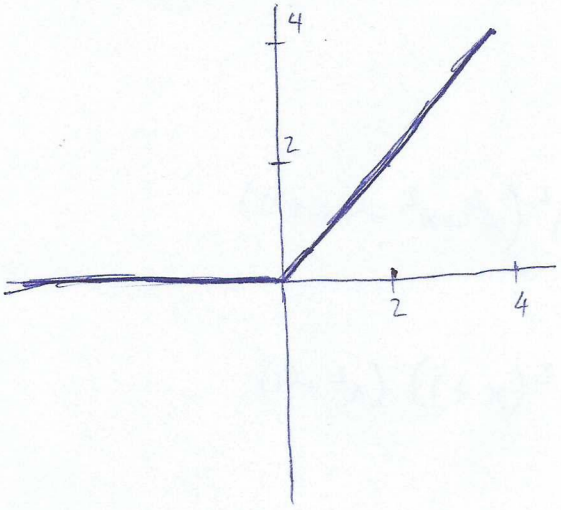
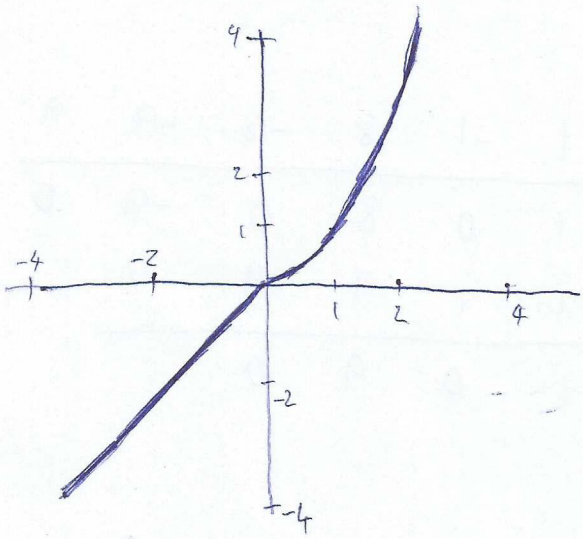
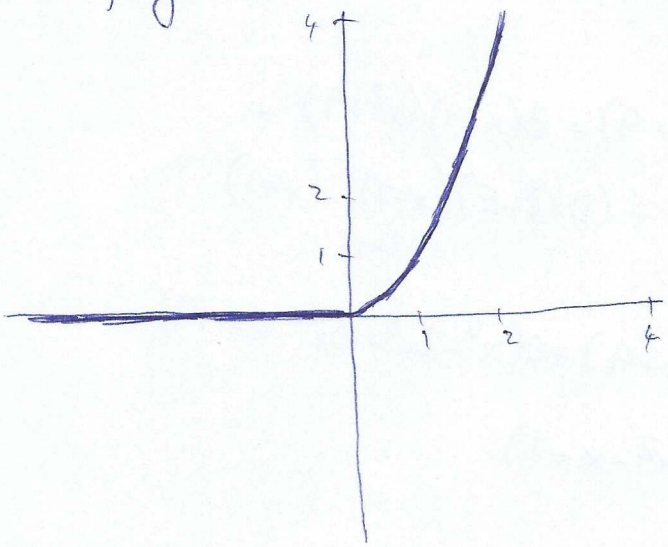
1. $f(x) = \frac{1}{2}(x + |x|)$



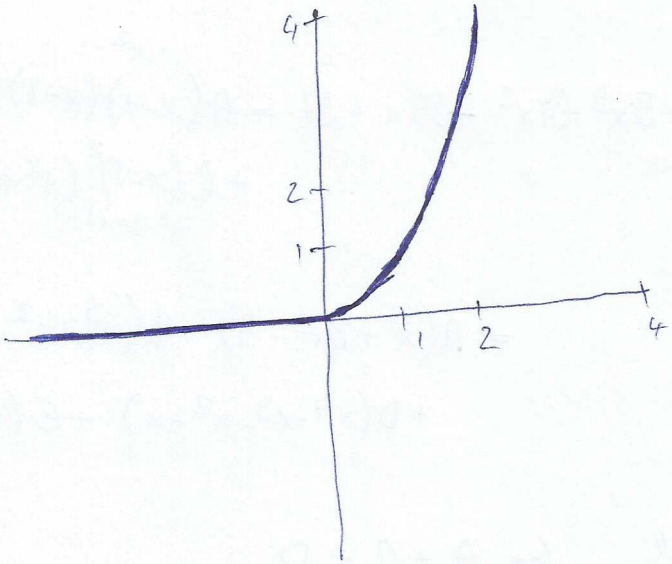
$g(x) = \begin{cases} x & x < 0 \\ x^2 & x \geq 0 \end{cases}$



