

⑤ what are the collapsing rules? solve by considering the instance: weights are, $w: \{10, 15, 6, 9\}$, profits $\{2, 5, 8, 13\}$. The knapsack capacity is 25.

2 5 8 1

10 15 6 9

Purging Rule (Dominance Rule):-

If S^{i+1} contains (P_j, w_j) and (P_k, w_k) such that $P_j < P_k$ and $w_j >= w_k$, then (P_j, w_j) can be eliminated. (remove object with less profit and more weight).

Initially $S^0 = \{(0, 0)\}$ i.e. $P=0, w=0$.

$$S_1^0 = \{(0+2, 0+10)\} = \{(2, 10)\}$$

$$S_1^1 = \{S^0 \cup S_1^0\} = \{(0, 0), (2, 10)\}$$

$$S_1^2 = \{(0+5, 0+15), (2+5, 10+15)\} = \{(5, 15), (7, 25)\}$$

$$S_1^3 = \{S_1^1 \cup S_1^2\} = \{(0, 0), (2, 10), (5, 15), (7, 25)\}$$

$$S_1^4 = \{(0+8, 0+6), (2+8, 10+6), (5+8, 15+6), (7+8, 25+6)\}$$

$$= \{(8, 6), (10, 16), (13, 21), (15, 31)\}$$

$$S_1^5 = \{S_1^4 \cup S_1^2\} = \{(0, 0), (2, 10), (5, 15), (7, 25), (8, 6), (10, 16), (13, 21), (15, 31)\}$$

exceeding knapsack capacity

tuple $(15, 31)$ eliminated due to exceeding knapsack capacity.

By applying Purging rule
 $(7,25), (8,6)$ $\rightarrow (7,25)$ is eliminated.

$$\therefore S^3 = \{(0,0), (2,10), (5,15), \cancel{(8,6)}, (10,16), (13,21)\}$$

By applying Purging rule on $(5,15), (8,6)$
 $\Rightarrow (5,15)$ is eliminated

$$S^3 = \{(0,0), (2,10), \cancel{(8,6)}, (10,16), (13,21)\}$$

By applying Purging rule on $(2,10), (8,6)$
 $\Rightarrow (2,10)$ is eliminated

$$S^3 = \{(0,0), (8,6), (10,16), (13,21)\}$$

$$S_1^3 = \{(0+1, 0+9), (8+1, 6+9), (10+1, 16+9), (13+1, 21+9)\}$$

$$= \{(1,9), (9,15), (11,25), (14,30)\}$$

$$S^4 = \{S^3 \cup S_1^3\}$$

$$= \{(0,0), \cancel{(1,9)}, \underline{(8,6)}, (9,15), (10,16), (11,25), (13,21), (14,30)\}$$

By applying Purging rule on $(1,9), (8,6) \Rightarrow (1,9)$ is eliminated.

$$S^4 = \{(0,0), (8,6), (9,15), (10,16), \underline{(11,25)}, (13,21), (14,30)\}$$

By applying Purging rule on $(11,25), (13,21)$

$\Rightarrow (11,25)$ is eliminated.

$S^4 = \{(0,0), (8,6), (9,15), (10,16), (13,21), (14,30)\}$

↓
exceeding knapsack capacity

$$\therefore S^4 = \{(0,0), (8,6), (9,15), (10,16), (13,21)\}$$

The last tuple in S^4 is $(13,21)$. $(13,21) \in S^3$ so $x_4 = 0$
 $(13,21) \notin S^2 \Rightarrow x_3 = 1$

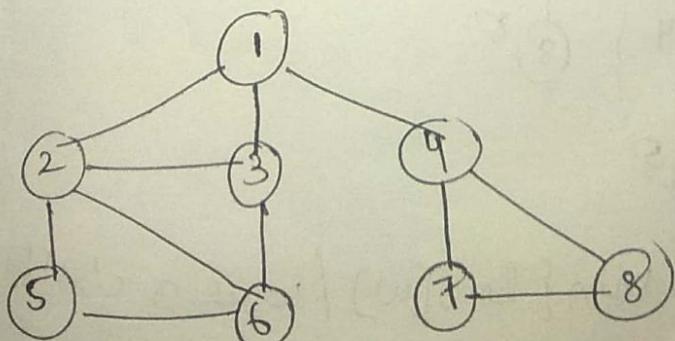
$(13,21)$ came from $(5,15)$. $(5,15) \in S^2$

$$(5,15) \notin S^1 \Rightarrow x_2 = 1$$

$(5,15)$ came from $(0,0)$. $(0,0) \in S^1$ and $(0,0) \in S^0$
 $\Rightarrow x_1 = 0$

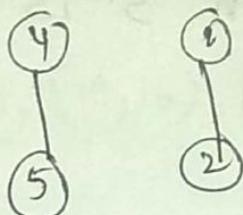
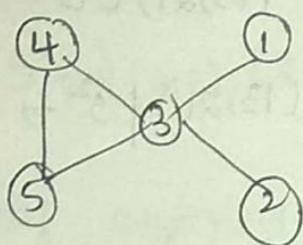
∴ optimal solution is $(x_1, x_2, x_3, x_4) = (0, 1, 1, 0)$.

