

INFLATION INSTRUMENTS: ZERO-COUPON SWAPS AND BONDS

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ABSTRACT. The most common inflation instruments are described.

1. INTRODUCTION

Inflation derivatives pay cash-flows linked to widely-available price indices, e.g. the European HICP x, the US CPI-U and the UK RPI.

The price indices are published on a monthly basis. Usually, they are published two weeks after month end.

The price indices are used in inflation bonds and derivatives with a certain lag. The lag is currently three months in USD, EUR and GBP, and was previously eight months in GBP (for bonds issued before 2005, the last of which expires in 2035).

The price index data are widely available:

US CPI: <http://www.bls.gov/cpi/tables.htm>

Euro HICP: <http://epp.eurostat.ec.europa.eu/portal/page/portal/hicp/data/database>.

UK RPI: <http://www.statistics.gov.uk/statbase/TSDdownload1.asp>

For index-linked Gilts an interesting page is <http://www.dmo.gov.uk/index.aspx?page=Gilts/Indexlinked>. It contains a list of all index-linked Gilts currently in issue and the details the methods used to compute the cash-flows.

2. DERIVATIVES

2.1. Zero-coupon swap. A zero-coupon swap is the exchange of two flows on one given date t_p , which is a certain number of years n after the start date t_s , on a reference notional N , with one flow a fixed amount. The amount is quoted through a compounded annual rate. For a rate R , the amount paid is

$$N((1 + R)^n - X)$$

with $X = 0$ if the notional is exchanged and 1 otherwise. The standard in the derivative market is no exchange of notional.

The inflation flow is given by the change of the price index. The amount paid is

$$N \left(\frac{\text{ReferenceIndex}_{\text{End}}}{\text{ReferenceIndex}_{\text{Start}}} - X \right).$$

The reference indices are linked to the coupon dates t_s and t_p . There is usually a *fixing lag* of several months.

2.1.1. Monthly index. In some cases, the reference index is the price index of a month, i.e.

$$\text{ReferenceIndex}_{\text{End}} = \text{PriceIndex}(r)$$

with r the reference month (first of month) linked to the payment date. The start reference index is computed in the same way using the start date.

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2.1.2. *Interpolated index.* In other cases, the reference index is linearly interpolated between two months. The interpolation is done with the number of days of the *payment* month (not the reference months). The end price index is given by

$$\begin{aligned}\text{ReferenceIndex}_{\text{End}} &= \text{PriceIndex}(r_1) + \left(\frac{d-1}{D}\right) (\text{PriceIndex}(r_2) - \text{PriceIndex}(r_1)) \\ &= \alpha \text{PriceIndex}_1 + (1 - \alpha) \text{PriceIndex}_2.\end{aligned}$$

where d is the day of the month of the payment date and D is the number of calendar days in the payment month. The reference dates are the first of two consecutive months. The start reference index is computed in the same way using the start date.

3. CURVES AND DISCOUNTING

The pricing of simple inflation instruments is done in a similar way to the multi-curve approach used for interest rate instruments. A standard discounting curve is used. The inflated cash-flows are priced using discounted estimated cash-flow with the estimation done with a *price index* curve. As for interest rate instruments, the price index curve is defined as the function for which the *estimate and discount* formulas hold.

The price index is a monthly index. The data for the month are stored on the first of month date.

3.1. **Start of the month index.** The price in t_0 of an inflated flow paying $\text{PriceIndex}_r / \text{ReferenceIndex}_{\text{Start}} - X$ in t_p satisfies

$$(1) \quad \left(\frac{\text{PriceIndex}(t_0, r)}{\text{PriceIndex}_{\text{Start}}} - X \right) P^D(t_0, t_p).$$

This formula defines the price index function.

When the curve is constructed with zero-coupons, the price index can be computed explicitly with

$$(2) \quad \text{PriceIndex}(t_0, r) = \text{PriceIndex}_{\text{Start}}(1 + R)^n.$$

3.2. **Interpolated index.** The price of an inflated flow paying $\alpha \text{PriceIndex}_{r_1} + (1 - \alpha) \text{PriceIndex}_{r_2} / \text{ReferenceIndex}_{\text{Start}} - X$ on t_p is given on t_0 by

$$(3) \quad \left(\frac{\alpha \text{PriceIndex}(t_0, r_1) + (1 - \alpha) \text{PriceIndex}(t_0, r_2)}{\text{PriceIndex}_{\text{Start}}} - X \right) P^D(t_0, t_p).$$

This formula does not fully define the price index function as there are two indices for one function.