$(f \circ g)(x)$



$(g \circ f)(x)$



(2)

$$R(x)=\frac{6x^4-5x^3+37x^2-35x+37}{x^5-x^4+8x^3-8x^2-9x+9}$$

	1	-1	8	-8	-9	9
1	1	0	8	0	-9	0
	1	1	9	9	0	
-1	1	0	9	0		

$$(x-1)^2(x^3+x^2+9x+9)$$

$$(x-1)^2(x+1)(x^2+9)$$

$$R(x)=\frac{A}{x-1}+\frac{B}{(x-1)^2}+\frac{C}{x+1}+\frac{Dx+E}{x^2+9}$$

$$6x^4-5x^3+37x^2-35x+37 = A(x-1)(x+1)(x^2+9)+B(x+1)(x^2+9)+C(x-1)^2(x^2+9)+(Dx+E)(x-1)^2(x+1)$$

(x^2-1; x^2-2x+1; x^2-2x+1)

$$=A(x^4+8x^2-9)+B(x^3+x^2+9x+9)+C(x^4-2x^3+10x^2-18x+9)+D(x^4-x^3-x^2+x)+E(x^3-x^2-x+1)$$

x^4: $6=A+C+D$

x^3: $-5=B-2C-D+E$

x^2: $37=8A+B+10C-D-E$

x^1: $35=9B-18C+D+E$

x^0: $37=-9A+9B+9C+E$

$$\begin{pmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & -2 & -1 & 1 \\ 8 & 1 & 10 & -1 & -1 \\ 0 & 9 & -18 & 1 & -1 \\ -9 & 9 & 9 & 0 & 1 \end{pmatrix} x = \begin{pmatrix} 6 \\ -5 \\ 37 \\ 35 \\ 37 \end{pmatrix}$$

$A=1$ $B=2$ $C=3$ $D=2$ $E=1$

$$R(x)=\frac{1}{x-1}+\frac{2}{(x-1)^2}+\frac{3}{x+1}+\frac{2x-1}{x^2+9}$$

③ $f(x) = x^2\left(\frac{\pi}{4} - \operatorname{arctg}\left(\frac{x^2}{x^2-4}\right)\right)$

asymptoty: $x = \pm 2$

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} x^2\left(\frac{\pi}{4} - \operatorname{arctg}\left(\frac{x^2}{x^2-4}\right)\right) = \lim_{x\to 2^-} x^2\left(\frac{\pi}{4} + \frac{\pi}{2}\right) = 3\pi$$

($\frac{x^2}{x^2-4} \to -\infty$)

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} x^2\left(\frac{\pi}{4} - \operatorname{arctg}\left(\frac{x^2}{x^2-4}\right)\right) = \lim_{x\to 2^+} x^2\left(\frac{\pi}{4} - \frac{\pi}{2}\right) = -\pi$$

