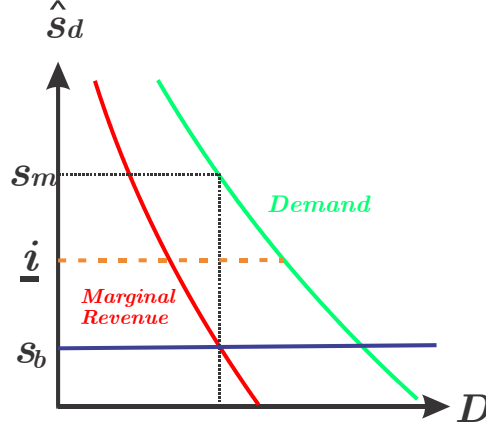


Figure 15: Determination of \hat{s}_d

The objective function is single-peaked in \hat{s}_d and hence a unique interior solution to the first-order condition exists and is equal to

$$s_m \equiv \frac{a\sigma + s_b}{1 - a}.$$

Since the solution s_m might exceed \underline{i} , the bank's optimal choice of \hat{s}_d equals to the minimum of the interior solution and \underline{i} , namely

$$\hat{s}_d = \min \{s_m, \underline{i}(i, s_b, \theta, \alpha, \sigma)\}.$$

Since the first argument in the right side is constant in θ and the second argument rises in θ , $\hat{s}_d = s_m$ when θ is large and $\hat{s}_d = \underline{i}(i, s_b, \theta, \alpha, \sigma)$ when θ is small. We summarize these results with the following proposition:

Proposition 11 (Optimal linear contract under private information.) Assume $u(y) = y^{1-a}/(1-a)$ with $a \in (0, 1)$. The solution to the banker's problem (139) is

$$\hat{s}_d = \min \left\{ \frac{a\sigma + s_b}{1 - a}, \underline{i}(i, s_b, \theta, \alpha, \sigma) \right\}$$

and there exists $\hat{\theta}$ such that

$$\begin{aligned} \hat{s}_d &= \frac{a\sigma + s_b}{1 - a} \quad \text{for } \theta > \hat{\theta} \\ \hat{s}_d &= \underline{i} \quad \text{for } \theta \leq \hat{\theta}. \end{aligned}$$

If $i > (a\sigma + s_b)/(1 - a)$, then $\hat{\theta} \in (0, 1)$ and otherwise $\hat{\theta} > 1$. The deposit is

$$d(\varepsilon) = \left(\frac{\varepsilon\sigma}{\hat{s}_d + \sigma} \right)^{1/a}$$

and the bank's flow profits are

$$\phi(\varepsilon) = d(\varepsilon)(\hat{s}_d - s_b).$$

Proposition 11 implies that, if $\theta > \hat{\theta}$, then the IR constraint (137) is slack and a change in i has no impact on the spread or deposits. But if $\theta \leq \hat{\theta}$, then the IR constraint binds and banks choose $\hat{s}_d = \underline{i}(i, s_b, \theta, \alpha, \sigma)$. We illustrate the latter case by Figure 15. The average demand of deposits, D , is represented by the green line and it decreases in the interest rate, \hat{s}_d , chosen by the bank. The opportunity cost of providing each unit of deposits, s_b , is denoted by the blue line. The bank's profits are given by $\Phi = (\hat{s}_d - s_b)D$. By the standard monopoly pricing logic, the bank would like to choose $\hat{s}_d = s_m$ to equate the marginal revenue and the marginal cost. But the outside option of the consumers act as a price ceiling for the banks and hence they cannot choose any $\hat{s}_d > \underline{i}$. If θ is small, then $s_m > \underline{i}$ and the bank's optimal choice is the corner solution \underline{i} . Since \underline{i} increases in the policy rate, the passthrough is positive and aggregate deposits fall in i .

E Multiple deposit categories

In this appendix we generalize our model to incorporate dual deposit categories. Type-1 deposits are liquid and has a spread s_b^1 where Type-2 deposits are less-liquid but have a lower spread $s_b^2 < s_b^1$.

E.1 Complete information

We generalize our complete-information model by introducing two categories of bank deposits, d^1 and d^2 . Type-1 deposits are invested by the bank in assets with spread $s_b^1 \leq i$ and can be liquidated on demand. Type-2 deposits are invested in assets with a lower spread, $s_b^2 < s_b^1$ but can only be liquidated on demand with probability $\chi_2 < 1$. With complement probability, $\chi_1 = 1 - \chi_2$, a consumer can only use her type-1 deposits to finance her consumption. We think of type-1 deposits as checkable accounts and type-2 deposits as time deposits.

The joint surplus of a consumer and a bank, $\mathcal{S}^b \equiv V^b - V^u + \Pi$, solves:

$$(\rho + \delta) \mathcal{S}^b = U^b(s_b^1, s_b^2) - (\rho + \delta) V^u, \quad (140)$$

where the utility flow of the banked consumer is

$$U^b(s_b^1, s_b^2) = \max_{d^1, d^2 \geq 0} \{-s_b^1 d^1 - s_b^2 d^2 + \sigma [\chi_1 v(d^1) + \chi_2 v(d^1 + d^2)]\}. \quad (141)$$

The portfolio of deposits, (d^1, d^2) , is chosen to maximize the joint surplus, \mathcal{S}^b . Assuming interior solutions, the first-order conditions are:

$$v'(d^1) = \frac{s_b^1 - s_b^2}{\sigma \chi_1} \quad (142)$$

$$v'(d^1 + d^2) = \frac{s_b^2}{\sigma \chi_2}. \quad (143)$$

If $d^2 > 0$, then $v'(d^1 + d^2) < v'(d^1)$, which requires $s_b^2 < \chi_2 s_b^1$. From (143), if s_b^2 is independent of i , then total deposits, $d^1 + d^2$, are unaffected by monetary policy. Market tightness and deposit spread are given by (16) and (17) where $U^b(s_b)$ is replaced with $U^b(s_b^1, s_b^2)$ defined in (141). The average deposit spread is defined as:

$$\hat{s}_d = \frac{d^1 s_b^1 + d^2 s_b^2 + \phi}{d^1 + d^2}, \quad (144)$$

where, from (15),

$$\phi = \theta [U^b(s_b^1, s_b^2) - U(i)] - (1 - \theta) \kappa \tau.$$

We now turn to the transmission of monetary policy. We assume that liquid deposits (type 1) are invested into fiat money (or reserves) while type-2 deposits are invested into loans with a real rate of return of r_ℓ and a spread equal to $s_\ell = \rho - r_\ell < i$. Hence, $s_b^1 = i$ and $s_b^2 = s_\ell$.

Proposition 12 (Transmission of monetary policy with multiple deposit categories.) Assume $u(y) = y^{1-a}/(1-a)$ with $a \in (0, 1)$. Suppose $s_b^2 < \chi_2 s_b^1$. An increase in i generates:

1. A decrease in liquid deposits, $\partial d^1/\partial i < 0$; no change in total deposits, $\partial(d^1 + d^2)/\partial i = 0$; and an increase in less-liquid deposits, $\partial d^2/\partial i > 0$;
2. An increase of the tightness of the deposit market, $\partial\tau/\partial i > 0$;
3. An increase in the average deposit spread, \hat{s}_d .

Proof of Proposition 12.

Part 1. From (142)-(143),

$$\begin{aligned} d^1 &= \left[\frac{\sigma\chi_1}{\sigma\chi_1 + i - s_\ell} \right]^{\frac{1}{a}} \\ d^1 + d^2 &= \left[\frac{\sigma\chi_2}{\sigma\chi_2 + s_\ell} \right]^{\frac{1}{a}}. \end{aligned}$$

It follows immediately that $\partial d^1/\partial i < 0$ and $\partial(d^1 + d^2)/\partial i = 0$. Holdings of type-2 deposits are

$$d^2 = \left[\frac{\sigma\chi_2}{\sigma\chi_2 + s_\ell} \right]^{\frac{1}{a}} - \left[\frac{\sigma\chi_1}{\sigma\chi_1 + i - s_\ell} \right]^{\frac{1}{a}}.$$

Hence, $\partial d^2/\partial i > 0$.

