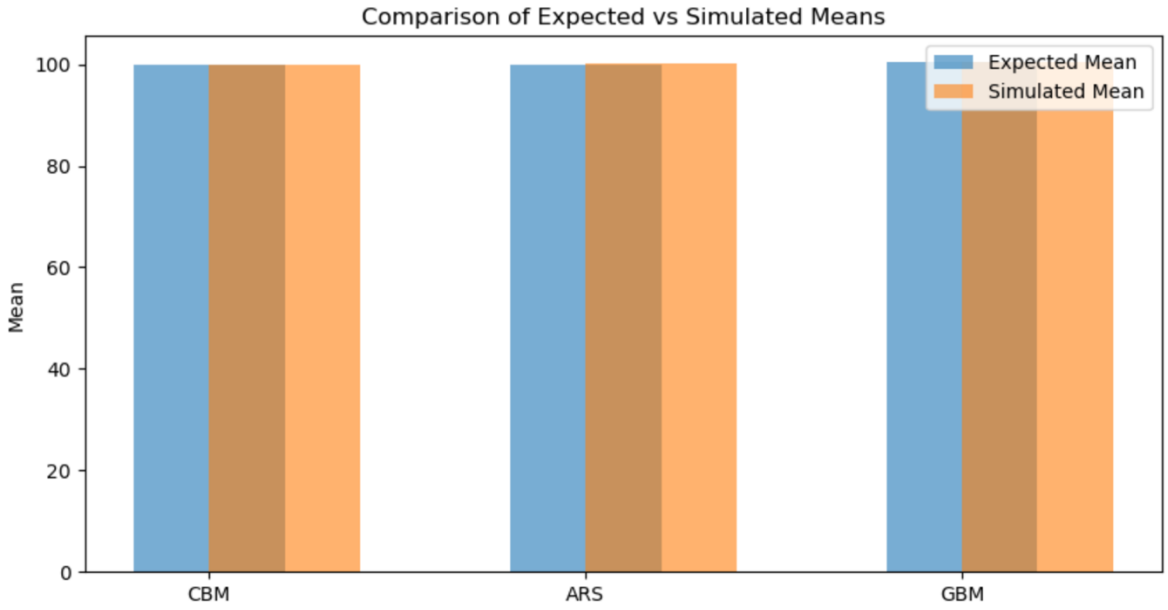
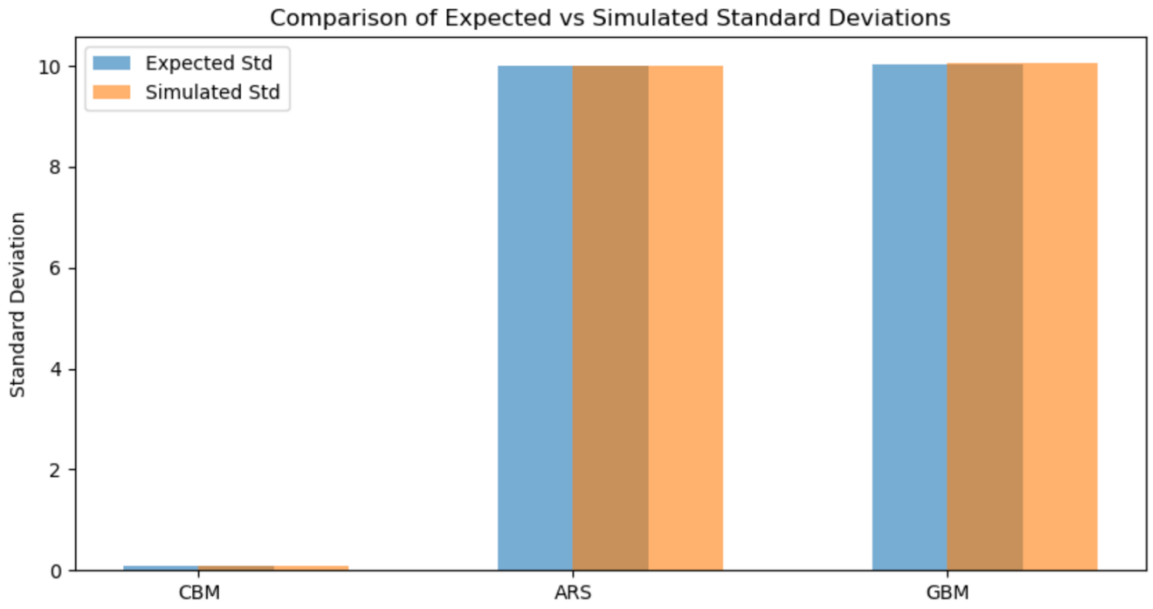
## Problem 1

```
===== Comparison of Results =====
Classical Brownian Motion (CBM):
  Expected Mean: 100, Simulated Mean: 100.00043474493789
  Expected Std: 0.1, Simulated Std: 0.09994562349697608

Arithmetic Return System (ARS):
  Expected Mean: 100, Simulated Mean: 100.04347449378847
  Expected Std: 10.0, Simulated Std: 9.99456234969761

Geometric Brownian Motion (GBM):
  Expected Mean: 100.5012520859401, Simulated Mean: 100.54417910146658
  Expected Std: 10.025052161544073, Simulated Std: 10.068485384172567
```