$$\lim_{x\to -2^-} f(x) = -\pi$$

$$\lim_{x\to -2^+} f(x) = 3\pi$$

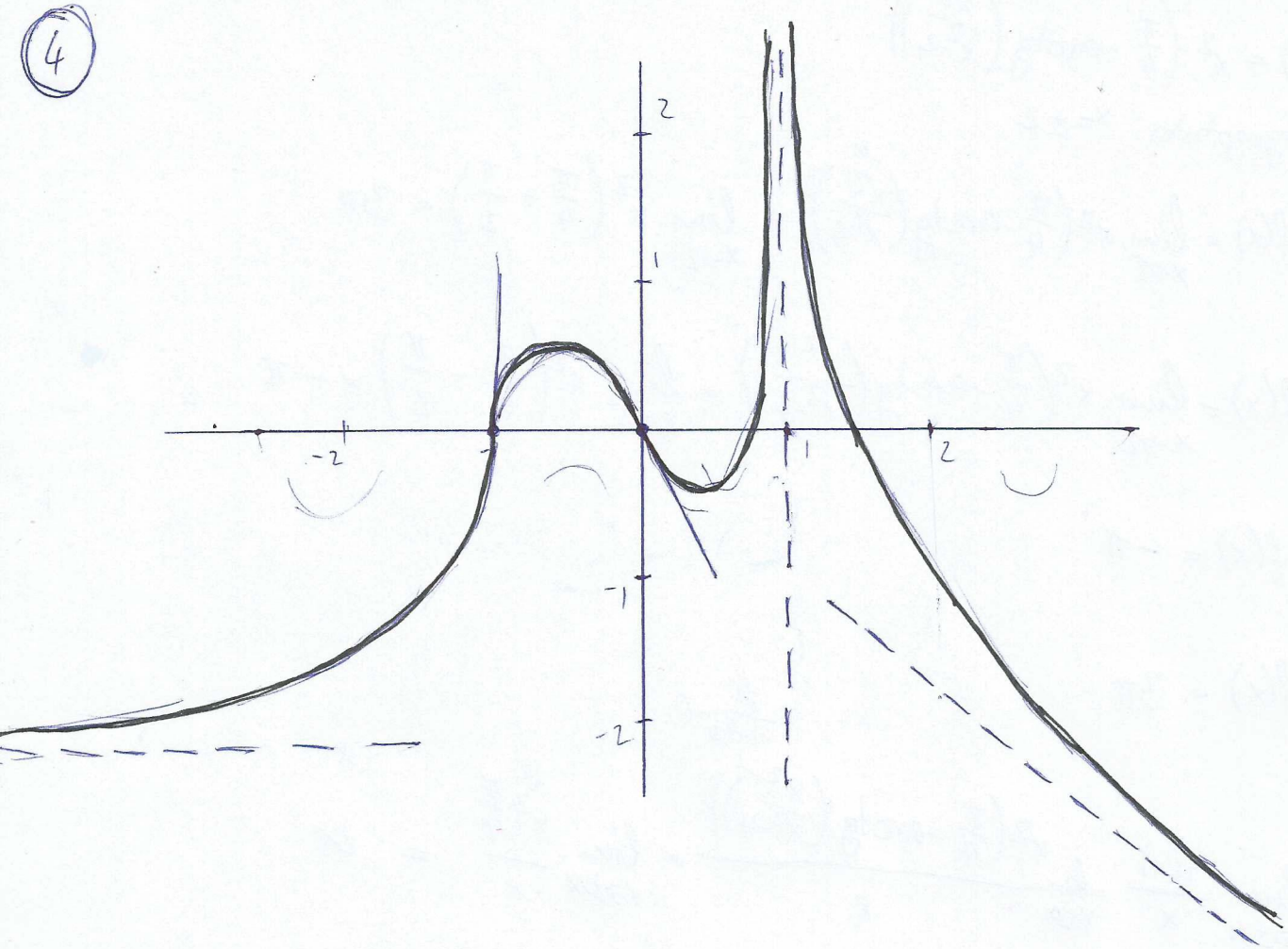
$\frac{x^2}{x^2-4} = \frac{1}{1-\frac{4}{x^2}} \to 0$

$$a = \lim_{x\to\pm\infty} \frac{f(x)}{x} = \lim_{x\to\pm\infty} \frac{x^2\left(\frac{\pi}{4} - \operatorname{arctg}\left(\frac{x^2}{x^2-4}\right)\right)}{x} = \lim_{x\to\pm\infty} \frac{x^2\frac{\pi}{4}}{x} = \infty$$

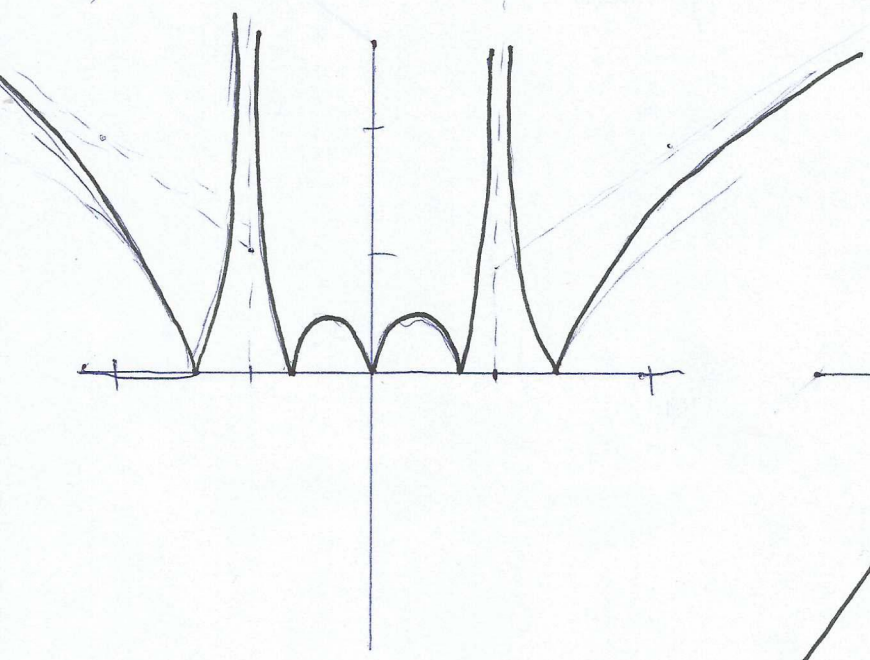
diverguje $\Rightarrow$ nemá v $\pm\infty$ asymptotu

asymptoty: $x = \pm 2$

4



$|f(|x|)|$



$f(-x)$