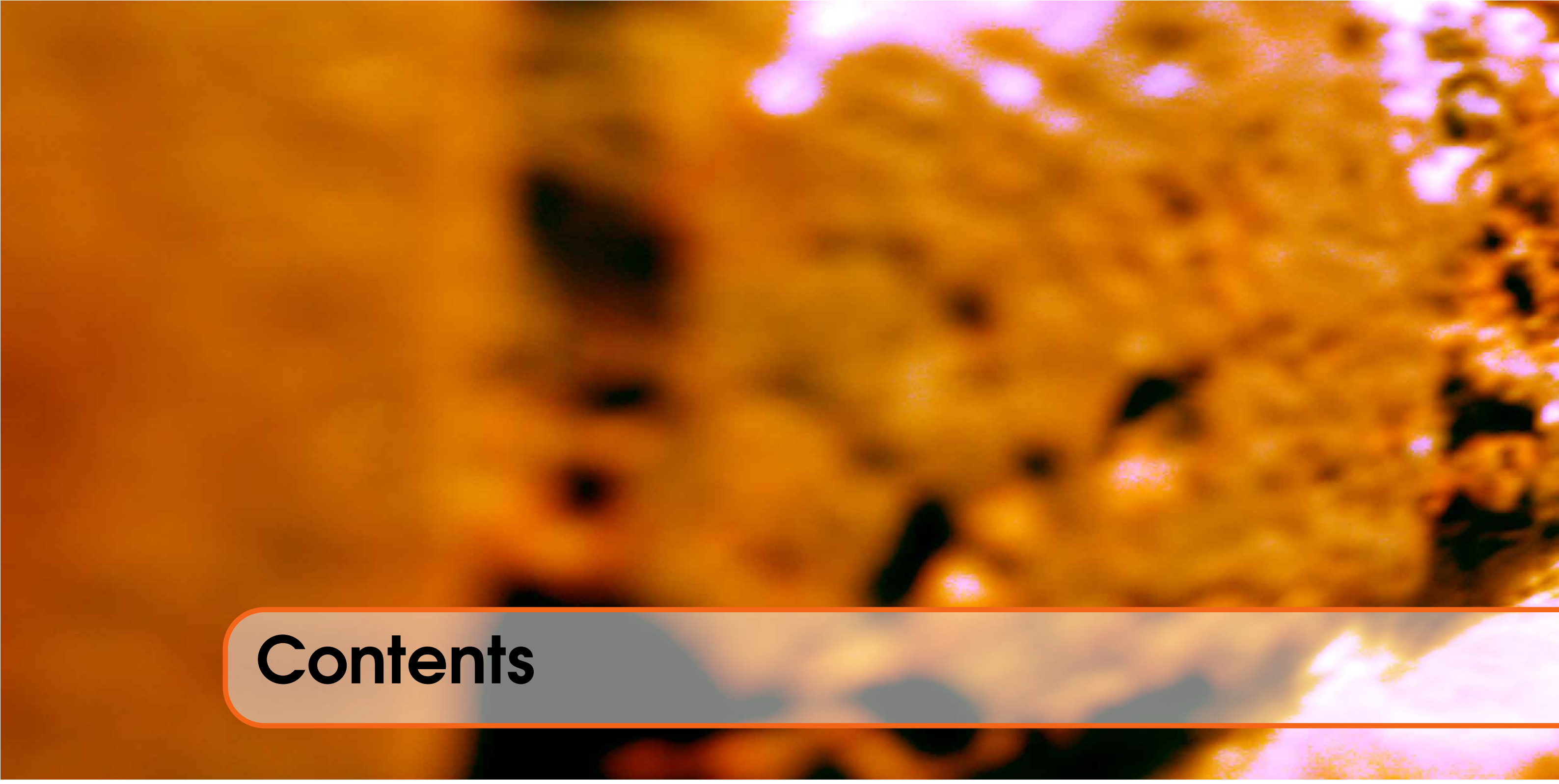
# Calculus

# Solution



# Contents

# 1. Functions

## 1.1 Functions

### 1.1.1 Function

Some types of functions: linear, parabolas, ...

The domain of a function $f$ is the set of all valid input values. The range consists of the set of all output values that can be reached using those domain values.

### 1.1.2 Rational Function

- basic form: $y = \frac{1}{x}$
- vertical asymptote at $x = 0$
- horizontal asymptote at $y = 0$

### 1.1.3 Root Function

- basic form: $y = \sqrt[n]{x}$
- $n$ even: undefined when the root is negative
- $n$ odd: $x \in \mathbb{R}$

