Gemunustic

Euclidean coordinates

- Usual form
$$\mathrm{d}\ell^2 = \mathrm{d}x^2 + \mathrm{d}y^2$$
- Matrix form
$$\mathrm{d}\ell^2 = \begin{pmatrix} \mathrm{d}x & \mathrm{d}y \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \mathrm{d}x \\ \mathrm{d}y \end{pmatrix}$$
- Concise form
$$\mathrm{d}\vec{r} = \begin{pmatrix} \mathrm{d}x \\ \mathrm{d}y \end{pmatrix}$$
$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Polar coordinates

- Usual form
$$\mathrm{d}\ell^2 = \mathrm{d}r^2 + r^2\,\mathrm{d}\theta^2$$
- Matrix form
$$\mathrm{d}\ell^2 = \begin{pmatrix} \mathrm{d}r & \mathrm{d}\theta \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & r \end{pmatrix} \begin{pmatrix} \mathrm{d}r \\ \mathrm{d}\theta \end{pmatrix}$$
- Concise form
$$\mathrm{d}\vec{r} = \begin{pmatrix} \mathrm{d}r \\ \mathrm{d}\theta \end{pmatrix}$$
$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 \\ 0 & r \end{pmatrix}$$

Common

- Concise form
$$\mathrm{d}\ell^2 = \mathrm{d}\vec{r}^{\top} g_{\mu\nu}\,\mathrm{d}\vec{r}$$
- Einstein form
$$\mathrm{d}\ell^2 = g_{ij}\,\mathrm{d}x^i\,\mathrm{d}x^j$$

Spherical coordinates (2D)

- Usual form
$$\mathrm{d}\ell^2 = R^2\,\mathrm{d}\theta^2 + R^2 \sin^2\theta\,\mathrm{d}\varphi^2$$
- Matrix form
$$\mathrm{d}\ell^2 = \begin{pmatrix} \mathrm{d}\theta & \mathrm{d}\varphi \end{pmatrix} \begin{pmatrix} R & 0 \\ 0 & R\sin\theta \end{pmatrix} \begin{pmatrix} \mathrm{d}\theta \\ \mathrm{d}\varphi \end{pmatrix}$$
- Concise form
$$\mathrm{d}\vec{r} = \begin{pmatrix} \mathrm{d}\theta \\ \mathrm{d}\varphi \end{pmatrix}$$
$$g_{\mu\nu} = \begin{pmatrix} R & 0 \\ 0 & R\sin\theta \end{pmatrix}$$

Γ for euclidean coordinates

$$\Gamma^{k}{}_{ij} = \frac{1}{2} g^{kl} \left(g_{li,j} + g_{lj,i} - g_{ij,l} \right) = \frac{1}{2} g^{k\hat{x}} \left(g_{\hat{x}i,j} + g_{\hat{x}j,i} - g_{ij,x} \right) + \frac{1}{2} g^{k\hat{y}} \left(g_{\hat{y}i,j} + g_{\hat{y}j,i} - g_{ij,y} \right)$$

$$\begin{tikzcd}
\Gamma^{\hat{x}}{}_{\hat{x}j} = \frac{1}{2} g^{\hat{x}\hat{x}} \left(g_{\hat{x}\hat{x},j} + g_{\hat{x}j,x} - g_{\hat{x}j,x} \right) + \frac{1}{2} g^{\hat{x}\hat{y}} \left(g_{\hat{y}\hat{x},j} + g_{\hat{y}j,x} - g_{\hat{x}j,y} \right) & \Gamma^{\hat{x}}{}_{ij} = \frac{1}{2} g^{\hat{x}\hat{x}} \left(g_{\hat{x}i,j} + g_{\hat{x}j,i} - g_{ij,x} \right) + \frac{1}{2} g^{\hat{x}\hat{y}} \left(g_{\hat{y}i,j} + g_{\hat{y}j,i} - g_{ij,y} \right) \arrow[l, "i \leftarrow \hat{x}"'] \arrow[r, "i \leftarrow \hat{y}"] & \Gamma^{\hat{x}}{}_{\hat{y}j} = \frac{1}{2} g^{\hat{x}\hat{x}} \left(g_{\hat{x}\hat{y},j} + g_{\hat{x}j,y} - g_{\hat{y}j,x} \right) + \frac{1}{2} g^{\hat{x}\hat{y}} \left(g_{\hat{y}\hat{y},j} + g_{\hat{y}j,y} - g_{\hat{y}j,y} \right) \\
& \Gamma^{k}{}_{ij} = \frac{1}{2} g^{k\hat{x}} \left(g_{\hat{x}i,j} + g_{\hat{x}j,i} - g_{ij,x} \right) + \frac{1}{2} g^{k\hat{y}} \left(g_{\hat{y}i,j} + g_{\hat{y}j,i} - g_{ij,y} \right) \arrow[u, "k \leftarrow \hat{x}"] \arrow[d, "k \leftarrow \hat{y}"'] & \\
\Gamma^{\hat{y}}{}_{\hat{x}j} = \frac{1}{2} g^{\hat{y}\hat{x}} \left(g_{\hat{x}\hat{x},j} + g_{\hat{x}j,x} - g_{\hat{x}j,x} \right) + \frac{1}{2} g^{\hat{y}\hat{y}} \left(g_{\hat{y}\hat{x},j} + g_{\hat{y}j,x} - g_{\hat{x}j,y} \right) & \Gamma^{\hat{y}}{}_{ij} = \frac{1}{2} g^{\hat{y}\hat{x}} \left(g_{\hat{x}i,j} + g_{\hat{x}j,i} - g_{ij,x} \right) + \frac{1}{2} g^{\hat{y}\hat{y}} \left(g_{\hat{y}i,j} + g_{\hat{y}j,i} - g_{ij,y} \right) \arrow[l, "i \leftarrow \hat{x}"'] \arrow[r, "i \leftarrow \hat{y}"] & \Gamma^{\hat{y}}{}_{\hat{y}j} = \frac{1}{2} g^{\hat{y}\hat{x}} \left(g_{\hat{x}\hat{y},j} + g_{\hat{x}j,y} - g_{\hat{y}j,x} \right) + \frac{1}{2} g^{\hat{y}\hat{y}} \left(g_{\hat{y}\hat{y},j} + g_{\hat{y}j,y} - g_{\hat{y}j,y} \right)
\end{tikzcd}$$