Part 2. From (16), market tightness solves

$$(\rho + \delta)\kappa = \frac{\alpha(\tau)}{\tau}\theta \{U^b(i, s_\ell) - U(i)\} - \alpha(\tau)(1 - \theta)\kappa.$$

Market tightness increases with the term between squared brackets on the right side, $U^b(s_b^1, s_b^2) - U(i)$. We differentiate this term with respect to i to obtain:

$$\frac{\partial [U^b(i, s_\ell) - U(i)]}{\partial i} = m^u - d^1,$$

where m^u solves $v'(m^u) = i/\sigma$, i.e.,

$$m^u = \left(\frac{\sigma}{\sigma + i} \right)^{\frac{1}{a}}.$$

The comparison with

$$d^1 = \left[\frac{\sigma}{\sigma + (i - s_\ell)/\chi_1} \right]^{\frac{1}{a}}$$

shows that $m^u > d^1$ if $i < (i - s_\ell)/\chi_1$, i.e., $s_\ell < \chi_2 i$. This condition is satisfied by assumption. It follows that $U^b(i, s_\ell) - U(i)$ increases with i , and hence $\partial\tau/\partial i > 0$.

Part 3. Finally, the average spread can be rewritten as

$$\hat{s}_d = \frac{d^1(i - s_\ell) + (d^1 + d^2)s_\ell + \phi}{d^1 + d^2}.$$

Using, from (142), that $i - s_\ell = \sigma\chi_1 v'(d^1)$, it can be reexpressed as

$$\hat{s}_d = \frac{\sigma\chi_1 v'(d^1)d^1 + (d^1 + d^2)s_\ell + \phi}{d^1 + d^2}.$$

If i is small, $v'(d^1)$ is also small and $[v'(d^1)d^1]' \approx v''(d^1)d^1 < 0$. Hence $v'(d^1)d^1$ is decreasing in d^1 . Using that d^1 decreases with i and ϕ increases with i , it follows that \hat{s}_d increases in i . ■

E.2 Posting under private information

We now introduce two deposit categories in our model of positing under private information of Section 4.

The flow utility of a banked consumer is now

$$\nu(\varepsilon) \equiv \max_{\varepsilon'} \left\{ -\phi(\varepsilon') - (s_b^1 + \sigma)d^1(\varepsilon') - (s_b^2 + \sigma\chi_2)d^2(\varepsilon') + \sigma\chi_1\varepsilon u[d^1(\varepsilon')] + \sigma\chi_2\varepsilon u[d^1(\varepsilon') + d^2(\varepsilon')] \right\}, \quad (145)$$

where we assumed that liquidity constraints, $y_1 \leq d^1$ and $y_2 \leq d^1 + d^2$, always bind since $s_b^2 > 0$. The envelope condition gives the law of motion for $\nu(\varepsilon)$ as

$$\nu'(\varepsilon) = \sigma\chi_1 u(d^1) + \sigma\chi_2 u(d^1 + d^2). \quad (146)$$

The contract must also satisfy the constraint according to which banked consumers have no incentive to accumulate cash,

$$\sigma\chi_1\varepsilon u'(d^1) + \sigma\chi_2\varepsilon u'(d^1 + d^2) \leq i + \sigma. \quad (147)$$

The problem of the bank consists in maximizing $\int_0^{\bar{\varepsilon}} \phi(\varepsilon) d\Upsilon(\varepsilon)$ subject to (146), (147), and the participation constraints, $\nu(\varepsilon) \geq U(\varepsilon; i)$.

Proposition 13 (*Optimal banking contract with two deposit categories.*) Assume $s_b^2 < \chi_2 s_b^1 < \chi_2 i$.

Let $\tilde{\varepsilon} < \bar{\varepsilon}$ be the unique solution to

$$\left[1 - \frac{1 - \Upsilon(\tilde{\varepsilon})}{\tilde{\varepsilon}\gamma(\tilde{\varepsilon})} \right] = \frac{s_b^1 + \sigma}{i + \sigma}. \quad (148)$$

The optimal menu of contracts is such that:

1. For all $\varepsilon \geq \tilde{\varepsilon}$, $[d^1(\varepsilon), d^2(\varepsilon)]$ solves

$$\left[\varepsilon - \frac{1 - \Upsilon(\varepsilon)}{\gamma(\varepsilon)} \right] \chi_1 \sigma u'(d^1) = s_b^1 - s_b^2 + \sigma\chi_1 \quad (149)$$

$$\left[\varepsilon - \frac{1 - \Upsilon(\varepsilon)}{\gamma(\varepsilon)} \right] \sigma\chi_2 u'(d^1 + d^2) = s_b^2 + \sigma\chi_2. \quad (150)$$

Moreover, $\partial d^1 / \partial i = \partial d^2 / \partial i = 0$, $\partial d^1 / \partial s_b^1 < 0$, $\partial d^2 / \partial s_b^1 > 0$.

2. For all $\varepsilon < \tilde{\varepsilon}$, $[d^1(\varepsilon), d^2(\varepsilon)]$ solves

$$\left\{ \varpi(\varepsilon) \chi_1 \sigma d^1 - (s_b^1 - s_b^2 + \sigma\chi_1) (d^1)^{1+a} \right\} \chi_2 = \left\{ \varpi(\varepsilon) \sigma\chi_2 \ell - (s_b^2 + \sigma\chi_2) (\ell)^{1+a} \right\} \chi_1 \quad (151)$$

$$\sigma\chi_1 \varepsilon (d^1)^{-a} + \sigma\chi_2 \varepsilon (\ell)^{-a} = i + \sigma, \quad (152)$$

where $\ell = d^1 + d^2$. Moreover, $\partial d^1 / \partial i < 0$, $\partial d^2 / \partial i < 0$, $\partial d^1 / \partial s_b^1 < 0$, $\partial d^2 / \partial s_b^1 > 0$.

Proof of Proposition 13. The proof is organized as follows. We first write down the Hamiltonian and the necessary conditions for an optimum. Second, we partition the menu of contracts into tiers depending on which incentive constraints bind. Finally, we turn to comparative statics with respect to i and s_b^1 .

Hamiltonian and optimality conditions. The Hamiltonian corresponding to the bank's optimal control problem is:

$$\begin{aligned} H(\nu, d^1, d^2, \mu, \xi) \equiv & [-\nu - (s_b^1 - s_b^2 + \sigma\chi_1) d^1 - (s_b^2 + \sigma\chi_2) (d^1 + d^2) + \sigma\chi_1 \varepsilon u(d^1) + \sigma\chi_2 \varepsilon u(d^1 + d^2)] \gamma(\varepsilon) \\ & + \xi [\nu - U(\varepsilon; i)] + \mu \sigma [\chi_1 u(d^1) + \chi_2 u(d^1 + d^2)] \\ & + \varsigma [i + \sigma - \sigma\chi_1 \varepsilon u'(d^1) - \sigma\chi_2 \varepsilon u'(d^1 + d^2)], \end{aligned} \quad (153)$$

where μ is the costate variable, ξ is the Lagrange multiplier associated with the participation constraint, and ς is the Lagrange multiplier associated with the incentive constraint (147). The complementary slackness conditions are:

$$\xi(\varepsilon) [\nu - U(\varepsilon; i)] = 0 \quad \text{for all } \varepsilon \quad (154)$$

$$\varsigma(\varepsilon) \{ \sigma\chi_1 \varepsilon u' [d^1(\varepsilon)] + \sigma\chi_2 \varepsilon u' [d^1(\varepsilon) + d^2(\varepsilon)] - i - \sigma \} = 0 \quad \text{for all } \varepsilon \quad (155)$$

and the nonnegativity of the Lagrange multipliers, $\xi(\varepsilon) \geq 0$ and $\varsigma(\varepsilon) \geq 0$, must hold.

The necessary condition for a maximum are:

$$[\varepsilon\gamma(\varepsilon) + \mu] \chi_1 \sigma u'(d^1) = (s_b^1 - s_b^2 + \sigma\chi_1) \gamma(\varepsilon) + \varsigma(\varepsilon) \sigma\chi_1 \varepsilon u''(d^1) \quad (156)$$

$$[\varepsilon\gamma(\varepsilon) + \mu] \sigma\chi_2 u'(d^1 + d^2) \leq (s_b^2 + \sigma\chi_2) \gamma(\varepsilon) + \varsigma(\varepsilon) \sigma\chi_2 \varepsilon u''(d^1 + d^2) \quad (157)$$

$$\mu'(\varepsilon) = \gamma(\varepsilon) - \xi(\varepsilon), \quad (158)$$

where the inequality in (157) holds as an equality if $d^2 > 0$. In the following we partition the menu of contracts according to whether (147) and participation constraints bind or not.

