习题(40)

40.1 对正态总体的数学期望μ进行假设检验,如果在显著性水平$\alpha = 0.05$下,接受原假设$H_0: \mu = \mu_0$.那么在显著性水平$\alpha = 0.01$下,下列结论正确的是 【 】

(A) 必接受H_0. (B) 可能接受也可能拒绝H_0.

(C) 必拒绝H_0. (D) 不接受,也不拒绝H_0.

40.2 设$X_1, X_2, \cdots, X_{16}$是来自总体$N(\mu,4)$的样本,$\overline{X}$为样本均值,μ未知.考虑如下检验问题:

$H_0: \mu = 0$; $H_1: \mu = -1$.

1) 试证相应于下列三个集合为拒绝域的检验方法所犯第一类错误的概率都是0.05:

$V_1 = \{2\overline{X} \le -1.65\}$;

$V_2 = \{1.5 \le 2\overline{X} \le 2.125\}$;

$V_3 = \{2\overline{X} \le -1.96$,或$2\overline{X} \ge 1.96\}$.

2) 通过计算它们犯第二类错误的概率,说明哪个检验方法最好?

40.3 对于正态总体$X \sim N(\mu, \sigma^2)$,σ^2已知,样本为$X_1, X_2, \cdots, X_n$,样本均值为$\overline{X}$,关于假设问题

$H_0: \mu \ge \mu_0$; $H_1: \mu < \mu_0$

在显著性水平α下,我们已指出拒绝H_0的拒绝域:$\dfrac{\overline{X} - \mu_0}{\sigma/\sqrt{n}} < -u_{1-\alpha}$. 试证:

$$P\{\frac{\overline{X} - \mu_0}{\sigma/\sqrt{n}} < -u_{1-\alpha} \mid H_0\text{真}\} \le \alpha.$$

习题(40)参考解答

40.1 解: 在$\alpha = 0.05$下,记拒绝H_0的拒绝域为W_1,对应的接受域为$\overline{W_1}$;在$\alpha = 0.01$下,记拒绝H_0的拒绝域为W_2,对应的接受域为$\overline{W_2}$.则

$$W_2 \subset W_1 \qquad \overline{W_1} \subset \overline{W_2}.$$

由此得本题答案应为(A). ♣

40.2 解： 1) 若H_0为真,由

$$\overline{X} \sim N(0, \frac{4}{16}) \qquad \overline{X} \sim N(0, \frac{1}{4}) \qquad 2\overline{X} \sim N(0, 1).$$

则在H_0为真时,犯第一类错误的概率分别为

$$\alpha_1 = P\{(X_1, X_2, \cdots, X_{16}) \in V_1\} = P\{2\overline{X} \le -1.645\} = \Phi(-1.645)$$

$$= 1 - \Phi(1.645) = 0.05 \text{ (查表得)};$$

$$\alpha_2 = P\{(X_1, X_2, \cdots, X_{16}) \in V_2\} = P\{1.5 \le 2\overline{X} \le 2.125\}$$

$$= \Phi(2.125) - \Phi(1.5) = 0.9832 - 0.9332 = 0.05;$$

$$\alpha_3 = P\{(X_1, X_2, \cdots, X_{16}) \in V_3\} = P\{2\overline{X} \le -1.96, \text{或} 2\overline{X} \ge 1.96\}$$

$$= P\{2\overline{X} \le -1.96\} + P\{2\overline{X} \ge 1.96\} = \Phi(-1.96) + 1 - \Phi(1.96)$$

$$= 2(1 - \Phi(1.96)) = 2(1 - 0.975) = 0.05.$$

2) 若H_1为真,由

$$\overline{X} \sim N(-1, \frac{4}{16}) \qquad u \triangleq 2\overline{X} + 2 \sim N(0, 1).$$

则在H_1为真时,犯第二类错误的概率分别为

$$\beta_1 = P\{(X_1, X_2, \cdots, X_{16}) \notin V_1\} = P\{2\overline{X} > -1.645\} = P\{2\overline{X} + 2 > -1.645 + 2\}$$

$$= P\{u > 0.355\} = 1 - \Phi(0.355) \approx 0.3613 \text{ (查表得)};$$

$$\beta_2 = P\{(X_1, X_2, \cdots, X_{16}) \notin V_2\} = 1 - P\{1.5 + 2 \le 2\overline{X} + 2 \le 2.125 + 2\}$$

$$= 1 - P\{1.5 + 2 \le u \le 2.125 + 2\} = 1 - \Phi(4.125) + \Phi(3.5) \approx 1;$$

$$\beta_3 = P\{(X_1, X_2, \cdots, X_{16}) \notin V_3\} = P\{|2\overline{X}| < 1.96\}$$

$$= P\{-1.96 < 2\overline{X} < 1.96\} = P\{-1.96 + 2 < 2\overline{X} + 2 < 1.96 + 2\}$$

$$= P\{0.04 < u < 3.96\} = \Phi(3.96) - \Phi(0.04)$$

$$\approx 1-0.5160=0.484.$$

由此可见,拒绝域为$V_1=\{2\overline{X}\le -1.645\}$的检验方法最好. ♣

40.3 证: 由$\dfrac{\overline{X}-\mu}{\sigma/\sqrt{n}}\sim N(0,1)$,则当$H_0$为真,即$\mu\ge\mu_0$时,有

$$P\{\frac{\overline{X}-\mu_0}{\sigma/\sqrt{n}}<-u_{1-\alpha}\}=P\{\frac{\overline{X}-\mu}{\sigma/\sqrt{n}}<-u_{1-\alpha}+\frac{\mu_0-\mu}{\sigma/\sqrt{n}}\}$$

$$=\Phi(-u_{1-\alpha}+\frac{\mu_0-\mu}{\sigma/\sqrt{n}})\le\Phi(-u_{1-\alpha})=\Phi(u_\alpha)=\alpha.$$ ♣