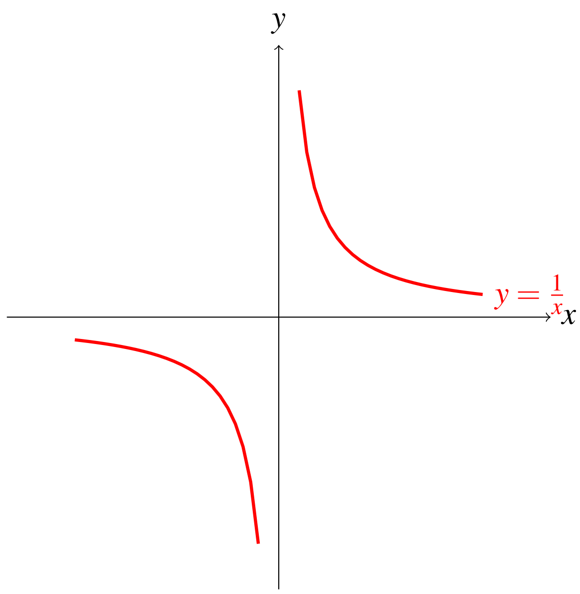



Figure 1.1: rational function

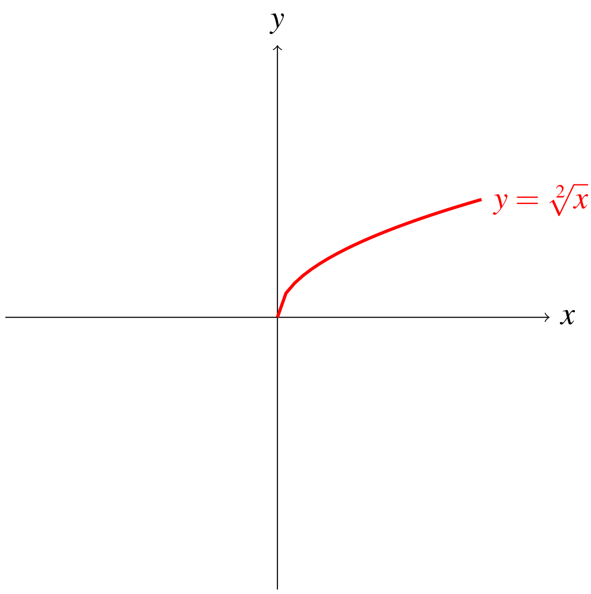



Figure 1.2: root function when $n$ is even

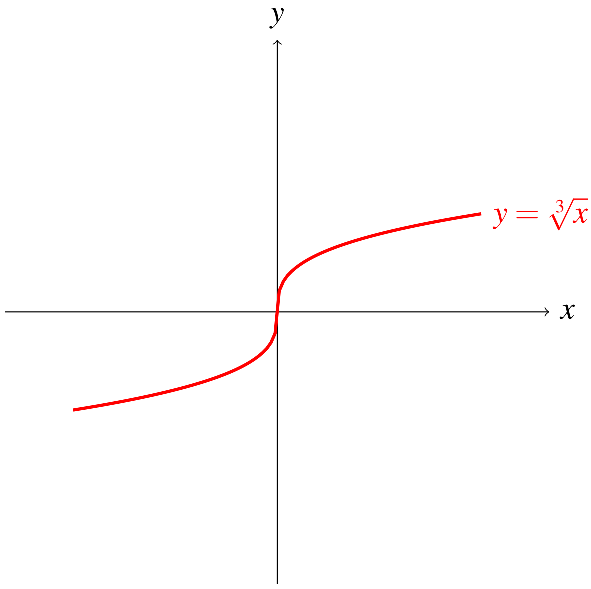



Figure 1.3: root function when $n$ is odd

### 1.1.4 Higher-degree of Polynomial Function

- basic form: $y = x^n$
- domain: $x \in \mathbb{R}$
- $n$ even: both ends of the function tend to $+\infty$ or both tend to $-\infty$
- $n$ odd: one end tends to $+\infty$ while the other tends to $-\infty$

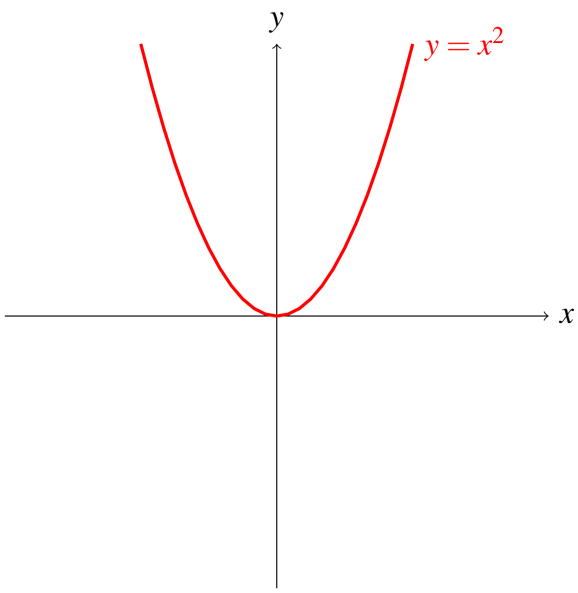



Figure 1.4: polynomial function when $n$ is even

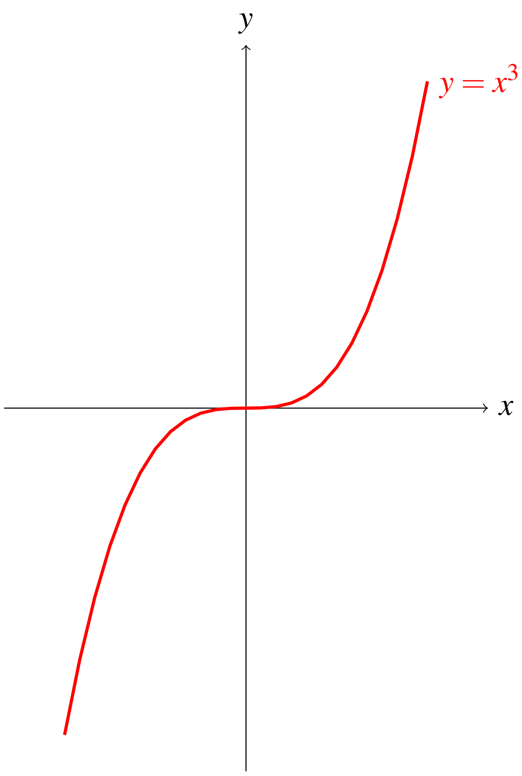



Figure 1.5: polynomial function when $n$ is odd

## 1.2 Angles, Degrees, and Radians

### 1.2.1 Angle

An angle is created by two rays that intersect at a common endpoint. We use Greek letter $\theta$ to denote angles.

An angle that opens counterclockwise from the x-axis is positive.

An angle that opens clockwise from the x-axis is negative.

### 1.2.2 Degree and Radian

Angles can be measured in 2 ways:

1. A degree is a measure of the angle formed by $\frac{1}{360}$ of one complete rotation of a circle.

2. A radian is a measure of the angle formed by the arc of a circle whose length is equal to the circle's radius.

**Theorem 1.2.1 — Degree.**

$$\theta = \frac{s}{r} = \frac{arclength}{radius} \tag{1.1}$$

How are radians and degrees related?