Tier #1: $\xi(\varepsilon) = \varsigma(\varepsilon) = 0$. If the constraints are slack, the FOCs (156)-(157) can be rewritten as (149)-(150), where we used (158) according to which $\mu'(\varepsilon) = \gamma(\varepsilon)$ and the terminal condition, $\mu(\bar{\varepsilon}) = 0$, to obtain $\mu(\varepsilon) = -[1 - \Upsilon(\varepsilon)]$. We can solve for d^1 and $\ell = d^1 + d^2$ independently. We define $\tilde{\varepsilon}$ the threshold such that (147) binds when d^1 and d^2 solve (149)-(150). It solves (148). Under the assumption of log concavity of $1 - \Upsilon(\varepsilon)$ and using that $s_b^1 < i$, there is a unique $\tilde{\varepsilon} < \bar{\varepsilon}$ solution to (148). So for all $\varepsilon > \tilde{\varepsilon}$ the constraint (147) is slack. We will check the participation constraints are satisfied, $\nu(\varepsilon) \geq U(\varepsilon; i)$, later in the proof.

Tier #2: $\xi(\varepsilon) = 0$ and $\varsigma(\varepsilon) > 0$. From (157) at equality:

$$\varsigma(\varepsilon) = \frac{[\varepsilon\gamma(\varepsilon) + \mu] \sigma\chi_2 u'(\ell) - (s_b^2 + \sigma\chi_2) \gamma(\varepsilon)}{\sigma\chi_2 \varepsilon u''(\ell)}, \quad (159)$$

where $\ell = d^1 + d^2$ represents the total deposits of the consumer. We substitute $\varsigma(\varepsilon)$ into (156) to obtain:

$$\varpi(\varepsilon) \chi_1 \sigma u'(d^1) - (s_b^1 - s_b^2 + \sigma\chi_1) = \{ \varpi(\varepsilon) \sigma\chi_2 u'(\ell) - (s_b^2 + \sigma\chi_2) \} \frac{\chi_1 u''(d^1)}{\chi_2 u''(\ell)},$$

where $\varpi(\varepsilon) \equiv \varepsilon + \mu(\varepsilon)/\gamma(\varepsilon)$ and $\mu(\varepsilon) = -[1 - \Upsilon(\varepsilon)]$. The second condition to determine the pair (d^1, ℓ) is (147) at equality. Under a CRRA utility function, $u'(y) = y^{-a}$, we rewrite the two conditions that determine (d^1, ℓ) as (151)-(152). In order to analyze (151) we will distinguish two cases, $\varpi(\varepsilon) > 0$ and $\varpi(\varepsilon) \leq 0$.

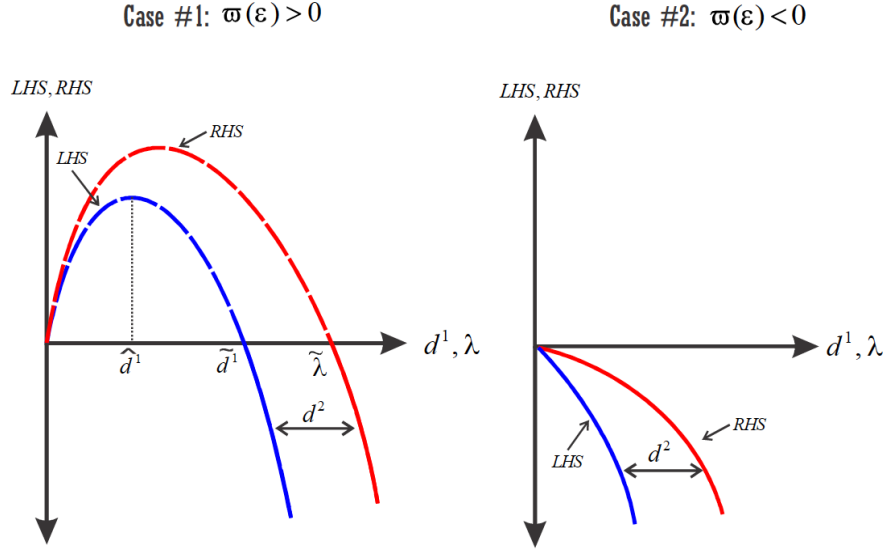


Figure 16: Representation of Equation (151)

Case #1: $\varpi(\varepsilon) > 0$. We define \hat{d}^1 as the unique maximizer of the left side of (151), i.e.,

$$\hat{d}^1 = \left[\frac{\varpi(\varepsilon)\chi_1\sigma}{(1+a)(s_b^1 - s_b^2 + \sigma\chi_1)} \right]^{\frac{1}{a}}.$$

Over the interval $[\hat{d}^1, +\infty)$ the left side of (151) is decreasing in d_1 . For all $d_1 \in [\hat{d}^1, +\infty)$, there is a unique $\ell > d_1$ solution to (151). To see this, note that if $\ell = d_1$, then the right side is larger than the left side if $\chi_2(s_b^2 + \sigma\chi_2) < \chi_1(s_b^1 - s_b^2 + \sigma\chi_1)$, i.e., $s_b^2 < \chi_1 s_b^1$. Moreover, both the left and right sides are single-peaked in d_1 and ℓ , respectively, with the maximizer of the right side being larger than the maximizer of the left side, and they tend to $-\infty$ as $d^1 \rightarrow +\infty$ and $\ell \rightarrow +\infty$, respectively. Hence, for given $d_1 \in [\hat{d}^1, +\infty)$, there is a unique $\ell > d^1$ solution to (151) as illustrated by the left panel of Figure 16. Moreover, the solution of ℓ increases in d_1 .

From (159), the condition $\varsigma(\varepsilon) > 0$ can be reexpressed as $\ell > \tilde{\ell}$ where

$$\tilde{\ell} = \left[\frac{\varpi(\varepsilon)\sigma\chi_2}{s_b^2 + \sigma\chi_2} \right]^{\frac{1}{a}}. \quad (160)$$

Since we have assumed $\varpi(\varepsilon) > 0$, this expression is the unique $\ell > 0$ such that the right side of (151) is 0. Similarly, we define \tilde{d}^1 as the unique $d_1 > 0$ such that the left side of (151) is 0, i.e.,

$$\tilde{d}^1 = \left[\frac{\varpi(\varepsilon)\chi_1\sigma}{s_b^1 - s_b^2 + \sigma\chi_1} \right]^{\frac{1}{a}}. \quad (161)$$

So, in order for a solution with $\varsigma(\varepsilon) > 0$ to exist, the curve representing (152) must be located above the point $(\tilde{d}^1, \tilde{\ell})$ in the space (d^1, ℓ) , as illustrated in Figure 17 where (151) is labelled FOCs in red and (152) is labelled ICM in purple. Formally,

$$\sigma\chi_1\varepsilon(\tilde{d}^1)^{-a} + \sigma\chi_2\varepsilon(\tilde{\ell})^{-a} \geq i + \sigma.$$

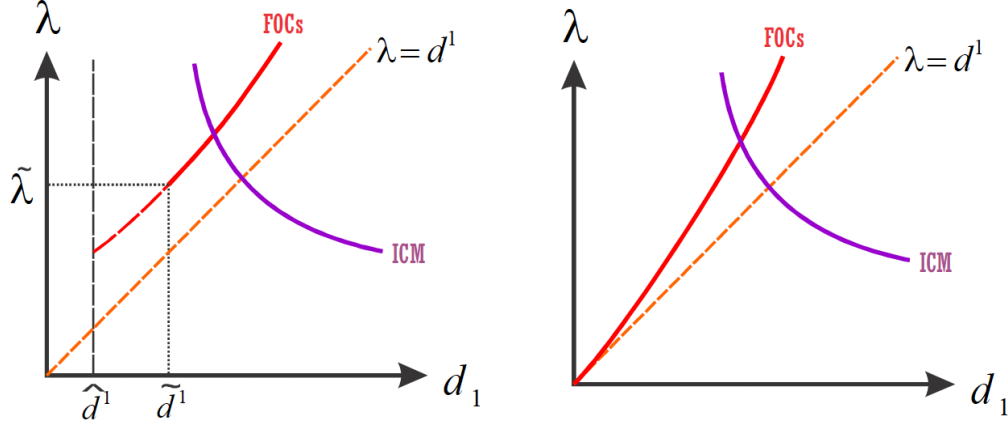


Figure 17: Determination of (d_1, ℓ)

Substituting \tilde{d}^1 and $\tilde{\ell}$ by their expressions given by (160) and (161) we obtain:

$$\frac{\varepsilon}{\varpi(\varepsilon)} (s_b^1 + \sigma) \geq i + \sigma. \quad (162)$$

This condition is the complement of the condition for Regime 1, $\varepsilon > \tilde{\varepsilon}$ where $\tilde{\varepsilon}$ solves (148).

