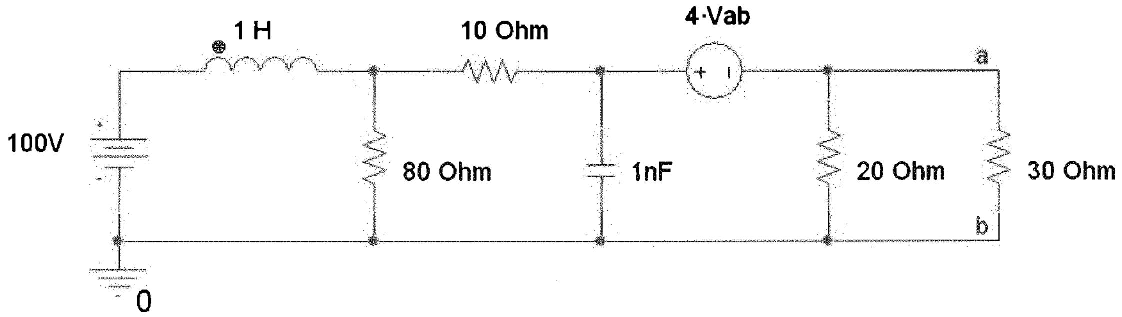
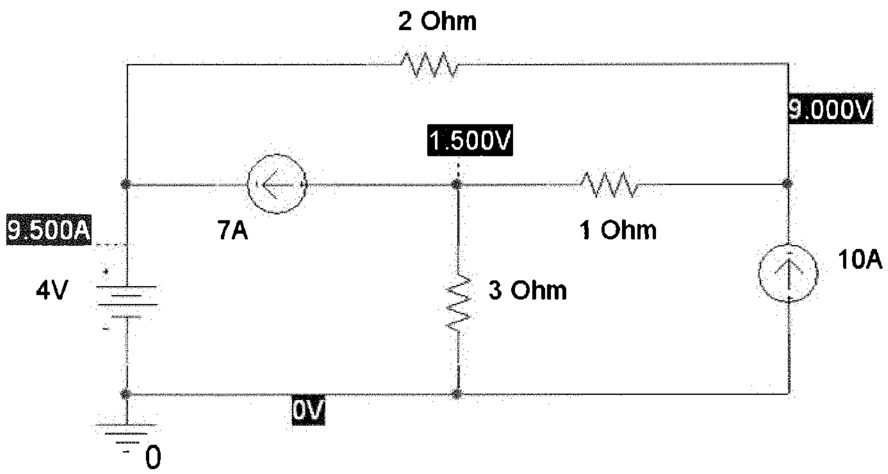
Examen de Análisis de Circuitos. Grado en Ingeniería de Tecnologías de Telecomunicación.
12 de febrero de 2015.

NOMBRE: **DNI:**

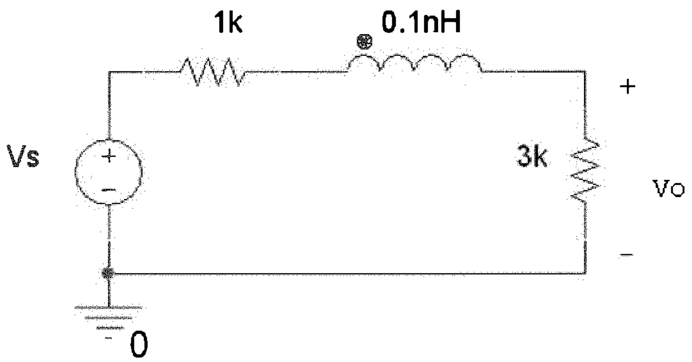
1. Determine el equivalente Thévenin del siguiente circuito entre los terminales A y B (en DC) . **(2 puntos)**



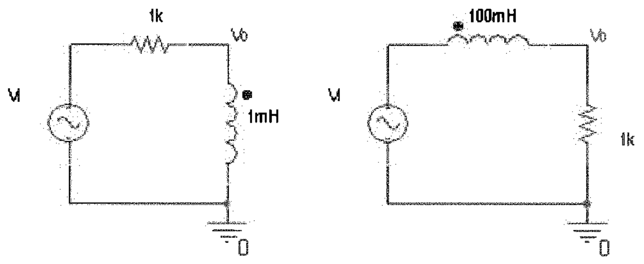
2. Dado el siguiente circuito, en el que se muestran las tensiones en todos los nudos y las corrientes que circulan, calcule **(1.5 puntos)**:
   - La potencia disipada en cada elemento.
   - Comprobar que la potencia total disipada es igual que la potencia total generada por las fuentes.



