Estatística Computacional - aluna: Giullia Rodrigues
Primeira Lista de Exercícios - matrícula: 11621BCC016
Revisão de Probabilidade

Exercício 1.

$\Omega = \{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),$
$(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),$
$(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}$ $n = 36$

$6 \times 6 = 36$

P(A):

$P(A) = \frac{n(A)}{n(\Omega)} = \frac{33}{36} = \frac{11}{12} = 0.9166$

A = {soma maior ou igual a 4}

$A = \{(1,3),(1,4),(1,5),(1,6),(2,2),(2,3),(2,4),(2,5),(2,6),(3,1),(3,2),(3,3),(3,4),$
$(3,5),(3,6),(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),(5,1),(5,2),(5,3),(5,4),(5,5)$
$(5,6),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}$ $n = 33$

P(B|C):

$P(B|C) = \frac{P(B \cap C)}{P(C)} = \frac{6/36}{11/36} = \frac{6}{36} \times \frac{36}{11} = \frac{216}{396} = \frac{6}{11}$

C = {um dos lançamentos foi 2}

$C = \{(1,2),(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),(3,2),(4,2),(5,2),(6,2)\}$ $n = 11$

B = {soma ímpar}

$B = \{(1,2),(1,4),(1,6),(2,1),(2,3),(2,5),(3,2),(3,4),(3,6),(4,1),(4,3),(4,5),(5,2),(5,4)$
$(5,6),(6,1),(6,3),(6,5)\}$ $n = 18$

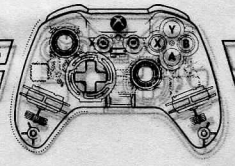
$B \cap C = \{(1,2),(2,1),(2,3),(2,5),(3,2),(5,2)\}$ $n = 6$

P(A∩D):

$P(A \cap D) = \frac{5}{36}$

D = {o máximo das faces é 3}.

$D = \{(1,3),(2,3),(3,1),(3,2),(3,3)\}$ $n = 5$



AND = {(1,3), (2,3), (3,1), (3,2), (3,3)} $n = 5$

P(CUD):

$P(CUD) = \frac{14}{36} = \frac{7}{18}$

CUD = {(1,2), (1,3), (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (3,1), (3,2), (3,3), (4,2), (5,2), (6,2)} $n = 14$

Exercício 2

95% detecção da doença

1% dos positivos são na verdade saudáveis

0,5% da população tem a doença

A = { a pessoa estar doente }

B = { o teste da pessoa ter dado positivo }

C = { a pessoa estar saudável }

$P(B|A) = 0,95$

$P(A) = 0,005$

$P(B|C) = 0,01$

$P(A|B) = \frac{P(A \cap B)}{P(B)}$

como dito no exercício $P(A \cap B) = P(B|A) \cdot P(A)$

$P(B) = P(B \cap A) + P(B \cap C)$

$P(B) = P(A) \cdot P(B|A) + P(C) \cdot P(B|C)$

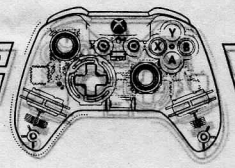
$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(A) \cdot P(B|A) + P(C) \cdot P(B|C)}$

$P(C) = 1 - 0,005 = 0,995$

substituindo:

$P(A|B) = \frac{0,005 \cdot 0,95}{0,005 \cdot 0,95 + 0,995 \cdot 0,01} = \frac{0,00475}{0,0147} = 0,03231 = 32,31\%$

A probabilidade de uma pessoa ter a doença dado que o resultado de seu exame foi positivo é de 32,31%.



Exercício 3

	Prata	Bronca	Vermelha	
1	6	3	4	$n = 13$
2	3	5	2	$n = 10$
3	4	2	2	$n = 8$

$$\begin{cases} 5 \Rightarrow \text{urna 1} \\ 1,4,6 \Rightarrow \text{urna 2} \\ 2,3 \Rightarrow \text{urna 3} \end{cases} \qquad n = 6$$

a) Bola ser vermelha

$$P(V) = P(V \cap U_1) + P(V \cap U_2) + P(V \cap U_3)$$

$$= \frac{1}{6} \cdot \frac{4}{13} + \frac{3}{6} \cdot \frac{2}{10} + \frac{2}{6} \cdot \frac{2}{8}$$

