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Atmospheric Tides and the Resonant Rotation of Venus

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The observed spin–orbit resonance of Venus, whereby the same side of Venus faces the Earth at each inferior conjunction, cannot be explained adequately by gravitational interaction with the Earth alone. The expected solar tidal drag on the solid body of Venus would easily overwhelm the Earth's couple upon any reasonable permanent deformation of Venus. If there exists, however, a solar atmospheric tide, partly thermally induced and similar to that known on the Earth, its torque may counteract that due to the solar solid body tide at a particular rotation period. The small interaction with the Earth is then sufficient to lock the period to one of the resonances in the vicinity of that angular velocity.

INTRODUCTION

Venus is now known to rotate very precisely with an angular velocity that causes it to face the Earth with the same side at each inferior conjunction. This astonishing apparent domination of the rotation rate of Venus by the Earth has not so far been adequately explained. The spin–orbit resonance between the two planets is undoubtedly due to the action of the Earth upon the net permanent deformation (or transverse quadrupole moment) of Venus. But in order for this rather weak torque, which is significant only near inferior conjunction, to overcome the constant drag of solar tidal friction on the body of Venus, the permanent quadrupole moment required for Venus would have to be at least an order of magnitude larger than comparison with the Earth would suggest. If this permanent bulge is assumed to be no larger than in the case of the Earth, the solar drag could not be balanced and Venus could not have been arrested in the resonance. The solution to this dilemma may lie in the atmospheric tides, which could work to counteract the solar solid body tide and, because of a different dependence on the rotation rate, could balance it precisely at some particular rotation period. In the vicinity of such a balance a much smaller couple exerted by the Earth may then nevertheless dominate.

It was first shown by Lord Kelvin (Thompson, 1882) that the Earth's atmospheric tides transfer angular momentum and energy from the orbit to the rotation and tend to accelerate the latter. Holmberg (1952) suggested that the associated production of mechanical energy might be sufficient to balance the dissipation of oceanic and solid body tides, so that the Earth's present rotation rate would be one of stable equilibrium over geological time. Subsequent studies, however, appear to confirm the older view that the Earth is in fact slowing down; i.e., that the energy loss in conventional tides *exceeds* the input through atmospheric tides. Nevertheless, the notion that a planet's rotation rate might be stabilized in the balance between the opposing two kinds of tides is perhaps the key to the resonant rotation of Venus.

DESCRIPTION OF RESONANCES AND TIDES

Venus is observed to spin in the retrograde sense and with the longest known sidereal period of any body in the solar system. It appears likely (as will be shown) that the planet in fact originated with a retrograde spin, although probably more rapid than the present one. As it slowed down over the age of the solar system, Venus must have passed through a series of spin states possessing a successively decreasing whole or half-integer number of diurnal rotations fitting the duration of the almost fixed synodic period. Such resonant spin states, in which the same or alternating hemispheres of Venus are presented toward the Earth at successive inferior conjunctions, are potentially stable; i.e., the rotation can be "captured" at such values in spin-orbit coupling (Goldreich and Peale, 1966).

Increasingly more accurate radar measurements of the Venus solid body rotation over the past few years (obtained through radar observations of the motion of identified regions seen repeatedly at the Arecibo and Goldstone installations) have reduced the error and give a very precise period (Fig. 1). The best current value is 243.00 ±

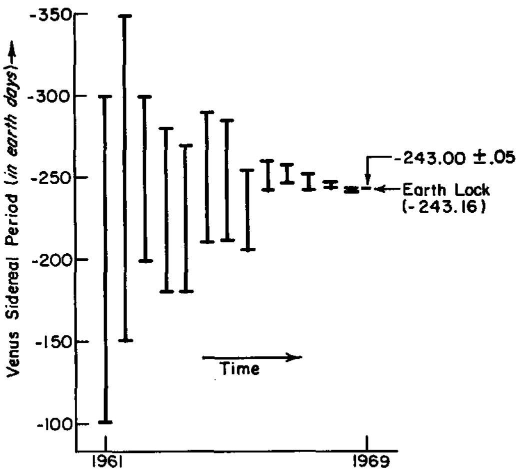

FIG. 1. Radar determination of the retrograde sidereal period of Venus. With increasing accuracy, the values have converged toward the resonant period of −243.16 days. The next nearest resonant periods are at −201 and −308 days. Figure adapted from Eshleman (1967); see also Shapiro (1967).

0.05 days retrograde (Jurgens, 1970), which strongly suggests that the time averaged period is 243.16 days retrograde, one of the resonant values. The measurement is improbably close for this fit to be accidental, the next nearest resonant periods being at 201 and 308 days retrograde. (However it is sufficiently removed from the precise time averaged resonant value—by about three standard deviations —to suggest a physical libration.)

Now Venus, like the other terrestrial planets, is not a perfect spheroid and must possess some small but relatively permanent net longitudinal asymmetry giving rise to a transverse quadrupole moment. This might be due to a deformation in the shape or to density inhomogeneities arising from mantle convection (Runcorn, 1967; Goldreich and Toomre, 1969) or from the presence of mascons (O'Leary, Campbell,and Sagan, 1969). For the Earth, the equatorial axes of the equivalent triaxial ellipsoid differ by about 140 m (Caputo, 1967), and we assume for the present that Venus possesses a net permanent deformation of the same order. The gravitational torque exerted by the Earth on this Venus quadrupole moment, averaged over a synodic period, has a sign that tends to spin up or spin down the planet toward a resonant rotation state once it is close to one.