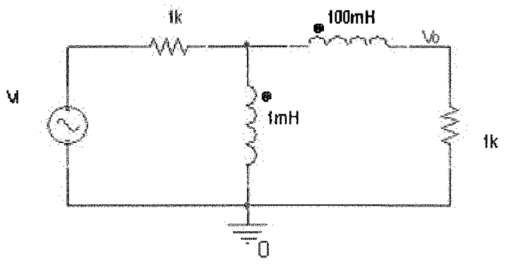
3. En la fuente de tensión de la figura se produce un escalón en t = 0, que hace que la tensión de entrada al circuito pase de 2 V a 4 V. Calcule **(2 puntos)**:
   - Los valores estacionarios de v_0 antes y después del salto de tensión.
   - Determinar i(t) en el transitorio que tiene lugar entre los valores estacionarios y el tiempo de transición entre ellos (tiempo de subida).



4. En la figura (a) se representan dos redes RL que se comportan como filtros paso-alta y paso-baja, respectivamente.
   - Obtenga la función de transferencia y represente el diagrama de bode de cada uno de los circuitos de la figura (a) **(1 punto)**.
   - Si se conectan en cascada, tal como muestra la figura (b), obtenga la función de transferencia y el diagrama de Bode **(1 punto)**.

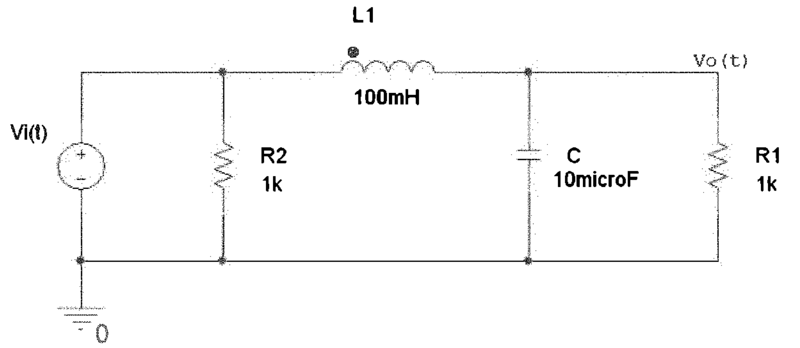


(A)



(B)

5. Para el circuito de la figura, determine:
   - La función de transferencia T(s) **(1 punto)**
   - La tensión de salida $v_o(t)$ cuando $v_i(t) = \delta(t)$ **(0.75 puntos)**
   - Si $v_i(t) = [5\cos(10^3 t) + 10\cos(10^5 t)]$ V, determine $v_o(t)$. **(0.75 puntos)**

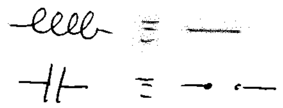


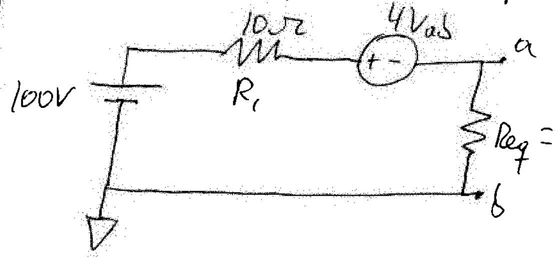
AN ABBREVIATED LIST OF LAPLACE TRANSFORM PAIRS

$f(t)(t > 0^-)$	TYPE	$F(s)$
$\delta(t)$	(impulse)	1
$u(t)$	(step)	$\dfrac{1}{s}$
t	(ramp)	$\dfrac{1}{s^2}$
e^{-at}	(exponential)	$\dfrac{1}{s+a}$
$\sin \omega t$	(sine)	$\dfrac{\omega}{s^2+\omega^2}$
$\cos \omega t$	(cosine)	$\dfrac{s}{s^2+\omega^2}$
te^{at}	(damped ramp)	$\dfrac{1}{(s+a)^2}$
$e^{-at}\sin \omega t$	(damped sine)	$\dfrac{\omega}{(s+a)^2+\omega^2}$
$e^{-at}\cos \omega t$	(damped cosine)	$\dfrac{s+a}{(s+a)^2+\omega^2}$

1-. $V_T = V_{ab}\Big)_{I_S=0}$

↳ Circuito equivalente para calcular V_{ab}:

DC: 

