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THE DEMAND FOR MONEY, NEAR-MONEY, AND TREASURY BONDS

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ABSTRACT

Bank-created money, shadow-bank money, and Treasury bonds all satisfy investors' demand for a liquid transaction medium and safe store of value. We measure the quantity of these three forms of liquidity and their corresponding liquidity premium over a sample from 1934 to 2016. We empirically examine the links between these different assets, estimating the extent to which they are substitutes, and the amount of liquidity per unit delivered by each asset. Treasury bonds and bank deposits are imperfect substitutes, in contrast to the findings of perfect substitutes of Nagel (2016). This result is directly relevant to the monetary transmission mechanism running through shifts in asset supplies, such as quantitative easing policies. Our results on the imperfect substitutability of bank and shadow-bank money also inform analyses of the coexistence of the shadow-banking and regulated banking system. We construct a new broad monetary aggregate based on our estimates and show that it helps resolve the money-demand instability and missing-money puzzles of the monetary economics literature.

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In a modern financial system, many financial assets provide the liquidity services traditionally associated with money.¹ One of the lessons from the Global Financial Crisis of 2008 is that money creation can migrate outside the commercial banking system, with securities such as repurchase agreements and asset-backed commercial paper filling the traditional roles of money (Gorton (2010); Lucas and Stokey (2011)). Academic research has begun to formally explore the expanded universe of monetary financial assets. Krishnamurthy and Vissing-Jorgensen (2012) show that U.S. government bonds satisfy investors' need for trading liquidity and a safe store-of-value, two of the central functions of money. Greenwood, Hanson and Stein (2015) show that these services are particularly high for short-term government bonds. Gorton, Lewellen and Metrick (2012), Krishnamurthy and Vissing-Jorgensen (2015) and Greenwood, Hanson and Stein (2015) show that privately issued safe assets, including bank debt and commercial paper, are a substitute for the services provided by government debt. Nagel (2016) shows that monetary policy impacts the liquidity premium on U.S. government bonds. Drechsler, Savov and Schnabl (2018) show that monetary policy impacts not just the level of deposit rates but also the spreads on deposits relative to other rates, while Li, Ma and Zhao (2019) show that the supply of government bonds impacts these deposit spreads.

This paper contributes to this literature. Following Nagel (2016) we consider a liquidity aggregate of the form:

$$Q_t = \left((1 - \lambda_t) \left(\frac{D_t}{P_t} \right)^\rho + \lambda_t \left(\frac{B_t}{P_t} \right)^\rho \right)^{\frac{1}{\rho}}. \quad (1)$$

Here D_t are nominal bank deposit holdings (checking plus savings), B_t are nominal government bond holdings, P_t is the price level, and λ_t is the fraction of liquidity services provided by each asset. The first exercise in this paper is to estimate ρ which measures the substitutability between bank deposits and government bonds. We also estimate λ_t which measures the liquidity services per unit of asset of government bonds relative to bank deposits. We then turn to a broader aggregate that measures the quantity of financial sector debt issued by banks and shadow banks, but excluding traditional bank deposits. This measure includes wholesale deposits, repurchase agreements, commercial paper, and money-market fund deposits and is from Krishnamurthy and Vissing-Jorgensen (2015). Denote this measure as D_t^{NB} and consider an aggregate Q'_t :

$$Q'_t = \left((1 - \lambda_t) \left(\frac{D_t}{P_t} \right)^\rho + \lambda_t \left(\frac{B'_t}{P_t} \right)^\rho \right)^{\frac{1}{\rho}}. \quad (2)$$

¹Throughout this paper, we use the word “liquidity services” as short-hand for the services provided by financial assets as a transaction medium, a safe store of value, and collateral services.

where,

$$B'_t = \left((1 - \mu_t) \left(\frac{D_t^{NB}}{P_t} \right)^{\eta} + \mu_t \left(\frac{B_t}{P_t} \right)^{\eta} \right)^{\frac{1}{\eta}}. \quad (3)$$

We estimate ρ , η , as well as μ_t and λ_t .

We report three principal results. First, ρ is estimated to be around 0.6 for a sample running from 1934 to 2016. The value is also quite stable over this long period. Bank-issued money and government bonds provide similar but not identical services. The result is in contrast to [Nagel \(2016\)](#) who estimates a value of $\rho = 1$ (i.e. perfect substitutes). A way of understanding this result is to note that bank deposits provide transaction services which government bonds do not, while government bonds provide collateral services which bank deposits do not.

Second, η is estimated to be near one in a sample from 1974 to 2016. That is, non-transaction financial sector debt and Treasury bonds are near perfect substitutes. One way of understanding the $\eta = 1$ result is that both non-transactional financial sector debt and Treasury bonds are a safe store of value. Note that $\eta = 1$ does not imply that these assets provide the same quantity of liquidity services per unit of asset. That characteristic is measured by μ , which is estimated to be on average 0.57, indicating that per unit of asset, government bonds provide roughly 1.5 times the liquidity services of non-transaction financial sector debt.

Our last result is that estimating a money demand function using either Q_t or Q'_t as the monetary aggregate delivers stable estimates over the entire sample. As noted by [Goldfeld and Sichel \(1990\)](#), [Teles and Zhou \(2005\)](#), and [Lucas and Nicolini \(2015\)](#), the stability of the money demand function breaks down around 1980, when money is measured as M1 (currency plus transaction deposits). These authors tie the breakdown to the growth of money market funds. Our results build on this point by considering the liabilities of the shadow-bank sector, which includes money market funds but is broader, and the quantity of government bonds.²

These estimates are obtained from examining the comovement between quantities of bank deposits and Treasuries and the price of liquidity services from these moneys, effectively tracing out money and bond demand curves. We measure the liquidity premium on Treasuries using the yield spread between Treasury bills and Bankers Acceptances, as a less liquid asset. We measure the liquidity premium on bank deposits using the spread between the federal funds rate and bank deposit rates. The principal challenge in the estimation is a simultaneity

²Our work also builds on the literature on how to aggregate different components of money. See [Fisher \(1922\)](#), [Chetty \(1969\)](#), [Barnett \(1980\)](#), and [Barnett, Offenbacher and Spindt \(1984\)](#). The monetary search literature provides a foundation for the imperfect substitution across different types of money, as in [Lagos and Wright \(2005\)](#) and [Geromichalos and Herrenbrueck \(2016\)](#).

problem: unobserved shocks to the demand for deposits/Treasuries will affect both the price and the quantity variables. We present instrumental variables regressions using the total quantity of Treasuries, driven by fiscal policy considerations, as well as seasonal variation in Treasury bill supply ([Greenwood, Hanson and Stein, 2015](#)), driven by tax receipts. The exclusion restriction is that unobserved shifts in the relative demand for Treasuries versus bank deposits are orthogonal to the instruments. Our estimated $\rho \approx 0.65$ is stable across a variety of specifications and different sample periods. That is, we find strong evidence that Treasury bonds and bank deposits are partial substitutes.

These empirical results inform theoretical work. First, quantitative easing changes the relative supplies of government bonds, bank deposits, and private assets held by investors. In theoretical models such as [Williamson \(2012\)](#), [Gorton and He \(2016\)](#), [Rocheteau, Wright and Xiao \(2018\)](#), [Piazzesi and Schneider \(2018\)](#), and [Diamond \(2020\)](#), shifts in the supplies of these assets are central to the monetary transmission mechanism. Our results provide quantitative guidance on which assets are the relevant ones to be considered, and the degree of substitutability of these assets. Our empirical estimates can be used to discipline the money/bond demand curves in these theoretical models. If, for example, $\rho = 1$, then quantitative easing policies that aim to impact the liquidity premium on bonds can be replicated via conventional monetary policy. For the empirically relevant case of $\rho \approx 0.65$, the supplies of bonds, deposits, and the path of the nominal interest rate all factor in determining bond market liquidity premiums. Our results provide quantitative guidance on the strength of each of these factors.

Likewise, the optimal mix between Treasury bills, the Fed's reverse repo facility and bank reserves, as in the analysis of [Greenwood, Hanson and Stein \(2015\)](#) and [Duffie and Krishnamurthy \(2016\)](#), depends on the substitutability of these assets. Thus, the values of ρ and η are important to modeling the monetary transmission mechanism. Second, as discussed in [Lucas \(2001\)](#), the quantitative values of the interest and income elasticity of money demand are needed to answer questions such as, what is the optimal growth rate of money, and, what is the welfare cost of inflation. For example, the welfare cost of inflation, as in [Lucas \(2001\)](#), will depend in part on the ability of private agents to use alternatives to currency for liquidity needs. We provide new estimates based on a broad liquidity aggregate which delivers a stable money demand function and is thus better suited to answering these questions. Last, for a collection of issues surrounding banking regulation, the substitutability of bank deposits, non-bank debt, and government bonds is central. If government bonds and bank deposits are perfect substitutes for providing liquidity services, then there is less reason for government policy to encourage the issuance of private bank deposits. Arguments in favor of narrow banks and increasing bank capital requirements are thus strengthened.

Our results more broadly inform analyses of the role of banks and shadow-banks in creating money ([Gorton and Metrick \(2012\)](#), [Krishnamurthy and Vissing-Jorgensen \(2015\)](#), [Sunderam \(2015\)](#), [Moreira and Savov \(2017\)](#), [Xiao \(2020\)](#), [dAvernas and Vandeweyer \(2020\)](#), [Cipriani and La Spada \(2021\)](#), [Egan, Lewellen and Sunderam \(2021\)](#)) and studies of the coexistence of the shadow-banking and regulated banking system, as in [Hanson et al. \(2015\)](#) and [Begenau and Landvoigt \(2021\)](#).

The remainder of the paper is structured as follows: Section 1 presents the model and empirical specification. Section 2 estimates the level of substitution between bank deposits and Treasuries. Section 3 broadens the liquidity aggregate to include shadow-bank deposits and estimates a demand function for this aggregate. Section 4 expands on the implications of our estimates for sizing quantitative easing as well as calibrating models of multiple moneys. Section 5 explains the differences between our findings and those of [Nagel \(2016\)](#). Section 6 concludes. An appendix detailing data sources and providing a number of robustness checks of our results follows.

1. The Model

In this section, we present our model. Then we discuss how to map the model to the data, and the data sources.

1.1. Model Setup

The model is composed of investors, commercial banks, a government, and a central bank. We model a representative investor, a stand-in for the non-financial sector, that chooses consumption and investment, receiving utility from its holdings of bank deposits and government bonds. Investors directly hold the majority of deposits and a large fraction of Treasuries either directly or indirectly through passthrough institutions. The liquidity demand is modeled along the lines of [Sidrauski \(1967\)](#) money-in-the-utility function formulation. As in [Barnett \(1980\)](#), we consider the case where the investor receives utility over a liquidity aggregate. The model formulation is closely related to [Nagel \(2016\)](#) and the liquidity aggregate contains Treasuries. Specifically, we strip out the decision problems by the government and the central bank, and adopt a general utility function $u(C_t, Q_t)$ (consumption C_t and liquidity Q_t), while [Nagel \(2016\)](#) uses log utility (all of the results also go through under log utility).

The representative investor has utility function:

$$E_0\left[\sum_{t=1}^{\infty} \beta^t u(C_t, Q_t)\right], \quad (4)$$

which consists of both real consumption C_t and a liquidity aggregate Q_t . The liquidity aggregate is composed of both deposits and bonds,

$$Q_t = \left((1 - \lambda_t) \left(\frac{D_t}{P_t}\right)^{\rho} + \lambda_t \left(\frac{B_t}{P_t}\right)^{\rho} \right)^{\frac{1}{\rho}}, \quad (5)$$

where D_t is the nominal deposits holding, B_t is the nominal bond holdings (we will use “bonds” interchangeably with “Treasuries” to correspond to the notation B_t in the model), and P_t is the price level. Note that we will omit currency holdings in our definition of D_t . We are interested in how financial assets such as Treasury bonds as well as financial sector debt are used to meet liquidity and portfolio needs, and less interested in the narrow role of currency to purchase small-ticket goods. [Piazzesi and Schneider \(2018\)](#) document the massive payment volume in financial sector claims.

The λ_t component reflects time-varying liquidity of bonds relative to bank deposits. For example, in a “flight-to-safety”, large investors may prefer bonds to holding uninsured bank deposits. The substitution between bonds and deposits is, in general, more stable than these demand fluctuations, and thus modeled as a constant³ $\rho \in (-\infty, 1]$. The elasticity of substitution is equal to $1/(1 - \rho)$, and is thus monotone in ρ . It is numerically convenient to work with ρ rather than the elasticity of substitution.

When $\rho = 1$, deposits and bonds are perfect substitutes, and the liquidity aggregate is linear in both deposits and bonds,

$$Q_t|_{\rho=1} = (1 - \lambda_t) \left(\frac{D_t}{P_t}\right) + \lambda_t \left(\frac{B_t}{P_t}\right). \quad (6)$$

When $\rho \rightarrow 0$, deposits and bonds are neither substitutes nor complements,

$$Q_t|_{\rho=0} = \left(\frac{D_t}{P_t}\right)^{1-\lambda_t} \left(\frac{B_t}{P_t}\right)^{\lambda_t}. \quad (7)$$

In this Cobb-Douglas case, the investor’s portfolio allocation into deposits and bonds are fixed fractions of the investor’s wealth. Thus, the return on bonds or deposits is irrelevant to the portfolio holdings. When $\rho < 0$, bonds and deposits are complements, with a limit of the Leontief function as $\rho \rightarrow -\infty$.

³In the empirical exercise, we will also estimate ρ across different subsamples.

We note that our model, and the elasticity parameter ρ , describe the aggregate demand curve for the liquidity provided by bank deposits and government bonds. There is likely considerable heterogeneity underlying this aggregate demand curve. At one extreme, there are many households whose only liquidity is obtained from bank deposits. There are also households who own both bank deposits and Treasury bills (directly or indirectly via money market funds) as liquidity. At the other extreme are large institutions such as foreign exchange reserve managers and corporate treasuries whose primary liquidity is from directly-held Treasury bonds, but who also own wholesale bank deposits. It is apparent then that for at least some investors, deposits and Treasuries are not substitutes. Thus in order for $\rho = 1$ in the aggregate, shifts in demand from these investors must be elastically accommodated by other investors and banks. If there is imperfect accommodation, then the aggregate demand curve will reflect a $\rho < 1$. Our analysis aims to pin down the aggregate elasticity.

Investors are subject to the budget constraint

$$\underbrace{D_{t-1}(1 + i_{t-1}^d)}_{\text{deposit return}} + \underbrace{B_{t-1}(1 + i_{t-1}^b)}_{\text{bond return}} + \underbrace{A_{t-1}(1 + i_{t-1})}_{\text{risk-free lending return}} + \underbrace{I_t}_{\text{income}} = \\ \underbrace{P_t C_t}_{\text{consumption}} + \underbrace{D_t + B_t + A_t}_{\text{investment}} + \underbrace{T_t}_{\text{transfer}}, \quad (8)$$

where i_t is the risk-free borrowing and lending rate, i_t^d is the deposit rate, and i_t^b is the one-period bond yield. In each period t , the investor collects returns from deposits, bonds, and lending, and earns income. Then the investor consumes, lends, and invests in liquid assets, including deposits and bonds. The final term is a non-distortionary transfer from the government to the investors, which can be either positive or negative.

From the budget constraint, it is clear that given a level of consumption, by substituting one unit of lending into deposits at time t , the investor earns a marginal liquidity benefit $\partial u(C_t, Q_t)/\partial Q_t \cdot \partial Q_t/\partial D_t$, but at the opportunity cost of losing $(i_t - i_t^d)$ on asset returns. Similarly, by substituting one unit of lending into bonds, the investor earns an additional liquidity benefit $\partial u(C_t, Q_t)/\partial Q_t \cdot \partial Q_t/\partial B_t$, but at the opportunity cost of losing $(i_t - i_t^b)$. Thus the first-order conditions on holdings of deposits and bonds are:

$$u_Q'(C_t, Q_t)Q_t^{1-\rho}(1 - \lambda_t)(\frac{D_t}{P_t})^{\rho-1} = u_C'(C_t, Q_t)\frac{i_t - i_t^d}{1 + i_t}, \quad (9)$$

$$u_Q'(C_t, Q_t)Q_t^{1-\rho}\lambda_t(\frac{B_t}{P_t})^{\rho-1} = u_C'(C_t, Q_t)\frac{i_t - i_t^b}{1 + i_t}. \quad (10)$$

By dividing the first-order condition of bonds by the first-order condition of deposits on both

sides and rewriting, we find:

$$\ell_t^b = \frac{\lambda_t}{1 - \lambda_t} \left(\frac{B_t}{D_t} \right)^{\rho-1} \ell_t^d, \quad (11)$$

where,

$$\ell_t^b = i_t - i_t^b \quad \text{and}, \quad \ell_t^d = i_t - i_t^d. \quad (12)$$

We will estimate (11) instead of the first order conditions in (9) and (10). Note equation (11) does not depend on the utility specification $u(C, Q)$. The economic relation described by (11) comes from assuming bonds and money in the utility function.

We have not specified the government's and the central bank's policy functions because they are not required to estimate ρ . The equality (11) is a first-order condition that holds irrespective of these policy functions and the underlying drivers of the liquidity premium. We discuss identification challenges in Section 2.3.

1.2. Data

We collect data on the quantity of deposits, Treasury bonds, bank holdings of Treasury bonds, the Treasury liquidity premium and the deposit spread. After 1952, we construct the sample at a quarterly frequency matching the frequency of all the data. Prior to 1952, some of the quantity series are only available annually. We linearly interpolate the quantity data to fill in the quarterly observations.

The Treasury liquidity premium measure follows Nagel (2016) in construction, and we extend the measure to 2016 so that our sample covers 1920–2016. For the sample after 1991, the liquidity premium is measured as the yield spread between the 3-month general collateral (GC) repo rate and the 3-month Treasury bill rate. The 3-month repo loan is collateralized by Treasuries and thus is virtually free of credit risk. Compared with a three-month Treasury bill, a 3-month Treasury-backed repo is less liquid. For the period before 1991, the three-month repo data are not available. We use 3-month Banker's Acceptance, which contains minimal default risk because they are backed by the credit of both banks and the borrowing firm. Therefore, before 1991 the liquidity premium is measured as the yield spread between the 3-month banker's acceptance and the 3-month Treasury bill. Finally, we winsorize the liquidity premium at 0.5% and 99.5% quantiles so that neither extremely large nor small values affect our results.