A resonant rotation state can only be maintained if the mean effect of the Earth is large enough to beat the sum total of the solar tidal torques exerted on Venus. For the solid body tides, the semidiurnal tidal bulges lag behind the Sun (and anti-sun) and the consequent restoring couple always works to decelerate the planet's spin. The tidal disturbance has a maximum height on the order of only 30 cm as compared to the proposed 140-m net permanent deformation. On the other hand, it is the gravitational field of the Sun that acts on the 30-cm bulge all the time, compared with the Earth's gravitational field acting on the 140-m bulge essentially only near inferior conjunction. As a result, the solar tidal drag is sufficient to overwhelm the Earth's torque. It is presumed that the solar tidal drag has slowed Venus from its assumed more rapid initial retrograde spin, overriding a number of resonant states until capture occurred into the present one.

The present resonance is stronger than any that Venus would have traversed in the past (the strength increases as the number of diurnal half-turns in a synodic period decreases; i.e., with slower spin). There appears to be nothing special about the present resonance. Rather, as shown by Goldreich and Peale (1967), it is simply one of the first resonances of sufficient strength to exceed the total solar tidal torque.

Quantitatively, however, this explanation is inadequate if one uses for the solar torque that calculated for the solid body tide alone. For the observed resonance, the Earth's average torque upon the Venus quadrupole moment is then much too weak to establish resonant rotation, even with the assumption of the lowest tidal dissipation in the body of Venus that one might reasonably expect. If one wished to overcome the difficulty by simply assuming a large enough quadrupole moment for Venus, then one would be required to explain a permanent deformation at least a factor of 10 larger than that possessed by the Earth; i.e., the equatorial radii of the equivalent triaxial ellipsoid for Venus would have to differ by at least 1.4 km.

Now radar studies of Venus at the Arecibo Observatory place an upper bound upon the low-latitude surface relief of about 3 km (Dyce, Campbell, and Jurgens, 1969). The Earth's relief is a few times this amount. Assuming that surface relief and gravitational quadrupole moment scale in even roughly similar fashion for the two planets, this would indicate a Venus quadrupole moment no larger than the Earth's, i.e., an equivalent longitudinal bulge for Venus of order $\lesssim 140$ m. Another indirect measure of this quantity is provided by analysis of the Mariner V tracking data (Anderson and Efron, 1969) which yields a polar "flattening" coefficient for Venus of $J_2 = (-5 \pm 10) \times 10^{-6}$. The negative value itself is improbable as it would imply rotation about an axis remote from that of greatest moment of inertia and least energy. However, the positive part of the large uncertainty in J_2 may be taken to estimate an upper bound on the equivalent deviation from sphericity along the polar axis of at most a few hundred meters. (Hydrostatic rotational flattening, which goes as the square of angular velocity, should only amount to about 40 cm for Venus. It is worth noting that the polar flattening of the Earth differs from the equilibrium shape by an amount that implies an internal rigidity comparable to or greater than that implied by the Mariner V results for Venus.) For the Earth, the rotation axis appears to be guided by the axis of greatest moment of inertia (Gold, 1955; Goldreich and Toomre, 1969); the same should be true for the case of Venus, and any deformation in longitude (or corresponding mass asymmetry) is not likely to exceed the measured upper bound for the polar discrepancy, and certainly not by a factor of more than 10.

Such a large asymmetry would indicate internal stresses far exceeding those in the Earth. The high surface temperature on Venus, on the other hand, would be expected to result in a lower strength of the outer mantle (Mueller, 1969), and thus a smaller permanent deformation. Goldreich and Peale (1968) suggest the possibility that large convective cells in the Venus mantle provide the gravitational asymmetry required. This, however, is unlikely if the Mariner V results are correct. They also (1966) allude to the suggestion made by T. Gold in 1964 that atmospheric tides cause the anomalous rotation of Venus, an idea discussed by MacDonald (1964) and Öpik (1965). It is this suggestion that we intend to explore.

In the Earth's atmosphere there is observed a semidiurnal tide, partly of solar thermal origin. Its phase leads the Sun by about 2 hours. The solar gravitational couple acting upon this atmospheric tide is of comparable magnitude to the solar couple on the solid body tide. They do not balance, however, because the latter is joined by lunar ocean and body tides that cause a deceleration of the Earth, and since the Moon does not stay in phase with the heating cycle of the atmosphere, its gravitational interaction does not contribute to the tidal couple of opposite sign. But if the Venus atmosphere had a semidiurnal tide comparable to the Earth's (with a similar phase lead), the associated torque would be able to nearly balance the solid body tidal couple due to the Sun, while, of course, not having to compete with a decelerating lunar torque.

These arguments merely show that for Venus the solid body and the atmospheric tidal couples may well be of comparable magnitude and opposite sign. It is, of course, most unlikely that they would precisely balance at all angular velocities. It is not unlikely, however, that the two tides have a different dependence on the angular velocity, especially since the thermal atmospheric tide is clearly related to thermal time constants. Thus if the ratio of the strengths of the two tides crosses unity at some particular angular velocity, then Earth resonances in the neighborhood of that angular velocity will dominate, even with a permanent deformation for Venus considerably smaller than that of the Earth. We now proceed to a quantitative examination of the above statements.