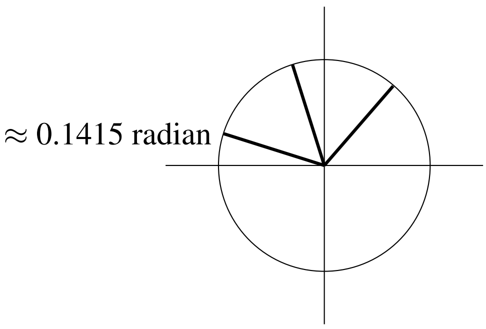



Figure 1.6: radian

**Theorem 1.2.2 — Degrees and radians.**

$$180^\circ = \pi \; radians \tag{1.2}$$

$$1^\circ = \frac{\pi}{180} \; radians \tag{1.3}$$

$$1 \; radian = \frac{180^\circ}{\pi} \tag{1.4}$$

This relationship provides us with a way to easily convert between the two measures.

**Exercise 1.1** Convert from degrees to radians.

(a) $30^\circ = \frac{\pi}{180}(30) = \frac{\pi}{6}$

(b) $220^\circ = \frac{\pi}{180}(220) = \frac{11\pi}{9}$ ■

**Exercise 1.2** Convert from radians to degrees.

(a) $\frac{\pi}{4} = \frac{180^\circ}{\pi} = 45^\circ$

(b) $\frac{5\pi}{6} = \frac{5}{6} \cdot 180^\circ = 150^\circ$ ■

Given any angle $\theta$, what are these equivalent angles?

**Theorem 1.2.3 — Equivalent angles.**

$$\theta + 2k\pi \ (k \in \mathbb{Z}) \tag{1.5}$$

## 1.3 Trigonometric Functions

### 1.3.1 Trigonometric Functions

Let $O$ be the origin and $P(x,y)$ be a point on the unit circle so that the radius $OP$ forms an angle of $\theta$ radians with respect to the positive x-axis.

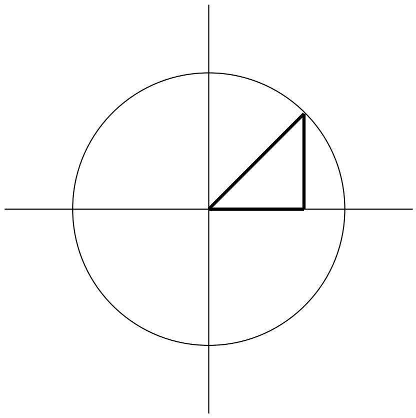
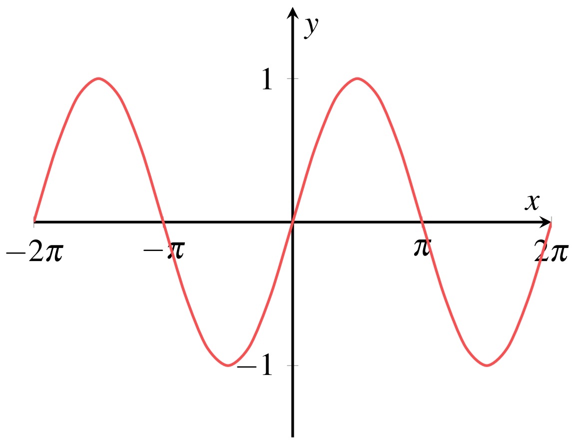


Figure 1.7: radian

**Theorem 1.3.1 — sin / cos.**

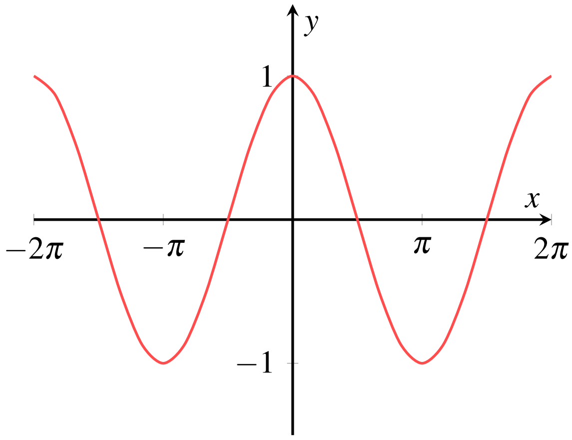
$$x = cos(\theta)$$
$$y = sin(\theta)$$

Here are the three most common trigonometric functions and their reciprocals.



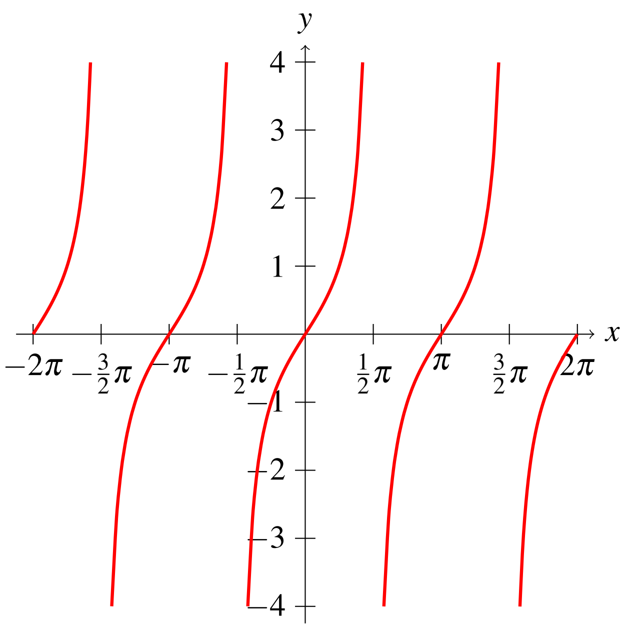

| | |
|---|---|
| range | $-1 \leq sin(\theta) \geq 1$ |
| doamin | $\theta \in \mathbb{R}$ |
| $sin(\theta) = 0$ when | $\theta = k\pi,\ k \in \mathbb{Z}$ |