Figure 1 plots the Treasury liquidity premium series. We note that the liquidity premium looks relatively constant during the World War II period. During this period, the Fed was a key factor in price determination in the Treasury market. The Fed promised to buy (or sell) Treasury bills at 3/8% (substantially below typical peacetime rates of 2% to 4%). The

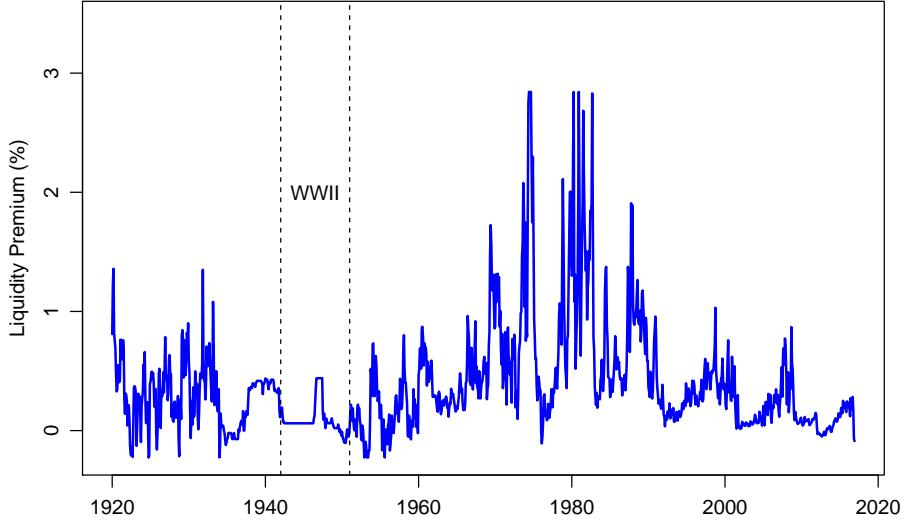


Fig. 1. **Treasury Liquidity Premium.** This figure plots the spread between 3-month general collateral (GC) repo rate and 3-month Treasury bill rate from 1991 to 2016, and 3-month general Banker’s Acceptance rate and 3-month Treasury bill rate from 1920 to 1991.

Fed also offered discount loans to banks against Treasury collateral at 50 basis points below the general discount rate. Both steps likely had a significant influence on both Treasury bill rates and other money market rates. These measures also incentivized banks to buy government debt. See [Krishnamurthy and Vissing-Jorgensen \(2015\)](#) and [Whittesley \(1943\)](#) for details. We include the WWII period in our main specifications, but present a robustness check where we drop the WWII period as well as restrict the sample to the period after the Treasury-Federal Reserve Accord of 1951.

We consider two measures of Treasury quantities. We measure “Total Treasury”, which is the total quantity of Treasury debt, at par value, excluding intra-governmental holdings, but including bank and Federal Reserve holdings. The data frequency is yearly from 1920 to 1952, and quarterly from 1953 to 2016. We use this variable primarily as an instrument in some specifications as we outline in the next section. From equation (11), we note that the quantity variable should be B_t/D_t where B_t corresponds to the market value of Treasury securities held by the agent for whom the model applies. We take this agent to reflect the non-bank sector, who demands Treasuries and bank deposits to satisfy liquidity needs. Thus, the relevant quantity measure for Treasuries is the non-bank private sector’s holdings of Treasuries. We construct this measure as the total quantity of Treasury debt, excluding intra-governmental holdings such as the Social Security Trust Fund, minus bank and Federal Reserve holdings. We construct this measure using book values, and then multiply by a factor equal to the ratio of market to book value of all Treasuries outstanding at each date.

This multiple is available beginning in 1942, and varies from 0.90 to 1.10. Prior to 1942, we use a multiple of one. While our results are not sensitive to using market or par values, the economics of the model call for using the market value. We call this measure the “net Treasury” quantity. The data on Federal Reserve and bank holdings are from the flow of funds after 1953, and from FRASER historical data before 1952. The data frequency is yearly before 1952, and quarterly from 1953 to 2016. We linearly interpolate the annual series to fill-in the quarterly observations.

Next, we discuss our measure of D_t . Bank deposits are the primary liquid asset for most investors in the U.S.⁴ We measure bank deposits as the sum of checking, savings (including money market deposit accounts), and small time deposits. We obtain monthly data after 1959 from *Federal Reserve Economic Data (FRED)* and yearly data covering 1934 to 1959 from the *Federal Deposit Insurance Corporation (FDIC)* historical bank data. Thus,

$$D_t = D_{\text{checking},t} + D_{\text{saving},t} + D_{\text{small time},t}. \quad (13)$$

We plot this series in Figure 2.

Given interest rates on checking, savings, and small time deposits, we construct a weighted average deposit spread to measure ℓ_t^d :

$$i_t - i_t^d = \frac{D_{\text{checking},t}}{D_t} (i_t - i_{\text{checking},t}) + \frac{D_{\text{saving},t}}{D_t} (i_t - i_{\text{saving},t}) + \frac{D_{\text{small time},t}}{D_t} (i_t - i_{\text{small time},t}). \quad (14)$$

Figure 3 plots the individual deposit rate data in the left panel and the corresponding deposit quantities in the right panel. Note that the deposit rate data are only available after 1987.

We also consider a broader measure of deposits, based on Krishnamurthy and Vissing-Jorgensen (2015). Their measure includes large-time deposits, wholesale bank deposits such as repos, Eurodollars, and commercial paper, as well as short-term debt issued by shadow banks, such as money market funds and broker/dealers. The construction of the measure nets out intra-financial sector holdings of debt, so that the measure corresponds to the non-financial sector holding of short-term financial sector debt. We refer to this measure as “KVJ deposits.” After 1952, we construct this series at a quarterly frequency from the Federal Reserve’s Financial Accounts (formerly referred to as the “Flow of Funds”). Prior to 1952, we use the equivalent series from Krishnamurthy and Vissing-Jorgensen (2015) who construct the series at an annual frequency. We linear interpolate the annual series to fill-in

⁴We exclude currency in our measurement. In Jan 2021, the total amount of deposits is about \$16 trillion while total currency in circulation is \$2 trillion. A significant quantity of the currency in circulation is held outside the U.S., such as in developing countries where the banking system is underdeveloped or not trusted. Refer to this [Fed Blog](#) for more information.

the quarterly observations. For the broader measure of D_t , we also reconstruct B_t to also exclude the broader financial sector’s holdings of Treasuries.

Figure 2 plots the four quantity series we use, all plotted as fractions of GDP. The two measures for Treasury supply comove, with gaps noticeable during the World War II period, when banks purchased a considerable quantity of Treasury debt, and during the last decade, when changes in financial regulation have incentivized banks to hold more Treasury debt and the Fed significantly increased its holding of Treasuries due to unconventional monetary policies. The two deposit measures also move together, but they clearly begin to separate post-1980 with the rise of non-bank intermediation. The KVJ Deposits measure includes both the bank and the non-bank or “shadow bank” sector. We note that these quantities vary at a low frequency. This observation is important in estimation, as we will make clear.

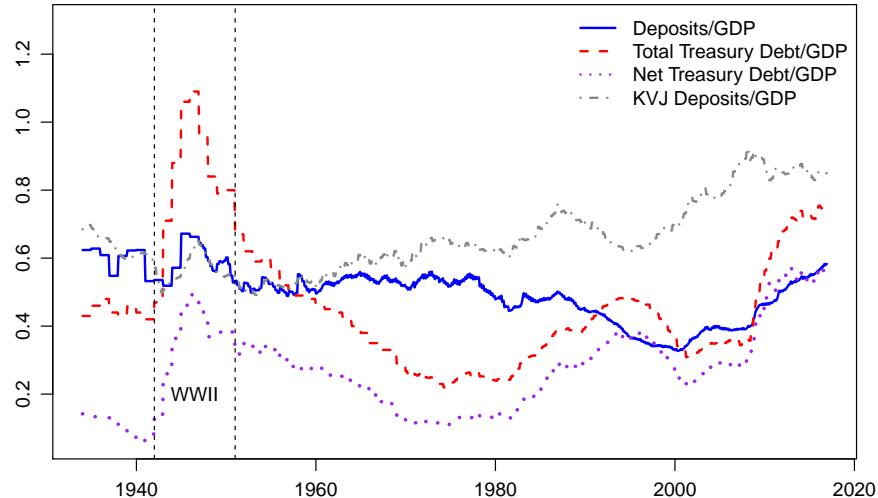


Fig. 2. Treasury Debt and Financial Sector Deposits. This figure plots two measures of B_t , relative to GDP, as Total Treasury Debt/GDP and Net Treasury Debt/GDP. We also plot our two measures of D_t , as the sum of checking, savings, and time deposits (Deposits) and the broad measure of financial sector debt (KVJ deposits), again as a ratio to GDP. For all lines, we use the highest frequency available. Sample period is 1934–2016.

Finally, we use monthly *Chicago Board Options Exchanges Volatility Index* (VIX) to measure the relative liquidity preference parameter λ_t , following Nagel (2016). The idea here is that Treasuries are fully guaranteed by the government, while checking and savings deposits are not insured above the FDIC insurance limit. In a crisis, the “flight-to-quality” effect drives up the liquidity share for Treasury, and thus the Treasury liquidity premium. The VIX data are only available since 1990. For the periods before 1990 when the VIX index is not available, we use a linear projection of VIX on realized volatility of the S&P 500 index, where the projection coefficients are estimated using the post-1990 data. We extend

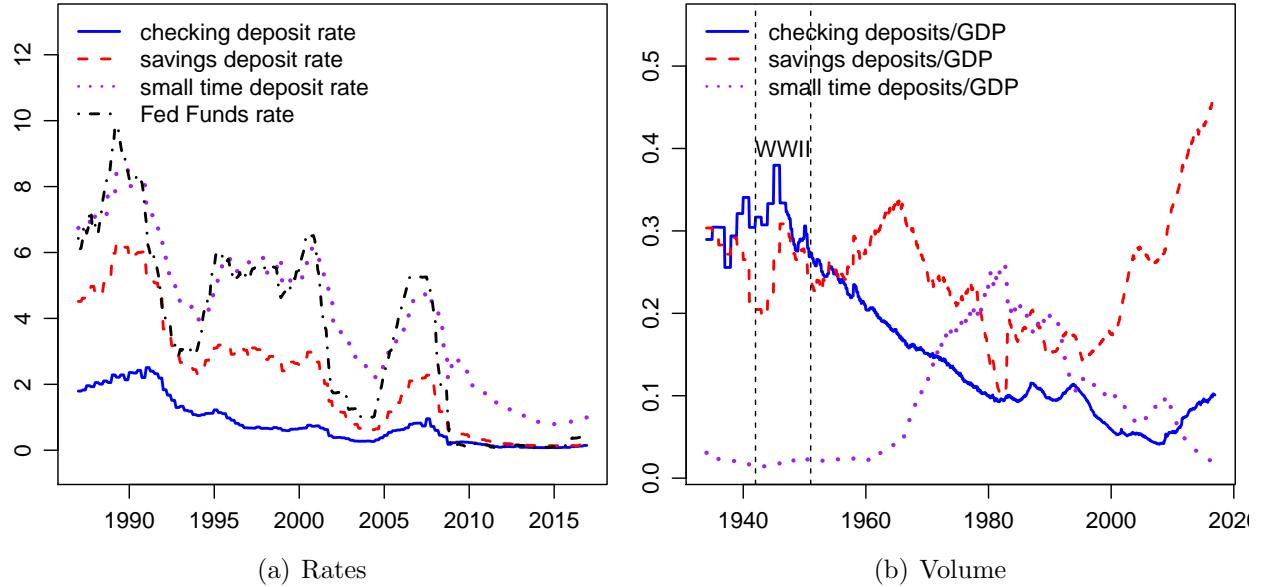


Fig. 3. Comparison of Deposit Rates and Volume. Panel (a) compares different deposit rates and the Fed Funds rate at a quarterly frequency. Deposits rates are calculated as interest expenses over the total amount of deposits for each category, using the Call Report data. Panel (b) compares the quantities of different deposits as fractions of GDP. Data are from Flow of Funds at a monthly frequency. The sample period is 1987–2016 for panel (a) and 1934–2016 for panel (b).

the sample back to 1934 in this fashion.

2. Model Estimation

In this section, we estimate ρ , the coefficient of substitution between Treasuries and deposits in equation (11) as well as λ_t , which parameterizes the liquidity services-per-unit-of-asset of Treasuries and bank deposits.

2.1. Specification

For estimation purpose, we introduce unobserved shocks to the demand for both Treasuries and deposits,

$$Q_t = \left((1 - \lambda_t) \left(\frac{D_t}{P_t} \right)^\rho \exp(\epsilon_t^d) + \lambda_t \left(\frac{B_t}{P_t} \right)^\rho \exp(\epsilon_t^b) \right)^{\frac{1}{\rho}} \quad (15)$$

where ϵ_t^d captures unobservable demand shocks for deposits, and ϵ_t^b captures unobservable demand shocks for Treasuries. The first-order-conditions give:

$$u'_Q(C_t, Q_t)Q_t^{1-\rho}(1 - \lambda_t)(\frac{D_t}{P_t})^{\rho-1} \exp(\epsilon_t^d) = u'_C(C_t, Q_t)\frac{i_t - i_t^d}{1 + i_t}, \quad (16)$$

and,

$$u'_Q(C_t, Q_t)Q_t^{1-\rho}\lambda_t(\frac{B_t}{P_t})^{\rho-1} \exp(\epsilon_t^b) = u'_C(C_t, Q_t)\frac{i_t - i_t^b}{1 + i_t}. \quad (17)$$

Define $\epsilon_t \equiv \epsilon_t^b - \epsilon_t^d$. Combining first-order-conditions (16) and (17) yields

$$\ell_t^b = \frac{\lambda_t}{1 - \lambda_t} \cdot (\frac{B_t}{D_t})^{\rho-1} \ell_t^d \cdot \exp(\epsilon_t). \quad (18)$$

We note that only the relative demand shock ϵ_t enters equation (18).

We assume that the relative preference for bank deposits and Treasuries is represented by VIX_t ,

$$\frac{\lambda_t}{1 - \lambda_t} = \beta_\lambda \cdot \text{VIX}_t. \quad (19)$$

With this parameterization, the model in equation (11) prescribes the following empirical relationship:

$$\ell_t^b = \beta_\lambda \text{VIX}_t (\frac{B_t}{D_t})^{\rho-1} \ell_t^d \cdot \exp(\epsilon_t). \quad (20)$$

The unobserved shock, ϵ_t , is the error term in the estimation. We will estimate the two parameters β_λ and ρ .

We take two approaches to the estimation of (20). A natural approach given the form of (20) is to take logs of both sides and run a linear regression. A drawback however is that the liquidity premium is occasionally negative, so that when running the log regressions we drop these observations, which can potentially bias the results. Thus, our main approach is to estimate the relation directly using GMM.

2.2. Deposit spread projection

Deposit rate data on checking, savings, and small time deposits are only available after 1987. Prior to 1987, deposit rates have to be imputed from bank deposit expenses which induces measurement error in the deposit spread and creates some error when pasting the pre- and post-1987 series. The appendix reports the results from using these series, but in our main specification we take a different approach.

We assume that the deposit rate i_t^d is a linear function of the nominal interest rate i_t . Such a connection can be microfounded by bank monopoly power or bank regulation. For

example, in Nagel (2016), regulatory reserve requirements pin down,

$$i_t^d = \delta i_t \quad \delta < 1. \quad (21)$$

In Drechsler, Savov and Schnabl (2017), the relation stems from local monopoly power of commercial banks. We are agnostic on the microfoundation and will instead use data to estimate δ . We project the deposits spread defined in equation (14) on the federal funds rate, thus computing a deposit spread back to 1934. The projection coefficient is obtained by a linear regression of the monthly deposit spread data after 1987 onto the federal funds rate (without constant term)⁵. We find that the R^2 in this regression is about 80%. Therefore, we use the following linear approximation of the deposit spread:

$$i_t - i_t^d \approx 0.34i_t \quad (22)$$

In addition to data availability, there are also economic reasons to use the deposit spread projection. First, before the banking deregulation of the early 1980s, Regulation Q restricted the payment of explicit interest on demand deposits, but banks often paid *implicit interest* to sidestep the restriction. Thus an explicit computation of deposit expense will omit this cost, while the projection approach can account for such implicit interest.⁶ Second, the deposit spread is likely endogenous to deposit quantities. We next discuss identification.

2.3. Identification

Taking logs of the estimation equation, we have:

$$\log(\ell_t^b) = (\log(\beta_\lambda) + E[\epsilon_t]) + \log(VIX_t) + (\rho - 1) \log\left(\frac{B_t}{D_t}\right) + \log(\ell_t^d) + (\epsilon_t - E[\epsilon_t]) \quad (23)$$

If the residual term, $\epsilon_t - E[\epsilon_t]$, is uncorrelated with the explanatory variables, including $\log(VIX_t)$, $\log\left(\frac{B_t}{D_t}\right)$, and $\log(\ell_t^d)$, then a simple linear regression yields an unbiased estimate of ρ . However, this condition is likely violated. The ratio, B_t/D_t , represents the ratio of Treasuries held by investors over the total volume of bank deposits, both of which are affected by the choices of banks due to market clearing. Thus, both B_t and D_t are likely endogenous to the unobserved relative demand shocks ϵ_t . Similarly, the deposit spread ℓ_t^d , may also be

⁵If we include the constant term, the coefficient is similar and the constant term is insignificant. Our results are similar using this alternative projection.

⁶Startz (1979) finds that accounting for this implicit interest results in a deposits spread half of the nominal interest rate, which is similar to our estimated sensitivity in (22). Therefore, we will use the approximation in (22) throughout the sample period. Our main results are robust to having a different approximation before and after regulation Q. Refer to Appendix C.3 for details.

correlated with the relative demand shocks.

We use total Treasury/GDP as an instrument variable in the estimation. We also use the projected deposit spread as in (22), which is a function of the level of the nominal rate, i_t , rather than the measured deposit spread. First, as we report in the appendix, total Treasury supply is strongly correlated with B_t/D_t , so that the instrument is relevant. Second, the exclusion restriction is that monetary and fiscal policy are not driven by the unobserved relative demand shock. That is, we require that fiscal policy considerations rather than the unobserved relative demand shock drive total Treasury supply. Likewise, we require that monetary policy considerations (e.g., inflation, unemployment) drive i_t rather than the relative demand shock. Finally, we note from equation (23), our relative-demand exclusion restriction is only imposed on ϵ_t , which is weaker than an exclusion restriction based on both the demand shock to Treasuries ϵ_t^b and the demand shock to bank deposits ϵ_t^d .