- ⑥ Refine biconnected component, articulation point. With
 the help of algorithm identify the articulation point
 & draw the biconnected components of the given graph.



Articulation points- A vertex v in a connected graph G is an articulation point iff, the deletion of vertex v together with all edges incident to v disconnects the graph into two or more non empty components.

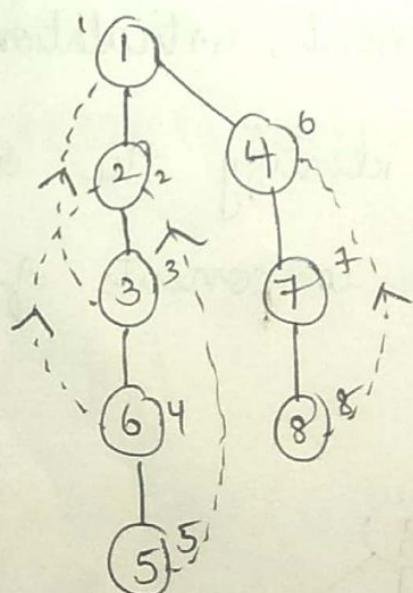
Ex:



vertex 3 is an articulation point

- Two Biconnected components can have atmost one common vertex and that vertex is an articulation point.
- A maximal biconnected subgraph of given graph is called as biconnected component.

Step ① & ②



$\text{low}[u] = \min \{ \text{dfn}[u], \min \{ \text{low}[w] \mid w \text{ is a child of } u \} \}$
 $\text{min} \{ \text{dfn}[v] \mid (u,v) \text{ is back edge} \}$

vertex	8	7	4	5	6	3	2	1
dfn	8	7	6	5	4	3	2	1
low	6	6	6	2	2	1	1	1

$$\text{low}[8] = \min \left\{ \text{dfn}[8], \min \left\{ \underset{\substack{\downarrow \\ \text{no child}}}{-}, \min \left\{ \text{dfn}[4] \right\} \right\} \right\}$$

$$= \min \{ 8, -, 6 \}$$

$$= \boxed{6}$$

$$\text{low}[7] = \min \left\{ \text{dfn}[7], \min \left\{ \text{low}[8] \right\} \right\}, \min \{ - \}$$

$$= \min \{ 7, \underset{\substack{\downarrow \\ 6}}{6}, - \}$$

$$= \boxed{6}$$

$$\text{low}[4] = \min \left\{ \text{dfn}[4], \min \left\{ \text{low}[7] \right\}, \min \{ - \} \right\}$$

$$= \min \{ 6, \underset{\substack{\downarrow \\ 6}}{6}, - \}$$

$$= \boxed{6}$$

$$\text{low}[5] = \min \left\{ \text{dfn}[5], \min \{ \cancel{\text{low}[6]} - \}, \min \left\{ \text{dfn}[2] \right\} \right\}$$

$$= \min \{ 5, -, \underset{\substack{\downarrow \\ 2}}{2} \}$$

$$= \boxed{2}$$

$$\text{low}[6] = \min \left\{ \text{dfn}[6], \min \{ \text{low}[5] \}, \min \{ \text{dfn}[2] \} \right\}$$

$$= \min \{ 4, \underset{\substack{\downarrow \\ 2}}{2}, \underset{\substack{\downarrow \\ 2}}{2} \} = \boxed{2}$$

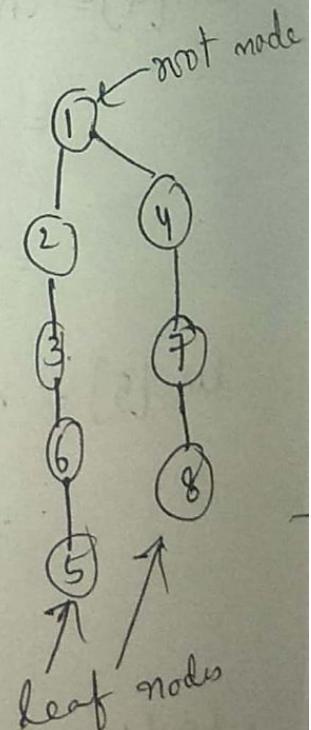
$$\begin{aligned}
 \text{low}[3] &= \min\{\text{dfn}[3], \min\{\text{low}[6]\}, \min\{\text{dfn}[1]\}\} \\
 &\quad \downarrow_2 \qquad \qquad \downarrow_1 \\
 &= \min\{3, 2, 1\} \\
 &= \boxed{1}
 \end{aligned}$$

$$\begin{aligned}
 \text{low}[2] &= \min\{\text{dfn}[2], \min\{\text{low}[3]\}, \min\{-3\}\} \\
 &\quad \downarrow_1 \\
 &= \min\{2, 1, -3\} \\
 &= \boxed{1}
 \end{aligned}$$

$$\begin{aligned}
 \text{low}[1] &= \min\{\text{dfn}[1], \min\{\text{low}[2]\}, \min\{-3\}\} \\
 &\quad \downarrow_1 \\
 &= \min\{1, 1, -3\} \\
 &= \boxed{1}
 \end{aligned}$$

finding an articulation point

vertex	1	2	3	4	5	6	7	8
dfn	1	2	3	6	5	4	7	8
low	1	1	1	6	2	2	6	6



* Leaf nodes are not articulation points.