Figure 1.8: $y = sin(\theta)$



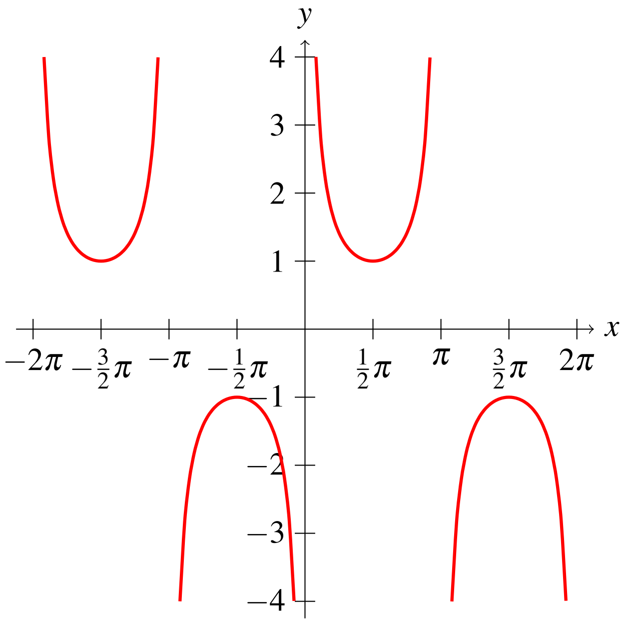

| | |
|---|---|
| range | $\theta \in \mathbb{R}$ |
| doamin | $-1 \leq cos(\theta) \leq 1$ |
| $cos(\theta) = 0$ when | $\theta = \frac{(2k+1)\pi}{2},\ k \in \mathbb{Z}$ |

Figure 1.9: $y = cos(\theta)$



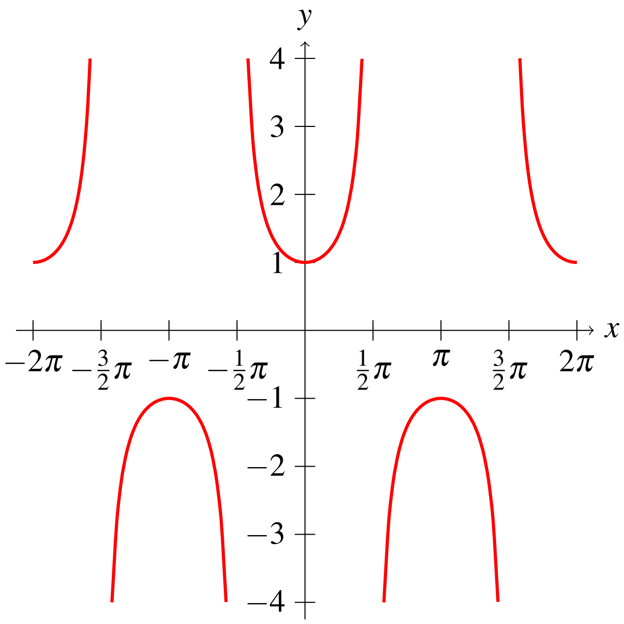

| | |
|---|---|
| range | $\theta \in \mathbb{R}, \theta \neq \frac{(2k+1)\pi}{2}, k \in \mathbb{Z}$ |
| doamin | $tan(\theta) \in \mathbb{R}$ |
| $tan(\theta) = 0$ when | $\theta = k\pi,\ k \in \mathbb{Z}$ |

Figure 1.10: $y = tan(\theta)$



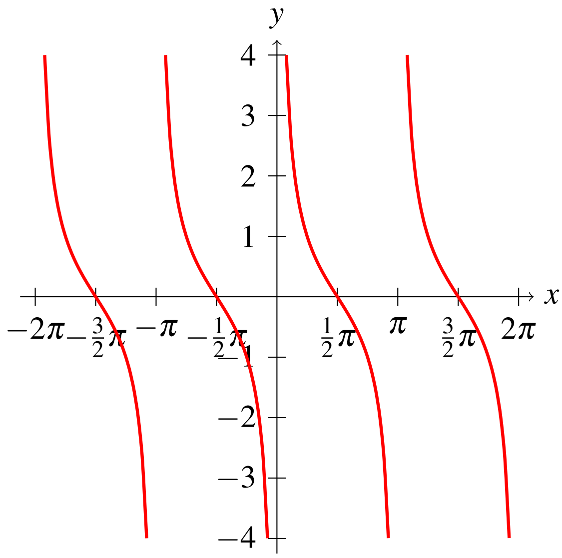

| | |
|---|---|
| range | $\theta \in \mathbb{R},\ \theta \neq k\pi,\ k \in \mathbb{Z}$ |
| doamin | $csc(\theta) \geq 1$ or $csc(\theta) \leq -1$ |
| $csc(\theta) = 0$ when | *never* |

Figure 1.11: $y = csc(\theta)$




| | |
|---|---|
| range | $\theta \in \mathbb{R},\ \theta \neq \frac{(2k+1)\pi}{2},\ k \in \mathbb{Z}$ |
| doamin | $sec(\theta) \geq 1$ or $sec(\theta) \leq -1$ |
| $sec(\theta) = 0$ when | *never* |

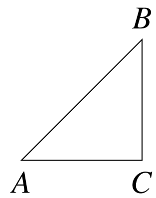
Figure 1.12: $y = sec(\theta)$

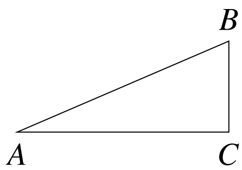



| | |
|---|---|
| range | $\theta \in \mathbb{R},\ \theta \neq k\pi, k \in \mathbb{Z}$ |
| doamin | $cot(\theta) \in \mathbb{R}$ |
| $cot(\theta) = 0$ when | $\theta = \frac{(2k+1)\pi}{2},\ k \in \mathbb{Z}$ |

