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Unscented Kalman Filtering for SINS Attitude Estimation

Lin Zhao, Qi Nie, Qiufen Guo
College of Automation
Harbin Engineering University
Harbin, China
zhaolin@hrbeu.edu.cn

***Abstract*—This paper presents a nonlinear error model based on the quaternion for attitude estimation of the strapdown inertial navigation system (SINS). Extended kalman filter (EKF) is widely applied in the attitude estimation problem of SINS. The unscented kalman filter (UKF) is an extension of the classic EKF to nonlinear process and measurement models. It is noted that for the nonlinear system UKF uses a carefully selected set of sample points to map the probability distribution more accurately than the linearization of the standard EKF. Then, unscented attitude filter is designed to achieve the nonlinear filter based on the proposed model. The attitude kinematics error model is described by a quaternion because no singularities are present and kinematics equation is bilinear. Monte Carlo simulation is made to compare the new filter with the standard EKF. The results and analysis indicate that UKF has the faster convergence rate, the higher filtering accuracy, and more stable estimation performance in attitude estimation of SINS.**

Keywords-SINS; attitude estimation; quaternion; EKF;UKF

I. INTRODUCTION

Attitude estimation of SINS is a basic nonlinear estimation problem, in which the object of investigation in the system consisting of noisy nonlinear state equation and corresponding measurement equation. The algorithm solution to this problem varies in terms of accuracy, stability and computation effectiveness. The task of attitude estimation is that of determining the orientation of the vehicle relative to the reference frame. The mathematical representations of attitude are diverse, such as Euler angle [1], quaternion [2], modified Rodrigues parameters [3], and even the rotation vector [4]. Quaternion is proved very useful in attitude representation. A well-know attractive feature of quaternion representation is that the formulation of attitude dynamic in terms of the quaternion is linear and nonsingular [5]. Moreover with only one redundant parameter, quaternion is the minimal nonsingular attitude parameter.

Several approaches have addressed the issue of attitude estimation. Optimal algorithm has been developed over the last three decades following the two main approaches: classical least-squares approach and kalman filter approach. The first approach is introduced in 1967, in the so-called Wahba's problem, which is a constrained least-squares minimization problem of finding the attitude matrix [6]. Later in 1971, Davenport solves the Wahba's problem in terms of attitude quaternion. The algorithm developed is known as q-method, whose advantage is that it yields a close-form optimal estimation of the quaternion while explicitly preserving the unit-norm property [7]. Under ideal circumstance, numerous approaches based on basic q-method work extremely well. On the other hand, the kalman filter approach yields sequential quaternion estimator that is minimum-covariance. However, Quaternion must obey the normalization constraint. Kalman filer is not designed to preserve constraints imposed on the estimated state variables. The most general approach to overcome the difficulty in EKF is using the multiplicative error quaternion, where neglecting higher-order terms, the four-component quaternion can effectively by replaced by three-component error vector. So well known EKF is wildly used in attitude estimation problem.

In this paper, the attitude estimation requires high estimation accuracy for SINS. A new attitude estimation approach, based on a filter firstly developed by Julier et al., is shown as an alternative to EKF in a variety of application, including navigation and parameter estimation for road vehicle [8], neural network training [9] and visual contour hand tracking [10]. This filter approach, which is called as UKF, has several advantages over EKF, including 1) the expected error of UKF is lower than that of the standard EKF, 2) UKF can be applied to non-differentiable functions, 3) UKF is unnecessary to linearize the system in prediction stage, so no linearization error is introduced and the derivation of Jacobian matrices is avoided, and 4) UKF is valid to higher-order expansions than the standard EKF. UKF can be adapted for the nonlinear system as well as linear system. UKF works on the premise that with a fixed number of parameters it should be easier to approximate a Guassian distribution than to approximate an arbitrary nonlinear function [11]. Also, UKF uses the standard kalman form in the posterior update, but uses a different propagation of the covariance and priori measurement update with no iterations. Therefore, in this paper, we explore the potential benefits of UKF over the standard EKF in attitude estimation of SINS.

