

Figure 5.1. Das FGE (2005).

CHANGE IN HEAD FROM POINTS A & B (Δh)

$$\Delta h = h_A - h_B$$

$$\Delta h = \left(\frac{u_A}{\gamma_w} + Z_A\right) - \left(\frac{u_B}{\gamma_w} + Z_B\right)$$

Δh can be expressed in non-dimensional form

$$i = \frac{\Delta h}{L}$$

Where:

i = Hydraulic Gradient

L = Length of Flow between Points A & B

VELOCITY (v) vs. HYDRAULIC GRADIENT (i)

General relationship shown in Figure 5.2

Three Zones:

- 1. Laminar Flow (I)
- 2. Transistion Flow (II)
- 3. Turbulent Flow (III)

For most soils, flow is laminar. Therefore:

 $V \propto i$

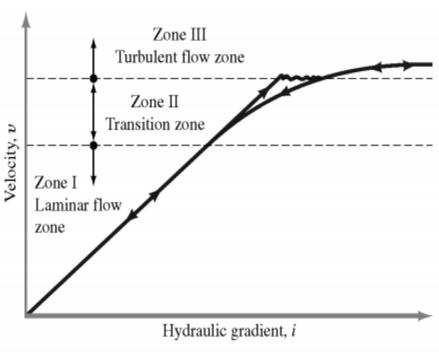


Figure 5.2. Das FGE (2005).

DARCY'S LAW (1856)

$$v = ki$$

Where:

v = Discharge Velocity (i.e. quantity of water in unit time through unit cross-sectional area at right angles to the direction of flow)

k = Hydraulic Conductivity (i.e. coefficient of permeability)

i = Hydraulic Gradient

^{*} Based on observations of flow of water through clean sands

HYDRAULIC CONDUCTIVITY (k)

Typical Values of k per Soil Type

$$k = \frac{\gamma_w}{\eta} \overline{K}$$

Where:

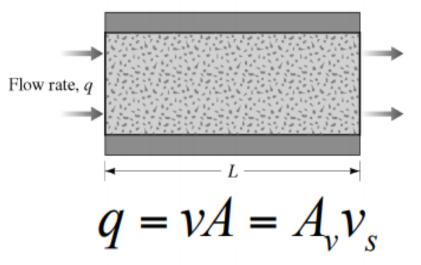
 η = Viscosity of Water

K = Absolute
Permeability
(units of L²)

Soil Type	k	k
	(cm/sec)	(ft/min)
Clean Gravel	100-1	200-2
Coarse Sand	1-0.01	2-0.02
Fine Sand	0.01-0.001	0.02-0.002
Silty Clay	0.001-0.0000 1	0.002-0.0000 2
Clay	< 0.000001	<0.000002

after Table 5.1. Das FGE (2005)

DISCHARGE & SEEPAGE VELOCITIES



Where:

q = Flow Rate
 (quantity of water/unit time)

A = Total Cross-sectional Area

 A_v = Area of Voids

 v_s = Seepage Velocity

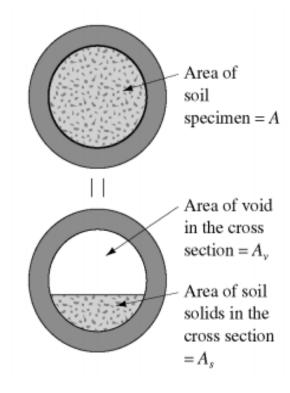
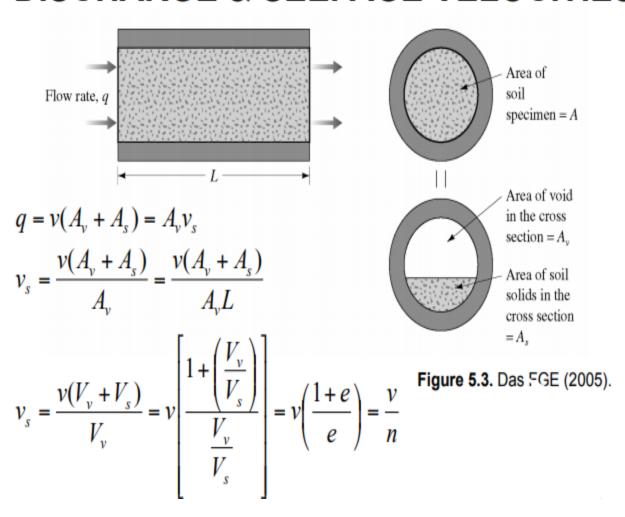


