习题(29)

29.1 设$X_1, X_2, \cdots, X_n$是来自总体X 的样本,记

$$\overline{X}_n = \frac{1}{n}\cdot\sum_{i=1}^{n} X_i\,,\quad S_n^2 = \frac{1}{n}\cdot\sum_{i=1}^{n}(X_i - \overline{X}_n)^2\,.$$

现增加一个数据X_{n+1},再记

$$\overline{X}_{n+1} = \frac{1}{n+1}\cdot\sum_{i=1}^{n+1} X_i\,,\quad S_{n+1}^2 = \frac{1}{n+1}\cdot\sum_{i=1}^{n+1}(X_i - \overline{X}_{n+1})^2\,.$$

证明下列递推公式:

$$\overline{X}_{n+1} = \overline{X}_n + \frac{1}{n+1}\cdot(X_{n+1} - \overline{X}_n)\,,$$

$$S_{n+1}^2 = \frac{n}{n+1}[S_n^2 + \frac{1}{n+1}\cdot(X_{n+1} - \overline{X}_n)^2]\,.$$

29.2 设有总体X 的 10 个独立观察值:

19.1，20.0，21.2，18.8，19.6，20.5，22.0，21.6，19.4，20.3

试求样本均值$\overline{X}$,样本方差S^2和样本二阶中心矩S_n^2.

29.3 设$X_1, X_2, \cdots, X_n$是来自总体X 的样本,$\overline{X}$ 为样本均值,S^2为样本方差.求证:

$$S^2 = \frac{1}{n-1}\cdot(\sum_{i=1}^{n} X_i^2 - n\cdot\overline{X}^2)\,.$$

29.4 设总体$X \sim b(1, p)$,$X_1, X_2, \cdots, X_n$为X 的样本,$\overline{X}$ 为样本均值,S^2为样本方差,S_n^2为样本二阶中心矩.

1) 求证:$S_n^2 = \overline{X}(1 - \overline{X})$.

2) 求$\sum_{i=1}^{n} X_i^2$ 的分布律;

3) 试求$E(\overline{X})$,$D(\overline{X})$及$E(S^2)$.

习题(29)参考解答

29.1 证: 由 $\overline{X}_{n+1}=\frac{1}{n+1}\cdot\sum_{i=1}^{n+1}X_i$,则

$$\overline{X}_{n+1}=\frac{1}{n+1}\cdot\left(\sum_{i=1}^{n}X_i+X_{n+1}\right)=\frac{n}{n+1}\cdot\overline{X}_n+\frac{1}{n+1}\cdot X_{n+1}$$

$$=\overline{X}_n+\frac{1}{n+1}\cdot(X_{n+1}-\overline{X}_n),$$

且 $\overline{X}_{n+1}-\overline{X}_n=\frac{1}{n+1}\cdot(X_{n+1}-\overline{X}_n)$.则

$$S_{n+1}^2=\frac{1}{n+1}\cdot\sum_{i=1}^{n+1}(X_i-\overline{X}_{n+1})^2=\frac{1}{n+1}\cdot\sum_{i=1}^{n+1}(X_i-\overline{X}_n+\overline{X}_n-\overline{X}_{n+1})^2$$

$$=\frac{1}{n+1}\cdot\left[\sum_{i=1}^{n+1}(X_i-\overline{X}_n)^2+2\sum_{i=1}^{n+1}(X_i-\overline{X}_n)(\overline{X}_n-\overline{X}_{n+1})+\sum_{i=1}^{n+1}(\overline{X}_n-\overline{X}_{n+1})^2\right]$$

$$=\frac{1}{n+1}\cdot[n\cdot S_n^2+(X_{n+1}-\overline{X}_n)^2]+\frac{2}{n+1}\cdot(\overline{X}_n-\overline{X}_{n+1})\cdot\sum_{i=1}^{n+1}(X_i-\overline{X}_n)+(\overline{X}_n-\overline{X}_{n+1})^2$$

