Introduction to Robotics

1D Point Dynamics, PID Controller, Gradient Descent Search

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Outline

- So far we saw how the motion of joints is related to motions of the rigid bodies of a robot

- We assumed we could command arbitrary joint level trajectories, which would be faithfully executed by the real-world robot

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- So far we saw how the motion of joints is related to motions of the rigid bodies of a robot
- We assumed we could command arbitrary joint level trajectories, which would be faithfully executed by the real-world robot
- Most robots are driven by electrical, pneumatic or hydraulic actuators, which apply torques (or for linear actuators forces)
- The *dynamics* of a robot manipulator describes how the robot moves in response to these actuator forces.

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Dynamics

- The **dynamics** of a system describes how the controls u_t influence the change-of-state of the system

$$x_{t+1} = f(x_t, u_t)$$

  – The notation x_t refers to the *dynamic state* of the system: e.g., joint positions *and velocities* $x_t = (q_t, \dot{q}_t)$.
  – f is an arbitrary function, often smooth

- We define a **nonholonomic system** as one with **differential constraints**:

$$\dim(u_t) < \dim(x_t)$$

$\Rightarrow$ *Not all degrees of freedom are directly controllable*

Toussaint

- Examples:
  – An air plane flying: You cannot command it to hold still in the air, or to move straight up.
  – A car: you cannot command it to move side-wards.
  – Your arm: you cannot command it to throw a ball with arbitrary speed (force limits).
  – A *torque controlled* robot: You cannot command it to apply arbitrary torques.

- What all examples have in comment:
  – One can set **controls** u_t (air plane's control stick, car's steering wheel, your muscles activations, torque/voltage/current send to a robot's motors)
  – But these controls only indirectly influence the **dynamics of state**, $x_{t+1} = f(x_t, u_t)$

Toussaint

The Math Behind

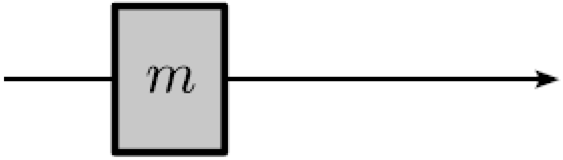
- In general the calculation can be done by summing up all the forces acting on the coupled rigid bodies of the robot

- We shall rely on the Lagrangian to derive the system dynamics, requiring only the potential and kinematic engeries of the system to be computed.

Proportional Control

- The simplest possible example:

- Task: Control a force *u(t)* at time *t* to move a 1-D point mass *m* towards from a given position x towards a certain position x*

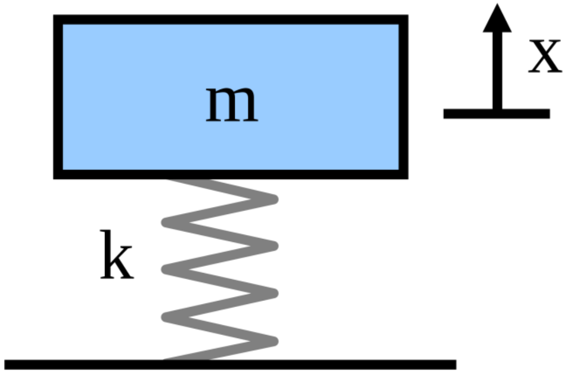


- Proportional Control: the bigger the error *(x* - x)*, the bigger should be *u(t)*

- Physical analogy: Mass-Spring System

Natural Systems

- Mass-Spring System
- Conservative System
- Assume no gravity
- Position and velocity $x\,,\,\dot{x}$



- System has kinetic energy (K) and potential energy (V)

- Lagrangian: *L* = *K* – *V*

$$\frac{d}{dt}\left(\frac{\partial(K-V)}{\partial\dot{x}}\right)-\frac{\partial(K-V)}{\partial x}=0$$

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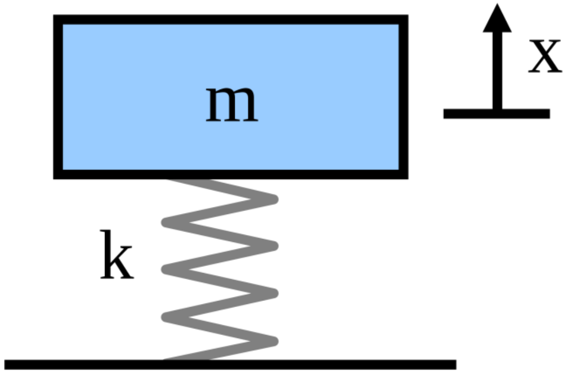
Mass-Spring System