$$= \frac{4}{78} + \frac{6}{60} + \frac{4}{48}$$

$$= \frac{160 + 312 + 260}{3120}$$

$$= \frac{732}{3120}$$

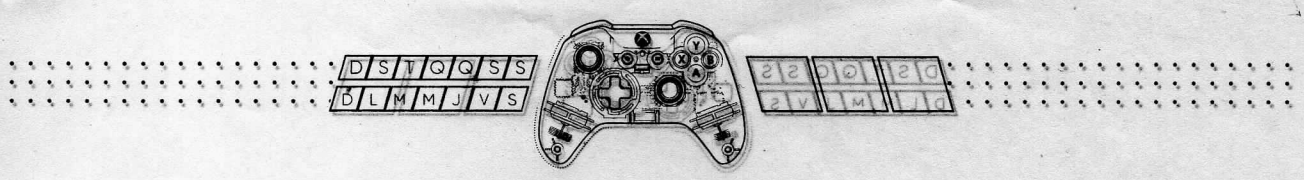
$$= 0{,}2346$$

b) $P(U_2|V)$

$$P(U_2|V) = \frac{P(U_2) \cdot P(V|U_2)}{P(V)}$$

$$= \frac{3/6 \cdot 2/10}{0{,}2346}$$

$$= \frac{0{,}1}{0{,}2346} \qquad = 0{,}4262$$



Exercício 4

$P(X = \text{cara}) = 0.8$ $\quad 1/5 = 0,20$

$P(X = \text{coroa}) = 0.2$

a)

$\Omega = \{(\text{cara}, \text{cara}), (\text{cara}, \text{coroa}), (\text{coroa}, \text{cara}), (\text{coroa}, \text{coroa})\}$

b) $\{(\text{cara}, \text{coroa}), (\text{coroa}, \text{cara})\}$

$$P(B) = \frac{4}{5} \cdot \frac{1}{5} + \frac{1}{5} \cdot \frac{4}{5}$$

$$= \frac{4}{25} + \frac{4}{25}$$

$$= \frac{8}{25}$$

$$= 0,32$$

c) $\{(\text{cara}, \text{coroa}), (\text{cara}, \text{cara}), (\text{coroa}, \text{cara})\}$

$$P(C) = \frac{4}{5} \cdot \frac{5}{5} + \frac{1}{5} \cdot \frac{4}{5}$$

$$= \frac{20}{25} + \frac{4}{25}$$

$$= \frac{24}{25}$$

$$= 0,96$$

d) $\{(\text{cara}, \text{cara}), (\text{coroa}, \text{coroa})\}$

$$P(D) = \frac{4}{5} \cdot \frac{4}{5} + \frac{1}{5} \cdot \frac{1}{5}$$

$$= \frac{16}{25} + \frac{1}{25}$$

$$= \frac{17}{25}$$

$$= 0,68$$

Exercício 5

Como no jogo temos 4 pessoas, ficaria:

$4 \times 3 \times 2 \times 1 = 4! = 24$

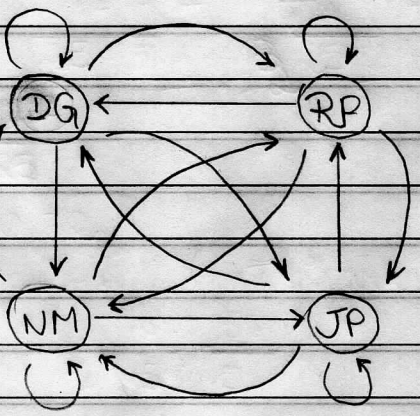
↳ segue o mesmo raciocínio do ③

↳ 3 opções, já que 1 delas foi escolhido pelo jogador 1

↳ 4 opções (total)

Vamos dividir em 2 casos, já visando que quero sucesso na escolha. considere:

$\frac{3}{4} \cdot \frac{1}{3} \cdot \frac{1}{2} \cdot \frac{1}{1} = \frac{3}{24}$



③ com isso só sobrou nick escolher Jimmy e vice versa

② Robert escolheu David.