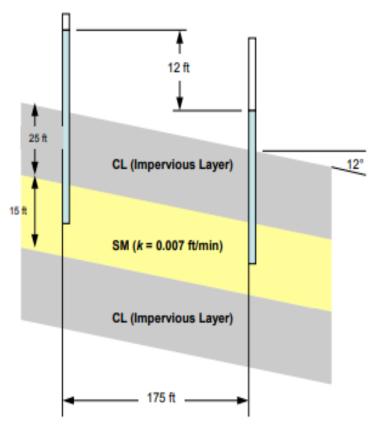
Figure 5.3. Das FGE (2005).

DISCHARGE & SEEPAGE VELOCITIES



EXAMPLE PROBLEM

GIVEN: REQUIRED:

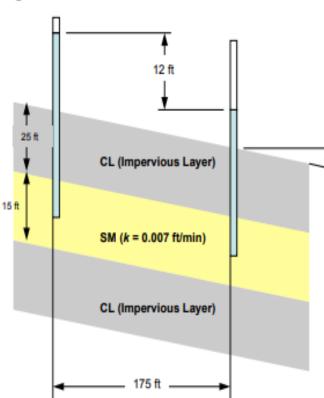


Find Hydraulic Gradient (i) and Flow Rate (q)

EXAMPLE PROBLEM – FIND i, q

GIVEN:

SOLUTION:



Hydraulic Gradient (i):

$$i = \frac{\Delta h}{L} \qquad i = \frac{12ft}{\left(\frac{175ft}{\cos 12^{\circ}}\right)} = 0.067$$

Rate of Flow per Time (q):

$$q = kiA$$

$$q = 0.007 \frac{ft}{\min} (0.067)(15 ft)(\cos 12^{\circ})(1 ft)$$

$$q = 6.9 \times 10^{-3} ft^{3} / \min / ft$$

LABORATORY TESTING OF HYDRAULIC CONDUCTIVITY

Constant Head (ASTM D2434)

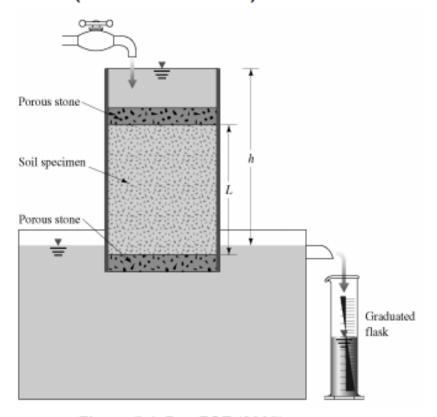


Figure 5.4. Das FGE (2005).

Falling Head (no ASTM)

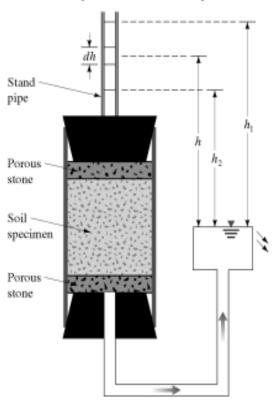


Figure 5.5. Das FGE (2005).