Conventional Resonant Stability

The orbits of Venus and the Earth can be approximated by coplanar circles. In the usual notation the principal moments of inertia for Venus are $A < B < C$, where C is the moment about the spin axis, assumed normal to the orbit plane. Following the procedure of Goldreich and Peale (1966), let α be the heliocentric angular separation between Venus and the Earth and let β be the longitude of the subsolar point with respect to a fixed meridian on Venus (Fig. 2). Then in general, $\langle\dot{\beta}\rangle = q\langle\dot{\alpha}\rangle$, where $|q|$

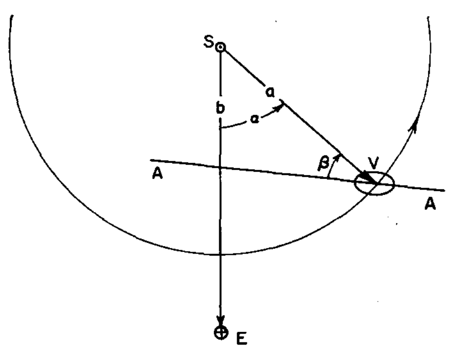

Fig. 2. Motion of Venus (V) in the frame of a fixed Earth (E) and Sun (S). The line AA marks the axis of least moment of inertia.

is in fact the ratio of the synodic period (583.92 days) to the length of the solar day on Venus. For resonant rotation, q will have an integer or half-integer value; $q = -5$ for the observed resonance, the minus sign indicating a retrograde rotation.

The stabilizing Earth torque, averaged over a synodic period, upon the permanent deformation of Venus, is calculated by Goldreich and Peale (1966) for the qth resonance and can be expressed as

$$T_q = \frac{GM_\oplus MR^2}{2b^3}\frac{B-A}{C}K(q), \qquad (1)$$

where G is the gravitational constant, $M_\oplus$ is the Earth's mass, M and R are the mass and radius of Venus, b is the astronomical unit, $(B-A)/C$ is a measure of the Venus transverse quadrupole moment, and the dimensionless $K(q)$ are tabulated. In particular, the relative strength of the observed resonance is $K(-5) = 2.53$.

For the earth, the measured difference in the A and B equivalent radii of about 140 m leads to a $(B-A)/C$ of about 2.2×10^{-5}. If we provisionally adopt this value for Venus, then Eq. (1) can be evaluated as

$$T_q = 1.1 \times 10^{26} K(q)\,(B-A)/C$$
$$= 2.4 \times 10^{21} K(q).$$

(All torque magnitudes in this paper refer to cgs units.) According to Goldreich and Peale (1966), the $K(q)$ and hence the T_q of the resonances slowly increase over the range $q = -8$ to $q = -2.5$. But the whole series of T_q scale together in proportion to the adopted $(B-A)/C$.

In the lower part of Fig. 3, we have plotted the resonant torque magnitudes for $q = -8.0, -7.5, \ldots, -0.5$ against rotation frequency with respect to the Sun, given by q itself. A corresponding abscissa shows the length of the diurnal period $D = 584/q$ in Earth days. Finally the equivalent sidereal rotation period is given in Earth days as $S = 584/(2.6 + q)$. Note that inertial nonrotation occurs at $q = -2.6$.

For simplicity, only the resonant maxima, T_q, of the Earth torques are indicated (as spikes). The typical behavior of the permanent deformation torques *between* the resonant q values is that they are negative for spin faster than resonance (tending to spin down Venus) and positive for spin slower (tending to spin up the planet), with the maximum strength changing sign across the resonance. Most of the resonances shown in Fig. 3 are of this kind and, if there were no disturbing torques, would mark the loci of stable rotational equilibria for Venus.