The organization of the reminder of this paper proceeds as follows. First, basic UKF theory is reviewed in section II. Then, Section III represents a nonlinear error model which is designed to estimate SINS attitude. Then, section IV designs unscented attitude filter using quaternion update for the proposed error model. Finally, Section V contains numerical simulations and results, which are proved the efficiency of the proposed algorithm compared with EKF. Section VI concludes the paper.

II. UNSCENTED KALMAN FILTERING

In this section, the unscented kalman filter is reviewed. The filter presented is derived for discrete-time nonlinear equations, where the system model is given by

$$x_{k+1} = f(x_k, k) + G_k w_k \tag{1}$$

$$\tilde{y}_k = h(x_k, k) + v_k \tag{2}$$

where x_k is the $n \times 1$ state vector and $\tilde{y}_k$ is the $m \times 1$ measurement vector. Note that a continuous-time model can always be expressed in the form of (1) through an appropriate numerical integration scheme. We assume that the process noise w_k is zero-mean Gaussian noise processes with covariance given by Q_k and R_k, respectively. The standard Kalman filter update equations are first rewritten as

$$\hat{x}_k^+ = \hat{x}_k^- + K_k v_k \tag{3}$$

$$P_k^+ = P_k^- - K_k P_k^{vv} K_k^T \tag{4}$$

where $\hat{x}_k^-$ and P_k^- are the posterior update state estimate and covariance, respectively, and $\hat{x}_k^+$ and P_k^+ are the posterior update state estimate and covariance, respectively. The innovation v_k is given by

$$v_k = \tilde{y}_k - y_k^- = \tilde{y}_k - h(x_k^-, k) \tag{5}$$

The covariance of v_k is denoted by P_k^{vv}. The gain K_k is computed by

$$K_k = P_k^{xy} (P_k^{vv})^{-1} \tag{6}$$

where P_k^{xy} is the cross-correlation matrix between $\hat{x}_k^-$ and $\hat{y}_k^-$.

UKF uses a different propagation than the standard EKF. Given an n×n covariance matrix P, a set of 2n sigma points can be generated from the columns of the matrice $\pm\sqrt{(n+\lambda)P}$. The set of points is zero mean, but if the distribution has mean μ, then simply adding μ to each of the points yields a symmetric set of 2n points having the desired mean and covariance. Because of the symmetric nature of this set, its odd central moments are zero, and so its first three moments are the same as the original Gaussian distribution. The scalar λ is a convenient parameter for exploiting knowledge about the higher moments of the given distribution. In scalar system, for example, for n=1, a value of λ=2 leads to errors in the mean and variance that are sixth order. For higher-dimensional systems, choosing λ=3-n minimized the mean-squared-error up to the fourth order. However, caution should be exercised when λ is negative because a possibility semidefinite. If this is a major concern, then another approach can be used that allows for scaling of the sigma points, which guarantees a positive semidefinite covariance matrix [12]. Also, it can be shown that when $n+\lambda$ tends to zero the mean tends to that calculated by the truncated second-order filter. This is the foundation for UKF. And the principle of UKF for mean and covariance is shown in Fig.1 [13].

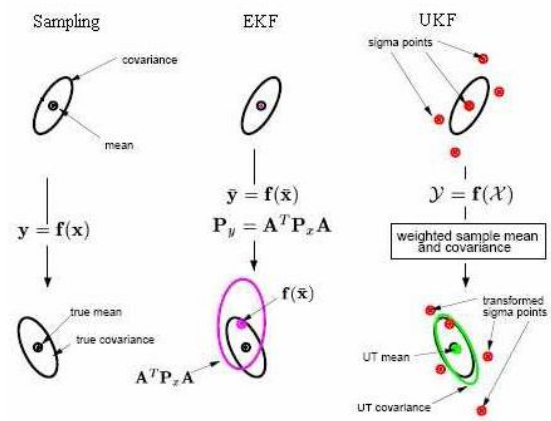

Figure 1. Principle of UKF for mean and covariance propagation

The general formulation for the propagation equations is given as follows. First, compute the following set of sigma points:

$$\sigma_k \leftarrow \text{2n columns from} \pm\sqrt{(n+\lambda)(P_k^+ + \overline{Q}_k)} \tag{7a}$$

$$x_k(0) = \hat{x}_k^+ \tag{7b}$$

$$x_k(i) = \sigma_k + x_k^+ \tag{7c}$$

where $\overline{Q}_k$ is relative to the process noise covariance. One efficient method to compute the matrix square root is the Cholesky decomposition. Next step is to sample points from the priori state distribution, that is, to generate the sigma points. Alternatively, the sigma points can be selected to lie along the eigenvector of the covariance matrix. Note that there are a total of 2n for σ_k (the positive and negative square root).

After the sigma points are generated, they are propagated through nonlinear model by

$$\chi_{k+1}(i) = f[\chi_k(i), k] \tag{8}$$

The predicted mean is defined by

$$\hat{x}_{k+1}^- = \frac{1}{n+\lambda}[\lambda\chi_{k+1}(0) + \frac{1}{2}\sum_{i=1}^{2n}\chi_{k+1}(i)] \tag{9}$$

The predicted covariance is defined by

$$\begin{aligned} P_{k+1}^- = &\frac{1}{n+\lambda}\{\lambda[\chi_{k+1}(0) - \hat{x}_{k+1}^-][\chi_{k+1}(0) - \hat{x}_{k+1}^-]^T\} \\ &+ \frac{1}{2}\sum_{i=1}^{2n}\{[\chi_{k+1}(i) - \hat{x}_{k+1}^-][\chi_{k+1}(i) - \hat{x}_{k+1}^-]^T\} + \overline{Q}_k \end{aligned} \tag{10}$$

The mean observation is defined by

$$\hat{y}_{k+1}^- = \frac{1}{n+\lambda}[\lambda\gamma_{k+1}(0) + \frac{1}{2}\sum_{i=1}^{2n}\gamma_{k+1}(i)] \tag{11}$$

where $\gamma_{k+1} = h[\chi_{k+1}(i), k]$ (12)

The output covariance is defined by

$$\begin{aligned} P_{k+1}^{yy} = &\frac{1}{n+\lambda}\{\lambda[\gamma_{k+1}(0) - \hat{y}_{k+1}^-][\gamma_{k+1}(0) - \hat{y}_{k+1}^-]^T\} \\ &+ \frac{1}{2}\sum_{i=1}^{2n}\{[\gamma_{k+1}(i) - \hat{y}_{k+1}^-][\gamma_{k+1}(i) - \hat{y}_{k+1}^-]^T\} \end{aligned} \tag{13}$$

Then, the innovation covariance is defined by

$$P_{k+1}^{vv} = P_{k+1}^{yy} + R_{k+1} \tag{14}$$

Finally, the cross-correlation matrix is determined using

$$\begin{aligned} P_{k+1}^{xy} &= \frac{1}{n+\lambda}\{\lambda[\chi_{k+1}(0)-\hat{x}_{k+1}^{-}][\gamma_{k+1}(0)-\hat{y}_{k+1}^{-}]^{T}\} \\ &+\frac{1}{2}\sum_{i=1}^{2n}\{[\chi_{k+1}(i)-\hat{x}_{k+1}^{-}][\gamma_{k+1}(i)-\hat{y}_{k+1}^{-}]^{T}\} \end{aligned} \tag{15}$$

The filter gain is then computed using (4) and the state vector can now be updated using (2). Even though 2n+1 propagations are required for UKF, especially if the continuous-time covariance equations need to be integrated and a numerical Jacobian matrix is evaluated. Since the propagations can be performed in parallel, UKF is ideally suited for parallel computation architecture.

