自由电子薛定谔方程、

$$\begin{cases} -\frac{\hbar^2}{2m_0}\frac{d^2\psi(x)}{dx^2}+V(x)\psi(x)=E\psi(x) \\ V(x)=0 \end{cases}$$

$\Rightarrow \psi(x)=Ae^{ikx}$

$$\Rightarrow \cancel{\begin{cases} \psi(x)=u_k(x)e^{ikx} \\ \psi(x)=\psi(x+na) \end{cases}}$$

晶体中电子…

$$\begin{cases} -\frac{\hbar^2}{2m_0}\frac{d^2\psi(x)}{dx^2}+V(x)\psi(x)=E\psi(x) \\ V(x)=V(x+sa) \end{cases}$$

$$\Rightarrow \begin{cases} \psi(x)=u_k(x)e^{ikx} \\ u_k(x)=u_k(x+na) \end{cases}$$

X 自由电子状态推导

$$\begin{cases} \text{波动性}\ p=\hbar k\ (\text{德布罗意关系}) & E=h\nu \\ \text{粒子性}\ p=mv & E=\frac{p^2}{2m_0} \end{cases} \Rightarrow E=\frac{\hbar^2k^2}{2m_0}$$

X 晶体中电子状态推导

将 $E(k)$ 在 $k=0$ 处泰勒展开

$$E(k)=E(0)+\left(\frac{dE}{dk}\right)_{k=0}k+\frac{1}{2}\left.\frac{d^2E}{dk^2}\right|_{k=0}k^2+\cdots$$

$\because$ $k=0$ 时 能量极小 $\left(\frac{dE}{dk}\right)_{k=0}=0$

$$\Rightarrow E(k)=E(0)+\frac{1}{2}\left(\frac{d^2E}{dk^2}\right)_{k=0}k^2$$

$$\Rightarrow E(k)-E(0)=\frac{1}{2}\left(\frac{d^2E}{dk^2}\right)_{k=0}k^2$$

令 $\frac{1}{m^*}=\frac{1}{\hbar^2}\left(\frac{d^2E}{dk^2}\right)_{k=0}$

$$\Rightarrow E(k)-E(0)=\frac{\hbar^2k^2}{2m^*}$$

X $v=\frac{1}{\hbar}\frac{dE}{dk}$ 推导

$E=h\nu$

$v=\frac{d\nu}{dk}$

$=\frac{1}{\hbar}\frac{dE}{dk}$

X 电子加速度 a 和 $\frac{dk}{dt}$ 推导

$$dE=f\cdot ds=+fv\,dt=f\cdot\frac{1}{\hbar}\frac{dE}{dk}\cdot dt \Rightarrow \frac{dk}{dt}=\frac{f}{\hbar}$$

$$a=\frac{dv}{dt}=\frac{d\left(\frac{1}{\hbar}\frac{dE}{dk}\right)}{dt}=\frac{1}{\hbar}\frac{dE}{dk\,dt}=\frac{1}{\hbar}\frac{d^2E}{dk^2}\cdot\frac{dk}{dt}=\frac{1}{\hbar^2}\frac{d^2E}{dk^2}\cdot\frac{f}{1}=\frac{f}{m^*}$$

X 空穴推导

① 满价带中有一 k 状态电子进入导带，留下一空态。在外电场作用下，电子和空态的运动状态是一致的。所以 $\frac{dk}{dt}=\frac{f}{\hbar}$ 速度相同运动，因此可以等效

② 设价带中剩余电子所产生电流为 J，若将一电子填入空态，其电流为 $J_1=-qv(k)$.

③ 由于满带不导电 $\Rightarrow J_1+J_2=0$ $\Rightarrow J_1=+qv(k)$ $\Rightarrow$ 所以价带中剩余大量电子可用正电荷式等效

ⅸ $m_p^* = -m_n^*$ 推导

由于价带的剩余电子和导带都在电场运动 所以两者加速度一致

∵ 电子 $a = \frac{-qE}{m_n^*}$ ∵ $m_n^* < 0$ ⇒ $a > 0$

∴ 空穴 $a = \frac{+qE}{m_p^*} > 0$ ⇒ $m_p^* = -m_n^*$

Ⅹ 回旋频率 $\omega_c = \frac{qB}{m_n^*}$ 推导

将样本位于磁感应强度为 $\vec{B}$ 的磁场中，电子速度 $\vec{v}$ 和 $\vec{B}$ 夹角为 θ.

则电子受力 $\vec{f} = -q\vec{v}\times\vec{B}$

$|q| = qvB\sin\theta = qv_\perp B$ 电子在 $\begin{cases}\text{与}B\text{平行方向做匀速直线运动} \\ \text{与}B\perp\text{方向做圆周运动}\end{cases}$ ⇒ 螺旋式运动

各向同性时

设回旋半径为 r，角频率为 ω_c. ⇒ $v_\perp = \omega_c \cdot r$

则向心加速度 $a = \frac{v_\perp^2}{r} = \frac{f}{m_n^*} = \frac{qvB\sin\theta}{m_n^*} = \frac{qv_\perp B}{m_n^*}$

$$\Rightarrow \frac{v_\perp^2}{r} = \frac{qv_\perp B}{m_n^*} \Rightarrow \omega_c = \frac{qB}{m_n^*}$$

各向异性时

$$\vec{f} = -q\vec{v}\times\vec{B} = (f_x\vec{i} + f_y\vec{j} + f_z\vec{k})$$

$$= -q(v_x\vec{i} + v_y\vec{j} + v_k\vec{k})\times B(\alpha\vec{i} + \beta\vec{j} + \gamma\vec{k}) \qquad \begin{cases}\alpha = \cos\theta_1 \\ \beta = \cos\theta_2 \\ \gamma = \cos\theta_3\end{cases}$$

以 证明k空间中k取值不连续

在一维晶体中，由波恩卡曼边界条件 晶体是首尾相连的

$\because \begin{cases} \psi_k(0)=\psi_k(L) \\ L=Na \end{cases} \Rightarrow u_k(0)=u_k(L)$

又$\because$ $u_k(0)=u_k(L)e^{i2\pi kL} \Rightarrow e^{i2\pi kL}=1 \Rightarrow 2\pi kL=2\pi n \Rightarrow k=\frac{n}{L}$ $(n=0,\pm1,\pm2\cdots)$

$\therefore$ 是不连续的

以 k空间中量子态密度推导

由波恩卡曼边界条件 $\Rightarrow k=\frac{n}{L}$

三维中、$\begin{cases} k_x=\frac{n_x}{L_1} \\ k_y=\frac{n_y}{L_2} \\ k_z=\frac{n_z}{L_3} \end{cases} \Rightarrow$ 每个允许的k值在k空间所占体积为 $\frac{1}{L_1L_2L_3}=\frac{1}{V}$

$\therefore$ k空间量子态密度为 $\frac{1}{\frac{1}{V}}=V$.

