# Problem Set 2

Due Date: April 6, 2018 in class.
Name: ____________________
Date: ____________________

## Instructions

Answer all the questions in the Homework part and hand in your solutions by the due date. You are strongly encouraged to try the exercise part as well.

## Homework

1. Let $x, y_1, \cdots, y_n$ be elements in a Boolean algebra, prove the following formula using induction method

$$x + (y_l \cdot y_2 \cdots y_n) = (x + y_1) \cdot (x + y_2) \cdots (x + y_n).$$

2. Count (exactly) the number of critical operations in the following algorithm, and then find good big-$\Theta$ reference functions for the number of critical operations.

(a)
```
i = 1
sum = 0
while i < n
    for j = 1 to i
        sum = sum + a[i,j]
    i = i + 1
```

(b)
```
for i = 1 to n
    for j = 1 to m
        c[i,j] = 0
        for k = 1 to p
            c[i,j] = a[i,k] * b[k,j]
```

(c)
```
product = 1
for i = 1 to n
    sum = 0
    for j = 1 to i
        factorial = 1
        for k = 1 to j
            factorial = k * factorial
        sum = sum + factorial
    product = product*sum
```

## Exercise

1. Let $p \in \mathbb{Z}$. Prove that the following are equivalent.
   (a) $p$ is a prime
   (b) for all $a, b \in \mathbb{Z}$, $p|(ab) \rightarrow [(p|a) \vee (p|b)]$
   (c) for all $d \in \mathbb{Z}$ with $1 < d < p^2$, $(d|p^2) \rightarrow (d = p)$

2. Prove: If $p$ is a prime with $p > 2$, then $p + 1$ is not prime.

3. Prove: If $n$ is an integer and $3|n^2$, then $3|n$.

4. Consider an alternative form of induction that is a compromise between mathematical induction and complete induction. Prove that this new form and mathematical induction and complete induction are all equivalent.

$$P(1) \wedge P(2) \wedge (\forall i,\ P(i) \wedge P(i+1) \rightarrow P(i+2)) \rightarrow (\forall k,\ P(k)).$$

5. Suppose the text consists of $n - 1$ “a”’s, followed by one “b”. Let the pattern be $m - 1$ “a”’s, followed by one “b”. Produce an exact count (in terms of $n$ and $m$) of the number of comparisons required by the obvious algorithm.

6. Build the Boyer-Moore shift table and last table for the following pattern matching (including table construction) for a search for the pattern
   (a) “connecticut”, {1c, e, i, n, o, t, u}
   (b) “mississippi”, {i, m, p, s}
   (c) “hawaii”, {a, h, i, w}