Figure 1.13: $y = cot(\theta)$

### 1.3.2 Special Triangles




**Exercise 1.3** Evaluate each of the following.

(a) $sin(\frac{\pi}{4}) = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

(b) $cos(\frac{\pi}{4}) = \frac{1}{2}$

(c) $csc(\frac{\pi}{4}) = \frac{\sqrt{2}}{1} = \sqrt{2}$ ■

**Exercise 1.4** Find all values of $\theta$ satisfying the following.

(a) $tan(\theta) = \frac{1}{\sqrt{3}}$

$$\theta = \frac{\pi}{6}$$

(b) $sec(\theta) = \sqrt{2}$

$$\theta = \frac{\pi}{4}$$

(c) $cot(\theta) = \sqrt{3}$

$$\theta = \frac{\pi}{6}$$

■

### 1.3.3 Trigonometric Identities

**Theorem 1.3.2 — Trigonometric Identities.**

$$sin^2(\theta) + cos^2(\theta) = 1 \tag{1.6}$$
$$sin(-\theta) = -sin(\theta) \tag{1.7}$$
$$cos(-\theta) = -cos(\theta) \tag{1.8}$$
$$tan(\theta) = \frac{sin(\theta)}{cos(\theta)} \tag{1.9}$$
$$cot(\theta) = \frac{cos(\theta)}{sin(\theta)} \tag{1.10}$$
$$tan^2(\theta) + 1 = sec^2(\theta) \tag{1.11}$$
$$cot^2(\theta) + 1 = csc^2(\theta) \tag{1.12}$$
$$sin(\theta) = cos(\theta - \frac{\pi}{2}) \tag{1.13}$$
$$sin(a \pm b) = sin(a)cos(b) \pm cos(a)sin(b) \tag{1.14}$$
$$cos(a \pm b) = cos(a)cos(b) \mp sin(a)sin(b) \tag{1.15}$$
$$sin(2\theta) = 2sin(\theta)cos(\theta) \tag{1.16}$$
$$cos(2\theta) = cos^2(\theta) - sin^2(\theta) \tag{1.17}$$
$$sin^2(\theta) = \frac{1 - cos(2\theta)}{2} \tag{1.18}$$
$$cos^2(\theta) = \frac{1 + cos(2\theta)}{2} \tag{1.19}$$

Reminder: $trig^n(x)$ is a notation often used to indicate $(trig(x))^n$.

## 1.4 Exponential Functions

### 1.4.1 Exponential Functions

Exponential functions are of the form $y = a^x$, where $a$ is a positive number and $x$ is any real number. You might see these sorts of functions when studying population growth, economic growth, global temperature, monetary value, etc.

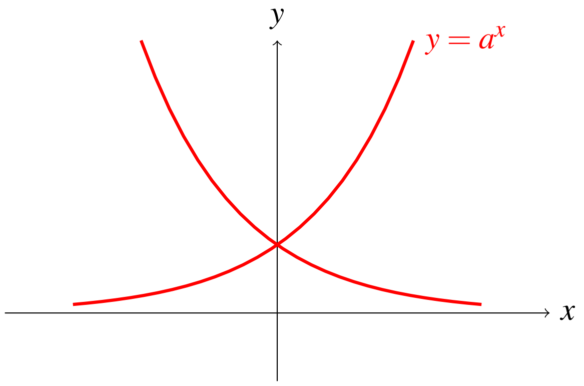



Figure 1.14: exponential function

- Domain: $x \in \mathbb{R}$
- Range: $y > 0$
- The graph $y = a^x$ always passes through $(0,1)$ and $(1,a)$.
- If $a > 1$ then the graph of $y = a^x$ is increasing.
- If $0 < a < 1$ then the graph of $y = a^x$ is decreasing.
- $y = 0$ is always a horizontal asymptote of $y = a^x$.

### 1.4.2 Exponent Rules

**Theorem 1.4.1 — Exponent Rules.**

$$a^{-x} = \frac{1}{a^x} \tag{1.20}$$
$$\frac{1}{a^{-x}} = a^x \tag{1.21}$$
$$(ab)^x = a^x b^x \tag{1.22}$$
$$(\frac{a}{b})^x = \frac{a^x}{b^x} \tag{1.23}$$
$$a^{kx} = (a^k)^x = (a^x)^k \tag{1.24}$$
$$a^m a^n = a^{m+n} \tag{1.25}$$
$$\frac{a^m}{a^n} = a^{m-n} \tag{1.26}$$
$$a^{1/n} = \sqrt[n]{a} \tag{1.27}$$
$$a^{m/n} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m \tag{1.28}$$

### 1.4.3 The Base e

A very special exponential function is $y = e^x$, where $e$ is just a content with a non-terminating decimal like $\pi$.

$$e = 2.718281845...$$

What is so special about an exponential function with base $e$?

At any point on the graph, the height of the exponential function is equal to the slope of the tangent line to the graph at that point.

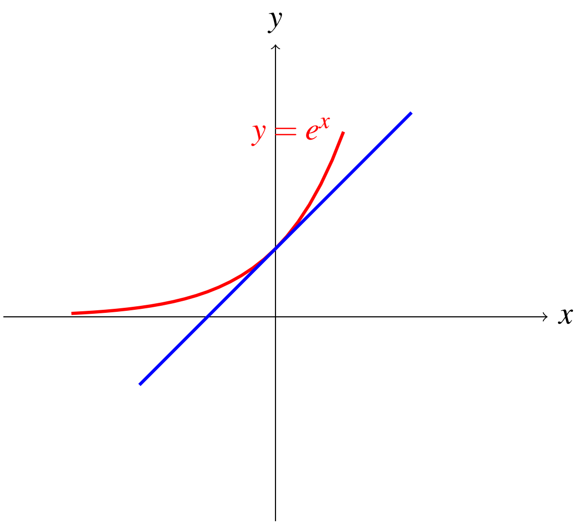



Figure 1.15: $y = e^x$

## 1.5 Logarithmic Functions

### 1.5.1 Logarithmic Functions

Logarithms are the inverse of exponential functions. Let $a > 0$, then we define a logarithm (log) as follows:

**Theorem 1.5.1 — Logarithmic Functions.**

$$y = log_a(x) \tag{1.29}$$

$$a^y = x \tag{1.30}$$

If no base $a$ is shown, a base of 10 is assumed.

For example:

$$log(x) = log_{10}(x)$$

**Exercise 1.5** Evaluate each of the following.

(a) $log_2 8 = 3$

(b) $log(100) = 2$

(c) $log_5 \frac{1}{25} = -2$

(d) $log_8 1 = 0$ ■

Since any positive number to the power of 0 is equal to 1, we have the property that $log_a(1)$, no matter what the base $a$ is.

