Problem Set 3

Due Date: April 20, 2018 in class.
Name: ________________________
Date: ________________________

Instructions

Answer all the questions in the Homework part and hand in your solutions by the due date. You are strongly encouraged to try the exercise part as well.

Homework

1. Find a closed-form formula for a_n, if $a_0 = 1$, $a_l = 2$, $a_2 = 3$ and $a_n = 4a_{n-1} + a_{n-2} - 4a_{n-3}$ for $n > 3$.
2. Find generating functions for the following sequences (1) $1, 2, 3, 4, \cdots$, (2) squares: $1, 2^2, 3^2, 4^2, \cdots$, (3) cubes: $1, 2^3, 3^3, 4^3, \cdots$.
3. pp348, 8-1 (8).
4. pp359, 8-2 (6).
5. pp395, 8-6 (2).

Exercise

1. Let f_n be the n-th Fibonacci number. Prove that (1) $f_n = f_k f_{n-k} + f_{k-1} f_{n-k-1}$ for $k \in \{1, 2, \cdots, n-1\}$; and (2) $\lim_{n\to\infty} \frac{f_n}{f_{n-1}} = \frac{1+\sqrt{5}}{2}$, the golden ratio.
2. Find a closed-form formula for a_n, if $a_0 = 3$, $a_1 = 7$, and $a_n = 6a_{n-1} + 3a_{n-2}$ for $n \geq 2$.
3. Use generating function method and character equation method to solve the following recurrence relations. (1) $a_0 = 1$, and $a_n = 3a_{n-1} + n$ for $n \geq 1$, (2) $a_0 = 4$, $a_1 = 20$ and $a_n = 4a_{n-1} - 4a_{n-2}$ for $n \geq 2$, (3) $a_0 = 8$, and $a_n = 24a_{n-1} - 144$ for $n \geq 1$.
4. Find coefficients of following generating functions, simplify as far as possible. (1) $(\sum_{k=0}^{\infty} 2^k z^k)(\sum_{k=0}^{\infty} 5^k z^k)$ and (2) $(\sum_{k=0}^{\infty} z^k)(\sum_{k=0}^{\infty} k z^k)$.
5. Find out the equation satisfied by the generating function of the sequence $\{a_n\}$, which is defined by the recursive relation $a_n = n^2 a_{n-1} + n$ for $n \geq 2$ and $a_0 = 0$.
6. pp348, 8-1 (7), (9).
7. pp358, 8-2 (1), (3).
8. pp394, 8-6 (1), pp395, 8-6 (5).