计入自旋后为 $2V$.

同理，一维为 $2L$ 二维为 $2S$.

✓ k空间中量子态的推导 $E=\frac{h^2k^2}{2m_n^*} \Rightarrow k=\sqrt{\frac{2m_n^*E}{h^2}}$

一维：① 量子态密度为 $2L$

② 体积为 $k\cdot 1$

③ $Z(E)=2L\cdot k=\frac{2L\sqrt{2m_n^*E}}{h}$

二维 ① $2S$

② $\pi k^2=\frac{2m_n^*\pi E}{h^2}$

③ $Z(E)=2S\cdot\pi k^2=\frac{4Sm_n^*\pi E}{h^2}$

三维 ① $2V$

(同) ② $\frac{4}{3}\pi k^3$ (同) $\cancel{\frac{4}{3}\pi k_1k_2k_3\ (异)}$

③ $Z(E)=\frac{4}{3}\pi\frac{(2m_n^*E)^{\frac{3}{2}}}{h^3}\cdot 2V=\frac{8\pi V}{3}\frac{(2m_n^*)^{\frac{3}{2}}}{h^3}\cdot E^{\frac{3}{2}}$

✗ 吸收峰判断推导

以Si为例，从6个旋转椭球等能面的取向轴沿[001]的一个为例。

再设一坐标系 $k_1k_2k_3$，以[001]上的椭球中心为原点，使 k_3 与[001]重合，再旋转 k_1k_2 使 $\vec{B}$ 恰位于 k_1k_3 平面内，则 $\Rightarrow \begin{cases} \alpha = \sin\theta \\ \beta = 0 \\ \gamma = \cos\theta \end{cases}$ 且 $m_{k_1} = m_{k_2} = m_t \quad m_{k_3} = m_l$

$$\because \ E(k) = \frac{\hbar^2}{2}\left[\frac{k_1^2+k_2^2}{m_t} + \frac{k_3^2}{m_l}\right]$$

$$\therefore \ \frac{1}{m_n^*} = \sqrt{\frac{m_x^*\alpha^2 + m_y^*\beta^2 + m_z^*\gamma^2}{m_x^* m_y^* m_z^*}} = \sqrt{\frac{m_t\sin^2\theta + m_l\cos^2\theta}{m_t^2 m_l}}$$

当 $\vec{B}$ 沿[1 1 1]方向时

B与[100] [010] [001]夹角为θ　与[$\bar{1}$00] [0$\bar{1}$0] [00$\bar{1}$]夹角为$\pi-\theta$

$\sin^2\theta = \sin^2(\pi-\theta) = \frac{2}{3} \quad \cos^2\theta = \cos^2(\pi-\theta) = \frac{1}{3} \quad \Rightarrow$ 只有一个 $m_n^* = \sqrt{\frac{3m_t}{2m_t+2m_l}} \cdot m_l$　1个峰。

当 $\vec{B}$ 沿[1 1 0]方向时

B与[100] [010] [$\bar{1}$00] [0$\bar{1}$0] 夹角为 $\frac{\pi}{4}$ 或 $\pi-\frac{\pi}{4}$ $\Rightarrow \cos^2\theta = \sin^2\theta = \frac{1}{2}$

$\Rightarrow m_n^* = m_t\sqrt{\frac{2m_l}{m_t+m_l}}$　　2个峰

B与[001] [00$\bar{1}$] 夹角为 $\frac{\pi}{2}$ $\Rightarrow \cos^2\theta = 0 \quad \sin^2\theta = 1 \quad \Rightarrow m_n^* = \sqrt{m_t m_l}$

当 $\vec{B}$ 沿[100]方向时

B与[100] [$\bar{1}$00] 夹角为 0 或 π $\Rightarrow \cos^2\theta = 1 \quad \sin^2\theta = 0$

B与[010] [001] [0$\bar{1}$0] [00$\bar{1}$] 夹角为 $\frac{\pi}{2}$ $\Rightarrow \cos^2\theta = 0 \quad \sin^2\theta = 1$　$\Rightarrow$ 2个峰。

当 B 沿任意方向 $\Rightarrow$ 3个峰。

✗ 类氢模型，用于估计 ΔE_D、ΔE_A

$$E_0 = \frac{m_0 q^4}{8\varepsilon_0^2 h^2} \qquad E = \frac{m_n^* q^4}{8\varepsilon_0\varepsilon_r h^2}$$

$$\Delta E_D = \frac{m_n^*}{m_0}\frac{E_0}{\varepsilon_r^2}$$

氢原子半径 $r_0 = \frac{h^2\varepsilon_0}{\pi q^2 m_0}$

类氢原子半径 $r = \frac{h^2\varepsilon_0\varepsilon_r}{\pi q^2 m_n^*}$

√费米分布 和 波尔兹曼分布

电子占据概率 $f(E)=\frac{1}{1+\exp(\frac{E-E_F}{k_0T})}$ 　空穴占据概率 $1-f(E)=\frac{1}{1+\exp(\frac{E_F-E}{k_0T})}$

当 $E-E_F \gg k_0T$ ⇒ 为波尔兹曼分布 ⇒ $f_B(E)=\exp(-\frac{E-E_F}{k_0T})$

$1-f_B(E)=\exp(-\frac{E_F-E}{k_0T})$

导带中电子浓度推导 (非简并) + (简并).

