习题(5)

5.1 口袋中有$2n-1$只白球,$2n$只黑球,一次取出n只球,发现都是同色球,问这种颜色是黑色的概率为多少?

5.2 据以往资料表明,某一3口之家,患某种传染病的概率有以下规律:

$P\{\text{孩子得病}\}=0.6$,

$P\{\text{母亲得病}\mid\text{孩子得病}\}=0.5$,

$P\{\text{父亲得病}\mid\text{母亲及孩子得病}\}=0.4$.

求母亲及孩子得病但父亲未得病的概率.

5.3 设两两相互独立的三事件A,B,C满足条件:$ABC=\phi$,$P(A)=P(B)=P(C)$,且已知

$P(A\cup B\cup C)=9/16$,则$P(A)=$_____.

5.4 今有甲、乙两名射手轮流对同一目标进行射击,甲、乙命中的概率分别为$p_1,\,p_2$,甲先射,谁先命中谁得胜.问甲、乙两人获胜的概率各为多少?

习题(5)参考解答

5.1 解:记事件$A=\{\text{所取}n\text{个球为同一种颜色}\}$,$B=\{\text{所取}n\text{个球全为黑球}\}$,要求:

$P(B\mid A)$?

则

$$P(B\mid A)=\frac{P(AB)}{P(A)}=\frac{\binom{2n}{n}\Big/\binom{4n-1}{n}}{\left[\binom{2n-1}{n}+\binom{2n}{n}\right]\Big/\binom{4n-1}{n}}$$

$$=\frac{\binom{2n}{n}}{\binom{2n-1}{n}+\binom{2n}{n}}=\frac{\dfrac{(2n)!}{n!\times n!}}{\dfrac{(2n-1)!}{n!\times(n-1)!}+\dfrac{(2n)!}{n!\times n!}}=\frac{2}{3}.$$ ♣

5.2 解 记事件$C=\{孩病\}, M=\{母病\}, F=\{父病\}$,要求:$P(CM\overline{F})$？已知

$$P(C)=0.6,\ P(M\mid C)=0.5,\ P(F\mid CM)=0.4 \Rightarrow P(\overline{F}\mid CM)=0.6.$$

则

$$P(CM\overline{F})=P(CM)\cdot P(\overline{F}\mid CM)=P(C)\cdot P(M\mid C)\cdot P(\overline{F}\mid CM)$$
$$=0.6\times0.5\times0.6=0.18.$$

♣

5.3 解:设$P(A)=x$,由

$$\frac{9}{16}=P(A\cup B\cup C)=P(A)+P(B)+P(C)-P(AB)-P(AC)-P(BC)+P(ABC)$$

且事件两两独立,及题中已知条件,则

$$\frac{9}{16}=3x-3x^2 \qquad x^2-x+\frac{3}{16}=0.$$

解得 $x=\frac{1}{4}$,或 $\frac{3}{4}$.

又由$P(A)\le P(A\cup B\cup C)=\frac{9}{16}$ $\quad x=\frac{1}{4}$,则$P(A)=\frac{1}{4}$. ♣

5.4 解:记事件$A_i=\{第i轮甲命中目标\}, B_i=\{第i轮乙命中目标\}, i=1,2,\cdots$.由

$$\{甲获胜\}=A_1\cup\overline{A}_1\overline{B}_1A_2\cup\overline{A}_1\overline{B}_1\overline{A}_2\overline{B}_2A_3\cup\cdots,$$

则

$$P\{甲获胜\}=P(A_1\cup\overline{A}_1\overline{B}_1A_2\cup\overline{A}_1\overline{B}_1\overline{A}_2\overline{B}_2A_3\cup\cdots)$$
$$=P(A_1)+P(\overline{A}_1\overline{B}_1A_2)+P(\overline{A}_1\overline{B}_1\overline{A}_2\overline{B}_2A_3)+\cdots$$
$$=P(A_1)+P(\overline{A}_1)\cdot P(\overline{B}_1)\cdot P(A_2)+P(\overline{A}_1)\cdot P(\overline{B}_1)\cdot P(\overline{A}_2)\cdot P(\overline{B}_2)\cdot P(A_3)+\cdots$$
$$=p_1+(1-p_1)\cdot(1-p_2)\cdot p_1+[(1-p_1)\cdot(1-p_2)]^2\cdot p_1+\cdots$$
$$=\frac{p_1}{1-(1-p_1)\cdot(1-p_2)}=\frac{p_1}{p_1+p_2-p_1\cdot p_2}.$$

由于$\{乙获胜\}=\overline{A}_1B_1\cup\overline{A}_1\overline{B}_1\overline{A}_2B_2\cup\overline{A}_1\overline{B}_1\overline{A}_2\overline{B}_2\overline{A}_3B_3\cup\cdots$,则

$$P\{乙获胜\}=P(\overline{A}_1B_1\cup\overline{A}_1\overline{B}_1\overline{A}_2B_2\cup\overline{A}_1\overline{B}_1\overline{A}_2\overline{B}_2\overline{A}_3B_3\cup\cdots)$$
$$=P(\overline{A}_1B_1)+P(\overline{A}_1\overline{B}_1\overline{A}_2B_2)+P(\overline{A}_1\overline{B}_1\overline{A}_2\overline{B}_2\overline{A}_3B_3)+\cdots$$

$$= (1-p_1)\cdot p_2 + (1-p_1)^2\cdot(1-p_2)\cdot p_2 + (1-p_1)^3\cdot(1-p_2)^2\cdot p_2 + \cdots$$

$$= \frac{(1-p_1)\cdot p_2}{1-(1-p_1)\cdot(1-p_2)} = \frac{(1-p_1)\cdot p_2}{p_1+p_2-p_1\cdot p_2}$$

(或 $= 1 - P\{\text{甲获胜}\} = 1 - \frac{p_1}{p_1+p_2-p_1\cdot p_2} = \frac{(1-p_1)\cdot p_2}{p_1+p_2-p_1\cdot p_2}$). ♣