

Solving mathematical problems

Chaque vérité que je trouvois étant une règle qui me servoit après à en trouver d'autres [Each truth that I discovered became a rule which then served to discover other truths]. ([René Descartes](#), “[Discours de la Méthode](#)”)

Problem solving, from homework problems to unsolved problems, is certainly an important aspect of mathematics, though definitely [not the only one](#). Later in your research career, you will find that problems are mainly solved by knowledge (of [your own field](#) and [of other fields](#)), experience, [patience](#) and [hard work](#); but for the type of problems one sees in school, college or in mathematics competitions one needs a slightly different set of problem solving skills. I do have a [book on how to solve mathematical problems](#) at this level; in particular, the [first chapter](#) discusses general problem-solving strategies. There are of course several other problem-solving books, such as Polya’s classic “[How to solve it](#)”, which I myself learnt from while competing at the Mathematics Olympiads.

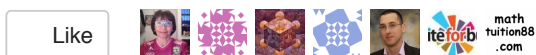
Solving homework problems is an essential component of *really* learning a mathematical subject – it shows that you can “walk the walk” and not just “talk the talk”, and in particular identifies any specific weaknesses you have with the material. It’s worth persisting in trying to understand how to do these problems, and not just for the immediate goal of getting a good grade; if you have a difficulty with the homework which is not resolved, it is likely to cause you further difficulties later in the course, or in subsequent courses.

I find that “playing” with a problem, even after you have solved it, is very helpful for understanding the underlying mechanism of the solution better. For instance, one can try removing some hypotheses, or trying to prove a stronger conclusion. See “[ask yourself dumb questions](#)”.

It’s also best to keep in mind that obtaining a solution is only the short-term goal of solving a mathematical problem. The long-term goal is to increase your understanding of a subject. A good rule of thumb is that if you cannot adequately explain the solution of a problem to a classmate, then you haven’t really understood the solution yourself, and you may need to think about the problem more (for instance, by covering up the solution and trying it again). For related reasons, one should [value partial progress on a problem](#) as being a stepping stone to a complete solution (and also as an important way to deepen one’s understanding of the subject).

See also Eric Schechter’s “[Common errors in undergraduate mathematics](#)”. I also have a post on [problem solving strategies in real analysis](#).

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[29 October, 2008 at 3:31 am](#) Dear Terry

fnasim

I am Farhan Nasim from Bangladesh. Thanks for your advice on *Solving*



mathematical problems.

I have solved a STEP problem. The solution is [here](#). Please make a comment on the solution.

Note: My English is quite poor, you may experience this in the solution.

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[31 December, 2008 at 6:43 am](#)Hi,

Anonymous



Not to be rude, but a translation of Descartes that captures the original poetry of his phrase better might be:

Each truth I discovered was a rule that then served to discover other truths.

[Corrected, thanks – T.]

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[1 January, 2009 at 3:46 pm](#)Dear Professor Tao,

[thomasteepe](#)



here are two articles on the benefits of clever note-taking for math problem solving:

http://www.artofproblemsolving.com/Resources/AoPS_R_A_Mistakes.php

with a strong emphasis on math competitions

and

with a number of ideas on how to use mind maps in math.

Best regards,
Thomas

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[25 January, 2011 at 2:19 pm](#) I've outlined some more ideas math problem solving:

[thomasteepe](#)

The 8 page paper contains a problem solving layout I find very useful personally (and indeed m u c h better than the previous mind map approach), and a densely packed one-page collection of essential problem solving tools.



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[6 January, 2015 at 6:05 am](#) [thomasteepe](#) I've spent more time thinking about combinations of math problem solving and note-making.
Here's the current state of thought:





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[18 January, 2009 at 4:14 am](#) Hi Thomas

[fnasim](#)



Thanks for your useful links. Would you mind reviewing a problem solved by me. Please go [here](#).

Best Regards

F Nasim

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[12 March, 2009 at 8:04 pm](#) Hi dear Professor Tao,

[analgeomatrica](#)



I am very interested in elementary geometry and higher dimension Euclidean geometry, could you please upload chapter 4 in your problem book (I see it is about geometry), thank you very much.

I hope you are interested in elementary geometry, too, nice to meet you here!

Best regards,

Tran Quang Hung.

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[23 March, 2009 at 4:10 am](#) Hi Prof Tao,

Undergrad



As an undergraduate student I often face the problem of deciding how many textbooks problems I should do before moving on, for example, Is ten questions per chapter of Rudin's Principles of Math Analysis adequate? The more problems I do on a specific topic the slower it takes to reach graduate level mathematics. On the other hand, if I just do the homework problems I feel I won't be fast enough for answering questions during exams.