Case #2: $\varpi(\varepsilon) \leq 0$. The equilibrium condition (151) can be rewritten as:

$$\left\{ \varpi(\varepsilon) \chi_1 \sigma d^1 - (s_b^1 - s_b^2 + \sigma \chi_1) (d^1)^{1+a} \right\} \chi_2 = \left\{ \varpi(\varepsilon) \sigma \chi_2 \ell - (s_b^2 + \sigma \chi_2) (\ell)^{1+a} \right\} \chi_1 \quad (163)$$

The left side is a negative, decreasing and concave function of d^1 while the right side is a negative, decreasing and concave function of ℓ . Provided that $s_b^2 < \chi_1 s_b^1$, the left side is located below the right side as illustrated in the right panel of Figure 16. Hence, for any $d^1 > 0$, there is a unique $\ell > d^1$ that solves the equation above. Moreover, the solution of ℓ increases with d^1 . It is $\ell = 0$ when $d^1 = 0$ and explodes when $\ell \rightarrow +\infty$. We illustrate this relationship in the right panel of Figure 17 where (163) is labelled. Recall that (152) defines a negative relationship between d^1 and ℓ which is labelled as ICM in Figure 17. When $d^1 = [(\sigma \chi_1 \varepsilon)/(i + \sigma)]^{1/a}$, $\ell = +\infty$. As $d^1 \rightarrow +\infty$, $\ell \downarrow [(\sigma \chi_2 \varepsilon)/(i + \sigma)]^{1/a}$. Hence a unique intersection point of FOCs and ICM exists.

Tier #3 and participation constraints: $\xi(\varepsilon) > 0$ and $\varsigma(\varepsilon) > 0$. If the participation constraints bind over some interval for ε , then $\nu(\varepsilon) = U(\varepsilon; i)$ and $\nu'(\varepsilon) = \partial U(\varepsilon; i)/\partial \varepsilon$ over that interval, i.e., $\sigma \chi_1 u(d^1) + \sigma \chi_2 u(d^1 + d^2) = \sigma u(m^u)$ where $\sigma \varepsilon u'(m^u) = i + \sigma$. Moreover, if the constraint (147) binds then

$$\sigma \chi_1 \varepsilon u'(d^1) + \sigma \chi_2 \varepsilon u'(d^1 + d^2) = i + \sigma = \sigma \varepsilon u'(m^u).$$

Under CRRA preferences, $u(y) = y^{1-a}/(1-a)$, the two equations can be rewritten as:

$$\begin{aligned} \chi_1 (d^1)^{1-a} + \chi_2 (d^1 + d^2)^{1-a} &= (m^u)^{1-a} \\ \left(\frac{m^u}{d^1} \right) \chi_1 (d^1)^{1-a} + \left(\frac{m^u}{d^1 + d^2} \right) \chi_2 (d^1 + d^2)^{1-a} &= (m^u)^{1-a} \end{aligned}$$

The only solution is $d^1 = m^u$ and $d^2 = 0$. From the FOCs, (156)-(157), it requires

$$\frac{s_b^2 + \sigma\chi_2}{\chi_2} \geq \frac{s_b^1 - s_b^2 + \sigma\chi_1}{\chi_1} \iff s_b^2 \geq \chi_2 s_b^1,$$

which violates the assumption $s_b^2 < \chi_2 s_b^1$.

We now show that the participation constraints, $\nu(\varepsilon) \geq U(\varepsilon; i)$, hold for all contracts in tiers 1 and 2. For contracts such that $\varepsilon < \tilde{\varepsilon}$, (147) holds at equality, i.e.,

$$\left(\frac{m^u}{d^1}\right) \chi_1 (d^1)^{1-a} + \left(\frac{m^u}{d^1 + d^2}\right) \chi_2 (d^1 + d^2)^{1-a} = (m^u)^{1-a}.$$

Using that $(d^1 + d^2)^{1-a} \geq (d^1)^{1-a}$ and $0 < m^u/(d^1 + d^2) \leq 1 \leq m^u/d^1$, it follows that

$$\chi_1 (d^1)^{1-a} + \chi_2 (d^1 + d^2)^{1-a} \geq (m^u)^{1-a}.$$

Hence, $\nu'(\varepsilon) \geq \partial U(\varepsilon; i)/\partial \varepsilon$ for all $\varepsilon < \tilde{\varepsilon}$. Together with $\nu(0) \geq U(0; i)$, it implies $\nu(\varepsilon) \geq U(\varepsilon; i)$ for all $\varepsilon < \tilde{\varepsilon}$.

Next, we turn to contracts such that $\varepsilon > \tilde{\varepsilon}$. From the first-order conditions (149)-(150),

$$\chi_1 (d^1)^{1-a} + \chi_2 (d^1 + d^2)^{1-a} = \left[\varepsilon - \frac{1 - \Upsilon(\varepsilon)}{\gamma(\varepsilon)} \right]^{\frac{1-a}{a}} \left\{ \chi_1 \left[\frac{\sigma}{\frac{s_b^1 - s_b^2}{\chi_1} + \sigma} \right]^{\frac{1-a}{a}} + \chi_2 \left[\frac{\sigma}{\frac{s_b^2}{\chi_2} + \sigma} \right]^{\frac{1-a}{a}} \right\}.$$

From Jensen's inequality and the fact that $x \mapsto [\sigma/(x + \sigma)]^{(1-a)/a}$ is strictly convex in x for all $a \in (0, 1)$,

$$\chi_1 \left[\frac{\sigma}{\frac{s_b^1 - s_b^2}{\chi_1} + \sigma} \right]^{\frac{1-a}{a}} + \chi_2 \left[\frac{\sigma}{\frac{s_b^2}{\chi_2} + \sigma} \right]^{\frac{1-a}{a}} > \left(\frac{\sigma}{s_b^1 + \sigma} \right)^{\frac{1-a}{a}}.$$

Hence,

$$\chi_1 (d^1)^{1-a} + \chi_2 (d^1 + d^2)^{1-a} > \left[\varepsilon - \frac{1 - \Upsilon(\varepsilon)}{\gamma(\varepsilon)} \right]^{\frac{1-a}{a}} \left(\frac{\varepsilon \sigma}{s_b^1 + \sigma} \right)^{\frac{1-a}{a}}.$$

From (148), $\varepsilon > \tilde{\varepsilon}$ is equivalent to

$$\left[1 - \frac{1 - \Upsilon(\varepsilon)}{\varepsilon \gamma(\varepsilon)} \right]^{\frac{1-a}{a}} > \left(\frac{s_b^1 + \sigma}{i + \sigma} \right)^{\frac{1-a}{a}}.$$

Hence,

$$\chi_1 (d^1)^{1-a} + \chi_2 (d^1 + d^2)^{1-a} > \left(\frac{\varepsilon \sigma}{i + \sigma} \right)^{\frac{1-a}{a}} = u(m^u).$$

This proves that $\nu'(\varepsilon) > \partial U(\varepsilon; i)/\partial \varepsilon$ for all $\varepsilon > \tilde{\varepsilon}$. Given $\nu(\tilde{\varepsilon}) > U(\tilde{\varepsilon}; i)$, then $\nu(\varepsilon) > U(\varepsilon; i)$ for all $\varepsilon > \tilde{\varepsilon}$.

So, the participation constraints hold for all ε .

Piecing the tiers together and sufficiency. We showed that the only two possible tiers are tiers 1 and 2. When $\varepsilon \geq \tilde{\varepsilon}$ where $\tilde{\varepsilon}$ is the unique solution to (148), then (d^1, d^2) are uniquely determined by (149)-(150). When $\varepsilon < \tilde{\varepsilon}$, (d^1, ℓ) with $\ell = d^1 + d^2$ is the unique solution to (151)-(152).