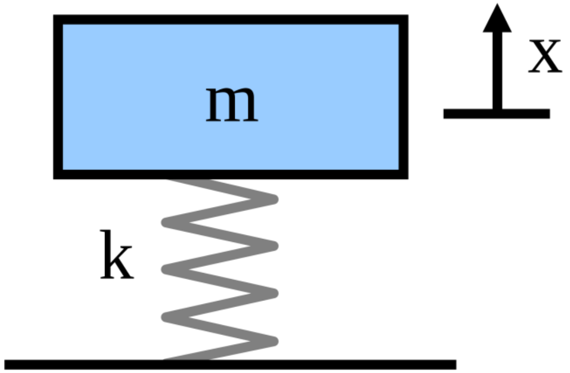
- Pull the spring and let it go: oscillation starts.
- While oscillating, kinetic energy is being transformed to potential energy and back.
- *m* is the mass, $\ddot{x}$ the acceleration
- *k* is spring constant *(F / x)*

$$K = \frac{1}{2} m \dot{x}^2 \qquad \text{Kinectic Energy}$$

$$V = \text{Work} = \int_x^0 (-kx)\mathrm{dx} = \frac{1}{2} k x^2 \qquad \text{Potential Spring Energy}$$

Mass-Spring System

$$\boxed{K = \frac{1}{2} m \dot{x}^2} \qquad \boxed{V = \frac{1}{2} k x^2}$$



$$\frac{d}{dt}\left(\frac{\partial(K-V)}{\partial \dot{x}}\right) - \frac{\partial(K-V)}{\partial x} = 0$$

$$\leftrightarrow$$

$$\frac{d}{dt}\left(\frac{\partial(K)}{\partial \dot{x}}\right) - \frac{\partial(K)}{\partial x} = -\frac{\partial(V)}{\partial x}$$

$$\leftrightarrow$$

$$\leftrightarrow$$

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Mass-Spring System

$$\boxed{K = \frac{1}{2} m\dot{x}^2} \qquad \boxed{V = \frac{1}{2} k x^2}$$

$$\frac{d}{dt}\left(\frac{\partial(K-V)}{\partial \dot{x}}\right) - \frac{\partial(K-V)}{\partial x} = 0$$

$$\leftrightarrow$$

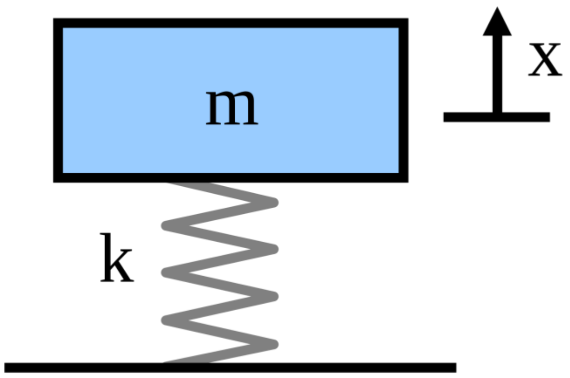
$$\frac{d}{dt}\left(\frac{\partial(K)}{\partial \dot{x}}\right) - \frac{\partial(K)}{\partial x} = -\frac{\partial(V)}{\partial x}$$

$$\leftrightarrow$$

$$m\ddot{x} = ma = F = -kx$$

$$\leftrightarrow$$

$$\boxed{m\ddot{x} + kx = 0}$$

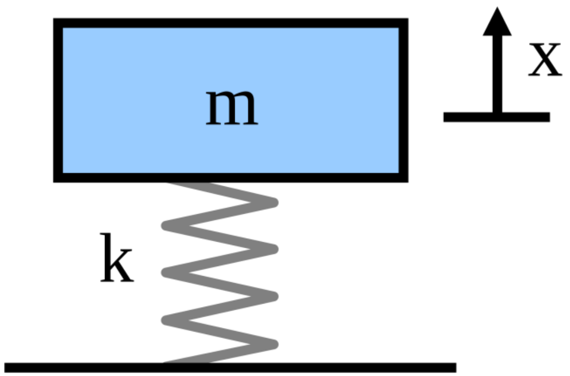


Mass-Spring System

- Conservative Systems

$$\boxed{m\ddot{x} + kx = 0} \quad \leftrightarrow \quad \ddot{x} + \frac{k}{m}x = 0$$

- What‘s the frequency, given *k* and *m*



Mass-Spring System

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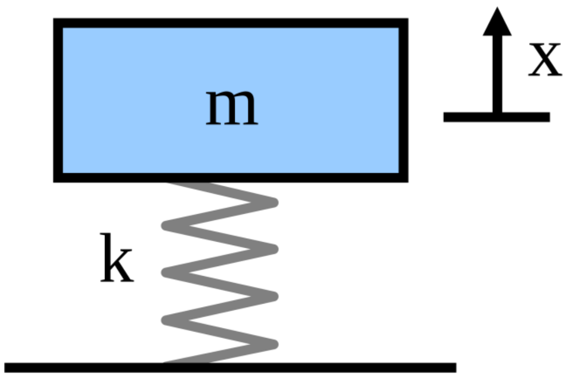