III. Attitude Kinematics Error Meodel

Generally SINS utilize quaternion to update the attitude of the vehicle due to the advantages of its nonsingularity, simplicity, and computation efficiency. Therefore we use quaternion to obtain attitude error model in this paper. The navigation attitude error equation is mechanized in a local level wander azimuth implementation.

The quaternion differential equation is defined between body frame and navigation frame in SINS as follows

$$\begin{aligned} \dot{Q}_b^n &= \frac{1}{2}[Q_b^n]\omega_{nb}^b = \frac{1}{2}[Q_b^n](\omega_{ib}^b-\omega_{in}^b) \\ &= \frac{1}{2}[Q_b^n]\omega_{ib}^b - \frac{1}{2}[Q_b^n]\omega_{in}^b \\ &= \frac{1}{2}\langle\omega_{ib}^b\rangle Q_b^n - \frac{1}{2}[Q_b^n]([Q_b^n]^{-1}\omega_{in}^n[Q_b^n]) \\ &= \frac{1}{2}\langle\omega_{ib}^b\rangle Q_b^n - \frac{1}{2}[\omega_{in}^n]Q_b^n \end{aligned} \tag{16}$$

where Q_b^n denotes the quaternion from the body frame to navigation frame; ω_{vb}^b denotes angular rates of body frame with respect to inertial frame expressed in body frame; ω_{vn}^n denotes angular rate of navigation frame with respect to inertial frame expressed in navigation frame.

$\langle\omega_{ib}^b\rangle$ and $[\omega_{in}^n]$ are given by below

$$\langle\omega_{ib}^b\rangle = \begin{bmatrix} 0 & -\omega_x & -\omega_y & -\omega_z \\ \omega_x & 0 & \omega_z & -\omega_y \\ \omega_y & -\omega_z & 0 & \omega_x \\ \omega_z & \omega_y & -\omega_x & 0 \end{bmatrix} \tag{17}$$

$$[\omega_{in}^n] = \begin{bmatrix} 0 & -\omega_E & -\omega_N & -\omega_U \\ \omega_E & 0 & -\omega_U & \omega_N \\ \omega_N & \omega_U & 0 & -\omega_E \\ \omega_U & -\omega_N & \omega_E & 0 \end{bmatrix} \tag{18}$$

Since the true ω_{ib}^b and ω_{in}^n are not available due to gyro drift and position and velocity error. The actual quaternion error is computed by angular velocity as following

$$\hat{\omega}_{ib}^b = \omega_{ib}^b + \delta\omega_{ib}^b \tag{19a}$$

$$\hat{\omega}_{in}^n = \omega_{in}^n + \delta\omega_{in}^n \tag{19b}$$

where $\hat{\omega}_{ib}^b$ denotes measured angular velocity; $\hat{\omega}_{in}^n$ denotes computed angular velocity; $\delta\omega_{ib}^b$ and $\delta\omega_{in}^n$ represent the error included in $\hat{\omega}_{ib}^b$ and $\hat{\omega}_{in}^n$ respectively.

In fact, differential equation for quaternion correction in navigation system is expressed

$$\dot{\hat{Q}}_b^n = \frac{1}{2}\langle\hat{\omega}_{ib}^b\rangle\hat{Q}_b^n - \frac{1}{2}[\hat{\omega}_{in}^n]\hat{Q}_b^n \tag{20}$$

So it is supposed that the quaternion Q_b^n is computed using $\hat{\omega}_{ib}^b$ and $\hat{\omega}_{in}^n$.