Given that $\xi(\varepsilon) = 0$ for all ε , the Hamiltonian in (153) can be rewritten (with a slight abuse of notation) as:

$$\begin{aligned} \frac{H(\nu, d^1, d^2, \varepsilon)}{\gamma(\varepsilon)} &\equiv -\nu - (s_b^1 - s_b^2 + \sigma\chi_1) d^1 - (s_b^2 + \sigma\chi_2) (d^1 + d^2) \\ &\quad + \sigma\chi_1 \left[\varepsilon - \frac{1 - \Upsilon(\varepsilon)}{\gamma(\varepsilon)} \right] u(d^1) + \sigma\chi_2 \left[\varepsilon - \frac{1 - \Upsilon(\varepsilon)}{\gamma(\varepsilon)} \right] u(d^1 + d^2) \\ &\quad + \varsigma(\varepsilon) [i + \sigma - \sigma\chi_1 \varepsilon u'(d^1) - \sigma\chi_2 \varepsilon u'(d^1 + d^2)], \end{aligned}$$

For all ε such that $\varepsilon - [1 - \Upsilon(\varepsilon)]/\gamma(\varepsilon) \geq 0$, $H(\nu, d^1, d^2, \varepsilon)$ is jointly concave in (ν, d^1, d^2) . When $\varepsilon - [1 - \Upsilon(\varepsilon)]/\gamma(\varepsilon) < 0$, H is not jointly concave but we showed the existence of a unique solution to the necessary conditions that is independent of ν . Hence, $\max_{(d^1, d^2)} H(\nu, d^1, d^2, \varepsilon)$ is linear in ν and by the Arrow sufficiency theorem our solution is a maximum.

Comparative statics. We consider the effects of an increase in i . From (149)-(150) the deposit levels in contracts such that $\varepsilon > \tilde{\varepsilon}$ are unaffected. From (148), and using that the left side is increasing in $\tilde{\varepsilon}$ while the right side is decreasing in i , the threshold $\tilde{\varepsilon}$ is decreasing in i . Finally, when $\varepsilon < \tilde{\varepsilon}$, then d^1 and ℓ are decreasing in i . Graphically, in Figure 17, the curve labelled ICM representing (152) moves inward as i increases.

We now turn to the effects of i on d^2 . Suppose $\varpi(\varepsilon) < 0$. From (151),

$$\chi_2 (s_b^1 - s_b^2 + \sigma\chi_1) (d^1)^{1+a} - \chi_1 (s_b^2 + \sigma\chi_2) (d^1 + d^2)^{1+a} = -\varpi(\varepsilon) \sigma\chi_1 \chi_2 d_2$$

The left side is decreasing in d_2 and the right side is linear and increasing. The derivative of the left side with respect to d^1 is:

$$\frac{\partial LHS}{\partial d^1} = \frac{(1+a)\chi_2 d_2 (d^1)^a}{(d^1 + d^2)} \left[\frac{-\varsigma(\varepsilon)}{\gamma(\varepsilon)} \sigma\chi_1 \varepsilon u''(d^1) \right].$$

So, if $\varpi(\varepsilon) < 0$, the left side increases with d^1 , and hence d^2 increases with d^1 . By a similar logic, we show that the same comparative statics hold if $\varpi(\varepsilon) > 0$, i.e., $\partial d^2 / \partial i < 0$.

We now study the effects of a change in s_b^1 on deposits. For contracts such that $\varepsilon > \tilde{\varepsilon}$, from (149)-(150), an increase in s_b^1 reduces d^1 , does not affect $\ell = d^1 + d^2$, and hence raises d^2 . For contracts such that $\varepsilon < \tilde{\varepsilon}$, an increase in s_b^1 reduces the left side of (151). Hence, for given d^1 , ℓ increases. In Figure 16, the curve LHS shifts downward. In Figure 17, the curve FOCs shifts up. So d^1 decreases, ℓ increases, and d^2 increases. ■

For consumers with high liquidity needs, $\varepsilon > \tilde{\varepsilon}$, the incentive constraint, (147), is slack. Holdings of liquid deposits depend on $s_b^1 - s_b^2$ while total deposits only depend on s_b^2 . For consumers with low liquidity needs, $\varepsilon < \tilde{\varepsilon}$, the incentive constraint, (147), binds. In that case, an increase in i lowers both d^1 and d^2 . So the deposits channel extends to both types of deposits. The participation constraints are slack for all consumers with $\varepsilon > 0$.⁴⁶

⁴⁶This result depends on the assumptions on the trading protocol. In Appendix E we generalize the multiple-deposit-categories model by assuming the terms of trade are determined via a bargaining game. In that version there is a cutoff $\tilde{\varepsilon} \in (0, \bar{\varepsilon})$ such that the participation constraints bind for consumers with $\varepsilon < \tilde{\varepsilon}$.

E.3 Bargaining under private information

We assume that banks and consumers trade according to the bargaining game introduced in Section 5. For an unbanked consumer, by the definition of $U(\varepsilon; i)$ and $u(y) = y^{1-a}/(1-a)$,

$$U(\varepsilon; i) = \varepsilon^{\frac{1}{a}} \frac{a}{1-a} \sigma \left(\frac{\sigma}{i + \sigma} \right)^{\frac{1-a}{a}}.$$

We denote $\tilde{U}(\varepsilon; s_b^1, s_b^2)$ as the utility flow of a consumer when the consumer makes a take-it-leave-it offer to the bank. By a similar logic as above,

$$\tilde{U}(\varepsilon; s_b^1, s_b^2) = \varepsilon^{\frac{1}{a}} \frac{a}{1-a} \left[\sigma \chi_1 \left(\frac{\sigma \chi_1}{s_b^1 - s_b^2 + \sigma \chi_1} \right)^{\frac{1-a}{a}} + \sigma \chi_2 \left(\frac{\sigma \chi_2}{s_b^2 + \sigma \chi_2} \right)^{\frac{1-a}{a}} \right].$$

A type- ε consumer accepts a bank's offer if the surplus from the contract, $\nu(\varepsilon)$, exceeds that of the outside option. Given $U(\varepsilon; i)$ and $\tilde{U}(\varepsilon; s_b^1, s_b^2)$ and by the logic leading to (42), the consumer's participation condition can be written as

$$\nu(\varepsilon) \geq \underline{\nu}(\varepsilon) \equiv \frac{(1-\theta)(\rho + \delta + \alpha)\tilde{U}(\varepsilon; s_b^1, s_b^2) + \theta(\rho + \delta)U(\varepsilon; i)}{(1-\theta)(\rho + \delta + \alpha) + \theta(\rho + \delta)}. \quad (164)$$

We define \underline{i} as the solution to $U(\varepsilon; \underline{i}) = \underline{\nu}(\varepsilon)$. One can interpret \underline{i} as the interest rate implied by the consumer's outside option. Using the expressions for $U(\varepsilon; i)$ and $\tilde{U}(\varepsilon; s_b^1, s_b^2)$, we can express \underline{i} as

$$\underline{i} = \left\{ \frac{(1-\theta)(\rho + \delta + \alpha) \left[\chi_1 \left(\frac{\chi_1}{s_b^1 - s_b^2 + \sigma \chi_1} \right)^{\frac{1-a}{a}} + \chi_2 \left(\frac{\chi_2}{s_b^2 + \sigma \chi_2} \right)^{\frac{1-a}{a}} \right] + \theta(\rho + \delta) \left(\frac{1}{\underline{i} + \sigma} \right)^{\frac{1-a}{a}}}{(1-\theta)(\rho + \delta + \alpha) + \theta(\rho + \delta)} \right\}^{-\frac{a}{1-a}} - \sigma. \quad (165)$$

A lower \underline{i} represents a higher outside option payoff for the consumers and the parameters θ and α affect the banking contract only via \underline{i} . It is easy to show that the implicit interest rate \underline{i} rises in i and θ and falls in α .

Given \underline{i} , the bank's problem is identical to that of Section 4.3, except the interest rate associated with the participation constraint is \underline{i} , and not i . As illustrated by Proposition 13, when $\underline{i} = i$, the participation constraint never binds. The following proposition argues that when $\underline{i} < i$ the participation constraint binds if and only if an agent has low liquidity needs. We denote $[d_u^1(\varepsilon), d_u^2(\varepsilon)]$ as the size of deposits under the (unconstrained) solution described by Proposition 13.

