

The Bank Capital Channel of Monetary Policy

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This version: December 2002

Abstract

This paper examines the role of bank lending in the transmission of monetary policy in the presence of capital adequacy regulations. I develop a dynamic model of bank asset and liability management that incorporates risk-based capital requirements and an imperfect market for bank equity. These conditions imply a failure of the Modigliani-Miller theorem for the bank: its lending will depend on the bank's financial structure, as well as on lending opportunities and market interest rates. Combined with a maturity mismatch on the bank's balance sheet, this gives rise to a 'bank capital channel' by which monetary policy affects bank lending through its impact on bank equity capital. This mechanism does not rely on any particular role of bank reserves and thus falls outside the conventional 'bank lending channel'. I analyze the dynamics of the new channel. An important result is that monetary policy effects on bank lending depend on the capital adequacy of the banking sector; lending by banks with low capital has a delayed and then amplified reaction to interest rate shocks, relative to well-capitalized banks. Other implications are that bank capital affects lending even when the regulatory constraint is not momentarily binding, and that shocks to bank profits, such as loan defaults, can have a persistent impact on lending.

(JEL Classification numbers: E440, E520, G280)

* Email: vdheuvel@wharton.upenn.edu. I would like to thank Christopher Sims, William Brainard and Stefan Krieger for valuable advice and comments. I would also like to thank Giancarlo Corsetti, Gaston Gelos, Joao Gomes, George Hall, Michael Krause, Stephen Morris, Nagpurnanand Prabhala, Jeremy Stein, Eric Swanson, and numerous seminar participants for valuable comments.

1. Introduction

Traditional monetary theory has largely ignored the role of bank equity. Bank-centered accounts of how monetary policy affects the real economy usually focus on the role of reserves in determining the volume of demand deposits. In addition, the ‘bank lending channel’ thesis maintains that monetary policy actions can also alter the supply of bank loans by changing bank reserves. While reserve requirements play a central role in these theories, bank capital¹ and capital regulations are at best discussed as an afterthought. This paper constitutes an attempt to fill this gap by taking the risk-based capital requirements of the Basle Accord explicitly into account. It provides a framework for analyzing the consequences of these regulations for bank lending and the response of lending to monetary policy actions in a dynamic setting.

The incorporation of bank capital effects is motivated by two sets of considerations. First, it is generally agreed that bank capital is an important factor in bank asset and liability management and that its importance has likely increased since the implementation of the risk-based capital requirements of the 1988 Basle Accord. The implementation of these regulations, along with other factors, has often been blamed for a perceived credit crunch immediately prior and during the 1990-91 recession. In fact, the term ‘capital crunch’ has been suggested as a more apt description for the reduction in lending during this episode, in view of the role of bank capital.² The evidence from state and bank level data shows that low bank capital has been associated with sluggish lending during this period.³ In addition, there is some more general evidence that the cost of loans depends on bank capital. Using a matched sample of individual loans, borrowers and banks, Hubbard, Kuttner, and Palia (2001) find that higher bank capital lowers the rate charged on loans, even after controlling for borrower characteristics, other bank characteristics and loan contract terms.⁴ If bank capital is a significant determinant of

¹ I use the terms bank equity and bank capital interchangeably to refer to the book value of bank equity. I will be more precise about the regulatory definitions below.

² Syron (1991). See also Bernanke and Lown (1991).

³ See Sharpe (1995) for an overview. In his judgement, the research has been less successful in determining whether this association is due to a causal effect of bank capital on loan supply or due to the effect of persistent variation in loan demand on capital.

⁴ In addition, Peek, Rosengren and Tootell (1999 and 2000) show that confidential supervisory bank ratings, which partly reflect capital adequacy, predict GDP growth even when controlling for commercial

loan supply, then it is important to examine the consequences of this dependence for the dynamics of bank lending and the monetary transmission mechanism.⁵

Second, taking into account capital adequacy regulations allows us to address the question what role bank lending plays in the monetary transmission in a world in which banks are increasingly able to issue nonreservable liabilities. According to the bank lending channel thesis, monetary policy affects the real economy at least in part through a direct effect on the supply of bank loans. A necessary condition for this channel to be operative is that banks change their loan supply in reaction to shocks to their reserves.⁶ If banks can frictionlessly issue nonreservable liabilities, then the bank lending channel, at least in its standard formulation, disappears. Romer and Romer (1990), among others, have claimed that banks can in fact switch fairly easily to alternative sources of finance that carry no or lower reserve requirements, such as CDs, and this is one reason the bank lending channel thesis is controversial. While the precise extent to which banks insulate their loan supply from shocks to their reserves is still in debate, it does seem safe to say that the ability of banks to raise nonreservable liabilities has increased in the past decade. According to the standard theory, this would suggest a diminished role of bank lending in the transmission of monetary policy. An important point of this paper is that in the presence of capital requirements bank lending plays a special role in the transmission of monetary policy even if banks face a perfect market for some nonreservable liability. The model demonstrates that capital requirements generate a mechanism by which monetary policy shifts the supply of bank loans through its effect on bank capital, rather than reserves. This new mechanism relies on an imperfect market for bank equity rather than

or Fed forecasts of growth. Interestingly, they also find that this information is used by the FOMC in decisions on the Federal Funds target rate.

⁵ In separate paper (Van den Heuvel (1999)), I examine empirically how the effect of monetary policy on output depends on bank capital. Using annual data from U.S. states, I find that when a state's banking sector starts out with a low capital-asset ratio, its subsequent output growth is more sensitive to changes in the Federal funds rate.

⁶ For an overview of the theory and empirical evidence relating to the bank lending channel see Kashyap and Stein (1994). Bernanke and Blinder (1988) provide a statement of the lending channel in terms of an IS/LM type model. Stein (1998) provides a 'micro-founded' adverse selection model of bank asset and liability management that generates a lending channel. It should be noted that the 'bank lending channel' is only a part of the body of research that recognizes that assets other than money and bonds may play a role in the monetary transmission mechanism. For example, Brainard (1964) models the monetary transmission mechanism using a whole array of asset prices.

bank debt. The model thus provides an alternative to the prevailing account of the bank lending channel that does not rely on any special role of reserves.

Besides capital adequacy regulations and an imperfect market for bank equity, a crucial feature of the model is the maturity transformation performed by banks. Bank loans are assumed to have longer maturity on average than the bank's nonequity liabilities. A consequence of this is that a rise of the short-term interest rate affects bank profits negatively, which will in time deteriorate the bank's capital adequacy. If banks cannot readily issue new equity, then this can have a persistent effect on bank lending.

The model analyzes an individual bank's asset and liability management problem in an infinite horizon setting. Each period the bank decides how many new loans to make, how much to invest in marketable securities and how much to pay out to shareholders as dividends, subject to regulatory and financial constraints. The bank faces uncertainty with respect to the fraction of outstanding loans that goes bad each period. In addition, the short interest rate is variable reflecting monetary policy actions, among other factors. Although the analysis is not formally embedded in a general equilibrium, some of the issues that would arise in such an exercise are discussed in the paper.

The model has several main implications. First, bank lending depends on the financial structure of the bank. Shocks to bank profitability, such as loan defaults, have a persistent impact on lending. Moreover, bank capital affects lending even when the regulatory constraint is not momentarily binding. Second, the model generates a 'bank capital channel' by which monetary policy actions can affect the supply of bank loans through their impact on bank capital. Third, the strength of this channel depends on the capital adequacy of the banking sector and the distribution of capital across banks. In particular, lending by banks with low capital has a delayed and then amplified reaction to interest rate shocks, relative to well-capitalized banks.

The rest of the paper is organized as follows. The next section discusses how the model relates to the literature. Section 3 presents the model, starting with a general discussion of the main assumptions, followed by the detailed assumptions and some analytical preliminaries. Section 4 discusses the calibration of the model as well as the solution methods employed. The main results are presented in the following three sections. Section 5 discusses the value function and optimal policy to the bank's decision

problem. Section 6 uses the results to analyze the dynamic response of bank lending to monetary policy shocks, and how this impact depends on the financial structure of the bank in question. Section 7 analyzes the impact of loan default shock. Section 8 devotes some thought to general equilibrium considerations and the final section concludes.

2. Related Literature

The idea that banks may be subject to financial frictions, just like nonfinancial firms, is shared by this work with many papers, especially in the literature on the microeconomics of banking. This paper follows Kashyap and Stein (1995) and Stein (1998) in arguing that these frictions can give rise to a balance sheet channel for financial intermediaries, just as for ordinary firms. The specific frictions in Kashyap and Stein's model, which give rise to the lending channel, are different from the ones in this paper, but in both cases the implication is that economic shocks, such as monetary policy actions, can affect the supply of bank loans at least in part through their effect on the quantity of some of the sources of finance of banks (reservable liabilities in Kashyap and Stein's model; equity in this paper).

The model presented here differs from the existing literature by incorporating capital requirements in a dynamic setting with endogenous bank equity. Most dynamic banking models with an imperfect market for bank liabilities focus on the distinction between insured deposits and other bank liabilities. Stein (1998) presents a two period adverse selection model of bank asset and liability management, which, as mentioned, can be viewed as providing microfoundations for the lending channel thesis.⁷ The key distinction in Stein's model is between reservable and nonreservable bank liabilities. Lending is affected by reserve shocks only if *all* nonreservable bank liabilities are subject to the adverse selection problem (i.e. are exposed to bank risk or, in the case of *insured* nonreservable liabilities, some other friction).⁸ ⁹ There is no separate role for bank equity.¹⁰

⁷ Kashyap and Stein (1995) contains a simplified version of the model.

⁸ As mentioned in the introduction, doubt about the importance of such frictions has been a reason for skepticism about the importance of a bank lending channel. In the model I take this criticism head on by

Schneider's (1998) infinite horizon model also focuses on the bank's limited access to money market funds. Since this limited access is derived from moral hazard associated with strategic default by the entrepreneurial banker, inside equity does play a role in the model, as it mitigates the moral hazard problem. Thus, the bank is in effect subject to market based, rather than regulatory, capital requirements. Because inside equity can only be increased through retained earnings, this generates a 'financial accelerator' on the part of the bank, which also arises in my model.¹¹

Furfine (2001) presents a dynamic model of bank asset and liability management with capital adequacy regulations, captured through an exogenously specified continuous cost function of the risk-based capital ratio and the leverage ratio.¹² Furfine estimates the model parameters using bank level data and then simulates the effect on bank portfolios of stricter capital adequacy regulation. He finds that the simulated effects fit the facts of the 1990-91 credit crunch qualitatively well, whereas as well as other candidate explanations for bank behavior in that period do not perform as well. In an earlier version of the paper (1995), Furfine also simulates the impact of the Basle capital regulations on the response of bank lending to a monetary policy shock. He finds that the response is much weaker in the presence of the capital requirements. As we will see, the present paper obtains a very different conclusion. The reason for this is the presence of duration transformation in my model, which exposes the bank to interest rate risk.

assuming that the bank faces a perfect market for some nonreservable liability. Instead of an imperfect market for *all* nonreservable liabilities, I make the less restrictive assumption that the market for bank equity is imperfect.

⁹ Edwards and Vegh (1997) show that, even in the absence of financial frictions in the market for nonreservable liabilities, a type of lending channel arises if there are cost complementarities between deposit taking and lending. The complementarity could be due to the informational content of borrowers' deposits.

¹⁰ However, it has been argued informally that bank capital can affect the strength of the lending channel. The argument is as follows. Better capitalized banks are less risky, which means that nonreservable and uninsured bank liabilities are less subject to asymmetric information problems, which improves the bank's access to these liabilities and diminishes the dependence of lending on reserves by that bank. See, e.g., Kashyap and Stein (2000).

¹¹ Besides the different nature of the capital requirements, key differences with this paper are that in my model bank loans are assumed to have a longer maturity than bank deposits, creating interest rate risk, and dividends are chosen optimally.

¹² Based on existing regulations, I assume below that the capital requirements impose direct restrictions on the bank's choice set when binding. An implication of my approach is that the cost of having a given degree of inadequate capital is endogenous. For example, low capital is more costly when lending opportunities are more profitable.

Bernanke and Gertler (1987), Holmstrom and Tirole (1997), Bolton and Freixas (2000) and Chen (2001) present macroeconomic equilibrium models with a nontrivial role for financial intermediaries. Due to imperfect information problems, the intermediaries are themselves subject to credit constraints. Consequently, their capital is an important determinant of lending and economic activity. In addition, Bolton and Freixas analyze the monetary transmission mechanism in such a context. In their model, contractionary monetary policy lowers the spread on bank loans and this may cause banks to repurchase their outside equity if it is no longer profitable to incur the associated dilution costs (which arise due to asymmetric information). If this happens, a ‘credit crunch’ occurs, in which bank lending declines severely. Thus, as in this paper, monetary policy can have an amplified effect on bank lending through its effect on bank capital.

The focus of this paper is not on the informational microfoundations that give rise to the existence of banks, bank-dependent borrowers and the longer maturity of bank loans. Rather, the emphasis is on a careful treatment of the joint dynamics of the bank’s balance sheet components and the implications thereof for bank lending and monetary policy. There is a large literature on the microeconomics of banking that deals with the former set of issues and that motivates some of the specific assumptions made in the model. Battacharya and Thakor (1993) and Freixas and Rochet (1997) present overviews of this literature. A common thread is that informational asymmetries generate a special role for bank-like intermediaries in the allocation of credit and create a class of borrowers that are to some degree dependent on bank-intermediated loans, a requirement for the macroeconomic relevance of bank lending. The dependence of firms on a *specific* bank arises in the context of a long-term bank-borrower relationship, if monitoring by the relationship bank generates an informational advantage relative to other intermediaries.¹³

Thakor (1996) employs an asymmetric information model of bank lending to assess the impact of the risk-based capital requirements of the Basle Accord. Assuming a (fixed) higher required return on bank equity relative to deposits, the risk-weighted capital requirements raise the required return on loans relative to other bank assets, so that the requirements increase credit rationing. Moreover, Thakor finds that a monetary

¹³ Sharpe (1990) and Rajan (1992). See Petersen and Rajan (1994) for empirical evidence on the benefits of relationship banking for small firms. Kashyap and Stein (1994) provide a discussion of the issues discussed in this paragraph in relation to the bank lending channel thesis.

policy expansion can *lower* bank lending, if it lowers the rate on bank deposits without changing other rates of return by much. This result relies on holding constant the total assets of the bank; for this reason, it does not arise in my model.

The rationale for capital requirements and the prudential regulation of banks in general has been explored extensively. Giammarino, Lewis and Sappington (1993) show that capital requirements arise naturally in the presence of an agency problem between the bank and a public deposit insurance system. In a comprehensive treatment employing the incomplete contract paradigm, Dewatripont and Tirole (1994) argue that prudential regulation can be viewed as a representation of small depositors, who due to a free rider problem cannot be expected to intervene themselves as effectively as large bondholders. Just as for nonbank firms, the risk-reducing intervention in the event of weak performance is necessary to provide management with the right *ex-ante* incentives. Interestingly, Hellman, Murdock and Stiglitz (2000) show that, in a dynamic context, capital requirements can also have a perverse effect on bank incentives: by increasing the banks' cost of funding, higher capital requirements lower the franchise value of the bank, which makes excessive risk-taking more attractive.¹⁴ The model proposed here will show that there is a separate source of risk-loving behavior induced by capital requirements, which is associated not with the possibility of bank failure, but with regulatory interventions at the minimum capital ratio.

3. The Model

A main feature of the model is the incorporation of capital adequacy regulations. Capital regulations have existed in the United States in some form since the national bank act of 1863.¹⁵ The current regulatory regime was shaped primarily by the 1988 international Basle Accord and the 1991 Federal Deposit Insurance Corporation

¹⁴ Diamond and Rajan's (1999) analysis points to a different possible cost of capital requirements: in their model a fragile bank capital structure improves the bank's ability to create liquidity.

¹⁵ For concreteness, this discussion focuses on the U.S. The Basle Accord was signed by all G-10 countries, although there is some variation in how strict the guidelines are implemented. The Basle framework has also been adopted by an increasing number of developing countries, often with interesting modifications.

Improvement Act (FDICIA).¹⁶ The Basle Accord established minimum capital requirements as ratios of two aggregates of accounting capital to risk weighted assets (and certain off-balance sheet activities). The risk weights are supposed to reflect credit risk. For example, commercial and industrial loans have weight one, while U.S. government bonds have zero weight, and consequently do not require any regulatory capital. Primary or tier 1 capital is required to exceed 4 percent of risk weighted assets, while total (tier 1 plus tier 2) capital must be at least 8 percent.¹⁷ The FDICIA prohibits insured banks from making any capital distributions, including dividends, which would have the effect of leaving the institution undercapitalized. In addition, the FDICIA's Prompt Corrective Action provision mandates specific interventions by bank regulators when banks fall short of their capital requirements. These include the need to submit and implement a capital restoration plan, limits on asset growth, and restrictions on new lines of business, as well as more stringent restrictions for successively higher degrees of undercapitalization. Banks that have less than 2 percent tangible capital are deemed 'critically undercapitalized' and face receivership within 90 days.¹⁸

Another principal assumption of the model is that the bank cannot readily issue new equity. Some such assumption is necessary if the bank is to hold any prudential capital above the regulatory minimum and we are to examine the possible dependence of the bank's lending decisions on its capital position. If banks were always able to costlessly raise new equity, no bank would ever forgo profitable lending opportunities and the capital requirements would be irrelevant except insofar as equity has a higher required rate of return. There is a sizable theoretical (e.g. Myers and Majluf (1984), and

Arguably, the imperfections in the market for bank equity that make capital requirements relevant are even more important in developing economies.