① David pode escolher RP, NM, JP já exclui ele escolher ele mesmo

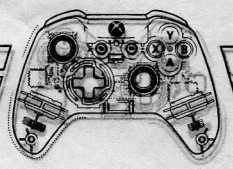
mas além disso:

$\frac{3}{4} \cdot \frac{2}{3} \cdot \frac{1}{2} \cdot \frac{1}{1} = \frac{6}{24}$

$\frac{3}{24} + \frac{6}{24} = \frac{9}{24}$

Resp: A probabilidade do amigo oculto não dar errado é $\frac{9}{24} = 37,5\% = 0,375$

↳ Robert poderia ter escolhido nick ou Jimmy



Exercício 6

$P(X=1) = \frac{1}{6}$ $P(X=3) = \frac{2}{6}$ $P(X=5) = \frac{3}{6}$

a) $P(X>2)$

$P(X>2) = P(X=3) + P(X=5)$

$= \frac{2}{6} + \frac{3}{6}$

$= \frac{2+3}{6}$

$= \frac{5}{6}$

b) $P(X \leq 2)$

$P(X \leq 2) = P(X=1)$

$= \frac{1}{6}$

c) $E[X]$ e $Var[X]$

$E[X] = \sum_i x_i \cdot P(X=x_i)$

$= \frac{1 \cdot 1}{6} + \frac{3 \cdot 2}{6} + \frac{5 \cdot 3}{6}$

$= \frac{1}{6} + \frac{6}{6} + \frac{15}{6}$

$= \frac{1+6+15}{6}$

$= \frac{22}{6}$

$= \frac{11}{3}$

$Var[X] = E[X^2] - (E[X])^2$

$$E[X^2] = 1^2 \cdot \frac{1}{6} + 3^2 \cdot \frac{2}{6} + 5^2 \cdot \frac{3}{6}$$

$$= 1 \cdot \frac{1}{6} + 9 \cdot \frac{2}{6} + 25 \cdot \frac{3}{6}$$

$$= \frac{1}{6} + \frac{18}{6} + \frac{75}{6}$$

$$= \frac{94}{6} //$$

$$Var[X] = E[X^2] - (E[X])^2$$

$$= \frac{94}{6} - \left(\frac{22}{6}\right)^2$$

Deixei assim, melhor para o mmc

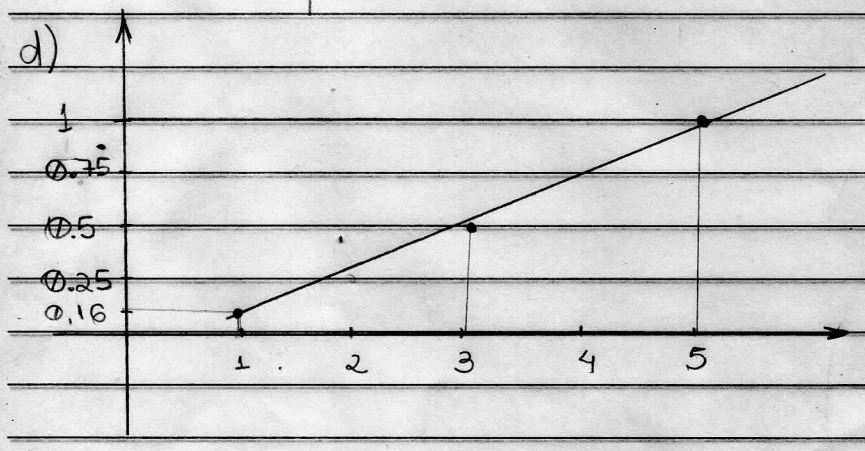
$$= \frac{94}{6} - \frac{484}{36}$$

$$= \frac{564 - 484}{36}$$

$$= \frac{80}{36}$$

$$= \frac{20}{9}$$

d)



Exercício 7

2 dados : (i,j) = maior

$\Omega = \{(1,1)=1, (1,2)=2, (1,3)=3, (1,4)=4, (1,5)=5, (1,6)=6$

$(2,1)=2, (2,2)=2, (2,3)=3, (2,4)=4, (2,5)=5, (2,6)=6$

$(3 \quad \ldots$

$(4$

$(5$

$(6$

a) X pode assumir os valores $\{1,2,3,4,5,6\}$

b)

$P(X=1)$:

$P(X=1) = \{(1,1)\}$

$= \frac{1}{6} \times \frac{1}{6}$

$= \frac{1}{36} = 0.027$

$P(X=2)$

$P(X=2) = (i=1 \text{ e } j=2) \text{ ou } (i=2 \text{ e } j \leq 2)$

$= \frac{1}{6} \times \frac{1}{6} + \frac{1}{6} \times \frac{2}{6}$

$= \frac{1}{36} + \frac{2}{36}$

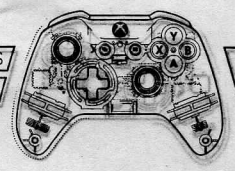
$= \frac{3}{36}$

$= 0.083$

$P(X=3)$

$P(X=3) = (i \leq 2 \text{ e } j=3) \text{ ou } (i \leq 3 \text{ e } j \leq 3)$

$= \frac{2}{6} \times \frac{1}{6} + \frac{1}{6} \times \frac{3}{3}$



$= \frac{2}{36} + \frac{3}{36}$

$= \frac{5}{36}$

$= 0.138$

$P(X=4)$

$P(X=4) = (i \leq 3 \text{ e } j = 4) \text{ ou } (i = 4 \text{ e } j \leq 4)$

$= \frac{3}{6} \times \frac{1}{6} + \frac{1}{6} \times \frac{4}{6}$

$= \frac{3}{36} + \frac{4}{36}$

$= \frac{7}{36}$

$= 0.194$

$P(X=5) = (i \leq 4 \text{ e } j = 5) \text{ ou } (i = 5 \text{ e } j \leq 5)$

$= \frac{4}{6} \times \frac{1}{6} + \frac{1}{6} \times \frac{5}{6}$

$= \frac{4}{36} + \frac{5}{36}$

$= \frac{9}{36}$

$= 0.25$

$P(X=6) = (i \leq 5 \text{ e } j = 6) \text{ ou } (i = 6 \text{ e } j \leq 6)$

$= \frac{5}{6} \times \frac{1}{6} + \frac{1}{6} \times \frac{6}{6}$

$= \frac{5}{36} + \frac{6}{36}$

$= \frac{11}{36}$

$= 0.305$

c) $P(X<3) = P(X=1) + P(X=2)$

$= \frac{1+3}{36}$

$= \frac{4}{36} =$

$P(X \geq 3) = P(X=3) + P(X=4) + P(X=5) + P(X=6)$

$= \frac{5+7+9+11}{36}$

$= \frac{32}{36} =$

d) Probabilidade de $X > 2$ dado que $X < 5$

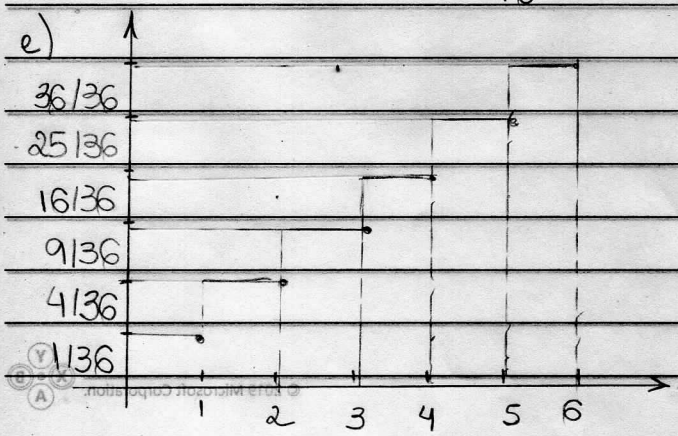
$P(X > 2 \mid X < 5) = \frac{P(X>2 \cap X<5)}{P(X<5)}$

$= \frac{P(X=3) + P(X=4)}{P(X=1) + P(X=2) + P(X=3) + P(X=4)}$

$= \frac{\left(\frac{5+7}{36}\right)}{\left(\frac{1+3+5+7}{36}\right)}$

$= \frac{12}{16} =$

e)



Exercício 8

$\sigma^2 = 16$

$\sigma = 4 \quad \mu = 5$

a) $P(X \leq 13)$

$Z = \frac{X-5}{4}, X \leq 13 \Rightarrow Z = \frac{13-5}{4} = 2$

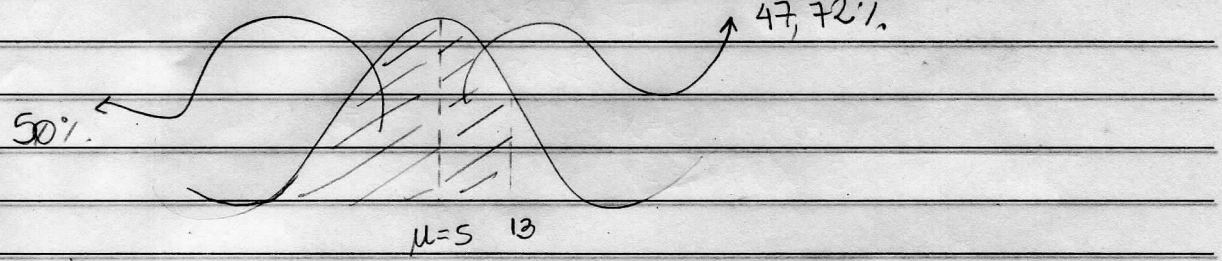
$P(X \leq 13) = P(Z \leq 2) = 50\% + 47,72\% = 97,72\%$