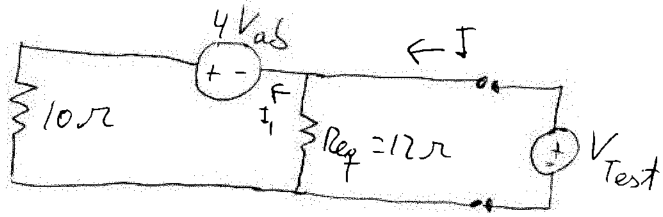


$R_{eq} = 20 \| 30\Omega = 12\Omega$

$$100V = I\cdot(R_1 + R_{eq}) + 4V_{ab} = I\cdot(R_1 + R_{eq}) + 4\cdot I\cdot R_{eq}$$

$$I = \frac{100\,V}{R_1 + 5R_{eq}} = \frac{100V}{10\Omega + 60\Omega} = 1'429\,A \Rightarrow \underline{V_T = V_{ab}}\Big|_{I_S=0} = I\cdot R_{eq} = \underline{\underline{17'14V}}$$

• ~~Req~~ $R_T = R\Big)_{V_T=0}$



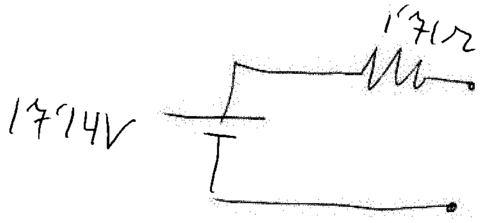
$$R_T = \frac{V_{Test}}{I}$$

$$V_{Test} = -4V_{Test} + I_1\cdot 10\Omega \Rightarrow$$

$$\Rightarrow I_1 = \frac{5V_{Test}}{10\Omega} = \frac{V_{Test}}{2\Omega}$$

$$I = \frac{V_{Test}}{R_{eq}} + \frac{V_{Test}}{2\Omega} = \frac{V_{Test}}{R_{eq}\|2\Omega} \Rightarrow \underline{R_T} = R_{eq}\|2\Omega = \underline{\underline{1'71\Omega}}$$

• Circuito eq. Th.:



2.-

2- a) 3 NUDOS PRINCIPALES (A PARTE DE $\perp$)

1 TIENE TENSIÓN FIJADA $\Rightarrow$ SÓLO DOS INCÓGNITAS

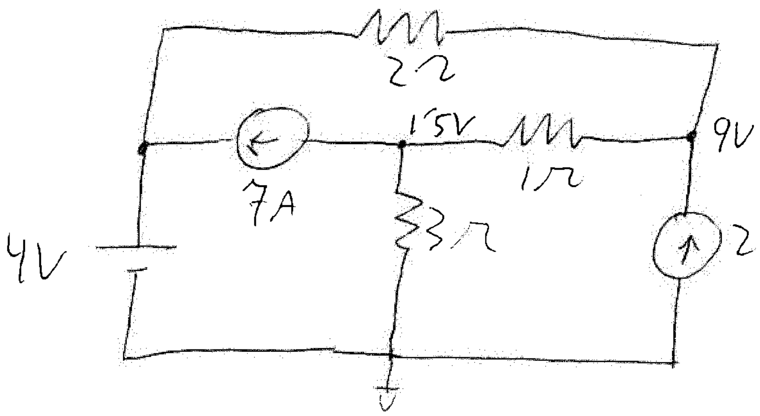
$$-7A + \frac{0 - V_B}{3\Omega} + \frac{V_C - V_B}{1\Omega} = 0$$

$$\frac{V_B - V_C}{1\Omega} + 2(V_C - 4V) + \frac{4V - V_C}{2\Omega} = 0$$

Solució:

$$V_B = 1'5V \; ; \; V_C = 9V$$

b)