2.4. Log Specification

Before turning to our first set of estimation results, we offer a word of caution with the log specification. The model prescribes that since Treasury bonds offer more liquidity services than the less liquid repo and Banker's Acceptance, the liquidity premium should be positive. In the data, the liquidity premium is occasionally negative, likely due to measurement error. If this measurement error is additive, then when taking logs, the measurement error blows up for liquidity premia near zero. In the table that follows, we drop observations with a nonpositive liquidity premium. The results are similar if we winsorize nonpositive liquidity premiums. We present a GMM estimation of equation (20) in Section 2.5 that avoids these issues (and significantly improves the model's fit).

The results are shown in Table 1. The supply variables in log terms are highly significant, as can be seen in columns (4) and (5). From equation (20), the coefficient on the log term of supply corresponds to $\rho - 1$. Thus, for the bank deposit measure, we find that $\rho = 0.65$, with a confidence interval of [0.52, 0.78], while for the broad financial sector debt measure, we find that $\rho = 0.48$, with a confidence interval of [0.30, 0.66]. In both cases, the results indicate that money and Treasury bonds are imperfect substitutes.

We also note that the coefficient on log(FFR) is below one, which does not accord with the predictions of the theory. It is possible that because we have winsorized or eliminated negative liquidity premium, we have introduced a bias that shrinks the coefficients. A formal way to address this problem is to use GMM and estimate (20) directly. We follow this approach in Section 2.5.

We next turn to the IV regressions using the total supply of Treasuries, set by fiscal

Table 1: Log Regressions Using Different Debt/Deposits Measures

	Dependent variable: $\log(\text{Treasury liquidity premium}_t)$				
	(1)	(2)	(3)	(4)	(5)
$\log\left(\frac{\text{Net Tsy}_t}{\text{Deposits}_t}\right)$		-0.69 (0.24)		-0.35 (0.13)	
$\log\left(\frac{\text{Net Tsy}_t}{\text{KVJ Deposits}_t}\right)$			-1.02 (0.29)		-0.52 (0.18)
$\log(\text{FFR}_t)$	0.59 (0.09)			0.54 (0.09)	0.51 (0.09)
$\log(\text{VIX}_t)$	0.82 (0.27)	0.18 (0.32)	-0.08 (0.31)	0.73 (0.28)	0.57 (0.28)
Constant	-4.28 (0.81)	-2.34 (0.93)	-2.13 (0.90)	-4.21 (0.81)	-3.98 (0.80)
Observations	300	300	300	300	300
R ²	0.40	0.13	0.20	0.43	0.44

Notes: Data are at quarterly frequency from 1934 to 2016. We remove observations with negative liquidity premium due to the log transformation. The liquidity premium is measured as the spread between three-month bankers acceptance and three-month T-bill before 1991, and the spread between the three-month GC term repo and three-month T-bill afterward. FFR is the effective federal funds rate in percentage points. VIX represents CBOE S&P 500 implied volatility index. Before 1990, the index is not available, so we calculate a proxy index as the projection of VIX on the realized volatility of the S&P 500 index's daily return in each month before 1990, where the projection coefficients are estimated with data after 1990. Quarterly VIX is the average of monthly VIX. Total Tsy is the total market value of government debt, excluding intra-governmental holdings. Net Tsy is the total market value of Treasuries excluding intra-governmental holdings, minus bank and Federal Reserve holdings. Deposits include checking, savings, and small-time deposits. “KVJ deposits” are the amount of financial sector short-term liabilities as in Krishnamurthy and Vissing-Jorgensen (2015). Newey-West standard errors with one-year lag (4 lags) are shown in parentheses.

policy, as the instrument. It is unlikely that this total supply responds to the liquidity premium, so that the endogeneity concern is allayed with this instrument. There is still a possibility of omitted variable bias, to the extent that both fiscal policy and changes in relative liquidity demand are correlated with the business cycle. We also follow [Greenwood, Hanson and Stein \(2015\)](#) and [Nagel \(2016\)](#) by running a difference specification where we use seasonal variation in tax receipts as an instrument for Treasury supply, and using changes in Federal Funds future as an instrument for monetary policy shocks.

Table 2 presents the results using Total Treasury/GDP and $\log(\text{Total Treasury}/\text{GDP})$ as the instruments for the quantity ratios. Note that while we have argued that this regressor is not the appropriate measure for the non-bank holdings of Treasury securities in equation (11), it is the regressor used in [Nagel \(2016\)](#). The first stage in the instrumental variable regression, reported in the appendix establishes relevance: shifts in total Treasury supply strongly correlate with shifts in the debt-to-deposits ratio and the F -statistic is well above 10. We use both $\log(\text{Total Tsy}_t/\text{GDP}_t)$ and $\text{Total Tsy}_t/\text{GDP}_t$ as instruments to capture a non-linear relationship in the first-stage.

In Table 2, columns (1) and (2) are the same OLS regressions as columns (4) and (5) of Table 1. Columns (3) and (4) present the IV regressions. The estimate of ρ is 0.13 in column (3) and 0.28 in column (4), but the standard error ranges are large and both include zero. The broader deposit measure is a closer Treasury substitute than the narrow deposit measure. We further investigate this possibility in the GMM estimations next section.

We next follow [Greenwood, Hanson and Stein \(2015\)](#) and construct an instrument using monthly dummies that capture the strong cyclical variations in T-bill supply. As these authors show, the cyclicalities is driven by calendar cycles in tax receipts. The instrument can additionally deal with any omitted variable biases in our estimation. To be able to apply the monthly IV, we change the frequency of our analysis to monthly in this case, and interpolate quarterly quantities to monthly frequency. We instrument FFR changes with expected changes from the federal funds futures market as in [Nagel \(2016\)](#). Specifically, we take the average price difference in month $t - 2$ of federal funds futures for month $t - 1$ and t to instrument for the actual changes of FFR from $t - 1$ to t . The sample period is 1991 to 2016.

The results are in Table 3. These regression results use the monthly differences of variables given the nature of the instrument. The first stage in these regressions is reported in the appendix. We follow our theoretical model and use the ratio of Treasuries to deposits as the quantity measure, while continuing to use the same instruments. We note again the strong statistical significance of the quantity variable, and the larger-magnitude coefficient on the IV result compared to the OLS result.

Table 2: OLS and IV Regressions in Logs

	<i>Dependent variable: log(Treasury liquidity premium_t)</i>			
	OLS		IV	
	(1)	(2)	(3)	(4)
$\log\left(\frac{\text{Net Tsy}_t}{\text{Deposits}_t}\right)$	-0.35 (0.13)		-0.87 (0.39)	
$\log\left(\frac{\text{Net Tsy}_t}{\text{KVJ Deposits}_t}\right)$		-0.52 (0.18)		-0.72 (0.36)
$\log(\text{FFR}_t)$	0.54 (0.09)	0.51 (0.09)	0.46 (0.13)	0.47 (0.10)
$\log(\text{VIX}_t)$	0.73 (0.28)	0.57 (0.28)	0.59 (0.34)	0.46 (0.30)
Constant	-4.21 (0.81)	-3.98 (0.80)	-4.11 (0.96)	-3.86 (0.82)
Observations	300	300	300	300
R ²	0.43	0.44	0.36	0.44

Notes: Data are at quarterly frequency from 1934 to 2016. We remove observations with negative liquidity premium due to the log transformation. Variable explanations are provided in the notes of Table 1. We instrument $\log(\text{Debt}/\text{Deposits})$ by $\log(\text{Total Tsy}/\text{GDP})$ and $(\text{Total Tsy}/\text{GDP})$ in columns (3) and (4). Newey-West standard errors with 4 lags are shown in parentheses.

Table 3: Difference Regressions: OLS and IV.

	Dependent variable: $\Delta \text{liquidity premium}_t$							
	OLS				IV			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$\Delta \log\left(\frac{\text{T-bill}_t}{\text{Deposits}_t}\right)$		-45.88	-40.50	-38.66		-108.68	-106.06	-98.52
		(13.52)	(14.92)	(14.86)		(20.24)	(20.95)	(21.71)
$\Delta \log\left(\frac{\text{T-bill}_{t-1}}{\text{Deposits}_{t-1}}\right)$				37.89				66.54
				(16.83)				(26.06)
ΔFFR_t	9.40		7.18	8.81	9.36		7.04	8.17
	(3.78)		(3.33)	(4.22)	(3.33)		(3.36)	(3.23)
ΔVIX_t	0.53	0.59	0.58	0.55				
	(0.17)	(0.20)	(0.20)	(0.18)				
R ²	0.08	0.09	0.11	0.13				
Weak instruments test								
CD statistic					19.59	12.96	12.42	10.36
Critical value					6.56	6.56	6.23	5.87
Observations	307	307	307	306	306	306	306	306

Notes: Data are at monthly frequency from 1991 to 2016. T-bill is the total value of T-bills outstanding. Definitions of other variables are provided in the notes of Table 1. In the weak instrument tests, a higher Crag-Donald statistic means that the instrument is less likely to be weak. The critical values for the tests are from Table I of Stock and Yogo (2002). Newey West standard errors with 12 lags (one-year lags) are reported in the parentheses.

2.5. GMM Estimation

We next turn to a GMM estimation of the model. The GMM results are broadly consistent with the results we have presented thus far, with a value of ρ slightly larger than one-half. Additionally, the standard errors of the estimate of ρ are lower, while the regression R^2 s are higher than our earlier specifications.

Define the GMM residual as:

$$u_t = \ell_t^b - \beta_\lambda \text{VIX}_t \cdot \left(\frac{B_t}{D_t}\right)^{\rho-1} (\delta i_t). \quad (24)$$

The moment conditions are:

$$\begin{cases} E[u_t] &= 0 \\ E[u_t \cdot \frac{\text{Total Tsy}_t}{GDP_t}] &= 0 \\ E[u_t \cdot (\frac{\text{Total Tsy}_t}{GDP_t})^2] &= 0 \\ E[u_t \cdot i_t] &= 0 \\ E[u_t \cdot \text{VIX}_t] &= 0 \end{cases} \quad (25)$$

where the inclusion of both $\frac{\text{Total Tsy}_t}{GDP_t}$ and $(\frac{\text{Total Tsy}_t}{GDP_t})^2$ deal with both the identification issue and non-linearities in the underlying relationships.

For comparison to the log-linear OLS regressions, we also implement a variation of (25) by replacing the total Treasury IV moments,

$$\begin{cases} E[u_t \cdot \frac{\text{Total Tsy}_t}{GDP_t}] &= 0 \\ E[u_t \cdot (\frac{\text{Total Tsy}_t}{GDP_t})^2] &= 0 \end{cases} \quad (26)$$

with the following moment on B_t/D_t ,

$$E[u_t \cdot \frac{B_t}{D_t}] = 0. \quad (27)$$

In moment conditions (25), there are two parameters (β_λ, ρ) to be estimated but five moment conditions. Therefore, the above orthogonality conditions result in an over-identified system. We check the model fit with the standard GMM J-test.

We can understand the orthogonality conditions being assumed in (25) in terms of underlying unobserved shocks as follows. We consider shocks ϵ_t , e_t^b , and e_t^d so that,

$$\ell_t^b = \beta_\lambda \text{VIX}_t \cdot \left(\frac{B_t}{D_t}\right)^{\rho-1} (\delta i_t + e_t^d) \cdot \exp(\epsilon_t) + e_t^b. \quad (28)$$

Here, ϵ_t is the relative-demand shock, e_t^d is the shock to the deposit spread ℓ_t^d orthogonal to i_t , and e_t^b is a measurement error on the liquidity premium. All shocks are mean zero and independent. Combining equation (24) and (28), and applying a first-order approximation, we obtain

$$u_t = \beta_\lambda \text{VIX}_t \cdot \left(\frac{B_t}{D_t} \right)^{\rho-1} (e_t^d + \delta i_t \epsilon_t) + e_t^b, \quad (29)$$

where we have omitted the higher-order term $e_t^d \epsilon_t$. Given the multiplicative factor on these error terms, the identification assumptions are slightly more stringent than in the log-linear model. Higher moments of the shocks also need to be uncorrelated with the instruments. Otherwise, the economics of identification is the same and requires monetary and fiscal policies to be chosen independent of the shocks.

Results are shown in Table 4. The first two columns use B_t/D_t as the quantity instrument, for comparison to the OLS results, while columns (3) and (4) use Total Treasury/GDP and $(\text{Total Treasury}/\text{GDP})^2$ as the quantity instrument following moment conditions (25). We apply the two-step GMM method and a heteroskedasticity and autocorrelation consistent (HAC) residual covariance structure of 4 lags (1-year horizon). The model fits the data well, as the J-tests have large p -values. In the appendix, we show that increasing the number of lags does not have significant impact on our estimations.

The estimates of ρ are around 0.65, consistent with earlier estimates. There are no statistically significant differences across the four specifications in the table, and the instruments have no detectable effect on the estimate of ρ . This last point is perhaps not surprising given that the regression R^2 is about 70%, so the likelihood of unmodeled demand shocks and model measurement error is also low. All of the estimates are statistically different from both zero and one. Thus, we conclude that both bank money and the broad financial sector debt measure are imperfect substitutes for Treasuries.

We also note that the non-linear model has a much higher R^2 and thus fits the data much better than the log-linear model. Therefore, the GMM estimation captures meaningful variation in the liquidity premium that is missed by the log-linear model.

To provide a sense for how much each component of the model contributes to the explanatory power of the liquidity premium, we list the explained fraction of variation due to different model components in Table 5. The fraction of variation is calculated as 1 - residual variance / total variance. With only Net Treasury/Deposits data as inputs (FFR and VIX are set as their time-series averages, respectively), the fraction of variation explained is 8.6%. With both Net Treasury/Deposits and FFR, the model explains 63.4% of the liquidity premium variation. With all of the independent variables, the model explains 69.7% of the

Table 4: GMM Estimation of ρ

	Measurement of B/D			
	Net Tsy Deposits	Net Tsy KVJ Deposits	Net Tsy Deposits	Net Tsy KVJ Deposits
	(1)	(2)	(3)	(4)
ρ	0.685 (0.102)	0.671 (0.212)	0.657 (0.189)	0.625 (0.214)
β_λ	0.012 (0.001)	0.011 (0.003)	0.012 (0.002)	0.010 (0.003)
p-value of J-test	0.859	0.871	0.953	0.943
Total Treasury IV?	No	No	Yes	Yes
Variation explained	69.5%	68.9%	69.7%	69.2%
Observations	332	332	332	332

Notes: Data are at quarterly frequency from 1934 to 2016. Refer to Table 1 for variable definitions. In column (3) and (4), we estimate the GMM system as in equation (25). In column (1) and (2), we replace the Treasury IV moments in equation (26) with the B_t/D_t moment in equation (27). HAC standard errors with 4 lags are reported in parentheses.

Table 5: Model Explanatory Power from Different Components

Model Inputs	Fractions of Variation Explained
Only Net Tsy/Deposits	8.6%
Only FFR	58.0%
Only VIX	0.7%
Net Tsy/Deposits + FFR	63.4 %
FFR + VIX	64.3%
Full Model	69.7%

Notes: Refer to Table 1 for variable definitions. We calculate the variation explained by various model components, using parameters estimated from column (3) of Table 4. For example, in the first case, among the input data, we set VIX_t and deposit spread s_t to their mean values and only allow the variations in B_t/D_t . Then we calculate the fraction of variation explained by these partial inputs.

variation in the liquidity premium.

In the left panel of Figure 4, we plot the model generated liquidity premium using all three explanatory variables as well as the actual liquidity premium. The federal funds rate component of the model in equation (11) helps explain the high-frequency ups and downs, while the quantity of Treasuries/deposits modifies the general shape of the predicted liquidity premium and helps explain the liquidity premium at a low frequency. Finally, the variation in VIX, capturing the flight-to-quality effects, helps explain spikes in the liquidity premium. The right panel of the Figure plots the model’s predicted federal funds rate, using the liquidity premium, VIX, and the supply variable as explanatory variables. The model fit is also quite good for the federal funds rate. A point worth making is that the fit, estimated over the entire sample, works as well pre-1980 as post-1980. The fit for the federal funds rate suggests that our model may help lead to a stable estimate for money demand. We explore this possibility in Section 3.

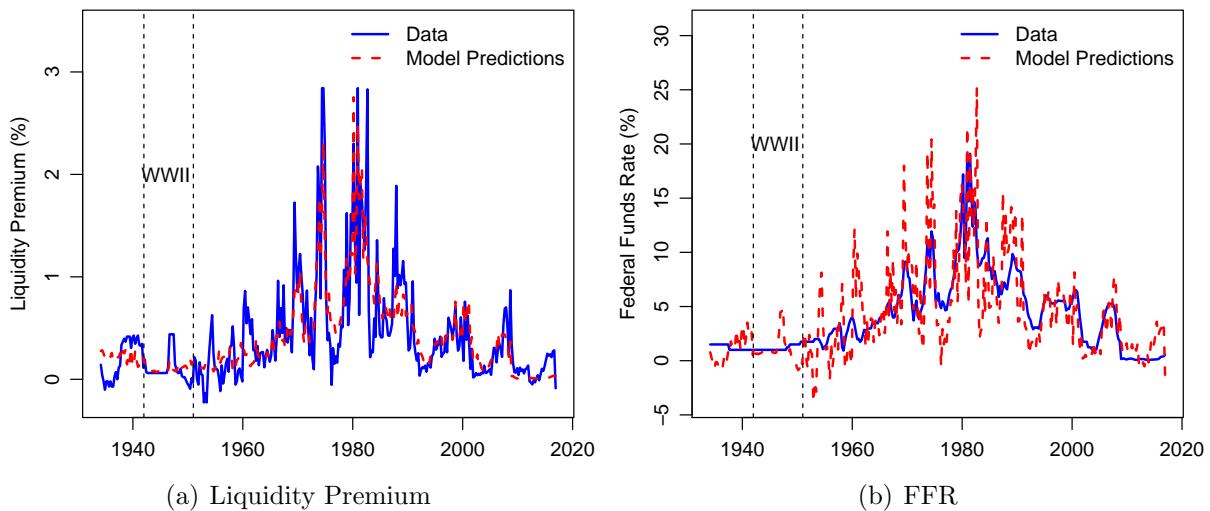


Fig. 4. Model Predictions versus Data. This figure compares the model predicted Treasury liquidity premium and federal funds rate versus the data counterparts. The model generates the liquidity premium as $\ell_t^b = s_t \beta_\lambda \text{VIX}_t (B_t/D_t)^{\rho-1}$, where s_t is the deposits spread, VIX_t is the constructed VIX, B_t/D_t is the Treasury/deposits ratio, and (β_λ, ρ) are set to the estimated values in column 3 of Table 4. The model generates the federal funds rate as $r_t = \ell_t^b / (\beta_\lambda \text{VIX}_t (B_t/D_t)^{\rho-1}) / \delta$, where δ is the projection coefficient of deposits spread on FFR. Data are at quarterly frequency from 1934 to 2016.