LABORATORY TESTING OF HYDRAULIC CONDUCTIVITY

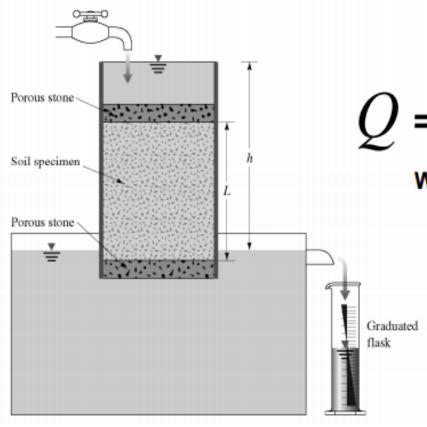


Figure 5.4. Das FGE (2005).

Constant Head (ASTM D2434)

$$Q = Avt = A(ki)t$$

Where:

Q = Quantity of water collected over time t

t = Duration of water collection

$$Q = A \left(k \frac{h}{L} \right) t$$
$$k = \frac{QL}{Aht}$$

LABORATORY TESTING OF HYDRAULIC CONDUCTIVITY

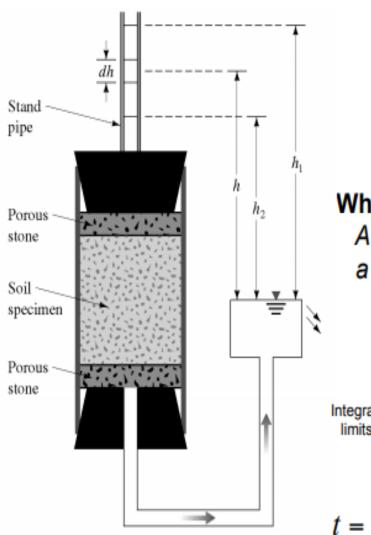


Figure 5.5. Das FGE (2005).

Falling Head

(No ASTM)

$$q = k \frac{h}{L} A = -a \frac{dh}{dt}$$

Where:

A = Cross-sectional area of Soil

a = Cross-sectional area of Standpipe

after rearranging above equation

Integrate from limits 0 to t
$$dt = \frac{aL}{Ak} \left(-\frac{dh}{h} \right)$$
 Integrate from limits h₁ to h₂

after integration

$$t = \frac{aL}{Ak} \log e \frac{h_1}{h_2} \text{ or } k = 2.303 \frac{aL}{At} Log_{10} \frac{h_1}{h_2}$$

EMPIRICAL RELATIONSHIPS FOR HYDRAULIC CONDUCTIVITY

Uniform Sands - Hazen Formula (Hazen, 1930):

$$k(cm/\sec) = cD_{10}^2$$

Where:

c = Constant between 1 to 1.5 D_{10} = Effective Size (in mm)

Sands – Kozeny-Carman

(Loudon 1952 and Perloff and Baron 1976):

$$k = C_1 \frac{e^3}{1 + e}$$

Where:

C = Constant (to be determined) e = Void Ratio

Sands - Casagrande

(Unpublished):

$$k = 1.4e^2 k_{0.85}$$

Where:

e = Void Ratio $k_{0.85}$ = Hydraulic Conductivity @ e = 0.85

Normally Consolidated Clays

(Samarasinghe, Huang, and Drnevich, 1982):

$$k = C_2 \left(\frac{e^n}{1 + e} \right)$$

Where:

C₂ = Constant to be determined experimentally
 n = Constant to be determined experimentally
 e = Void Ratio

EXAMPLE - ESTIMATION OF HYDRAULIC CONDUCTIVITY (NORMALLY CONSOLIDATED CLAYS)

GIVEN:

Normally consolidated clay with e and k measurements from 1D Consolidation Test.

Void Ratio (e)	k (cm/sec)
1.2	0.6 x 10-7
1.52	1.52 x 10-7

REQUIRED:

Find k for same clay with a void ratio of 1.4.