$$2V_x = 2\frac{A}{V} \cdot (9V - 4V) = 10\,A$$

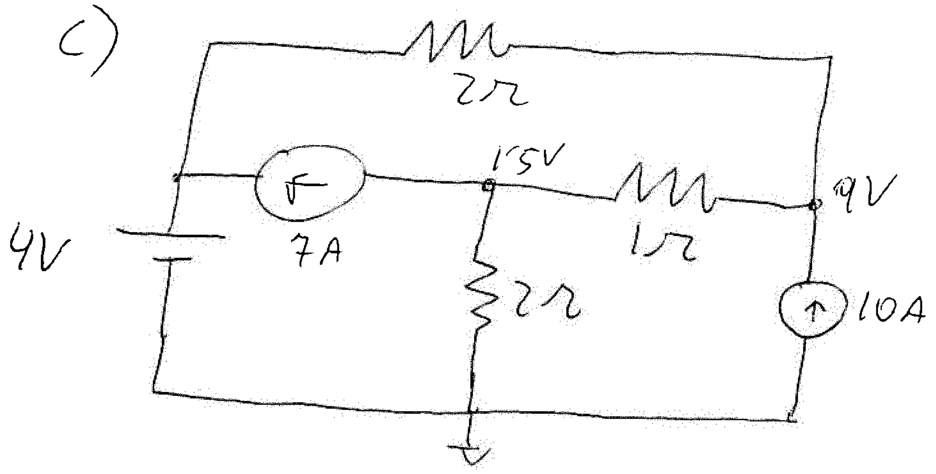
$$P_{2\Omega} = \frac{(5V)^2}{2\Omega} = \frac{25V^2}{2\Omega} = 12'5\,W$$

$$P_{1\Omega} = \frac{(9V - 1'5V)^2}{1\Omega} = 56'25\,W$$

$$P_{3\Omega} = \frac{(1'5V)^2}{3\Omega} = 0'75\,W.$$

TOTAL DISIPADO RESISTENCIAS: 69'5 W

c)



POTENCIA (DISIPADA) EN FUENTES:

- $P_{4V} = V \cdot I = 4V \cdot 9'5A = 38\ W$ ¡NO GENERA, DISIPA!

$$I = 7A + \frac{5V}{2\Omega} = 9'5\ A$$

- $P_{7A} = 7A \cdot V = 7A \cdot (1'5V - 4V) = 7A \cdot (-)2'5V = -17'5\ W$

¡GENERA!

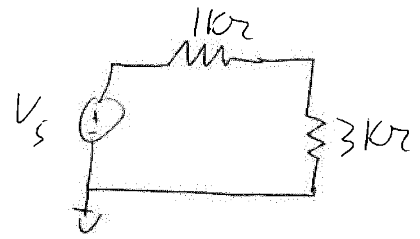
- $P_{10A} = 10A \cdot (0 - 9V) = -90\ W$

¡GENERA!

TOTAL GENERADO POR FUENTES: $+107'5\ W$

TOTAL DISIPADO EN CIRCUITO: $69'5W + 38\ W = 107'5$

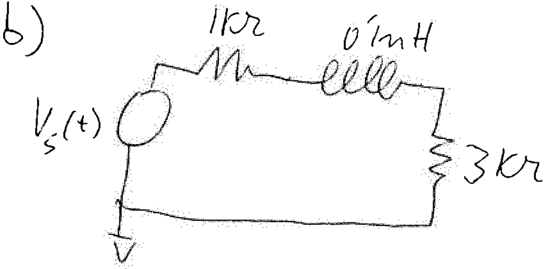
3.- a) DC



$$V_o = V_s \cdot \frac{3k\Omega}{3k\Omega + 1k\Omega}$$

- $V_o(0^-) = V_s(0^-) \cdot 0'75 = 1'5V$
- $V_o(t\to\infty) = V_s(t\to\infty) \cdot 0'75V = 3V$

b)



$$V_s(t) = i \cdot (1k\Omega + 3k\Omega) + L\frac{di}{dt}$$

Ponemos en la forma:

$$\tau \frac{dx}{dt} + x = K$$

$$\frac{L}{4k\Omega}\frac{di}{dt} + i = \frac{V_s(t)}{4k\Omega} = 1mA$$

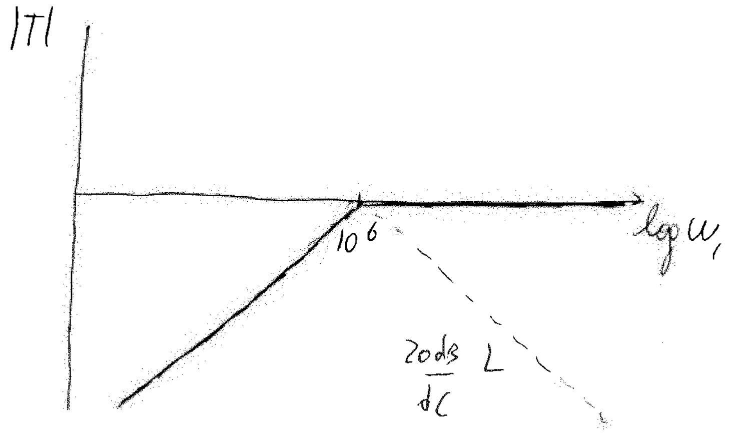
$\Rightarrow$ $\tau = \frac{L}{4k\Omega} = 0'025\,ps$