The λ_t in the model measures the liquidity services provided by one unit of Treasury bonds, while $1 - \lambda_t$ measures the liquidity services provided by one unit of bank deposits. Given that deposits are generally perceived as a more convenient form of transactions, we may expect that the share of Treasury liquidity should be below 50%, i.e., $\lambda_t < 50\%$. In

Figure 5, we plot the estimated λ_t , which comes from the measurement equation

$$\frac{\lambda_t}{1 - \lambda_t} = \beta_\lambda \text{VIX}_t \quad (30)$$

using the estimated $\beta_\lambda = 0.012$. As shown in Figure 5, despite the sizable variation in the liquidity premium, the measured λ_t is never above 0.5, consistent with our assertion. The 10-year moving average of measured λ_t is also quite stable. Over the entire sample, λ_t averages 0.18.

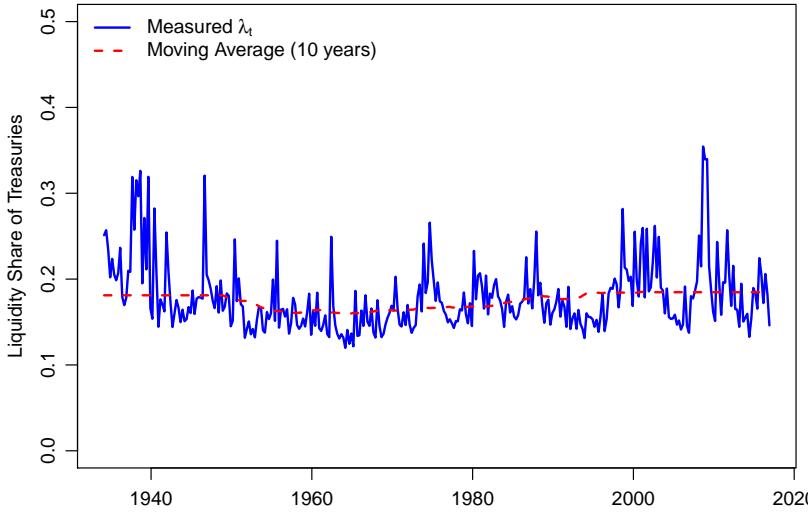


Fig. 5. The Liquidity Share of Treasuries (λ_t in the model)

2.6. Robustness Checks

We present results for different subsamples in Table 6. In columns (1) and (2), we drop the WWII period when the Fed played an unusually large role in setting prices in the money market. In columns (3) and (4), we consider only the sample post Fed-Treasury accord of 1951. This regression also drops the period surrounding the Great Depression. In columns (5) and (6), we consider only the sample after 1980, with the ending of Regulation Q. We present results for both measures of money. The analysis paints a consistent picture: the value of ρ is around 0.6 across samples and for both measures of deposits.

In the appendix, we present further robustness checks. (1) We use a measure of the deposit spread imputed from deposit expenses rather than the project deposit spread approach; (2) We consider alternative measure of the Treasury liquidity premium; (3) We consider alternative measures of λ_t (other than VIX); (4) We construct a Treasury quantity measure that only measures Treasury bills; (5) We consider alternative subsets of the five

Table 6: Subsample Analyses

	No WWII		Post Accord		Post 1980	
	<u>Net Tsy</u> Deposits	<u>Net Tsy</u> KVJ Deposits	<u>Net Tsy</u> Deposits	<u>Net Tsy</u> KVJ Deposits	<u>Net Tsy</u> Deposits	<u>Net Tsy</u> KVJ Deposits
	(1)	(2)	(3)	(4)	(5)	(6)
ρ	0.647 (0.163)	0.584 (0.196)	0.683 (0.137)	0.602 (0.172)	0.645 (0.117)	0.521 (0.171)
β_λ	0.012 (0.002)	0.010 (0.003)	0.012 (0.002)	0.010 (0.002)	0.011 (0.001)	0.008 (0.002)
p-value of J-test	0.843	0.81	0.768	0.787	0.201	0.097
Variations explained	68.7%	68.3%	69%	68.7%	60.5%	60.4%
Observations	292	292	260	260	144	144

Notes: The full sample is at quarterly frequency from 1934 to 2016. Refer to Table 1 for variable definitions. This table shows subsample GMM estimations of parameters ρ and β_λ . Columns of “No WWII” excludes the period around WWII (1942–1951), and columns of “Post Accord” restrict the sample to post-1951. In all columns, we use total Treasury/deposits and $(\text{total Treasury}/\text{deposits})^2$ as instruments in place of B/D . HAC standard errors with 4 lags are reported in parentheses.

GMM moments; (6) We consider more lags in the HAC estimator. The results are all broadly consistent.

3. Money, Near-Money, and Bonds

In this section, we expand our results in two directions. First, we consider how to expand our estimation to incorporate other assets that provide liquidity services. Our results here should be seen as a step in the direction of tracing out the full set of assets that provide liquidity services. Second, we use these results to construct a new monetary aggregate and revisit existing questions regarding the stability of the money demand equation.

3.1. Near Money

We have shown that Treasuries and bank deposits are imperfect substitutes, providing different liquidity services. Where do other money-market assets fall? In earlier sections, we report that the coefficient estimate for ρ for KVJ-deposits was somewhat higher than for bank deposits. The KVJ-deposit measure includes assets such as money market funds, repos, and commercial paper. These are likely more similar to Treasury bonds than traditional transaction deposits. In this section, we investigate a nested demand system that places Treasury bonds and non-transaction financial sector debt in an inner layer, and bank deposits in the outer layer.

Agents derive utility over C_t and Q_t as in our baseline model, but we redefine Q_t . The aggregate liquidity bundle is,

$$Q'_t = \left((1 - \lambda_t) \left(\frac{D_t}{P_t} \right)^\rho + \lambda_t \left(\frac{B'_t}{P_t} \right)^\rho \right)^{\frac{1}{\rho}}. \quad (31)$$

where,

$$B'_t = ((1 - \mu_t) \cdot (D_t^{NB})^\eta + \mu_t \cdot B_t^\eta)^{1/\eta}. \quad (32)$$

Here D_t is the total amount of deposits as defined in previous sections, B'_t is the amount of “composite bonds”, which is a CES aggregator over non-transaction financial sector debt, D_t^{NB} , and Treasuries, B_t . We measure D_t as checking, savings, and time deposits. We measure D_t^{NB} as KVJ-deposits minus D_t .

Denote the yield on Treasury bonds as i_t^{Tsy} and the yield on non-transaction financial sector debt as i_t^{NB} . Then the first order condition over Treasury bonds and non-transaction

financial sector debt implies

$$i_t - i_t^{\text{Tsy}} = \frac{\mu_t}{1 - \mu_t} \left(\frac{B_t}{D_t^{NB}} \right)^{\eta-1} (i_t - i_t^{NB}). \quad (33)$$

Using the notation $\ell_t^{\text{Tsy}} = i_t - i_t^{\text{Tsy}}$ and $\ell_t^{NB} = i_t - i_t^{NB}$, we can rewrite this equation as

$$\ell_t^{\text{Tsy}} = \frac{\mu_t}{1 - \mu_t} \left(\frac{B_t}{D_t^{NB}} \right)^{\eta-1} \ell_t^{NB}. \quad (34)$$

Moreover, we define the liquidity premium index for the bundled “bond” as

$$\ell_t^{B'} = \left((1 - \mu_t)^{-\frac{1}{\eta-1}} (\ell_t^{NB})^{\frac{\eta}{\eta-1}} + \mu_t^{-\frac{1}{\eta-1}} (\ell_t^{\text{Tsy}})^{\frac{\eta}{\eta-1}} \right)^{(\eta-1)/\eta} \quad (35)$$

With this definition, then we arrive at another equation:

$$\ell_t^{B'} = \frac{\lambda_t}{1 - \lambda_t} \left(\frac{B'_t}{D_t} \right)^{\rho-1} \ell_t^D \quad (36)$$

We proxy for the relative demand ratio, $\lambda_t/(1 - \lambda_t)$ and $\mu_t/(1 - \mu_t)$ using VIX:

$$\frac{\lambda_t}{1 - \lambda_t} = \beta_\lambda \text{VIX}_t$$

$$\frac{\mu_t}{1 - \mu_t} = \beta_\mu \text{VIX}_t$$

In total, we need to estimate four parameters, ρ , η , β_λ , and β_μ .

For estimation purposes, we consider shocks to the relative demand for Treasuries vs bank deposits, ϵ_t , as well as measurement errors on the Treasury and bundled bond liquidity premium. That is, the estimation equations are:

$$\ell_t^{B'} = \beta_\lambda \text{VIX}_t \left(\frac{B'_t}{D_t} \right)^{\rho-1} \ell_t^D \cdot \exp(\epsilon_t) + \varepsilon_t^B. \quad (37)$$

where ℓ_t^D is the projected deposit spread. This equation is the same as in the baseline case except that the bundled bond liquidity premium is defined in (35). The second equation is:

$$\ell_t^{\text{Tsy}} = \beta_\mu \text{VIX}_t \left(\frac{B_t}{D_t^{NB}} \right)^{\eta-1} \ell_t^{NB} + \varepsilon_t^{\text{Tsy}}. \quad (38)$$

Note that we have not included a relative demand shock in this equation. With only one instrument, total Treasury supply, our estimation approach cannot handle unobserved shocks

to relative demands in both demand conditions.

Define u_t as the error from the bundled-bond liquidity premium equation:

$$u_t = \ell_t^{B'} - \beta_\lambda \text{VIX}_t \left(\frac{B'_t}{D_t} \right)^{\rho-1} \ell_t^D \quad (39)$$

Define v_t as the error from the Treasury liquidity premium equation:

$$v_t = \ell_t^{Tsy} - \beta_\mu \text{VIX}_t \left(\frac{B_t}{D_t^{NB}} \right)^{\eta-1} \ell_t^{NB} \quad (40)$$

Then, the GMM conditions are:

$$E[v_t \cdot \begin{pmatrix} 1 \\ B_t/D_t^{NB} \\ \ell_t^{NB} \\ \text{VIX}_t \end{pmatrix}] = 0, \quad E[u_t \cdot \begin{pmatrix} 1 \\ \text{Total Tsy}_t/GDP_t \\ (\text{Total Tsy}_t/GDP_t)^2 \\ i_t \\ \text{VIX}_t \end{pmatrix}] = 0 \quad (41)$$

We also redefine the measures of the liquidity premium, defining the Treasury liquidity premium as the spread between 90-day P2CP rate and 90-day T-bill rate. In previous sections, we measure the Treasury liquidity premium as the spread between repo (and banker's acceptances) and Treasuries. However, to the extent that repo is also priced to reflect liquidity services, repo rates will also reflect a liquidity premium, and this measure will underestimate the full Treasury liquidity premium. We use P2-rated commercial paper (P2CP) as the benchmark rate for an asset that does not offer any liquidity services, and construct the Treasury liquidity premium as the spread between 90-day P2CP and 90-day Treasury bills. Note that a disadvantage of using this spread is that it will also reflect credit risk and thus add some noise to our estimation procedure. This is one reason that we have used repo rates as the benchmark in previous sections. The other reason is that P2CP data is only available monthly from 1974 onwards. We obtain this data from the Federal Reserve.

We construct the non-transaction financial sector liquidity premium in two different ways:

- The spread between 90-day P2-rated commercial paper (P2CP) and 90-day P1-rated (or “AA-rated”) commercial paper (P1CP).
- The spread between 90-day P2CP rate and the average money-market mutual fund (MMF) rate.

The MMF rate data are from [Xiao \(2020\)](#) at a monthly frequency. The MMF data are only available from 1987. Additionally, while the P2CP-P1CP spread is always positive, the

P2CP-MMF rate has three negative observations in our sample, likely due to measurement error. We set these observations to zero so that the liquidity premium index in (35) is well-defined.

We measure the deposit spread in a similar manner as in earlier sections. We first regress the P2CP-deposit spread (the spread between 90-day P2CP rate and the average deposit rate) on the federal funds rate using the data we have on the deposit spread post-1987. We use the regression result to project the deposit spread for the entire period, including the pre-1987 period. The projection approach also guarantees that the deposit spread remains positive.⁷

As a first step, we estimate the “inner CES aggregator” between Treasuries and non-transaction financial sector debt, as specified by equation (38). As in other specifications, we instrument the quantity ratio moment by Total Treasury/GDP and (Total Treasury/GDP)² to alleviate endogeneity concerns.

Results are reported in Table 7. We see that the estimated values of η are much closer to 1 compared to the estimated value of ρ in the previous section. The value of $\eta = 1$ for the MMF regression arises because we restrict that $\eta \leq 1$ in our estimation.

Next, we turn to the full model, described by the four parameters, η , ρ , β_μ , and β_λ , estimated from the moment conditions in (41). Because the results may be sensitive to the initial conditions for the GMM minimization algorithm, we use all permutations of point estimates from Table 7 and Table 4 (in total there are $4 \times 4 = 16$ different combinations) as initial values for the GMM algorithm and select the result that minimizes the GMM objective function. The estimation results are in Table 8. We find a value of η near one across all the specifications, consistent with the results in Table 7. The standard errors of estimated η are tight for all of the specifications. The use of Treasury IV seems more important in the nested estimations, since we observe a much smaller ρ estimation after we use the IV. The estimated value of ρ in column (3) is smaller than the 0.6 of earlier estimates, while the estimation in column (4) yields similar results. Additionally, the standard errors are wider in the versions where we use total Treasury as instruments. Note that our estimation of the more complicated model of this section is based on a shorter sample than earlier results. The table also reports goodness of fit measures for both of the liquidity premia explained by the model, the Treasury liquidity premium and the liquidity premium on the bundled bond (equation(37)).

Next, we use the estimated coefficients from column (1) of Table 8, β_λ and β_μ , and plot

⁷Figure 9 of the Appendix plots the deposit volumes, along with the volume of MMF outstanding, as well as the measures of the liquidity premia based on P2CP.

Table 7: GMM Estimation of the Substitution Between Non-bank Deposits and Treasuries

	Measurement of non-bank liquidity premium			
	P2CP–P1CP	P2CP–MMF	P2CP–P1CP	P2CP–MMF
	(1)	(2)	(3)	(4)
η	0.842 (0.127)	1.000 (0.181)	0.704 (0.077)	1.000 (0.210)
β_μ	0.120 (0.013)	0.030 (0.002)	0.092 (0.006)	0.027 (0.002)
Variation Explained	51.7%	78%	54.4%	78%
Total Treasury IV?	No	No	Yes	Yes
Observations	169	113	169	113

Notes: Data are at quarterly frequency from 1974 to 2016, while the availability of MMF deposit rate shrinks the sample size in columns (2) and (4). This table shows the GMM estimation of parameters η and β_μ . The GMM specification is in (41). We restrict $\eta \leq 1$ in the estimation so the results will be economically meaningful. In column (3) and (4), we use total Treasury/GDP and (total Treasury/GDP)² as instruments. HAC standard errors with 4 lags are reported in parentheses.

Table 8: GMM Estimation of the Nested Model

	Measurement of non-bank liquidity premium			
	P2CP–P1CP	P2CP–MMF	P2CP–P1CP	P2CP–MMF
	(1)	(2)	(3)	(4)
ρ	0.816 (0.155)	0.900 (0.327)	0.249 (0.161)	0.643 (0.432)
β_λ	0.024 (0.003)	0.059 (0.002)	0.018 (0.002)	0.060 (0.002)
η	0.959 (0.109)	0.997 (0.001)	0.855 (0.064)	0.997 (0.001)
β_μ	0.133 (0.009)	0.029 (0.001)	0.123 (0.006)	0.030 (0.001)
Total Treasury IVs?	No	No	Yes	Yes
R^2 for ℓ_t^{Tsy}	0.59	0.6	0.62	0.6
R^2 for $\ell_t^{B'}$	0.43	0.43	0.45	0.43
Observations	169	113	169	113

Notes: Data are at quarterly frequency from 1974 to 2016, while the availability of MMF deposit rate shrinks the sample size in columns (2) and (4). This table shows the GMM estimation of parameters ρ , β_λ , η , and β_μ . The GMM specification are in (41). In column (3) and (4), we use total Treasury/GDP and (total Treasury/GDP)² as instruments in the u_t equation. We report the R^2 fit of both the Treasury liquidity premium (equation (38)) and the composite bonds premium (equation (37)). HAC standard errors with 4 lags are reported in parentheses.

