

KI

Musterlösung 4. Übung

1. Aufgabe

a.

$$\forall Z \exists Y \forall X (f(X, Y) \Leftrightarrow (f(X, Z) \& \sim f(X, X))) \quad (1)$$

$$\forall Z \exists Y \forall X ((f(X, Y) \Rightarrow (f(X, Z) \& \sim f(X, X))) \& ((f(X, Z) \& \sim f(X, X)) \Rightarrow f(X, Y))) \quad (2)$$

$$\forall Z \exists Y \forall X ((\sim f(X, Y) \mid (f(X, Z) \& \sim f(X, X))) \& (\sim (f(X, Z) \& \sim f(X, X)) \mid f(X, Y))) \quad (3)$$

$$\forall Z \exists Y \forall X ((\sim f(X, Y) \mid (f(X, Z) \& \sim f(X, X))) \& ((\sim f(X, Z) \mid f(X, X)) \mid f(X, Y))) \quad (4)$$

$$\exists Y \forall X ((\sim f(X, Y) \mid (f(X, Z) \& \sim f(X, X))) \& ((\sim f(X, Z) \mid f(X, X)) \mid f(X, Y))) \quad (5)$$

$$\forall X ((\sim f(X, y(Z)) \mid (f(X, Z) \& \sim f(X, X))) \& ((\sim f(X, Z) \mid f(X, X)) \mid f(X, y(Z)))) \quad (6)$$

$$((\sim f(X, y(Z)) \mid (f(X, Z) \& \sim f(X, X))) \& ((\sim f(X, Z) \mid f(X, X)) \mid f(X, y(Z)))) \quad (7)$$

$$((\sim f(X, y(Z)) \mid f(X, Z)) \& (\sim f(X, y(Z)) \mid \sim f(X, X))) \& (\sim f(X, Z) \mid f(X, X) \mid f(X, y(Z))) \quad (8)$$

$$\{\sim f(X, y(Z)) \mid f(X, Z), \sim f(X, y(Z)) \mid \sim f(X, X), \sim f(X, Z) \mid f(X, X) \mid f(X, y(Z))\} \quad (9)$$

b.

$$\forall X \forall Y (q(X, Y) \Leftrightarrow \forall Z (f(Z, X) \Leftrightarrow f(Z, Y))) \quad (10)$$

$$\forall X \forall Y (q(X, Y) \Leftrightarrow \forall Z ((f(Z, X) \Rightarrow f(Z, Y)) \& (f(Z, Y) \Rightarrow f(Z, X)))) \quad (11)$$

$$\begin{aligned} \forall X \forall Y ((q(X, Y) \Rightarrow \forall Z ((f(Z, X) \Rightarrow f(Z, Y)) \& (f(Z, Y) \Rightarrow f(Z, X)))) \\ \& (q(X, Y) \Leftarrow \forall Z ((f(Z, X) \Rightarrow f(Z, Y)) \& (f(Z, Y) \Rightarrow f(Z, X))))) \end{aligned} \quad (12)$$

$$\begin{aligned} \forall X \forall Y ((\sim q(X, Y) \mid \forall Z ((\sim f(Z, X) \mid f(Z, Y)) \& (\sim f(Z, Y) \mid f(Z, X)))) \\ \& (q(X, Y) \mid \sim \forall Z ((\sim f(Z, X) \mid f(Z, Y)) \& (\sim f(Z, Y) \mid f(Z, X))))) \end{aligned} \quad (13)$$

$$\begin{aligned} \forall X \forall Y ((\sim q(X, Y) \mid \forall Z ((\sim f(Z, X) \mid f(Z, Y)) \& (\sim f(Z, Y) \mid f(Z, X)))) \\ \& (q(X, Y) \mid \exists Z \sim ((\sim f(Z, X) \mid f(Z, Y)) \& (\sim f(Z, Y) \mid f(Z, X))))) \end{aligned} \quad (14)$$

$$\begin{aligned} \forall X \forall Y ((\sim q(X, Y) \mid \forall Z ((\sim f(Z, X) \mid f(Z, Y)) \& (\sim f(Z, Y) \mid f(Z, X)))) \\ \& (q(X, Y) \mid \exists Z (\sim (\sim f(Z, X) \mid f(Z, Y)) \mid \sim (\sim f(Z, Y) \mid f(Z, X))))) \end{aligned} \quad (15)$$