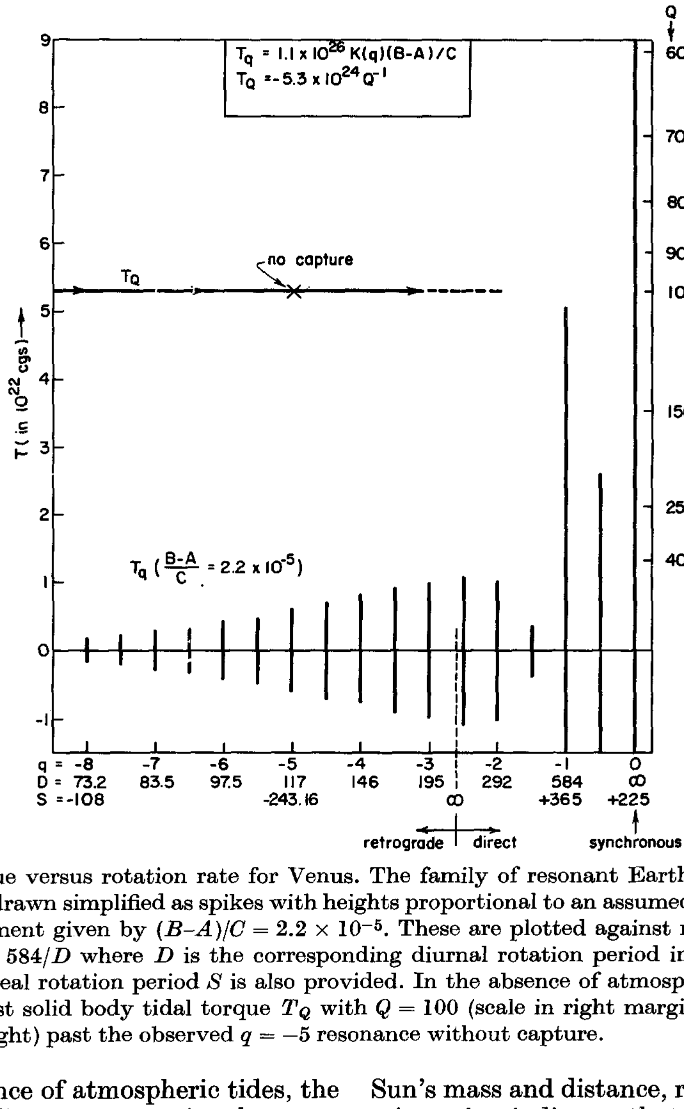

FIG. 3. Torque versus rotation rate for Venus. The family of resonant Earth torques T_q upon Venus are here drawn simplified as spikes with heights proportional to an assumed Venus transverse quadrupole moment given by $(B-A)/C = 2.2 \times 10^{-5}$. These are plotted against rotation frequency as given by $q = 584/D$ where D is the corresponding diurnal rotation period in Earth days. The equivalent sidereal rotation period S is also provided. In the absence of atmospheric tidal torque, even the weakest solid body tidal torque T_Q with $Q = 100$ (scale in right margin), will spin down Venus (left to right) past the observed $q = -5$ resonance without capture.

In the absence of atmospheric tides, the chief destabilizing torque is the conventional couple due to the Sun's action upon the solid body tide it raises. This torque is discussed by Jeffreys (1962) and by Goldreich and Peale (1968). For our purposes it may be written as

$$T_Q = -\frac{3}{2}\frac{k}{Q}\frac{GM_\odot{}^2 R^5}{a^6}, \qquad (2)$$

where k is the tidal Love number, which monotonically increases with decreasing planetary rigidity; Q is the quality factor, an inverse measure of the bodily tidal energy dissipation in Venus (see Goldreich and Soter, 1966); and $M_\odot$ and a are the Sun's mass and distance, respectively. The minus sign indicates that body tides tend to decrease the rotation rate, despinning Venus toward synchronous rotation. To evaluate (2), we will assume $k = 0.25$, the terrestrial value as scaled to Venus; in fact, k may be slightly larger if the mantle of Venus is less rigid due to high temperatures. We have then, approximately,

$$T_Q = -5.3 \times 10^{24} Q^{-1}. \qquad (3)$$

For the Earth the tidal effective Q is about 13, but this low Q is largely due to extensive dissipation in shallow seas. If the Earth had no surface water, its tidal Q would be on the order of 100 (Lagus and

Anderson, 1968). The high temperatures in Venus are too extreme for surface water but may well cause interior melting or near melting of silicates if the mantle composition is similar to the Earth's. This would provide an increased source of tidal energy dissipation and tend to reduce the effective Q (MacDonald, 1962). We therefore assume that the amount of tidal dissipation for Venus falls between that for the actual Earth and that for a waterless Earth; thus $10 \lesssim Q \lesssim 100$. The T_Q associated with the upper limit is the weakest tidal torque admissible under the above assumptions, and we denote its magnitude as $T_{100} = 5.3 \times 10^{22}$.

The final path of the Venus rotation state from an assumed more rapid initial retrograde spin is indicated in Fig. 3. Venus enters the diagram from the left driven by the (assumed) constant tidal torque (shown for the T_{100} case). If one of the resonant torque spikes were strong enough, it could act to prevent the planet's overshooting the resonant spin rate. However, the T_q are all too weak to oppose the despinning of Venus. The planet will traverse the observed $q = -5$ resonance (indicated by the X) and spin down to a yet slower rotation. Even T_{100} is much too large to be intercepted by any of the Earth resonance T_q spikes. These resonant torques as drawn are based on the assumption of a terrestrial $(B\text{–}A)/C$. (They would be systematically raised by an order of magnitude if we accepted for Venus $(B\text{–}A)/C = 2.2 \times 10^{-4}$. The resulting $q = -5$ spike would then intercept T_{100} at the point X and the resonance would just be stable. But this is with the assumption of an excessive quadrupole moment and the maximum acceptable value of Q).

Including Atmospheric Tides

Let us begin by considering the form of the Earth's atmospheric tides. An excellent review of observations and theory is given by Haurwitz (1964). At the surface of the Earth the atmospheric tides are detected as barometric fluctuations. There is a pronounced pressure oscillation (of about 10^{-3} atm) with a period of 12 solar hours (semidiurnal), and a smaller 24-hour wave (diurnal), as well as smaller higher harmonics and a very weak component with a period of 12 lunar hours. The lunar tide is, of course, of gravitational origin and is analogous to solid body tides. However, the predominating solar atmospheric tides are of thermal origin, and we wish to generalize the terrestrial description of these thermal tides for application to the planet Venus.

