Gemunustic

Euclidean coordinates

- Usual form
$$\mathrm{d}\ell^2 = \mathrm{d}x^2 + \mathrm{d}y^2$$
- Matrix form
$$\mathrm{d}\ell^2 = \begin{pmatrix}\mathrm{d}x & \mathrm{d}y\end{pmatrix}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}\mathrm{d}x\\ \mathrm{d}y\end{pmatrix}$$
- Concise form
$$\mathrm{d}\vec{r} = \begin{pmatrix}\mathrm{d}x\\ \mathrm{d}y\end{pmatrix}$$
$$g_{\mu\nu} = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$$

Polar coordinates

- Usual form
$$\mathrm{d}\ell^2 = \mathrm{d}r^2 + r^2\,\mathrm{d}\theta^2$$
- Matrix form
$$\mathrm{d}\ell^2 = \begin{pmatrix}\mathrm{d}r & \mathrm{d}\theta\end{pmatrix}\begin{pmatrix}1 & 0\\ 0 & r\end{pmatrix}\begin{pmatrix}\mathrm{d}r\\ \mathrm{d}\theta\end{pmatrix}$$
- Concise form
$$\mathrm{d}\vec{r} = \begin{pmatrix}\mathrm{d}r\\ \mathrm{d}\theta\end{pmatrix}$$
$$g_{\mu\nu} = \begin{pmatrix}1 & 0\\ 0 & r\end{pmatrix}$$

Common

- Concise form
$$\mathrm{d}\ell^2 = \mathrm{d}\vec{r}^{\top} g_{\mu\nu}\,\mathrm{d}\vec{r}$$
- Einstein form
$$\mathrm{d}\ell^2 = g_{ij}\,\mathrm{d}x^i\,\mathrm{d}x^j$$

Spherical coordinates (2D)

- Usual form
$$\mathrm{d}\ell^2 = R^2\,\mathrm{d}\theta^2 + R^2\sin^2\theta\,\mathrm{d}\varphi^2$$
- Matrix form
$$\mathrm{d}\ell^2 = \begin{pmatrix}\mathrm{d}\theta & \mathrm{d}\varphi\end{pmatrix}\begin{pmatrix}R & 0\\ 0 & R\sin\theta\end{pmatrix}\begin{pmatrix}\mathrm{d}\theta\\ \mathrm{d}\varphi\end{pmatrix}$$
- Concise form
$$\mathrm{d}\vec{r} = \begin{pmatrix}\mathrm{d}\theta\\ \mathrm{d}\varphi\end{pmatrix}$$
$$g_{\mu\nu} = \begin{pmatrix}R & 0\\ 0 & R\sin\theta\end{pmatrix}$$

Γ for euclidean coordinates

$$
\Gamma^{k}{}_{ij} = \frac{1}{2} g^{kl}\left(g_{li,j} + g_{lj,i} - g_{ij,l}\right) = \frac{1}{2} g^{k\hat{x}}\left(g_{\hat{x}i,j} + g_{\hat{x}j,i} - g_{ij,x}\right) + \frac{1}{2} g^{k\hat{y}}\left(g_{\hat{y}i,j} + g_{\hat{y}j,i} - g_{ij,y}\right)
$$

$$
g_{\mu\nu} = \begin{pmatrix} g_{\hat{x}\hat{x}} & g_{\hat{x}\hat{y}} \\ g_{\hat{y}\hat{x}} & g_{\hat{y}\hat{y}} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}
$$

$$
g^{\mu\nu} = \begin{pmatrix} g^{\hat{x}\hat{x}} & g^{\hat{x}\hat{y}} \\ g^{\hat{y}\hat{x}} & g^{\hat{y}\hat{y}} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}
$$

$$
\Gamma^{\hat{x}}{}_{\hat{x}\hat{x}} = \frac{1}{2} 1\left(1_{,x} + 1_{,x} - 1_{,x}\right) + \frac{1}{2} g^{\hat{x}\hat{y}}\left(g_{\hat{y}\hat{x},x} + g_{\hat{y}\hat{x},x} - g_{\hat{x}\hat{x},y}\right)
$$