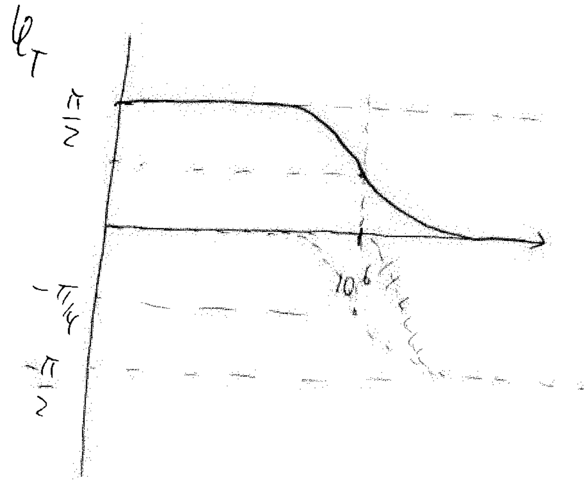
$K = 1mA$

(4.-) a.1.

$$V_o(s) = \frac{sL_1 \cdot V_i(s)}{sL_1+R} = \frac{sL_1/R\ V_i}{s\frac{L_1}{R}+1} = \frac{s/w_1}{\frac{s}{w_1}+1} V_i \quad \text{con } w_1 = R/L_1 = 10^6 \text{ rad/s}$$

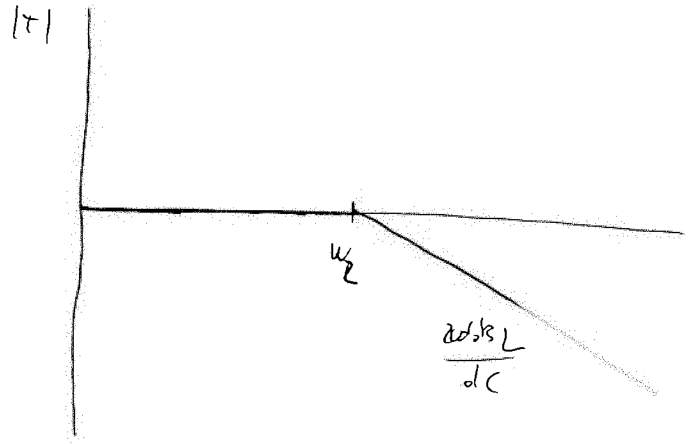
$$AC \rightarrow \frac{V_o(jw)}{V_i(jw)} = \frac{j\frac{w}{w_1}}{\frac{jw}{w_1}+1}$$

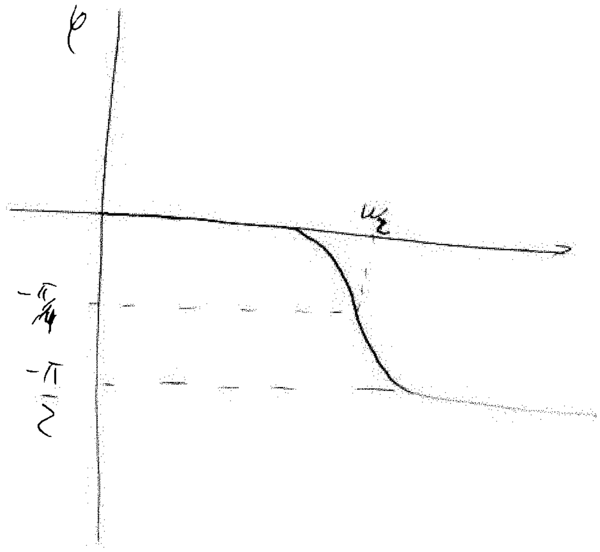




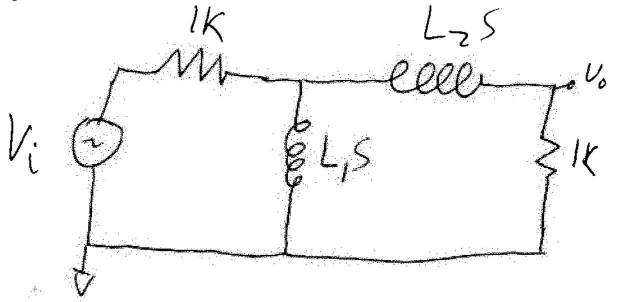
a.2. $$V_o(s) = \frac{R}{R+sL_2} \cdot V_i(s) = \frac{1}{\frac{s}{w_2}+1} V_i(s) \quad \text{con } w_2 = R/L_2 = 10^4 \frac{\text{rad}}{s}$$