$$n_0=\frac{1}{V}\int_{E_c}^{E_c'} g_c(E)\cdot f_B(E)\,dE$$ 非简并

$$=\frac{1}{V}\int_{E_c}^{E_c'}\frac{4\pi V(2m_n^*)^{\frac{3}{2}}}{h^3}(E-E_c)^{\frac{1}{2}}\exp(-\frac{E-E_F}{k_0T})\,dE$$

$$=\frac{4\pi(2m_n^*)^{\frac{3}{2}}}{h^3}\int_{E_c}^{E_c'}(E-E_c)^{\frac{1}{2}}\cdot\exp(-\frac{E-E_c}{k_0T})\exp(-\frac{E_c-E_F}{k_0T})\,dE$$ 令 $\frac{E-E_c}{k_0T}=x$

$$=\frac{4\pi(2m_n^*)^{\frac{3}{2}}}{h^3}\exp(-\frac{E_c-E_F}{k_0T})\cdot\int_0^{+\infty}(k_0T)^{\frac{1}{2}}x^{\frac{1}{2}}\exp(-x)\cdot k_0T\,dx$$

$$\because\int_0^{+\infty}x^{\frac{1}{2}}\exp(-x)\,dx=\Gamma(\tfrac{3}{2})=\Gamma(\tfrac{1}{2})=\frac{\sqrt{\pi}}{2}$$

$$\Rightarrow\frac{4\pi(2m_n^*k_0T)^{\frac{3}{2}}}{h^3}\cdot\frac{\sqrt{\pi}}{2}\exp(-\frac{E_c-E_F}{k_0T})=N_c\exp(-\frac{E_c-E_F}{k_0T})$$

$$n_0=\frac{1}{V}\int_{E_c}^{E_c'}g_c(E)f(E)\,dE$$

$$=\frac{4\pi(2m_n^*)^{\frac{3}{2}}}{h^3}\int_{E_c}^{E_c'}(E-E_c)^{\frac{1}{2}}\frac{1}{1+\exp(\frac{E-E_c+E_c-E_F}{k_0T})}\,dE$$ 令 $\frac{E-E_c}{k_0T}=x$ 　$\frac{E_F-E_c}{k_0T}=\xi$

$$=\frac{4\pi(2m_n^*k_0T)^{\frac{3}{2}}}{h^3}\cdot\int_0^{+\infty}x^{\frac{1}{2}}\frac{1}{1+\exp(x-\xi)}\,dx.$$ 记 $\int_0^{+\infty}\frac{x^{\frac{1}{2}}}{1+e^{x-\xi}}dx=F_{\frac{1}{2}}(\xi)$

$$=\frac{2(2\pi m_n^*\cdot k_0T)^{\frac{3}{2}}}{h^3}\cdot\frac{2}{\sqrt{\pi}}F_{\frac{1}{2}}(\xi).$$

注 ① $2V$
(另)
② $\frac{4}{3}\pi k_1k_2k_3$ $\quad k_1=\sqrt{\frac{2m_x^*E}{h^2}}$ $\quad k_2=\sqrt{\frac{2m_y^*E}{h^2}}$ $\quad k_3=\sqrt{\frac{2m_z^*E}{h^2}}$

③ $$Z(E)=2V\frac{4}{3}\pi k_1k_2k_3$$
$$=\frac{8\pi V}{3}\cdot\frac{\sqrt{8m_x^*m_y^*m_z^*}}{h^3}(E-E_c)^{\frac{3}{2}}$$

✓✗ 旋转椭球面方程

$$\frac{(k_x-k_{0x})^2}{\frac{2m_x^*(E-E_c)}{h^2}}+\frac{(k_y-k_{0y})^2}{\frac{2m_y^*(E-E_c)}{h^2}}+\frac{(k_z-k_{0z})^2}{\frac{2m_z^*(E-E_c)}{h^2}}=1$$

由 $E(k)-E_c=\frac{h^2}{2}\left(\frac{(k_x-k_{0x})^2}{m_x^*}+\frac{(k_y-k_{0y})^2}{m_y^*}+\frac{(k_z-k_{0z})^2}{m_z^*}\right)$ 推导得

✓✗ Si导带底状态密度和状态密度有效质量推导.

① $$Z(E)=\frac{8}{3}\pi V\cdot\frac{\sqrt{8m_x^*m_y^*m_z^*}}{h^3}(E-E_c)^{\frac{3}{2}}\cdot s$$ $\quad s\begin{cases}Si: 6\\ Ge: 4\end{cases}$

$$g_c(E)=\frac{4\pi V\sqrt{8m_x^*m_y^*m_z^*}}{h^3}(E-E_c)^{\frac{1}{2}}\cdot s$$

② 令 $m_x^*=m_y^*=m_t$ $\quad m_z^*=m_l$

$$g_c(E)=\frac{4\pi Vs\sqrt{8m_t^2m_l}}{h^3}\cdot(E-E_c)^{\frac{1}{2}}$$

令 $$g_c(E)=\frac{4\pi V(2m_n^*)^{\frac{3}{2}}}{h^3}(E-E_c)^{\frac{1}{2}}$$

$\Rightarrow (2m_n^*)^{\frac{3}{2}}=s\sqrt{8m_t^2m_l}$

$\Rightarrow m_n^*=s^{\frac{2}{3}}(m_t^2m_l)^{\frac{1}{3}}$

✗ Si价带顶同上.