When the curve is constructed with zero coupons and no seasonal adjustment are made, one can choose for each of the two months

$$(4) \quad \text{PriceIndex}(t_0, r_i) = \text{PriceIndex}_{s_i}(1 + R)^n.$$

This keeps a certain symmetry in the construction and prices the base instruments correctly.

3.3. **Seasonal adjustment.** The indices are usually adjusted for seasonal (monthly) variations.

The seasonal adjustment we describe here is multiplicative in two ways. The adjustment is a multiplicative factor on the non-adjusted price and the adjustments themselves are stored as a multiplicative factor from one month to the next.

A factor f_i is associated with each month. The factor represents the multiplicative seasonal adjustment from that month to the next. The first month is January. The seasonal adjustment is

$$\text{Adj}(t_0, \text{Month } i + 1) = f_i \text{Adj}(t_0, \text{Month } i).$$

The cumulative seasonal adjustment over the year is 1, i.e.

$$\prod_{i=1}^{12} f_i = 1.$$

The price index is given by

$$\text{PriceIndex}(t, m) = \text{Adj}(m) \text{PriceIndex}_{\text{NoAdj}}(t, m).$$

With that extra constraint on the value of consecutive indices, the curves are constructed by solving the equation

$$(5) \quad \alpha \text{PriceIndex}_{\text{NoAdj}}(r_1) \text{Adj}(r_1) + (1 - \alpha) \text{PriceIndex}_{\text{NoAdj}}(r_2) \text{Adj}(r_2) = \text{PriceIndex}_{\text{Start}}(1 + R)^n.$$

If r_2 is chosen as the node and the value in r_1 is interpolated, it means that we have one equation for each node on the $\text{PriceIndex}_{\text{NoAdj}}$. From there the curve can be constructed with any interpolation scheme.

4. BONDS

The two most common forms of inflation bonds are the *Capital Indexed Bond* and the *Interest Indexed Bond*.

4.1. Capital indexed bond. Those bonds have a fixed real coupon and real notional. The payments are the real coupons and notional multiplied by the ratio of reference indices.

The amounts paid for a notional N and a real coupon C are, at each coupon date,

$$CN \frac{\text{ReferenceIndex}_{\text{EndCoupon}}}{\text{ReferenceIndex}_{\text{StartBond}}}$$

and at maturity

$$N \frac{\text{ReferenceIndex}_{\text{EndBond}}}{\text{ReferenceIndex}_{\text{StartBond}}}.$$

The inflation bonds issued by France, UK and US all are in this category.

Each coupon and the notional are (in pricing terms) like the inflation leg of a zero-coupon swap with notional exchange.

4.2. Interest indexed bond. At each coupon date, the amount paid is the real coupon plus the indexation of the notional over the coupon period, i.e.

$$N \left(C + \frac{\text{ReferenceIndex}_{\text{EndCoupon}}}{\text{ReferenceIndex}_{\text{StartCoupon}}} \right).$$

The notional payment at maturity is not adjusted for inflation.

Each coupon is (in pricing terms) similar to the inflation leg of a year-on-year swap. The final notional is a fixed amount.

4.3. Prices. Prices are quoted with different conventions in different market.

4.3.1. US TIPS. Real clean price. Rounding rules not taken into account.

Quoted price $\text{Price}_{\text{Quoted}}$ for settlement on s with reference index $\text{ReferenceIndex}(s)$.

$$(\text{Price}_{\text{Quoted}} + \text{Accrued}) \text{ReferenceIndex}(s)$$

4.4. Main markets. The main inflation bond markets features are given in Table 1.

Country	Currency	Price Index	Lag	Reference Index
United States	USD	CPI-U	3m	Interpolated
United Kingdom (before 2005)	GBP	RPI	8m	Monthly
United Kingdom (after 2005)	GBP	RPI	3m	Interpolated
France	EUR	Euroland HICP-x	3m	Interpolated
France	EUR	France HICP-x	3m	Interpolated

TABLE 1. Inflation market

5. IMPLEMENTATION

The zero inflation coupon with first-of-month reference index (no interpolation) is in the class `CouponInflationZeroCouponFirstOfMonth`. The zero inflation coupon with reference index interpolated between two month indices is in the class `CouponInflationZeroCouponInterpolation`.

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