FX OPTION - VALUE V.S. INTRINSIC VALUE

Setup

- $X(t)$ - FX rate
- $B(t)$ - Cash account
- $Z(t)$ - Discount curve
- Domestic measure - $\mathbb{Q}^d$, associated numeraire $M(t) = B^d(t)$
$$\mathbb{E}^d\left[\frac{1}{B^d(t)}\right] = Z^d(t)$$
- Foreign measure - $\mathbb{Q}^f$, associated numeraire $N(t) = X(t)B^f(t)$
$$\mathbb{E}^f\left[\frac{1}{B^f(t)}\right] = Z^f(t)$$
- RN-derivative
$$\frac{d\mathbb{Q}^f}{d\mathbb{Q}^d} = \frac{M(0)N(t)}{M(t)N(0)} = \frac{X(t)B^f(t)}{B^d(t)X(0)}$$

Linear

$$\mathbb{E}^d\left[\frac{X(t)}{B^d(t)}\right] = \int_\Omega \frac{X(t)}{B^d(t)}d\mathbb{Q}^d = \int_\Omega \frac{X(0)}{B^f(t)}d\mathbb{Q}^f = X(0)\mathbb{E}^f\left[\frac{1}{B^f(t)}\right] = X(0)Z^f(t)$$

Call option (put option follows the same)

$$\begin{aligned}
\mathbb{E}^d\left[\frac{(X(t)-K)^+}{B^d(t)}\right] &= \mathbb{E}^d\left[\left(\frac{X(t)}{B^d(t)} - \frac{K}{B^d(t)}\right)^+\right] \\
&\geq \left(\mathbb{E}^d\left[\frac{X(t)}{B^d(t)} - \frac{K}{B^d(t)}\right]\right)^+ \\
&= \left(X(0)Z^f(t) - KZ^d(t)\right)^+ \\
&= Z^d(t)\left(\frac{Z^f(t)}{Z^d(t)}X(0) - K\right)^+
\end{aligned}$$

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