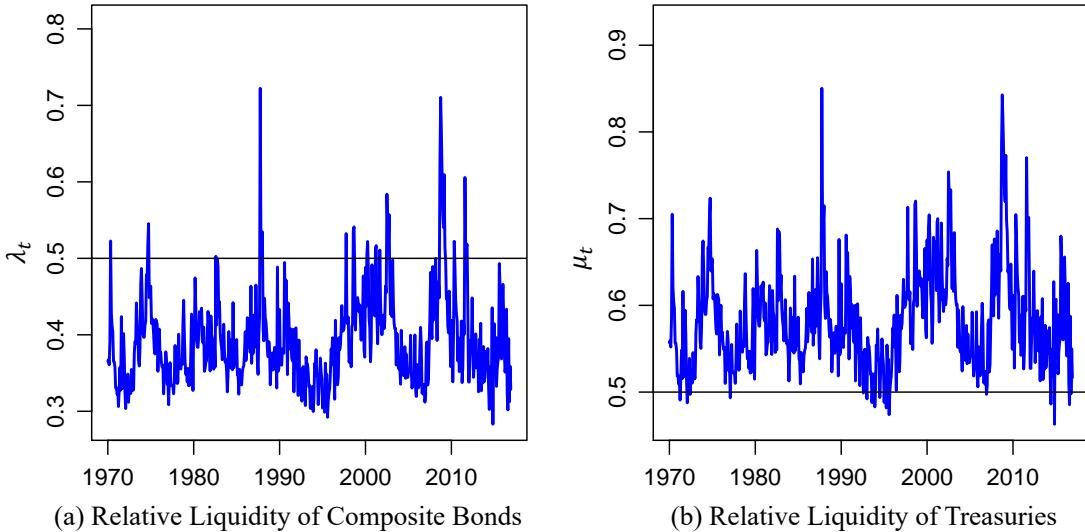


Fig. 6. Time Series of Estimated Relative Liquidity

the implied λ_t and μ_t series in Figure 6:

$$\lambda_t = \frac{\beta_\lambda \text{VIX}_t}{1 + \beta_\lambda \text{VIX}_t}$$

$$\mu_t = \frac{\beta_\mu \text{VIX}_t}{1 + \beta_\mu \text{VIX}_t}.$$

Here λ_t measures the relative liquidity per unit of asset of composite bonds and deposits. Similarly, μ_t measures the relative liquidity of Treasuries and non-transaction financial sector debt. From Figure 6, we see that most of the time $\lambda_t < 0.5$ with an average value of 0.42, so the composite bonds offer less liquidity compared to deposits. On the other hand, $\mu_t > 0.5$ in the majority of periods with an average value of 0.58, indicating that Treasuries offer more liquidity services compared to non-transaction financial sector debt.

3.2. A New Liquidity Aggregate

There is a large literature estimating money-demand equations, in which a measure of money (typically M1/P or M2/P) is regressed against the opportunity cost of money (typically the nominal commercial paper rate) and GDP. The literature aims to measure the interest-rate and income elasticity of money demand. See [Goldfeld and Sichel \(1990\)](#)'s chapter in the Handbook of Monetary Economics. The literature is interested in these elasticity estimates because they are needed to answer questions such as, what is the optimal growth rate of money, and, what is the welfare cost of inflation (e.g., see [Lucas \(2001\)](#)).

Many authors have commented on the instability of traditional money demand functions (see [Goldfeld and Sichel \(1990\)](#), [Teles and Zhou \(2005\)](#), [Lucas and Nicolini \(2015\)](#)). The finding in the literature is that while there is a stable relation between real money demand, the nominal interest rate, and real income, in the period before 1980, this relation breaks down post-1980. The most prominent puzzle in the literature is the “missing money” of the post-1980 period, when interest rates fell and money balances rose, but not as strongly as earlier estimates would have predicted. Resolutions of this puzzle have centered on recognizing that, prompted by the banking deregulation of the 1980s, a broader set of assets than checkable deposits offer monetary services. [Teles and Zhou \(2005\)](#) and [Lucas and Nicolini \(2015\)](#) expand the definition of money to include money market deposit accounts and measure the interest rate spread on these accounts as the opportunity cost of money. They show that growth in these accounts are the “missing-money” from other estimates, and including these accounts leads to a stable money-demand curve. [Krishnamurthy and Vissing-Jorgensen \(2013\)](#) note that accounting for the impact of Treasury supply on bank money may help with the missing money puzzle.

In this section, we construct a new broad liquidity aggregate based on our analysis and show that the demand for this aggregate is stable despite financial innovation. Our approach is similar to [Teles and Zhou \(2005\)](#) and [Lucas and Nicolini \(2015\)](#) in that we broaden the monetary aggregate. We are different in that we consider Treasury bonds along with the KVJ measure of non-bank deposits as the new components of the monetary aggregate. We also drop currency from the aggregate, in keeping with our focus on traded financial assets. The appendix reports results where we include currency. Finally, central to the results of this paper, we allow that these assets may be imperfect substitutes.

We specialize the model to a functional form that is common in the monetary economics literature:

$$u(C_t, Q_t) = \frac{C_t^{1-\gamma_C}}{1-\gamma_C} + \frac{Q_t^{1-\gamma_Q}}{1-\gamma_Q} \quad (42)$$

Then the FOC for the deposit spread (for $\rho \neq 0$) gives,

$$\log\left(\frac{i_t - i_t^d}{1 + i_t}\right) = \gamma_C \log(C_t) - (1 - \rho) \log\left(\frac{D_t}{P_t}\right) + \frac{1}{\rho}(1 - \rho - \rho\gamma_Q) \log(Q_t) + \log(1 - \lambda_t). \quad (43)$$

We rewrite this equation to describe the demand over the liquidity aggregate, Q_t :

$$\frac{1}{\rho}(\gamma_Q + \rho - 1) \log(Q_t) = -\log\left(\frac{i_t - i_t^d}{1 + i_t}\right) + \gamma_C \log(C_t) - (1 - \rho) \log\left(\frac{D_t}{P_t}\right) + \log(1 - \lambda_t). \quad (44)$$

We note the typical case studied in the money-demand literature sets $Q_t = m_t$ (real value

of money), $\rho = 1$, and omits the relative demand term $\log(1 - \lambda_t)$, in which case equation (43) is:

$$\gamma_Q \log(m_t) = -\log\left(\frac{i_t - i_t^d}{1 + i_t}\right) + \gamma_C \log(C_t) \quad (45)$$

The literature typically runs regressions of $\log(m_t/\text{real GDP}_t)$ on $\log((i_t - i_t^d)/(1 + i_t))$ and $\log(\text{real GDP}_t)$ in order to estimate γ_Q and γ_C .

We construct two monetary aggregates, Q_t and Q'_t . The first aggregate is from our baseline, defining Q_t as in (5), with B_t equal to net Treasury supply, D_t is the deposit measure, and ρ set to be 0.60, which is the common value across our different specifications. We construct the aggregate with λ_t varying over time with VIX. The second aggregate is based on the nested specification, defining Q'_t as in (31) and using the average value of ρ and η from our nested estimations in Table 8 to construct Q'_t . We again consider time-varying weights for μ_t and λ_t .⁸ As in the last section, we use the projection δi_t for the deposits spread $i_t - i_t^d$ in the model.

The first and second columns of Table 9 present the instability finding in the literature. We regress the log of real money balances to GDP, where money is measured as M1 (currency and checking deposits), on the log of the deposit spread and the log of real GDP. The estimate for the interest elasticity in the first row changes considerably from the pre-1980 sample to the post-1980 sample, in line with the instability finding in the literature. The first panel of Figure 7 presents a scatter plot of annual averages of $\log(m_t/\text{real GDP}_t)$ against the $\log(i_t)$. As the spread falls in the post-1980 sample, we would have expected based on the pre-1980 results that money balances would have risen. But they do not, resulting in the “missing money” puzzle.

Our results illustrated in Panels (b) and (c) of Figure 7 indicate that the expanded liquidity aggregates, Q_t and Q'_t , help resolve the missing money puzzle. Columns (3) through (6) of Table 9 present these results in regression form. We note that the interest rate elasticity of money demand is now quite similar across the two sub-samples for both liquidity aggregates. From columns (3) and (4), we see that the income elasticity however still changes across the sub-samples.

In the appendix, we also report the impact of including currency in our new monetary aggregates. We include currency in D_t and otherwise aggregate non-bank debt and Treasury bonds following the same procedure. The results are consistent with those we report in this section: the demand for our new broad aggregate is stable compared to that of the demand for M1. See Table 23 of the appendix.

⁸The Appendix reports these results where we construct Q_t and Q'_t based on holding μ_t and λ_t constant over time.

Table 9: Money Demand Regressions

	Dependent variable:					
	$\log(m_t/\text{real GDP}_t)$		$\log(Q_t/\text{real GDP}_t)$		$\log(Q'_t/\text{real GDP}_t)$	
	pre 1980 (1)	post 1980 (2)	pre 1980 (3)	post 1980 (4)	pre 1980 (5)	post 1980 (6)
$\log\left(\frac{\delta i_t}{1+i_t}\right)$	-0.299 (0.057)	-0.086 (0.055)	-0.247 (0.096)	-0.125 (0.014)	-0.040 (0.091)	-0.054 (0.021)
$\log(\text{real GDP}_t)$	-0.200 (0.095)	-0.509 (0.253)	0.178 (0.086)	-0.104 (0.151)	0.076 (0.094)	0.188 (0.134)
$\log(\text{VIX}_t)$			-0.426 (0.112)	-0.183 (0.058)	-0.321 (0.130)	-0.065 (0.023)
Constant	0.254 (0.757)	2.720 (2.397)	-1.288 (0.859)	0.473 (1.357)	-0.954 (0.964)	-2.594 (1.237)
Observations	184	148	184	148	184	148
R ²	0.888	0.410	0.492	0.760	0.479	0.848

Notes: Data are at quarterly frequency and the whole sample is from 1934 to 2016. This table presents the money demand regressions with different definitions of money, where m_t is the real quantity of money including currency and checking deposits, Q_t is the real value of liquidity bundle as in equation (5), and Q'_t is the real value of liquidity bundle as in equation (31). Newey-West standard errors with 4 lags are reported in parentheses.

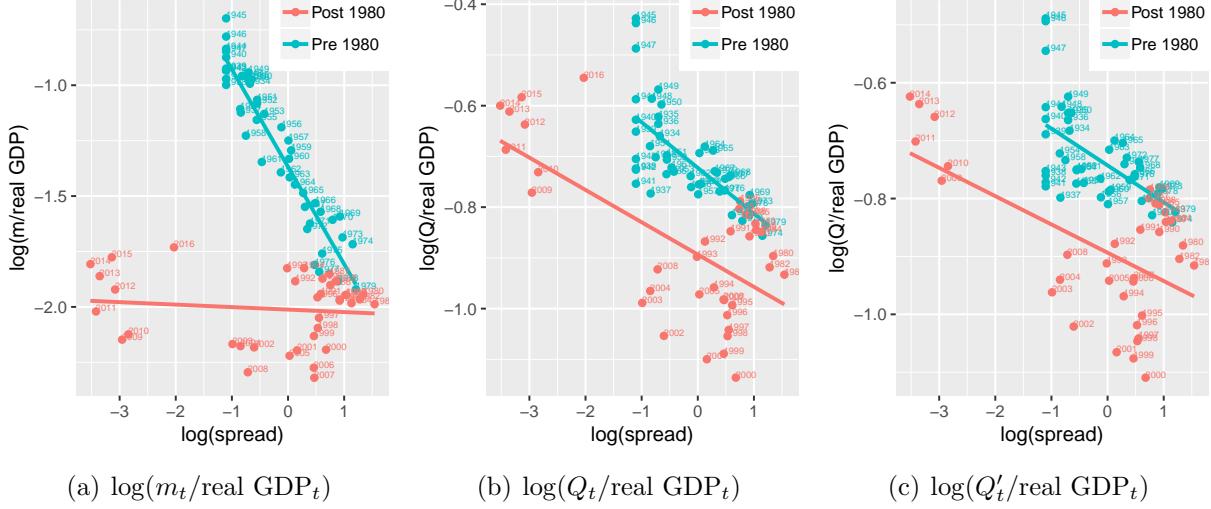


Fig. 7. Quantity of Liquidity and the Opportunity Cost of Holding Liquidity. The x-axis is $\log(\delta i_t/(1 + \text{FFR}_t))$ across all three panels. Data are annual averages from 1934 to 2016, with the years marked on the figure.

4. Implications

This section discusses the implications of our results for models of money and banking.

4.1. Quantitative Easing

Quantitative easing policies aim to impact the interest rate on long-term bonds, both government and private bonds that are close substitutes to the government bonds, via changes in the supply of government bonds and money. Let us first derive the liquidity premium on long-term zero-coupon bonds of unit face value in our setting. Denote the price of a T period zero-coupon bond with convenience services as $V_{t,T}$. The analogous condition to (10) for holding a long-term bond from period t to $t+1$ is:

$$u'_Q(C_t, Q_t) Q_t^{1-\rho} \lambda_t \left(\frac{B_t}{P_t} \right)^{\rho-1} = u'_C(C_t, Q_T) \left(1 - E_t[\beta \frac{P_t}{P_{t+1}} \frac{u'_C(C_{t+1}, Q_{t+1})}{u'_C(C_t, Q_t)} \frac{V_{t+1,T-1}}{V_{t,T}}] \right) \quad (46)$$

Combining with (9), we have that,

$$1 - E_t[\beta \frac{P_t}{P_{t+1}} \frac{u'_C(C_{t+1}, Q_{t+1})}{u'_C(C_t, Q_t)} \frac{V_{t+1,T-1}}{V_{t,T}}] = \frac{\lambda_t}{1 - \lambda_t} \left(\frac{B_t}{D_t} \right)^{\rho-1} \frac{i_t - i_t^d}{1 + i_t} \quad (47)$$

We make the simplifying assumption that the economy has no uncertainty and drop the expectation operator. In this case, we can iterate on (47) from period t to T and combine

with Appendix equations (A-2) and (A-3) to find:

$$\frac{1}{1+i_t} \times \dots \times \frac{1}{1+i_{T-1}} \times \frac{1}{V_{t,T}} = \left(1 - \frac{\lambda_t}{1-\lambda_t} \left(\frac{B_t}{D_t}\right)^{\rho-1} \frac{i_t - i_t^d}{1+i_t}\right) \times \dots \times \left(1 - \frac{\lambda_T}{1-\lambda_T} \left(\frac{B_T}{D_T}\right)^{\rho-1} \frac{i_T - i_T^d}{1+i_T}\right),$$

where we have used the terminal condition $V_{t+T,0} = 1$. Using the definition of $1/V_{t,T} = (1+i_{t,T}^b)^T$, $(1+i_{t,T})^T = (1+i_t) \times (1+i_{t+1}) \times \dots \times (1+i_{T-1})$, and with a first-order Taylor approximation, we find,

$$i_{t,T} - i_{t,T}^b = \frac{1}{T} \left(\frac{\lambda_t}{1-\lambda_t} \left(\frac{B_t}{D_t}\right)^{\rho-1} \frac{i_t - i_t^d}{1+i_t} + \dots + \frac{\lambda_T}{1-\lambda_T} \left(\frac{B_T}{D_T}\right)^{\rho-1} \frac{i_T - i_T^d}{1+i_T} \right). \quad (48)$$

where $i_{t,T} - i_{t,T}^b$ is the liquidity premium on a T -period Treasury bond. Equation (48) is an “expectations hypothesis” expression to determine the liquidity premium on a long-term bond. Our empirical results pin down this functional form as well as the parameters governing ρ and λ_t .

Next consider our analysis’ implications for quantitative easing policies. First, we see that the impact of policies will depend on ρ . If $\rho = 1$, the perfect substitutes case as argued by Nagel (2016), then QE policies will have no effect on liquidity spreads. We have found a value of ρ around 0.65, which helps parameterize the impact of a given change in bond supply. We also see that the impact of a given change in bond supply depends inversely on the quantity of deposits. If the quantity of deposits in the economy is doubled, then the size of QE needs to double to effect the same change in liquidity spreads. For example, this result is a way of understanding why QE in 2008 had a bigger impact than later rounds of QE, at which point the quantity of deposits had increased substantially. Another result apparent from this expression which can also shed light on the diminishing impact of QE is the dependence on liquidity spreads, $i_t - i_t^d$. When these spreads are high, as in 2008, QE has a larger impact. Our empirical results offer both qualitative and quantitative guidance on these points.

4.2. Models with multiple moneys

Our results can be used to quantitatively discipline monetary economics models with multiple forms of money, even in cases where these models do not impose a money-in-the-utility function relation as we have in 5. For example, in Rocheteau, Wright and Xiao (2018), agents use both money and government bonds as payment instruments. Differential search and pledgeability frictions mean that money and bonds are not perfect substitutes. Their micro-founded money model traces out the quantity and asset pricing effects of varying the

relative supplies of these assets. Our empirical results can be used to discipline the price relation in their analysis. This same point can be applied to other recent macro-finance models with multiple modes of payment, including [Piazzesi and Schneider \(2018\)](#) and [Gorton and He \(2016\)](#). Even though the models vary in their microeconomic foundations, they imply asset pricing relations between different asset quantities and interest rates, analogous to the relation we have estimated.

[Begenau and Landvoigt \(2021\)](#) study a related macro-banking question. They consider a model where banks and shadow banks both provide deposits that are used by the non-bank sector as money, and use the funds to provide credit to the real economy. A key parameter in their quantitative model is the elasticity of substitution between bank and shadow-bank deposits. Our empirical results in Table 8 provides an estimate of this parameter. More generally, any macroeconomic analysis of the coexistence of shadow and traditional banks has to take a stand on the differential services provided by bank and shadow-bank money, and our results are directly applicable to these models.

5. Comparison with Nagel (2016)

[Nagel \(2016\)](#) shows theoretically and empirically that the liquidity premium on Treasury bonds is impacted by monetary policy. The paper has been important because of this monetary policy link: it takes the quantity/liquidity-premium findings of [Krishnamurthy and Vissing-Jorgensen \(2012\)](#) and broadens the research to include monetary policy. Our paper builds closely on [Nagel \(2016\)](#). However a key point of difference in our results is that we estimate $\rho \approx 0.65$ while the results from [Nagel \(2016\)](#) indicate $\rho \approx 1$. In this section, we explain the sources of differences between these results and provide further support in favor of our approach and the $\rho \approx 0.65$ estimate.

5.1. Replication

We first replicate the results of [Nagel \(2016\)](#), which estimates a linearized version of equation (20). The first three columns of Table 10 should be compared to Table III of [Nagel \(2016\)](#). The estimates are quite close to that table, and the discrepancies are likely due to slight differences in data and sample.⁹ In the first column, we see that the federal funds rate strongly comoves with the liquidity premium. In column (2), we see that Treasury supply has significant explanatory power for the liquidity premium, as shown in [Krishnamurthy and](#)

⁹Our sample starts from 1926 when S&P 500 index is available in WRDS, while [Nagel \(2016\)](#) further goes back to 1920 (without using deposits volume data). We also winsorize the liquidity premium series.

[Vissing-Jorgensen \(2012\)](#). In the third column, the [Krishnamurthy and Vissing-Jorgensen \(2012\)](#) result is overturned: Treasury supply loses its explanatory power if the federal funds rate is included in the regression. Indeed, the coefficient of $\log(\text{Total Tsy}/\text{GDP})$ in column (3) is near zero and statistically insignificant.