$$\begin{aligned} \forall X \forall Y ((\sim q(X, Y) \mid \forall Z ((\sim f(Z, X) \mid f(Z, Y)) \& (\sim f(Z, Y) \mid f(Z, X)))) \\ \& (q(X, Y) \mid \exists Z ((f(Z, X) \& \sim f(Z, Y)) \mid (f(Z, Y) \& \sim f(Z, X))))) \end{aligned} \quad (16)$$

$$\begin{aligned} \forall X \forall Y ((\sim q(X, Y) \mid \forall Z ((\sim f(Z, X) \mid f(Z, Y)) \& (\sim f(Z, Y) \mid f(Z, X)))) \\ \& (q(X, Y) \mid \exists A ((f(A, X) \& \sim f(A, Y)) \mid (f(A, Y) \& \sim f(A, X))))) \end{aligned} \quad (17)$$

$$\begin{aligned} (\sim q(X, Y) \mid ((\sim f(Z, X) \mid f(Z, Y)) \& (\sim f(Z, Y) \mid f(Z, X)))) \\ \& (q(X, Y) \mid ((f(a(X, Y), X) \& \sim f(a(X, Y), Y)) \mid (f(a(X, Y), Y) \& \sim f(a(X, Y), X)))) \end{aligned} \quad (18)$$

$$\begin{aligned} ((\sim q(X, Y) \mid \sim f(Z, X) \mid f(Z, Y)) \& (\sim q(X, Y) \mid \sim f(Z, Y) \mid f(Z, X))) \\ \& (q(X, Y) \mid ((f(a(X, Y), X) \mid f(a(X, Y), Y)) \& (\sim f(a(X, Y), Y) \mid f(a(X, Y), Y)) \\ \& (f(a(X, Y), X) \mid \sim f(a(X, Y), X)) \& (\sim f(a(X, Y), Y) \mid \sim f(a(X, Y), X)))) \end{aligned} \quad (19)$$

$$\begin{aligned} (\sim q(X, Y) \mid \sim f(Z, X) \mid f(Z, Y)) \& (\sim q(X, Y) \mid \sim f(Z, Y) \mid f(Z, X)) \\ \& (q(X, Y) \mid f(a(X, Y), X) \mid f(a(X, Y), Y)) \& (q(X, Y) \mid \sim f(a(X, Y), Y) \mid f(a(X, Y), Y)) \\ \& (q(X, Y) \mid f(a(X, Y), X) \mid \sim f(a(X, Y), X)) \& (q(X, Y) \mid \sim f(a(X, Y), Y) \mid \sim f(a(X, Y), X))) \end{aligned} \quad (20)$$

$$\begin{aligned} \{ \sim q(X, Y) \mid \sim f(Z, X) \mid f(Z, Y), \sim q(X, Y) \mid \sim f(Z, Y) \mid f(Z, X) \\ , q(X, Y) \mid f(a(X, Y), X) \mid f(a(X, Y), Y), q(X, Y) \mid \sim f(a(X, Y), Y) \mid f(a(X, Y), Y) \\ , q(X, Y) \mid f(a(X, Y), X) \mid \sim f(a(X, Y), X), q(X, Y) \mid \sim f(a(X, Y), Y) \mid \sim f(a(X, Y), X) \} \end{aligned} \quad (21)$$

c.

$$\forall X \exists Y ((p(X, Y) \Leftarrow \forall X \exists T q(Y, X, T)) \Rightarrow r(Y)) \quad (22)$$

$$\forall X \exists Y (\sim (p(X, Y) \mid \sim \forall X \exists T q(Y, X, T)) \mid r(Y)) \quad (23)$$

$$\forall X \exists Y (\sim (p(X, Y) \mid \exists X \forall T \sim q(Y, X, T)) \mid r(Y)) \quad (24)$$

$$\forall X \exists Y ((\sim p(X, Y) \& \forall X \exists T q(Y, X, T)) \mid r(Y)) \quad (25)$$

$$\forall X \exists Y ((\sim p(X, Y) \& \forall A \exists T q(Y, A, T)) \mid r(Y)) \quad (26)$$

$$(\sim p(X, y(X)) \& q(y(X), A, t(X, A))) \mid r(y(X)) \quad (27)$$

$$(\sim p(X, y(X)) \mid r(y(X))) \& (q(y(X), A, t(X, A)) \mid r(y(X))) \quad (28)$$

$$\{\sim p(X, y(X)) \mid r(y(X)), q(y(X), A, t(X, A)) \mid r(y(X))\} \quad (29)$$

d.