* The root node of DFS tree is an articulation point if it has atleast 2 children.

1 is articulation point

5, 8 are not articulation points

vertex 2:

$$\text{low}[3] \geq \text{dfn}[2]$$

$$1 \geq 2 \text{ (false)}$$

∴ 2 is not an articulation point.

vertex 3:

~~$$\text{low}[6] \geq \text{dfn}[3]$$~~

$$2 \geq 3 \text{ (false)}$$

∴ 3 is not an articulation point

vertex 4:

$$\text{low}[7] \geq \text{dfn}[4]$$

$$6 \geq 6 \text{ (true)}$$

∴ 4 is articulation point

vertex 6:

$$\text{low}[5] \geq \text{dfn}[6]$$

$$2 \geq 4 \text{ (false)}$$

∴ 6 is not an articulation point

vertex 7:

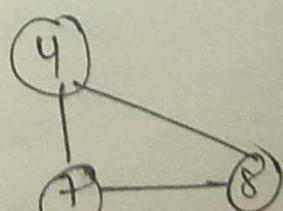
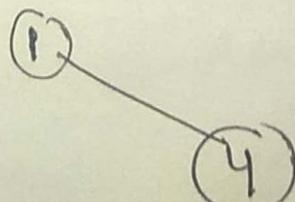
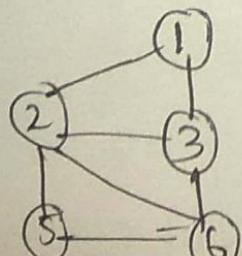
$$\text{low}[8] \geq \text{dfn}[7]$$

$$6 \geq 7 \text{ (false)}$$

∴ 7 is not an articulation point

∴ 1, 4 are articulation points

Biconnected components of given graph are:



④ Explain travelling salesperson problem using DP and

solve problem

	0	2	3	4	5
1	0	8	13	18	20
2	3	0	7	8	10
3	4	11	0	10	7
4	6	6	7	0	11
5	10	6	2	1	0

$$\begin{array}{r} 8 \\ 7 \\ 10 \\ 11 \\ \hline 10. \\ \hline 31 \\ 15 \\ \hline 46 \end{array}$$

$$\begin{array}{r} 8 \\ 10 \\ 1. \\ 7. \\ \hline 19 \\ 11 \\ \hline 30 \end{array}$$

$$\text{As } g(i, \phi) = c_{i1} \quad 1 \leq i \leq n$$

$$g(2, \phi) = c_{21} = 3 \quad g(4, \phi) = c_{41} = 6$$

$$g(3, \phi) = c_{31} = 4 \quad g(5, \phi) = c_{51} = 10$$

we know that

$$g(i, S) = \min \{c_{ij} + g(j, S - \{j\})\} \quad j \in S$$

\Rightarrow for $|S|=1$

$$g(2, \{3\}) = c_{23} + g(3, \phi) = 7 + 4 = 11$$

$$g(2, \{4\}) = c_{24} + g(4, \phi) = 8 + 6 = 14$$

$$g(2, \{5\}) = c_{25} + g(5, \phi) = 10 + 10 = 20$$

$$g(3, \{2\}) = c_{32} + g(2, \phi) = 11 + 3 = 14$$

$$g(3, \{4\}) = c_{34} + g(4, \phi) = 10 + 6 = 16$$

$$g(3, \{5\}) = c_{35} + g(5, \phi) = 7 + 10 = 17$$

$$g(4, \{2\}) = c_{42} + g(2, \phi) = 6 + 3 = 9$$

$$g(4, \{3\}) = c_{43} + g(3, \phi) = 7 + 4 = 11$$

$$g(4, \{5\}) = c_{45} + g(5, \phi) = 11 + 10 = 21$$

$$g(5, \{2\}) = c_{52} + g(2, \phi) = 6 + 3 = 9$$

$$g(5, \{3\}) = c_{53} + g(3, \phi) = 2 + 4 = 6$$

$$g(5, \{4\}) = c_{54} + g(4, \phi) = 1 + 6 = 7$$

for $|S|=2$ if $i \neq 1, i \notin S, i \notin s$

$$g(2, \{3, 4\}) = \min \{c_{23} + g(3, \{4\}), c_{24} + g(4, \{3\})\}$$
$$= \min \left\{ \underset{23}{7+16}, \underset{19}{8+11} \right\}$$
$$= 19$$

$$g(2, \{3, 5\}) = \min \left\{ \underset{24}{7+17}, \underset{16}{10+6} \right\}$$
$$= 16$$

