Gemunustic

Euclidean coordinates

- Usual form
$$\mathrm{d}\ell^2 = \mathrm{d}x^2 + \mathrm{d}y^2$$
- Matrix form
$$\mathrm{d}\ell^2 = \begin{pmatrix} \mathrm{d}x & \mathrm{d}y \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \mathrm{d}x \\ \mathrm{d}y \end{pmatrix}$$
- Concise form
$$\mathrm{d}\vec{r} = \begin{pmatrix} \mathrm{d}x \\ \mathrm{d}y \end{pmatrix}$$
$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Polar coordinates

- Usual form
$$\mathrm{d}\ell^2 = \mathrm{d}r^2 + r^2\,\mathrm{d}\theta^2$$
- Matrix form
$$\mathrm{d}\ell^2 = \begin{pmatrix} \mathrm{d}r & \mathrm{d}\theta \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & r \end{pmatrix} \begin{pmatrix} \mathrm{d}r \\ \mathrm{d}\theta \end{pmatrix}$$
- Concise form
$$\mathrm{d}\vec{r} = \begin{pmatrix} \mathrm{d}r \\ \mathrm{d}\theta \end{pmatrix}$$
$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 \\ 0 & r \end{pmatrix}$$

Common

- Concise form
$$\mathrm{d}\ell^2 = \mathrm{d}\vec{r}^{\top} g_{\mu\nu}\,\mathrm{d}\vec{r}$$
- Einstein form
$$\mathrm{d}\ell^2 = g_{ij}\,\mathrm{d}x^i\,\mathrm{d}x^j$$

Spherical coordinates (2D)

- Usual form
$$\mathrm{d}\ell^2 = R^2\,\mathrm{d}\theta^2 + R^2 \sin^2\theta\,\mathrm{d}\varphi^2$$
- Matrix form
$$\mathrm{d}\ell^2 = \begin{pmatrix} \mathrm{d}\theta & \mathrm{d}\varphi \end{pmatrix} \begin{pmatrix} R & 0 \\ 0 & R\sin\theta \end{pmatrix} \begin{pmatrix} \mathrm{d}\theta \\ \mathrm{d}\varphi \end{pmatrix}$$
- Concise form
$$\mathrm{d}\vec{r} = \begin{pmatrix} \mathrm{d}\theta \\ \mathrm{d}\varphi \end{pmatrix}$$
$$g_{\mu\nu} = \begin{pmatrix} R & 0 \\ 0 & R\sin\theta \end{pmatrix}$$