¹⁶ The following is based on Macey and Miller (1997), especially chapter 3E, Dewatripont and Tirole (1994), Sinkey (1998) and Aggarwal and Jacques (1998). These references contain more detailed information about the Basle Accord and the FDICIA.

¹⁷ Tier 1 capital consists of common stockholders' equity plus noncumulative perpetual preferred stock and any related surplus plus minority interest in the equity accounts of consolidated subsidiaries less goodwill and other intangibles. Tier 2 capital consists of the allowance for loan lease losses, cumulative perpetual, long-term and convertible preferred stock; perpetual debt and other hybrid debt/equity instruments; intermediate term preferred stock and term subordinated debt. In addition to the risk-based capital requirements, U.S. banks are also required to meet a leverage ratio requirement of 4%. The leverage ratio is tier 1 capital divided by average assets.

¹⁸ Tangible capital is tier 1 capital plus cumulative preferred stock and related surplus less intangibles except qualifying purchased-mortgage-servicing rights. The tangible capital ratio is calculated by dividing by total assets less intangibles except qualifying purchased-mortgage-servicing rights.

Stein (1998) for banks) and empirical literature (e.g. Calomiris and Hubbard (1995), and Cornett and Tehranian (1994) for banks) to support the assumption that issuing new equity can be quite costly. As a starting point we make the extreme assumption that this cost is infinite, so that the model bank is unable to issue any new equity. Equity capital is still endogenous since it reflects retained earnings. The bank has some control over it through the choice of dividends.

However, the bank's inability to issue new equity is not a sufficient assumption for capital regulations to materially influence the bank's real decisions. The bank could simply retain its earnings until its assets are only financed by capital, making it unlikely that the bank will ever encounter a series of bad shocks sufficient to wipe out its capital. Banks do not in fact hold such extreme levels of prudential capital. The reason is that equity capital, even when obtained through retained earnings, is considered a relatively expensive form of funding. There are a number of reasons for the cost disadvantage of equity, including agency costs, but also, and more mundanely, the tax advantage of debt.¹⁹ The latter feature is present in the model. It is assumed that accounting profits, operating income minus interest payments and write-offs on loans, are taxed at a constant rate $\tau > 0$.

Banking theory recognizes as one of the main functions of banks "the transformation of securities with short maturities, offered to depositors, into securities with long maturities that borrowers desire." (Freixas and Rochet (1997), p. 5). The model bank is assumed to perform this maturity transformation. The resultant maturity mismatch on the balance sheet exposes the bank to interest rate risk. For example, if the interest rate rises unexpectedly, the bank's interest costs will rise faster than its interest revenue, resulting in a profits squeeze. In addition, compared to other firms, banks are highly leveraged, so that a given size profits shock results in a larger percentage change in equity. This exposure to interest rate risk will play a crucial role in shaping the dynamics of bank lending in response to a monetary policy shock. The evidence from bank stock return generally supports the view that banks are exposed to interest rate risk.

¹⁹ There are also some bank-specific considerations regarding the relative cost of debt finance, such as deposit insurance and reserve requirements. These considerations are taken up below. The bank's shareholders may also demand a risk premium, but this is not, in a Modigliani-Miller world, a factor in limiting the desired level of capital, as the risk premium declines with lower leverage (MM proposition II).

Flannery and James (1984), Yourougou (1990) and Bae (1990) all find a significant negative effect of interest rate innovations on bank stock returns that is not captured by a broad stock market index. In addition, across banks, this sensitivity is positively related to measures of the bank's maturity or duration gap (Flannery and James (1984), Akella and Greenbaum (1992)). Moreover, Robinson (1995) and Grenadier and Hall (1995) present evidence that banks' interest rate risk may have increased since the implementation of the (credit) risk-based capital standards.²⁰

3.1 Detailed assumptions

The bank's objective is to maximize shareholder value by making loans and investing in tradable securities. These assets are financed by equity capital and debt instruments, such as time and demand deposits, bonds, etc. The bank's balance sheet at the beginning of period t is given below:

Assets		Liabilities	
Loans	L_t	Debt (including deposits)	B_t
Securities	S_t	Equity	E_t
Total Assets	A_t		A_t

I now discuss the assumptions regarding each of the components of the balance sheet.

Loans: At the beginning of each period the bank makes N_t new loans to its clients. The model abstracts from the details of the maturity structure by assuming that the average maturity of the bank's loan portfolio is constant. Each period a fraction $\bar{\delta}$ of outstanding loans becomes due, so that $1/\bar{\delta}$ is a measure of the loan portfolio's average maturity. The fact that bank loans typically have longer maturity than bank deposits can be captured by choosing $\bar{\delta} < 1$. While a constant fraction of loans becomes due each period, actual repayments are allowed to deviate from this rate in a stochastic fashion. δ_{t+1}

²⁰ In spite of the excess sensitivity of bank stock returns to interest rate risk, studies by Flannery (1981 and 1983) using accounting data do not provide evidence of a large maturity gap or a negative effect of short interest rates on bank profits. A more recent study by Robinson (1995) does find a significant and sizable effect of interest rate movements on bank profits.

denotes the fraction of loans actually repaid at the end of period t . (The subscript $t+1$ is used to maintain the convention that variables dated t are known at the beginning of period t which is when time t decisions are made. Any variables realized during or at the end of period t are therefore dated $t+1$.) δ_{t+1} may differ from $\bar{\delta}$ to allow for changes in the bank's loan portfolio that are best modeled as beyond the control of the bank, rather than as part of N_t . For example, some loans could be repaid early ($\delta_{t+1} > \bar{\delta}$) or the bank may be forced to agree to deferments for some clients ($\delta_{t+1} < \bar{\delta}$). Also, new loans made under a loan commitment from an older contract can be captured in this way.

Loans are a risky investment for the bank. During each period a stochastic fraction ω_{t+1} of outstanding loans L_t goes bad and must be written off by the bank. In addition, a stochastic fraction b_{t+1} of new loans N_t goes bad. (The reason for the different charge-off rates will be clear shortly.) In accounting terms, $\omega_{t+1}L_t + b_{t+1}N_t$ represents net charge-offs on loans and leases during period t . By causing a loss to the bank these charge-offs directly reduce regulatory capital.²¹

Loans thus evolve according to

$$L_{t+1} = (1 - \delta_{t+1} - \omega_{t+1})L_t + (1 - \delta_{t+1} - b_{t+1})N_t \quad (1)$$

Note that some of the new loans are assumed to become due in one period.

The bank faces a downward sloping demand curve for its loanable funds. That is, the more it lends in a given period, the lower the average expected return on these loans. This assumption reflects either some market power on the part of the bank or convex screening costs. Since the focus of this model is not on the micro-foundations of bank lending, these factors are not modeled explicitly. The market power can be due to geographical specialization of the bank or an informational advantage stemming from

²¹ Technically, net charge-offs are made against the allowance for loan lease losses, which is part of tier 2 bank capital. If the bank decides to keep the allowance constant in the face of these charge-offs, a provision for loan lease losses of equal size must be made. The net result would be that tier 2 capital is unchanged, but tier 1 capital is reduced by the amount of the provision, which lowers the bank's net income. Either way, total capital (tier 1 plus tier 2) is reduced. (The allowance for loan lease losses is supposed to be adequate to absorb expected loan and lease losses, based upon management's evaluation of the bank's current loan and lease portfolio.)

long-term relations with its clients. Limited screening capacity will lead to lower average quality loans when the resources are spread over more candidate projects or when more of the screened projects are accepted. One way to model the dependence of the expected return on loans is through the contractual interest payments. Another is through expected defaults. The model accommodates both approaches.

First, the average contractual interest rate on new loans is a decreasing function, $\rho(N_t)$, of the amount of new loans made, N_t . The average contractual interest rate on all outstanding loans in period t , $\bar{\rho}_t$, is then given recursively by

$$\bar{\rho}_{t+1} = \frac{\{(1 - \delta_{t+1} - \omega_{t+1})L_t\}\bar{\rho}_t + \{(1 - \delta_{t+1} - b_{t+1})N_t\}\rho(N_t)}{L_{t+1}} \quad (2)$$

with

$$\rho'(N) \leq 0 \quad (3)$$

Second, the charge-offs on the new loans are an increasing function of the amount of new loans made:

$$b_{t+1} = b(N_t, \omega_{t+1}) \quad (4)$$

with

$$\frac{\partial b(N, \omega)}{\partial N} \geq 0 \quad (5)$$

Furthermore, at least one of the inequalities in assumptions (3) and (5) must be strict. It is convenient to assume that $\rho(N)N$ is a weakly concave function of N and that $b(N, \omega)$ is weakly convex in N .

Note that b_{t+1} is allowed to be stochastic through its dependence on ω_{t+1} . This formulation can capture both a higher expected value and a higher variance of charge-offs as a result of intensive lending during a given period.²²

Bank Debt and Securities: It is assumed that the bank's debt liabilities B are all short. Since loans have maturity exceeding one, the model bank performs its traditional role of maturity transformation. Despite the maturity mismatch on the bank's balance sheet, the model abstracts from liquidity problems on the part of the bank: the model bank can borrow freely at the market interest rate r_t . All debt liabilities are assumed to be fully insured in a government run fixed rate deposit insurance scheme. This justifies the absence of a risk premium on the bank's debt. For simplicity, the insurance premium is assumed to be zero.²³ In addition, the bank can invest in tradable securities S , yielding the same riskless return r_t . Because the bank has perfect access to both markets, its securities and debt level are indeterminate; only net debt $B - S$ is determined.²⁴ For this reason, without loss of generality, S is simply set equal to zero hereafter. The abstraction from liquidity problems makes the role of capital explicit. It is not argued that liquidity management is an unimportant consideration for banks. A few comments about these assumptions are in order.

First, there is no special role for reservable and/or demand deposits in the model. This is an unusual assumption from the viewpoint of banking and monetary theory. However, as long as the bank can borrow and lend freely at the market interest rate r , then this rate is the relevant margin for the lending decision. Even if the bank can attract some deposits at lower cost, as is often assumed, this cannot be the marginal source of funding – the bank would always want to take all the below-market rate deposits it can get and put them in securities. This would simply add an amount to the bank's profits determined only by the size and profitability of the bank's market for deposits. Of course, to the extent that these 'cheap' deposits are subject to reserve requirements, the interest forgone on the reserves should be deducted from this amount. In the model this component of profits is simply held constant.²⁵ Second, the same logic suggests that the

²² The dependence of charge-offs on new loans is resolved within one period. This avoids the need for an additional state variable in solving the model.

²³ The premium could also be lump-sum. As the model is set up, a positive premium rate would simply lead the bank to hold no securities given that there are no liquidity problems. In reality, the premium is positive but very low and banks do generally hold securities, in part for liquidity management.

²⁴ As will be explained below, neither B nor S enter the risk-based capital requirement.

²⁵ Suppose the bank funds itself in part with deposits M with a below market return. Let the required return on M , plus any transaction, reserves, and deposit insurance costs, be $i = r - \varphi < r$. Obviously, M cannot be chosen without limit, otherwise $M = \infty$ and i would be the market rate. Assuming securities are the closest substitutes to M for consumers/firms, write $M = M(\varphi)$. Then the component in the bank's profits from the

assumption that all bank debt is fully insured is an immaterial simplification, if the bank can at the margin obtain insured deposits at market rates. The point is that the market interest rate is the relevant margin for lending decisions as long as the bank has access to nonreservable and risk-free (insured) debt liabilities and can trade in risk-free securities.

The assumption of perfect access to insured and nonreservable deposits contrasts strongly with a precondition for the existence of a bank lending channel: the inability of banks to shield their loan supply from shocks to reserves.²⁶ Thus, the model takes head on the critique by Romer and Romer (1990), among others, of the bank lending channel, that the bank's lending decisions should not depend on shocks to reserves as long as banks have perfect access to nonreservable liabilities. This is true in my model, but there is a different channel. If the monetary authority increases the short interest rate, the bank's cost of funding increases, and, given the maturity mismatch on the bank's balance sheet, its profits decrease. This in turn worsens the bank's future capital position, which, finally, increases the likelihood that its lending will be constrained by inadequate capital.

Equity: The accounting value of equity, or capital, is a residual: $E_t = L_t - B_t$. Equity evolves according to:

$$E_{t+1} = E_t - D_t + (1 - \tau)\pi_{t+1} \quad (6)$$

where D_t denotes dividends paid out to shareholders at the beginning of period t and π_{t+1} is pre-tax accounting profits made during period t . (Again, the subscript $t+1$ is used because the profits are not known at the beginning of the period t .) Dividends are a decision variable for the bank. Banks are constrained from issuing new equity; if they could issue new equity, D_t would be negative and $(-D_t)$ would represent the value of newly issued equity. It is assumed that accounting profits are the same for regulatory and

rent on M is $\pi^M = \varphi M(\varphi)$. If φ is constant, or subject to choice by the bank, then π^M is also constant. If φ increases with r , as is the case if $i = 0$ for example, then $\partial \pi^M / \partial r < (>) 0$, if the elasticity $-(\varphi / M) \partial M / \partial \varphi > (<) 1$.

²⁶ See, for example, Kashyap and Stein (1994) and Bernanke and Blinder (1988). Stein (1998) provides a micro-founded model of the bank lending channel in which some banks are dependent on insured and reservable deposits, and thus on reserves, for financing loans.

tax purposes and that they are taxed at the constant rate τ . Accounting profits are given by:

$$\pi_{t+1} = \bar{\rho}_t(1 - \omega_{t+1})L_t + \rho(N_t)(1 - b_{t+1})N_t - r_t(L_t - E_t + N_t + D_t) - (\omega_{t+1}L_t + b_{t+1}N_t) + \pi^F \quad (7)$$

The balance sheet identity $B_t \equiv L_t - E_t$ is used here to eliminate B_t from the equation. The first two terms represent the interest income from loans, including new loans, during period t . Recall that ω_{t+1} (b_{t+1}) is the fraction of loans (new loans) that goes bad during period t . The third term is the interest payment on the bank's debt. New loans and dividends increase the bank's demand for nonequity funding. The fourth term represents the net charge-offs on loans (see footnote 21 above). Finally, the constant term π^F represents any other components of profits, such as fixed operating costs and, possibly, rents on intra-marginal deposits.

It is useful to see what the bank's balance sheet looks like after accounting for its decisions on new loans and dividends, but prior to defaults and repayments. For comparability, we momentarily forget the $S = 0$ normalization.

Assets		Liabilities	
Loans	$L_t + N_t$	Debt	$B_t + N_t + \Delta S_t + D_t$
Securities	$S_t + \Delta S_t$	Equity	$E_t - D_t$
Total Assets	$A_t + N_t + \Delta S_t$		$A_t + N_t + \Delta S_t$

For completeness, we now also state the law of motion for bank debt, taking into account any tax effects:

$$B_{t+1} = (1 + (1 - \tau)r_t)(B_t + N_t + D_t) - \delta_{t+1}(L_t + N_t) - (1 - \tau)(\bar{\rho}_t(1 - \omega_{t+1})L_t + \rho(N_t)(1 - b_{t+1})N_t + \pi^F) - \tau(\omega_{t+1}L_t + b_{t+1}N_t)$$

The tax advantage of debt is apparent from the $(1-\tau)$ factor by which the interest costs are discounted. The remaining terms reflect the repayments, interest income and tax shield from the charge-offs on loans, in that order.

Capital Regulation: At the beginning of each period capital requirements are enforced by the regulatory authorities. In calculating the risk weighted capital asset ratio all loans are assumed to be in the highest risk category in the sense of the Basle Accord, with a risk weight of 100%. This category includes all claims to the non-bank private sector, except for mortgages on residential property, which receive a risk weight of 50% (see Hall's (1993) table 8.4 for an overview of the asset categories). The riskless securities are in the lowest risk category, with weight zero. Typical examples are Treasury bills and short loans to other depository institutions. Note that, given the assumptions of the model, the bank would never hold securities with a positive risk weight unless they yield an above market return. Thus, for the model bank, the risk-adjusted capital asset ratio (*RACAR*) at the beginning of the period is given by:

$$RACAR_t = E_t / L_t$$

The capital adequacy regulations place restrictions on the bank's activities as soon as this ratio falls below the regulatory minimum γ . In the Basle Accord, γ is equal to 0.04 for tier 1 capital and 0.08 for total capital. In view of the discussion above, these restrictions are modeled in the following way: When at the beginning of the period t the *RACAR* exceeds the regulatory minimum, the bank is free to pay dividends and invest in new loans, as long as these actions do not have the immediate consequence of pushing the *RACAR* below γ . Put more succinctly,

$$E_t \geq \gamma L_t \Rightarrow E_t - D_t \geq \gamma(L_t + N_t) \quad (8)$$

If at the beginning of the period the bank is already undercapitalized, then the regulator prevents the bank from paying out any dividends or making any new loans:

$$E_t < \gamma L_t \Rightarrow N_t = 0 \text{ and } D_t = 0 \quad (9)$$

The prohibition of dividends can be found directly in the FDICIA, while the no new loans restriction captures the imposed limits on asset growth. The main cost of undercapitalization to the bank is thus that it must forgo profitable lending opportunities.