① $$Z(E)=\frac{8}{3}\pi V\frac{(2m_p^*)^{\frac{3}{2}}}{h^3}(E_v-E)^{\frac{3}{2}}\cdot s$$

$$g_v(E)=4\pi V\cdot\frac{(2m_p^*)^{\frac{3}{2}}}{h^3}\cdot(E_v-E)^{\frac{1}{2}}\cdot s$$

② 对于重空穴带 $g_{vh}(E)=4\pi Vs\frac{(2m_{ph}^*)^{\frac{3}{2}}}{h^3}(E_v-E)^{\frac{1}{2}}$

$\because g_v(E)=g_{vh}(E)+g_{vl}(E)$

对于轻空穴带 $g_{vl}(E)=4\pi Vs\frac{(2m_{pl}^*)^{\frac{3}{2}}}{h^3}(E_v-E)^{\frac{1}{2}}$

$\Rightarrow m_p^{*\frac{3}{2}}=m_{ph}^{*\frac{3}{2}}+m_{pl}^{*\frac{3}{2}}$ $\quad\Rightarrow m_p^*=(m_{ph}^{*\frac{3}{2}}+m_{pl}^{*\frac{3}{2}})^{\frac{2}{3}}$

价带空穴浓度推导 ·（非简并）+（简并）

$$p_0 = \frac{1}{V}\int_{E_V'}^{E_V} g_V(E)(1-f_B(E))\,dE$$

$$= \frac{1}{V}\int_{E_V'}^{E_V} \frac{4\pi V(2m_p^*)^{\frac{3}{2}}}{h^3}(E_V-E)^{\frac{1}{2}} \exp\left(-\frac{E_F-E}{k_0T}\right)dE$$

$$= \frac{4\pi(2m_p^*)^{\frac{3}{2}}}{h^3}\exp\left(-\frac{E_F-E_V}{k_0T}\right)\int_{E_V'}^{E_V}(E_V-E)^{\frac{1}{2}}\exp\left(-\frac{E_V-E}{k_0T}\right)dE \qquad 令\ x=\frac{E_V-E}{k_0T}$$

$$= \frac{4\pi(2m_p^*)^{\frac{3}{2}}}{h^3}\cdot(k_0T)^{\frac{3}{2}}\exp\left(-\frac{E_F-E_V}{k_0T}\right)\int_0^{+\infty}x^{\frac{1}{2}}\exp(-x)\,dx$$

$$= \frac{2(2\pi m_p^* k_0T)^{\frac{3}{2}}}{h^3}\exp\left(-\frac{E_F-E_V}{k_0T}\right) = N_V\exp\left(-\frac{E_F-E_V}{k_0T}\right)$$

$$p_0 = \frac{1}{V}\int_{E_V'}^{E_V} g_V(E)(1-f(E))\,dE$$

$$= \frac{4\pi(2m_p^*)^{\frac{3}{2}}}{h^3}\cdot\int_{E_V'}^{E_V}(E_V-E)^{\frac{1}{2}}\frac{1}{1+\exp\left(\frac{E_F-E_V+E_V-E}{k_0T}\right)}dE \qquad 令\ x=\frac{E_V-E}{k_0T} \quad \xi=\frac{E_V-E_F}{k_0T}$$

$$= \frac{4\pi(2m_p^*\cdot k_0T)^{\frac{3}{2}}}{h^3}\int_0^{+\infty}x^{\frac{1}{2}}\frac{1}{1+\exp(x-\xi)}dx \qquad 令\ F_{\frac{1}{2}}(\xi)=\int_0^{+\infty}\frac{x^{\frac{1}{2}}}{1+e^{x-\xi}}dx$$

$$= N_V\cdot\frac{2}{\sqrt{\pi}}F_{\frac{1}{2}}(\xi)$$

本征载流子浓度推导

$$n_i^2 = n_0p_0 = N_C\exp\left(-\frac{E_C-E_F}{k_0T}\right)\cdot N_V\exp\left(-\frac{E_F-E_V}{k_0T}\right)$$

$$= N_CN_V\exp\left(-\frac{E_g}{k_0T}\right)$$

$$n_i = \sqrt{n_0p_0} = \sqrt{N_CN_V}\exp\left(-\frac{E_g}{2k_0T}\right)$$

本征E_F与禁带中线位置.

$$n_0=p_0=n_i \Rightarrow N_C\exp\left(-\frac{E_C-E_F}{k_0T}\right) = N_V\exp\left(-\frac{E_F-E_V}{k_0T}\right)$$

$$\Rightarrow \exp\left(\frac{2E_F-E_C-E_V}{k_0T}\right) = \frac{N_V}{N_C}$$

$$\Rightarrow E_F = \frac{E_C+E_V}{2}+\frac{1}{2}k_0T\ln\frac{N_V}{N_C} \qquad \because \frac{N_V}{N_C}=\left(\frac{m_p^*}{m_n^*}\right)^{\frac{3}{2}}$$

$$\Rightarrow \underset{(E_i)}{E_F} = \frac{E_C+E_V}{2}+\frac{3}{4}k_0T\ln\frac{m_p^*}{m_n^*}$$

✓ $E_g(0)$ 的求解推导（$\ln n_i \sim \frac{1}{T}$ 关系）.

设 $E_g = E_g(0) + \beta T \quad \therefore \beta = \frac{dE_g}{dT}$

$$n_i = \sqrt{N_c N_v}\, \exp\left(-\frac{E_g}{2k_0T}\right) = \sqrt{N_c N_v}\, \exp\left(-\frac{E_g(0)}{2k_0T} - \frac{\beta T}{2k_0T}\right)$$

$$= \sqrt{N_c(300)N_v(300k)} \cdot T^{\frac{3}{2}} \exp\left(-\frac{E_g(0)}{2k_0T}\right) \exp\left(-\frac{\beta}{2k_0}\right)$$

$$\Rightarrow \ln\left(n_i \cdot T^{-\frac{3}{2}}\right) = \frac{1}{2}\ln\left[N_c(300)N_v(300)\right] - \frac{E_g(0)}{2k_0T} - \frac{\beta}{2k_0}$$

$$= A - \frac{E_g(0)}{2k_0} \cdot \frac{1}{T}$$

作出 $\ln\left(\frac{n_i}{T^{\frac{3}{2}}}\right) \sim \frac{1}{T}$ 图像，则 斜率 $= -\frac{E_g(0)}{2k_0}$

$\Rightarrow E_g(0) = -$ 斜率 $\cdot 2k_0$

✓ 杂质半导体 电子占据能级的概率

$$f_D(E) = \frac{1}{1 + \frac{1}{2}\exp\left(\frac{E_D - E_F}{k_0T}\right)}$$

✓ …… 空穴占据能级的概率

$$f_A(E) = \frac{1}{1 + 4\exp\left(\frac{E_F - E_A}{k_0T}\right)}$$

✓ 施主能级上电子浓度和受主能级上空穴概率

$$n_D = N_D \cdot f_D(E) = \frac{N_D}{1 + \frac{1}{2}\exp\left(\frac{E_D - E_F}{k_0T}\right)} \qquad p_A = N_A f_A(E) = \frac{N_A}{1 + 4\exp\left(\frac{E_F - E_A}{k_0T}\right)}$$