Proposition 14 (Bargaining with two deposit categories.) *Assume $\underline{i} < i$. There exists a cutoff $\hat{\varepsilon} \in (0, \bar{\varepsilon})$ such that a type- ε consumer's participation constraint binds if and only if $\varepsilon \leq \hat{\varepsilon}$. For $\varepsilon > \hat{\varepsilon}$,*

$$d^1(\varepsilon) = d_u^1(\varepsilon),$$

$$d^2(\varepsilon) = d_u^2(\varepsilon).$$

For $\varepsilon \leq \hat{\varepsilon}$,

$$d^1(\varepsilon) = [\chi_1 + \chi_2 (1+A)^{1-a}]^{-\frac{1}{1-a}} \left(\frac{\varepsilon}{\underline{i}/\sigma + 1} \right)^{\frac{1}{a}}, \quad (166)$$

$$d^2(\varepsilon) = A[\chi_1 + \chi_2 (1+A)^{1-a}]^{-\frac{1}{1-a}} \left(\frac{\varepsilon}{\underline{i}/\sigma + 1} \right)^{\frac{1}{a}}, \quad (167)$$

where the constant $A = \min\{B, C\}$, $B > 0$ solves

$$[\chi_1 + \chi_2(1+B)^{-a}][\chi_1 + \chi_2(1+B)^{1-a}]^{\frac{a}{1-a}} = \left(\frac{i+\sigma}{\underline{i}+\sigma}\right) \quad (168)$$

and

$$C = \left[\frac{(s_b^1 - s_b^2 + \sigma\chi_1)\chi_2}{(s_b^2 + \sigma\chi_2)\chi_1} \right]^{\frac{1}{a}} - 1 > 0.$$

There exists a cutoff $i' < i$ such that if $\underline{i} \in (i', i]$, then $B < C$, $\hat{\varepsilon} < \tilde{\varepsilon}$ and $\hat{\varepsilon}$ solves

$$\frac{\varpi(\hat{\varepsilon})}{\hat{\varepsilon}} = \left[(s_b^2 + \sigma\chi_2)(1+B)^{1+a}\chi_1 - (s_b^1 - s_b^2 + \sigma\chi_1)\chi_2 \right] \left(\frac{\chi_1 + \chi_2(1+B)^{-a}}{\chi_1\chi_2 B(i+\sigma)} \right). \quad (169)$$

If $\underline{i} \leq i'$, then $B \geq C$, $\hat{\varepsilon} \geq \tilde{\varepsilon}$ and $\hat{\varepsilon}$ solves

$$\frac{\varpi(\hat{\varepsilon})}{\hat{\varepsilon}} = \frac{(s_b^1 - s_b^2 + \sigma\chi_1)}{\chi_1\sigma} [\chi_1 + \chi_2(1+C)^{1-a}]^{-\frac{a}{1-a}} \left(\frac{1}{\underline{i}/\sigma + 1} \right). \quad (170)$$

Proof of Proposition 14. Hamiltonian and optimality conditions. The Hamiltonian corresponding to the bank's optimal control problem is:

$$H(\nu, d^1, d^2, \mu, \xi) \equiv [-\nu - (s_b^1 - s_b^2 + \sigma\chi_1)d^1 - (s_b^2 + \sigma\chi_2)(d^1 + d^2) + \sigma\chi_1\varepsilon u(d^1) + \sigma\chi_2\varepsilon u(d^1 + d^2)]\gamma(\varepsilon) \\ + \xi[\nu - U(\varepsilon; \underline{i})] + \mu\sigma[\chi_1 u(d^1) + \chi_2 u(d^1 + d^2)] \quad (171)$$

$$+ \varsigma[i + \sigma - \sigma\chi_1\varepsilon u'(d^1) - \sigma\chi_2\varepsilon u'(d^1 + d^2)], \quad (172)$$

where μ is the costate variable, ξ is the Lagrange multiplier associated with the participation constraint, and ς is the Lagrange multiplier associated with the incentive constraint (172). The complementary slackness conditions are:

$$\xi(\varepsilon)[\nu - U(\varepsilon; \underline{i})] = 0 \quad \text{for all } \varepsilon \quad (173)$$

$$\varsigma(\varepsilon)\{\sigma\chi_1\varepsilon u'(d^1(\varepsilon)) + \sigma\chi_2\varepsilon u'(d^1(\varepsilon) + d^2(\varepsilon)) - i - \sigma\} = 0 \quad \text{for all } \varepsilon \quad (174)$$

and the nonnegativity of the Lagrange multipliers, $\xi(\varepsilon) \geq 0$ and $\varsigma(\varepsilon) \geq 0$, must hold. The necessary condition for a maximum are the same as (156)-(158). In the following we partition the menu of contracts into four tiers according to whether (172) and participation constraints bind or not. The characterization of Tier 1, defined by $\xi(\varepsilon) = \varsigma(\varepsilon) = 0$, and Tier 2, defined by $\xi(\varepsilon) = 0$ and $\varsigma(\varepsilon) > 0$, are the same as that in the proof of Proposition 13.

Tier #3: $\xi(\varepsilon) > 0$ and $\varsigma(\varepsilon) > 0$. If the participation constraints bind over some interval for ε , then $\nu(\varepsilon) = U(\varepsilon; \underline{i})$ and $\nu'(\varepsilon) = \partial U(\varepsilon; \underline{i})/\partial \varepsilon$ over that interval, i.e.,

$$\sigma\chi_1 u(d^1) + \sigma\chi_2 u(d^1 + d^2) = \sigma u(\underline{m}^u)$$

where \underline{m}^u solves $\varepsilon u'(\underline{m}^u) = \underline{i}/\sigma + 1$. Moreover, if the constraint (147) binds then

$$\sigma\chi_1\varepsilon u'(d^1) + \sigma\chi_2\varepsilon u'(d^1 + d^2) = i + \sigma.$$

Under CRRA preferences, $u(y) = y^{1-a}/(1-a)$, the two equations can be rewritten as:

$$\chi_1 (d^1)^{1-a} + \chi_2 (d^1 + d^2)^{1-a} = \left(\frac{\varepsilon}{\underline{i}/\sigma + 1} \right)^{\frac{1-a}{a}} \quad (175)$$

$$\chi_1 \varepsilon (d^1)^{-a} + \chi_2 \varepsilon (d^1 + d^2)^{-a} = \underline{i}/\sigma + 1. \quad (176)$$

Each equation defines a negative relationship between d^1 and d^2 while the magnitude of $\partial d^1 / \partial d^2$ is smaller under the second equation. Hence, there is at most one solution for (d^1, d^2) . We conjecture the solution satisfies $d^2(\varepsilon) = B d^1(\varepsilon)$, where $B > 0$ is a constant. Under this conjecture, the constraints above can be rewritten as

$$(d^1)^a [\chi_1 + \chi_2 (1+B)^{1-a}]^{\frac{a}{1-a}} = \frac{\varepsilon}{\underline{i}/\sigma + 1} \quad (177)$$

$$(d^1)^{-a} [\chi_1 + \chi_2 (1+B)^{-a}] = \left(\frac{\varepsilon}{\underline{i}/\sigma + 1} \right)^{-1}. \quad (178)$$

Multiplying the two equations yields

$$[\chi_1 + \chi_2 (1+B)^{-a}] [\chi_1 + \chi_2 (1+B)^{1-a}]^{\frac{a}{1-a}} = \left(\frac{\underline{i} + \sigma}{\underline{i}} \right), \quad (179)$$

which corresponds to (168). The left side rises in B . It equals 1 when $B = 0$ and it explodes to ∞ when B explodes. Therefore, for $\underline{i} < \underline{i}$, a unique solution with $B > 0$ exists and it rises as \underline{i} falls. Since the solution of d^1 and d^2 to (175)-(176) is unique, by (178) it must take the form

$$d_{T3}^1(\varepsilon) = [\chi_1 + \chi_2 (1+B)^{-a}]^{\frac{1}{a}} \left(\frac{\varepsilon}{\underline{i}/\sigma + 1} \right)^{\frac{1}{a}} \quad (180)$$

$$d_{T3}^2(\varepsilon) = B [\chi_1 + \chi_2 (1+B)^{-a}]^{\frac{1}{a}} \left(\frac{\varepsilon}{\underline{i}/\sigma + 1} \right)^{\frac{1}{a}}, \quad (181)$$

where the subscript “T3” refers to Tier 3 of the contract. When $\underline{i} = \underline{i}$, $B = 0$ by (179) and $d_{T3}^1 = m^u$ and $d_{T3}^2 = 0$. As \underline{i} falls, B rises, d_{T3}^1 drops and d_{T3}^2 rises.