Financial Constraint: As discussed, the bank is unable to issue new equity. This implies that dividends are constrained to be nonnegative:

$$D_t \geq 0 \quad (10)$$

Finally, it is assumed that shareholders do not price the risk of the bank's shares.²⁷ (Alternatively, we could assume that the entire analysis is conducted under the risk neutral probability measure of the arbitrage-free economy.) The market value of the bank's equity, and the bank's objective, is then given by:

$$V_t = \max_{\{N_{t+s}, D_{t+s}\}_{s=0}^{\infty}} E_t \left[\sum_{s=0}^{\infty} \left(\prod_{u=0}^{s-1} (1 + r_{t+u})^{-1} \right) D_{t+s} \right] \quad (11)$$

The maximization in (11) is subject to the laws of motion in (1), (2), (4), (6) and (7), the regulatory constraints (8) and (9) and the financial constraint (10), as well as an initial balance sheet position, as given above.²⁸ The bank takes the stochastic processes of r_t , δ_t and ω_t as given. It is convenient to make some technical assumptions with regard to these processes.

²⁷ There is no inconsistency between risk neutrality and rational expectations and the imperfect market for bank equity, if the financial constraint is due to asymmetric information problems. Most agency models of imperfect financial markets do not rely on departures from either risk neutrality or rational expectations (e.g. Myers and Majluf, 1984).

²⁸ I adopt the convention that $\prod_{t=a}^b x_t = 1$ whenever $b < a$.

Exogenous shocks: r_t , δ_t and ω_t follow a joint Markov process with bounded support. Moreover, the time t conditional distribution of $(r_{t+1}, \delta_{t+1}, \omega_{t+1})$ depends on time t information only through r_t according to a density $F(r_{t+1}, \delta_{t+1}, \omega_{t+1}; r_t)$. It is also assumed that F is continuous with respect to its second and third argument.

The Markov assumption allows the maximization problem in (11) to be formulated as a dynamic programming problem. Assuming that all r_t is a sufficient statistic for the time t conditional distribution of $(r_{t+1}, \delta_{t+1}, \omega_{t+1})$ avoids having to include the shocks δ_t and ω_t in the vector of state variables. The vector $(E_t, L_t, \bar{\rho}_t, r_t)$ is thus a valid state for the bank's decision problem at time t . The associated Bellman equation is:

$$V(E_t, L_t, \bar{\rho}_t, r_t) = \max_{D_t, N_t} \left\{ D_t + \frac{1}{1+r_t} E_t[V(E_{t+1}, L_{t+1}, \bar{\rho}_{t+1}, r_{t+1})] \right\} \quad (12)$$

s.t. (1), (2), (4), (6), (7), (8), (9) and (10)

For comparison, it is useful to consider what the bank's optimal policy would be, if it faced no financial constraint, but could raise new equity without cost at the beginning of each period. With D_t denoting *net* dividends, *i.e.*, gross dividends minus the value of newly issued equity, we do not need to modify the law of motion for capital, (6). (Obviously, net dividends are the same as dividends for the bank that is unable to issue any new equity.) Also, the expression in (11) then defines the bank's *pre-issue* shareholder value. Thus, it remains the appropriate objective function for the unconstrained bank as it represents the interests of the existing shareholders when the time t decisions are taken.²⁹ Let V^U define the value function for this 'financially unconstrained' problem. With D_t denoting net dividends, the Bellman equation for V^U differs only in the constraints:

²⁹ Formulating the problem in terms of choosing net dividends alone economizes on notation. Given the model assumptions it is easy to see that optimal gross dividends and new equity issues are indeterminate for the 'financially unconstrained' bank. Only the difference, D_t , is determined in the maximization.

$$V^U(E_t, L_t, \bar{\rho}_t, r_t) = \max_{D_t, N_t} \left\{ D_t + \frac{1}{1+r_t} E_t [V^U(E_{t+1}, L_{t+1}, \bar{\rho}_{t+1}, r_{t+1})] \right\} \quad (13)$$

s.t. (1), (2), (4), (6), (7), and

$$E_t - D_t \geq \gamma(L_t + N_t)$$

The regulatory constraint has been altered since the bank can now always maintain the minimum risk adjusted capital adequacy ratio by issuing new equity.

Even though equity is still more costly due to the tax effect, the absence of the financial constraint leads to a decoupling of the bank's lending decision from its financial position. New loans depend only on the lending opportunities and the cost of funding. Essentially, the Modigliani-Miller theorem now applies in the sense that lending is independent of the bank's capital structure at the beginning of the period. The following proposition formalizes these statements. Let $s_t \equiv (E_t, L_t, \bar{\rho}_t, r_t)$ denote the vector of states and let $N^U(s_t)$ and $D^U(s_t)$ be the optimal policy functions associated with the unconstrained problem in (13). Then the following proposition applies.

Proposition 1: The unconstrained bank's lending decisions are a function of the path of interest rates only: $N^U(s_t) = N^U(r_t)$, where $N^U(\cdot)$ is a time-invariant function. Furthermore, the bank's optimal financial policy is to keep the risk adjusted capital ratio exactly at the regulatory minimum: $D^U(s_t) = E_t - \gamma(L_t + N^U(r_t))$. Finally, the unconstrained bank's value function is of the following form:

$$V^U(E_t, L_t, \bar{\rho}_t, r_t) = E_t + a_L(r_t)L_t + a_{\rho L}(r_t)\bar{\rho}_t L_t + a_0(r_t)$$

Proof: see Appendix A.

While the capital regulation does affect the after-tax cost of funding for new loans, which is equal to $(1-\gamma)(1-\tau)r_t + \gamma r_t$, the financial position of the bank at the beginning of the period is irrelevant to its lending decision, because the bank can always raise sufficient funds to finance profitable lending opportunities. For the same reason, there is no need

for the bank to hold a costly precautionary buffer of capital above the regulatory minimum.

A bank that faces the financial constraint is able to lend $N^U(r_t)$, the optimal level for a financially unconstrained bank, only if it starts the period with sufficient capital, that is, only if $E_t \geq \gamma(L_t + N^U(r_t))$. Suppose the bank were to attempt to keep its lending equal to its unconstrained counterpart. It could achieve this only by maintaining a high buffer of prudential capital through retained earnings. That is, it would have to leave $E_{t-1} - D_{t-1}$ sufficiently high to ensure that $E_t \geq \gamma(L_t + N^U(r_t))$ for all realizations of r_t , δ_t and ω_t . While this may be feasible, holding the additional equity is costly due to the unfavorable tax treatment of equity funding. The bank will trade off the benefit of holding more equity (a smaller likelihood that the financial constraint will bind and limit lending) against the tax disadvantage. The optimal trade-off involves a positive probability of being financially constrained, because at $N_t = N^U(r_t)$ a marginal reduction in lending has no effect on the value of the bank's equity. The following proposition formalizes this. The condition $\Pr[D^U(s_t) < 0] > 0$ simply states that the bank's problem is sufficiently challenging so that an unconstrained bank, which does not keep any excess capital, will sometimes want to issue new equity.

Proposition 2: Suppose $\Pr[D^U(s_t) < 0] > 0$. Then a bank that faces the financial constraint will in some periods deviate from the unconstrained optimal level of lending, even when it is feasible for the bank to always lend at this level. That is, for all s_0 , $N(s_t) \neq N^U(s_t) (= N^U(r_t))$ for some t .

Proof: see Appendix B.

It may seem surprising that we cannot prove a stronger proposition, namely that the financially constrained bank will always lend weakly less ($N(s_t) \leq N^U(r_t)$ for all t), but that is because it is not true, as we will see.

4. Calibration and Solution Methods

The size of the bank is implicitly determined by the bank's loan demand functions $b(N, \omega)$ and $\rho(N)$. Computations of versions with a strictly downward sloping $\rho(\cdot)$ function resulted in only very minimal variation in the average contractual interest rate, $\bar{\rho}_t$, reflecting the longer maturity of loans. In light of this, I proceed with a constant contractual interest rate, ρ^0 , so that the bank's market power (or limited screening capacity) is completely captured through charge-offs $b(N, \omega)$. The advantage of this approach is that it reduces the number of states by one. This greatly reduces the computational burden. It should be stressed that even though ρ is fixed, the expected net return on bank loans is variable and will move with the market rate r .

Table 1.1. Parameters values (at annual rates):

γ	0.08		
τ	0.40		
ρ^0	0.08		
α	4.00		
η	8.00		
π^F	- 0.094		
<i>Distribution of:</i>	<i>Min:</i>	<i>Mode:</i>	<i>Max:</i>
ω	- 0.005	0.005	0.040
δ	0.200	0.250	0.300

Table 1.2. Interest rate process

<i>Transition probabilities: $\Pr[r_{t+1} r_t]$</i>				
	$r_{t+1}=0.04$	$r_{t+1}=0.05$	$r_{t+1}=0.06$	$r_{t+1}=0.07$
$r_t = 0.04$	0.8	0.2	0.0	0.0
$r_t = 0.05$	0.1	0.8	0.1	0.0
$r_t = 0.06$	0.0	0.1	0.8	0.1
$r_t = 0.07$	0.0	0.0	0.2	0.8

4.1 Calibration

The model is calibrated for a quarterly time period. Table 1 gives some key parameter values. The required risk adjusted capital asset ratio, γ , is set to 0.08, the

relevant value for total capital. The corporate income tax τ is 0.4. The interest rate is assumed to have finite support. This makes exact integration with respect to this shock possible. As a starting point, we allow r to have four values, ranging from 0.04 to 0.07 annually, as given in table 1.2 (top row). The interest rate r thus follows a Markov process in a finite space with transition probabilities as given in the table. The values chosen imply a quarterly autocorrelation of 0.9, which is close to the empirical value for the Federal funds rate.

To separate the monetary policy effects most clearly the charge-off rate, ω , and loan repayments, δ , are assumed to be independent of the interest rate r . Without this assumption, movements in lending would partly reflect shifts in loan demand. Each of the two shocks has a tent-shaped density, with minimum, mode and maximum as given in table 1.1, again expressed at annual rates. The continuity assumption requires that the densities are zero at the minimum and maximum of the support.³⁰ The mean of the net charge-off rate ω is 1.3% annually, which is close to the historic average for commercial and industrial and consumer loans.³¹ The average maturity of loans ($1/\bar{\delta}$) is set at four years. According to Wright and Houpt (1996), for the median commercial bank in 1995, the proportion of bank assets maturing or repricing in more than five years is 0.1. Since in the model all securities are short, and taking a loans-to-assets ratio of 0.6 (Berger et al., 1995), this would imply a value of $\bar{\delta} = 0.3$, slightly higher than the value used here.

Given the processes of r , ω and δ , the value of the contractual interest rate, ρ^0 , must be set sufficiently high to ensure that some lending is profitable for all values of r . The value chosen of 0.08 per annum accomplishes this. I assume the following functional form for $b(N, \omega)$:

$$b(N, \omega) = \alpha N^\eta \omega$$

³⁰ Net charge-offs can be negative if recoveries on loans exceed gross charge-offs.

³¹ Residential mortgages have historically had much lower charge-off rates in the U.S. (about 0.1% for 1-4 family homes). Consequently, they have been placed in a lower risk category. Commercial real estate loans are more risky, with a 1.7% average charge-off rate, compared to 1.1% for C&I loans and 1.3% for consumer loans (Grenadier and Hall (1995), 1976-1993 data).

The parameter α normalizes the bank's size. The values of ρ^0 and η jointly determine the slope of the bank's loan demand curve. At the chosen values, the elasticity of loan demand with respect to the market interest rate, measured as $-(r/N^U)\partial N^U/\partial r$, is approximately unity for permanent changes in the short rate r , and 0.4 for changes of the calibrated persistence. With this loan demand curve, the response of loans to interest rate shocks turns out to be order-of-magnitude comparable to the response of loans in identified VARs to a monetary policy shock that moves the Federal funds rate by the same amount.³²

Finally, the value of π^F , the fixed component of bank profits, is set to ensure that the bank's problem is sufficiently challenging to be interesting, while ensuring that the market-to-book-value ratio of the bank's equity, $q = V/E$, is above one most of the time. (In the simulations, the mean average q is 1.16).

Since there are only two endogenous states left, the value function reduces to the form: $V^U(E_t, L_t, r_t) = E_t + a_L(r_t)L_t + a_0(r_t)$. The numerical values of a_L and a_0 are given in table 2, as is the policy function for new loans of the financially unconstrained bank. At the highest level of lending ($N = 0.9$) the marginal new loan is expected to have 5% more charged off over its lifetime than the marginal loan at $N = 0$.

Table 2. Unconstrained value and policy functions

	$r = 0.04$	$r = 0.05$	$r = 0.06$	$r = 0.07$
$N^U(r)$	0.897	0.868	0.803	0.722
$a_0(r)$	0.111	0.060	- 0.031	- 0.077
$a_L(r)$	0.030	0.022	0.011	0.003

³² I do not argue that the model impulse response functions should match the VAR results with a high degree of precision. The VAR responses reflect in part shifts in loan demand due to changes in economic activity following a monetary shock, whereas in the model the loan demand curve is held fixed to isolate the supply effect. If loan demand depends positively on economic activity, then the model should produce somewhat smaller responses of loans to a given change in the Funds rate than identified VARs (but see footnote 47). For what it's worth, Bernanke and Blinder's (1992) results imply that a monetary policy shock that increases the Funds rate by one percentage point produces a 1.9 (4.4) percent decline in bank loans at a one (two) year horizon, compared to 1.6 (2.5) percent in the model. While identified VARs usually produce a more transitory movement in the Funds rate following a monetary policy shocks than the model's interest rate process, the 18 variable VAR of Leeper, Sims and Zha (1996) shows that a monetary policy shock with roughly the same persistence as the model's produces nearly the same effect on bank loans during the first two years as the more transitory shock.

4.2 Solution Methods

A value iteration method is used to obtain an approximate solution to the bank's problem. Because the effect of bank capital on lending is expected to be highly nonlinear, linearization techniques are undesirable for solving this model. The algorithm employs modified policy iteration on a finite grid. The problem is not discretized, however. The choice variables remain continuous and simplicial interpolation is used to evaluate the value function at non-grid points. The grid is denser around $E = \gamma L$ as the value function is highly nonlinear in this area. The initial guess for the value function is V^U , which is easily solved for using proposition 1. Appendix C provides additional computational details. The results presented are computed using a grid with 28,800 points.

Table 3. Simulated moments

<i>Variable</i>	<i>Expected value</i>	<i>Standard deviation</i>	<i>Autocorrelation</i>
$Z = E - \gamma L$	0.0958	0.0243	0.5971
L	11.4519	1.1502	0.9940
N	0.8056	0.1151	0.8442
D	0.0161	0.0186	0.5439
π	0.0267	0.0411	0.4135
$Z' = Z - D - \gamma N$	0.0153	0.0157	0.8082

5. Results

Before delving into a detailed discussion of the results, table 3 gives some key moments from the simulation of the solved model. Histograms of key variables can be found in figure 1. The tables and graphs refer to excess capital, which is defined as equity capital minus the regulatory minimum. Beginning of period excess capital, Z_t , is given by

$$Z_t \equiv E_t - \gamma L_t$$

After the new lending and dividends decisions have been made, excess capital reduces to $Z'_t \equiv E_t - D_t - \gamma(L_t + N_t)$. A positive Z' implies that the regulatory constraint (8) is slack. It is noted that the bank violates capital adequacy regulations about 0.43 percent of the

periods, while bankruptcy occurs only about 0.02 percent of the time. (In 1993, the year following the full implementation of the FDICIA, 0.26 percent of bank assets was in undercapitalized banks, while 0.03 percent was in critically undercapitalized banks.)

The remainder of this section characterizes the value function and policy function of the model bank. The next sections will analyze the bank's response to monetary and loan default shocks.

5.1 The Value Function

Figure 2 depicts the values of the constrained and unconstrained³³ banks as functions of equity capital and for different levels of the interest rate. The numbers on the horizontal axis actually refer to excess capital. In the figure L is held constant at its average value (11.5). The range of excess capital in the graph represents 98% of the simulated values of Z . As predicted by proposition 1, the value function of the unconstrained bank is linear in equity with slope equal to 1. For a given interest rate, the constrained bank's value function lies below the unconstrained bank's and is nonlinear in equity. The nonlinearity is more clearly displayed in the bottom panel of the figure, which shows the marginal value of equity, $\partial V(s_t)/\partial E_t$, at the same levels of excess capital. As can be seen from the discontinuity, the value function V contains a kink at $Z = 0$, the point at which beginning of period equity capital is exactly equal to the regulatory minimum. At this point the bank is unable to make any new loans, as both the regulatory and the financial constraints are binding. The kink arises because at this point a marginal increase in equity allows the bank to take on the most profitable lending opportunities, which accounts for the higher right derivative. The left derivative is lower because a decrease in equity at $Z = 0$ does not change the bank's policy. (The regulator does not force the bank to liquidate its loans when capital is inadequate, as long as equity remains positive.)