✓ 电离施主和受主浓度

$$n_D^+ = N_D\left[1 - f_D(E)\right] = N_D - n_D = \frac{N_D}{1 + 2\exp\left(-\frac{E_D - E_F}{k_0T}\right)}$$

$$p_A^- = N_A - p_A = \frac{N_A}{1 + 4\exp\left(-\frac{E_F - E_A}{k_0T}\right)}$$

√ 低温弱电离区推导

$n_0 = n_D^+$

$\Rightarrow N_c\exp\left(-\frac{E_c-E_F}{k_0T}\right) = \frac{N_D}{1+2\exp\left(-\frac{E_D-E_F}{k_0T}\right)}$ $\quad\because\quad E_D-E_F \gg k_0T$

$\Rightarrow N_c\exp\left(-\frac{E_c-E_F}{k_0T}\right) = \frac{1}{2}N_D\exp\left(\frac{E_D-E_F}{k_0T}\right)$

$\Rightarrow \exp\left(\frac{2E_F-E_c-E_D}{k_0T}\right) = \frac{1}{2}\frac{N_D}{N_c}$

$\Rightarrow E_F = \frac{E_c+E_D}{2} + \frac{1}{2}k_0T\ln\frac{N_D}{2N_c}$

√ 低温时 E_F 和 T 的关系推导

$$E_F = \frac{E_c+E_D}{2} + \frac{1}{2}k_0T\ln\frac{N_D}{2N_c(300K)\cdot T^{\frac{3}{2}}} = \frac{E_c+E_D}{2} + \frac{1}{2}k_0T\ln(AT^{-\frac{3}{2}})$$

$$\frac{dE_F}{dT} = \frac{k_0}{2}\ln\frac{N_D}{2N_c} + \frac{1}{2}k_0T\left[\ln A - \ln T^{\frac{3}{2}}\right]'$$

$$= \frac{k_0}{2}\ln\frac{N_D}{2N_c} - \frac{1}{2}k_0T\cdot\frac{3}{2}\cdot\frac{1}{T}$$

$$= \frac{k_0}{2}\left[\ln\frac{N_D}{2N_c} - \frac{3}{2}\right]$$

$\therefore$ 当 $\ln\frac{N_D}{2N_c} = \frac{3}{2} \Rightarrow E_F$ 达到最大值.

✗ 求电离能的推导 ΔE_D.

$n_0 = n_D^+ = \frac{N_D}{1+2\exp\left(-\frac{E_D-E_F}{k_0T}\right)} \Rightarrow \frac{1}{2}N_D\exp\left(\frac{E_D-E_F}{k_0T}\right)$

将 $E_F = \frac{E_c+E_D}{2} + \frac{1}{2}k_0T\ln\frac{N_D}{2N_c}$ 代入

$n_0 = \frac{1}{2}N_D\exp\left(\frac{E_D-\frac{E_c+E_D}{2}-\frac{1}{2}k_0T\exp\left(-\frac{E_D-E_F}{k_0T}\right)}{k_0T}\right)$

$n_0 = \frac{1}{2}N_D\exp\left(\frac{E_D-E_c}{2k_0T} - \frac{1}{2}\exp\left(-\frac{E_D-E_F}{k_0T}\right)\right)$

$\ln n_0 = \ln\frac{N_D}{2} + \frac{E_D-E_c}{2k_0T} - \frac{1}{2}\exp\left(-\frac{E_D-E_F}{k_0T}\right)$

$n_0 = N_c\exp\left(-\frac{E_c-E_F}{k_0T}\right)$

$\Rightarrow n_0 = N_c\exp\left(-\frac{E_c-\frac{E_c+E_D}{2}-\frac{k_0T}{2}\ln\frac{N_D}{2N_c}}{k_0T}\right)$

$\Rightarrow n_0 = N_c\exp\left(-\frac{E_c-E_D}{k_0T} + \frac{1}{2}\ln\frac{N_D}{2N_c}\right)$

$\Rightarrow n_0 = N_c\exp\left(-\frac{\Delta E_D}{k_0T}\right)\left(\frac{N_D}{2N_c}\right)^{\frac{1}{2}}$

$\Rightarrow n_0 = \left(\frac{N_cN_D}{2}\right)^{\frac{1}{2}}\exp\left(-\frac{\Delta E_D}{k_0T}\right)$

$\Rightarrow \ln n_0 = \frac{1}{2}\ln\frac{N_cN_D}{2} - \frac{\Delta E_D}{k_0T}$

$\Rightarrow \ln n_0T^{-\frac{3}{4}} = -\frac{\Delta E_D}{k_0T} + A$

$\therefore$ 斜率 $\times k_0 = \Delta E_D$.

以 中间电离区：

$E_D = E_F \Rightarrow \exp\left(-\frac{E_D - E_F}{k_0 T}\right) = 1$

$\cancel{E_F = \frac{E_D + E_C}{2} + \frac{1}{2} k_0 T \ln \frac{N_D}{2N_C}}$

$n_0 = n_D^+ = \frac{N_D}{1 + 2\exp\left(-\frac{E_D - E_F}{k_0 T}\right)} = \frac{1}{3} N_D$. 弓电离

V 强电离区 E_F 位置

$n_0 = N_D$

$\Rightarrow N_C \exp\left(-\frac{E_C - E_F}{k_0 T}\right) = N_D$

$\Rightarrow E_F = E_C + k_0 T \ln \frac{N_D}{N_C}$

$p_0 = N_A$

$\Rightarrow N_V \exp\left(-\frac{E_F - E_V}{k_0 T}\right) = N_A$

$\Rightarrow E_F = E_V + k_0 T \ln \frac{N_V}{N_A}$

以 未电离百分比推导

$n_D = \frac{N_D}{1 + \frac{1}{2}\exp\left(\frac{E_D - E_F}{k_0 T}\right)}$ 代入 $E_F = E_C + k_0 T \ln \frac{N_D}{N_C}$ 且 $E_D - E_F \gg k_0 T$