- What's the frequency, given *k* and *m*

$$\text{assume:} \quad x = a + be^{\omega(t+\varphi)}$$

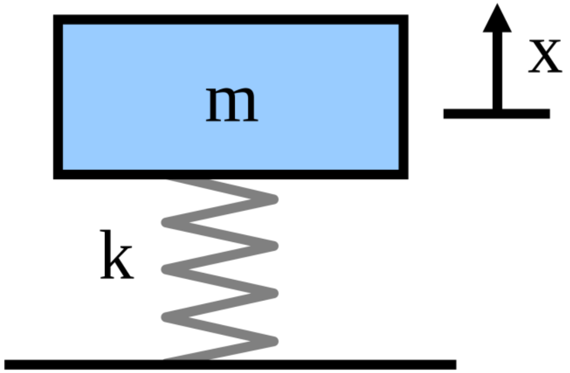
$$\begin{aligned}
\left[a + b\ddot{e}^{\omega(t+\varphi)}\right] + \left[\frac{k}{m}a + be^{\omega(t+\varphi)}\right] &= 0 \\
\omega^2 be^{\omega(t+\varphi)} + \frac{k}{m}be^{\omega(t+\varphi)} + \frac{k}{m}a &= 0 \quad \text{assume: a} = 0 \\
\omega^2 &= -\frac{k}{m}
\end{aligned}$$

Mass-Spring System

$$\omega^2 = -\frac{k}{m}$$

$$\omega = i\sqrt{\frac{k}{m}}$$

Natural Frequency: $\boxed{\omega_n = \sqrt{\frac{k}{m}}}$



Natural Frequency increases with stiffness and inverse mass

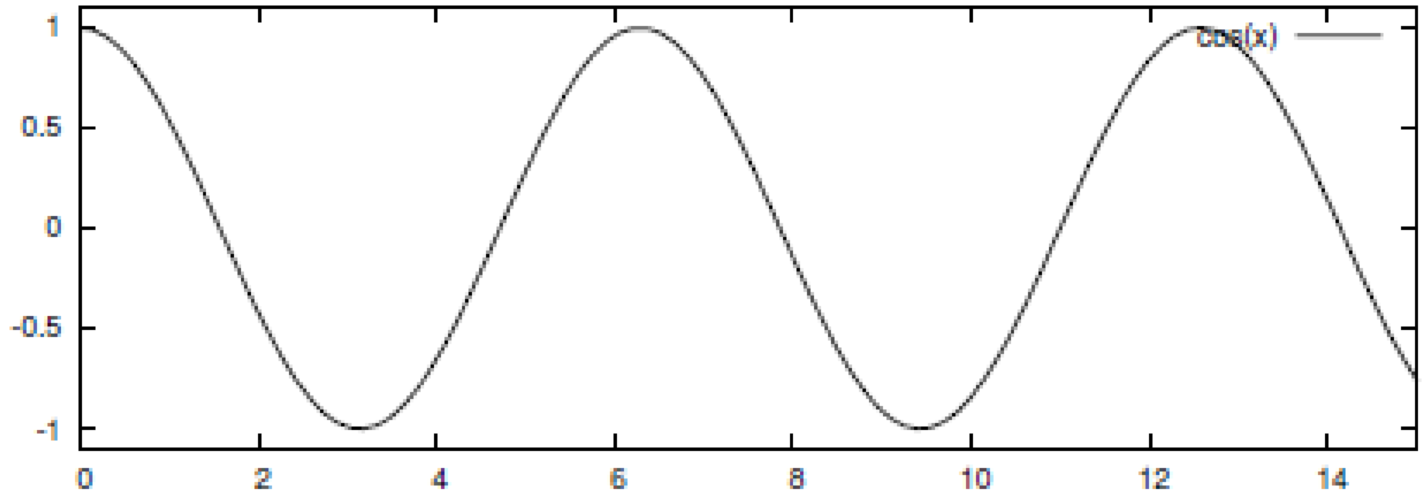
$$x(t) = a + be^{i\sqrt{\frac{k}{m}}(t-\varphi)}$$

$$\text{real part: } x(t) = a + b\cos(\omega_n(t-\varphi))$$

Mass -Spring

$$x(t) \quad = \quad a + b\cos(\omega_n(t-\varphi))$$

- Oscillation around *a* with amplitude *b,* phase shift *φ* and natural frequency: $$\omega_n = \sqrt{\frac{k}{m}}$$



P-Controller

$$
\begin{aligned}
V(x) &= \frac{1}{2}K_P(x_t^* - x_t)^2 \\
f &= -\frac{\partial V}{\partial x} \\
0 &= m\ddot{x} + K_P(x_t^* - x_t)
\end{aligned}
$$

$$u(t) = K_p(x_t^* - x_t)$$

x^* = a (target position)

K_p = Spring parameter

$u(t)$ = resulting output (e.g. force / torque)