$$a_n = n^2 a_{n-1} + n,\quad n \geq 1$$
$$a_0 = 0.$$

$$G(z) = \sum_{n=0}^{\infty} a_n z^n \xlongequal{a_0=0} \sum_{n=1}^{\infty} a_n z^n$$

$$= \sum_{n=1}^{\infty} n^2 a_{n-1} z^n + \sum_{n=1}^{\infty} n \cdot z^n$$

$$= \sum_{n=3}^{\infty} (n^2-3n+2)\, a_{n-1} z^n + \sum_{n=3}^{\infty} 3(n-1)\, a_{n-1} z^n + \sum_{n=3}^{\infty} a_{n-1} z^n$$

$$+ \sum_{n=1,2} n^2 a_{n-1} z^n + \frac{z}{(1-z)^2}$$

由 $(n^2-3n+2)\, a_{n-1} z^n = z^3 (a_{n-1} z^{n-1})''$ $(n \geq 3)$.

$(n-1)\, a_{n-1} z^n = z^2 (a_{n-1} z^{n-1})'$ $(n \geq 2)$.

$$G(z) = z^3 \cdot \sum_{n=3}^{\infty} (a_{n-1} z^{n-1})'' + 3z^2 \sum_{n=3}^{\infty} (a_{n-1} z^{n-1})' + \sum_{n=3}^{\infty} a_{n-1} z^n$$

$$+ a_0 + 4a_1 z^2 + \frac{z}{(1-z)^2}$$

由 $G(z) = a_0 + a_1 z + \sum_{n=3}^{\infty} a_{n-1} \cdot z^{n-1}$

$$\sum_{n=3}^{\infty} a_{n-1} z^{n-1} = G(z) - z.$$

$$G(z) = z^3 G''(z) + 3z^2 (G'(z) - 1) + z \cdot (G(z) - z) + 4z^2 + \frac{z}{(1-z)^2}$$

$$= z^3 G''(z) + 3z^2 G'(z) + z \cdot G(z) + \frac{z}{(1-z)^2}$$

$$\Rightarrow z^3 G''(z) + 3z^2 G'(z) + (z-1)\, G(z) + \frac{z}{(1-z)^2} = 0.$$

初/边值条件: $G(0) = a_0 = 0$

$G'(0) = a_1 = 1$.