$\Rightarrow n_D = 2N_D \exp\left(-\frac{E_D - E_C - k_0 T \ln \frac{N_D}{N_C}}{k_0 T}\right)$

$= 2N_D \exp\left(-\frac{E_D - E_C}{k_0 T}\right)\left(\frac{N_D}{N_C}\right)$

$= 2N_D \left(\frac{N_D}{N_C}\right) \exp\left(+\frac{\Delta E_D}{k_0 T}\right)$

$\Rightarrow D_- = \frac{n_D}{N_D} = 2\left(\frac{N_D}{N_C}\right) \exp\left(\frac{\Delta E_D}{k_0 T}\right)$

$p_A = \frac{N_A}{1 + \frac{1}{4}\exp\left(\frac{E_F - E_A}{k_0 T}\right)}$

$= 4N_A \exp\left(-\frac{E_F - E_A}{k_0 T}\right)$ 代入 $E_F = E_V + k_0 T \ln \frac{N_V}{N_A}$

$= 4N_A \exp\left(-\frac{E_F - E_V}{k_0 T}\right) \exp\left(-\ln \frac{N_V}{N_A}\right)$

$= 4N_A \left(\frac{N_A}{N_V}\right) \exp\left(\frac{\Delta E_A}{k_0 T}\right)$

$D_+ = \frac{p_A}{N_A} = 4\left(\frac{N_A}{N_V}\right) \exp\left(\frac{\Delta E_A}{k_0 T}\right)$

过渡区 E_F 推导

$$\begin{cases} n_0 = N_D + p_0 \\ n_0 p_0 = n_i^2 \end{cases} \Rightarrow n_0 = \frac{N_D}{2} + \sqrt{\left(\frac{N_D}{2}\right)^2 + n_i^2}$$

$$n_0 = N_D + p_0$$

$$\Rightarrow n_i \exp\left(-\frac{E_i - E_F}{k_0 T}\right) = N_D + n_i \exp\left(-\frac{E_F - E_i}{k_0 T}\right)$$

$$\Rightarrow N_D = n_i\left[\exp\left(-\frac{E_i - E_F}{k_0 T}\right) - \exp\left(-\frac{E_F - E_i}{k_0 T}\right)\right] = 2 n_i \, sh\left(\frac{E_F - E_i}{k_0 T}\right).$$

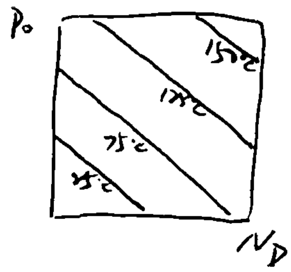
$$\Rightarrow E_F = E_i + k_0 T \, arcsh\left(\frac{N_D}{2 n_i}\right)$$

平衡少子浓度和T及杂质浓度的关系推导(强电离)

n型　$p_0 = \frac{n_i^2}{N_D}$

$\because n_i^2 \sim T^3 \exp\left(-\frac{E_g}{k_0 T}\right)$　$\therefore p_0$ 随 $T\uparrow$ 而 $\uparrow$

且 p_0 随 $N_D \uparrow$ 而 $\downarrow$



X Si 电子有效质量的推导

因为Si导带有6个旋转椭球等能面，

∴沿 x, y, z 方向的迁移率分别为 $\mu_1 = \frac{q\tau_n}{m_t}$ $\mu_2 = \frac{q\tau_n}{m_t}$ $\mu_3 = \frac{q\tau_n}{m_l}$

又因为有6个极值，每个能谷单位体积中有 $\frac{n}{6}$ 个电子.

设电场强度 E_x 沿 x 方向，电流密度 J_x 为

$$J_x = \frac{n}{3}q\mu_1 E_x + \frac{n}{3}q\mu_2 E_x + \frac{n}{3}q\mu_3 E_x$$

$$= \frac{nq}{3}(\mu_1+\mu_2+\mu_3)E$$

令 $J_x = nq\mu_c E_x$

$\therefore \mu_c = \frac{1}{3}(\mu_1+\mu_2+\mu_3)$ 且 $\mu_c = \frac{q\tau_n}{m^*}$

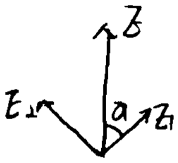
$$\therefore \frac{1}{m^*} = \frac{1}{3}\left(\frac{2}{m_t} + \frac{1}{m_l}\right)$$

X Ge 电子有效质量的推导

设电场 E 沿 [001] 方向，与 [111] 方向夹角为 α. $\cos^2\alpha = \frac{1}{3}$ $\sin^2\alpha = \frac{2}{3}$

Ge 有 8个半个旋转椭球等能面，所以每个能谷单位体积中有 $\frac{n}{4}$ 个电子

把电场分解为 $E_1 = E\cos\alpha$ $E_2 = E\sin\alpha$



则 E_1 和 E_2 方向上的电流密度为

$$J_1 = \frac{n}{4}q\mu_1 E\cos\alpha \qquad J_2 = \frac{n}{4}q\mu_2 E\sin\alpha$$

又因为总电流密度 $J = J_1\cos\alpha + J_2\sin\alpha$

$$= \frac{n}{4}q\mu_1 E\cos^2\alpha + \frac{n}{4}q\mu_2 E\sin^2\alpha$$

$$= \frac{n}{4}qE\left(\frac{\mu_1}{3} + \frac{2\mu_2}{3}\right)$$

令 $J = \frac{n}{4}q\mu_c E$

$\therefore \mu_c = \frac{\mu_1}{3} + \frac{2\mu_2}{3}$ 且 $\mu_c = \frac{q\tau_n}{m_c}$ $\mu_1 = \frac{q\tau_n}{m_l}$ $\mu_2 = \frac{q\tau_n}{m_t}$

$$\therefore \frac{1}{m_c} = \frac{1}{3}\left(\frac{1}{m_l} + \frac{2}{m_t}\right)$$