SOLUTION:

Using (Samarasinghe, Huang, and Drnevich, 1982) Equation:

$$\frac{k_1}{k_2} = \frac{C_2 \left[\frac{e_1^n}{1 + e_1} \right]}{C_2 \left[\frac{e_2^n}{1 + e2} \right]} \quad \begin{array}{l} \text{Substituting} \\ \text{known} \\ \text{quantities} \end{array} \quad \frac{0.6cm / \sec}{1.52cm / \sec} = \left(\frac{1.2}{1.52} \right)^n \left(\frac{2.52}{2.2} \right)$$

EXAMPLE - ESTIMATION OF HYDRAULIC CONDUCTIVITY (NORMALLY CONSOLIDATED CLAYS) (continued)

$$\frac{0.6cm/\sec}{1.52cm/\sec} = \left(\frac{1.2}{1.52}\right)^n \left(\frac{2.52}{2.2}\right) \therefore n = 4.5$$

$$k_1 = C_2 \left(\frac{e_1^n}{1 + e_1} \right)$$

$$0.6x10^{-7} cm / sec = \left(\frac{1.2^{4.5}}{1+1.2}\right)$$

$$C_2 = 0.581x10^{-7} cm / sec$$

$$k_{e=1.4} = (0.581x10 - 7cm / sec) \left(\frac{1.4^{4.5}}{1+1.4}\right) = 1.1x10^{-7} cm / sec$$

EQUIVALENT HYDRAULIC CONDUCTIVITY IN STRATIFIED SOILS – HORIZONTAL DIRECTION

Considering cross-section of Unit Length 1.

Total flow through cross-section can be written as:

$$q = v \cdot 1 \cdot H$$

$$q = v_1 \cdot 1 \cdot H_1 + v_2 \cdot 1 \cdot H_2 + ... + v_n \cdot 1 \cdot H_n$$

Where:

v = Average Discharge Velocity v_1 = Discharge Velocity in Layer 1

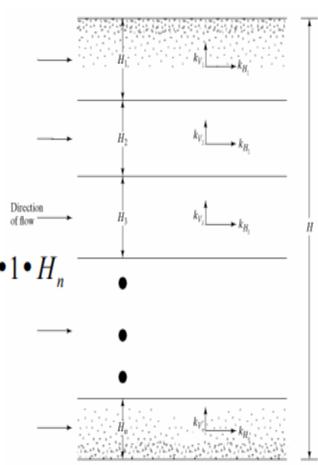


Figure 5.7. Das FGE (2005).

EQUIVALENT HYDRAULIC CONDUCTIVITY IN STRATIFIED SOILS – HORIZONTAL DIRECTION

Substituting *v=ki* into q equation and using H to denote Horizontal Direction

$$v = k_{H(eq)}i_{eq}$$

 $v_1 = k_{H1}i_1; v_2 = k_{H2}i_2; ...; v_n = k_ni_n$
Noting that $i_{eq} = i_1 = i_2 = ... = i_n$

$$k_{H(eq)} = \frac{1}{H}(k_{H1}H_1 + k_{H2}H_2 + ... + k_{Hn}H_n)$$

Where $k_{H(eq)}$ = Equivalent Hydraulic Conductivity in Horizontal Direction

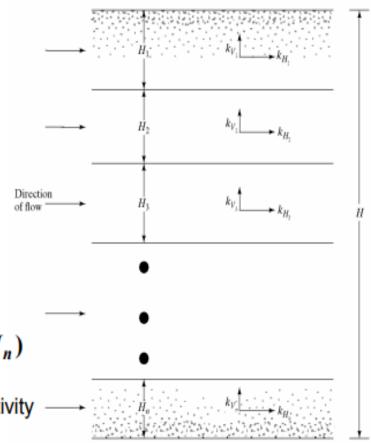


Figure 5.7. Das FGE (2005).