Columns (4) and (5) replace the quantity variable in the regression. In column (4), we follow the theory and use the net Treasury variable to correspond to the Treasury holdings of the non-bank sector. In column (5), we use Net Treasury/Deposits as the measure of $\frac{B_t}{D_t}$, as prescribed by equation (20). Using the theoretically motivated quantity variable improves the fit in the regression: the R^2 rises from 60% in (3) to 64% in (5). Additionally, the quantity variable is a statistically significant driver of the liquidity premium, in line with [Krishnamurthy and Vissing-Jorgensen \(2012\)](#).

Table 10: Replication of Nagel (2016) Table III

	<i>Dependent variable:</i> liquidity premium _t				
	(1)	(2)	(3)	(4)	(5)
FFR _t	10.78 (1.08)		10.76 (1.15)	10.43 (1.04)	10.40 (0.99)
VIX _t	1.25 (0.20)	0.40 (0.27)	1.25 (0.21)	1.10 (0.22)	1.40 (0.33)
$\log\left(\frac{\text{Total Debt}_t}{\text{GDP}_t}\right)$		-48.65 (14.18)	-0.29 (4.60)		
$\log\left(\frac{\text{Net Tsy}_t}{\text{GDP}_t}\right)$				-8.47 (3.84)	
$\log\left(\frac{\text{Net Tsy}_t}{\text{Deposits}_t}\right)$					-12.86 (3.72)
Constant	-29.43 (6.76)	-12.89 (11.35)	-29.56 (7.37)	-38.18 (8.70)	-39.92 (9.72)
Observations	1,032	1,032	1,032	1,032	936
R ²	0.60	0.20	0.60	0.61	0.64

Notes: We use monthly data and use the sample from 1926–2011 to make results in this table comparable with [Nagel \(2016\)](#). Variable explanations are provided in the notes of Table 1. HAC standard errors with 12 lags (one-year lags) are shown in parentheses.

5.2. Non-linearity and Interaction Effects

Table 11 shows that accounting for the non-linear terms in equation (20) is important in the estimation. Relative to Table 10 we extend the sample to 2016. The first three columns show the linear regression with the extended sample. The coefficient on the Treasury supply variable changes slightly and remains significant. The rest of the columns include non-linear terms. In column (5), we include both $\log(\text{Net Tsy}/\text{GDP})$ and the interaction between the federal funds rate and $\log(\text{Net Tsy}/\text{GDP})$. The coefficient on the federal funds rate falls by a half relative to column (3), while the interaction term is negative and highly significant. We also note that the regression R^2 rises from 61% in column (1) to 65% in column (5).

We can interpret these higher-order terms by expanding (11). To a first-order approximation around the average value of B_t/D_t (denoted by x_0), we get

$$\left(\frac{B_t}{D_t}\right)^{\rho-1} \approx x_0^{\rho-1} + x_0^{\rho-1}(\rho - 1) \left(\log\left(\frac{B_t}{D_t}\right) - \log(x_0) \right). \quad (49)$$

In this case, the coefficient on $\log(B_t/D_t)$ is proportional to $x_0^{\rho-1}(\rho - 1)$, and the Nagel (2016) result that supply has no explanatory power for the liquidity premium leads to the conclusion that ρ is not statistically different than one. If we include more terms from the expansion of (20), we find

$$\ell_t^b \propto (1 - (\rho - 1) \log(x_0)) \cdot \text{VIX}_t i_t + (\rho - 1) \cdot \text{VIX}_t \log\left(\frac{B_t}{D_t}\right) i_t \quad (50)$$

In column (6) of Table 11, we report a regression based on the higher-order terms in equation (50). Including these interaction terms substantially improves the fit of the model: the R^2 in column (6) is 65%, despite only including two explanatory variables. The coefficients on the triple-interaction term are also significantly negative as the theory suggests. The significant and negative coefficient on the triple-interaction term is robust to including the individual terms, FFR_t , VIX_t , or $\log(\text{Net Tsy}/\text{Deposits})$ in the regression.

In terms of economics, when $\rho \neq 1$, the interaction terms capture that the marginal impact of a change in the deposit spread depends on the quantity of bonds and deposits. For example, with more bonds outstanding, liquidity premia are smaller and the marginal impact of a change in monetary policy is likewise smaller. The same logic applies in reverse for a change in the quantity of bonds. Thus, including the interaction terms is economically meaningful.

In statistical terms, a linear regression with the federal funds rate and bond supply will ascribe most of the explanatory power for the liquidity premium to the federal funds rate. In the data, there is considerable high frequency variation in both the federal funds rate

Table 11: Non-Linear Interaction Effect

	Dependent variable: Liquidity Premium _t					
	(1)	(2)	(3)	(4)	(5)	(6)
FFR _t	10.51 (1.04)		10.19 (1.03)	2.37 (2.69)	0.08 (3.50)	
VIX _t		1.21 (0.20)	0.33 (0.29)	1.09 (0.21)	0.99 (0.20)	1.09 (0.22)
$\log\left(\frac{\text{Net Tsy}_t}{\text{GDP}_t}\right)$			-26.49 (8.89)	-5.78 (3.52)		8.79 (5.85)
$\text{FFR}_t * \log\left(\frac{\text{Net Tsy}_t}{\text{GDP}_t}\right)$					-4.36 (1.59)	-5.85 (2.20)
$\text{VIX}_t * \text{FFR}_t$						0.20 (0.10)
$\text{VIX}_t * \text{FFR}_t * \log\left(\frac{\text{Net Tsy}_t}{\text{GDP}_t}\right)$						-0.17 (0.06)
Constant	-26.74 (6.19)	-7.84 (10.84)	-31.91 (7.57)	-18.79 (5.62)	-8.23 (8.00)	1.66 (2.30)
Observations	1,092	1,092	1,092	1,092	1,092	1,092
R ²	0.61	0.12	0.61	0.64	0.65	0.65

Notes: We use monthly data and the sample from 1926–2016 to make results in this table more comparable with Nagel (2016). Variable explanations are provided in the notes of Table 1. HAC standard errors with 12 lags (one-year lags) are shown in parentheses.

and the liquidity premium. The maximum value of the federal funds rate is more than 100 times its minimum value. In contrast, the Treasury supply is slow-moving. Thus in a linear regression, it will be difficult to pick-out a relationship between Treasury supply and the liquidity premium. The next section makes this point explicit via regressions on simulated data.

5.3. Comparison of Estimation Methods

Our estimation implies that ρ is around 0.6. Applying the Wald test to the null hypothesis,

$$H0 : \rho = 1, \quad (51)$$

across the four settings in Table 4, we find an average Wald statistic $W = 4.6$, with an average p -value of 0.04. The model thus rejects the null hypothesis that deposits and Treasuries are perfect substitutes, contradicting the conclusion in Nagel (2016) that Treasuries and bank deposits are close to perfect substitutes. Testing the hypothesis $H0 : \rho = 0$ yields an even smaller p -value of 0.0009, indicating that we can also reject that ρ is zero. In what follows, we further explore the statistical power of our approach to estimating ρ .

Probability of False Positive

Suppose that the true model has $\rho = 1$, what is the probability that the GMM estimation wrongly finds $\hat{\rho} \leq 0.66$ (column (3) of Table 4)? For this purpose, we first estimate the model

$$lp_t = \beta_\lambda s_t VIX_t + \tilde{\varepsilon}_t \quad (52)$$

Next, we apply a stationary block bootstrap on $\tilde{\varepsilon}_t$ 5,000 times to generate 5,000 different time series of the liquidity premium. With the GMM estimation and an underlying model of (11), we generate 5,000 different estimates of ρ . Then we calculate the probability of finding $\hat{\rho}$:

$$P(\hat{\rho} \leq 0.66 | \rho = 1) = \frac{\# \text{ of estimations smaller than } 0.66}{\# \text{ of estimations}} = 1.5\% \quad (53)$$

We conclude that it is quite unlikely that the true ρ equals to 1 but our estimates reveal a ρ less than 0.66.

Sources of Differences

There are two main differences between our approach and Nagel (2016). First, we use $\log(\text{Net Treasuries}/\text{Deposits})$ instead of $\log(\text{Total Treasuries}/\text{GDP})$. As noted, the former

variable more closely aligns with the theory, and as we have shown, using this variable sharpens our results. The second difference has to do with the interaction effect we have discussed. Theoretically, the nominal interest rate and Treasury supply interact in driving the liquidity premium, as illustrated by equation (11), and this interaction term is missed in Nagel (2016)'s linearized model.

To understand how these differences contribute to the different estimate in this paper, we compare three estimation methods:

1. Linear regression using bond/GDP ratio: liquidity premium $\sim \text{FFR} + \log(\text{Total Debt}/\text{GDP}) + \text{VIX}$. This regression is the same as Nagel (2016).
2. Linear regression using bond/deposit ratio: liquidity premium $\sim \text{FFR} + \log(\text{Net Debt}/\text{Deposit}) + \text{VIX}$.
3. GMM: direct estimation of equation (11) using GMM with moment conditions (25).

Using the average of the estimated values across columns in Table 4 and equation (11), we bootstrap the residuals and then summarize the statistical properties of the different estimation methods.

There are different ways to implement the bootstrap. A simple bootstrap with replacement works for i.i.d. residuals. However, in the data, the residuals of the liquidity premium have a strong time-series correlation that is well represented by a stationary autoregressive process. In the statistics literature, the method designed for this case is stationary bootstrapping (Politis and Romano, 1994), which preserves stationarity and time series correlation, based on the assumption that the time series is stationary and weakly dependent.

We repeat the stationary bootstrap 5,000 times,¹⁰ each with a different random seed. For each bootstrap, the three methods are used to estimate the impact of the supply variable, which is the coefficient of $\log(\text{Net Debt}/\text{GDP})$ in the first approach, the coefficient of $\log(\text{Net Debt}/\text{Deposits})$ in the second approach, and $\rho - 1$ in the third approach.

Figure 8 presents the results. It is apparent that method 3 implies higher confidence in $\rho < 1$ than method 2 and method 1. The average t -stat is about -0.8 using the first linear regression method, -2.9 using the second linear regression method, -3.8 using GMM. Furthermore, with method 1, the probability of finding that ρ is above 1 is 6.5%, while such a probability drops to essentially zero for method 3.

Thus, we are more likely to wrongly conclude that bonds and deposits are almost perfect substitutes based on the supply factor's insignificance using the first linear regression method, even if the underlying truth is far from perfect substitutes.

¹⁰Results are similar if we use more than 5,000 rounds.

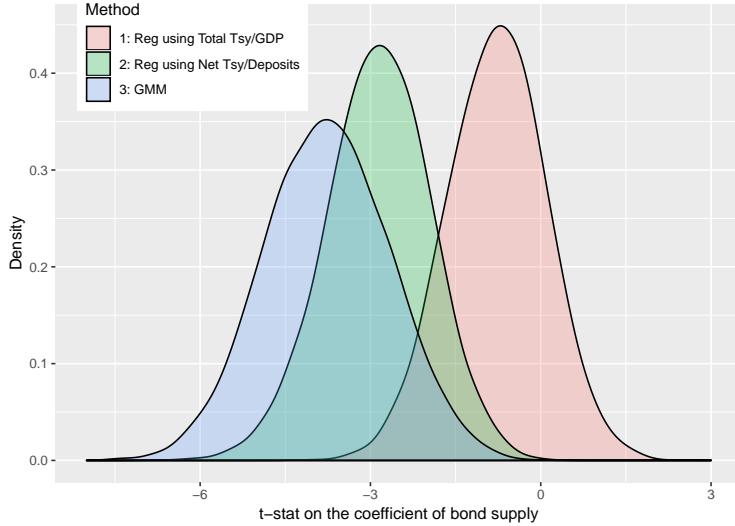


Fig. 8. Comparing t-stats under Three Different Methods. This figure shows the density of t -stats on the estimated coefficient of bond supply, which is the coefficient of $\log(\text{Net Debt}/\text{GDP})$ in method 1, the coefficient of $\log(\text{Net Debt}/\text{Deposits})$ in method 2, and $\rho - 1$ in method 3. Block-bootstrap residuals are used for simulation analysis and we assume the underlying model has ρ equal to the average estimation results in Table 4. The GMM moments are in equation (25).

6. Conclusion

Bank deposits, non-bank short-term safe debt, and Treasury bonds all provide liquidity services to investors. They provide different types of these services, as evidenced by our results that bank deposits and Treasury bonds are imperfect substitutes. They also provide different amounts of liquidity services per-unit-of-asset, as evidenced by our estimates of μ and λ , with Treasury bonds notably providing a larger amount of liquidity services than non-transaction financial sector debt.

Our analysis furthers a research area which has been active over the last decade of broadening the definition of money to grapple with the complexities of a modern financial system. This research is directly relevant for understanding many monetary and banking issues, including the transmission of monetary policy and regulation of the banking system.

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Appendix A. Model Derivations

In this section, we derive the baseline first-order conditions of the representative investor in (9) and (10). Then we show how to incorporate other types of assets.

A.1. Baseline Results

The Lagrangian of the investor optimization problem is

$$E\left[\sum_{t=1}^{\infty} \beta^t \left(u(C_t, Q_t) + \lambda_t \begin{pmatrix} D_{t-1}(1+i_{t-1}^b) + B_{t-1}(1+i_{t-1}^b) + A_{t-1}(1+i_{t-1}) \\ + I_t - (P_t C_t + D_t + B_t + A_t + T_t) \end{pmatrix} \right)\right] \quad (\text{A-1})$$

The first order condition on consumption is

$$\frac{\partial u(C_t, Q_t)}{\partial C_t} = \lambda_t P_t. \quad (\text{A-2})$$

The first order condition on lending is

$$E_t[\beta \lambda_{t+1}](1+i_t) - \lambda_t = 0. \quad (\text{A-3})$$

The first order condition on deposit holding is

$$\frac{\partial u(C_t, Q_t)}{\partial Q_t} \frac{\partial Q_t}{\partial D_t} - \lambda_t + E_t[\beta \lambda_{t+1}](1+i_t^d) = 0. \quad (\text{A-4})$$

The first order condition on bond holding is

$$\frac{\partial u(C_t, Q_t)}{\partial Q_t} \frac{\partial Q_t}{\partial B_t} - \lambda_t + E_t[\beta \lambda_{t+1}](1+i_t^b) = 0. \quad (\text{A-5})$$

Combining Equations (A-2), (A-3), and (A-4), we get

$$\frac{\partial u(C_t, Q_t)}{\partial D_t} = \frac{\partial u(C_t, Q_t)}{C_t} \frac{1}{P_t} \frac{i_t - i_t^d}{1+i_t}. \quad (\text{A-6})$$

Similarly, from (A-2), (A-3), and (A-5), we have

$$\frac{\partial u(C_t, Q_t)}{\partial B_t} = \frac{\partial u(C_t, Q_t)}{C_t} \frac{1}{P_t} \frac{i_t - i_t^b}{1+i_t}. \quad (\text{A-7})$$

When $\rho \neq 0$, by expanding the definition of Q_t , we have

$$\frac{\partial u(C_t, Q_t)}{\partial D_t} = \frac{\partial u(C_t, Q_t)}{\partial Q_t} Q_t^{1-\rho} (1 - \lambda_t) \left(\frac{D_t}{P_t}\right)^{\rho-1} \frac{1}{P_t}. \quad (\text{A-8})$$

$$\frac{\partial u(C_t, Q_t)}{\partial B_t} = \frac{\partial u(C_t, Q_t)}{\partial Q_t} Q_t^{1-\rho} \lambda_t \left(\frac{B_t}{P_t}\right)^{\rho-1} \frac{1}{P_t}. \quad (\text{A-9})$$

When $\rho = 0$,

$$Q_t = \left(\frac{D_t}{P_t}\right)^{1-\lambda_t} \left(\frac{B_t}{P_t}\right)^{\lambda_t}, \quad (\text{A-10})$$

and therefore the derivatives are

$$\frac{\partial u(C_t, Q_t)}{\partial D_t} = \frac{\partial u(C_t, Q_t)}{\partial Q_t} (1 - \lambda_t) \frac{Q_t}{(D_t/P_t)} \frac{1}{P_t}, \quad (\text{A-11})$$

$$\frac{\partial u(C_t, Q_t)}{\partial B_t} = \frac{\partial u(C_t, Q_t)}{\partial Q_t} \lambda_t \frac{Q_t}{(B_t/P_t)} \frac{1}{P_t}. \quad (\text{A-12})$$

Combining (A-8) and (A-9) with (A-11) and (A-12), we have

$$\frac{\partial u(C_t, Q_t)}{\partial Q_t} Q_t^{1-\rho} (1 - \lambda_t) \left(\frac{D_t}{P_t}\right)^{\rho-1} = \frac{\partial u(C_t, Q_t)}{\partial C_t} \frac{i_t - i_t^d}{1 + i_t}, \quad (\text{A-13})$$

$$\frac{\partial u(C_t, Q_t)}{\partial Q_t} Q_t^{1-\rho} \lambda_t \left(\frac{B_t}{P_t}\right)^{\rho-1} = \frac{\partial u(C_t, Q_t)}{\partial C_t} \frac{i_t - i_t^b}{1 + i_t}, \quad (\text{A-14})$$

for all $\rho \in \mathbb{R}$.

A.2. Allowing for Other Assets in Liquidity

The first order conditions may remain the same when we introduce other assets into the CES aggregator of liquidity. For example, suppose we include cash M_t that change the aggregator Q_t into

$$Q_t = \left((1 - \lambda_t) \left(\frac{D_t}{P_t}\right)^\rho + \lambda_t \left(\frac{B_t}{P_t}\right)^\rho + \xi_t \left(\frac{M_t}{P_t}\right)^{\rho\chi} \right)^{\frac{1}{\rho}}. \quad (\text{A-15})$$

Then the derivatives of Q_t over D_t and B_t are still going to be the same as (A-8) (A-9) for $\rho \neq 0$, and (A-11) (A-12) for $\rho = 0$. The opportunity costs for holding bonds and deposits are still the same. Therefore, the first order conditions are still the same as (A-13) and (A-14).

In general, we can add more liquid assets into the liquidity substitution bundle without affecting the estimates of the elasticity of substitution between Treasuries and bank deposits.