Define quaternion error δQ

$$\delta Q = \hat{Q}_b^n - Q_b^n = [\delta q_0 \quad \delta q_1 \quad \delta q_2 \quad \delta q_3]^T \tag{21}$$

Substituting (19) into (20) and integrating above (16) and (21), the attitude error equation can be derived as

$$\delta\dot{Q} = \frac{1}{2}\langle\omega_{ib}^b\rangle\delta Q - \frac{1}{2}[\omega_{in}^n]\delta Q + \frac{1}{2}\langle\delta\omega_{ib}^b\rangle\hat{Q}_b^n - \frac{1}{2}[\delta\omega_{in}^n]\hat{Q}_b^n \tag{22}$$

Using the following transform as

$$\langle\delta\omega_{ib}^b\rangle\hat{Q}_b^n = U(\hat{Q}_b^n)\delta\omega_{ib}^b \tag{23}$$

$$[\delta\omega_{in}^n]\hat{Q}_b^n = Y(\hat{Q}_b^n)\delta\omega_{in}^n \tag{24}$$

where $U(\dot{Q}_b^n) = U = \begin{bmatrix} -q_1 & -q_2 & -q_3 \\ q_0 & -q_3 & q_2 \\ q_3 & q_0 & -q_1 \\ -q_2 & q_1 & q_0 \end{bmatrix}$ (25a)

$$Y(\dot{Q}_b^n) = Y = \begin{bmatrix} -q_1 & -q_2 & -q_3 \\ q_0 & q_3 & -q_2 \\ -q_3 & q_0 & q_1 \\ q_2 & -q_1 & q_0 \end{bmatrix} \tag{25b}$$

From the U and Y expressions, it is easy to validate the following relationships

$$U^T U = I_{3\times3} \tag{26a}$$

$$Y^T Y = I_{3\times3} \tag{26b}$$

$$Y^T U = C_b^n \tag{26c}$$

Equation (22), which can be rewritten by above (25), is of a form

$$\delta\dot{Q} = \frac{1}{2}\langle\omega_{ib}^b\rangle\delta Q - \frac{1}{2}[\omega_{in}^n]\delta Q + \frac{1}{2}U(\hat{Q}_b^n)\delta\omega_{ib}^b - \frac{1}{2}Y(\hat{Q}_b^n)\delta\omega_{in}^n \tag{27}$$

Consequently, (27) describes the differential equation of the quaternion error. So this equation can exactly reflect propagation characteristics of attitude errors.

IV. Unscented Attitude Filter Design

In this section, unscented attitude filter is designed for the attitude estimation of SINS. The approach for the design of filter using quaternion kinematics equation straightforward makes no guarantees that the resulting quaternion has the unit norm since the predicted quaternion mean is derived using the averaged sum of quaternion [14]. This makes direct implementation of UKF for the quaternion estimation difficult.

However, the filter can be designed in the approach where the quaternion is normalized by brute force. The procedure of this approach is as follow. First, the attitude quaternion $\hat{q}_0^+$, covariance matrix $\hat{P}_0^+$ and gyro bias estimation $\hat{\varepsilon}_0^+$ are initialized. The derivation of matrix $\overline{Q}_k$ assumes that the approximation $\|\Delta t\hat{\omega}_k^+\| \langle\langle 1$ is valid, which is adequately for computing the process noise [15]. With this approximation the state transition matrix is approximated by

$$\phi(\Delta t) = \begin{bmatrix} 0 & -\Delta t I_{3\times3} \\ I_{3\times3} & I_{3\times3} \end{bmatrix} \tag{28}$$

So the discrete process noise covariance is given by

$$\overline{Q}_k = \frac{\Delta t}{2}\begin{bmatrix} (\varepsilon^2 - \frac{1}{6}\varepsilon^2\Delta t)I_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & \varepsilon^2 I_{3\times3} \end{bmatrix} \tag{29}$$

Then $\overline{Q}_k$ will be used in (7) and (10). The sigma points can be chosen by (7). Next, the corresponding error quaternion is calculated by choosing the parameter a and the set f=2a+1.