Is there any way of deciding this question?

many many thanks

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[23 March, 2009 at 9:52 am](#) Hi Professor Tao,

Daj

<http://www.mathlinks.ro/viewtopic.php?t=266042>



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[12 October, 2010 at 8:09 am](#)

Depression and problem solving in mathematics: the art of staying upbeat « Republic of Mathematics



[...] attitude to problem solving and to mathematics in general. Terry wrote a helpful blog post "Solving mathematical problems". He recommends playing with a [...]

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[8 January, 2011 at 9:43 pm](#) Dear Prof. Tao:

hezhi gang



I have translated this essay into Chinese, I'm sorry I couldn't translate it well enough, as my ability in English is as poor as mathematics. Your blog is Gospel to those who are interested in Professional mathematics.

译文url: <http://article.yeeyan.org/view/99083/121600>

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[2 March, 2011 at 7:44 pm](#)

Leif



My name is Leif and I wanted to ask you on your opinion on something-forgive me if this seems strange or random. As someone who has apparently fit the profile of "child prodigy" and "genius" (e.g. the fact that you graduated high school and university and earned your doctorate so young), I'm wondering if you think you possess something that only a few others have in terms of intellectual ability or not. I have always believed that if someone applies himself and puts in enough time, effort, concentration, and perseverance they can accomplish whatever they set their mind to. And I greatly appreciated the article on your page where you say that someone does NOT have to be a genius to be a mathematician. Does that mean you don't consider yourself a genius or you don't really see a distinction between yourself and others who apply themselves and are ambitious? I am not saying I don't believe some people are naturally more gifted at certain things or develop stronger skills or have stronger talents than others. I just think it's possible to achieve levels of excellence and brilliance even in your weak areas through consistent

practice. That's why I disagree with this post by astronomer Julianne Dalcanton <http://blogs.discovermagazine.com/cosmicvariance/2007/02/25/the-cult-of-genius/> which i found linked from your page where she doesn't believe most people can reach the level of Feynman-Einstein-Hawking smart. I'm wondering why not? With enough time and effort. After all, they made their share of mistakes and setbacks but did not give up and they were not superhuman or genetically enhanced as far as I know.. And isn't part of their fame due to circumstance and perhaps even chance-not their intellectual ability but I mean their status and the fact that their discoveries happened to be earth-shattering or were given more attention by the public at-large beyond the scientific community? Surely some of the scientists working today will make equally groundbreaking or insightful discoveries or develop innovative theories and thus can fairly be labeled "genius" or as having the same level of smarts?.

I think I have developed a stronger aptitude for language than for math and due to suffering from depression in high school and middle school I didn't push myself nearly as much as I could and lost much of my motivation. However, I don't see why it's not possible for me to develop mathematical abilities as strong as my linguistic abilities or even pursue a career in astronomy (which I love) or physics or even pure mathematics. And I know you say similar things on your career advice blog, and I know it's important to be realistic and plan for graduate school and beyond I just really don't like how people put this label of genius or prodigy on certain people to (in my opinion) make them seem able to achieve things that most other people cannot—even the levels of Einstein or Mozart. And I don't think that's arrogant or unrealistic..but I wanted to get your honest opinion. Thanks in advance for your time.

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[15 March, 2011 at 7:35 pm](#) Hey Leif,

Amateur

This book might be useful in pursuing the answer for your question:

[http://www.amazon.com/Talent-Overrated-Separates-World-Class-](http://www.amazon.com/Talent-Overrated-Separates-World-Class-Performers/dp/1591842247)

[Performers/dp/1591842247](http://www.amazon.com/Talent-Overrated-Separates-World-Class-Performers/dp/1591842247)

Disclaimer: I just read the summary and reviews of that book. I plan to read it and a few related books later. I'm, at the moment, too busy with studying Maths stuff.

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[11 April, 2011 at 3:20 pm](#) Thanks Amateur for the link I appreciate that you replied to me since Mr. Tao still hasn't..

Anonymous

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[23 April, 2011 at 3:01 am](#) Dear Prof. Tao,

Oneminus

I am a high school student,I loved math got good grades in my middle school years.But I find math hard and i often make many mistakes now.In fact ,i think i can work out many problems while doing my homework .But i am very nervous during my math exams and i almost forget everything i have learnt.

Now i still love math but i am afraid of exams.What should i do?

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[3 June, 2011 at 8:11 am](#)

[...] Terence Tao's take on problem-solving

[How to Solve It and some proposed analogies for problem-solving « Boundless Rationality](#)[...]

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[27 September, 2011 at 9:21 pm](#) Dear Professor Tao,

Sabysachi Mukherjee

I am a fifteen year old student currently in high school. I am currently self-studying some non-rigorous calculus. (I have 2 more years in high school).

I am curious to know what I should study next. Should I study some analysis or is it group theory that you recommend? I am confused.