$$
\begin{tikzcd}
\Gamma^{\hat{x}}{}_{\hat{x}\hat{x}} = \frac{1}{2} g^{\hat{x}\hat{x}}\left(g_{\hat{x}\hat{x},x} + g_{\hat{x}\hat{x},x} - g_{\hat{x}\hat{x},x}\right) + \frac{1}{2} g^{\hat{x}\hat{y}}\left(g_{\hat{y}\hat{x},x} + g_{\hat{y}\hat{x},x} - g_{\hat{x}\hat{x},y}\right) & & \Gamma^{\hat{x}}{}_{\hat{y}\hat{x}} = \frac{1}{2} g^{\hat{x}\hat{x}}\left(g_{\hat{x}\hat{y},x} + g_{\hat{x}\hat{x},y} - g_{\hat{y}\hat{x},x}\right) + \frac{1}{2} g^{\hat{x}\hat{y}}\left(g_{\hat{y}\hat{y},x} + g_{\hat{y}\hat{x},y} - g_{\hat{y}\hat{x},y}\right) \\
\Gamma^{\hat{x}}{}_{\hat{x}j} = \frac{1}{2} g^{\hat{x}\hat{x}}\left(g_{\hat{x}\hat{x},j} + g_{\hat{x}j,x} - g_{\hat{x}j,x}\right) + \frac{1}{2} g^{\hat{x}\hat{y}}\left(g_{\hat{y}\hat{x},j} + g_{\hat{y}j,x} - g_{\hat{x}j,y}\right) \arrow[u, "j/\hat{x}"'] \arrow[d, "j/\hat{y}"'] & \Gamma^{\hat{x}}{}_{ij} = \frac{1}{2} g^{\hat{x}\hat{x}}\left(g_{\hat{x}i,j} + g_{\hat{x}j,i} - g_{ij,x}\right) + \frac{1}{2} g^{\hat{x}\hat{y}}\left(g_{\hat{y}i,j} + g_{\hat{y}j,i} - g_{ij,y}\right) \arrow[l, "i/\hat{x}"'] \arrow[r, "i/\hat{y}"] & \Gamma^{\hat{x}}{}_{\hat{y}j} = \frac{1}{2} g^{\hat{x}\hat{x}}\left(g_{\hat{x}\hat{y},j} + g_{\hat{x}j,y} - g_{\hat{y}j,x}\right) + \frac{1}{2} g^{\hat{x}\hat{y}}\left(g_{\hat{y}\hat{y},j} + g_{\hat{y}j,y} - g_{\hat{y}j,y}\right) \arrow[u, "j/\hat{x}"'] \arrow[d, "j/\hat{y}"'] \\
\Gamma^{\hat{x}}{}_{\hat{x}\hat{y}} = \frac{1}{2} g^{\hat{x}\hat{x}}\left(g_{\hat{x}\hat{x},j} + g_{\hat{x}j,x} - g_{\hat{x}j,x}\right) + \frac{1}{2} g^{\hat{x}\hat{y}}\left(g_{\hat{y}\hat{x},j} + g_{\hat{y}j,x} - g_{\hat{x}j,y}\right) & & \Gamma^{\hat{x}}{}_{\hat{y}\hat{y}} = \frac{1}{2} g^{\hat{x}\hat{x}}\left(g_{\hat{x}\hat{y},y} + g_{\hat{x}\hat{y},y} - g_{\hat{y}\hat{y},x}\right) + \frac{1}{2} g^{\hat{x}\hat{y}}\left(g_{\hat{y}\hat{y},y} + g_{\hat{y}\hat{y},y} - g_{\hat{y}\hat{y},y}\right) \\
 & \Gamma^{k}{}_{ij} = \frac{1}{2} g^{k\hat{x}}\left(g_{\hat{x}i,j} + g_{\hat{x}j,i} - g_{ij,x}\right) + \frac{1}{2} g^{k\hat{y}}\left(g_{\hat{y}i,j} + g_{\hat{y}j,i} - g_{ij,y}\right) \arrow[uu, "k/\hat{x}"'] \arrow[dd, "k/\hat{y}"'] & \\