As equity is increased from the regulatory minimum, its marginal value drops gradually to 1. The reason for this is that, with equity close to the regulatory minimum,

³³ Throughout the paper I use the phrase 'constrained bank' for a bank that faces the financial constraint (10), whether that constraint is currently binding or not. The value function of this bank is denoted V and is defined in (12). The phrase 'unconstrained bank' refers to the bank's problem without the financial constraint. Its value function is given by V^U as defined in (13).

both the financial and regulatory constraints are binding so that a marginal addition to equity allows the bank to undertake two potentially value increasing actions. First, an additional dollar of equity allows the bank to invest $1/\gamma$ dollars more in new loans. The bank selects the most profitable lending opportunities first, but new lending is subject to diminishing returns, reflecting the bank's market power (formalized in the assumptions (3) and (5)). As new lending increases, the marginal profitability of new loans declines and this accounts, at least in part, for the declining marginal value of equity. At the lower market interest rate lending is more profitable, which results in a higher marginal value of equity for small but positive excess capital.

The second reason why the marginal value of equity can be higher than 1 for low but positive values of excess capital lies in the possibility that the bank faces a binding financial constraint next period. If lending conditions are expected to be good in the future, but there is a positive probability of losses, then the bank has an incentive to maintain equity capital in excess of the regulatory minimum this period in order to lower the expected cost of future capital inadequacy. The regulatory constraint (8) is then slack. The bank weighs this benefit of holding excess capital against the tax cost of holding the additional equity. Thus, the bank will never choose to retain so much equity so as to drive the probability of facing a binding financial constraint to zero.

At a sufficiently high level of excess capital, the bank will have exhausted all profitable lending opportunities and retained the optimal amount of excess capital, and at this point all remaining excess capital is paid out to shareholders as dividend. It is immediate from the Bellman equation that, if dividends are strictly positive, the marginal value of equity is equal to one.

For negative levels of excess capital the marginal value of equity can in principle be either smaller or larger than one. To see why this is so it is useful to differentiate the Bellman equation (12) with respect to equity, holding the policy variable constant (as an undercapitalized bank is required to do):

$$\frac{\partial V(s_t)}{\partial E_t} = \frac{1 + (1 - \tau)r_t}{1 + r_t} E_t \left[\frac{\partial V(s_{t+1})}{\partial E_{t+1}} \right]$$

If the bank is certain to have enough capital next period to pay out dividends, then the marginal value of equity this period is less than one due to the tax effect. However, if there is a positive probability that the financial constraint (10) will be binding next period then the current marginal value of equity may be greater than one. In the calibrated model, the second effect outweighs the first in by far most regions of the state-space, so that the marginal value of equity is usually greater than one for undercapitalized banks.

The nonlinearity of the value function has interesting consequences for the bank's derived preferences toward risk. While the unconstrained bank is risk-neutral, the constrained bank's value function's curvature has both risk-averse and risk-loving characteristics. The decreasing marginal value of equity at positive but below-average levels of excess capital implies that the bank is locally risk averse with respect to small equity gambles in this region. Around $Z = 0$, however, the bank is risk-loving, due to the kink in the value function. As explained, the source of the kink and the induced preference for risk is the nature of the capital regulation. It has nothing to do with the deposit insurance or any 'gamble-for-resurrection' phenomenon associated with bankruptcy.

The interest rate also affects the curvature of the value function. Both the kink at $Z = 0$ and the concavity at positive levels of excess capital are more pronounced at lower levels of the interest rate, as a lower interest rate increases the profitability of new lending. The implication is that deviations from risk neutrality are more likely to affect bank behavior when interest rates are low.

Figure 3 shows the difference between the bank's value and the value of its unconstrained counterpart ($V(s) - V^U(s)$) as a function of the endogenous state variables, excess capital and loans. The range of loans represents 98% of the simulated values. This figure shows *the negative of* how much a constrained bank would be willing to pay to have its financial constraint lifted permanently, depending on its balance sheet. (Note that the vertical axis has only negative values; this presentation yields the clearest view of the nonlinearity of V .) Higher values thus correspond to lower expected cost of capital inadequacy.³⁴ While the bank value is lowered by the financial constraint everywhere, the

³⁴ To be precise, these costs include both the loss due to forgone lending opportunities and the tax cost of holding of excess capital, which the bank holds to limit this loss.

cost is highest for banks with low levels of capital and low outstanding loans. The magnitude of the cost is economically significant. For example, a bank with average book values of loans and equity (11.5 and 1.01, respectively) has expected capital inadequacy costs ranging from 0.013 for $r = 0.04$ to 0.019 for $r = 0.07$, which is about 1 to 2% of the bank's market value of equity at these interest rates. In contrast, an undercapitalized bank with excess capital of -0.01 , but the same amount of loans, has expected capital inadequacy costs of 0.040 and 0.034 at these interest rates.

In addition, figure 3 fully documents the nonlinearity of the bank's value function in E and L , since V^U is linear in these states. Two findings emerge in addition to the results already discussed. For a given level of excess capital, both the marginal value of equity and the marginal value of loans are higher when the amount of outstanding loans is low. The reason for this is that loans are profitable, reflecting the bank's market power, so that with fewer loans outstanding, the bank's expected profits are lower (even as fraction of loans outstanding, due to the presence of fixed operating costs). For a given buffer stock of excess capital, this implies a greater likelihood that the financial constraint will be binding in the future, which in turn increases the marginal value of equity in the current period. The next subsection will show that banks with low loans do indeed hold a larger buffer stock of capital when feasible.

5.2 The Policy Function

Figure 4 depicts the bank's optimal policies as a function of excess capital, again given an average loan portfolio. Both new loans N (scaled by $\gamma = 0.08$) and dividends D are shown, at two levels of the interest rate: $r1 = 0.05$ and $r2 = 0.06$. The third variable shown, Z' , is the slack in the regulatory constraint (8):

$$Z'_t \equiv E_t - D_t - \gamma(L_t + N_t) \equiv Z_t - D_t - \gamma N_t$$

Although Z' is obviously not an independent decision variable, given N and D , it is useful to include it in the discussion of the bank's optimal policy.

For a bank with high levels of excess capital at the beginning of the period, the bank's decisions resemble those of its unconstrained counterpart. For an abundantly

capitalized bank new lending is unaffected by (small) changes in equity which only affect dividends. The main difference is that, while the financially unconstrained bank maintains no excess capital after the new loans and dividends have been paid out, an adequately capitalized bank that faces a financial constraint will in general decide to maintain a buffer of excess capital to lower the expected cost of future capital inadequacy. In this case the regulatory constraint (8) is slack and Z' is strictly positive.³⁵ It is not surprising that new loans depend negatively on the short interest rate. This is a straightforward cost-of-funding effect, which occurs for a financially unconstrained bank as well.

For lower levels of excess capital the financial constraint is binding first. Dividends are zero and the bank responds to further reductions in equity capital by both lowering the buffer of excess capital Z' and, to a lesser extent, by cutting new lending. Although the effect is not very strong for a bank with average loans, the bank cuts new lending in response to a negative capital shock even when the regulatory constraint is slack, so that it is still feasible for the bank to isolate its lending decisions from the capital shock. In doing so, the bank forgoes profitable lending opportunities now in order to lower the risk that capital inadequacy will force it to forgo even more profitable lending opportunities in the future. Figure 5a, which displays the lending decision as a function of both excess capital and outstanding loans, shows that this precautionary reduction in lending is much stronger when loans are low or the short rate is high. Again, the reason is that expected profits are lower, so the bank values a greater buffer stock of excess capital more. (The dividend graphs in figure 5b confirm that no dividends are paid out in the regions of reduced lending). The precautionary reduction in new lending is especially strong when the short interest rate is at its highest level. New lending actually falls to zero for a moderately capitalized bank with low loans. This reflects not only low expected profits, but also the fact that, at this high interest rate, r is expected to decrease, so that lending is likely to be more profitable in the future. Conditional on the regulatory constraint being slack, a precautionary cut in N occurs about 40% of the time and the mean cut is 5.6%.

³⁵ One other difference is that even a well-capitalized bank does not in general lend at exactly the same level as its unconstrained counterpart: $N \neq N^U(r)$. This is discussed in more detail below.

As equity is lowered even further toward the regulatory minimum, the regulatory constraint eventually becomes binding ($Z' = 0$). This situation occurs about 19 percent of the periods in the simulation. With the financial constraint already binding, a unit reduction in equity results in a $1/\gamma$ ($= 12.5$) unit reduction in lending. The bank's behavior is also radically different with respect to interest rate shocks: these have no effect within the period.

Table 4. Distribution of over- and underlending

<i>Overlending</i>	<i>Frequency (%)</i>
$n = -1$	0.9
$-1 < n \leq -0.2$	5.2
$-0.2 < n \leq -0.1$	4.3
$-0.1 < n \leq -0.05$	3.6
$-0.05 < n \leq -0.01$	6.1
$-0.01 < n \leq 0$	4.8
$0 < n \leq 0.01$	50.8
$0.01 < n \leq 0.05$	24.2
$n > 0.05$	0.0

There is one surprising aspect of the bank's lending decisions that is not apparent from these figures: for any given interest rate, the model bank often lends *more* than its unconstrained counterpart. Table 4 gives the distribution of new loans relative to new loans by a financially unconstrained bank. The variable n is defined as:

$$n(s_t) \equiv (N(s_t) - N^U(r_t)) / N^U(r_t)$$

I refer to n as overlending (and a negative n as underlending) attributable to the combined effect of the capital regulation and the financial constraint.³⁶ Because N^U does not depend on bank-specific states, $n(s_t)$ is the difference between lending by a constrained bank with state s_t and lending by an unconstrained bank with the same state (or any other state with the same interest rate). While it is not surprising in light of the discussion above that underlending occurs frequently, and sometimes by large amounts, the fact that

³⁶ Due to the tax distortion overlending is not necessarily sub-optimal.

constrained banks will often overlend may seem puzzling. Almost one quarter of the time the bank will overlend by more than a percentage point. While the effect is not very large, it is not economically insignificant either.³⁷

What causes this phenomenon? How can loans that are not profitable to the unconstrained bank be profitable to a bank that faces a financial constraint? To inspect the mechanism at work it is useful to examine a special case of the model by assuming full and certain repayment of all loans in one quarter: $\delta_t \equiv 1$ and $\omega_t \equiv 0 = b(N, \omega)$. In that special case, the first order conditions with respect to new loans simplifies to:³⁸

$$E_t \left[\frac{\partial V(E_{t+1}, r_{t+1})}{\partial E_{t+1}} \right] (1 - \tau) (\text{MRN}(N_t) - r_t) = \gamma \psi_t (1 + r_t)$$

where $\text{MRN}(N) \equiv d(\rho(N)N) / dN = \rho'(N)N + \rho(N)$ is the marginal revenue from new loans and $\psi (\geq 0)$ is the Kuhn-Tucker multiplier associated with the regulatory constraint, (8). Note that there is L and $\bar{\rho}$ are no longer required as state variables due to the full repayment of loans within the quarter. The above first order condition applies equally to the constrained and unconstrained banks.

For the *unconstrained* bank, we know from proposition 1 that the marginal value of equity is 1 and that the capital requirement is always binding, so that $\psi_t > 0$. In fact, Appendix A shows that $\psi_t = \tau r_t / (1 + r_t)$. Hence, for this special case, optimal lending for the unconstrained bank satisfies:

$$\text{MRN}(N_t^U) = r_t + \gamma \tau r_t / (1 - \tau)$$

For the *constrained* bank the capital requirement may or may not be binding. Suppose that it is momentarily nonbonding, so that $\psi_t = 0$. In the context of this special

³⁷ The reader may be concerned about computational error. The overlending is large, however, compared to the accuracy of the optimization and integration routines. In addition, increasing the number of grid points and the number of Monte-Carlo draws does not reduce the overlending phenomenon.

³⁸ Assume $E_t > 0$.

case this can happen if the current short rate is high and there is a sufficiently high probability of a decline in r (so that not much capital is needed now but much more may be needed next period to satisfy the capital requirement). Optimal lending with $\psi_t = 0$ is given by

$$\text{MRN}(N_t) = r_t$$

Because MRN is a decreasing function of N , it follows that $N_t > N^U(r_t)$ whenever the capital requirement is slack for the bank that faces the financial constraint.

The intuition can be summarized as follows. The financial imperfection motivates the bank to hold costly equity in excess of the regulatory minimum most of the time (i.e. the regulatory constraint is slack). While this means that *on average* loans incur a larger cost of funding disadvantage than for an unconstrained bank, this need not be true *at the margin*. A bank that is already holding a buffer of excess capital can choose to fund an additional loan by less than a fraction γ in equity. This does not necessarily have an adverse impact on the distribution of future excess capital – which is what the bank cares about when the current constraint is slack – because the future capital requirement associated with the new loans will be less than γN , as the loans will be repaid gradually, reducing their book value.³⁹ In contrast to the special case above, in the calibrated version of the model this effect may be offset by the riskiness of loans, so that overlending does not *always* occur whenever the capital requirement is slack.⁴⁰

The overlending is essentially due to the trade-off between the beneficial role of equity in reducing the risk of future capital inadequacy and the higher cost of equity compared to debt. It thus occurs most frequently for banks that are in a position to exploit this trade-off: banks whose financial constraint is not binding and that pay dividends.

³⁹ In the special case above loans are fully repaid in one period. An appendix in a previous version of this paper analyzed the overlending in a simplified two period model with $\delta < 1$. This appendix is available from the author upon request.

⁴⁰ As discussed, the derived risk preferences of the financially constrained bank are complicated. However, in the regions of the state space where overlending occurs, risk aversion seems to be dominant, though not very strong. This is so because overlending occurs mostly by well-capitalized banks who are unlikely to be hit by a shock large enough to move it into the $Z < 0$ region next period, which would be the source of any risk-loving behavior.

6. Monetary Policy Effects

Monetary policy actions affect the bank's behavior in the model through movements of the short interest rate r . This section examines the dynamic response of the bank's balance sheet, lending and dividends to changes in the short rate, and how this response depends on the initial financial position of the bank.

For this purpose, impulse response functions are computed that show the difference between the expected paths of the variables with and without the shock. Since the model is nonlinear, this difference depends on the state of the bank at the time the shock occurs. Specifically, the impulse response function of, say, new loans to an interest rate shock is defined as:

$$\text{Res}_{N,r}(t; E_0, L_0, r_0) \equiv E_0[N_t | s_0 = (E_0, L_0, r_0 + \Delta r_0)] - E_0[N_t | s_0 = (E_0, L_0, r_0)], \quad t = 0, 1, 2, \dots$$

where Δr_0 is the size of the interest rate shock, referring to the period the shock occurs as period 0. Note that the response is not just a function of the time since the shock (t), but also of the initial state. For any given initial state, the expectation in $\text{Res}_{x,r}$ is computed through Monte-Carlo integration. (The usual method of equating all shocks in periods $t \geq 1$ to their expected values is not valid due to the nonlinear policy functions.)

Figure 6 shows the response of an 'average' bank to a one percentage point rise in the interest rate, starting from $r_0 = 0.05$ ($\text{Res}_{x,r}(t; E(E), E(L), 0.05)$). That is, the values for E_0 and L_0 are their unconditional means. In all impulse response diagrams the solid lines refer to the bank that faces a financial constraint and the dashed lines to the unconstrained bank. As can be seen in the figure, the unconstrained bank cuts lending in response to the interest rate hike. This response simply reflects the higher cost of funding the new loans, the conventional interest rate channel. The effect is persistent because of the positive serial correlation of the interest rate process. The response of the constrained bank's new loans is similar. The main difference is in the paths of dividends and excess capital. Apparently, the 'average' bank is able to adjust dividends to limit the decline in excess capital that follows the higher cost of funding. Consequently, the bank manages to

maintain a path of new loans largely similar to that of the unconstrained bank. Nonetheless, the response of N is somewhat smaller (in absolute value) in the first 3 quarters and larger after a year. The reason for these differences will become clear after we discuss the reactions of a bank that starts out with lower equity.

Figure 7 displays the impulse response functions of a bank that starts out with a binding regulatory constraint. The initial level of excess capital is set at 0.05, which is insufficient to fund all profitable lending opportunities, regardless of the interest rate, and at which level the bank's regulatory constraint is binding. (Loans remain set at the mean value. The figure thus shows $\text{Res}_{x,r}(t; \gamma E(L) + 0.05, E(L), 0.05)$). To give an idea of how extreme this value is, bank excess capital falls below 0.05 about 4% of the time. The response of the unconstrained bank is the same as in the previous figure because it is independent of the initial states E_0 and L_0 due to its linear policy functions. The response of the financially constrained bank is markedly different, however. In the first period, with $Z = 0.05$, both the regulatory and the financial constraints are binding, so the bank cannot make any new loans or pay out any dividends. This is why the response of both variables is nil in period 0. Hence, *monetary policy initially has no effect on bank lending*. This possibility was mentioned earlier by Kashyap and Stein (1994) in their survey of monetary policy and bank lending. *After the first quarter*, however, the reverse is true: *new lending 'overreacts'* to the contractionary monetary policy shock, relative to the financially unconstrained bank. This result is the combined effect of two forces. First, there is a positive probability that the bank will recover its equity position sufficiently so that the regulatory constraint is no longer binding. Conditional on this situation materializing, the response of N is approximately the same as that of the unconstrained bank. Thus, this first effect alone can at most generate a response that equals the unconstrained response. Second, the interest rate rise *lowers* the probability of such a recovery. Because of the maturity mismatch on the bank's balance sheet, the rise in the short rate increases the bank's interest cost by more than it increases its interest revenues. Consequently, profits are expected to be lower, which implies a lower probability of a recovery. The bank must rely on profits to rebuild its capital due to the inability of the bank to raise new equity.

