Problem Set 2

Due Date: April 8, 2018 in class.
Name: ____________________
Date: ____________________

Instructions

Answer all the questions in the Homework part and hand in your solutions by the due date. You are strongly encouraged to try the exercise part as well.

Homework

1. Let $x, y_1, \cdots, y_n$ be elements in a Boolean algebra, prove the following formula using induction method

$$x + (y_l \cdot y_2 \cdots y_n) = (x + y_1) \cdot (x + y_2) \cdots (x + y_n).$$

2. Count (exactly) the number of critical operations in the following algorithm, and then find good big-Θ reference functions for the number of critical operations.

(a)
```
i = 1
sum = 0
while i < n
   for j = 1 to i
      sum = sum + a[i,j]
   i = i + 1
```

(b)
```
for i = 1 to n
   for j = 1 to m
      c[i,j] = 0
      for k = 1 to p
         c[i,j] = c[i,j] + a[i,k] * b[k,j]
```

(c)
```
product = 1
for i = 1 to n
   sum = 0
   for j = 1 to i
      factorial = 1
      for k = 1 to j
         factorial = k * factorial
      sum = sum + factorial
   product = product*sum
```

Exercise

1. Let $p \in \mathbb{Z}$. Prove that the following are equivalent.

   (a) p is a prime

   (b) for all $a, b \in \mathbb{Z}$, $p|(ab) \to [(p|a) \vee (p|b)]$

   (c) for all $d \in \mathbb{Z}$ with $1 < d < p^2$, $(d|p^2) \to (d = p)$

2. Prove: If p is a prime with $p > 2$, then $p + 1$ is not prime.

3. Prove: If n is an integer and $3|n^2$, then $3|n$.

4. Consider an alternative form of induction that is a compromise between mathematical induction and complete induction. Prove that this new form and mathematical induction and complete induction are all equivalent.

$$P(1) \wedge P(2) \wedge (\forall i,\ P(i) \wedge P(i+1) \to P(i+2)) \to (\forall k,\ P(k)).$$

5. Suppose the text consists of $n - 1$ "a"'s, followed by one "b". Let the pattern be $m - 1$ "a"'s, followed by one "b". Produce an exact count (in terms of n and m) of the number of comparisons required by the obvious algorithm.

6. Build the Knuth-Morris-Pratt shift table $s[m]$ and the maximal border table $b[m]$ for the following pattern matching for a search for the pattern

   (a) "connecticut",

   (b) "mississippi",

   (c) "cincinnati".

7. In the Boyer-Moore algorithms, the tables $s[m]$ and $b[m]$ are similarly defined. The only difference is, since the comparison starts from the end of the pattern in a right-to-left manner, the suffix takes the place of the prefix. Try to build the Boyer-Moore shift table $s[m]$ and the maximal border table $b[m]$ for the pattern matching in the previous exercise.