**Based on the results and graphs, we can find that the simulated results and expected results are almost the same.**

## Problem 2

```
VaR (Normal Distribution): -0.038249838724583034
VaR (Exponentially Weighted Variance): -0.03099142422297602
VaR (MLE Fitted T-Distribution): -0.03242585900409048
VaR (Fitted AR(1) Model): -0.03811666178305521
VaR (Historical Simulation): -0.0016577131646031966
```

- The **Normal Distribution** and **Fitted AR(1) Model** provide similar and higher estimates of risk.
- The **Exponentially Weighted Variance** suggests that recent data shows a lower risk compared to assuming constant variance.
- The **MLE Fitted T-Distribution** captures tail risk better, but still gives a slightly lower risk than the normal distribution.
- The **Historical Simulation** suggests the lowest risk, indicating that recent actual historical performance shows much less potential loss than predicted by other models.

## Problem 3

```
VaR for Portfolio A (Discrete Returns): $16702.3876764
VaR for Portfolio A (Log Returns): $16739.3778306
VaR for Portfolio B (Discrete Returns): $17477.7995283
VaR for Portfolio B (Log Returns): $17487.4399111
VaR for Portfolio C (Discrete Returns): $21007.8554675
VaR for Portfolio C (Log Returns): $21058.9549963

Total VaR (Discrete Returns): $55188.0426722
Total VaR (Log Returns): $55285.7727380
```

**Discrete Returns vs. Log Returns:**

- **Portfolio A:**
  - Discrete VaR: $16,702
  - Log VaR: $16,739
  - The difference between the two methods is minimal, with log returns slightly higher. This indicates that the risk estimation between discrete and log returns is quite similar for this portfolio.
- **Portfolio B:**
  - Discrete VaR: $17,477
  - Log VaR: $17,487
  - Similar to Portfolio A, the difference between discrete and log returns is small. Both models predict a comparable level of risk.
- **Portfolio C:**
  - Discrete VaR: $21,007
  - Log VaR: $21,058
  - Again, the difference is very slight, with log returns being marginally higher. This suggests that Portfolio C has the highest level of risk among the three portfolios, but the overall risk difference between the models remains negligible.

**Total VaR for All Portfolios:**

- Total VaR (Discrete Returns): $55,188
- Total VaR (Log Returns): $55,285
- The total VaR calculated using both discrete and log returns is nearly the same, with the log returns method producing a slightly higher risk estimate. This suggests that, at the portfolio level, the choice between discrete and log returns does not drastically alter the overall risk profile.

**Conclusion on Method Choice:**

- The difference between the two models (discrete and log returns) is very small across all portfolios. This suggests that for the portfolios, either method is acceptable, as the risk profile is consistent.
- Log returns might be slightly more conservative, giving slightly higher risk estimates, but the difference is minor.

- Discrete returns offer a more straightforward approach with similarly accurate results.