Appendix B. Data Construction

B.1. Quantities

- Checking deposits: all checking accounts in commercial banks. Data series downloaded from FRED, with identifier “TCDSL”, from 1959-2016. Before 1959, we obtain data from the FDIC historical bank dataset, <https://banks.data.fdic.gov/explore/historical>, by selecting “Choose a report – Commercial Banks – Financial – Deposits – Domestic Demand”.
- Savings deposits: all saving accounts in commercial banks, including money market deposit accounts. Data series downloaded from FRED, with identifier “SAVINGSL”, from 1959-2016. Before 1959, we obtain data from the FDIC historical bank dataset with similar procedures as the checking deposits (select “Deposits – Domestic Savings”).
- Time deposits: all time deposits at banks and thrifts with balances less than \$100,000. Data series downloaded from FRED, with identifier “STDSDL”, from 1959-2016. Before 1959, we obtain data from the FDIC historical bank dataset with similar procedures as the checking deposits (select “Deposits – Domestic Time”).
- Total debt (book value): Treasury debt held by the public (FRED identifier “FYGFD-PUN”) from 1970 to 2016. Before 1970, we use the total debt measure in [Nagel \(2016\)](#), which originally comes from [Bohn \(2008\)](#). Both pre- and post- 1970 measures exclude Social Security Trust Funds holdings.
- Net Tsy (market value): We first calculate the book value as “Total debt’ minus financial-sector holding and Federal Reserve holdings of Treasuries, which leads to a measure of non-bank private sector holding of Treasuries. Then we translate the book value into market values using the market-to-book ratio of all marketable Treasury securities (Data on market and book values are provided by Federal Reserve Bank of Dallas, <https://www.dallasfed.org/research/econdata/govdebt#tab3>).).
- Nominal and real GDP: After 1947, quarterly data are downloaded from FRED with identifiers “GDP” and “GDPC1”. For 1929-1946, yearly data are downloaded from FRED with identifiers “GDPA” and “GDPCA”. Then we concatenate the two data series to get nominal and real GDP from 1929 to 2016.
- M1: a monetary aggregate that includes currency in circulation and checking deposits. Currency in circulation is from FRED with the identifier “CURRCIR”, available from 1917 to 2021 at a monthly frequency.

- KVJ deposits: the financial sector liability as measured by [Krishnamurthy and Vissing-Jorgensen \(2012\)](#). For the pre-1952 period, we use the [Krishnamurthy and Vissing-Jorgensen \(2015\)](#) construction of the “net short-term debt” of the financial sector. This series is the sum across each firm in the financial sector (banks and non-bank) of:

short-term debt liabilities – (short-term debt assets + government supplied liquid assets).

We use the net short-term debt measure and add back the government-supplied liquid assets, which then corresponds to the short-term debt liabilities of the financial sector held by the nonfinancial sector. The pre-1952 series is annual. For the period post-1952, we construct the series at a quarterly frequency using the Financial Accounts of the United States (formerly known as the Flow of Funds). We follow the procedure outlined in [Krishnamurthy and Vissing-Jorgensen \(2015\)](#). The resulting series has slight discrepancies relative to [Krishnamurthy and Vissing-Jorgensen \(2015\)](#), but these discrepancies do not affect our results.

B.2. Rates

- Federal funds rate (FFR): Monthly effective FFR in percentage, with FRED identifier “FEDFUNDS”, from 1954 to 2016. From 1920 to 1954, monthly data is from [Nagel \(2016\)](#).
- Liquidity premium: after 1991, liquidity premium is measured as the yield spread between three-month Repo (collateralized by Treasuries) and three-month Treasuries. Since the original repo series in Bloomberg discontinued in 2016, the liquidity premium series ends in 2016. From 1920 to 1991, liquidity premium is measured as the yield spread between three-month banker’s acceptance and three-month Treasury bills.
- 3 month T-bill rate: secondary-market rate for 3-month T-bill downloaded from FRED, with the identifier “TB3MS”. Data cover 1934 to 2021.
- Deposit rates: we obtain monthly data on commercial bank checking, saving, and small-time interest expenses from bank call reports covering 1986 to 2013. Checking deposit rates are calculated as the commercial bank checking interest expense over total checking volume. Saving and small-time deposit rates are defined in a similar way.
- P1CP: 90-day P1-rated (equivalent to AA-rated) commercial paper rates obtained from FRED. The data have two FRED identifiers: the first one is from 1971 to 1997 with

the identifier “WCP3M”, and the second one is from 1997 to 2021 with the identifier “RIFSPPNAAD90NB”.

- P2CP–P1CP spread: P2CP data are directly from FRED for the post-1998 period, with the identifier “RIFSPPNA2P2D90NB”. Before 1998, the data are from researchers at the Federal Reserve Board.
- Money market funds rate: we obtain the aggregate money-market rate from [Xiao \(2020\)](#) for the period 1987–2017. The original data in that paper are from iMoneyNet and cover almost all MMFs after 1987.

B.3. Plots of Near-Money Quantities and Spreads

In Figure 9, we plot the quantities and spreads used in Section 3. Quantities include KVJ net deposits, MMF, and bank deposits, where KVJ net deposits is defined as KVJ deposits (broadly-defined financial sector short-term liabilities in [Krishnamurthy and Vissing-Jorgensen \(2015\)](#)) minus bank deposits. Both KVJ net deposits and MMF are measures of the shadow banking sector. Spreads include P2CP–P1CP spread, P2CP–MMF spread, P2CP–T-bill spread, and P2CP–deposit spread. The P2CP–deposit spread is a projection of the original spread (P2CP 3 month rate minus average deposit rate) onto the FFR.

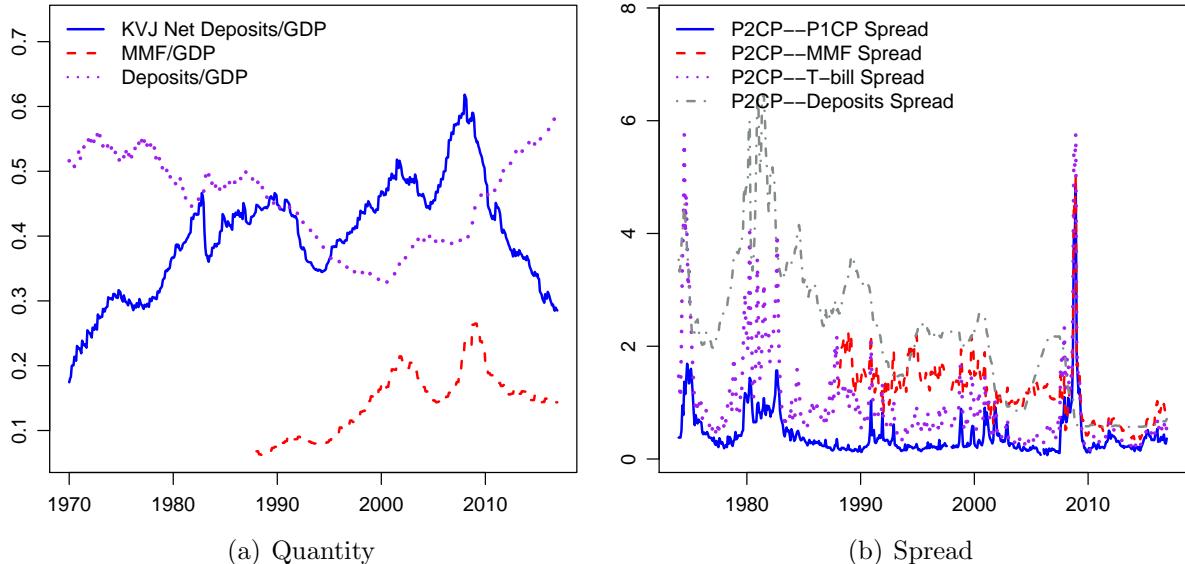


Fig. 9. Near-Money Quantity and Spread Measures.

Appendix C. Robustness Results

This section provides additional results and robustness checks.

C.1. First-Stage Regressions

In Table 12, we report the first stage for the level regressions of Table 2, where we use quarterly data. The F -statistic for both first-stages is well above the standard threshold of 10.

Table 12: First Stage for Level Regressions in Table 2

Dependent variable:		
	(1)	(2)
$\log\left(\frac{\text{Net Tsy}}{\text{Deposits}}\right)$	2.097 (0.319)	1.419 (0.213)
$\log\left(\frac{\text{Total Tsy}}{\text{GDP}}\right)$	-2.619 (0.629)	-0.898 (0.419)
Constant	2.279 (0.558)	0.626 (0.372)
Observations	300	300
R^2	0.309	0.565
F Statistic	66.284	193.034

In Table 13, we show the first stage for difference regressions in Table 3, where we use monthly data. We find that the monthly dummies are strongly related to the changes in T-bill ratios, while the federal funds futures IV (price difference in month $t - 2$ of federal funds futures for month $t - 1$ and t) is strongly related to changes in FFR from $t - 1$ to t .

C.2. Alternative Measures of the Deposits Spread

In our main text, we use the projection of deposit spread on FFR throughout the whole sample. In this subsection, we present our results with an alternative construction where we use the raw deposit spread data whenever data are directly available and the projection on

Table 13: First Stage for Difference Regressions in Table 3

	<i>Dependent variable:</i>				
	$\Delta \log\left(\frac{\text{T-bill}_t}{\text{GDP}_t}\right)$	$\Delta \log\left(\frac{\text{T-bill}_{t-1}}{\text{GDP}_{t-1}}\right)$	$\Delta \log\left(\frac{\text{T-bill}_t}{\text{Deposit}_t}\right)$	$\Delta \log\left(\frac{\text{T-bill}_{t-1}}{\text{Deposit}_{t-1}}\right)$	ΔFFR_t
	(1)	(2)	(3)	(4)	(5)
M1	0.035 (0.012)	0.006 (0.009)	0.025 (0.013)	0.017 (0.009)	-0.006 (0.035)
M2	0.028 (0.012)	0.041 (0.010)	0.018 (0.012)	0.041 (0.010)	0.036 (0.024)
M3	-0.063 (0.013)	0.033 (0.008)	-0.060 (0.013)	0.034 (0.008)	-0.005 (0.021)
M4	0.006 (0.010)	-0.057 (0.011)	-0.004 (0.010)	-0.044 (0.011)	-0.005 (0.025)
M5	-0.005 (0.009)	0.012 (0.008)	-0.015 (0.009)	0.012 (0.008)	0.007 (0.025)
M6	0.018 (0.010)	0.001 (0.006)	0.018 (0.011)	0.001 (0.006)	-0.039 (0.026)
M7	0.036 (0.012)	0.023 (0.008)	0.024 (0.013)	0.034 (0.008)	-0.025 (0.022)
M8	-0.008 (0.011)	0.041 (0.010)	-0.019 (0.011)	0.040 (0.010)	-0.010 (0.029)
M9	0.030 (0.012)	-0.003 (0.011)	0.031 (0.012)	-0.003 (0.011)	-0.083 (0.038)
M10	0.043 (0.012)	0.035 (0.012)	0.034 (0.012)	0.047 (0.011)	-0.034 (0.033)
M11	-0.005 (0.009)	0.049 (0.010)	-0.016 (0.009)	0.050 (0.010)	-0.086 (0.031)
FFfutures_IV	-0.021 (0.015)	-0.018 (0.013)	-0.009 (0.016)	-0.003 (0.012)	1.146 (0.143)
Constant	-0.009 (0.008)	-0.014 (0.005)	-0.003 (0.008)	-0.019 (0.005)	-0.018 (0.023)
Observations	335	336	335	336	335
R ²	0.352	0.352	0.330	0.329	0.403
F Statistic	14.5	14.6	13.2	13.2	18.1

FFR if not. We note that the construction via total deposit expense is much less accurate than the directly observed deposit rate. Here are the details for data construction:

- We use the FDIC historical bank data to construct the historical deposits spread for the period 1934–1986, and the data frequency is yearly. Deposit spread is defined as deposit expense over total deposit volume.
- After 1986, we use the call report data that are at a quarterly frequency. We use the deposits spread for average checking and savings deposits.

Then we redo the same regressions as Table 4 but with different number of lags in the construction of HAC standard errors. Results are shown in Table 14. Panel A uses 4 lags (i.e., 1-year horizon). Panel B uses 12 lags (i.e., 3-year horizon). Panel C uses 40 lags (i.e., 10-year horizon). Across all panels, the estimates of ρ are similar and reconfirm our main results that ρ is slightly above a half and significantly different from 1.

C.3. Varying the Projection of Deposits Spread on FFR before 1986

From 1990s onwards, deposit rate data is directly available. Therefore, the only uncertainty brought by the approximation of deposits spread via FFR is before 1990s. We examine how the projection coefficient δ affects our main results while keeping the post-1990 deposit spread data the same.

We vary the δ from 0.3 to 0.6 (including the actual projection coefficient 0.34) and show how estimates change accordingly in Table 15. We find that as δ increases, the estimated ρ increases, but the p -value and R^2 both decline, indicating a worse approximation. For δ between 0.3 and 0.5, the estimated coefficient is close to the main results in Table 4. For $\delta = 0.6$, the standard error on ρ increased significantly while the p -value drops.

In Figure 10, we show p -values and R^2 as a function of δ . We find that the post-1986 projection coefficient, $\delta = 0.34$, is also close to the best approximation of deposit spread in terms of the model's p -value and R^2 .

C.4. Alternative Measures of the Treasury Liquidity Premium

In this section, we redo the main estimation with alternative liquidity premium measures. We only consider alternative liquidity premium measures with at least 30 years of data to ensure statistical confidence. This leaves us the following choices:

1. CD/T-bill 3M spread, which is the yield spread between 3-month certificates of deposit and 3-month T-bill.

Table 14: GMM Estimations with Concatenated Deposit Spread

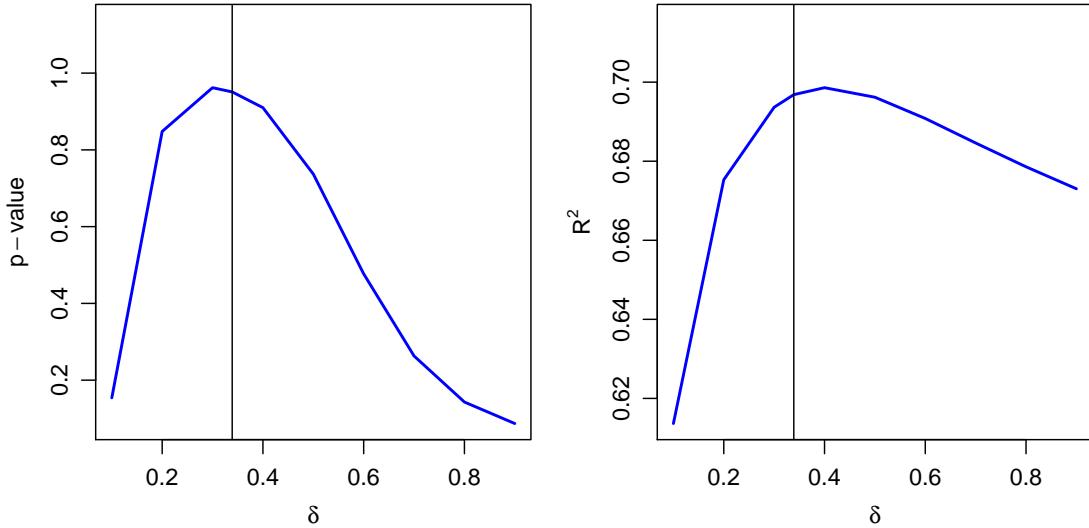
	Measurement of B/D			
	Net Tsy Deposits	Net Tsy KVJ Deposits	Net Tsy Deposits	Net Tsy KVJ Deposits
	(1)	(2)	(3)	(4)
Panel A (HAC standard errors with 4 lags)				
ρ	0.724 (0.100)	0.669 (0.190)	0.638 (0.219)	0.642 (0.214)
β_λ	0.008 (0.001)	0.007 (0.002)	0.007 (0.002)	0.006 (0.002)
p-value of J-test	0.54	0.186	0.219	0.199
Total Treasury IV?	No	No	Yes	Yes
Variations explained	66.3%	66.3%	66.3%	66.4%
Observations	332	332	332	332
Panel B (HAC standard errors with 12 lags)				
ρ	0.727 (0.110)	0.660 (0.167)	0.613 (0.204)	0.616 (0.197)
β_λ	0.008 (0.001)	0.007 (0.002)	0.007 (0.002)	0.006 (0.002)
p-value of J-test	0.626	0.301	0.331	0.286
Total Treasury IV?	No	No	Yes	Yes
Variations explained	66.3%	66.4%	66.3%	66.5%
Observations	332	332	332	332
Panel C (HAC standard errors with 40 lags)				
ρ	0.676 (0.107)	0.578 (0.158)	0.589 (0.220)	0.641 (0.197)
β_λ	0.007 (0.001)	0.005 (0.001)	0.006 (0.002)	0.006 (0.002)
p-value of J-test	0.55	0.164	0.271	0.164
Total Treasury IV?	No	No	Yes	Yes
Variations explained	66.3%	66%	65.9%	65.8%
Observations	332	332	332	332

Notes: Data are at quarterly frequency from 1934 to 2016. This table shows the two-step GMM estimations of parameters ρ and β_λ using the concatenated deposit spread measure. Column (3) and (4) follow model specifications in (25) while column (1) and (2) replace the total Tsy/GDP IV with Tsy/Deposits. Variable definitions are provided in Table 1. HAC standard errors with different lags are shown in parentheses.

Table 15: Adjusting the pre-1986 Deposit Spread Projection

	Projection Coefficient of Deposit Spread on FFR				
	$\delta = 0.3$	$\delta = 0.34$	$\delta = 0.4$	$\delta = 0.5$	$\delta = 0.6$
	(1)	(2)	(3)	(4)	(5)
ρ	0.607 (0.180)	0.661 (0.190)	0.723 (0.202)	0.799 (0.218)	0.855 (0.232)
β_λ	0.012 (0.002)	0.012 (0.002)	0.011 (0.002)	0.010 (0.002)	0.009 (0.002)
p-value of J-test	0.962	0.951	0.91	0.737	0.477
Variations explained	69.4%	69.7%	69.9%	69.6%	69.1%
Observations	332	332	332	332	332

Notes: Data are quarterly frequency from 1934 to 2016. This table shows the GMM estimations of parameters ρ and β_λ when we vary the projection coefficient δ for the period before 1990. We instrument the quantity ratios B_t/D_t (measured as deposits/net Treasury) by total Treasury/GDP and (total Treasury/GDP)². HAC standard errors with 4 lags are reported in parentheses.