EQUIVALENT HYDRAULIC CONDUCTIVITY IN STRATIFIED SOILS – VERTICAL DIRECTION

Total Head Loss = h h = Sum Head Loss in Each Layer

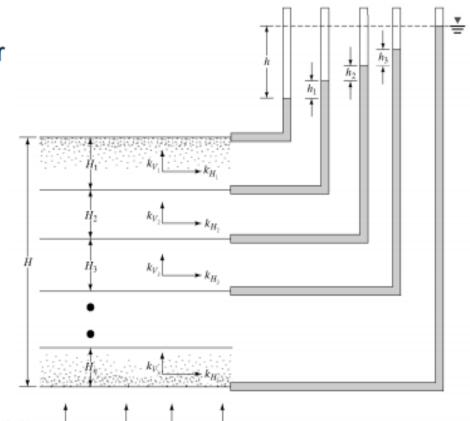
$$v = v_1 = v_2 = \dots = v_n$$
 and

$$h = h_1 = h_2 = \dots = h_n$$

Using Darcy's Law (v=ki) into v equation and using V to denote Vertical Direction

$$k_{V(eq)} \frac{h}{H} = k_{V1} i_1 = \dots = k_{Vn} i_n$$

Where $k_{V(eq)}$ = Equivalent Hydraulic Conductivity in Vertical Direction



FIELD PERMEABILITY TEST BY PUMPING WELLS

UNCONFINED PERMEABLE LAYER UNDERLAIN BY IMPERMEABLE LAYER

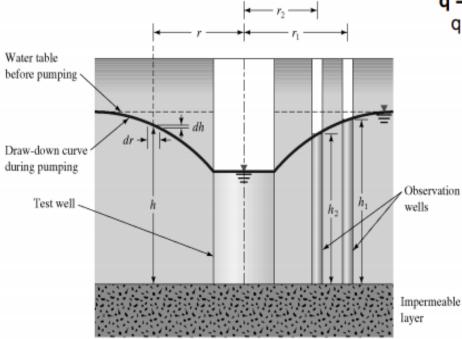


Figure 5.9. Das FGE (2005).

Field Measurements Taken:

q = Groundwater Flow into Well q also is rate of discharge from pumping

Equation:

$$q = k \left(\frac{dh}{dr}\right) 2\pi r h$$

can be re-written as

$$\int_{r_2}^{r_1} \frac{dr}{r} = \left(\frac{2\pi k}{q}\right) \int_{h_2}^{h_1} h dh$$

Solving Equation:

$$k_{field} = \frac{2.303q \log_{10} \left(\frac{r_1}{r_2}\right)}{\pi (h_1^2 - h_2^2)}$$

FIELD PERMEABILITY TEST BY PUMPING WELLS

WELL PENETRATING CONFINED AQUIFER

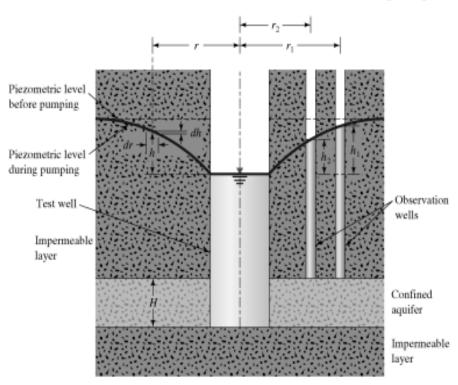


Figure 5.10. Das FGE (2005).