\Gamma^{\hat{y}}{}_{\hat{x}\hat{x}} = \frac{1}{2} g^{\hat{y}\hat{x}}\left(g_{\hat{x}\hat{x},x} + g_{\hat{x}\hat{x},x} - g_{\hat{x}\hat{x},x}\right) + \frac{1}{2} g^{\hat{y}\hat{y}}\left(g_{\hat{y}\hat{x},x} + g_{\hat{y}\hat{x},x} - g_{\hat{x}\hat{x},y}\right) & & \Gamma^{\hat{y}}{}_{\hat{y}\hat{x}} = \frac{1}{2} g^{\hat{y}\hat{x}}\left(g_{\hat{x}\hat{y},x} + g_{\hat{x}\hat{x},y} - g_{\hat{y}\hat{x},x}\right) + \frac{1}{2} g^{\hat{y}\hat{y}}\left(g_{\hat{y}\hat{y},x} + g_{\hat{y}\hat{x},y} - g_{\hat{y}\hat{x},y}\right) \\
\Gamma^{\hat{y}}{}_{\hat{x}j} = \frac{1}{2} g^{\hat{y}\hat{x}}\left(g_{\hat{x}\hat{x},j} + g_{\hat{x}j,x} - g_{\hat{x}j,x}\right) + \frac{1}{2} g^{\hat{y}\hat{y}}\left(g_{\hat{y}\hat{x},j} + g_{\hat{y}j,x} - g_{\hat{x}j,y}\right) \arrow[u, "j/\hat{x}"'] \arrow[d, "j/\hat{y}"'] & \Gamma^{\hat{y}}{}_{ij} = \frac{1}{2} g^{\hat{y}\hat{x}}\left(g_{\hat{x}i,j} + g_{\hat{x}j,i} - g_{ij,x}\right) + \frac{1}{2} g^{\hat{y}\hat{y}}\left(g_{\hat{y}i,j} + g_{\hat{y}j,i} - g_{ij,y}\right) \arrow[l, "i/\hat{x}"'] \arrow[r, "i/\hat{y}"] & \Gamma^{\hat{y}}{}_{\hat{y}j} = \frac{1}{2} g^{\hat{y}\hat{x}}\left(g_{\hat{x}\hat{y},j} + g_{\hat{x}j,y} - g_{\hat{y}j,x}\right) + \frac{1}{2} g^{\hat{y}\hat{y}}\left(g_{\hat{y}\hat{y},j} + g_{\hat{y}j,y} - g_{\hat{y}j,y}\right) \arrow[u, "j/\hat{x}"'] \arrow[d, "j/\hat{y}"'] \\
\Gamma^{\hat{y}}{}_{\hat{x}\hat{y}} = \frac{1}{2} g^{\hat{y}\hat{x}}\left(g_{\hat{x}\hat{x},y} + g_{\hat{x}\hat{y},x} - g_{\hat{x}\hat{y},x}\right) + \frac{1}{2} g^{\hat{y}\hat{y}}\left(g_{\hat{y}\hat{x},y} + g_{\hat{y}\hat{y},x} - g_{\hat{x}\hat{y},y}\right) & & \Gamma^{\hat{y}}{}_{\hat{y}\hat{y}} = \frac{1}{2} g^{\hat{y}\hat{x}}\left(g_{\hat{x}\hat{y},y} + g_{\hat{x}\hat{y},y} - g_{\hat{y}\hat{y},x}\right) + \frac{1}{2} g^{\hat{y}\hat{y}}\left(g_{\hat{y}\hat{y},y} + g_{\hat{y}\hat{y},y} - g_{\hat{y}\hat{y},y}\right)
\end{tikzcd}
$$