b) $P(X > 1)$

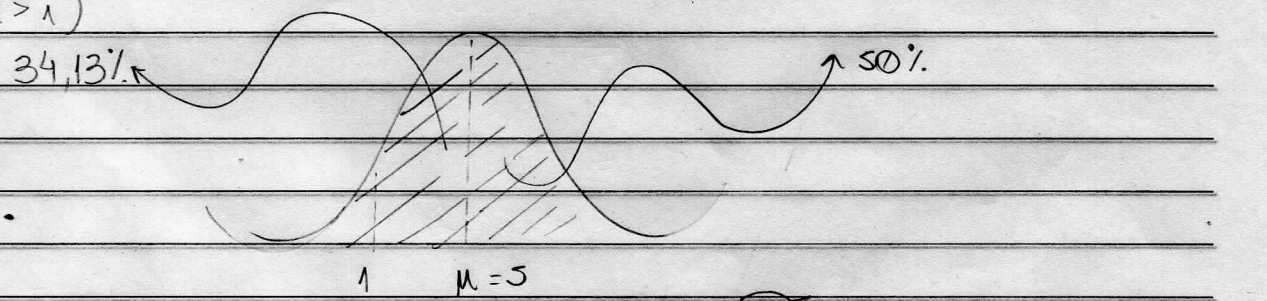
$Z = \frac{X-5}{4}, X > 1 \Rightarrow Z = \frac{1-5}{4} = -1$

$P(X > 1) = P(Z > -1) = 50\% + 34,13\% = 84,13\%$

c) $P(X \leq 13)$



$P(X > 1)$



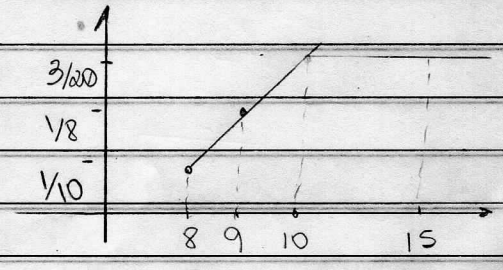
d) De acordo com a tabela da distribuição normal: $Z = 0,51595$

$Z = \frac{a-5}{4} \Rightarrow 0,51595 \cdot 4 + 5 = a$

$a = 7,06380$

Exercício 9

a)



$f(8) = \frac{4}{40} = \frac{1}{10}$

$f(9) = \frac{5}{40} = \frac{1}{8}$

$f(10) = \frac{6}{40} = \frac{3}{200}$

b) $$\int_{-\infty}^{\infty} f(t)\,dt = \int_{-\infty}^{8} f(t)\,dt + \int_{8}^{10} f(t)\,dt + \int_{10}^{15} f(t)\,dt + \int_{15}^{\infty} f(t)\,dt$$

$$= \int_{8}^{10} f(t)\,dt + \int_{10}^{15} f(t)\,dt$$

$$= \frac{1}{40}\int_{8}^{10} (t-4)\,dt + \frac{3}{200}\int_{10}^{15} 1\,dt$$

$$= \frac{1}{40}\left[\frac{t^2}{2} - 4t\right]_{8}^{10} + \frac{3}{200}\left[t\right]_{10}^{15}$$

$$= \frac{5}{200} + \frac{15}{200}$$

$$= \frac{20}{20}$$

$$= 1$$

c) $$P(0 < T \leq 12) = \int_{-\infty}^{8} f(t)\,dt + \int_{8}^{10} f(t)\,dt + \int_{10}^{15} f(t)\,dt + \int_{15}^{\infty} f(t)\,dt$$

$$= \frac{1}{40}\int_{8}^{10} (t-4)\,dt + \int_{10}^{15} 1\,dt$$

$$= \frac{1}{40}\left[\frac{t^2}{2} - 4t\right]_8^{10} + \frac{3}{20}\left[t\right]_{10}^{12}$$

$$= \frac{5}{20} + \frac{6}{20}$$

$$= \frac{11}{20}$$

d) $P(9 < T \leq 12) = \int_9^{12} f(t)\,dt = \int_9^{10} \frac{(t-4)}{40}\,dt + \int_{10}^{12} \frac{3}{20}\,dt$

$$= \frac{7}{16}$$