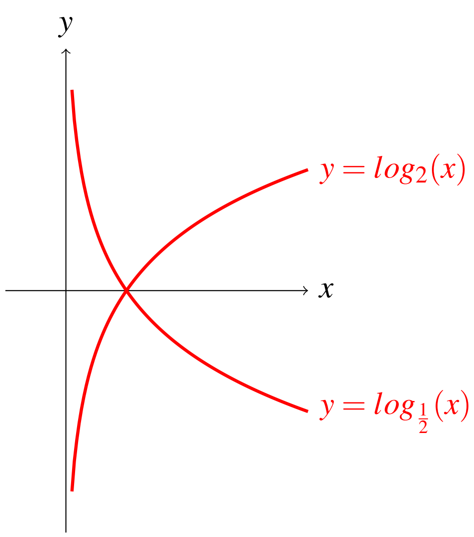



Figure 1.16: logarithmic function

- Domain: $0 < x < \infty$
- Range: $y \in \mathbb{R}$
- $y = log_a(x)$ always passes through $(a, 1)$ and $(1, 0)$.
- If $a > 1$ then the graph of $y = log_a(x)$ is increasing.
- If $0 < a < 1$ then the graph of $y = log_a(x)$ is decreasing.
- $x = 0$ is always a vertical asymptote of $y = log_a(x)$.

### 1.5.2 Logarithm Rules

**Theorem 1.5.2 — Logarithm Rules.**

$$log_a(a^x) = x \tag{1.31}$$
$$a^{log_a(x)} = x \tag{1.32}$$
$$log_a(xy) = log_a(x) + log_a(y) \tag{1.33}$$
$$log_a(\frac{x}{y}) = log_a(x) - log_a(y) \tag{1.34}$$
$$log_a(x^n) = nlog_a(x) \tag{1.35}$$

### 1.5.3 Change of Base Formula

We can switch between any two bases easily by using the formula:

**Theorem 1.5.3 — Change of Base Formula.**

$$log_a(x) = \frac{log_b(x)}{log_b(a)} \tag{1.36}$$

**Exercise 1.6** Proof

$$\because y = log_a(x) \leftrightarrow a^y = x$$
$$\begin{aligned} log_b(x) &= log_b(a^y) \\ &= ylog_b(a) \\ &= log_a(x) \cdot log_b(a) \end{aligned}$$
$$\therefore \frac{log_b(x)}{log_b(a)} = log_a(x)$$

■

**Exercise 1.7** Convert $log_4(x)$ into a logarithm with each of the following bases.

(a) base 3

$$log_3(x) = \frac{log_4(x)}{log_4 3}$$

(b) base 22

$$log_{22}(x) = \frac{log_4(x)}{log_4(22)}$$

■

### 1.5.4 The Natural Logarithm

A special logarithm is the natural logarithm, which is the logarithm with a base of $e$. Rather than write $log_e(x)$, we typically write $ln(x)$.

The natural logarithm has the exact same properties as any other logarithmic function.

**Theorem 1.5.4 — Natural Logarithm.**

$$ln(e^x) = x \tag{1.36}$$

$$e^{ln(x)} = x \tag{1.37}$$

**Exercise 1.8** Solve each of the following for $x$.

(a)

$$\begin{aligned} 2^x &= 2^{1-x} \\ x &= 1 - x \\ 2x &= 1 \\ x &= \frac{1}{2} \end{aligned}$$

(b)

$$\begin{aligned}
3^{\frac{1}{2}+10} &= 27 \\
3^{\frac{1}{2}+10} &= 3^3 \\
\frac{1}{2}x+10 &= 3 \\
\frac{1}{2}x &= -7 \\
x &= -14
\end{aligned}$$

(c)

$$\begin{aligned}
2^x &= 10 \\
ln(2^x) &= ln(10) \\
xln(2) &= ln(10) \\
x &= \frac{ln(10)}{ln(2)}
\end{aligned}$$

(d)

$$\begin{aligned}
log(x)-1 &= log(x-1) \\
-1 &= log(x-1)-log(x) \\
-1 &= log(\frac{x-1}{x}) \\
10^{-1} &= 10^{log(\frac{x-1}{x})} \\
\frac{1}{10} &= \frac{x-1}{x} \\
x &= 10(x-1) \\
-9x &= -10 \\
x &= \frac{10}{9}
\end{aligned}$$

(e)

$$\begin{aligned} log_2(x) + log_2(x^2) &= 6 \\ log_2(x \cdot x^2) &= 6 \\ log_2(x^3) &= 6 \\ 3log_2(x) &= 6 \\ log_2(x) &= 2 \\ x &= 4 \end{aligned}$$

(f)

$$\begin{aligned} log_2(x^4) + log_2(x^2) &= 6 \\ log_2(x^4 \cdot x^2) &= 6 \\ log_2(x^6) &= 6 \\ 2^{log_2(x^6)} &= 2^6 \\ x^6 &= 64 \\ x &= \pm 2 \end{aligned}$$

■

# 2. Inequalities

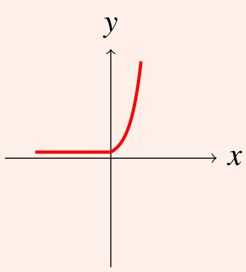
## 2.1 Piecewise Functions

### 2.1.1 Piecewise Functions

Piecewise functions typically feature one or more points at which the function changes from one form to another. To graph a piecewise function, simply graph each piece and then restrict it to its designated domain. Pay special attention when plotting the breaking point (closed circle includes the point, open circle excludes the point).