 Fig. 10. Model Fit under Different Deposit Spread Projection Coefficients (δ)

2. Note/bill spread, the yield spread between 3-month Treasury notes and bills.
3. GSW/T-bill 3M spread, which is the yield spread between the implied 3-month Treasury yield from the widely-used fitting algorithm in [Gürkaynak, Sack and Wright \(2007\)](#) and

3-month T-bill rate. We follow the same construction as [Lenel, Piazzesi and Schneider \(2019\)](#).

All of the above data are obtained from [Nagel \(2016\)](#). We report the GMM estimation results in Table 16. As shown in the first column, when we use CD/T-bill 3M as the liquidity premium measure, the estimate of ρ is 0.519. When we use note/bill spread (second column), the estimated ρ is 0.706 although the estimation fit is very poor, reflected in the large standard error and low R^2 . In the time series, the note/T-bill spread is more volatile and likely has other non-money demand factors driving its variation. For example, the note/T-bill spread is negative in around 22% of the sample. Finally, the estimate using the GSW/T-bill spread is in the range of our baseline estimates, although the estimation fit is relatively poor, with a smaller fraction of variations being explained.

Table 16: GMM Estimation with Alternative Measures of ℓ_t

Treasury Liquidity Premium Measures			
	CD-Tbill 3M	Note-Bill Spread	GSW-Tbill 3M
	(1)	(2)	(3)
ρ	0.519 (0.105)	0.706 (0.520)	0.449 (0.246)
β_λ	0.014 (0.001)	0.002 (0.001)	0.006 (0.001)
p-value of J-test	0.64	0.148	0.468
Variations explained	58.4%	10.3%	5.4%
Observations	211	144	223

Notes: This table shows the two-step GMM estimations of parameters ρ and β_λ with specification (25). We instrument quantity ratio by total Treasury/GDP and (total Treasury/GDP)². CD-Tsy 3M and Note-Bill spread are obtained from [Nagel \(2016\)](#), where “CD” denotes certificate of deposits. GSW-Tsy 3M is the spread between implied 3-month Treasury yield from the fitting algorithm in [Gürkaynak, Sack and Wright \(2007\)](#) and 3-month T-bill rate. Data are at quarterly frequency from 1961 to 2016, but the sample size can shrink due to data availability. HAC standard errors with 12 lags are reported in parentheses.

C.5. Alternative Measures of λ_t

We next consider other functions and data to proxy for VIX. A priori, there is no reason that the VIX should drive λ in the functional form prescribed by (30). We instead consider

Table 17: GMM Estimation with Alternative Measures of λ_t

Measurement of B/D	Proxy for λ_t					
	VIX		Credit Spread		Bank Leverage	
	Net Tsy Deposits	KVJ Deposits	Net Tsy Deposits	KVJ Deposits	Net Tsy Deposits	KVJ Deposits
	(1)	(2)	(3)	(4)	(5)	(6)
ρ	0.656 (0.187)	0.617 (0.219)	0.626 (0.249)	0.580 (0.289)	0.549 (0.245)	0.541 (0.276)
β_λ	0.009 (0.006)	0.010 (0.007)	0.207 (0.049)	0.174 (0.061)	0.098 (0.041)	0.080 (0.035)
κ	1.085 (0.237)	1.005 (0.250)	0.206 (0.147)	0.193 (0.159)	0.232 (0.142)	0.250 (0.142)
p-value of J-test	0.887	0.795	0.824	0.726	0.73	0.565
Variations explained	69.7%	69.2%	63.5%	63.6%	64.7%	65%
Observations	332	332	332	332	332	332

Notes: This table shows the two-step GMM estimations of parameters ρ , β_λ , and κ . Model specifications is in (25) and (C-1), but we instrument quantity ratio by total Treasury/GDP and (total Treasury/GDP)². Credit spread is the BAA–AAA corporate bond credit spread by Moody. Bank leverage is the inverse of intermediary capital ratio from He, Kelly and Manela (2017), where we extend the measure to before 1970s by projecting it to the daily returns of 49 industrial portfolios from Kenneth French’s website. The sample period is 1934–2016 for column 1,3,5, and 1934–2014 for column 2,4,6. HAC standard errors with 4 lags are reported in parentheses.

a functional form:

$$\frac{\lambda_t}{1 - \lambda_t} = \beta_\lambda (\text{proxy}_t)^\kappa \quad (\text{C-1})$$

where “proxy” here denotes either the VIX index, the BAA–AAA credit spread, which reflects credit risk premium in the corporate sector, or intermediary market leverage, which reflects bank vulnerability and has been widely used in the intermediary asset pricing literature as a factor of the pricing kernel. We construct bank market leverage as 1/bank capital ratio using data from He, Kelly and Manela (2017). These latter two variables capture the idea that the “safety” of deposits may vary over time and influence the relative moneyness of deposits and Treasuries.

Estimation results are presented in Table 17. As shown in column (1), the estimated $\kappa \approx 1$ using VIX, which supports our baseline specification where we implicitly impose

$\kappa = 1$. More importantly, across all of the specifications, the estimated ρ is around 0.6, and the regression R^2 are high and also quite similar across these alternative proxies, indicating that all of these proxies work in a similar fashion.

C.6. Substitution Between T-bills and Bank Deposits

Given that the total amount of Treasurys is mainly driven by longer-maturity bonds, the main result suggests a medium elasticity of substitution between longer-maturity bonds and deposits. To study the substitution between T-bills and bank deposits, in this subsection, we implement an estimation based on T-bill/deposits, instead of Treasuries/deposits. Results are shown in Table 18. We find that ρ estimated using “T-bill Tsy/Deposits” in column (3) is slightly larger than the baseline, while ρ estimated using “Net Tsy/KVJ Deposits” is slightly smaller than the baseline. However, such differences are insignificant.

Table 18: Estimation with T-bills

	Measurement of B/D			
	T-bills Deposits	T-bills KVJ Deposits	T-bills Deposits	T-bills KVJ Deposits
	(1)	(2)	(3)	(4)
ρ	0.457 (0.231)	0.587 (0.400)	0.838 (0.271)	0.322 (0.913)
β_λ	0.008 (0.003)	0.007 (0.006)	0.013 (0.005)	0.004 (0.007)
p-value of J-test	0.071	0.817	0.362	0.344
Total Treasury IV?	No	No	Yes	Yes
Variations explained	68.5%	66.3%	66.5%	66.3%
Observations	280	280	280	280

Notes: This table shows the two-step GMM estimations of parameters ρ and β_λ when we measure B_t as T-bills. Other variable explanations are provided in the notes of Table 1. Data are at a monthly frequency from 1947 to 2017. We instrument $\log(\text{Debt}/\text{Deposits})$ by Total Tsy/GDP and $(\text{Total Tsy}/\text{GDP})^2$ in column (3) and (4). HAC standard errors with 4 lags are reported in parentheses.

C.7. Robustness of Estimation Method

In the main text, we have used two-step GMM to estimate an over-identified system in (25). As robustness checks, we will use different combinations of moment conditions. We

list the individual moment conditions as follows:

$$E[u_t] = 0. \quad (\text{C-2})$$

$$E[u_t \cdot i_t] = 0, \quad (\text{C-3})$$

$$E[u_t \cdot \text{VIX}_t] = 0, \quad (\text{C-4})$$

$$E[u_t \cdot \left(\frac{\frac{\text{Total Tsy}_t}{\text{GDP}_t}}{\left(\frac{\text{Total Tsy}_t}{\text{GDP}_t} \right)^2} \right)] = 0 \quad (\text{C-5})$$

For robustness checks, we will use the same GMM methods, but with subsets of the moment conditions (C-2), (C-3), (C-4), and (C-5). Another method is the generalized least square that finds the best fit of the model for the data, with the objective

$$\min_{\rho, \beta_\lambda} \frac{1}{T} \sum_{t=1}^T \varepsilon_t^2 \quad (\text{C-6})$$

As a result, the first order conditions include

$$\frac{1}{T} \sum_{t=1}^T \varepsilon_t \cdot s_t \cdot \text{VIX}_t \left(\frac{B_t}{D_t} \right)^{\rho-1} = 0 \quad (\text{C-7})$$

$$\frac{1}{T} \sum_{t=1}^T \varepsilon_t \cdot s_t \cdot \text{VIX}_t \left(\frac{B_t}{D_t} \right)^{\rho-1} \log \left(\frac{B_t}{D_t} \right) = 0 \quad (\text{C-8})$$

Results are shown in Table 19. We find that the estimated values are, in general, quite similar to different combinations of the moment conditions in (C-2) to (C-5). The NLS estimations are also close to the GMM estimations in Table 4.

C.8. Additional Lags for the HAC Standard Errors

In Table 20 and 21, we repeat the baseline GMM estimation of Table 4 but with more lags for the HAC standard errors. We report results using 12 lags (3 years) and 40 lags (10 years). The estimates are fairly stable.

C.9. Money-Demand Regressions

In Section 3, we use the time-varying μ_t and λ_t . In this appendix section, we will show that the time-series variations in μ_t and λ_t is not important in the money-demand

Table 19: Robustness Checks with Different Moment Conditions and Estimation Methods

	Estimation Methods							
	GMM1	GMM2	GMM3	GMM4	GMM5	GMM6	GMM7	NLS
ρ	0.79 (0.35)	0.33 (0.67)	0.67 (0.20)	0.73 (0.33)	0.65 (0.18)	0.66 (0.21)	0.66 (0.19)	0.57 (0.19)
β_λ	0.01 (0.005)	0.01 (0.01)	0.01 (0.003)	0.01 (0.004)	0.01 (0.002)	0.01 (0.003)	0.01 (0.002)	0.01 (0.002)
Variation explained	0.69	0.68	0.7	0.69	0.7	0.7	0.7	0.7
p-value	NA	NA	NA	0.6	0.9	0.84	0.95	NA
Observations	332	332	332	332	332	332	332	332

Notes: Column GMM1 to GMM7 shows the estimations of ρ using GMM with different moment conditions. GMM1 uses (C-2) and (C-3). GMM2 uses (C-2) and (C-4). GMM3 uses (C-2) and (C-5). GMM4 uses (C-2), (C-3) and (C-4). GMM5 uses (C-2), (C-3) and (C-5). GMM6 uses (C-3), (C-4) and (C-5). GMM7 uses (C-2), (C-3), (C-4) and (C-5). The last column uses nonlinear least squares (NLS) with objective function (C-6). The data are at quarterly frequency from 1934 to 2016. HAC standard errors with 4 lags are reported in parentheses.

Table 20: GMM Estimation with 12 Lags for HAC Standard Errors

	Measurement of B/D			
	Net Tsy Deposits	Net Tsy KVJ Deposits	Net Tsy Deposits	Net Tsy KVJ Deposits
	(1)	(2)	(3)	(4)
ρ	0.678 (0.109)	0.672 (0.186)	0.646 (0.167)	0.626 (0.187)
β_λ	0.012 (0.001)	0.011 (0.003)	0.012 (0.002)	0.010 (0.003)
p-value of J-test	0.876	0.908	0.93	0.902
Total Treasury IV?	No	No	Yes	Yes
Variations explained	69.6%	68.9%	69.7%	69.1%
Observations	332	332	332	332

Notes: The setting and data are exactly the same as Table 4. HAC standard errors with 12 lags are reported in parentheses.

Table 21: GMM Estimation of 40 Lags for HAC Standard Errors

	Measurement of B/D			
	Net Tsy Deposits	Net Tsy KVJ Deposits	Net Tsy Deposits	Net Tsy KVJ Deposits
	(1)	(2)	(3)	(4)
ρ	0.655 (0.092)	0.638 (0.145)	0.652 (0.139)	0.651 (0.154)
β_λ	0.012 (0.001)	0.010 (0.002)	0.012 (0.002)	0.011 (0.002)
p-value of J-test	0.863	0.879	0.81	0.717
Total Treasury IV?	No	No	Yes	Yes
Variations explained	69.7%	69.1%	69.7%	69%
Observations	332	332	332	332

Notes: The setting and data are exactly the same as Table 4. HAC standard errors with 40 lags are reported in parentheses.

regressions. Then we will show the impact of including currency in circulation into the monetary aggregate.

In Table 22, we replicate the same regressions as in Table 9, but set λ_t and μ_t as constants and equal to their average values implied by β_λ , β_μ , and VIX_t . We find that results are very similar, so even without the time-varying relative liquidity proxies, we are still able to achieve a stable money-demand relationship using our new definitions of liquidity bundles.

To further illustrate this point, in Figure 11, we show the money demand relationships using constant λ_t and μ_t . We find that the plots are very similar to those in Figure 7.

Finally, we consider currency in circulation, which we have thus far omitted but is customary to include in monetary aggregates. We include currency in D_t and otherwise aggregate non-bank debt and Treasury bonds following the same procedure. These new aggregates are denoted as \bar{Q} and \bar{Q}' . Then we regress $\log(\bar{Q})$ and $\log(\bar{Q}')$ on the same regressors as in Table 9. We find that the inclusion of currency stabilizes the demand function further, as is evident in Table 23.

Table 22: Money Demand Regressions with Constant μ_t and λ_t

	Dependent variable:					
	log(m_t /real GDP $_t$)	log(Q_t /real GDP $_t$)	log(Q'_t /real GDP $_t$)	log(m_t /real GDP $_t$)	log(Q_t /real GDP $_t$)	log(Q'_t /real GDP $_t$)
	pre 1980 (1)	post 1980 (2)	pre 1980 (3)	post 1980 (4)	pre 1980 (5)	post 1980 (6)
log($\frac{\delta i_t}{1+i_t}$)	-0.299 (0.057)	-0.086 (0.055)	-0.240 (0.091)	-0.125 (0.016)	-0.034 (0.092)	-0.054 (0.035)
log(real GDP $_t$)	-0.200 (0.095)	-0.509 (0.253)	0.155 (0.084)	-0.117 (0.189)	0.054 (0.086)	0.173 (0.233)
log(VIX $_t$)			-0.144 (0.081)	-0.113 (0.065)	-0.002 (0.091)	0.006 (0.032)
Constant	0.254 (0.757)	2.720 (2.397)	-1.907 (0.818)	0.404 (1.660)	-1.687 (0.862)	-2.657 (2.124)
Observations	184	148	184	148	184	148
R ²	0.888	0.410	0.377	0.747	0.035	0.842

Notes: This table presents the money demand regressions with different definitions of money, where m_t is the real quantity of money including currency and checking deposits, Q_t is the real value of liquidity bundle as in equation (5), and Q'_t is the real value of liquidity bundle as in equation (31). We set λ_t and μ_t as constant and equal to their mean values. Newey-West standard errors with 4 lags are reported in parentheses.

Table 23: Money Demand Regressions (including currency in circulation)

	Dependent variable:					
	$\log(m_t/\text{real GDP}_t)$		$\log(\bar{Q}_t/\text{real GDP}_t)$		$\log(\bar{Q}'_t/\text{real GDP}_t)$	
	pre 1980	post 1980	pre 1980	post 1980	pre 1980	post 1980
	(1)	(2)	(3)	(4)	(5)	(6)
$\log\left(\frac{\text{deposit spread}_t}{1+i_t}\right)$	-0.299 (0.021)	-0.086 (0.010)	-0.108 (0.015)	-0.109 (0.006)	-0.053 (0.013)	-0.086 (0.006)
$\log(\text{real GDP}_t)$	-0.200 (0.028)	-0.509 (0.052)	0.004 (0.023)	-0.283 (0.034)	-0.027 (0.019)	-0.187 (0.031)
$\log(\text{VIX}_t)$			-0.177 (0.029)	-0.136 (0.024)	-0.162 (0.025)	-0.095 (0.022)
Constant	0.254 (0.227)	2.720 (0.480)	-0.185 (0.240)	2.215 (0.322)	-0.015 (0.204)	1.210 (0.294)
Observations	184	148	184	148	184	148
R ²	0.888	0.410	0.505	0.723	0.403	0.676

Notes: This table presents the money demand regressions with different definitions of money, where m_t is the real quantity of money including currency and checking deposits. \bar{Q}_t follows the same construction as equation (5) for Q_t but we incorporate currency in circulation into D_t . Another liquidity aggregate, \bar{Q}'_t , follows the same construction as equation (31) for Q'_t but we include currency in circulation into the construction of D_t . Newey-West standard errors with 4 lags are reported in parentheses.

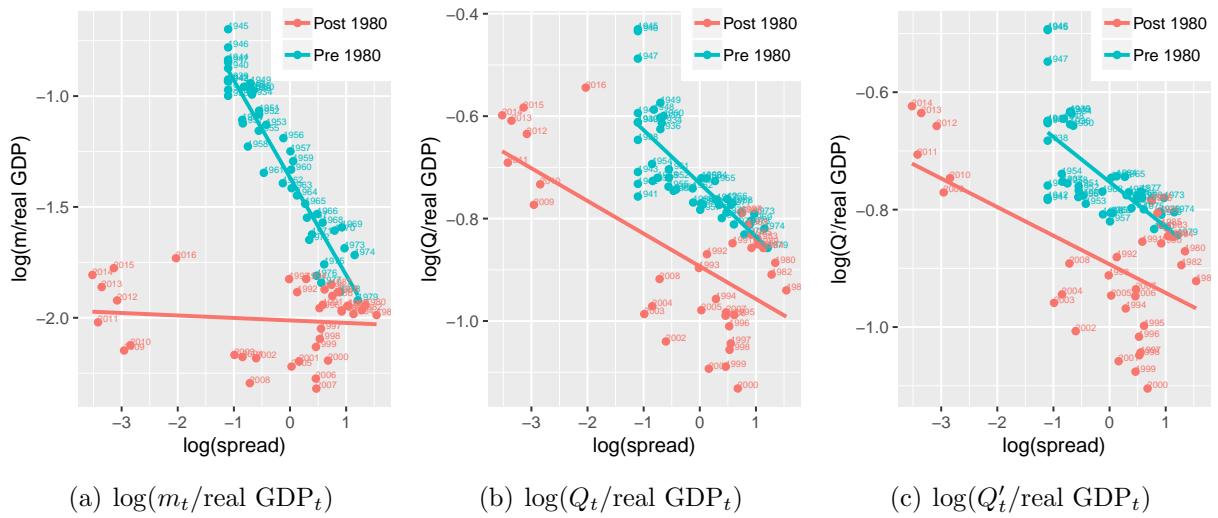


Fig. 11. Quantity of Liquidity and the Opportunity Cost of Holding Liquidity. The x-axis is $\log(\delta i_t/(1 + \text{FFR}_t))$ across all three panels. Data are annual averages from 1934–2016, with the years marked on the figure.