Künstliche Intelligenz

Hausaufgabe 6

N. Lehmann, T. Gleißner, A. Zubarev

27.05.2015

Aufgabe 1: Modallogik

there are tho agents a and b
ws_i means agent i has a white spot
$\Box_i x$ means agent i knows x
$\Box_{common} x$ means all agents know x

1. At least one agent has a white spot. This implies it is common knowledge that at least one agent has a white spot.

$$\Box_{common}(ws_a \vee ws_b) \tag{1}$$

2. If one agent has a white spot the other agent sees/knows this.

$$\Box_{common}(ws_a \implies \Box_b ws_a) \tag{2}$$
$$\Box_{common}(ws_b \implies \Box_a ws_b) \tag{3}$$

3. If one agent does not have a white spot the other agent sees/knows this.

$$\Box_{common}(\neg ws_a \implies \Box_b \neg ws_a) \tag{4}$$
$$\Box_{common}(\neg ws_b \implies \Box_a \neg ws_b) \tag{5}$$

4. $\Box_{common}$ is modelled as an S4 operator.

$$(\Box_{common} x) \implies x \quad \text{only things that are true can be known} \tag{6}$$
$$(\Box_{common} x) \implies (\Box_{common} \Box_{common} x) \quad \text{if x is commonly known, it is commonly known that x is commonly know} \tag{7}$$

5. $\Box_a, \Box_b$ are modelled as K operators.

$$(\Box_a x) \implies (\Box_a \Box_a x) \quad \text{a knows that a knows that x} \tag{8}$$
$$(\Box_b x) \implies (\Box_b \Box_b x) \quad \text{b knows that b knows that x} \tag{9}$$

6. Connect common knowledge with the knowledge of the agents.

$$(\Box_{common} x) \implies \Box_a x \quad \text{if x is common knowledge then a knows x too} \tag{10}$$
$$(\Box_{common} x) \implies \Box_b x \quad \text{if x is common knowledge then b knows x too} \tag{11}$$

7. When an agent does (not) know something the other agent knows that he does (not) know.

$$(\Box_a x) \implies \Box_b \Box_a x \qquad (\Box_b x) \implies \Box_a \Box_b x \tag{12}$$
$$(\neg\Box_a x) \implies \Box_b \neg\Box_a x \qquad (\neg\Box_b x) \implies \Box_a \neg\Box_b x \tag{13}$$

8. The first agent does not know whether he has a white spot.

$$\neg\Box_a ws_a \tag{14}$$

9. We can now prove: The second agent does know he has a whit spot (or the opposite).

$$\Box_b ws_b \tag{15}$$

Aufgabe 2: Tableau

2a $\Diamond(P \implies \Box P)$ in T

(1) $\neg(\Diamond(P \implies \Box P))$	negation	(1)
(1) $\neg(P \implies \Box P)$	from eq. 1: T rule	(2)
(1) P	from eq. 2: $\neg(\cdot \implies \cdot)$	(3)
(1) $\neg\Box P$	from eq. 2: $\neg(\cdot \implies \cdot)$	(4)
(1.1) $\neg P$	from eq. 4: prefix (1.1) new to b.	(5)
(1.1) $\neg(P \implies \Box P)$	from eq. 1: prefix (1.1) already occurs on path	(6)
(1.1) P	from eq. 6: $\neg(\cdot \implies \cdot)$	(7)
(1.1) $\neg\Box P$	from eq. 6: $\neg(\cdot \implies \cdot)$	(8)
		(9)

The only path is closed by (1.1) P (eq. 7) and (1.1) $\neg P$ (eq. 5).

2b $(\Box P \wedge \Box Q) \Rightarrow (\Box(\Box P \wedge \Box Q))$ in K4

(1) $\neg((\Box P \wedge \Box Q) \Rightarrow (\Box(\Box P \wedge \Box Q)))$		negation	(1)
(1) $\Box P \wedge \Box Q$		from eq. 1: $\neg(\cdot \Rightarrow \cdot)$	(2)
(1) $\neg\Box(\Box P \wedge Q)$		from eq. 1: $\neg(\cdot \Rightarrow \cdot)$	(3)
(1) $\Box P$		fom eq. 2: $\cdot \wedge \cdot$	(4)
(1) $\Box Q$		fom eq. 2: $\cdot \wedge \cdot$	(5)
(1.1) $\neg(\Box P \wedge Q)$		from eq. 3: prefix (1.1) new to b.: $\neg\Box\cdot$	(6)
(1.1) $\neg\Box P$	(1.1) $\neg Q$	from eq 6:$\neg(\cdot \wedge \cdot)$	(7)
(1.1) $\Box P$	(1.1) $\Box Q$	from eq. 4 and 5, 4 rule	(8)
	(1.1) Q	(1) $\Box Q$ already occurs on the branch	(9)

The left path is closed by (1.1) $\neg\Box P$ (eq. 7) and (1.1)$\Box P$ (eq.8).
The right path is closed by (1.1) $\neg Q$ (eq. 7) and (1.1)Q (eq. 9).

2c $\Box(P \implies Q) \vee \Box\neg\Box(\neg Q \implies \neg P)$ in S5

(1) $\neg(\Box(P \implies Q) \vee \Box\neg\Box(\neg Q \implies \neg P))$	negation	(1)
(1.1) $\neg((P \implies Q) \vee \Box\neg\Box(\neg Q \implies \neg P))$	from eq. 1: prefix (1.1) new to b.	(2)
(1.1) $\neg(P \implies Q)$	from eq. 2: $\neg(\cdot \vee \cdot)$	(3)
(1.1) $\neg\Box\neg\Box(\neg Q \implies \neg P)$	from eq. 2: $\neg(\cdot \vee \cdot)$	(4)
(1.1) P	from eq. 3: $\neg(\cdot \vee \cdot)$	(5)
(1.1) $\neg Q$	from eq. 3: $\neg(\cdot \vee \cdot)$	(6)
(1.2) $\neg\neg\Box(\neg Q \implies \neg P)$	from eq. 4: prefix (1.2) new to b.	(7)
(1.2) $\Box(\neg Q \implies \neg P)$	double negation	(8)
(1.1) $\Box(\neg Q \implies \neg P)$	4r rule	(9)
(1) $\Box(\neg Q \implies \neg P)$	4r rule	(10)
(1.1) $\neg Q \implies \neg P$	from eq. 10: $\Box\cdot$	(11)
(1.1) Q	from eq. 11: $\neg\cdot \implies \cdot$	(12)
(1.1) $\neg P$	from eq. 11: $\neg\cdot \implies \cdot$	(13)

The only path is closed by (1.1) P (eq. 5) and (1.1) $\neg P$ (eq. 13).