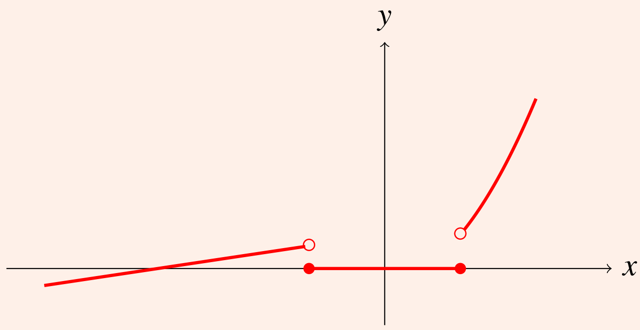
**Exercise 2.1** Graph the piecewise function given by

$$y = \begin{cases} 1 & x \leq 0 \\ 4^x & x > 0 \end{cases}$$



**Exercise 2.2** Graph the piecewise function given by

$$y = \begin{cases} \frac{1}{2}x + 3 & x < -2 \\ 0 & -2 \leq x \leq 2 \\ x^2 - 1 & x > 2 \end{cases}$$

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## 2.2 Absolute Value Functions

### 2.2.1 Absolute Value Functions

A very special and common piecewise function is the absolute value function.

- $y = |x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$
- $x \in \mathbb{R}$
- $y \geq 0$

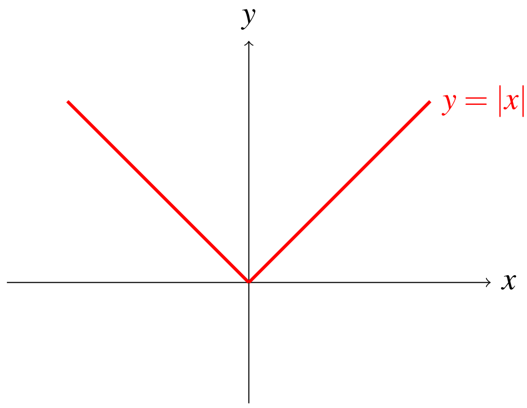



Figure 2.1: absolute value function

**Theorem 2.2.1 — Absolute Values.**

$$|ab| = |a| \cdot |b| \tag{2.1}$$

$$|\frac{a}{b}| = \frac{|a|}{|b|} \tag{2.2}$$

$$if\ |a| \leq b,\ then\ -b \leq a \leq b \tag{2.3}$$

$$if\ |a| \geq b,\ then\ a \geq b\ or\ a \leq -b \tag{2.4}$$

$$(Triangle\ Inequality)\ |a+b| \leq |a| + |b| \tag{2.5}$$

## 2.3 Inequalities Notation

### 2.3.1 Inequalities Notation

When solving equation we may get a single answer, or a number of answers that satisfy the equation.

Consider $3x - 5 = 1$, only one value satisfies this equation.

But if we consider $x^2 - 1 = 3$, more than one value satisfies this equation.

Inequalities notation like $1 \leq x < 3$, where the symbols $\leq$ and $\geq$ indicate inclusion of an endpoint, and $<$ and $>$ indicate exclusion of an endpoint.

A second notation is interval notation, for example, $x \in [1,3)$, where a square (or closed) bracket indicates inclusion of an endpoint, and a round (or open) bracket indicates exclusion of an endpoint.

The infinity symbol $\infty$ is always accompanied by round brackets.

**Exercise 2.3** Write each of the following in interval notation.

(a) $2 \leq x \leq 7$

$$x \in [2,7]$$

(b) $x < 9$

$$x \in (-\infty, 9)$$

(c) $-3 > x > 0$

$$x \in \emptyset$$

■

**Exercise 2.4** Write each of the following using inequalities.

(a) $x \in [3,6)$

$$3 \leq x < 6$$

(b) $x \in (-2,4)$

$$-2 < x < 4$$

(c) $x \in (-\infty, -1]$

$$x \leq -1$$

It is possible to have ranges of values that are disjoint. We use the union symbol $\cup$ to include all of the values in any of the disjoint ranges. For example, $[-1,4)\cup[7,10)$ meas $-1 \leq x < 4$ or $7 \leq x < 10$.

**Exercise 2.5** Express each of the following in interval notation.

(a) $-3 \leq x < \frac{1}{2}$ or $4 < x < 7$

$$x \in [-3, \frac{1}{2}] \cup (4,7)$$

(b) $1 \leq x < 5$ or $3 \leq x < 7$

$$x \in [1,7)$$

(c) $x \in [-2,6) \bigcup (0,5)$

$$x \in [-2,6)$$

Intersection symbol $\cap$ allows only the values that are common between intervals. For example, $[-1,6)\cap(2,7)$ means $(2,6)$.

**Exercise 2.6** Express each of the following in interval notation.

(a) $-2 < x \leq 6$ and $0 < x < 7$

$$x \in (0,6]$$

(b) $x \in [0,5] \bigcap [3,5]$

$$x \in [3,5]$$

(c) $-4 < x < 0$ and $3 < x < 7$

$$x \in \emptyset$$

### 2.3.2 Solving Inequalities

When solving inequalities, there are a few rules that we must follow:

1. When it comes to addition, subtraction, multiplication, and division, what you do to one side of the inequality, you must do to the other.

2. If you multiply or divide by a negative quantity, you must flip the inequality.

3. If both sides are positive or both sides are negative, then you can take the reciprocal of both sides, but you must flip the inequality.

**Exercise 2.7** Find all values of x that satisfy the following.

(a)

$$\begin{aligned} -6x+7 &\geq 8x \\ -14x &\geq -7 \\ x &\leq \frac{1}{2} \end{aligned}$$

(b)

$$\begin{aligned} -\frac{5}{2} &< 4-2x \leq 1 \\ -\frac{13}{2} &< -2x \leq -3 \\ \frac{13}{4} &> x \geq \frac{3}{2} \\ x &\in [\frac{3}{2}, \frac{13}{4}) \end{aligned}$$

(c)

$$\begin{aligned} 5x^3+27 &> -13 \\ 5x^3 &> -40 \\ x^3 &> -8 \\ x &> -2 \end{aligned}$$

(d)

$$\begin{aligned} 3x^2+2 &< -4 \\ 3x^2 &< -6 \\ x^2 &< -2 \\ x &\in \emptyset \end{aligned}$$

(e)

$$\begin{aligned}
\sqrt{x-1} &> 4 \\
x-1 &> 16 \\
x &> 17
\end{aligned}$$

(f)

$$\begin{aligned}
log_2(3x) &\leq -3 \\
2^{log_2(3x)} &\leq 2^{-3} \\
3x &\leq \frac{1}{8} \\
x &\leq \frac{1}{24}
\end{aligned}$$

## 2.4 The Case Method

### 2.4.1 The Case Method

Consider the following example: $\frac{x-3}{x-1} < 10$

You might be tempted to cross multiply, but be careful! The quantity $x-1$ is not always positive. If we multiply by $x-1$, the inequality needs to flip for some values of $x$. How might we deal with this?