$$AC \rightarrow T(w) = \frac{1}{j\frac{w}{w_2}+1}$$

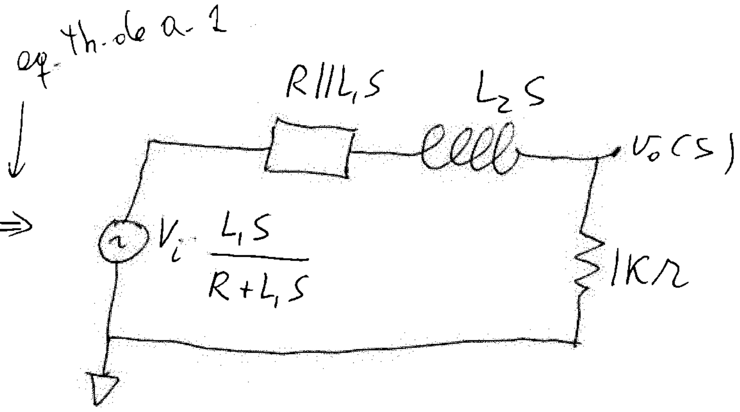




b.



$\Longrightarrow$



$$R \| L_1 s = \frac{R L_1 s}{R + L_1 s} = \frac{L_1 \cdot s}{\frac{s}{\omega_1} + 1}$$

Divisor de tensión:

$$V_o(s) = \frac{R}{R + L_2 s + \frac{R L_1 s}{R + L_1 s}} \cdot \frac{L_1 s}{R + L_1 s} \cdot V_i(s) \Rightarrow$$

$$\Rightarrow T(s) = \frac{R \cdot L_1 \cdot s}{R^2 + 2L_1 s \cdot R + L_2 s R + L_1 L_2 s^2} = \frac{R \cdot L_1 \cdot s}{L_1 L_2 s^2 + R(2L_1 + L_2)s + R^2} =$$

$$= \frac{R/L_2 \cdot s}{s^2 + \frac{R(2L_1 + L_2)}{L_1 L_2} s + \frac{R^2}{L_1 L_2}} = \frac{\cancel{A_0} \cdot s \cancel{A_{w_2}} \cdot \omega_2}{s^2 + 2\delta \omega_0 s + \omega_0^2}$$

Con $\omega_0^2 = \frac{R^2}{L_1 L_2} = \omega_1 \cdot \omega_2 = 10^{10} \left(\frac{rad}{s}\right)^2 \rightarrow \omega_0 = 10^5 \frac{rad}{s}$

$$2\delta\omega_0 = \frac{R(2L_1 + L_2)}{L_1 L_2} = 1k/0'999\,mH \simeq \omega_1 = 10^6\,rad/s$$

(exacto: $1'001 \cdot 10^6$ rad/s.)

$$\delta \simeq \frac{\omega_1}{2\omega_0} = \frac{10^6}{2\cdot 10^5} = \frac{10}{2} = 5$$

Dos raíces reales diferentes:

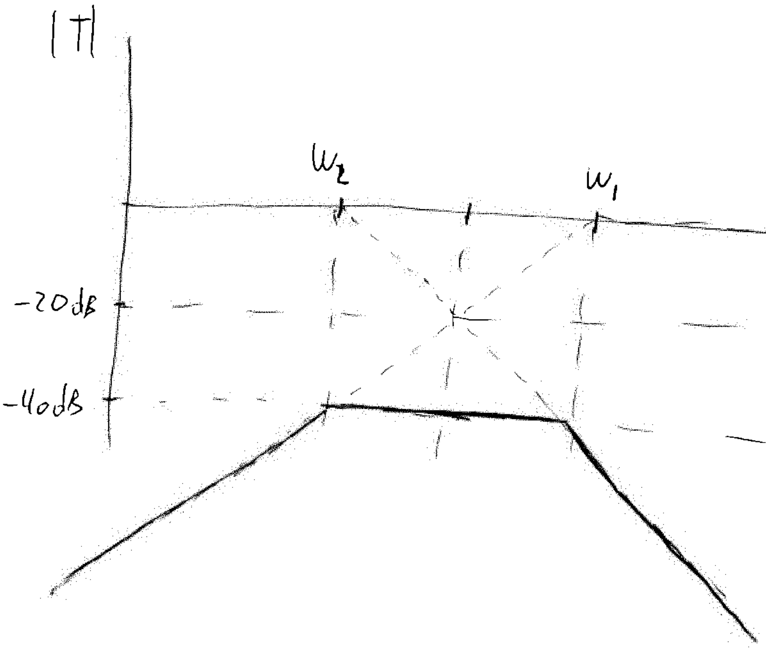
$$s_{1,2} = \frac{-2\delta\omega_0 \pm \sqrt{4\delta^2\omega_0^2 - 4\omega_0^2}}{2} = -\omega_0\left(\delta \pm \sqrt{\delta^2-1}\right) =$$