$$\forall X \forall Z (p(X, Z) \Rightarrow \exists Y \sim (q(X, Y) \mid \sim r(Y, Z))) \quad (30)$$

$$\forall X \forall Z (\sim p(X, Z) \mid \exists Y \sim (q(X, Y) \mid \sim r(Y, Z))) \quad (31)$$

$$\forall X \forall Z (\sim p(X, Z) \mid \exists Y (\sim q(X, Y) \& r(Y, Z))) \quad (32)$$

$$\sim p(X, Z) \mid (\sim q(X, y(X, Z)) \& r(y(X, Z), Z)) \quad (33)$$

$$(\sim p(X, Z) \mid \sim q(X, y(X, Z))) \& (\sim p(X, Z) \mid r(y(X, Z), Z)) \quad (34)$$

$$\{\sim p(X, Z) \mid \sim q(X, y(X, Z)), \sim p(X, Z) \mid r(y(X, Z), Z)\} \quad (35)$$

2. Aufgabe

Alle Variablen der **CNF Vorlage** mit `the_kangaroo` (bzw. `k`) ersetzen, da wir Unifikation an dieser Stelle noch nicht hatten. (Oder eben in jedem Schritt unifizieren, aber das kommt auf das gleiche Ergebnis raus.)

1. $in_house(k) \Rightarrow cat(k) \equiv \sim in_house(k) \mid cat(k)$ ($\sim I \mid C_1$)
2. $gaze_at_moon(k) \Rightarrow suitable_pet(k) \equiv \sim gaze_at_moon(k) \mid suitable_pet(k)$ ($\sim G \mid S$)
3. $detest(k) \Rightarrow avoid(k) \equiv \sim detest(k) \mid avoid(k)$ ($\sim D \mid A$)
4. $carnivorous(k) \Rightarrow prowl_at_night(k) \equiv \sim carnivorous(k) \mid prowl_at_night(k)$ ($\sim C_2 \mid P$)
5. $cat(k) \Rightarrow kill_mice(k) \equiv \sim cat(k) \mid kill_mice(k)$ ($\sim C_1 \mid K_1$)
6. $\sim take_to_me(k) \mid in_house(k)$ ($\sim T \mid I$)
7. $kangaroo(k) \Rightarrow \sim suitable_pet(k) \equiv \sim kangaroo(k) \mid \sim suitable_pet(k)$ ($\sim K_2 \mid S$)
8. $kill_mice(k) \Rightarrow carnivorous(k) \equiv \sim kill_mice(k) \mid carnivorous(k)$ ($\sim K_1 \mid C_2$)
9. $\sim take_to_me(k) \Rightarrow detest(k) \equiv take_to_me(k) \mid detest(k)$ ($T \mid D$)
10. $prowl_at_night(k) \Rightarrow gaze_at_moon(k) \equiv \sim prowl_at_night(k) \mid gaze_at_moon(k)$ ($\sim P \mid G$)

Conjecture:

$$kangaroo(k) \Rightarrow avoid(k) \equiv \sim kangaroo(k) \mid avoid(k) \quad (\sim K_2 \mid A)$$

negierte Conjecture:

$$kangaroo(k) \& \sim avoid(k) \quad (K_2 \& \sim A)$$

$$\Delta = \{\sim I \mid C_1, \sim G \mid S, \sim D \mid A, \sim C_2 \mid P, \sim C_1 \mid K_1, \sim T \mid I, \sim K_2 \mid S, \sim K_1 \mid C_2, T \mid D, \sim P \mid G, K_2, \sim A\}$$