Field Measurements Taken:

q = Groundwater Flow into Well q also is rate of discharge from pumping

Equation:

$$q = k \left(\frac{dh}{dr}\right) 2\pi r H$$

can be re-written as

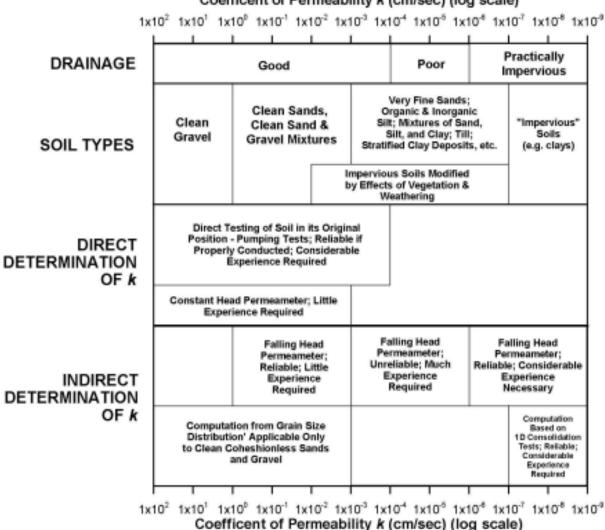
$$\int_{r_2}^{r_1} \frac{dr}{r} = \int_{h_2}^{h_1} \frac{2\pi kH}{q} dh$$

Solving Equation:

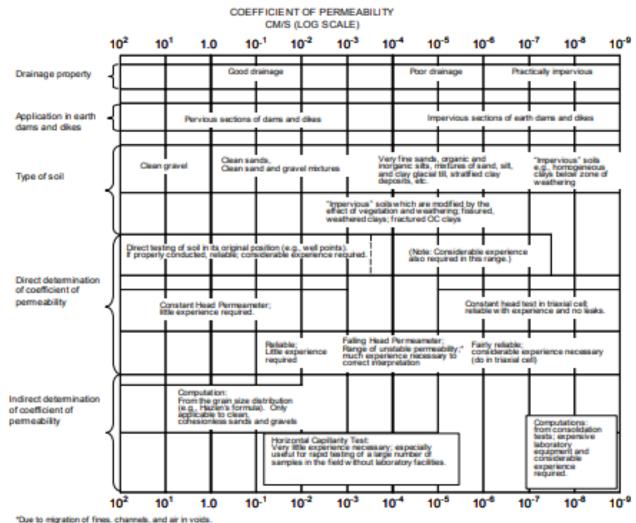
$$k_{field} = \frac{q \log_{10} \left(\frac{r_1}{r_2}\right)}{2.727H(h_1 - h_2)}$$

SOIL PERMEABILITY & DRAINAGE

Coefficent of Permeability k (cm/sec) (log scale)



SOIL PERMEABILITY & DRAINAGE



From FHWA IF-02-034 Evaluation of Soil and Rock Properties.

FLOW NETS DEFINITION OF TERMS

Flow Net: Graphical Construction used to calculate groundwater flow through soil. Comprised of Flow Lines and Equipotential Lines.

Flow Line: A line along which a water particle moves through a permeable soil medium.

Flow Channel: Strip between any two adjacent Flow Lines.

Equipotential Lines: A line along which the potential head at all points is equal.

NOTE: Flow Lines and Equipotential Lines must meet at right angles!

FLOW NETS

FLOW AROUND SHEET PILE WALL

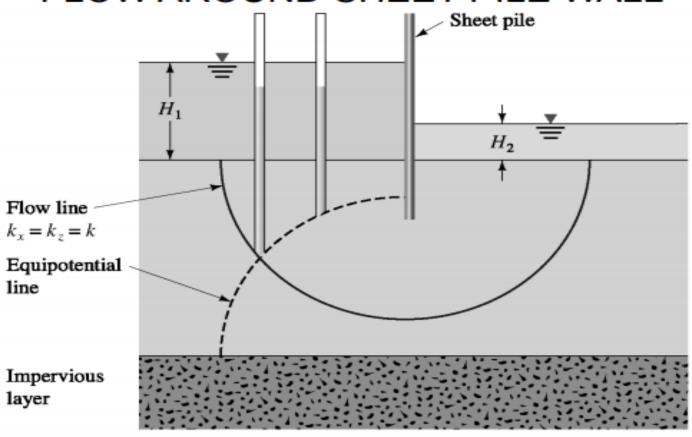


Figure 5.12a. Das FGE (2005).