Tier #4: $\xi(\varepsilon) > 0$ and $\varsigma(\varepsilon) = 0$. When $\varsigma(\varepsilon) = 0$, we can divide (156) by (157) to derive

$$(s_b^2 + \sigma \chi_2) \chi_1 u'(d^1) = (s_b^1 - s_b^2 + \sigma \chi_1) \chi_2 u'(d^1 + d^2). \quad (182)$$

This equation defines a positive relationship between d^1 and d^2 . The participation constraint (175) defines a negative relationship between d^1 and d^2 . Hence, there is a unique solution solving (175) and (182). We conjecture the solution takes the form $d^2(\varepsilon) = C d^1(\varepsilon)$ where $C > 0$ is a constant. Using (182), we have

$$C = \left[\frac{(s_b^1 - s_b^2 + \sigma \chi_1) \chi_2}{(s_b^2 + \sigma \chi_2) \chi_1} \right]^{\frac{1}{a}} - 1 > 0,$$

which corresponds to (14). The inequality holds because $s_b^2 < \chi_2 s_b^1$. Solving for d^1 and d^2 yields

$$d_{T4}^1(\varepsilon) = [\chi_1 + \chi_2 (1+C)^{1-a}]^{-\frac{1}{1-a}} \left(\frac{\varepsilon}{\underline{i}/\sigma + 1} \right)^{\frac{1}{a}} \quad (183)$$

$$d_{T4}^2(\varepsilon) = C [\chi_1 + \chi_2 (1+C)^{1-a}]^{-\frac{1}{1-a}} \left(\frac{\varepsilon}{\underline{i}/\sigma + 1} \right)^{\frac{1}{a}}, \quad (184)$$

where the subscript “T4” refers to Tier 4 of the contract. As \underline{i} decreases, C remains unchanged and both d_{T4}^1 and d_{T4}^2 increase.

Binding participation constraint. We argue that a contract can be composed of either Tier 3 or Tier 4, but not both in the same contract. By (177), we can rewrite (180) and (181) as

$$d_{T3}^1(\varepsilon) = [\chi_1 + \chi_2(1+B)^{1-a}]^{-\frac{1}{1-a}} \left(\frac{\varepsilon}{\underline{i}/\sigma + 1} \right)^{\frac{1}{a}} \quad (185)$$

$$d_{T3}^2(\varepsilon) = B[\chi_1 + \chi_2(1+B)^{1-a}]^{-\frac{1}{1-a}} \left(\frac{\varepsilon}{\underline{i}/\sigma + 1} \right)^{\frac{1}{a}}. \quad (186)$$

These equations are similar to (183) and (184), except C is replaced by B . If $C \geq B$, then $d_{T3}^1(\varepsilon) \geq d_{T4}^1(\varepsilon)$ and $d_{T3}^2(\varepsilon) \leq d_{T4}^2(\varepsilon)$. As discussed above, the curve defined by (175) cuts that of (176) in the space of (d^2, d^1) from above at the point $[d_{T3}^2(\varepsilon), d_{T3}^1(\varepsilon)]$. Therefore, the point $[d_{T4}^2(\varepsilon), d_{T4}^1(\varepsilon)]$ lies below the curve defined by (176) and hence it violates the money holding constraint (172). It follows that if the participation constraint binds, then the solution is not in Tier 4, and so must be in Tier 3. By a similar logic, if $C < B$, then when the participation constraint binds, so is the money holding constraint. Therefore, the solution is in Tier 4 when the participation constraint binds. Altogether, if $C \geq B$, then only Tier 3 is relevant and if $C < B$ then only Tier 4 is relevant.

Next, we argue that there exists a cutoff $i' < i$ such that $C \geq B$ if and only if $\underline{i} \geq i'$. By (179), if $\underline{i} = i$, then $C > B = 0$. It remains to show $C < B$ when \underline{i} is sufficiently small. If $\theta = 0$ or $\alpha \rightarrow \infty$, then \underline{i} approaches its lowest possible value, $\underline{\underline{i}}$. By (165) the lower bound $\underline{\underline{i}}$ is defined as

$$\underline{\underline{i}} = \left[\chi_1 \left(\frac{\chi_1}{s_b^1 - s_b^2 + \sigma \chi_1} \right)^{\frac{1-a}{a}} + \chi_2 \left(\frac{\chi_2}{s_b^2 + \sigma \chi_2} \right)^{\frac{1-a}{a}} \right]^{-\frac{a}{1-a}} - \sigma. \quad (187)$$

Using this equation, we can write the right side of (179) as

$$\frac{i + \sigma}{\underline{i} + \sigma} = (i + \sigma) \left[\chi_1 \left(\frac{\chi_1}{s_b^1 - s_b^2 + \chi_1} \right)^{\frac{1-a}{a}} + \chi_2 \left(\frac{\chi_2}{s_b^2 + \chi_2} \right)^{\frac{1-a}{a}} \right]^{\frac{a}{1-a}}.$$

Evaluate the left side of (179) at $B = C$, then it becomes

$$[\chi_1 + \chi_2(1+C)^{-a}][\chi_1 + \chi_2(1+C)^{1-a}]^{\frac{a}{1-a}} = (s_b^1 + \sigma) \left[\chi_1 \left(\frac{\chi_1}{s_b^1 - s_b^2 + \chi_1} \right)^{\frac{1-a}{a}} + \chi_2 \left(\frac{\chi_2}{s_b^2 + \chi_2} \right)^{\frac{1-a}{a}} \right]^{\frac{a}{1-a}}.$$

Since $i > s_b^1$ by assumption, the right side of (179) exceeds the left side when we evaluate it at $B = C$. It follows that the solution of B must exceed C as $\underline{i} \rightarrow \underline{\underline{i}}$.

Piecing the tiers together. We now show that the contract takes the following form: There is a cutoff $\hat{\varepsilon} \in (0, \bar{\varepsilon})$ such that the participation constraint binds for $\varepsilon < \hat{\varepsilon}$ and it follows the unconstrained contract, $[d_u^1(\varepsilon), d_u^2(\varepsilon)]$, for all larger ε .

First, we solve for the cutoff $\hat{\varepsilon}$, assuming $B < C$. Denote $[d_u^1(\varepsilon), d_u^2(\varepsilon)]$ as the (unconstrained) solution described by Proposition 13. To solve for the cutoff $\hat{\varepsilon}$ where $[d_u^1(\varepsilon), d_u^2(\varepsilon)]$ intersects $[d_{T3}^1(\varepsilon), d_{T3}^2(\varepsilon)]$, substitute the solution of $d_{T3}^1(\varepsilon)$ and $d_{T3}^2(\varepsilon)$, given by (180)-(181), into equation (151). The cutoff $\hat{\varepsilon}$ uniquely solves

the resulting equation, which is given by (169). It follows that the cutoff $\hat{\varepsilon} > 0$ if the following condition holds around $\varepsilon \approx 0$,

$$\left[(s_b^2 + \sigma\chi_2)(1+B)^{1+a}\chi_1 - (s_b^1 - s_b^2 + \sigma\chi_1)\chi_2 \right] \left(\frac{\chi_1 + \chi_2(1+B)^{-a}}{\chi_1\chi_2B(i+\sigma)} \right) > \frac{\varpi(\varepsilon)}{\varepsilon}.$$

Since $\lim_{\varepsilon \rightarrow 0} \varpi(\varepsilon)/\varepsilon \rightarrow -\infty$, the condition is always satisfied around $\varepsilon \approx 0$. Hence $\hat{\varepsilon} > 0$ when $B < C$. One can check that when $B = C$, the solution of $\hat{\varepsilon}$ coincides with $\tilde{\varepsilon}$. By (180) and (181), as B rises, $d_{T3}^1(\varepsilon)$ falls and $d_{T3}^2(\varepsilon)$ rises at each ε . Since $d_{T3}^2(\varepsilon)$ cuts $d_u^2(\varepsilon)$ from above at $\hat{\varepsilon}$ and $d_u^2(\varepsilon)$ rises in ε for $\varepsilon \leq \tilde{\varepsilon}$, the cutoff $\hat{\varepsilon}$ rises as B rises. Therefore, if $B < C$, then $\hat{\varepsilon} \in (0, \tilde{\varepsilon})$.