The second mechanism can be referred to as a ‘bank capital channel’ of monetary policy. The rise of the short interest rate lowers future bank capital, which on average reduces bank lending. It is this channel that accounts for the excess response to the monetary shock in the later quarters.

The ‘bank capital channel’ is also apparent from the response of excess capital. While the well-capitalized bank was able to adjust dividends to smooth the path of excess capital, the poorly capitalized bank cannot lower dividends below zero. Consequently, its equity is actually more volatile than the unconstrained and the well-capitalized banks’ equity.

The response of the stock of loans to the interest rate shock is easy to understand given the behavior of new lending. At first the decline is lower for the financially constrained bank. After about three quarters, the decline is larger, eventually by about 0.9 percentage points after 12 quarters.

The effect of a given size monetary policy shock may also depend on the initial interest rate. Figure 8 documents the response functions of the ‘average’ bank with $r_0 = 0.06$. With the higher initial interest rate, the described ‘bank capital channel’ is stronger. This is due to three reasons. First, expected profits are lower when the interest rate is high. This means that the bank is less likely to be able to adjust dividends to limit movements in excess capital when the interest rate rises even further. Second, a given size shock to bank equity has a larger effect on lending when equity is already low, which is more likely to be the case with the higher rates. Third, at $r = 0.07$, following the shock, lending opportunities can only get better, so precautionary lending cuts are more pronounced (see section 5.2). The importance of the second and third effects is apparent from the fact that the ‘excess response’ of loans is actually a bit larger than for the poorly capitalized bank at $r_0 = 0.05$ (1.1% versus 0.9% at 12 quarters), even though the decline in excess capital is *smaller*.⁴¹

⁴¹ Interestingly, for a *negative* interest rate innovation, starting from $r_0 = 0.06$, the response of new loans is much closer to the unconstrained bank’s (not shown). This is again due to the nonlinearity of the bank’s policy function, which results in a weaker bank capital channel for the interest rate decline. Dell’Ariccia and Garibaldi (1998) provide evidence that the response of bank lending to interest rate innovations is asymmetric in the U.S. and Mexico, with a more rapid response to increases. They provide an explanation based on a difference between the speed at which new loans become available and the speed at which banks can recall existing loans.

Finally, the model can be used to shed some light on the issue of monetary policy effectiveness during a ‘capital crunch’. Expansionary monetary policy was deemed by many to have been relatively ineffectual in stimulating bank lending during the 1990-91 recession, especially in New England. One explanation offered for this is that many banks were having trouble meeting their capital requirements, so that they could not take advantage of the better lending opportunities created by the lower cost of funding.⁴² Figure 9 shows how a significantly undercapitalized bank responds to expansionary monetary policy, according to the model. Z_0 is set at -0.11 , which corresponds to a risk-weighted capital asset ratio 1 percentage point below the regulatory minimum. Not surprisingly, the model predicts that the decrease in the interest rate initially has no effect on lending by the undercapitalized bank. However, after two quarters the easier monetary policy starts to have a very strong impact on lending. Thus, according to the model, if the slump in bank lending was due to a ‘capital crunch’, it would have lasted much longer without the easier monetary policy.

7. Loan Default Shocks

To examine the effect of loan defaults, we construct the impulse response functions to a loan default shock in an analogous fashion as for the interest rate innovations. For example, for new loans the response is defined as

$$\begin{aligned} \text{Res}_{N,\omega}(t; E_0, L_0, r_0) \equiv & E_0[N_t | s_0 = (E_0, L_0, r_0), \omega_1 = E_0(\omega_1) + \Delta\omega] \\ & - E_0[N_t | s_0 = (E_0, L_0, r_0), \omega_1 = E_0(\omega_1)] \quad t = 0, 1, 2, \dots \end{aligned}$$

As the loan default shock of size $\Delta\omega$ takes place during period zero, at the beginning of that period no balance sheet or decision variables are affected.

Figure 11 shows the response of an ‘average’ bank to a one standard deviation loan default rate shock ($\Delta\omega = 0.0024$), at $r_0 = 0.06$ ($\text{Res}_{x,\omega}(t; E(E), E(L), 0.06)$). Due to

⁴² See Bernanke and Lown (1991) for a discussion. See Peek and Rosengren (1995) for evidence on the ‘capital crunch’ for New England banks.

the negative effect of the loan defaults on bank capital, new lending declines in period 1 and stays lower for several periods. This contrasts with the financially unconstrained bank whose lending is unaffected by any charge-offs. Of course, the book value of both banks' loans declines due to the defaults. Due to the reduction in lending, the financially constrained bank's loans decline by an additional quarter percent, roughly doubling the decline.

Figure 12 shows how the impact of defaults on loans is amplified for a bank with lower, but still positive, excess capital in period 0: $Z_0 = 0.07$. New lending strongly declines in period 1 (compared to the size of the shock), and the cumulative decline in loans is over 1 percent after 6 quarters.

8. General Equilibrium Considerations

The model presented analyzes the decision problem of an individual bank. Several additional issues would arise in incorporating it in a general equilibrium framework. For example, loan demand and charge-offs may vary with economic activity and this could have a potentially important feedback on the bank's problem. I now discuss some of these issues and how they affect the way in which the model's implications on bank-level quantities translate into conclusions about aggregate variables.

First, banks compete for clients in making loans. By assuming that the loan demand curve facing the *individual* bank is independent of the short interest rate, I have abstracted from interbank competition. Suppose instead that there is perfect competition between banks because borrowers can switch costlessly from one bank to any other. In that case, only aggregate bank capital matters for aggregate lending, as borrowers would switch to the lowest cost lenders. In addition, if there are any financially unconstrained banks, the bank capital channel would cease to exist with zero switching costs.⁴³ One would not expect banks to hold any excess capital in this case, as lending is no longer strictly profitable, nor would bank-specific variables affect the cost of loans to firms.

⁴³ This is true of the bank lending channel as well. See Stein (1998).

Both implications are contradicted by the data.⁴⁴ As argued, banks do in fact have some market power due to customer relations, geographic specialization, etc. This means that switching costs are positive for at least a subset of borrowers. In the extreme case that all borrowers face infinite switching costs, any change in an individual bank's lending causes aggregate lending to move one for one. Due to the nonlinear effect of capital on lending, not only aggregate bank capital but also its distribution across banks affects aggregate lending in this case, with a mean preserving spread of excess capital lowering aggregate lending. Moreover, a bank capital channel exists, even if there are some banks with perfect access to the market for bank equity.

In the more realistic case where switching costs are positive but finite, part of the idiosyncratic fluctuations in individual bank lending will be picked up, at some cost, by other banks; the remainder will result in changes in aggregate lending. Low switching cost would diminish the importance of individual bank capital to aggregate lending, other things being equal. However, other things are not equal: lower switch costs imply that the loan demand curve faced by an individual bank becomes flatter.⁴⁵ Thus, intramarginal lending is less profitable and the bank will be less averse to reductions in lending due to capital inadequacy. Hence, the bank will hold less excess capital and will be more likely to be capital constrained, increasing idiosyncratic fluctuations in lending. A useful extension of the model would incorporate finite switching costs in an industry equilibrium.⁴⁶

A second issue that arises in a general equilibrium context is the endogeneity of *aggregate* loan demand. While in the model the loan demand schedule is held constant to trace out movements in loan supply most clearly, demand for bank loans is likely to move with economic conditions. A temporary increase in loan demand has two effects on the bank's problem. On the one hand, the bank will need more capital to satisfy the additional demand. On the other hand, with imperfect competition, the profitability of lending is increased. The first effect increases the risk of inadequate capital, the second

⁴⁴ See, for example, Hubbard, Kuttner and Palia (2001).

⁴⁵ It will also shift with aggregate conditions, including r .

⁴⁶ Den Haan, Ramey and Watson (1999) present a model of long-term credit relations, where the cost of switching for a borrower is the production forgone in the time it takes to find a match with a new lender. In their model the intermediaries face idiosyncratic liquidity shocks that can break up lending relations. In

effect lowers it. It is not clear a priori which effect dominates in equilibrium, but this points to a potential feedback on the bank's capital adequacy.⁴⁷

A third and important issue concerns the fact that net charge-offs on loans are countercyclical. By reducing economic activity, contractionary monetary policy may result in increased net charge-offs, which would in turn lower bank capital. This mechanism would clearly amplify the bank capital channel.

Finally, the analysis has been silent on movements in inflation. Since loan demand presumably depends primarily on the real interest rate, the model should be interpreted as real. Obviously, if inflation is constant, it can also be interpreted as nominal. Purely nominal changes in interest rates would still have roughly the same impact on the bank's balance sheet, since most bank loans are not indexed. Thus, an increase in expected inflation, accompanied by an equal increase in nominal rates, would worsen the bank's capital position and this could reduce its lending. Naturally, this 'debt-deflation in reverse' also has implications for the impact of monetary policy actions, as these produce both real movements in the Federal funds rate and, at longer horizons, changes in inflation.

9. Conclusion

This paper has analyzed a dynamic model of optimal bank lending that incorporates risk-based capital requirements and an imperfect market for bank equity. Under these conditions bank lending depends on the bank's capital structure, which in turn evolves endogenously. An interesting result is that well-capitalized banks frequently lend more than they would if they faced no financial constraint in future periods. An important implication of the relevance of the bank's capital adequacy is that shocks to

equilibrium, aggregate shocks are propagated by the breakup of credit relations which, because of diminishing returns at the firm level, in turn lowers efficiency and thus the level of investment.

⁴⁷ How this feedback would alter the response of lending to monetary policy shocks obviously also depends on the effect of these shocks on loan demand. While loans are procyclical, it may well be the case that the demand for credit initially increases in response to a contractionary monetary policy shock, reflecting a buildup of inventories. While not entirely conclusive, the empirical evidence from the commercial paper market (Kashyap, Wilcox and Stein (1993)) and lending under commitment (Morgan (1992)) seems to favor this view.

bank profits, such as loan defaults, can have a persistent impact on lending. This is akin to the ‘financial accelerator’ mechanism that arises in firm models with imperfect financial markets, such as Bernanke, Gertler and Gilchrist (1999). While these models usually focus on the firm’s debt capacity being limited by the level of inside equity, for banks the mechanism is operative even without an imperfect debt market, because the regulatory capital requirements make (inside or outside) equity itself indispensable.

Another somewhat special characteristic of banks is that they transform short liabilities into longer assets. Combined with the relevance of the bank’s capital adequacy, this gives rise to a ‘bank capital channel’ by which monetary policy can change the supply of bank loans through its impact on bank equity. An implication is that monetary policy effects on bank lending depend on the capital adequacy of the banking sector. In particular, lending by banks with low capital has a delayed and then amplified reaction to interest rate shocks, relative to well-capitalized banks.

The model also provides a useful framework for analyzing the effectiveness of capital regulations in preventing bank failures and limiting the costs of deposit insurance. To explore this issue in more detail, a useful extension of the model would give the bank an explicit choice with respect to the risk characteristics of its assets, since the capital regulations can generate both risk-averse and risk-loving preferences on the part of the bank.

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APPENDIX A. PROOF OF PROPOSITION 1.

It is convenient to work with a transformed state-space. Define $Q_t \equiv \bar{\rho}_t L_t$ and let

$$W(E_t, L_t, Q_t, r_t) \equiv V^U(E_t, L_t, Q_t / L_t, r_t)$$

The Bellman equation for W is:

$$W(E_t, L_t, Q_t, r_t) = \max_{D_t, N_t} \left\{ D_t + \frac{1}{1+r_t} E_t [W(E_{t+1}, L_{t+1}, Q_{t+1}, r_{t+1})] \right\} \quad (\text{A.14})$$

s.t.

$$L_{t+1} = (1 - \delta_{t+1} - \omega_{t+1})L_t + (1 - \delta_{t+1} - b(N_t, \omega_{t+1}))N_t$$

$$Q_{t+1} = (1 - \delta_{t+1} - \omega_{t+1})Q_t + (1 - \delta_{t+1} - b(N_t, \omega_{t+1}))\rho(N_t)N_t$$

$$E_{t+1} = E_t - D_t + (1 - \tau)\pi_{t+1}$$

$$\begin{aligned} \pi_{t+1} = & (1 - \omega_{t+1})Q_t + (1 - b(N_t, \omega_{t+1}))\rho(N_t)N_t - r_t(L_t - E_t + N_t + D_t) \\ & - (\omega_{t+1}L_t + b(N_t, \omega_{t+1})N_t) + \pi^F \end{aligned}$$

$$E_t - D_t \geq \gamma(L_t + N_t)$$

To find a solution to the Bellman equation, assume W is differentiable. Using shorthand notation, the first-order conditions are:

$$(\partial N) \frac{1}{1+r_t} E_t \left[\frac{\partial W_{t+1}}{\partial E_{t+1}} \left((1-\tau) \frac{\partial \pi_{t+1}}{\partial N_t} \right) + \frac{\partial W_{t+1}}{\partial L_{t+1}} \left(\frac{\partial L_{t+1}}{\partial N_t} \right) + \frac{\partial W_{t+1}}{\partial Q_{t+1}} \left(\frac{\partial Q_{t+1}}{\partial N_t} \right) \right] = \gamma \psi_t$$

$$(\partial D) \frac{1 + (1-\tau)r_t}{1+r_t} E_t \left[\frac{\partial W_{t+1}}{\partial E_{t+1}} \right] = 1 - \psi_t$$

where ψ_t is the Kuhn-Tucker multiplier associated with the regulatory constraint, $\partial W_{t+1} / \partial E_{t+1} \equiv \partial W(E_{t+1}, L_{t+1}, Q_{t+1}, r_{t+1}) / \partial E_{t+1}$, etc. The envelope condition with respect to equity is:

$$(\partial E) \frac{\partial W_t}{\partial E_t} = \frac{1 + (1-\tau)r_t}{1+r_t} E_t \left[\frac{\partial W_{t+1}}{\partial E_{t+1}} \right] + \psi_t = 1$$

where the last equality follows from the first-order condition with respect to D . This implies:

$$\psi_t = \pi_t / (1+r_t)$$

Using this we can write the envelope condition with respect to L as:

$$(\partial L) \frac{\partial W_t}{\partial L_t} = \frac{1}{1+r_t} \left(E_t \left[(1-\delta_{t+1} - \omega_{t+1}) \frac{\partial W_{t+1}}{\partial L_{t+1}} - (1-\tau)(r_t + \omega_{t+1}) \right] - \gamma \pi_t \right)$$

Let $Y_t \equiv (1-\delta_t - \omega_t) \frac{\partial W_t}{\partial L_t}$, $\theta_t \equiv \frac{1-\delta_t - \omega_t}{1+r_t}$, $X_t \equiv -(1-\tau)(r_t + E_t[\omega_{t+1}]) - \gamma \pi_t$. Then we have

$$Y_t = \theta_t (E_t[Y_{t+1}] + X_t)$$

Since $0 < \theta_t < 1$, the forward solution converges:

$$Y_t = E_t \left[\sum_{s=0}^{\infty} \left(\prod_{u=0}^s \theta_{t+u} \right) X_{t+s} \right]$$

so that

$$\frac{\partial W_t}{\partial L_t} = -\frac{1}{1+r_t} E_t \left[\sum_{s=0}^{\infty} \left(\prod_{u=1}^s \theta_{t+u} \right) ((1-\tau)(r_{t+s} + \omega_{t+1+s}) + \gamma \pi_{t+s}) \right] \equiv a_L(r_t)$$

That the right-hand side is a function of r_t only follows from the assumption that r_t is a sufficient statistic for the time t distribution of all future exogenous shocks. Applying the same steps to the envelope condition with respect to Q yields:

$$\frac{\partial W_t}{\partial Q_t} = \frac{1-\tau}{1+r_t} E_t \left[\sum_{s=0}^{\infty} \left(\prod_{u=1}^s \theta_{t+u} \right) (1-\omega_{t+s+1}) \right] \equiv a_{\rho L}(r_t)$$

Hence, W is given by:

$$W(E_t, L_t, Q_t, r_t) = E_t + a_L(r_t)L_t + a_{\rho L}(r_t)Q_t + a_0(r_t)$$

We can now rewrite the first-order condition with respect to N as:

$$E_t \left[(1-\tau) \frac{\partial \pi_{t+1}}{\partial N_t} + a_L(r_{t+1}) \frac{\partial L_{t+1}}{\partial N_t} + a_{\rho L}(r_{t+1}) \frac{\partial Q_{t+1}}{\partial N_t} \right] - \gamma \pi_t = 0$$

It is easy to verify that the left hand side depends only on N_t and r_t and, given the assumptions on $\rho(\cdot)$ and $b(\cdot)$, that it is decreasing in N_t . Hence, N_t is uniquely determined and is a function of r_t only. Therefore, if our candidate W is the true value function,

$$N^U(s_t) = N^U(r_t)$$

We have already established that $\psi > 0$, which implies that the regulatory constraint is binding. Thus,

$$D^U(s_t) = E_t - \gamma(L_t + N^U(r_t))$$

Given the policy functions, $a_0(r_t)$ can be solved for from the Bellman equation, again by solving forward. This also completely verifies that our candidate W satisfies the Bellman equation and that it is the only differentiable function that does. Standard arguments can be used to show that it is indeed the true value function (in the transformed state-space).

Finally, V^U is derived from W as:

$$V^U(E_t, L_t, \bar{\rho}_t, r_t) = W(E_t, L_t, \bar{\rho}_t L_t, r_t) = E_t + a_L(r_t)L_t + a_{\rho L}(r_t)\bar{\rho}_t L_t + a_0(r_t)$$

APPENDIX B. PROOF OF PROPOSITION 2.