1. Separate into 2 cases

| **Case 1** | **Case 2** |
|---|---|
| $x-1 > 0$ | $x-1 < 0$ |
| $x > 1$ | $x < 1$ |

2. Solve the original problem under each assumption

| **Case 1** | **Case 2** |
|---|---|
| $\frac{x-3}{x-1} < 10$ | $\frac{x-3}{x-1} < 10$ |
| $x-3 < 10(x-1)$ | $x-3 > 10(x-1)$ |
| $-9x < -7$ | $-9x > -7$ |
| $x > \frac{7}{9}$ | $x < \frac{7}{9}$ |

3. Find all common points between assumption and solution

| **Case 1** | **Case 2** |
|---|---|
| $x > 1$ *and* $x > \frac{7}{9}$ | $x < 1$ *and* $x < \frac{7}{9}$ |
| $x \in (1, \infty)$ | $x \in (-\infty, \frac{7}{9})$ |

4. Consolidate the 2 cases by taking the union

$$x \in (1, \infty) \cup (-\infty, \frac{7}{9})$$

**Exercise 2.8** Find all values of x such that $\frac{7x-2}{1-2x} \geq 4$.

1. Separate into 2 cases

| **Case 1** | **Case 2** |
|---|---|
| $1-2x > 0$ | $1-2x < 0$ |
| $x < \frac{1}{2}$ | $x > \frac{1}{2}$ |

2. Solve the original problem under each assumption

| Case 1 | | | Case 2 | | |
|---|---|---|---|---|---|
| $\frac{7x-2}{1-2x}$ | $\geq$ | $4$ | $\frac{7x-2}{1-2x}$ | $\geq$ | $4$ |
| $7x-2$ | $\geq$ | $4(1-2x)$ | $7x-2$ | $\leq$ | $4(1-2x)$ |
| $15x$ | $\geq$ | $6$ | $15x$ | $\leq$ | $6$ |
| $x$ | $\geq$ | $\frac{2}{5}$ | $x$ | $\leq$ | $\frac{2}{5}$ |

3. Find all common points between assumption and solution

| Case 1 | Case 2 |
|---|---|
| $x < \frac{1}{2}$ *and* $x \geq \frac{2}{5}$ | $x > \frac{1}{2}$ *and* $x \leq \frac{2}{5}$ |
| $x \in [\frac{2}{5}, \frac{1}{2})$ | $x \in \emptyset$ |

4. Consolidate the 2 cases by taking the union

$$x \in [\frac{2}{5}, \frac{1}{2})$$

## 2.5 The Number Line Method

### 2.5.1 The Number Line Method

Another method for solving inequalities uses the following basic logic:

$$(+)(+) = + \qquad \frac{(+)}{(+)} = +$$

$$(-)(-) = + \qquad \frac{(-)}{(-)} = +$$

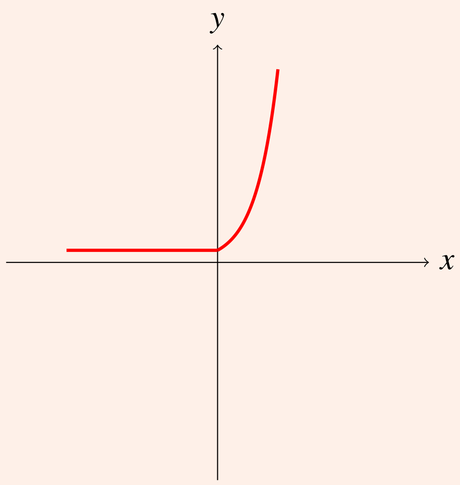
$$(+)(-) = - \qquad \frac{(+)}{(-)} = -$$

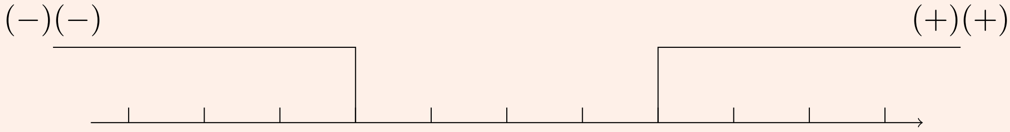
$$(-)(+) = - \qquad \frac{(-)}{(+)} = -$$

By manipulating expressions into factors that are multiplied and/or divided on one side of the inequality (with a zero appearing on the other side), we can simply consider the combinations of positive and negative factors to draw conclusions.

**Exercise 2.9** Find all values of x that satisfy $3x^2 - 13x > -10$.

$$\begin{aligned} 3x^2 - 13x &> -10 \\ 3x^2 - 13x + 10 &> 0 \\ 3x(x-1) - 10(x-1) &> 0 \\ (x-1)(3x-10) &> 0 \end{aligned}$$

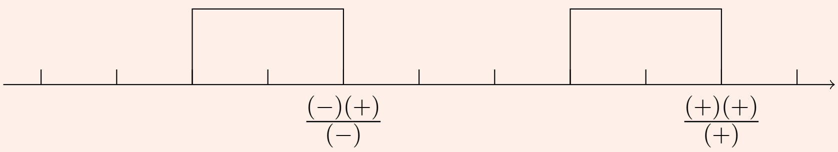





$$x \in (-\infty, 1) \cup (\frac{10}{3}, \infty)$$

**Exercise 2.10** Find all values of x that satisfy $x-2 \geq \frac{4}{x+1}$.

$$\begin{aligned}
x-2 &\geq \frac{4}{x+1} \\
\frac{(x-2)(x+1)-4}{x+1} &\geq 0 \\
\frac{x^2-x-6}{x+1} &\geq 0 \\
\frac{(x-3)(x+2)}{x+1} &\geq 0
\end{aligned}$$




$$x \in [-2, 1) \cup [3, \infty)$$