FLOW NETS

FLOW AROUND SHEET PILE WALL

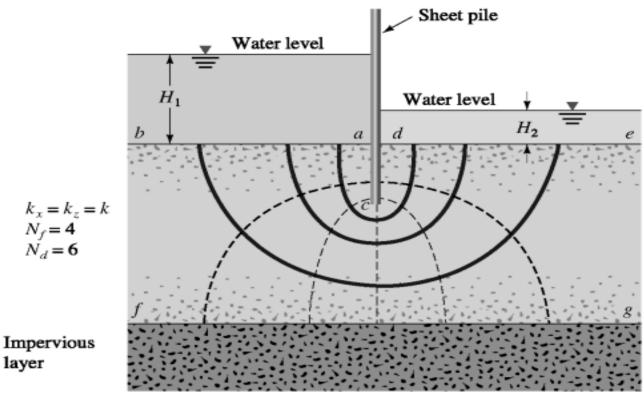


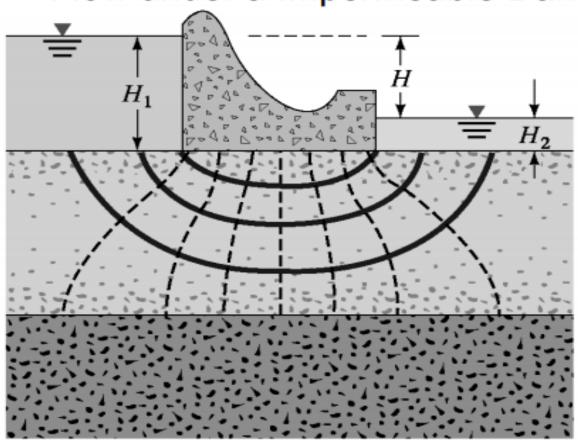
Figure 5.12b. Das FGE (2005).

FLOW NETS - BOUNDARY CONDITIONS

- The upstream and downstream surfaces of the permeable layer (i.e. lines ab and de in Figure 12b Das FGE (2005)) are equipotential lines.
- 2. Because *ab* and *de* are equipotential lines, all the flow lines intersect them at right angles.
- The boundary of the imprevious layer (i.e. line fg in Figure 12b Das FGE (2005)) is a flow line, as is the surface of the impervious sheet pile (i.e. line acd in Figure 12b Das FGE (2005)).
- 4. The equipontential lines intersect acd and fg (Figure 12b Das FGE (2005)) at right angles.

FLOW NETS

Flow under a Impermeable Dam



$$k_x = k_z = k$$

$$N_f = 4$$

$$N_d = 8$$

FLOW NETS

Seepage Calculations

 Δq

Rate of Seepage Through Flow Channel (per unit length):

$$\Delta q_1 = \Delta q_2 = \Delta q_3 = \dots = \Delta q_n$$

Using Darcy's Law (q=vA=kiA)

$$\Delta q = k \left(\frac{h_1 - h_2}{l_1} \right) l_1 = k \left(\frac{h_2 - h_3}{l_2} \right) l_2 = k \left(\frac{h_3 - h_4}{l_3} \right) l_3 = \dots$$

Figure 5.14. Das FGE (2005).

 Δq

Potential Drop

$$h_1 - h_2 = h_2 - h_3 = h_3 - h_4 = \dots = \frac{H}{N_d}$$

Where:

H = Head Difference N_d = Number of Potential Drops

FLOW NETS

FLOW AROUND SHEET PILE WALL EXAMPLE

GIVEN:

Flow Net in Figure 5.17.