Proof is by contradiction. Suppose $N(s_t) = N^U(r_t)$ for all t . For a given initial stock of loans, L_0 , let L_t^U denote the level of loans generated by this policy, a sequence of shocks, and the law of motion for loans (1). Define the generated average contractual interest rate $\bar{\rho}_t^U$ in a similar way, given an initial level $\bar{\rho}_0$ and its law of motion (2). Note that L_t^U and $\bar{\rho}_t^U$ are equal to the loans and the average contractual interest rate of an unconstrained bank with the same initial state and the same history of shocks. Define:

$$\underline{E}_t^T := \inf \left\{ E_t \in \mathbb{R}^+ : \Pr[E_{t+v} \geq \gamma(L_{t+v}^U + N^U(r_{t+v})) \mid s_t = (E_t, L_t^U, \bar{\rho}_t^U, r_t) \wedge (D_{t+u} \equiv 0, N_{t+u} \equiv N^U(r_{t+u}), u = 0, 1, \dots, v)] = 1 \quad \forall v \in \{0, 1, \dots, T\} \right\}$$

where the laws of motion (1), (2), (4), (6) and (7) apply as usual. \underline{E}_t^T is the minimum level of equity at the beginning of period t that makes it feasible for the bank to lend at the unconstrained level in periods t through $t+T$ with probability 1. If $E_t < \underline{E}_t^T$ then, with positive probability, the proposed policy ($N_t = N^U(r_t)$ for all t) would involve a violation of the capital requirement (8) even if dividends are kept at the minimum level of zero. Since \underline{E}_t^T is increasing in T , the following limit exists in $\mathbb{R}^+ \cup \{\infty\}$:

$$\underline{E}_t := \lim_{T \rightarrow \infty} \underline{E}_t^T$$

Claim 1: With the proposed lending policy ($N_t = N^U(r_t)$ for all t) the bank will choose $D_t = E_t - \underline{E}_t$.

Proof: (i) Suppose $D_t > E_t - \underline{E}_t$. Then, from the law of motion of equity (6) and the definition of \underline{E}_t , it is immediate that there exists a finite T such that, with $N_{t+u} = N^U(r_{t+u})$ and $D_{t+1+u} \geq 0$ (the financial constraint) for all $u \in \mathbb{N}$, $\Pr[E_{t+T} < \gamma(L_{t+T}^U + N^U(r_{t+T}))] > 0$, implying a violation of the capital requirement (8) with positive probability. As a corollary, the financial constraint implies that if $E_t - \underline{E}_t < 0$ for any t , the proposed policy is not feasible with positive probability.

Corollary 1: With the proposed lending policy, the following must hold: $E_t \geq \underline{E}_t$ for all t , almost surely.

(ii) Continuing the proof of claim 1, now suppose that for some t_0 we have $D_{t_0} < E_{t_0} - \underline{E}_{t_0}$. For all $t \geq t_0$ define ε_t by writing:

$$D_t + \varepsilon_t \equiv E_t - \underline{E}_t.$$

We have already established that, for the proposed lending policy to be feasible, it must be true that $\varepsilon_t \geq 0$ for all t , and we have supposed that $\varepsilon_{t_0} > 0$. Starting from the same s_{t_0} , consider the alternative financial policy of keeping post-dividends equity exactly equal to \underline{E}_t :

$$D_t^* = E_t^* - \underline{E}_t \text{ for all } t \geq t_0,$$

where E_t^* is defined recursively by $E_{t_0}^* = E_{t_0}$ and, for $t \geq t_0$,

$$E_{t+1}^* = E_t^* - D_t^* + (1 - \tau)\pi_{t+1}^*,$$

where π_{t+1}^* are the profits associated with this alternative financial policy. By construction, the lending policy, $N_t = N^U(r_t)$ for all t , remains feasible with D_t^* . From (7), note that:

$$\pi_{t+1}^* = \pi_{t+1} - r_t \varepsilon_t$$

As a result, for all $t \geq t_0$,

$$\begin{aligned} E_{t+1} - E_{t+1}^* &= (1 + (1 - \tau)r_t)\varepsilon_t \\ \Rightarrow D_t^* - D_t &= \varepsilon_t + (E_t^* - E_t) = \begin{cases} \varepsilon_t & \text{if } t = t_0 \\ \varepsilon_t - (1 + (1 - \tau)r_{t-1})\varepsilon_{t-1} & \text{if } t > t_0 \end{cases} \end{aligned}$$

Thus, the change in value resulting from the new dividend policy is

$$\begin{aligned} V_{t_0}^* - V_{t_0} &= E_{t_0} \left[\sum_{s=0}^{\infty} \left(\prod_{u=0}^{s-1} (1 + r_{t_0+u})^{-1} \right) (D_{t_0+s}^* - D_{t_0+s}) \right] \\ &= E_{t_0} \left[\sum_{s=0}^{\infty} \left(\prod_{u=0}^s (1 + r_{t_0+u})^{-1} \right) \tau r_{t_0+s} \varepsilon_{t_0+s} \right] \geq \frac{\tau r_{t_0} \varepsilon_{t_0}}{1 + r_{t_0}} > 0 \end{aligned}$$

Hence, the original choice, $D_{t_0} < E_{t_0} - \underline{E}_{t_0}$, cannot be optimal, which concludes the proof of claim 1.

Claim 2: With the proposed lending policy, the following must hold a.s. for all t : $D_t > 0$ and $\partial V(s_t)/\partial E_t = 1$.

Proof: First we prove $D_t > 0$. From the financial constraint, we already know that $D_t \geq 0$. It remains to be shown that, for any t , $\Pr[D_t = 0] = 0$. Claim 1 implies that this is

equivalent to $\Pr[E_t = \underline{E}_t] = 0$. A sufficient condition for this is $\Pr[E_t = \underline{E}_t | s_{t-1}] = 0$ for all s_{t-1} . Now

$$E_t = \underline{E}_{t-1} + (1-\tau)(\bar{\rho}_{t-1}^U(1-\omega_t)L_{t-1}^U + \rho(N_{t-1})(1-b(N_{t-1}, \omega_t)) \\ - r_{t-1}(L_{t-1}^U - \underline{E}_{t-1} + N_{t-1}) - (\omega_t L_{t-1}^U + b(N_{t-1}, \omega_t)N_{t-1}) + \pi^F$$

with $N_{t-1} = N^U(r_{t-1})$. This expression is strictly decreasing in ω_t . Conditional on s_{t-1} , ω_t is distributed according to a density without mass points (as F is continuous with respect to ω). It is true that \underline{E}_t may vary with ω_t as well, through L_t^U and $\bar{\rho}_t^U$, but if it does, then it will also vary with δ_t - see (1) and (2). Conditional on s_{t-1} and ω_t , δ_t is distributed according to a density without mass points (again, this is implied by the continuity of F). Hence, $\Pr[E_t = \underline{E}_t | s_{t-1}] = 0$ and $D_t > 0$, a.s.

We can now prove the second part of the claim. Since $D_t = E_t - \underline{E}_t$ and $E_t > \underline{E}_t$, almost surely, we have for any $-D_t < \Delta E < \infty$

$$V(E_t + \Delta E, L_t, \bar{\rho}_t, r_t) = V(E_t, L_t, \bar{\rho}_t, r_t) + \Delta E$$

a.s. (cf. (12)). Rewriting and taking the limit of $\Delta E \rightarrow 0$, we have a.s. $\partial V(s_t)/\partial E_t = 1$.

With the existence of a derivative of the value function with respect to equity, the optimal choice for dividends must satisfy the following first order necessary condition:

$$1 - \frac{1 + (1-\tau)r_t}{1 + r_t} E_t \left[\frac{\partial V(s_{t+1})}{\partial E_{t+1}} \right] + \mu_t - \psi_t = 0$$

where $\mu_t \geq 0$ is the Kuhn-Tucker multiplier associated with the financial constraint (10) and $\psi_t \geq 0$ is the Kuhn-Tucker multiplier associated with the capital requirement (8).

Since $D_t > 0$, $\mu_t = 0$, a.s. Moreover, $E_t [\partial V(s_{t+1})/\partial E_{t+1}] = 1$, so $\psi_t = \tau r_t / (1 + r_t) > 0$. This implies that the capital requirement binds, a.s., for all t : $E_t - D_t = \gamma(L_t + N_t)$ or

$D_t = E_t - \gamma(L_t^U + N^U(r_t)) = D^U(s_t)$, the dividends paid by an unconstrained bank with the same state. But this implies that with the proposed lending policy we have a.s.

$N(s_t) = N^U(r_t)$ and $D(s_t) = D^U(s_t)$ for all t . But this implies that the bank will make exactly the same choices, and have the same sequence of states, as an unconstrained bank with the same initial conditions s_0 and the same sequence of shocks. However, by assumption to the proposition this involves $\Pr[D^U(s_t) < 0] > 0$, implying a violation of the financial constraint with positive probability. This is a contradiction. Hence, $N_t = N^U(r_t)$ for all t cannot be optimal.

APPENDIX C. COMPUTATIONAL DETAILS

The main features of the algorithm are:

1. Modified policy iteration (see Judd (1998), pp. 416-417) is used on a finite grid. The choice variables remain continuous however, and simplicial interpolation is used to evaluate the value function at non-grid points.
2. The grid is uniform with respect to L . $Z = E - \gamma L$ is used as a state instead of E , as the value function has a kink at $Z = 0$. The grid is denser around $Z = 0$ as the value function is most nonlinear in that area.
3. The optimization routine uses a BFGS algorithm with line search, modified to deflect the search direction when either inequality constraint is hit. Specifically, when the BFGS line search moves into ε distance of a binding constraint, a new BFGS direction is computed without a Hessian update. If this new direction violates the inequality constraint, a steepest ascent direction, if necessary deflected to move along the inequality constraint, is used for the next linesearch. After a (deflected) steepest ascent search no Hessian update is performed.
4. To avoid nonglobal optima, local searches are preceded by a global search every n th value iteration. Otherwise, optimization uses as starting value the optimum from the previous value iteration at the same grid point.
5. Integration with respect to ω and δ , conditional on r , is Monte-Carlo. Since r is assumed to have finite support, integration with respect to r is exact.
6. For $Z \geq 0$ and $\Delta Z \geq 0$, $V(Z + \Delta Z, L, r) \geq V(Z, L, r) + \Delta Z$, since the additional capital can always be paid out as dividends (see (12)). This fact can be used to economize on computation. Let Z_{\max} be the highest level of Z on the grid and fix L and r . Then, for all $Z \geq Z_{\max} - D(Z_{\max}, L, r)$ on the grid, we have, at each iteration,
$$V(Z, L, r) = V(Z_{\max}, L, r) - \Delta Z, \quad D(Z, L, r) = D(Z_{\max}, L, r) - \Delta Z \quad \text{and} \\ N(Z, L, r) = N(Z_{\max}, L, r), \text{ where } \Delta Z = Z_{\max} - Z.$$
7. The initial guess for the value function is V^U , which is computed, using proposition 1, prior to the main algorithm.

Figure 1. Histograms of key variables

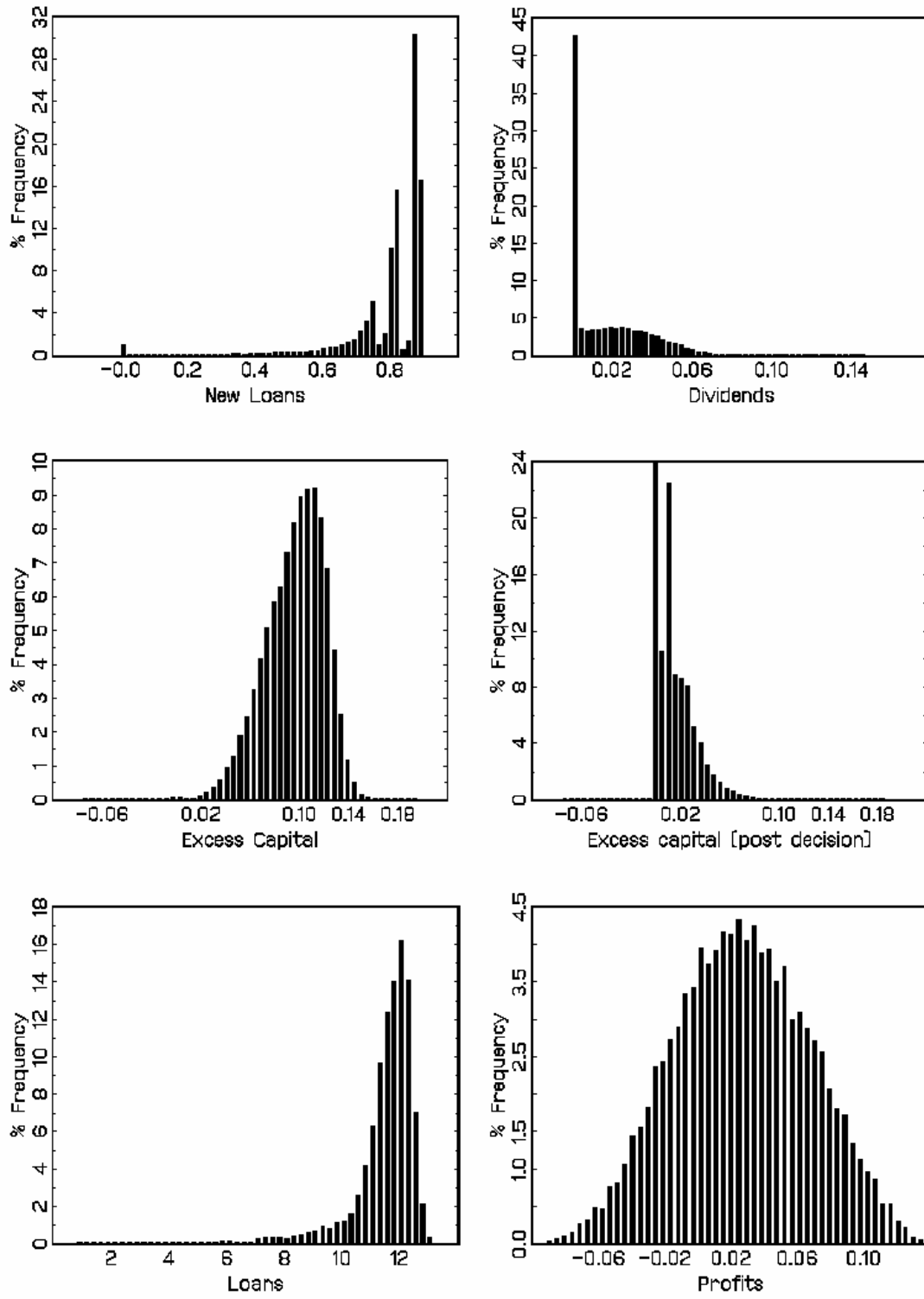


Figure 2. Bank value as a function of excess capital

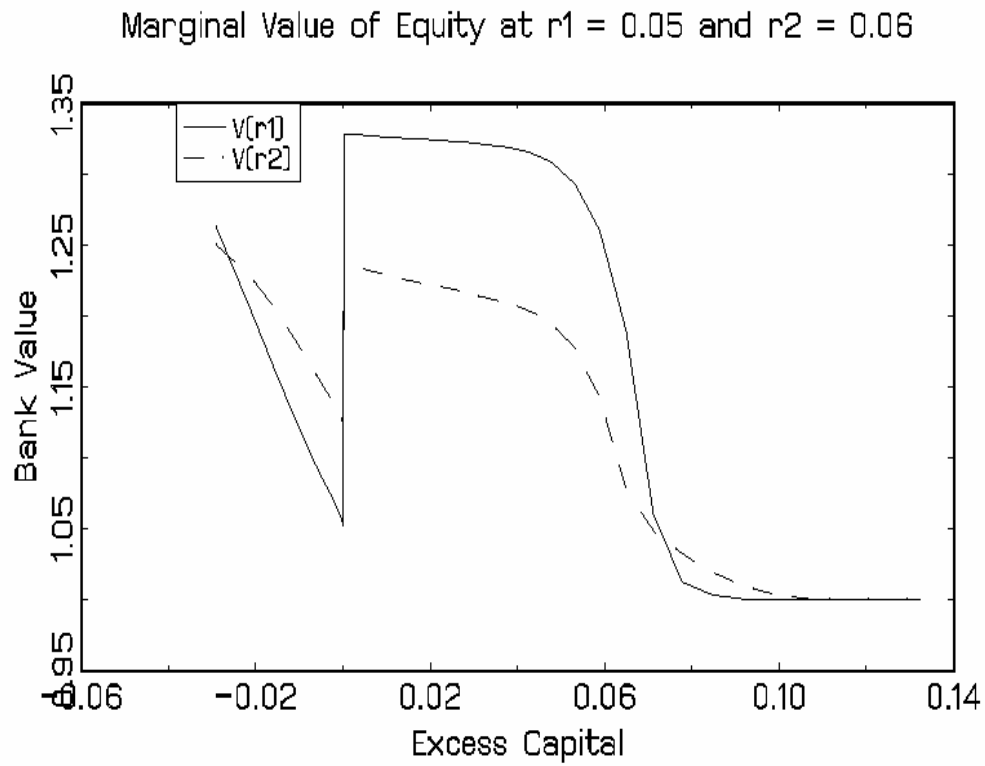
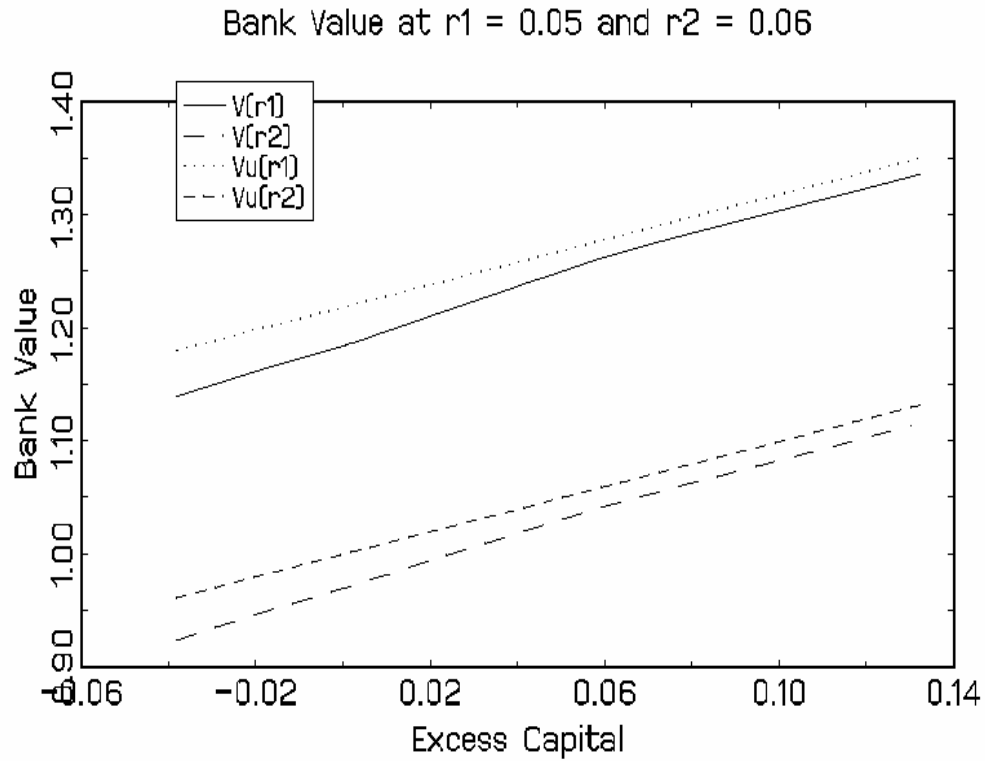