$$=\frac{n}{n+1}\cdot S_n^2+\frac{1}{n+1}\cdot(X_{n+1}-\overline{X}_n)^2+\frac{2}{n+1}\cdot(\overline{X}_n-\overline{X}_{n+1})\cdot(X_{n+1}-\overline{X}_n)+(\overline{X}_n-\overline{X}_{n+1})^2$$

$$=\frac{n}{n+1}\cdot S_n^2+\frac{1}{n+1}\cdot(X_{n+1}-\overline{X}_n)^2-\frac{2}{(n+1)^2}\cdot(X_{n+1}-\overline{X}_n)^2+\frac{1}{(n+1)^2}\cdot(X_{n+1}-\overline{X}_n)^2$$

$$=\frac{n}{n+1}\cdot S_n^2+\frac{1}{n+1}\cdot(X_{n+1}-\overline{X}_n)^2-\frac{1}{(n+1)^2}\cdot(X_{n+1}-\overline{X}_n)^2$$

$$=\frac{n}{n+1}\cdot\left[S_n^2+\frac{1}{n+1}\cdot(X_{n+1}-\overline{X}_n)^2\right].$$ ♣

29.2 解: 对于总体 X 的样本 $X_1,X_2,\cdots,X_n$, $n=10$,由

$$\overline{X}=\frac{1}{n}\sum_{i=1}^{n}X_i,\ S^2=\frac{1}{n-1}\cdot\sum_{i=1}^{n}(X_i-\overline{X})^2,\ S_n^2=\frac{1}{n}\cdot\sum_{i=1}^{n}(X_i-\overline{X})^2,$$

计算得: $\overline{X}=20.25$, $S^2=1.165$, $S_n^2=1.0485$. ♣

29.3 证: 由 $\overline{X}=\frac{1}{n}\sum_{i=1}^{n}X_i$, $S^2=\frac{1}{n-1}\cdot\sum_{i=1}^{n}(X_i-\overline{X})^2$,则

$$S^2=\frac{1}{n-1}\cdot\sum_{i=1}^{n}(X_i^2-2X_i\overline{X}+\overline{X}^2)=\frac{1}{n-1}\cdot\left(\sum_{i=1}^{n}X_i^2-2n\overline{X}\cdot\frac{1}{n}\sum_{i=1}^{n}X_i+n\cdot\overline{X}^2\right)$$

$$=\frac{1}{n-1}\cdot\left(\sum_{i=1}^{n}X_i^2-2n\cdot\overline{X}^2+n\cdot\overline{X}^2\right)=\frac{1}{n-1}\cdot\left(\sum_{i=1}^{n}X_i^2-n\cdot\overline{X}^2\right).$$ ♣

29.4 解: 1) 由 $X_i \sim b(1,p)$ $X_i^2 = X_i, i = 1,2,\cdots n$.则

$$\sum_{i=1}^{n} X_i^2 = \sum_{i=1}^{n} X_i ,$$

$$S_n^2 = \frac{1}{n}\cdot\sum_{i=1}^{n}(X_i - \overline{X})^2 = \frac{1}{n}\cdot(\sum_{i=1}^{n} X_i^2 - n\cdot\overline{X}^2) = \frac{1}{n}\cdot\sum_{i=1}^{n} X_i - \overline{X}^2$$

$$= \overline{X} - \overline{X}^2 = \overline{X}(1-\overline{X}) .$$

2) 由 $X_i \sim b(1,p)$ $X_i^2 = X_i, i = 1, 2, \cdots, \cdots n$.又由 $X_1, X_2, \cdots, X_n$ 相互独立,则

$$\sum_{i=1}^{n} X_i^2 = \sum_{i=1}^{n} X_i \sim b(n,p) .$$

3) 已知总体 $X \sim b(1,p)$ $E(X) = p, D(X) = p(1-p)$.则

$$E(\overline{X}) = E(X) = p\ ,\ D(\overline{X}) = \frac{1}{n}\cdot D(X) = \frac{1}{n}\cdot p(1-p) ,$$

$$E(S^2) = D(X) = p(1-p) .$$ ♣