The propagated sigma points can be calculated by

$$\chi_k^{\delta P}(0) = 0 \qquad \chi_k^{\delta P}(i) = f\,\frac{\delta \overline{Q}_k^-(i)}{a + \delta q_k^-(i)} \tag{30}$$

$$\delta q_k^+(i) = \frac{-a\|\chi_k^{\delta P}(i)\|^2 + f\sqrt{f^2 + (1-a)^2\|\chi_k^{\delta P}(i)\|^2}}{f^2 + \|\chi_k^{\delta P}(i)\|^2} \tag{31}$$

The quaternion is subsequently propagated forward in time by

$$\hat{q}_{k+1}^-(i) = \Omega[\hat{\omega}_k^+(i)]\hat{q}_k^+(i) \tag{32}$$

The propagated error quaternion is determined by

$$\delta q_{k+1}^-(i) = \hat{q}_{k+1}^-(i) \otimes [\hat{q}_{k+1}^-(0)]^{-1} \tag{33}$$

The predicted mean and covariance can be computed by (9) and (10). Stored the propagated quaternion from (32) we can calculate the mean observation by using (11) and (12) with

$$\gamma_{k+1}(i) = \begin{bmatrix} A[\hat{q}^-(i)]r_1 \\ A[\hat{q}^-(i)]r_2 \\ \vdots \\ A[\hat{q}^-(i)]r_N \end{bmatrix}_{k+1} \tag{34}$$

The output covariance, innovation covariance and cross-correlation matrix are using (13), (14) and (15).

Finally, the quaternion is updated using

$$\hat{q}_{k+1}^+ = \delta q_{k+1}^+ \otimes \hat{q}_{k+1}^-(i) \tag{36}$$

V. Simulations and results

The aim of this section is to compare mainly the estimation accuracy and the convergence rate of the attitude error using proposed model. This paper studies the medium accuracy fiber optic gyroscope inertial navigation system (FOG INS). List in table I is the specification for the inertial sensor model.

A. Simulations under the sway motion

In general, the sway motion model is represented as follow

TABLE I
SPECIFICATION OF THE INERTIAL SENSOR

	Accelerometer	Gyroscope
Random walk	< 0.1 m/s/ h^2	< 0.005 deg/h
Scale factor	<40 ppm	<80 ppm
Nonlinearity	<40 ppm	<80 ppm
Misalignment	<2 mrad	
Short term in bias stability	± 2 mg	<0.005 deg/h
Long term in bias stability		<0.025 deg/h

$$\left.\begin{aligned} \theta &= \theta_m \sin(\omega_\theta t + \theta_0) \\ r &= r_m \sin(\omega_r + r_0) \\ \varphi &= \varphi_m \sin(\omega_\varphi t) + \varphi_0 \end{aligned}\right\}$$

where θ, r, φ are pitch, roll and yaw respectively; the sway magnitudes θ_m, r_m, φ_m are 3°, 3°, and 10° respectively. The sway frequents ω_θ, ω_r, ω_φ are 0.1 Hz, 0.1Hz, and 0.05 Hz respectively. The initial phase angles θ_0, r_0 and φ_0 are 0°, 0°, and 45° respectively. All gyro constant drift are 0.01°/h and all the accelerometer bias are $10^{-4}g$. The initial quaternion equals [1, 0, 0, 0]. The random noise of the gyros and accelerometers are supposed as Gaussian white noise N (0, (0.001 deg/h)^2) and N (0, (10^{-6}g)^2) respectively.

With the above parameter, simulation is conducted by UKF and EKF. To compare the estimation performance of the two methods, the Monte Carlo simulation with 500 seconds run is used. The estimated state and covariance are obtained, and the simulation results are shown in Fig.2-4.