Figure 3. Bank Value: difference with unconstrained bank ($V - V^U$)

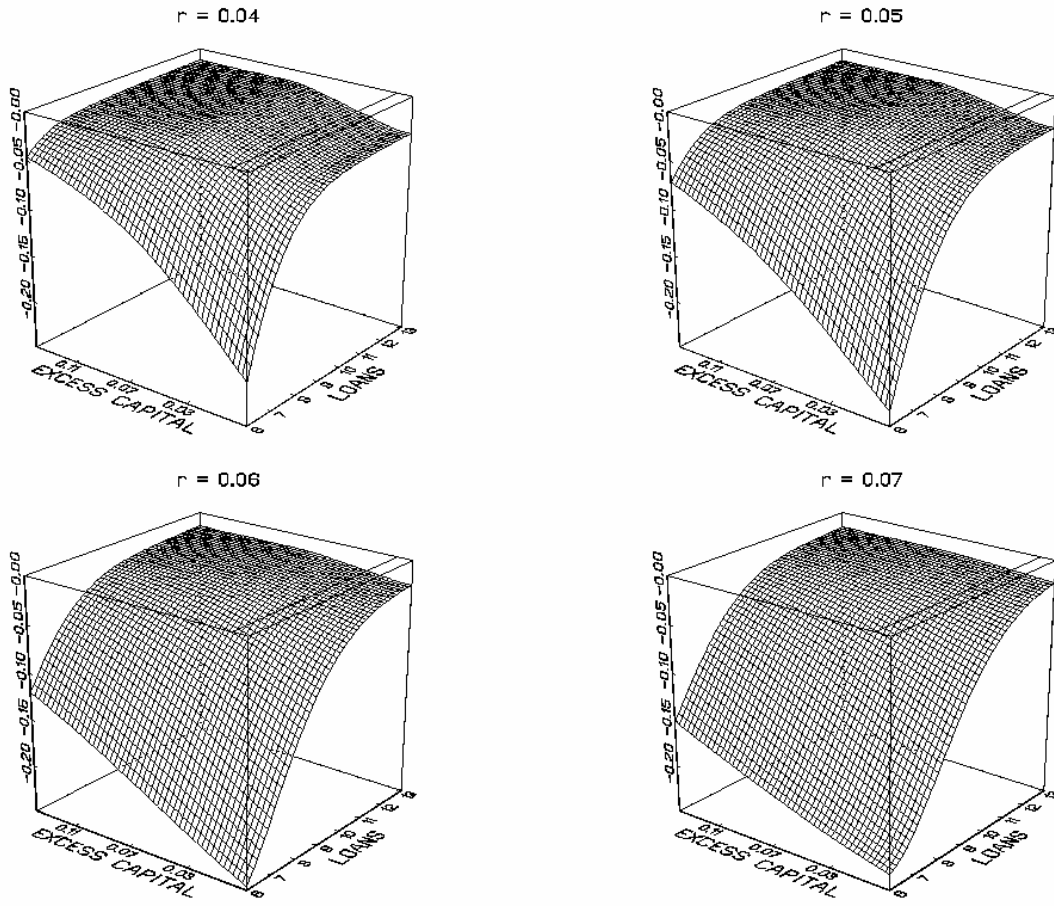


Figure 4. Optimal policy as a function of excess capital for $r_1 = 0.05$ and $r_2 = 0.06$