Si 空穴电导有效质量的推导

重空穴带上电流密度为 $J_1 = p_1 q \mu_{p1} \mathcal{E}$ $\quad \mu_{p1} = \frac{q\tau_p}{m_{ph}^*}$

轻空穴带上电流密度为 $J_2 = p_2 q \mu_{p2} \mathcal{E}$ $\quad \mu_{p2} = \frac{q\tau_p}{m_{pl}^*}$

又∵ $p_1 \propto N_{v1} \propto (m_{ph}^*)^{\frac{3}{2}} \Rightarrow J_1 = A(m_{ph}^*)^{\frac{1}{2}}$

$p_2 \propto N_{v2} \propto (m_{pl}^*)^{\frac{3}{2}} \Rightarrow J_2 = A(m_{pl}^*)^{\frac{1}{2}}$

∴ $J_{总} = J_1 + J_2 = A\left[(m_{ph}^*)^{\frac{1}{2}} + (m_{pl}^*)^{\frac{1}{2}}\right]$

又∵ $J_{总} = pq\mu_p\mathcal{E} \propto (m_{dp})^{\frac{3}{2}} \cdot \frac{1}{m_{cp}} \Rightarrow J_{总} = A\, m_{dp}^{\frac{3}{2}} \cdot \frac{1}{m_{cp}}$ $\quad m_{dp} = (m_{ph}^{\frac{3}{2}} + m_{pl}^{\frac{3}{2}})^{\frac{2}{3}}$

$\Rightarrow A \cdot m_{dp}^{\frac{3}{2}} \frac{1}{m_{cp}} = A\left[m_{pl}^{\frac{1}{2}} + m_{ph}^{\frac{1}{2}}\right]$

$$\Rightarrow \frac{1}{m_{cp}} = \frac{m_{ph}^{\frac{1}{2}} + m_{pl}^{\frac{1}{2}}}{m_{ph}^{\frac{3}{2}} + m_{pl}^{\frac{3}{2}}}$$

平均自由时间的推导

设有N个电子以v运动，N(t)表示在t时刻尚未遭散射的电子数

∴在 $t \sim t+\Delta t$ 时间内被散射的电子数为 $N(t)P\Delta t$ $\quad$ P为散射几率.

∴ $N(t) - N(t+\Delta t) = N(t)P\Delta t$

∴ $\frac{dN(t)}{dt} = \lim_{\Delta t \to 0} \frac{N(t+\Delta t) - N(t)}{\Delta t} = -N(t)P$ $\quad$ 解为 $N(t) = N_0 e^{-Pt}$

∴在 $t \sim t+dt$ 期间内被散射的电子数为

$P \cdot N(t) = N_0 P e^{-Pt} dt$

∴ $\tau = \frac{1}{N_0}\int_0^{+\infty} N_0 P e^{-Pt}\, t\, dt = \frac{1}{P}$.

X 迁移率与温度的关系推导

散射概率 P	平均自由时间	迁移率
$P_i \propto N_i T^{-\frac{3}{2}}$	$\tau_i \propto N_i^{-1} T^{\frac{3}{2}}$	$\mu_i \propto N_i^{-1} T^{\frac{3}{2}}$
$P_s \propto T^{\frac{3}{2}}$	$\tau_s \propto T^{-\frac{3}{2}}$	$\mu_s \propto T^{-\frac{3}{2}}$
$P_o \propto \frac{1}{[\exp(\frac{\hbar\omega}{kT})-1]}$	$\tau_o \propto [\exp(\frac{\hbar\omega}{kT})-1]$	$\mu_o \propto [\exp(\frac{\hbar\omega}{kT})-1]$

因为总有几种散射机构同时存在，故它们相互独立

$\therefore P_总 = P_I + P_{II} + P_{III} + \cdots$

$\therefore \tau = \frac{1}{P} = \frac{1}{P_I + P_{II} + P_{III} + \cdots}$

$\therefore \frac{1}{\tau} = P_I + P_{II} + P_{III} = \frac{1}{\tau_I} + \frac{1}{\tau_{II}} + \frac{1}{\tau_{III}} + \cdots$

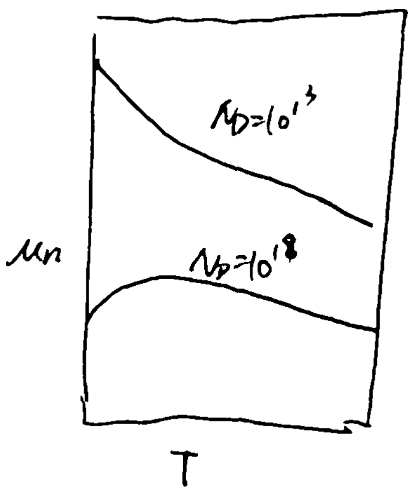
$\therefore \frac{1}{\mu} = \frac{1}{\mu_I} + \frac{1}{\mu_{II}} + \frac{1}{\mu_{III}} + \cdots$

对 Si Ge 等半导体，散射机构是电离杂质和声学波散射

$\mu_i \propto \frac{1}{BN_iT^{-\frac{3}{2}}}$ $\quad \mu_s \propto \frac{1}{AT^{\frac{3}{2}}}$

且 $\frac{1}{\mu} = \frac{1}{\mu_i} + \frac{1}{\mu_s}$

$\Rightarrow \mu \propto \frac{1}{AT^{\frac{3}{2}} + BN_iT^{-\frac{3}{2}}}$



当 $N_D=10^{13}\,cm^{-3}$ 时 即掺杂 N_i 不大，
晶格振动散射起主要作用，μ_n 随 $T\uparrow$ 而 $\downarrow$。
当 $N_D \approx 10^{18}\,cm^{-3}$ 时，
低温时，杂质散射起主要作用
高温时，晶格散射起主要作用
$\therefore$ 曲线先升高再降低