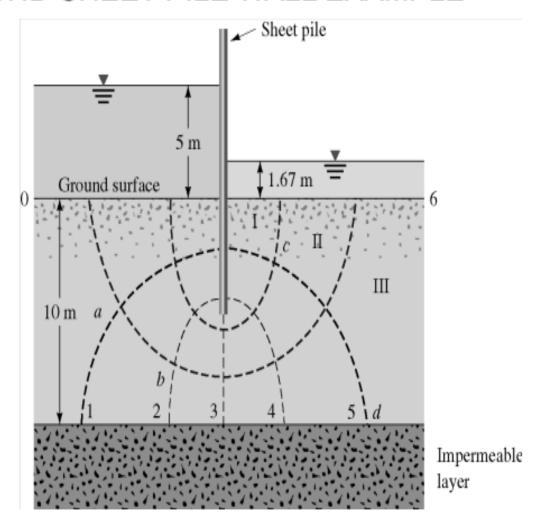
 $N_f = 3$

 $N_d = 6$

 $k_x = k_z = 5x10^{-3}$ cm/sec

DETERMINE:

- a. How high water will rise in piezometers at points a, b, c, and d.
- b. Rate of seepage through flow channel II.
- c. Total rate of seepage.



FLOW NETS

FLOW AROUND SHEET PILE WALL EXAMPLE

SOLUTION:

Potential Drop =
$$\frac{H}{N_d}$$

$$\frac{(5m-1.67m)}{6} = 0.56m$$

At Pt a:

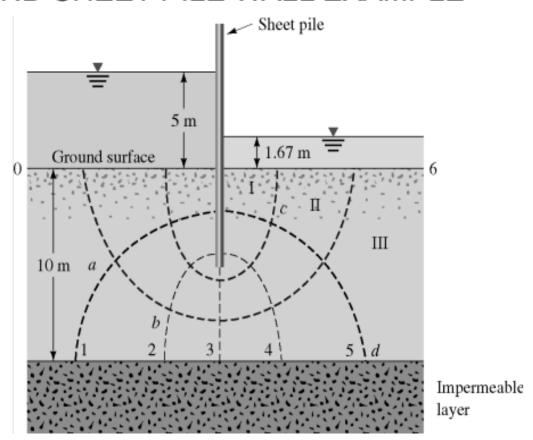
Water in standpipe = (5m - 1x0.56m) = 4.44m

At Pt b:

Water in standpipe = (5m - 2x0.56m) = 3.88m

At Pts c and d:

Water in standpipe = (5m - 5x0.56m) = 2.20m



FLOW NETS

FLOW AROUND SHEET PILE WALL EXAMPLE

SOLUTION:

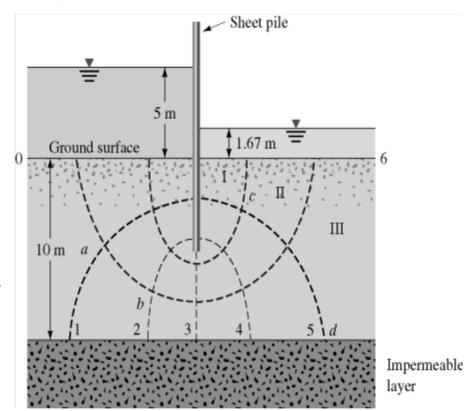
$$\Delta q = k \frac{H}{N_d}$$

 $k = 5x10^{-3}$ cm/sec = $5x10^{-5}$ m/sec

 $\Delta q = (5x10^{-5} \text{ m/sec})(0.56\text{m})$ $\Delta q = 2.8x10^{-5} \text{ m}^3/\text{sec/m}$

$$q = k \frac{HN_f}{N_d} = \Delta q N_f$$

 $q = (2.8 \times 10^{-5} \text{ m}^3/\text{sec/m}) * 3$ $q = 8.4 \times 10^{-5} \text{ m}^3/\text{sec/m}$



ALL FIGURES WERE TAKEN FROM THE BOOK ENTITLED "GEOTECHNICAL ENGINEERING" BY DAS (2012).