Problem 1

1. Mean: 1.0489703904839585
   Variance: 5.427220681881726
   Skewness: 0.8806086425277363
   Kurtosis: 23.12220078998973

2. Mean: 1.0489703904839585
   Variance: 5.427220681881727
   Skewness: 0.8806086425277364
   Kurtosis: 23.122200789989723

3. I use the hypothesis testing with the null hypothesis that the statistical package functions are not biased. Since the p-values of 4 moments are not all greater than alpha (0.05), **we need to reject the hypothesis that the statistical package functions are not biased**.

Problem 2

1.

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.346
Model:                            OLS   Adj. R-squared:                  0.342
Method:                 Least Squares   F-statistic:                     104.6
Date:                Sat, 14 Sep 2024   Prob (F-statistic):           5.59e-20
Time:                        01:26:30   Log-Likelihood:                -284.54
No. Observations:                 200   AIC:                             573.1
Df Residuals:                     198   BIC:                             579.7
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         -0.0874      0.071     -1.222      0.223      -0.228       0.054
x1             0.7753      0.076     10.226      0.000       0.626       0.925
==============================================================================
Omnibus:                       11.922   Durbin-Watson:                   2.023
Prob(Omnibus):                  0.003   Jarque-Bera (JB):               16.685
Skew:                           0.387   Prob(JB):                     0.000238
Kurtosis:                       4.184   Cond. No.                         1.09
==============================================================================
```

MLE Results
MLE Beta: [-0.08738446 0.7752741]
MLE Standard Deviation (Sigma): 1.008813058320225