$$g(2, \{4, 5\}) = \min \left\{ \underset{29}{8+21}, \underset{17}{10+7} \right\}$$
$$= 17$$

$$g(3, \{2, 4\}) = \min \left\{ \underset{25}{11+14}, \underset{19}{10+9} \right\} = 19$$

$$g(3, \{2, 5\}) = \min \left\{ \underset{31}{11+20}, \underset{16}{7+9} \right\} = 16$$

$$g(3, \{4, 5\}) = \min \left\{ \underset{31}{10+21}, \underset{14}{7+11} \right\} = 14$$

$$g(4, \{2, 3\}) = \min \left\{ \underset{17}{6+11}, \underset{14}{7+14} \right\} = 17$$

$$g(4, \{2, 5\}) = \min \left\{ \underset{20}{6+20}, \underset{11}{11+9} \right\} = 20$$

$$g(4, \{3, 5\}) = \min \left\{ \underset{17}{7+17}, \underset{11}{11+6} \right\} = 17$$

$$g(5, \{2, 3\}) = \min \left\{ \underset{16}{6+11}, \underset{16}{2+14} \right\} = 16$$

$$g(5, \{2, 4\}) = \min \left\{ \underset{10}{6+14}, \underset{9}{1+9} \right\} = 10$$

$$g(5, \{3, 4\}) = \min \left\{ \underset{12}{2+16}, \underset{11}{1+11} \right\} = 12$$

for $|S| = 3$

$$g(2, \{3, 4, 5\}) = \min \{c_3 + g(3, \{4, 5\}), c_{24} + g(4, \{3, 5\}), \\ c_{25} + g(5, \{3, 4\})\} \\ = \min \left\{ \frac{7+14}{21}, \frac{8+17}{25}, \frac{10+12}{22} \right\} \\ = 21$$

$$g(3, \{2, 4, 5\}) = \min \left\{ \frac{11+17}{28}, \frac{10+20}{30}, \frac{7+10}{17} \right\} = 17$$

$$g(4, \{2, 3, 5\}) = \min \left\{ \frac{6+16}{22}, \frac{7+16}{23}, \frac{11+16}{27} \right\} = 22$$

$$g(5, \{2, 3, 4\}) = \min \left\{ \frac{6+19}{25}, \frac{2+19}{21}, \frac{1+17}{18} \right\} = 18$$

for $|S| = 4$

$$g(1, \{2, 3, 4, 5\}) = \min \{c_2 + g(2, \{3, 4, 5\}), \\ c_3 + g(3, \{2, 4, 5\}), c_4 + g(4, \{2, 3, 5\}), \\ c_5 + g(5, \{2, 3, 4\})\} \\ = \min \left\{ \frac{8+21}{29}, \frac{13+17}{30}, \frac{18+22}{40}, \frac{20+18}{28} \right\} \\ = 29$$

$\therefore 1, 2, 3, 5, 4, 1$ is the shortest path.

4

Construct an OBST for identifiers.

$$(a_1 - a_4) = (A, B, C, D)$$

$$P(1:4) = (0.1, 0.2, 0.4, 0.3)$$

$$q(0:4) = (0.3, 0.2, 0.2, 0.1, 0.2)$$

$$\sum_{1 \leq i \leq n} P(i) * \text{level}(a_i) + \sum_{0 \leq i \leq n} q(i) * (\text{level}(E_i) - 1)$$

$$w(i, j) = P(j) + q(j) + w(i, j - 1)$$

$$\text{cost}(i, j) = \min_{i < k \leq j} \{ \text{cost}(i, k-1) + \text{cost}(k, j) \} + w(i, j)$$

$$w(i, i) = q(i), c(i, i) = \gamma(i, i) = 0$$

The 'k' value which gives minimum cost is chosen as

	0	1	2	3	4
j-i=0	w(0,0)=0.3 c(0,0)=0 γ(0,0)=0	w(1,1)=0.2 c(1,1)=0 γ(1,1)=0	w(2,2)=0.2 c(2,2)=0 γ(2,2)=0	w(3,3)=0.1 c(3,3)=0 γ(3,3)=0	w(4,4)=0.2 c(4,4)=0 γ(4,4)=0
j-i=1	w(0,1)=0.6 c(0,1)=0.6 γ(0,1)=1	w(1,2)=0.6 c(1,2)=0.6 γ(1,2)=2	w(2,3)=0.7 c(2,3)=0.7 γ(2,3)=3	w(3,4)=0.6 c(3,4)=0.6 γ(3,4)=4	
j-i=2	w(0,2)=1 c(0,2)=1.6 γ(0,2)=1,2	w(1,3)=1.1 c(1,3)=1.7 γ(1,3)=3	w(2,4)=1.2 c(2,4)=1.8 γ(2,4)=3		X
j-i=3	w(0,3)=1.5 c(0,3)=2.8 γ(0,3)=2	w(1,4)=1.6 c(1,4)=2.8 γ(1,4)=3			X
j-i=4	w(0,4)=2 c(0,4)=4.2 γ(0,4)=3		X		X

