习题(27)

27.1 已知二维随机变量$(X,Y)\sim N(-2,5,4,1,0.7)$,求$D(X+Y)$.

27.2 已知随机向量(X,Y)的协方差矩阵为$\begin{pmatrix}1 & 1\\ 1 & 4\end{pmatrix}$,求$\xi=X-2Y$与$\eta=2X-Y$的相关系数.

27.3 设随机变量X服从参数为λ的指数分布,求它的各阶矩$E(X^n)$,$n=1,2,\cdots$.

27.4 设随机变量X的分布函数$F(x)=\begin{cases}1-(\dfrac{\alpha}{x})^{\beta}, & x>\alpha\\ 0, & x\le\alpha\end{cases}$,其中$\alpha>0$,$\beta>0$.对于$0<p<\beta$,试求$E(X^p)$.

习题(27)参考解答

27.1 解: 由$D(X)=4$,$D(Y)=1$,$\rho_{XY}=0.7$,则

$$\begin{aligned}D(X+Y)&=D(X)+D(Y)+2\cdot\operatorname{cov}(X,Y)\\&=D(X)+D(Y)+2\cdot\rho_{XY}\cdot\sqrt{D(X)}\cdot\sqrt{D(Y)}\\&=4+1+2\times0.7\times2\times1=7.8.\end{aligned}$$

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27.2 解法1: 已知$D(X)=1$,$D(Y)=4$,$\operatorname{cov}(X,Y)=1$,则

$$D(\xi)=D(X-2Y)=D(X)+4\cdot D(Y)-4\cdot\operatorname{cov}(X,Y)=1+4\times4-4\times1=13,$$

$$D(\eta)=D(2X-Y)=4\cdot D(X)+D(Y)-4\cdot\operatorname{cov}(X,Y)=4\times1+4-4\times1=4,$$

$$\begin{aligned}\operatorname{cov}(\xi,\eta)&=\operatorname{cov}(X-2Y,2X-Y)\\&=2\operatorname{cov}(X,X)+2\operatorname{cov}(Y,Y)-\operatorname{cov}(X,Y)-4\operatorname{cov}(Y,X)\\&=2D(X)+2D(Y)-5\operatorname{cov}(X,Y)\\&=2+2\times4-5=5\end{aligned}$$

$$\rho_{\xi\eta}=\frac{\operatorname{cov}(\xi,\eta)}{\sqrt{D(\xi)}\cdot\sqrt{D(\eta)}}=\frac{5}{\sqrt{13}\cdot\sqrt{4}}=\frac{5}{26}\cdot\sqrt{13}.$$

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***解法 2**：由 $\xi=(1,-2)\begin{pmatrix}X\\Y\end{pmatrix}$，$\eta=(2,-1)\begin{pmatrix}X\\Y\end{pmatrix}$，且已知 $D\begin{pmatrix}X\\Y\end{pmatrix}=\begin{pmatrix}1&1\\1&4\end{pmatrix}$，则

$$D(\xi)=D((1,-2)\begin{pmatrix}X\\Y\end{pmatrix})=(1,-2)\cdot D\begin{pmatrix}X\\Y\end{pmatrix}\cdot\begin{pmatrix}1\\-2\end{pmatrix}=(1,-2)\cdot\begin{pmatrix}1&1\\1&4\end{pmatrix}\cdot\begin{pmatrix}1\\-2\end{pmatrix}=13,$$

$$D(\eta)=D((2,-1)\begin{pmatrix}X\\Y\end{pmatrix})=(2,-1)\cdot D\begin{pmatrix}X\\Y\end{pmatrix}\cdot\begin{pmatrix}2\\-1\end{pmatrix}=(2,-1)\cdot\begin{pmatrix}1&1\\1&4\end{pmatrix}\cdot\begin{pmatrix}2\\-1\end{pmatrix}=4,$$

$$\operatorname{cov}(\xi,\eta)=\operatorname{cov}((1,-2)\begin{pmatrix}X\\Y\end{pmatrix},\ (2,-1)\begin{pmatrix}X\\Y\end{pmatrix})=(1,-2)\cdot D\begin{pmatrix}X\\Y\end{pmatrix}\cdot\begin{pmatrix}2\\-1\end{pmatrix}$$

$$=(1,-2)\cdot\begin{pmatrix}1&1\\1&4\end{pmatrix}\cdot\begin{pmatrix}2\\-1\end{pmatrix}=5$$

$$\rho_{\xi,\eta}=\frac{\operatorname{cov}(\xi,\eta)}{\sqrt{D(\xi)}\cdot\sqrt{D(\eta)}}=\frac{5}{\sqrt{13}\cdot\sqrt{4}}=\frac{5}{26}\cdot\sqrt{13}.$$ ♣

27.3 解：由 X 的密度函数为 $f(x)=\begin{cases}\lambda\cdot e^{-\lambda x}&,\ x>0\\0&,\ x\le 0\end{cases}$，则

$$E(X^n)=\int_{-\infty}^{+\infty}x^n\cdot f(x)dx=\int_0^{+\infty}x^n\cdot\lambda e^{-\lambda x}dx$$ （令 $\lambda x=t$，得:）

$$=\frac{1}{\lambda^n}\int_0^{+\infty}t^n\cdot e^{-t}dt=\frac{1}{\lambda^n}\cdot\Gamma(n+1)=\frac{n!}{\lambda^n}.$$ ♣

27.4 解：由题意，得 X 的密度函数为 $f(x)=\begin{cases}\dfrac{\beta\cdot\alpha^\beta}{x^{\beta+1}}&,\ x>\alpha\\0&,\ x\le\alpha\end{cases}$，则

$$E(X^p)=\int_{-\infty}^{+\infty}x^p\cdot f(x)dx=\int_\alpha^{+\infty}x^p\cdot\frac{\beta\cdot\alpha^\beta}{x^{\beta+1}}dx=\beta\cdot\alpha^\beta\int_\alpha^{+\infty}x^{p-\beta-1}dx=\frac{\beta}{\beta-p}\cdot\alpha^p.$$ ♣