Comparison of Results:
OLS Beta: [-0.08738446 0.7752741]
OLS Standard Deviation of Errors: 1.008813058320225
Difference in Beta: [0. 0.]
Difference in Standard Deviation (Sigma): 0.0

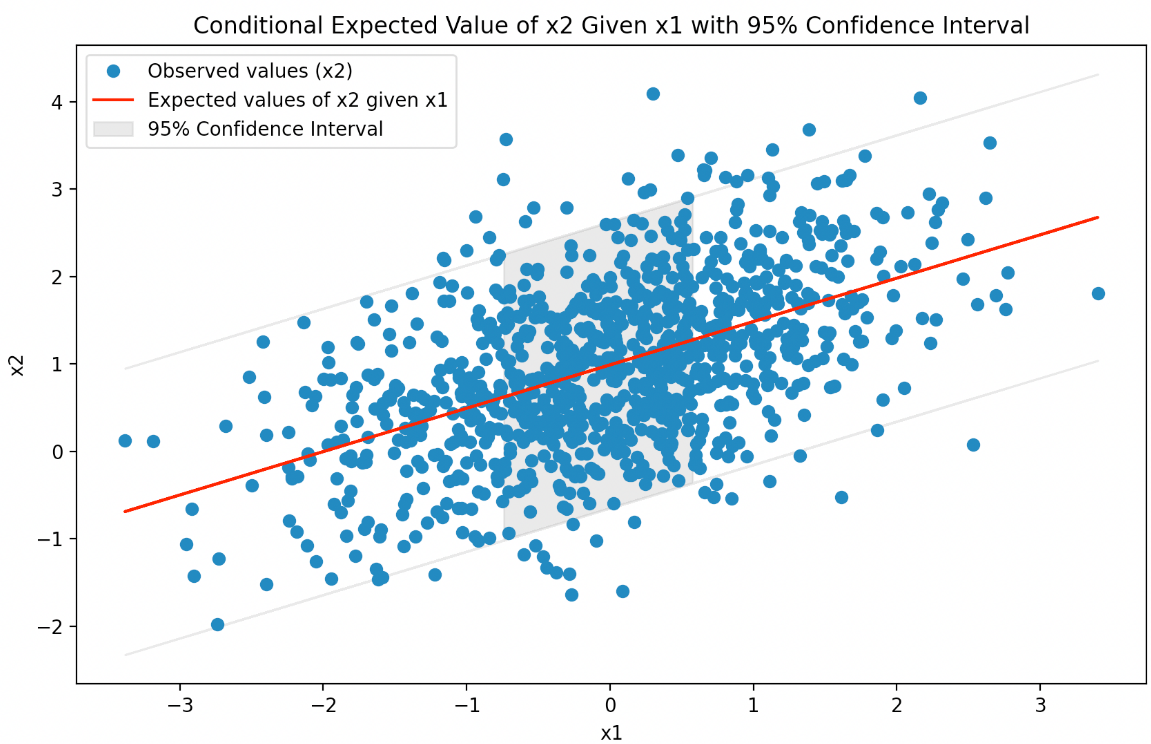
In a linear regression context, with the assumption of normally distributed errors, **the OLS estimators are also the MLE estimators**. This is why there is no difference between the coefficients (Beta) and the estimated standard deviation of errors (Sigma) when comparing results from the OLS and MLE methods.

2.

Comparison of Results
Adjusted R-squared MLE (Normal): 0.3389632986263845
Adjusted R-squared MLE (T-distribution): 0.3379809763066779

I use goodness of fit statistics to decide which model is the best fit. Although the adjusted R-squared for both models is almost the same, **MLE (Normal) would be the best fit in this case since it is slightly greater than MLE (T-distribution)**.

3.



Problem 3

Before comparing the models, we expect that the model with the lowest AIC value is considered the best fit because a lower AIC indicates a model that better balances goodness-of-fit with model complexity.

AR(1) AIC: 1644.6555047688475

AR(2) AIC: 1581.0792659049775

AR(3) AIC: 1436.6598066945892

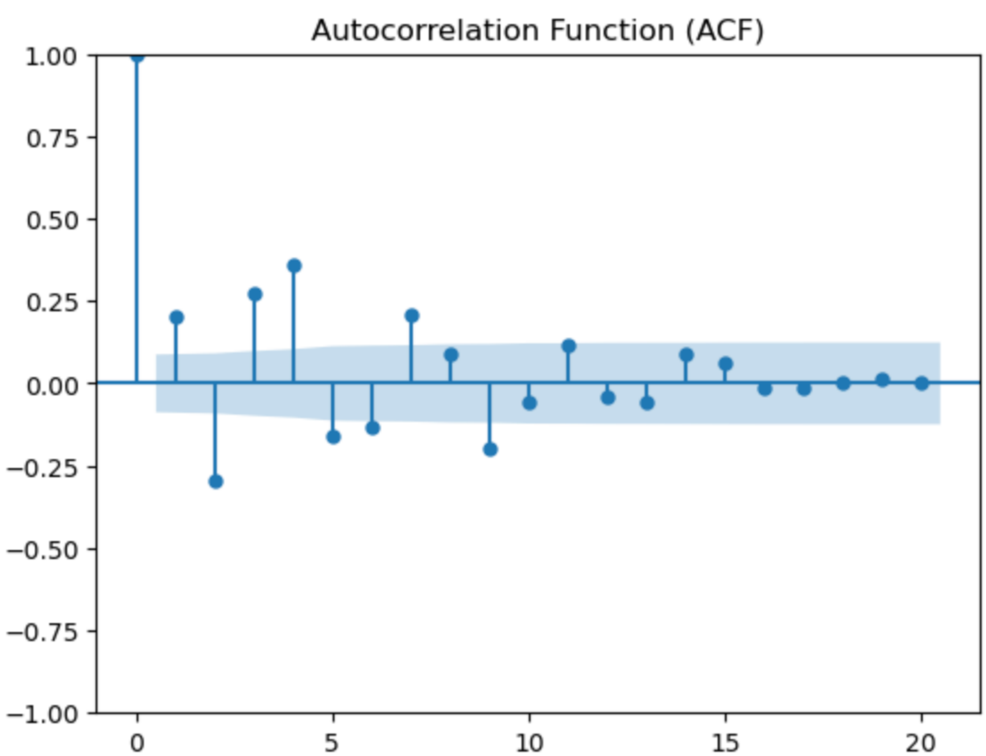
MA(1) AIC: 1567.4036263707874

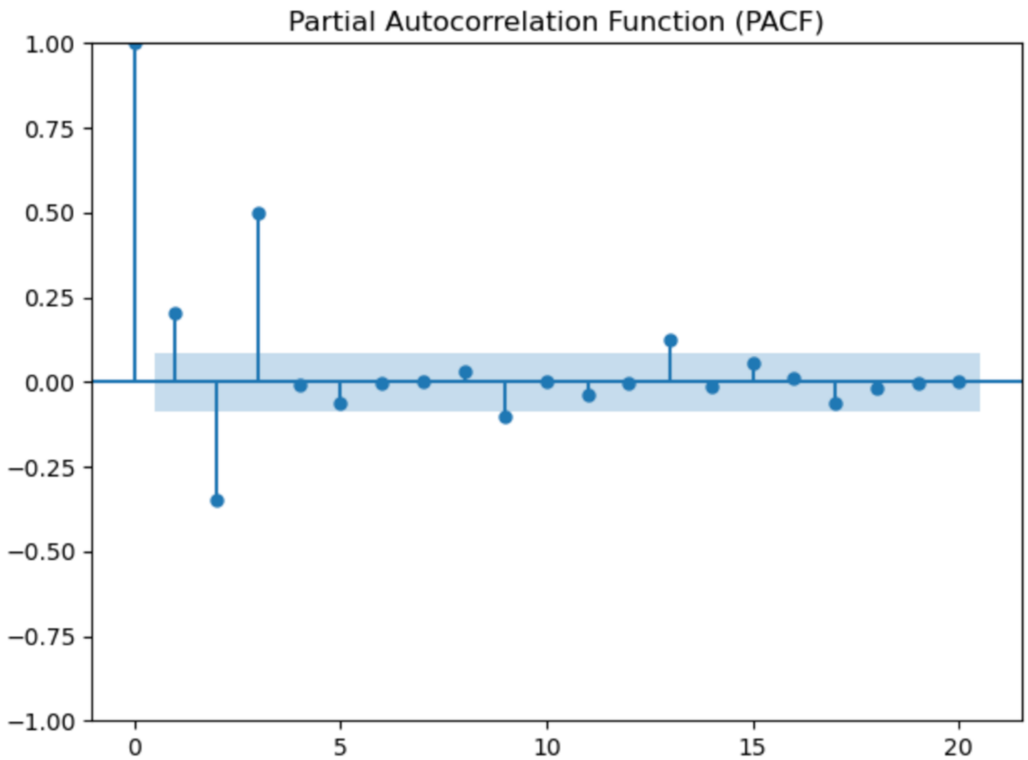
MA(2) AIC: 1537.941206380739

MA(3) AIC: 1536.8677087350304

The best model is: AR(3) with AIC: 1436.6598066945892

Also, by observing the graph, we can find that the autocorrelation value gradually decreases to 0 for the ACF graph. For the PACF graph, the partial autocorrelation value suddenly drops to 0. Therefore, we can infer that the AR(3) model might be the best fit.





Partial Autocorrelation Function (PACF)
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
0
5
10
15
20