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[30 October, 2011 at 10:46 am](#) Well, after calculus, one usually studies multivariable calculus. Try

Anonymous

[http://ocw.mit.edu/courses/mathematics/18-01sc-single-variable-calculus-fall-](http://ocw.mit.edu/courses/mathematics/18-01sc-single-variable-calculus-fall-2010/)

[2010/](http://ocw.mit.edu/courses/mathematics/18-01sc-single-variable-calculus-fall-2010/) to see if you've learned everything in regular calculus, and then go onto

<http://ocw.mit.edu/courses/mathematics/18-02sc-multivariable-calculus-fall-2010/> .

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[24 December, 2011 at 7:27 pm](#) [...] original: <https://terrytao.wordpress.com/career-advice/solving-mathematical-problems/> [...]

Learning Chinese « tummykung

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[11 December, 2013 at 5:23 pm](#) The link for the book doesn't work.

Tomas Kojar

[Fixed, thanks – T.]

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[2 February, 2014 at 1:17 pm](#) On a somewhat unrelated note: do you engage in light to moderate physical activity

Anonymous

in order to concentrate over extended periods of time?

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[16 April, 2014 at 4:15 am](#) i want more and more suggestions

palak

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[10 October, 2014 at 7:04 am](#) But i dont get time to read all the theories how can i directly solve the problems

Deepak kunar shukla

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[20 October, 2014 at 6:07 pm](#)

[...] Tao did a good job on Solving mathematical problems. This helped me a

Solving mathematical problems | RAMY's Technical Blog lot on my math [...]

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[6 February, 2015 at 5:26 pm](#)

[...] advice on how to solve mathematical problems,

Career Advice by Prof Terence Tao, Mozart of Mathematics | MScMathematics you can try my book on the [...]

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[14 May, 2015 at 5:15 am](#) Dear Dr. Tao, I am an undergraduate studying math. While trying to solve problems

Anonmath

from my text books (like Stein's Complex Analysis), I notice that very often I cannot solve the hardest problems from them. Since research is about hard problems, does that

mean I don't have what it takes to be a mathematician?



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[14 May, 2015 at 10:37 am](#) As you are still several years away from having to attack research-level mathematics problems, your *current* skill in solving such problems is not particularly relevant (much as the calculus-solving skill of, say, a seventh-grader, has much bearing on how good that seventh-grader will be at calculus when he or she encounters it at the college level). The more important consideration is the extent to which your problem-solving skills are *improving* over time. For instance, if after failing to solve a problem, you receive the solution and study it carefully, you may discover an insight or problem-solving technique that eluded you before, and will now be able to solve similar problems that were previously out of reach. One should also bear in mind that being able to partially solve a problem (e.g. to expand out the definitions, solve some special cases, and isolate key difficulties) is also a very important measure of progress (see [this previous post of mine](#) on this topic), as is the practice of constantly asking yourself “dumb” questions in the subject (as discussed in [this post](#)). One should also [not focus on the most difficult questions](#), but rather on [those just outside your current range](#). (More generally, you may find other links on my [career advice page](#) to be of use to you.)



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[14 May, 2015 at 7:12 pm](#) Dear Dr. Tao, how should I select those problems which are just outside my range?
Anonmath Thank you.



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[11 July, 2015 at 5:56 am](#) [...] 按：此篇是对陶哲轩的一篇博客 Solving mathematical problems 的翻译。翻译仅仅为我译：解决数学问题 by 陶哲轩 | 万里风云 的个人兴趣，水平有限，如有纰漏，烦请指正。您不得将其用于商业目的。 [...]

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[27 July, 2015 at 5:36 am](#) Dear sir ,
nabil I learn mathematics as a hobby because I did not have a chance to study college-level mathematics at any good educational institutions.



Do one best learn mathematics which one does not know anything about , say symplectic geometry , best by starting from page 1 of a book on symplectic geometry , and work out all (or most) the proofs and problems as he read. Or is it much more efficient to start from a problem in , say , classical mechanics and work out the mathematical structure of hamiltonian systems on his own from scratch using symplectic topology books as a guide ?

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[13 February, 2016 at 2:32 am](#) I am fond of mathematics and want to solve problems
subodh



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[18 June, 2016 at 5:24 am](#) Have doubts regarding msc mathematics
Sneha Datalu HS



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[12 December, 2016 at 1:27 pm](#) I love this
Anonymous



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[3 January, 2017 at 9:13 am](#)

[For Auld Lang Syne and Cosyne / Global Math Department](#)

[...] on the reasoning than on the procedure for answering a word problem. Terry Tao's quote from this post adds some light to my thinking: "Finding a solution is a short term goal and increasing [...]"

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[20 February, 2017 at 9:54 pm](#)

[Learning in a better way – My Hilbert Space](#)

[...] <https://terrytao.wordpress.com/career-advice/solving-mathematical-problems/> [...]

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[12 March, 2017 at 6:54 am](#)

Maths student

One could also try to use a slightly different notation than the usual one, and solve the problem in this radically new notation. Then perhaps one will work with the pseudo-visuals in the brain, instead of the symbols.



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