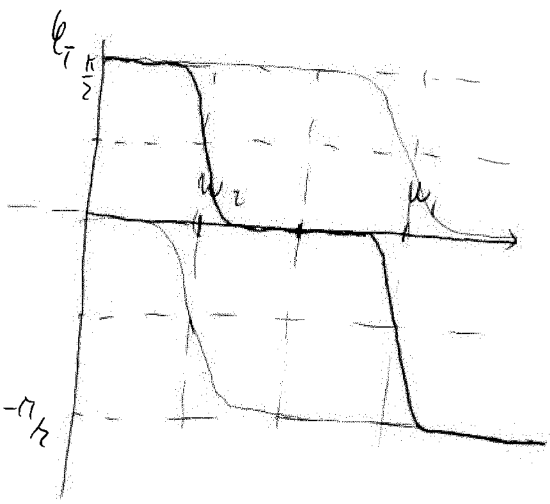
$$s_{1,2} = -\omega_0\cdot(5 \pm 4'89898) \rightarrow$$

$$\rightarrow s_1 = -9'8989\cdot 10^5 \text{ rad/s} \simeq -\omega_1$$

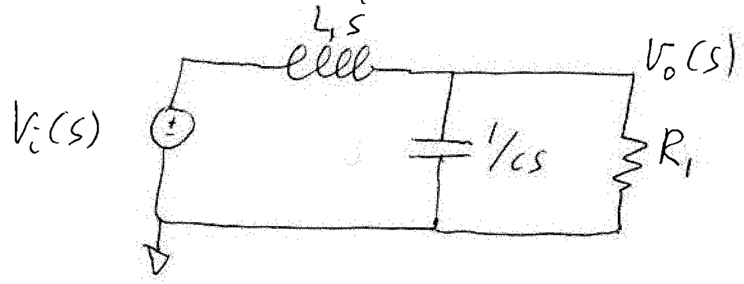
$$s_2 = -0'1010\cdot 10^5 \text{ rad/s} \simeq -\omega_2$$

$$T(s) \simeq \frac{\omega_2 \cdot s}{(s+\omega_2)(s+\omega_1)} = \frac{s/\omega_1}{\left(\frac{s}{\omega_2}+1\right)\left(\frac{s}{\omega_1}+1\right)} = T_{a1}(s)\cdot T_{a2}(s)$$





(5.-) *Circuito equivalente:*



$$R_1 \parallel \frac{1}{Cs} = \frac{R_1}{1+R_1Cs}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{\dfrac{R_1}{1+R_1Cs}}{\dfrac{R_1}{1+R_1Cs}+L_1s} = \frac{R_1}{R_1+L_1s+R_1L_1Cs^2} = \frac{R_1/L_1 \cdot \dfrac{1}{R_1C}}{s^2+\dfrac{1}{R_1C}s+\dfrac{1}{L_1C}}$$

$$= \frac{1/L_1C}{s^2+\dfrac{1}{R_1C}s+\dfrac{1}{L_1C}} \equiv \frac{\omega_o^2}{s^2+2\delta\omega_o s+\omega_o^2}$$

Con $\omega_o^2 = \dfrac{1}{L_1C} = 10^6\left(\dfrac{\text{rad}}{s}\right)^2 \rightarrow \omega_o = 10^3\ \text{rad}/s$

$$2\delta\omega_o = \frac{1}{R_1C} = 100 \Rightarrow \delta = \frac{100}{10^3}\cdot\frac{1}{2} = 0'05$$

RAÍCES COMPLEJO CONJUGADAS:

$$s_{1,2} = \frac{-2\delta\omega_o \pm\sqrt{4\delta^2\omega_o^2-4\omega_o^2}}{2} = -\delta\omega_o \pm \omega_o\sqrt{\delta^2-1} =$$

$$= -\omega_o\delta \pm j\,\omega_o\sqrt{1-\delta^2} \equiv -\sigma \pm \underset{\omega}{\omega} j\ \frac{\text{rad}}{s}$$