We give below a simplified description of the mechanism of thermal atmospheric tides, adapted from the advanced theory as found in Butler and Small (1963), Green (1965), and Lindzen (1968). A region in the Earth's atmosphere heated by direct absorption of solar radiation will expand, thus tending to create a mass increment in adjacent cooler regions. For a temperature change δT spread over a height Δz, the resulting pressure disturbance generated at the base of Δz would be

$$\delta p = -\rho g \Delta z \frac{\delta T}{T},$$

where ρ and T are the ambient density and temperature in the excited level and g is the acceleration of gravity.

Atmospheric temperature variations due to daily absorption of solar radiation and constant infrared cooling arise principally from two sources in the case of the Earth: ozone and water vapor. In the absence of motion, water vapor absorption in the troposphere would give rise to a temperature amplitude of $\sim 0.5°$K and ozone heating centered around altitude 50 km would produce $\sim 2°$K. The resulting pressure oscillation and phase will differ from that due directly to the excitation when the atmosphere's dynamic response is taken into account. But we can see immediately that thermal tides may be much larger than gravitational tides in the Earth's atmosphere. The latter give rise to a surface pressure wave $\delta p = -\rho U$, where U is the gravitational tidal potential due to the Sun or Moon. In both cases, U is of order 10^4 cm^2 sec^{-2}, and the resulting δp is equivalent to that of a thermal tide arising from a temperature deviation δT of only $10^{-2}°$K occurring over a range Δz of several kilometers.

The direct absorption of solar radiation by the Earth's atmosphere at any level follows a rectified sine wave (with insolation cut off during the night) which can be Fourier-analyzed into a principal diurnal forcing term as well as an appreciable semidiurnal term plus a number of weaker higher harmonics. Although the diurnal excitation is dominant, the associated tidal oscillation is small because most of the energy activates trapped modes which do not propagate to the ground. On the other hand the semidiurnal modes all propagate relatively unimpeded and are found to dominate the tidal barometric oscillation at the surface of the Earth.

The semidiurnal pressure wave is (as will be shown) the only component that can give rise to a tidal couple. For the Earth, this component may be represented empirically (Haurwitz, 1964) as

$$\delta p = 1.16 \sin^3\theta \cos 2(\lambda - 146°)\,\text{mb},$$

where θ is co-latitude and λ is longitude measured in the sense of rotation from the subsolar meridian. The phase angle indicates that pressure maxima occur just before 10 a.m. and 10 p.m. (Fig. 4). We can

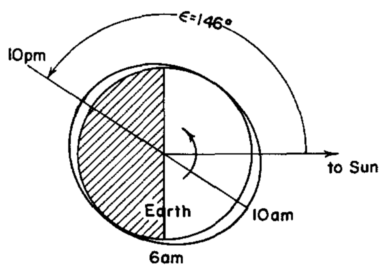

FIG. 4. Principal atmospheric tide of the Earth, with pressure maxima at about 10 a.m. and 10 p.m. These positions in the configuration are fixed as the Earth rotates through the semidiurnal pressure wave pattern.

generalize this semidiurnal term and write it as the second harmonic of a Fourier series; i.e.,

$$\delta p_2 = p_2 f_2(\theta) \cos 2(\lambda - \epsilon_2) \times 10^3 \text{ dynes cm}^{-2},$$

where, for the Earth, $f_2(\theta) = \sin^3\theta$ gives the empirical semidiurnal distribution, and where $p_2 = 1.16$ mb and $\epsilon_2 = 146°$ (or $-34°$). The factor 10^3 is simply a conversion of millibars to cgs units.

The resultant of all the periodic tidal waves in a planet's atmosphere, of which p_2 is the dominant term for the Earth, can now be written as

$$\delta p = 10^3 \sum_{n=1}^{\infty} f_n(\theta)\, p_n \cos n(\lambda - \epsilon_n),$$

where we allow for a different functional latitude dependence $f_n(\theta)$ for each harmonic.

Now an excess (or deficiency) in the surface pressure of δp corresponds to an atmospheric mass increment of

$$dm = (\delta p/g)\, dA,$$

where $dA = R^2 \sin\theta\, d\theta\, d\lambda$ is an element of the planet's surface area. The tide-raising potential for any surface point on the planet is, to lowest order (Jeffreys, 1962),

$$U = \tfrac{3}{4}(GM_\odot R^2/a^3) \sin^2\theta \cos 2\lambda.$$

The solar torque on the mass element dm is $dT_P = (\partial U/\partial\lambda)\, dm$. Inserting δp and dA into dm and integrating, the solar torque acting over the whole atmosphere is

$$T_P = -\tfrac{3}{2} \times 10^3 \frac{M_\odot}{M} \frac{R^6}{a^3} \int_0^\pi \sin^3\theta \int_0^{2\pi} \sin 2\lambda \sum_{n=1}^{\infty} f_n(\theta)\, p_n \cos n(\lambda - \epsilon_n)\, d\lambda\, d\theta.$$

It is easily verified that in the λ integral every term except the $n = 2$ term will vanish when integrated from 0 to 2π, thus leaving

$$\int_0^{2\pi} \sin 2\lambda f_2(\theta)\, p_2 \cos 2(\lambda - \epsilon_2)\, d\lambda = \pi f_2(\theta)\, p_2 \sin 2\epsilon_2.$$