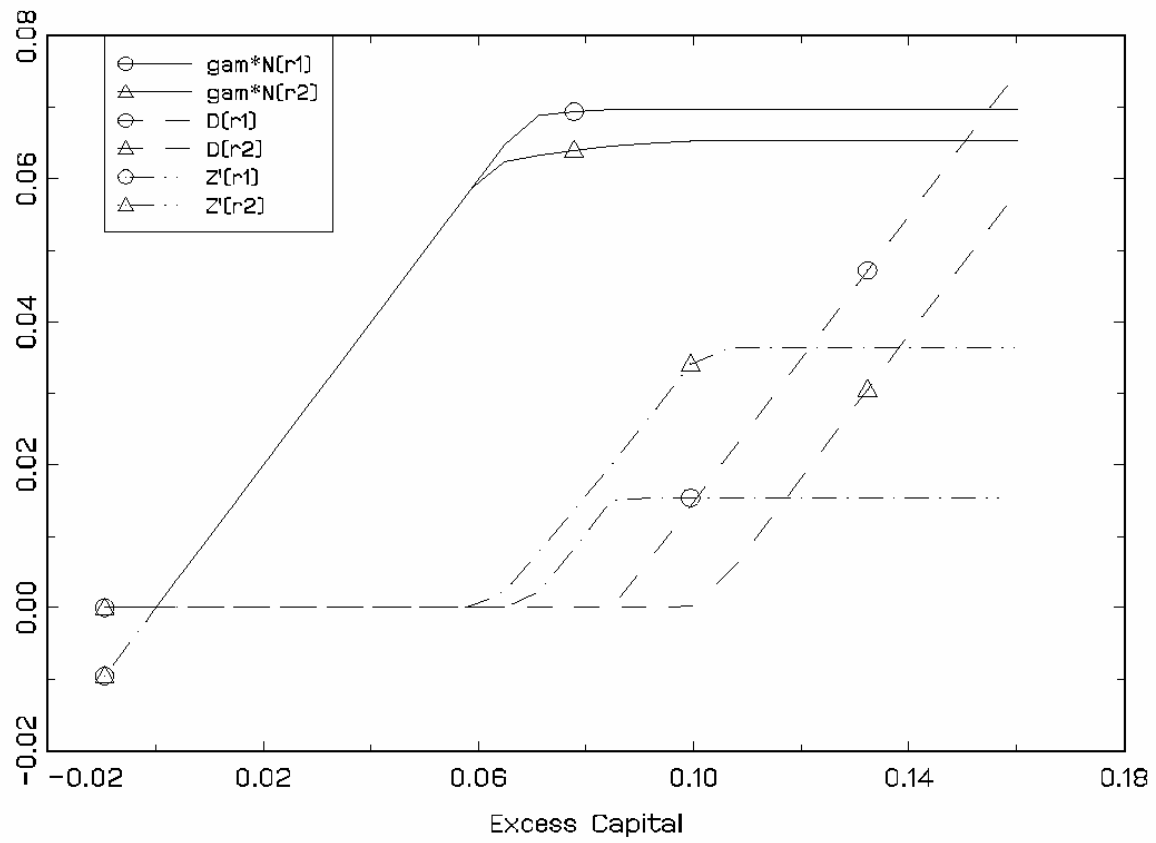


Figure 5a. Optimal Bank Policy: New Loans

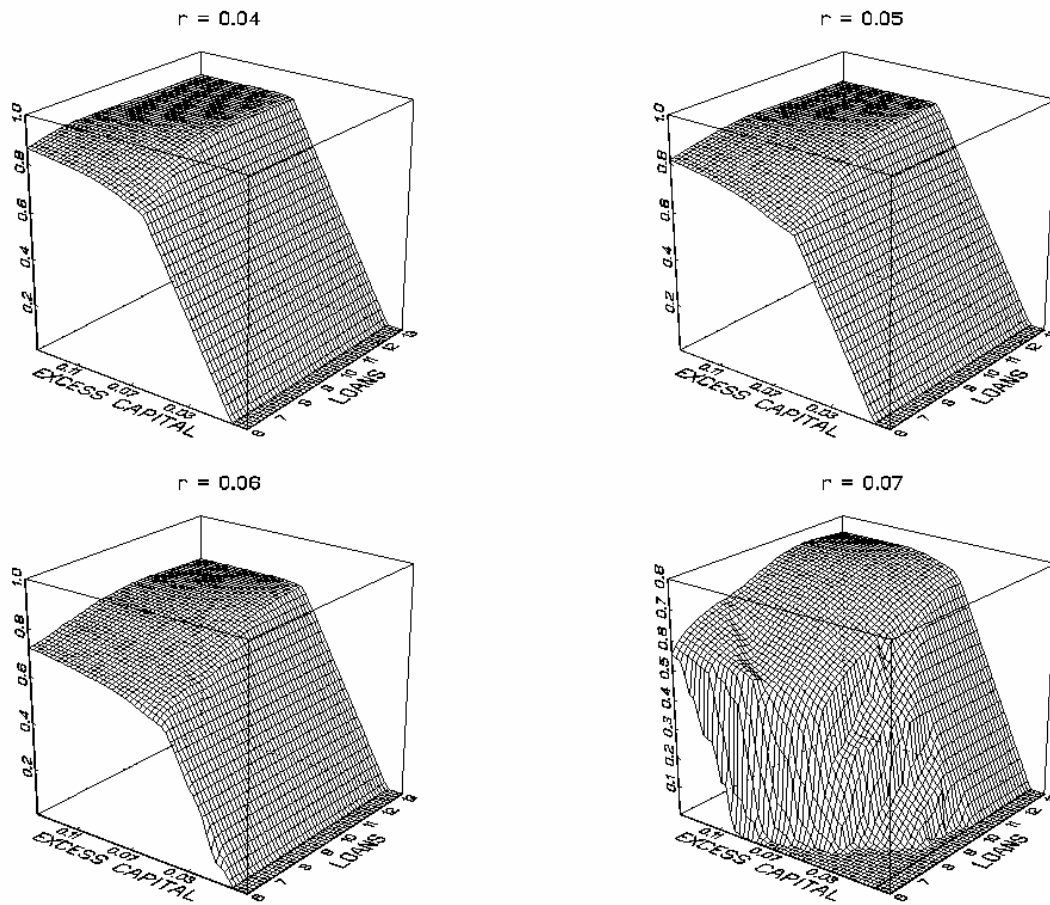


Figure 5b. Optimal Bank Policy: Dividends

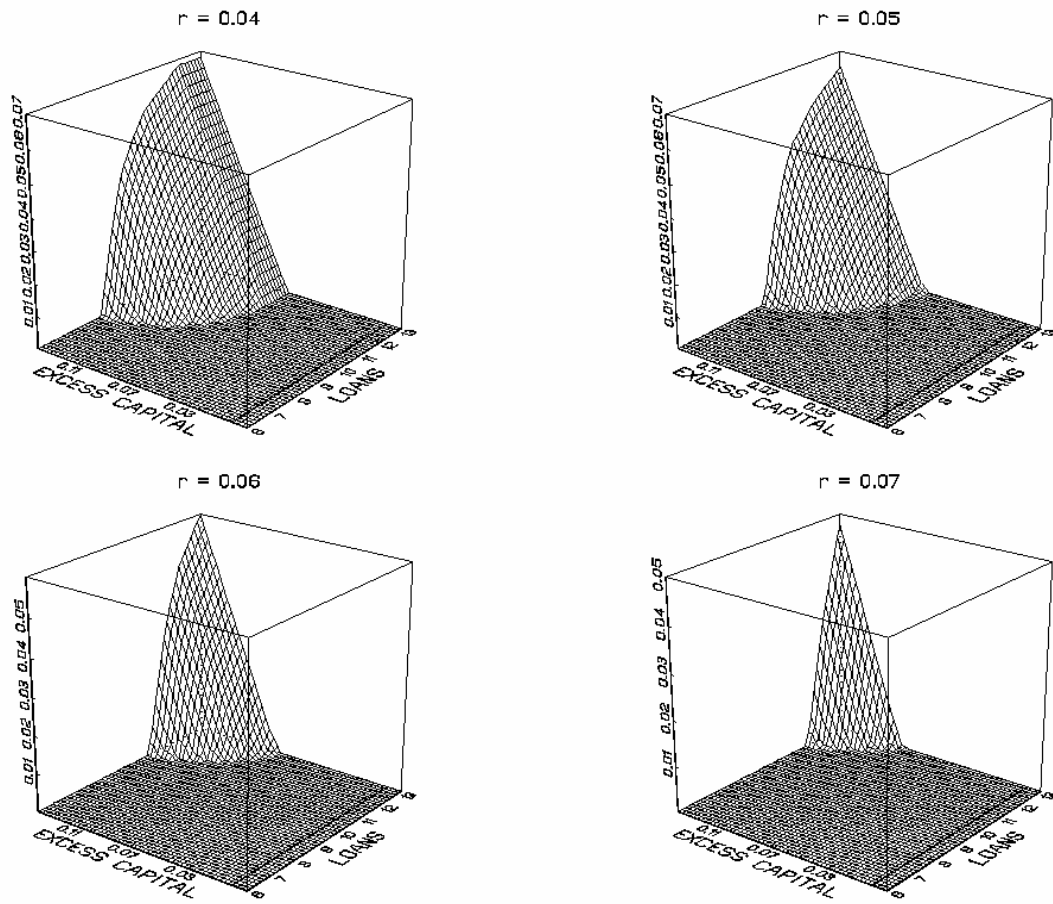


Figure 6. Response to interest rate shock: ‘average’ bank

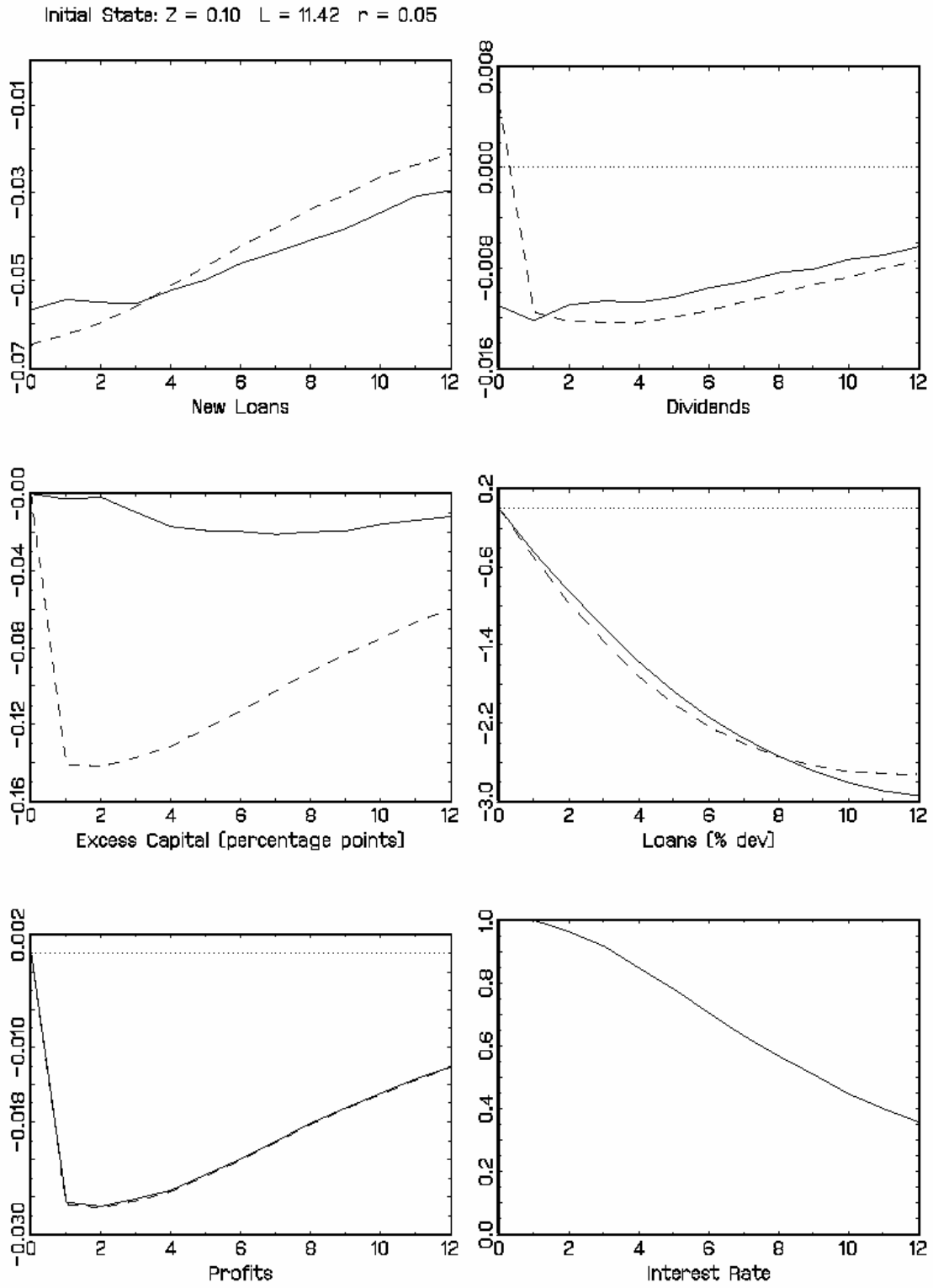
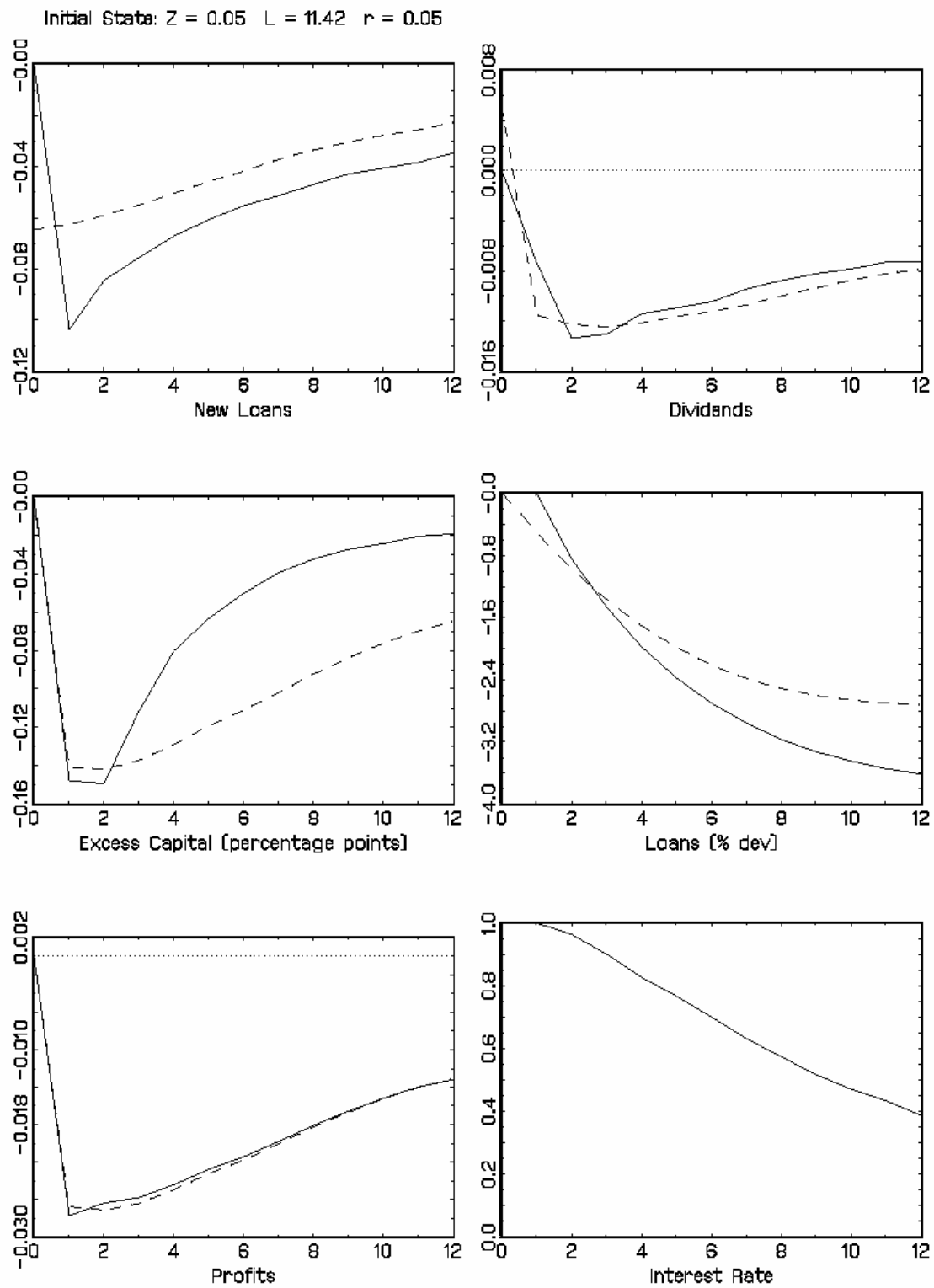


Figure 7. Response to interest rate shock: regulatory constraint binding



**Figure 8. Response to interest rate shock when initial rate is high:
'average' bank**

Initial State: $Z = 0.10$ $L = 11.42$ $r = 0.06$

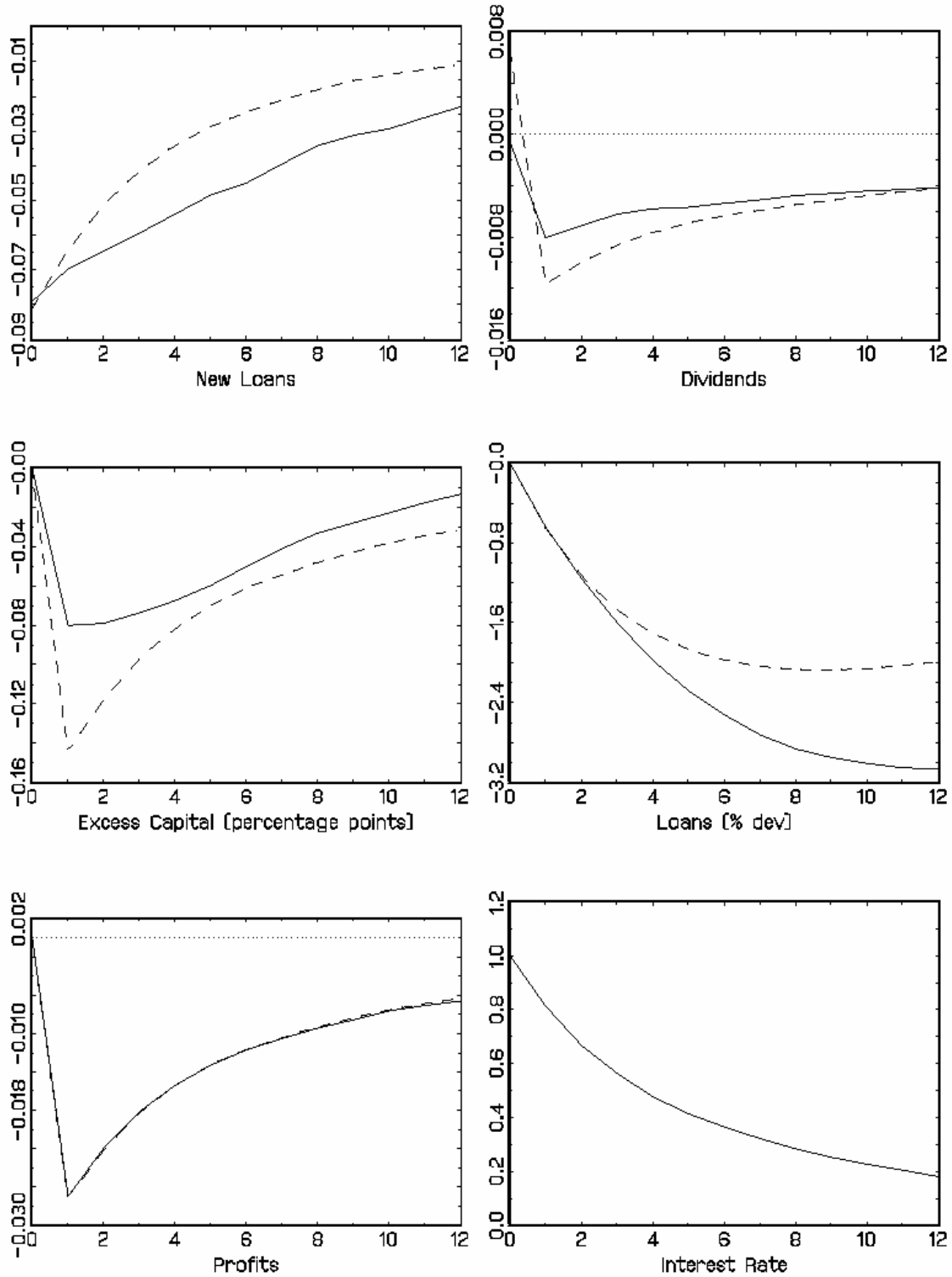


Figure 9. Response to expansionary monetary policy by undercapitalized bank

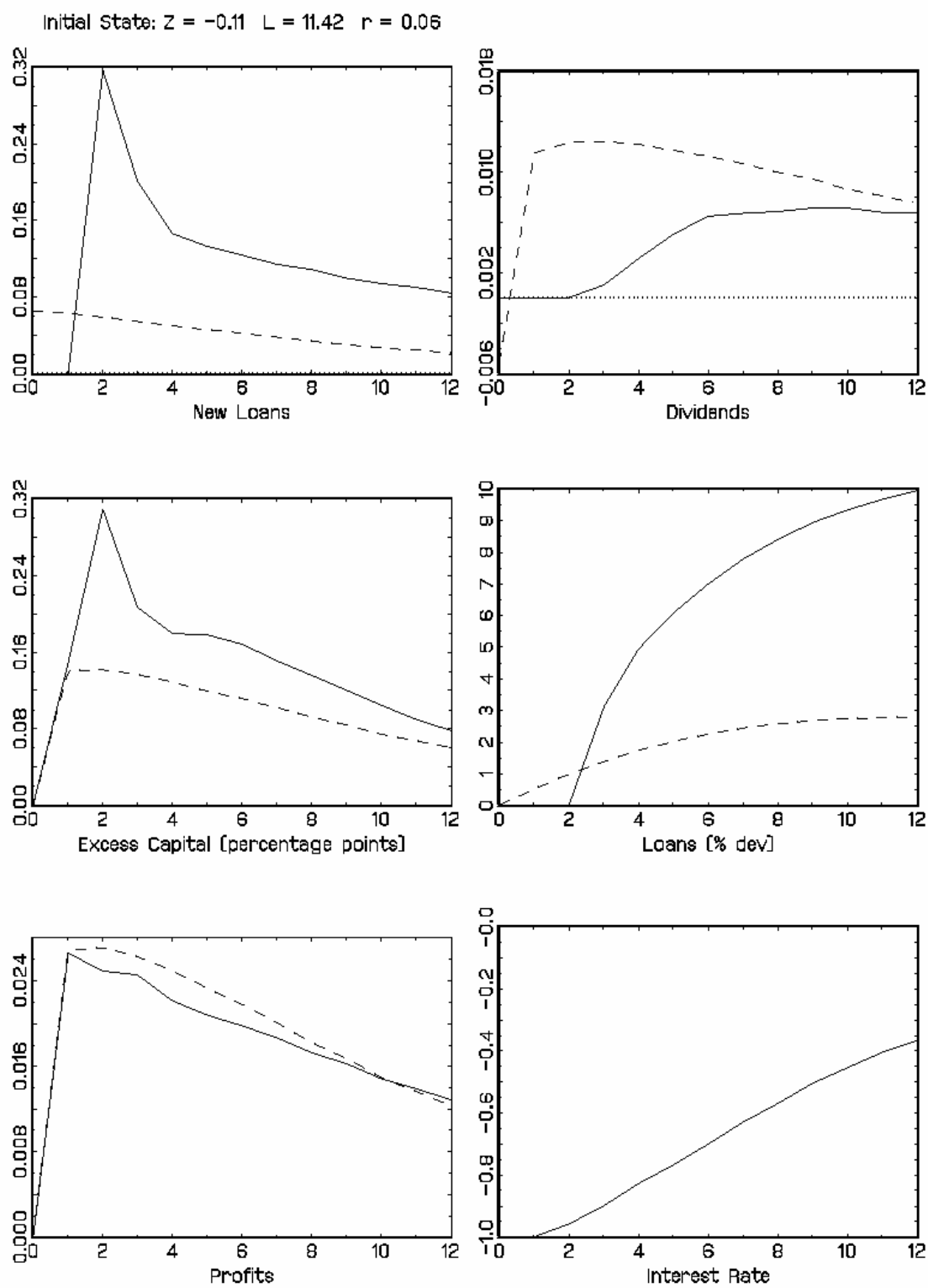


Figure 10. Response to loan default shock: ‘average’ bank

Initial State: $Z = 0.10$ $L = 11.42$ $r = 0.06$

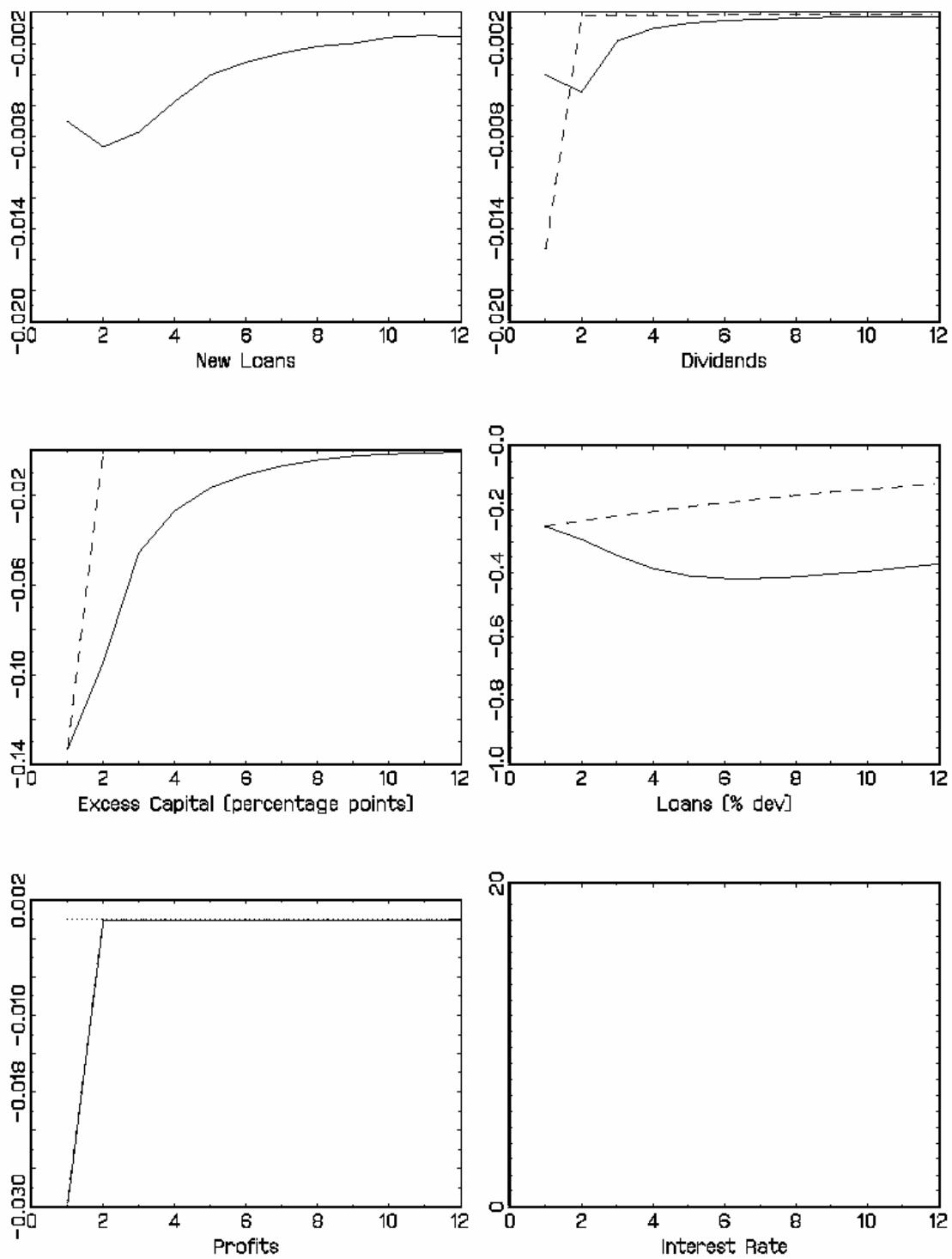


Figure 11. Response to loan default shock: poorly capitalized bank

Initial State: $Z = 0.07$ $L = 11.42$ $r = 0.06$