Con $\sigma = \omega_o\delta = 50$

$\omega = \omega_o\sqrt{1-\delta^2} = \omega_o\cdot 0'998 = 998$

$$s^2+2\delta\omega_o s+\omega_o^2 = (s+\sigma-\underset{\omega}{b}j)(s+\sigma+\underset{\omega}{b}j) = (s+\sigma)^2+\omega^2$$

Por tanto:

$$\underline{\underline{T(s)}} = \frac{w_0^2}{s^2+2\delta w_0 s+w_0^2} = \underline{\underline{\frac{w_0^2}{(s+\sigma)^2+w^2}}} \quad \text{con} \quad \begin{aligned} \sigma &= 50 \ rad/s \\ w &= 998 \ \frac{rad}{s} \end{aligned}$$

• Si $v_i(t) \Rightarrow v_o(t) = \mathcal{L}^{-1}\left[T(s)\right] \overset{(*)}{=} \mathcal{L}^{-1}\left[\frac{w_0}{\sqrt{1-\delta^2}} \cdot \frac{w_0 \sqrt{1-\delta^2}}{(s+\sigma)^2+w_0^2(1-\delta^2)}\right]$

Sabemos que $\mathcal{L}^{-1}\left[\frac{w}{(s+\sigma)^2+w^2}\right] = e^{-\sigma t} \cdot \operatorname{sen}(wt)$

Por tanto:

$$v_o(t) = \mathcal{L}^{-1}\left[\frac{w_0}{\sqrt{1-\delta^2}} \cdot \frac{w}{(s+\sigma)^2+w^2}\right] = \frac{w_0}{\sqrt{1-\delta^2}} \cdot e^{-\sigma t} \cdot \operatorname{sen}(wt) \cdot \frac{V}{rad/s}$$

(*) Más fácil:

$$\mathcal{L}^{-1}\left[\frac{w^2}{(\sqrt{1-\delta^2})^2} \cdot \frac{1}{(s+\sigma)^2+w^2}\right] = \mathcal{L}^{-1}\left[\frac{w}{(1-\delta^2)} \frac{w}{(s+\sigma)^2+w^2}\right] =$$

$$\rightarrow \boxed{v_o(t) = \frac{w}{1-\delta^2} \cdot e^{-\sigma t} \cdot \operatorname{sen}(wt) \ \frac{V}{rad/s}}$$

- $v_i(t) = \underbrace{5V\cos(w_1 t)}_{v_{i1}} + \underbrace{10V\cos(w_2 t)}_{v_{i2}}$ con $w_1 = 10^3$ rad/s

$w_2 = 10^5$ rad/s

Aplicamos superposición y trabajamos, para cada frec., con fasores: $\frac{\tilde{V}_o}{\tilde{V}_i} = T(w) = T(s=jw)$

$$\tilde{V}_1 \to \tilde{V}_o = T(w)\cdot\tilde{V}_i = \frac{w_o^2}{(jw)^2 + 2\delta w_o(jw) + w_o^2}\tilde{V}_i =$$

$$\tilde{V}_o = \tilde{V}_i \cdot \frac{1}{1 - \frac{w^2}{w_o^2} + j\frac{2\delta w}{w_o}} = \frac{1}{\left[\left(1-\frac{w^2}{w_o^2}\right)^2 + 4\delta^2\frac{w^2}{w_o^2}\right]^{1/2}} \angle -\operatorname{arctg}\left(\frac{2\delta w/w_o}{1-w^2/w_o^2}\right)$$

- $\underline{v_{o1}(t)}$ $(w_1 = 10^3$ rad/s$)$

$$\tilde{V}_{o1} = \tilde{V}_{i1}\cdot\frac{1}{2\delta}e^{-j\pi/2} = \frac{5V}{0'1}e^{-j\pi/2}$$

$$v_{o1}(t) = \operatorname{Re}\left[\tilde{V}_{o1}\cdot e^{jw_1 t}\right] = 50V\cos(w_1 t - \pi/2)$$

- $v_{o2}(t)$ $(w_2 = 10^5$ rad/s$) \gg w_o$

$$\tilde{V}_{o2} = \tilde{V}_{o2}\cdot\frac{1}{99'50}\cdot e^{-j3'13} \simeq \tilde{V}_{o2}\cdot\frac{1}{99'50}e^{-j\pi}$$

$$v_{o2}(t) = 100mV\cos(w_2 t - \pi)$$

—o—

$$\boxed{v_o(t) = 50V\cos(w_1 t - \pi/2) + 100mV\cos(w_2 t - \pi)}$$