A number of functions could be chosen for the semidiurnal co-latitude distribution $f_2(\theta)$ and inserted in the remaining calculation. Assuming that the terrestrial $f_2(\theta) = \sin^3\theta$ is sufficiently accurate for general use, the remaining integration becomes

$$\int_0^\pi \sin^6\theta\, d\theta = 5\pi/16.$$

Then the atmospheric torque is finally

$$T_P = -\tfrac{15}{32}\pi^2 \times 10^3 \frac{M_\odot R^6}{Ma^3} p_2 \sin 2\epsilon_2 = FP, \quad (4)$$

where F is a constant for a given planet, and where

$$P \equiv -p_2 \sin 2\epsilon_2$$

depends on the amplitude and phase of the semidiurnal tide. Note that for positive torque (working to speed up the planet), P is positive. This requires a phase *lead* ($\epsilon_2 > 90°$); i.e., the mass "bulges" precede the Sun and anti-Sun.

We see immediately that regardless of the components present in the pressure wave, only the second harmonic (semidiurnal term) contributes to the atmospheric tidal torque. Given p_2 and ϵ_2, we can specify T_P. For the Earth, the atmospheric pressure wave is predominantly semidiurnal and, consequently, p_2 and ϵ_2 are well determined. Using $p_2 = 1.16$ mb and $\epsilon_2 = 146°$, or $P = +1.075$, and $F = 3.1 \times 10^{22}$ for the Earth in (4), we obtain $T_P = 3.3 \times 10^{22}$. For Venus, noting that $F_♀ = 2.5F_\oplus$, we obtain

$$T_P = 7.7 \times 10^{22} P. \quad (5)$$

The net destabilizing torque on Venus is now assumed to be comprised of two terms: the conventional bodily tidal torque T_Q given by (3) and an atmospheric tidal torque (5). If ϵ_2 for Venus is a phase lead, then the atmospheric torque is positive so as to oppose T_Q. If these two torques are of comparable magnitude and opposite sign; i.e., $T_Q + T_P \approx 0$, then resonance stability is guaranteed. The condition for this is

$$QP \approx 70. \quad (6)$$

We do not know the strengths of the atmospheric pressure wave components on Venus. But suppose for the moment that the *second harmonic* term is comparable to that measured for the Earth, which is $P = +1.075$. Then the condition of ensured resonance stability (6) determines that $Q \approx 65$, and this turns out to be of the expected order of magnitude for Venus ($10 < Q < 100$). In other words, the dissipative solid body tidal torque on Venus with a reasonable Q could be balanced by an accelerating (retrograde) atmospheric tidal torque arising from a barometric semidiurnal term no larger than the Earth's!

We have thus shown that even in ignorance of the Venus atmospheric structure, it is not *a priori* unreasonable to expect that atmospheric tides provide the additional torque that permits the observed resonant spin stability. It depends, in fact, only upon the likelihood of P for the Venus atmosphere having the same sign and order of magnitude as for the Earth. For any given Q, a large range of pairs of p_2 and ϵ_2 will produce a value of $P = -p_2 \sin 2\epsilon_2 > 0$ sufficient to satisfy the condition (6) for the resonant rotation. But since p_2 and ϵ_2 are both determined theoretically from the atmospheric profiles of opacity and temperature, we have not here attempted to derive P from first first principles. There is no guarantee that the Venus atmospheric tidal response is analogous to that of the Earth, especially in light of the disparity in rotation periods. And there is apparently no way short of model calculations to determine whether the phase of such a response is a lead or a lag (although the latter would work with solid body tides to further upset the resonant stability theory). Here we simply suggest that the actual P for Venus is sufficient to account for the resonant rotation, but its exact value is left undetermined, as is that of Q.

In Fig. 5, we illustrate the hypothetical final path of the Venus rotation state under the combined influence of resonant, body tidal, *and* atmospheric tidal torques. Venus again enters the diagram from the left under the dominant influence of (an assumed constant) body tidal torque T_Q. The despinning is opposed by an atmospheric tidal torque T_P which tends to push Venus toward the left on the diagram. The strength of T_P is here taken to increase with increasing diurnal period (i.e., P is assumed frequency dependent) although this is not essential. The resonant torque spikes T_q work in concert with the atmospheric tidal torque and are now in reach of T_Q.