When $B \geq C$, one can solve for $\hat{\varepsilon}$ by equating the solution of $d_{T4}^1(\varepsilon)$, given by (183), with $d_u^1(\varepsilon)$, given by (149). The resulting equation corresponds to (170). One can check that the solution of $\hat{\varepsilon}$ falls in \underline{i} . When $\underline{i} = i'$, $B = C$ and in this case $\hat{\varepsilon}$ in (170) equals $\tilde{\varepsilon}$. For $\underline{i} \leq i'$, $B \geq C$ as discussed above and $\hat{\varepsilon} \geq \tilde{\varepsilon}$.

Finally, we argue that the participation constraint does not bind for $\varepsilon > \hat{\varepsilon}$.

Case 1: $i \in (i', i)$. If $i \geq i'$, then as argued above, $C > B$ and only Tier 3 is relevant. The contract is given by $[d_{T3}^1(\varepsilon), d_{T3}^2(\varepsilon)]$ for low ε , then intersects with $[d_u^1(\varepsilon), d_u^2(\varepsilon)]$ at some $\hat{\varepsilon}$, and is given by $[d_u^1(\varepsilon), d_u^2(\varepsilon)]$ for all $\varepsilon > \hat{\varepsilon}$. By (180)-(181), the pair $[d_{T3}^1(\varepsilon), d_{T3}^2(\varepsilon)]$ grows at the rate of $\varepsilon^{1/a}$ as ε increases. Evaluate equation (151) at $[d^1, d^2] = [d_{T3}^1(\varepsilon), d_{T3}^2(\varepsilon)]$. As ε increases from $\hat{\varepsilon}$, the left side of (151) becomes smaller than the right side. This implies $[d_u^1(\varepsilon), d_u^2(\varepsilon)]$, the solution of (151) and (152), is such that $d_u^1(\varepsilon) \leq d_p^1(\varepsilon)$ and $d_u^2(\varepsilon) \geq d_p^2(\varepsilon)$ for all $\varepsilon \in (\hat{\varepsilon}, \tilde{\varepsilon}]$. Since (175) cuts (176) from above in the space $[d^2, d^1]$ at $[d_{T3}^2(\hat{\varepsilon}), d_{T3}^1(\hat{\varepsilon})]$, $[d_u^1(\varepsilon), d_u^2(\varepsilon)]$ satisfies (175) for all $\varepsilon \in (\hat{\varepsilon}, \tilde{\varepsilon}]$. Since the participation constraint holds at $\hat{\varepsilon}$ and $[d_u^1(\varepsilon), d_u^2(\varepsilon)]$ satisfies (175) for all $\varepsilon \in (\hat{\varepsilon}, \tilde{\varepsilon}]$, the participation constraints hold for all $\varepsilon \in (\hat{\varepsilon}, \tilde{\varepsilon}]$.

For $\varepsilon > \tilde{\varepsilon}$, as ε rises, the utility $u(\underline{m}^u)$ rises at rate of $\varepsilon^{\frac{1-a}{a}}$ and $\chi_1 u(d_u^1) + \chi_2 u(d_u^1 + d_u^2)$ grows at rate $\left[\varepsilon - \frac{1-\Upsilon(\varepsilon)}{\gamma(\varepsilon)} \right]^{\frac{1-a}{a}}$ by (149) and (150). Since $\left[\varepsilon - \frac{1-\Upsilon(\varepsilon)}{\gamma(\varepsilon)} \right] / \varepsilon$ rises in ε , the growth rate of $\chi_1 u(d_u^1) + \chi_2 u(d_u^1 + d_u^2)$ is higher than that of $u(\underline{m}^u)$. Hence, the participation constraint is satisfied for all $\varepsilon > \tilde{\varepsilon}$.

Case 2. $i < i'$ If $i < i'$, then $C \leq B$ and only Tier 4 is relevant. The trajectory, $[d_{T4}^1(\varepsilon), d_{T4}^2(\varepsilon)]$, can only intersect with $[d_u^1(\varepsilon), d_u^2(\varepsilon)]$ at some $\hat{\varepsilon} > \tilde{\varepsilon}$. By a logic similar to the last part of the proof of case 1 (i.e. $\varepsilon > \tilde{\varepsilon}$), the participation constraint is satisfied for $\varepsilon > \hat{\varepsilon}$. ■

The bank's optimal contract is similar to that in Proposition 13, but now the participation constraint binds for consumers with low liquidity needs. If the value of the consumers' outside option is not high such that $\underline{i} > i'$, then both the participation constraint and the liquidity constraint (147) bind for consumers with $\varepsilon \leq \hat{\varepsilon}$. If the outside option becomes attractive enough such that $\underline{i} \leq i'$, then only the participation constraint binds for consumers with $\varepsilon \leq \hat{\varepsilon}$.

In Figure 18 we present a numerical example under the special case $s_b^1 = i$. The parameter values are the same as that in our calibrated example and additionally $\chi_1 = 0.2$ and $s_b^2 = 0$. As in Section 4.3, one interpretation is that banks offer a single type of deposits (d^2) that are imperfectly liquid while consumers can hold both cash (d^1) and deposits (d^2). Under this interpretation, the deposit spread is given

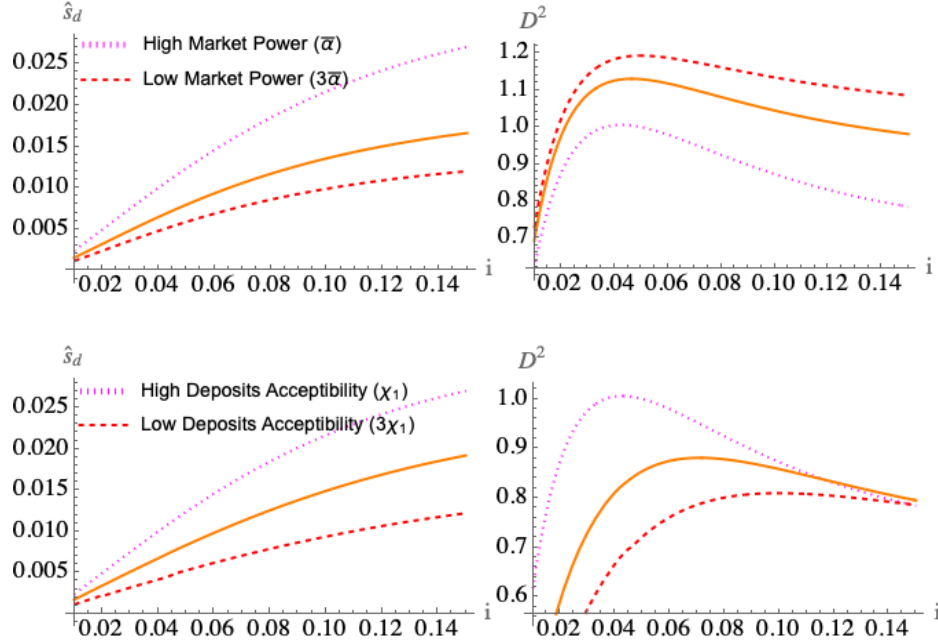


Figure 18: Changes in the average spread and aggregate deposits as i rises. (Top panels) The matching efficiencies are $\bar{\alpha}$, $2\bar{\alpha}$ and $3\bar{\alpha}$ for the purple dotted lines, yellow lines and red dashed lines, respectively. (Bottom panels) The probabilities of a cash meeting are χ_1 , $2\chi_1$ and $3\chi_1$ for the purple dotted lines, yellow lines and red dashed lines, respectively.

by $\hat{s}_d(\varepsilon) = s_b^2 + \phi(\varepsilon)/d^2(\varepsilon)$. As the policy rate i rises, the average spread, \hat{s}_d , rises. As shown in the top-left panel, the passthrough is larger when banks have more market power, which is consistent with our calibrated example. The aggregate deposits, D^2 , is non-monotone in i and the intuition is similar to that behind Figure 8. The bottom panels illustrate that the spread, the passthrough and the aggregate deposits increase when bank deposits become more liquid. Moreover, as bank deposits become more liquid, D^2 is more likely to decrease in i , which means the deposits channel documented in Drechsler et al. (2017) grows stronger.