$$w(i,j) = p(j) + q(j) + w(i,-1)$$

$$= 0 + 0.34 \approx 0.3$$

$$r(0,0) = 2$$

1st mw: $j-i=0$

$$w(i,j) = q(j)$$

$$\Rightarrow w(0,0) = q(0) = 0.3$$

$$w(1,1) = q(1) = 0.2 \quad w(2,2) = q(2) = 0.2$$

$$w(3,3) = q(3) = 0.1 \quad w(4,4) = q(4) = 0.2$$

2nd mw $j-i=1$

$$w(i,j) = p(j) + q(j) + w(i,j-1)$$

$$w(0,1) = p(1) + q(1) + w(0,0)$$

$$= 0.1 + 0.2 + 0.3 = 0.6$$

$$cost(i,j) = \min_{i < k \leq j} \{ cost(i,k-1) + cost(k,j) \} + w(i,j)$$

$$c(0,1) = \min_{0 < k \leq 1} \{ cost(0,0) + cost(1,1) \} + w(0,1)$$

$$k=1 \rightarrow$$

$$= \min \{ 0 + 0 \} + 0.6 \rightarrow 0 + 0.6 = 0.6$$

$$\Rightarrow \gamma = 1$$

$$w(1,2) = p(2) + q(2) + w(1,1)$$

$$= 0.2 + 0.2 + 0.6 = 1.0$$

$$c(1,2) = \min_{0 < k \leq 2} \{ cost(1,0) + cost(1,2), cost(1,1) + cost(2,2) \} + w(1,2)$$

$$= \min_{k=1,2} \{ \dots \}$$

$$c(1,2) = \min_{1 \leq k \leq 2} \{ c(1,1) + c(2,2) \} + w(1,2)$$

$$k=2$$

$$= \min \{ 0 + 0 \} + 0.6 \Rightarrow 0.6 \Rightarrow \tau = 2$$

$$w(2,3) = p(3) + q(3) + w(2,2)$$

$$= 0.4 + 0.1 + 0.2 = 0.7$$

$$c(2,3) = \min_{2 \leq k \leq 3} \{ c(2,2) + c(3,3) \} + w(2,3)$$

$$k=3$$

$$= (0+0) + 0.7 = 0.7 \Rightarrow \tau = 3$$

$$w(3,4) = p(4) + q(4) + w(3,3)$$

$$= 0.3 + 0.2 + 0.1 = 0.6$$

$$c(3,4) = \min_{3 \leq k \leq 4} \{ c(3,3) + c(4,4) \} + w(3,4)$$

$$k=4$$

$$= (0+0) + 0.6 = 0.6 \Rightarrow \tau = 4$$

3rd row j-i=2

$$w(0,2) = p(2) + q(2) + w(0,1)$$

$$= 0.2 + 0.2 + 0.6 = 1$$

$$c(0,2) = \min_{0 \leq k \leq 2} \{ c(0,0) + c(1,2), c(0,1) + c(2,2) \} + w(0,2)$$

$$k=1,2$$

$$= \min \left\{ \begin{matrix} 0 + 0.6 \\ 0.6 + 0 \end{matrix} \right\} + 1$$

$$= 0.6 + 1 = 1.6 \Rightarrow \text{Ans} = 1, 2$$

$$w(1,3) = p(3) + q(3) + w(1,2)$$

$$= 0.4 + 0.1 + 0.6 = 1.1$$

$$c(1,3) = \min_{1 \leq k \leq 3} \{ c(1,1) + c(2,k), c(1,2) + c(3,k) \} + w(1,3)$$

$$k=2, 3$$

$$= \min \left\{ \begin{matrix} 0 + 0.7 \\ 0.7 + 0 \end{matrix} \right\} + 1.1$$

$$= 0.6 + 1.1 = 1.7 \Rightarrow \text{Ans} = 3$$

$$w(2,4) = p(4) + q(4) + w(2,3)$$

$$= 0.3 + 0.2 + 0.7 = 1.2$$

$$c(2,4) = \min_{2 \leq k \leq 4} \{ c(2,2) + c(3,k), c(2,3) + c(4,k) \} + w(2,4)$$

$$k=3, 4$$

$$= \min \left\{ \begin{matrix} 0 + 0.6 \\ 0.6 + 0 \end{matrix} \right\} + 1.2$$

$$= 0.6 + 1.2 = 1.8 \Rightarrow \text{Ans} = 3$$

$$\frac{4}{\underline{\underline{mow}}} \quad \boxed{j-i=3}$$

$$w(0,3) = p(3) + q(3) + w(0,2)$$

$$= 0.4 + 0.1 + 1 = 1.5$$

$$\cancel{c(0,3)} = \min_{0 \leq k \leq 3} \{ c(0,0) + c(k,k) \}$$

$$c(0,3) = \min_{\substack{0 \leq k \leq 3 \\ k=1,2,3}} \left\{ c(0,0) + c(1,3), c(0,1) + c(2,3), \right. \\ \left. c(0,2) + c(3,3) \right\} + w(0,3)$$