$$\Delta = simplify(\Delta, K_2)$$

$$\Delta = \{\sim I \mid C_1, \sim G \mid S, \sim D \mid A, \sim C_2 \mid P, \sim C_1 \mid K_1, \sim T \mid I, S, \sim K_1 \mid C_2, T \mid D, \sim P \mid G, \sim A\}$$

$$\Delta = simplify(\Delta, S)$$

$$\Delta = \{\sim I \mid C_1, \sim G, \sim D \mid A, \sim C_2 \mid P, \sim C_1 \mid K_1, \sim T \mid I, \sim K_1 \mid C_2, T \mid D, \sim P \mid G, \sim A\}$$

$$\Delta = simplify(\Delta, \sim G)$$

$$\Delta = \{\sim I \mid C_1, \sim D \mid A, \sim C_2 \mid P, \sim C_1 \mid K_1, \sim T \mid I, \sim K_1 \mid C_2, T \mid D, \sim P, \sim A\}$$

$$\Delta = simplify(\Delta, \sim P)$$

$$\Delta = \{\sim I \mid C_1, \sim D \mid A, \sim C_2, \sim C_1 \mid K_1, \sim T \mid I, \sim K_1 \mid C_2, T \mid D, \sim A\}$$

$$\Delta = simplify(\Delta, \sim C_2)$$

$$\Delta = \{\sim I \mid C_1, \sim D \mid A, \sim C_1 \mid K_1, \sim T \mid I, \sim K_1, T \mid D, \sim A\}$$

$$\Delta = simplify(\Delta, \sim K_1)$$

$$\Delta = \{\sim I \mid C_1, \sim D \mid A, \sim C_1, \sim T \mid I, T \mid D, \sim A\}$$

$$\Delta = simplify(\Delta, \sim C_1)$$

$$\Delta = \{\sim I, \sim D \mid A, \sim T \mid I, T \mid D, \sim A\}$$

$$\Delta = simplify(\Delta, \sim I)$$

$$\Delta = \{\sim D \mid A, \sim T, T \mid D, \sim A\}$$

$$\Delta = simplify(\Delta, \sim T)$$

$$\Delta = \{\sim D \mid A, D, \sim A\}$$

$$\Delta = simplify(\Delta, D)$$

$$\Delta = \{A, \sim A\}$$

return "unsatisfiable"

3. Aufgabe

Herbrand Universum

$HU = \{$
 $max,$
 $vater_von(max), mutter_von(max),$
 $vater_von(vater_von(max)), vater_von(mutter_von(max)),$
 $mutter_von(vater_von(max)), mutter_von(mutter_von(max)),$
 $vater_von(vater_von(vater_von(max))), vater_von(vater_von(mutter_von(max))),$
 $vater_von(mutter_von(vater_von(max))),$
 $\dots\}$

Herbrand Basis

$HB = \{$
 $verheiratet(max, max),$
 $verheiratet(vater_von(max), max), verheiratet(max, vater_von(max)),$
 $verheiratet(vater_von(max), vater_von(max)),$
 $verheiratet(mutter_von(max), max), verheiratet(max, mutter_von(max)),$
 $verheiratet(mutter_von(max), mutter_von(max)),$
 $verheiratet(vater_von(max), mutter_von(max)), verheiratet(mutter_von(max), vater_von(max)),$
 $verheiratet(vater_von(vater_von(max)), max),$
 $\dots\}$

$\{verheiratet(max, max) \mapsto false,$
 $verheiratet(vater_von(max), max) \mapsto false,$
 $verheiratet(mutter_von(max), max) \mapsto false,$
 $verheiratet(max, vater_von(max)) \mapsto false,$
 $verheiratet(max, mutter_von(max)) \mapsto false,$
 $verheiratet(vater_von(max), mutter_von(max)) \mapsto true,$
 $verheiratet(mutter_von(max), vater_von(max)) \mapsto true,$
 $verheiratet(mutter_von(max), mutter_von(max)) \mapsto false,$
 $verheiratet(vater_von(max), vater_von(max)) \mapsto false,$
 $verheiratet(vater_von(vater_von(max)), max) \mapsto false,$
 $\dots\}$ bildet zusammen mit dem Herbrand Universum und der Identitätsfunktion ein Herbrand Modell.