B. Results and analysis

Fig.2 and Fig.3 reveal the estimation errors of the horizontal angle (so-called pitch and roll), respectively. Fig.4 shows that estimation error of azimuth angle (so-called yaw). From the previous two figures, the estimation errors of horizontal angle quickly converge in less than 50 seconds, regardless of EKF and UKF. Both of the horizontal estimation can converge almost at the same time, while the azimuth estimation of UKF shown in Fig.4 is obviously different from that of EKF. The estimation error of azimuth angle converges in less than 150 seconds using UKF. In contrast, the estimation error of azimuth angle by EKF can not converge within the 250 seconds. So in the sense, UKF is superior to EKF.

In order to compare the estimation accuracy further, we can carry on the simulation at the different sampling rate so as to get the root mean square error (RMS). For estimated quaternion error, RMS is defined by $RMS_q = \sqrt{\frac{1}{n}\sum_{i=1}^{n} e_i}$, in which e_i denotes each quaternion error. Table II shows RMS of quaternion error at different sampling frequency. These results show that UKF has higher accuracy than EKF at sampling rate of 10 Hz and 100 Hz. But the simulation, which run at the 25 Hz shows that there is slight improvement in EKF and UKF estimation accuracy. This result indicates that the 25 Hz sampling rate is probably not appropriate for applying the filtering algorithm in quaternion estimation. From the view of this point, sampling rate is the important factor inclination whether UKF performs better than EKF. In general, if the sampling rate is adequately high, the quaternion dynamics behaves in quasilinear term. With the small time steps, quaternion propagations only deviate from the unit sphere, making the linearity minimal.

In addition, the running time for two algorithms is studied. On the average, EKF takes 180.48 microseconds for per estimation while UKF takes 242.15 microseconds for per estimation. This seems that UKF also gets the advantages over EKF in the case of running time of the quaternion estimation. From the running time standpoint, UKF still is considered as the appropriate filtering in this case.

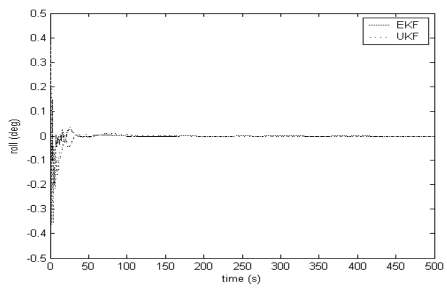

Figure 2. Pitch estimation from EKF and UKF.

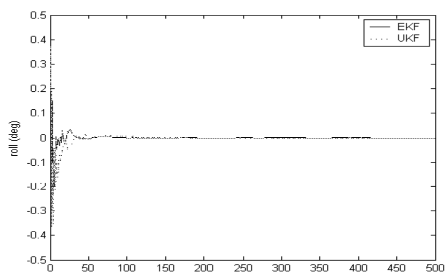

Figure 3. Roll estimation from EKF and UKF.

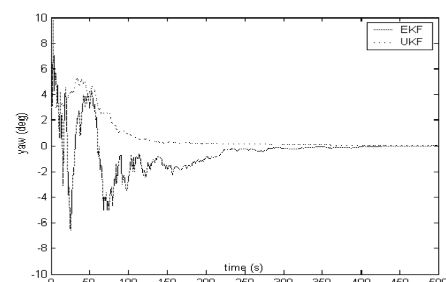

Figure 4. Yaw estimation from EKF and UKF.

TABLE II
RMS FOR EKF AND UKF AT DIFFERENT SAMPLING RATE

Sampling rate	EKF	UKF
10 Hz	0.6638	0.4457
25 Hz	0.5715	0.5688
100 Hz	0.2406	0.1029

VI. CONCLUSION

In this paper the nonlinear error model based on quaternion parameter for SINS attitude estimation has been presented. Then the unscented attitude filter is designed for nonlinear filtering. The comparisons between the UKF and EKF show that under the same conditions, the performance of the proposed algorithm far extends the standard EKF: rapid convergence rate, higher estimation accuracy, more stable performance. Therefore UKF is better choice for the task of attitude estimation in SINS application.

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