Key Points

5.0 Brownian Motion Representation & Black-Scholes Formula Using FK and assuming constant r, y, σ, we can immediately integrate the risk-neutral (Q) process for S to obtain:

$$\ln(S_T) - \ln(S_t) = \ln\left(\frac{S_T}{S_t}\right) \sim N\left[\left(r - y - \frac{\sigma^2}{2}\right)(T-t), \sigma^2(T-t)\right]$$

$$q\left[\ln\left(S_T/S_t\right), T - t\right] = \frac{1}{\sigma\sqrt{2\pi(T-t)}} \exp\left[\frac{-\left(\ln\left(S_T/S_t\right) - \left(r - y - \frac{\sigma^2}{2}\right)(T-t)\right)^2}{2\sigma^2(T-t)}\right]$$

- Alternatively, write S_T/S_t and payoff $C_T(S_T)$ in terms of a standard normal variate x:

$$\frac{S_T}{S_t} = \exp\left(\left(r - y - \frac{\sigma^2}{2}\right)(T-t) + \sigma\sqrt{T-t}\, x\right)$$

- Straightforward integration (completing the square) yields BS(M) call option formula:

$$C(S_t, t) = S_t\, e^{-y(T-t)} N[x_+] - K\, e^{-r(T-t)} N[x_-]$$

$$\text{with: } x_\pm = \frac{\ln\left(S_t/K\right) + (r - y)(T-t)}{\sigma\sqrt{T-t}} \pm \frac{\sigma\sqrt{T-t}}{2}$$

  $x_\pm$ can be thought of as standard normal variates

- Put valuation: change two things:

  1. Change sign of payoff function: $[S_T - K]^+ \to [K - S_T]^+$

  2. Change limits on integral: $\int_{x^*}^{\infty} dx \to \int_{-\infty}^{x^*} dx$

- Symmetric formula (Rubinstein):

$$\begin{Bmatrix} C \\ P \end{Bmatrix}(S_t, t) = \phi\left[Se^{-dT} N(\phi\, x_+) - Ke^{-rT} N(\phi\, x_-)\right]$$

  with $\phi = 1$ for calls and $\phi = -1$ for puts

6.0 Inputs

- Risky asset (e.g. **S**tock) price: $S = S_t = S(t)$

  - Units: (quantity of numeraire asset / unit of risky asset) e.g. \$/share
  - For currencies, sometimes use X ("e**X**change rate") for clarity

- Stri**K**e price: K, same units as risky asset

- Ma**T**urity: T (or $T-t$), with units of time (years, months, days...).

- Riskless **r**ate r, with units of time^{-1} (e.g. %/year). Be aware of issues:

  - Day count conventions
  - Traditions in academic literature (Treasury / sovereign debt) vs. industry (LIBOR)
  - Post-crisis "new normal" of OIS for forwards, credit-adjusted discounting (CVA/DVA/XVA)
  - Stochastic vs. constant vs. known future rates

- **y**ield y (q in some notations), with units of time^{-1} (e.g. %/year). Interpretations:

  – Dividend yield (also d or δ) for stocks & indices, also stock borrow/lending rate;
  – Foreign interest rate r_f for currencies;
  – Convenience yield and/or storage rate for commodities and real assets.

  Issues:

  1. Dividends are paid *discretely* (are lumpy) and aren't distributed evenly throughout the year→ sometimes need to model $d \sim \sum_i d_i \delta(t - t_i)$
  2. In the short term, dividend *amounts D* are sticky

Practice Exercises

1. Obtain the BS(M) value of a European put option following the procedure in the notes for a call option and thereby validate the Rubinstein symmetric formula.

2. Daily tasks to be prepared for interviews:
   (a) Where did the market close today (SP 500, but ideally also Dow and NASDAQ – the differences in their daily moves tell you about large vs. small cap behavior)? How much (relative or absolute) was the change since last close? Was this relatively large or small? What were the drivers? If you're interviewing in other markets, follow those indices too. If you're interviewing in currencies, then follow the major FX rates vs. USD, etc... How qualitatively would you describe the market behavior over the last week, month, quarter, and year (looking at price graphs helps build this intuition)? Trending up or down? Range trading? (Unusually) volatile or quiet? Why? What's the dividend yield?

   (b) Pick a favorite stock and follow it as in (a) above. Follow the major news headlines for that stock and use those to rationalize the stock price movements. Does it pay dividends? How much and how often? What are its major products? What is its outlook? What are its major competitors?

   (c) Follow the major headlines on Bloomberg, Yahoo Finance, and/or the WSJ and drill down if possible into stories that seem to be relevant to market behavior. Use those to start to develop a sense of how markets move and why. Use the WSJ headlines in particular to begin to develop a sense of "basic political economy": what's going on in the world at large and how does that move markets? The Economist (weekly) is also extremely good for that at a "big-picture" level.

   (d) Interest rates: Fed Funds: Treasury rates: short term – i.e. one-month T-bill, medium term (1-year T-bill, 2-year T-note), long-term (10-year note; 30-year bond). Swap rates (3-month LIBOR, 1-year swap, maybe some longer tenors, maybe OIS). How have these moved today, in the last month, quarter, and year? Why? What shape is the yield curve? Increasing (normal) with maturity? Decreasing or humped (inverted, sometimes considered a recessionary signal)? What is the outlook? Why?

   (e) Volatility: What's the VIX level? Is that high or low? Why? How has it changed today, last week, month, quarter, year, etc.? Why?

   (f) When do major economic news releases happen (payroll, unemployment, consumer/manu-facturer prices & inflation, FOMC meetings, housing starts & prices...) occur? What were the most recent results and how did they compare to expectations/consensus? How did markets react? What are the forthcoming expectations?