Venus will spin down in Fig. 5 until it is

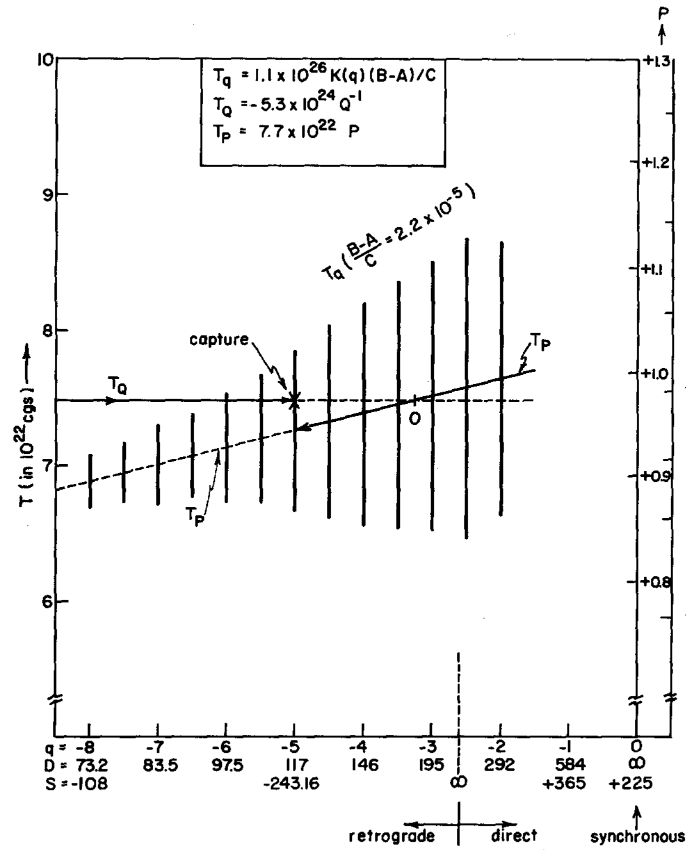

FIG. 5. Similar to Fig. 3 but including atmospheric tidal torque T_P proportional to the semidiurnal term P (in right margin). The resonant torques T_q are now augmented by the atmospheric torque whose effect is to spin up the planet (right to left). In this hypothetical example, we choose a constant T_Q with $Q \approx 70$. The two torques are assumed to be of comparable magnitude and opposite sign (arrows) so as to balance at some point 0. Venus is then captured in the third stable resonance that it encounters, which turns out to be the observed one at $q = -5$. The vertical scale is enhanced over that of Fig. 3 for clarity.

captured at the $q = -5$ resonance. Note that it has traversed the $q = -5.5$ and -6.0 resonances even though the net opposing torques there are sufficient to permit capture into stable equilibrium. Stability of a resonance is a necessary but not a sufficient condition for capture. According to Goldreich and Peale (1967), capture requires damping of the libration about a resonance by dissipation in a core–mantle interface. The probability that the requisite damping will occur while the rotation is passing through a given stable resonance depends upon initial conditions and may be rather small. Therefore the planet should be expected to pass through several stable resonances before finally being captured by one.

In the absence of any interaction with the Earth, Venus would asymptotically spin down to the rate where atmospheric torque just balanced body torque. In the example of Fig. 5, this occurs at the point marked 0. There it would remain, subject to zero net torque. However, it never reaches this point because it must first traverse the increasingly effective resonance torque spikes due to the permanent deformation, and is finally captured by the one at $q = -5$.

An infinite number of pairs of T_Q and T_P curves will result in the same capture probability for a given resonance. It is not even necessary that the two curves cross somewhere (i.e., that the net solar torque vanish) in order to permit capture by the resonant torque. We require only that T_Q and T_P have comparable magnitude (but opposite sign) and that their slopes are not too different in the vicinity of the desired capture. The hypothetical case in Fig. 5 involves a constant body torque (frequency independent Q) and an atmospheric torque that diminishes with increased spin. This was an arbitrary choice because it is not known how Q and P vary with rotation speed in this range. A wide range of other choices would also lead to a capture at the same resonance.

Let us now consider the case of initial *direct* ($q > 0$) rotation for Venus. In this case, Venus should enter Fig. 5 from the right, under the domination of an atmospheric torque T_P. However, to go from direct to its present retrograde spin would require Venus to have been pushed through several powerful resonances due to the *solar* torque on the permanent deformation. Solar resonances of this kind that Venus might encounter in spinning down from an initial retrograde rotation are entirely negligible, as a consequence of the small orbital eccentricity. But a few important ones are encountered in spinning down from initial direct rotation. The two of consequence in the range of Fig. 5 (but not shown) are at $q = -1.3$ and $q = 0$ (equivalent to 0.5 and 1.0 in Table 1 of Goldreich and Peale, 1966). And the latter, the synchronous resonance, is really formidable, with a maximum restoring torque of 2.0×10^{27}, assuming the same Earth-like quadrupole moment as before.

In order for the atmospheric tides to push Venus through the synchronous solar resonance and escape being captured there, the mass deformation in the atmospheric tide would have to be comparable to the planet's permanent deformation. Equivalently, setting $T_P \gtrsim 2.0 \times 10^{27}$, we would need $P \gtrsim 2.6 \times 10^4$, which amounts to a thermal pressure wave on Venus with an amplitude of more than 26 Earth atmospheres. After pushing Venus through $q = 0$ and the other sizable solar resonances, the atmospheric tide would have to diminish to terrestrial proportions so that its torque would nearly balance the body tidal torque and allow capture at the relatively weak $q = -5$ Earth resonance without overshooting it.

A very large inclination of the rotation axis to the orbit pole would reduce the strength of the solar resonant tide, but even then very improbable conditions would have to be assumed for Venus to have originated with a direct spin faster than solar synchronous rotation if tidal interactions are to have set up the present configuration. This virtually forces one to assume that Venus was assembled initially with a spin in the range between fast retrograde and direct but slower than solar synchronous. It seems very improbable that a planet could accumulate from the primeval material of the solar system and possess a very slow rotation; a fast retrograde rotation may merely signify that the chance element of some major collisions were involved in the process. Also, as is evident in Fig. 5, a despinning from a faster retrograde rotation towards the present value takes Venus into increasingly stronger Earth resonances, and a much wider range of circumstances would then allow capture at the $q = -5$ resonance than if the motion had been in the opposite sense. A faster retrograde spin therefore appears to be the most probable initial condition.