$$= \min_{\substack{1.7 \\ 1.3 \\ 1.6}} \left\{ 0 + 1.7, 0.6 + 0.7, 1.6 + 0 \right\} + 1.5 \\ = 1.3 + 1.5 = 2.8 \Rightarrow \gamma = 2$$

$$w(1,4) = p(4) + q(4) + w(1,3)$$

$$= 0.3 + 0.2 + 1.1 = 1.6$$

$$c(1,4) = \min_{\substack{1 \leq k \leq 4 \\ k=2,3,4}} \left\{ c(1,1) + c(2,4), c(1,2) + c(3,4), \right. \\ \left. c(1,3) + c(4,4) \right\} + w(1,4)$$

$$= \min_{\substack{1.8 \\ 1.2 \\ 1.7}} \left\{ 0 + 1.8, 0.6 + 0.6, 1.7 + 0 \right\} + 1.6$$

$$= 1.2 + 1.6 = 2.8 \Rightarrow \gamma = 3$$

$$\underline{\underline{5^4 \text{ row}}} \quad j-i=4$$

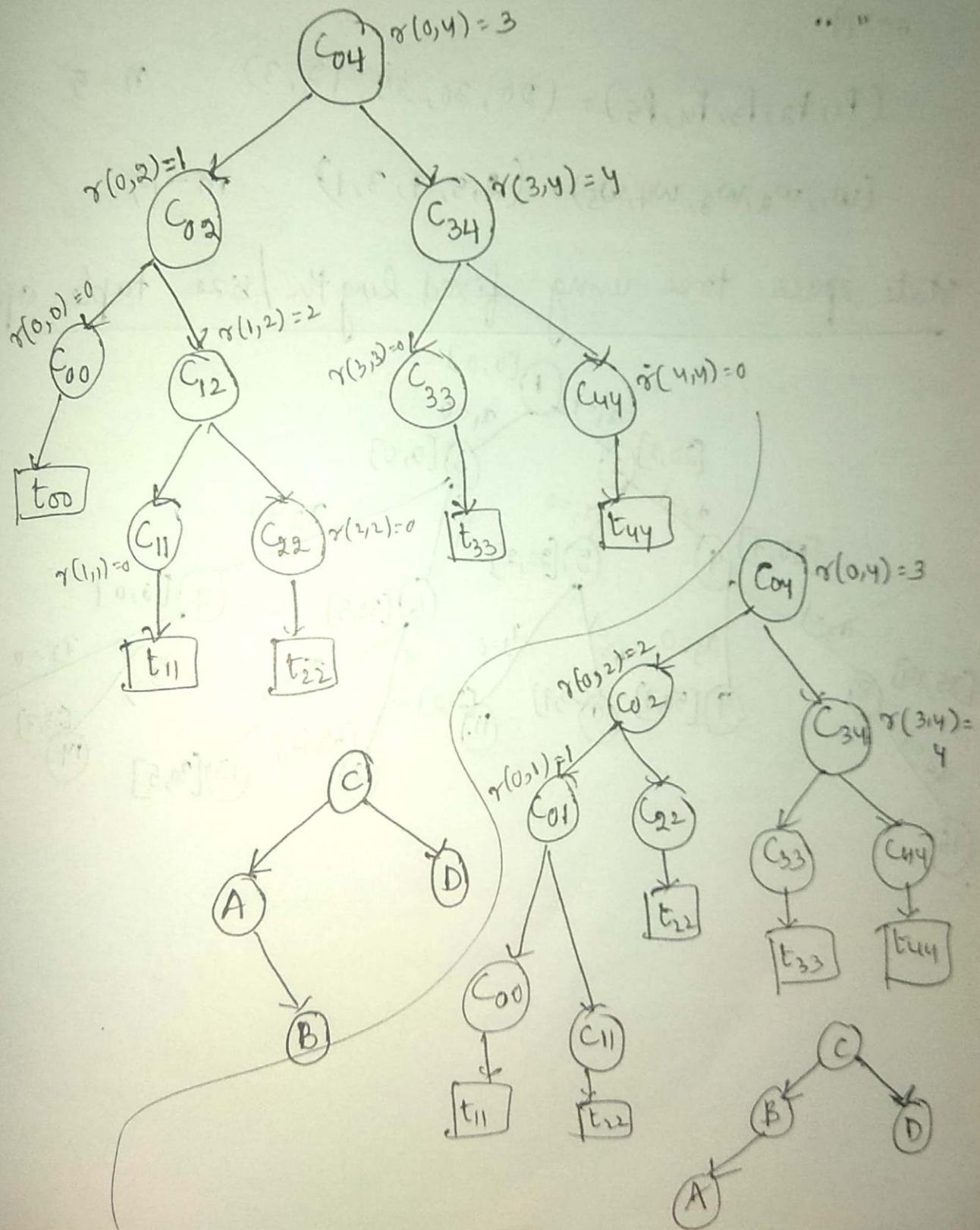
$$w(0,4) = p(4) + q(4) + w(0,3)$$

$$= 0.3 + 0.2 + 1.5 = 2$$

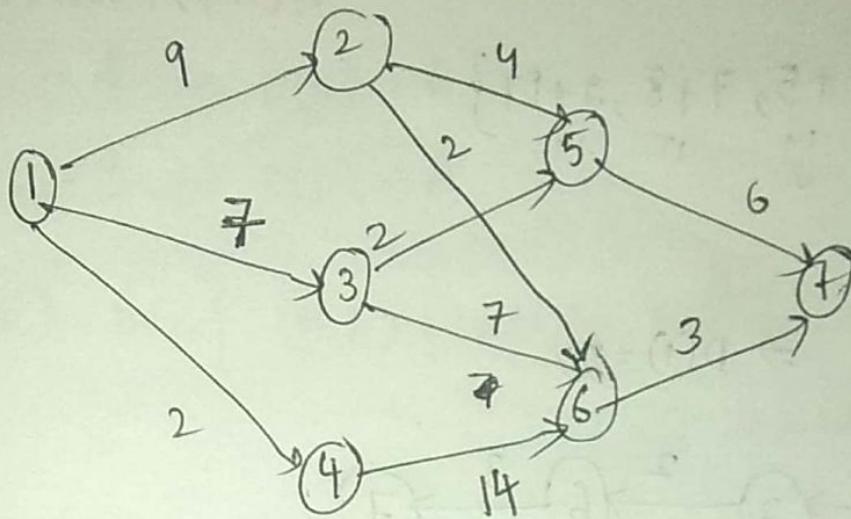
$$c(0,4) = \min_{\substack{0 \leq k \leq 4 \\ k=1,2,3,4}} \left\{ c(0,0) + c(1,4), c(0,1) + c(2,4), \right. \\ \left. c(0,2) + c(3,4), c(0,3) + c(4,4) \right\} \\ + w(0,4)$$

$$= \min \left\{ \begin{array}{l} 0 + 2.8 \\ 2.8 \end{array}, \begin{array}{l} 0.6 + 1.8 \\ 2.4 \end{array}, \begin{array}{l} 1.6 + 0.6 \\ 2.2 \end{array}, \begin{array}{l} 2.8 + 0 \\ 2.8 \end{array} \end{array} \right\} + 2$$