√ 电导率最小值的推导（对n型）

$$\begin{cases} \sigma = n_0 q \mu_n + p_0 q \mu_p \\ n_0 = \frac{n_i^2}{p_0} \end{cases} \Rightarrow \sigma = \frac{n_i^2}{p_0} q \mu_n + p_0 q \mu_p$$

$$\frac{d\sigma}{dp_0} = -\frac{n_i^2}{p_0^2} q \mu_n + q \mu_p = 0 \Rightarrow p_0 = n_i \sqrt{\frac{\mu_n}{\mu_p}}$$

同理 $n_0 = n_i \sqrt{\frac{\mu_p}{\mu_n}}$

$$\sigma_{min} = n_i \sqrt{\frac{\mu_p}{\mu_n}}\, q \mu_n + n_i \sqrt{\frac{\mu_n}{\mu_p}} \cdot q \mu_p = 2 n_i \sqrt{\mu_n \mu_p}$$

$\because \rho_{max} = \frac{1}{\sigma_{min}}$ 又$\because \mu_p < \mu_n$ $\therefore$ p型半导体的电阻率最大

X 附加光电导的测量方法

$$\Delta\sigma = \Delta p q(\mu_n+\mu_p)$$

而 $\Delta\rho = \frac{1}{\sigma} - \frac{1}{\sigma_0} = \frac{1}{(\sigma_0+\Delta\sigma)} - \frac{1}{\sigma_0}$ 且 $\sigma \approx \sigma_0$

$= \frac{-\Delta\sigma}{\sigma_0^2}$ $\therefore \Delta\rho \propto \Delta\sigma$

又: $\Delta r = \Delta\rho \frac{l}{S}$ $\therefore \Delta r \propto \Delta\sigma$

又: $\Delta V = I\Delta r$ $\therefore \Delta V \propto \Delta\sigma$

$\therefore$ 示波器上的电压变化直接反映附加电导率的变化

间接的检验了非平衡少数载流子的注入.

✓ 准费米能级公式

$$n = N_c \exp\left(-\frac{E_c - E_{Fn}}{k_0T}\right) = n_i \exp\left(-\frac{E_i - E_{Fn}}{k_0T}\right) = n_0 \exp\left(-\frac{E_F - E_{Fn}}{k_0T}\right)$$

$$p = N_v \exp\left(-\frac{E_{Fp} - E_v}{k_0T}\right) = n_i \exp\left(-\frac{E_{Fp} - E_i}{k_0T}\right) = p_0 \exp\left(-\frac{E_F - E_{Fp}}{k_0T}\right)$$

$$np = n_i^2 \exp\left(-\frac{E_{Fn} - E_{Fp}}{k_0T}\right)$$

$\therefore E_{Fn}$ 和 E_{Fp} 偏离的大小直接反映出 np 和 n_i^2 的相差程度

反映偏离热平衡态的程度

寿命 τ 的推导

lX 直接复合中的寿命推导

复合率 $R=\gamma np$　　产生率 $G=\gamma n_0 p_0=\gamma n_i^2$（热平衡时 $R=G$ 所得）

净复合率 $U_d=R-G=\gamma(np-n_i^2)$

$$=\gamma[(n_0+\Delta n)(p_0+\Delta p)-n_i^2]=\gamma[n_i^2+n_0\Delta p+p_0\Delta n+\Delta n\Delta p-n_i^2]$$

$$=\gamma(n_0+p_0)\Delta p+\gamma\Delta p^2$$

∴ 寿命为 $\tau=\dfrac{\Delta p}{U_d}=\dfrac{1}{\gamma(n_0+p_0)+\gamma\Delta p}$

小注入时 $\Delta p\ll(n_0+p_0)$ $\Rightarrow$ $\tau=\dfrac{1}{\gamma(n_0+p_0)}$　　对n型 $\Rightarrow$ $\tau=\dfrac{1}{\gamma n_0}$ $\begin{cases}T和N_i一定 & \tau近似为常数\\ 随 & n_0或\sigma\uparrow而\downarrow\end{cases}$

大注入时 $\Delta p\gg(n_0+p_0)$ $\Rightarrow$ $\tau=\dfrac{1}{\gamma\Delta p}$　随非平衡 Δp 浓度而变化、不是常数

lX 间接复合求 U_d 和 τ

甲：(俘电) $=\gamma_n n(N_t-n_t)$　　乙（发电）$=s_-n_t$

丙（俘空）$=\gamma_p p n_t$　　丁（发空）$=s_+(N_t-n_t)$

平衡时 甲=乙

$$\Rightarrow \gamma_n n_0(N_t-n_t)=s_-n_t$$

$$\Rightarrow \gamma_n\cdot n_0\frac{N_t\exp\left(\frac{E_t-E_F}{k_0T}\right)}{1+\exp\left(\frac{E_t-E_F}{k_0T}\right)}=s_-\frac{N_t}{1+\exp\left(\frac{E_t-E_F}{k_0T}\right)}$$

$$\Rightarrow s_-=\gamma_n N_c\exp\left(-\frac{E_c-E_t}{k_0T}\right)=\gamma_n n_1\qquad n_1=N_c\exp\left(-\frac{E_c-E_t}{k_0T}\right)$$

同理 $s_+=\gamma_p\cdot p_1$　　$p_1=N_v\exp\left(-\frac{E_t-E_v}{k_0T}\right)$

又∵ 甲+丁 为复合能级上电子的积累　乙+丙为电子的减少

∴ 甲+丁=丙+乙 $\Rightarrow$ $\gamma_n n(N_t-n_t)+\gamma_p p_1(N_t-n_t)=\gamma_n n_1 n_t+\gamma_p p n_t$

$$\Rightarrow n_t=N_t\frac{\gamma_n n+\gamma_p p_1}{\gamma_n(n+n_1)+\gamma_p(p+p_1)}$$

又∵ 净复合率 $U_d=$ 甲$-$乙$=$丙$-$丁　∴ $U_d=\dfrac{N_t\gamma_n\gamma_p(np-n_i^2)}{\gamma_n(n+n_1)+\gamma_p(p+p_1)}$

且 $n_1p_1=n_i^2$

$$\therefore \tau=\frac{\Delta p}{U_d}=\frac{\gamma_n(n_0+n_1)+\gamma_p(p_0+p_1)}{N_t\gamma_n\gamma_p(n_0+p_0)}$$