We conclude with a word on the long-term stability of the observed resonant rotation. The resonant lock as depicted in Fig. 5 depends upon the relative strengths of the body and atmospheric torques, and if these remain fixed, the resonance will be permanent. On the other hand, significant changes in the atmospheric and internal structure of Venus could alter the torque balance sufficiently to allow escape from the $q = -5$ resonance sometime in the future. Likewise, Venus might have occupied and escaped other resonant states in the past.

Summary

We have seen how without some agency such as atmospheric tides, the existence of the observed resonant rotation would require either a very large stabilizing permanent deformation for Venus or a very weak destabilizing solid body tidal torque, or both. The requisite $(B-A)/C$ and Q appear impossibly large in terms of what we would expect from the material strength and energy dissipation in Venus. It appears that some important further effect has to be included in the discussion, and the conspicuous atmosphere of Venus suggests itself as the additional agency at work to stabilize the resonance. If the atmospheric tide has a semidiurnal component of magnitude comparable to the Earth's and similarly with a phase lead, then the torque upon it will nearly counteract the solid body tidal drag. All that one need assume is that for fast retrograde spins the solid body tidal torque dominates, while for slower spins the atmospheric tidal torque is the greater and of opposite sign. There is then a particular angular velocity at which the torque vanishes. One of the many spin–orbit resonances with the Earth in the vicinity of this angular velocity would then dominate and could capture the rotation of Venus. This would remove the difficulty for the resonant rotation theory.

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References

Anderson, J. D., and Efron, L. (1969). Mass and dynamical oblateness of Venus. *Bull. Am. Astron. Soc.* **1**, 231–232.

Butler, S. T., and Small, K. A. (1963). The excitation of atmospheric oscillations. *Proc. Roy. Soc. (London)* **A274**, 91–121.

Caputo, M. (1967). "The Gravity Field of the Earth." Academic Press, New York.

Dyce, R., Campbell, D., and Jurgens, R. (1969). Private communication.

Eshleman, V. R. (1967). Radar astronomy. *Science* **158**, 585–597.

Gold, T. (1955). Instability of the earth's axis of rotation. *Nature* **175**, 526–529.

Goldreich, P., and Peale, S. J. (1966). Spin–orbit coupling in the solar system. *Astron. J.* **71**, 425–438.

Goldreich, P., and Peale, S. J. (1967). Spin–orbit coupling in the solar system. II. The resonant rotation of Venus. *Astron. J.* **72**, 662–668.

Goldreich, P., and Peale, S. J. (1968). The dynamics of planetary rotations. *Ann. Rev. Astron. Astrophys.* **6**, 287–320.

Goldreich, P., and Soter, S. (1966). Q in the solar system. *Icarus* **5**, 375–389.

Goldreich, P., and Toomre, A. (1969). Some remarks on polar wandering. *J. Geophys. Res.* **74**, 2555–2567.

Green, J. S. A. (1965). Atmospheric tidal oscillations: an analysis of the mechanics. *Proc. Roy. Soc. (London)* **A288**, 564–574.

Haurwitz, B. (1964). Atmospheric tides. *Science* **144**, 1415–1422.

Holmberg, E. R. R. (1952). A suggested explanation of the present value of the velocity of rotation of the earth. *Monthly Notices Roy. Astron. Soc., Geophys. Suppl.* **6**, 325–330.

Jeffreys, H. (1962). "The Earth," 4th ed. Cambridge Univ. Press, London.

Jurgens, R. F. (1970). Some preliminary results of the 70 cm radar studies of Venus. *Radio Science*, in press.

Lagus, P. L., and Anderson, D. L. (1968). Tidal dissipation in the earth and planets. *Phys. Earth Planetary Interiors* **1**, 505–510.

Lindzen, R. S. (1968). The application of classical atmospheric tidal theory. *Proc. Roy. Soc. (London)* **A303**, 299–316.

MacDonald, G. J. F. (1962). On the internal constitution of the inner planets. *J. Geophys. Res.* **67**, 2945–2974.

MacDonald, G. J. F. (1964). Tidal friction. *Rev. Geophys.* **2**, 467–541.

Mueller, R. F. (1969). Effect of temperature on the strength and composition of the upper lithosphere of Venus. *Nature* **224**, 354–356.

O'Leary, B. T., Campbell, M. J., and Sagan, C. (1969). Lunar and planetary mass concentrations. *Science* **165**, 651–657.

Öpik, E. (1965). Rotation of Mercury, Venus, the Moon, and tidal evolution. *Irish Astron. J.* **7**, 124–129.

Runcorn, S. K. (1967). Convection in the planets. *In* "The Mantles of the Earth and Terrestrial Planets" (S. K. Runcorn, ed.), pp. 513–524. Interscience, London.

Shapiro, I. I. (1967). Resonance rotation of Venus. *Science* **157**, 423–425.

Thompson, W. (1882). On the thermodynamic acceleration of the earth's rotation. *Proc. Roy. Soc. Edinburgh* **11**, 396–405.