$$= 2.2 + 2 = 4.2 \Rightarrow \Delta = 3$$



② @ Using forward approach



$$D(7)=0 \Leftrightarrow \text{cost}(7)=0$$

$$D(5)=7 \Leftrightarrow \text{cost}(5)=6$$

$$D(6)=7 \Leftrightarrow \text{cost}(6)=3$$

for forward approach

$$\text{cost}(i,j) = \min \{ c(j,l) + \text{cost}(i,l) \}$$

$$l \in V_{i+1}$$

$$(j,l) \in E$$

$$\begin{aligned} \text{cost}(4) &= \min \{ c(4,6) + \text{cost}(6) \} \\ &= \min \{ 14 + 3 \} = 17 \end{aligned}$$

$$\text{cost}(4) = 17 \Rightarrow D(4) = 6$$

$$\begin{aligned} \text{cost}(3) &= \min \{ c(3,6) + \text{cost}(6), c(3,5) + \text{cost}(5) \} \\ &= \min \{ 7 + 3, 2 + 6 \} \\ &\quad \begin{matrix} 10 & 8 \end{matrix} \end{aligned}$$

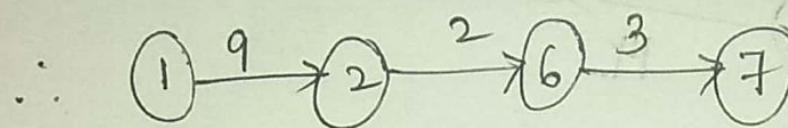
$$\text{cost}(3) = 8 \Rightarrow D(3) = 5$$

$$\begin{aligned} \text{cost}(2) &= \min \{ c(2,6) + \text{cost}(6), c(2,5) + \text{cost}(5) \} \\ &= \min \{ 2 + 3, 4 + 6 \} \\ &\quad \begin{matrix} 5 & 10 \end{matrix} \end{aligned}$$

$$\text{cost}(2) = 5 \Rightarrow D(2) = 6$$

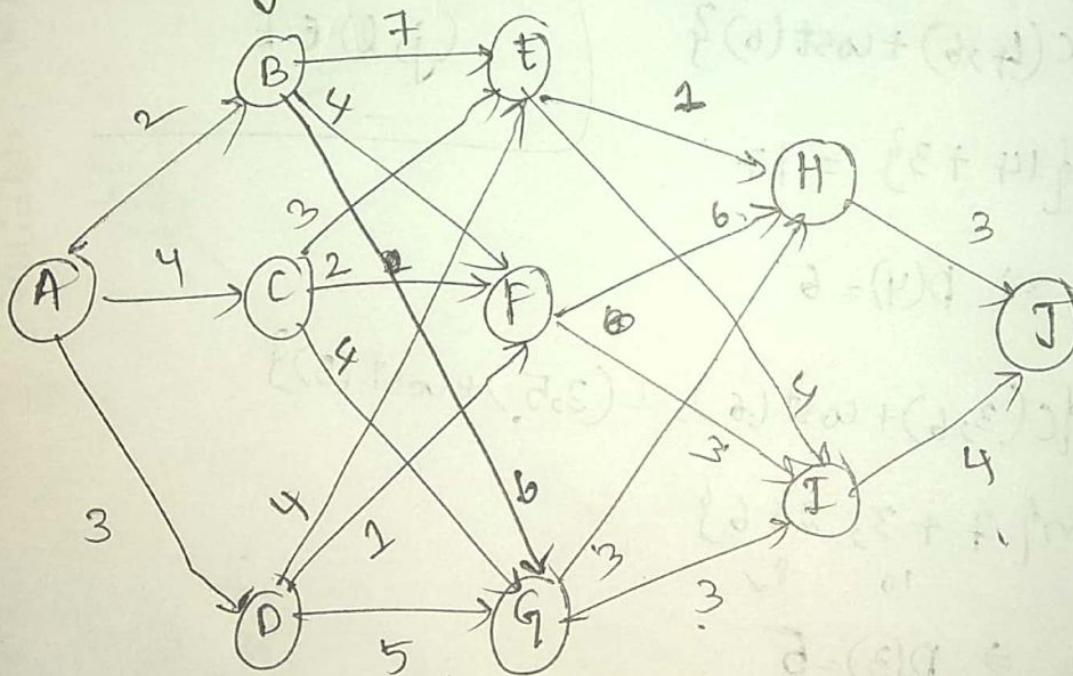
$$\begin{aligned}
 \text{cost}(1) &= \min \{ c(1,2) + \text{cost}(2), c(1,3) + \text{cost}(3), \\
 &\quad c(1,4) + \text{cost}(4) \} \\
 &= \min \left\{ \underset{14}{9+5}, \underset{15}{7+8}, \underset{19}{2+17} \right\} \\
 &= 14
 \end{aligned}$$

$$\text{cost}(1) = 14 \Rightarrow D(1) = 2$$



The cost is $9+2+3=14$

b) using backward approach in



$$\text{cost}(A) = 0 \Rightarrow D(A) = 0$$

$$\text{cost}(B) = 2 \Rightarrow D(B) = A$$

$$\text{cost}(C) = 4 \Rightarrow D(C) = A$$

$$\text{cost}(D) = 3 \Rightarrow D(D) = A$$

$$\text{cost}(E) = \min \{ C(B, E) + \text{cost}(B), \cancel{C(C, E) + \text{cost}(C)}, \\ \cancel{C(D, E) + \text{cost}(D)} \}$$

$$= \min \left\{ \begin{matrix} 7+2 \\ 9 \end{matrix}, \begin{matrix} 3+4 \\ 7 \end{matrix}, \begin{matrix} 4+3 \\ 7 \end{matrix} \right\}$$

$$= 7 \Rightarrow \text{cost}(E) = 7 \Rightarrow D(E) = C$$

$$\text{cost}(F) = \min \{ C(B, F) + \text{cost}(B), C(C, F) + \text{cost}(C),$$

$$C(D, F) + \text{cost}(D) \}$$

$$= \min \left\{ \begin{matrix} 4+2 \\ 6 \end{matrix}, \begin{matrix} 2+4 \\ 6 \end{matrix}, \begin{matrix} 1+3 \\ 4 \end{matrix} \right\}$$

$$= 4 \Rightarrow \text{cost}(F) = 4 \Rightarrow D(F) = D$$

$$\text{cost}(G) = \min \{ C(B, G) + \text{cost}(B), C(C, G) + \text{cost}(C),$$

$$C(D, G) + \text{cost}(D) \}$$

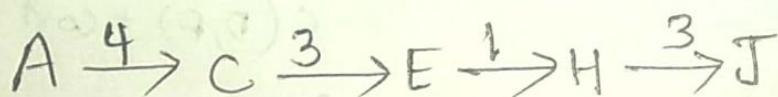
$$= \min \left\{ \begin{matrix} 6+2 \\ 8 \end{matrix}, \begin{matrix} 4+4 \\ 8 \end{matrix}, \begin{matrix} 5+3 \\ 8 \end{matrix} \right\}$$

$$= 8 \Rightarrow \text{cost}(G) = 8 \Rightarrow D(G) = B$$

$$\text{cost}(H) = \min \{ C(E, H) + \text{cost}(E), C(F, H) + \text{cost}(F),$$

$$C(G, H) + \text{cost}(G) \}$$

$$= \min \left\{ \begin{matrix} 1+7 \\ 8 \end{matrix}, \begin{matrix} 6+4 \\ 10 \end{matrix}, \begin{matrix} 3+8 \\ 11 \end{matrix} \right\} \Rightarrow \text{cost}(H) = 8 \Rightarrow D(H) = E$$



The cost is 11.