Diagrama de Bode

FFT

$$x(t) \longrightarrow \boxed{\text{Circuito Lineal}} \longrightarrow y(t)$$

Aplicando la Transformada de Laplace a la ecuación diferencial del circuito:

$$D(s)\,Y(s) - CI(s) = N(s)\,X(s)$$

$$Y(s) = \frac{N(s)}{D(s)}X(s) + \frac{CI(s)}{D(s)}$$

Respuesta total = a la debida a la entrada + la debida a las condiciones iniciales

Función de transferencia del circuito:

$$T(s) = \frac{N(s)}{D(s)} = \left.\frac{Y(s)}{X(s)}\right|_{CI=0}$$

Para ver la respuesta en frecuencia de un circuito, se usa el Diagrama de Bode, que es la representación gráfica del módulo (o magnitud) y la fase (o argumento) de la función de transferencia en s=iω , es decir, particularizando para señales armónicas.

Representa el módulo: $20\log_{10}|T(i\omega)|$ (dB)

Representa la fase: $\mathrm{arc}(T(i\omega))$ (radianes o grados)

Frente a la frecuencia: $\log_{10}(\omega)$ ó $\log_{10}(f)$ (rad/s ó Hz)

Util:

$20\log(\sqrt{2}) = 3$ dB	$20\log(2) = 6$ dB
$20\log\left(\frac{1}{\sqrt{2}}\right) = -3$ dB	$20\log\left(\frac{1}{2}\right) = -6$ dB
$20\log(10) = 20$ dB	$20\log(100) = 40$ dB
$20\log(0,1) = -20$ dB	$20\log(0,01) = -40$ dB

A recordar:

$$\omega = 2\pi f$$

Circuito lineal => el numerador y el denominador son polinomios

$$T(s) = k\,\frac{(s-z_1)(s-z_2)\ldots(s-z_m)}{(s-p_1)(s-p_2)\ldots(s-p_n)}$$

Circuito real => las raíces son reales o complejas conjugadas

$$T(s) = k\,\frac{\Pi_i(s-z_i)\ \ \Pi_k(s^2+2\delta_k s\omega_k+\omega_k^2)}{\Pi_j(s-p_j)\ \ \Pi_r(s^2+2\delta_r s\omega_r+\omega_r^2)} \qquad 0<\delta<1$$

La función de transferencia se reduce a esta forma (normalizada):

$$T(s) = k\,s^n\,\frac{\Pi_i\left(\frac{s}{\omega_i}+1\right)\ \ \Pi_k\left(\frac{s^2}{\omega_k^2}+2\delta_k\frac{s}{\omega_k}+1\right)}{\Pi_j\left(\frac{s}{\omega_j}+1\right)\ \ \Pi_r\left(\frac{s^2}{\omega_r^2}+2\delta_r\frac{s}{\omega_r}+1\right)}$$

Se representan los bloques básicos, ya que: $T = T_1 T_2 / T_3$

$$20\log(T) = 20\log(T_1)+20\log(T_2)-20\log(T_3)$$

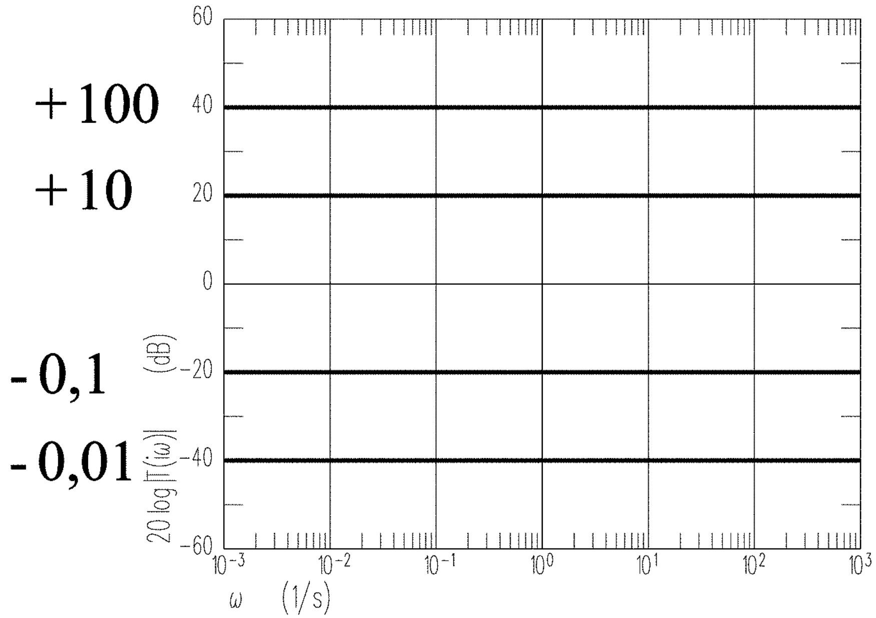
$$\mathrm{fase}(T) = \mathrm{fase}(T_1)+\mathrm{fase}(T_2)-\mathrm{fase}(T_3)$$

Diagrama de Bode de los bloques básicos:

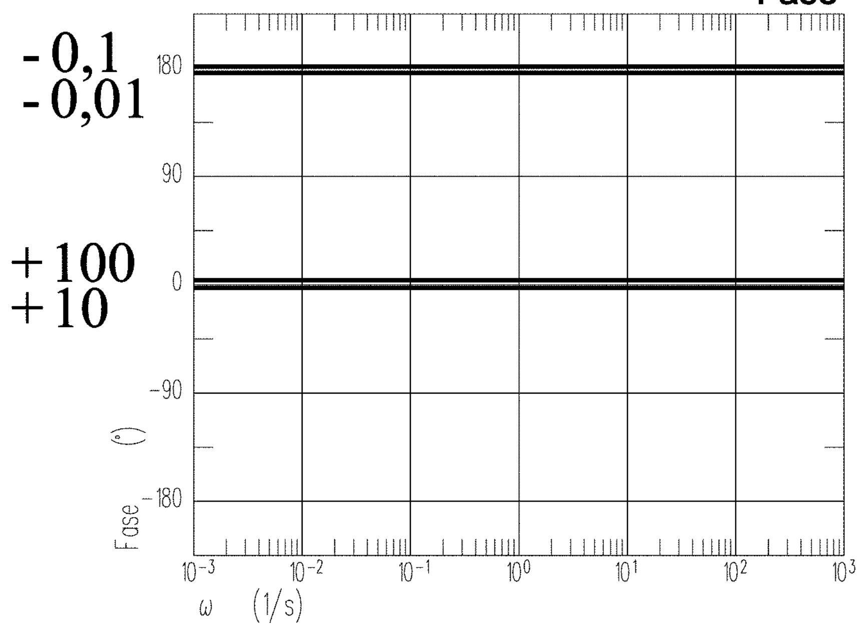
$T(s) = \mathbf{k}$ $\qquad T(i\omega) = \mathbf{k}$

Módulo	Fase
$20\log\lvert k\rvert$	0 si $k > 0$ π si $k < 0$

Módulo



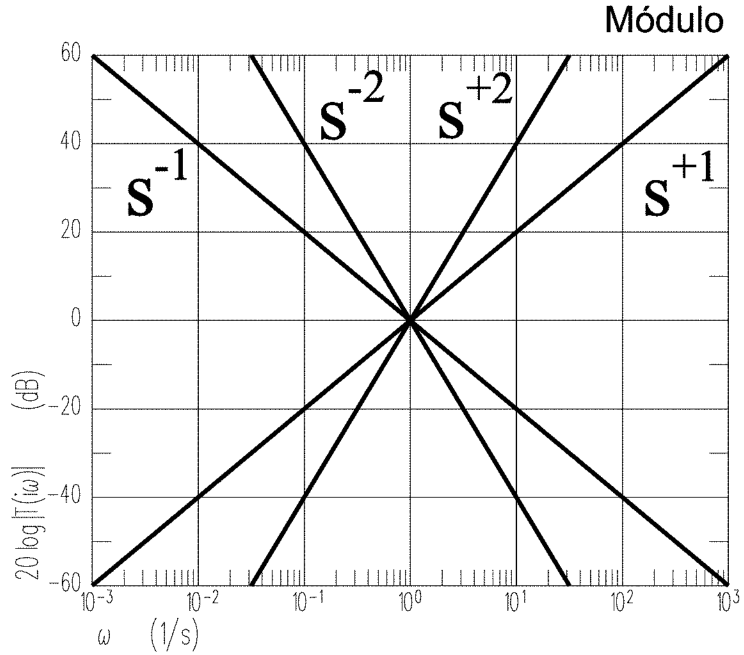
Fase

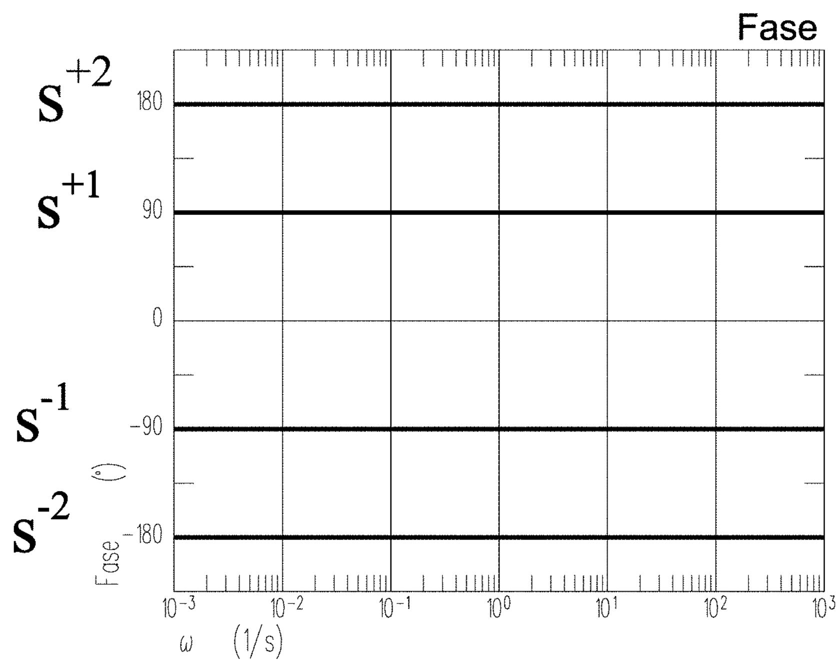


$T(s) = S^{\pm n}$ $\qquad$ $T(i\omega) = (i\omega)^{\pm n}$

Módulo $\qquad$ Fase

$\pm n\, 20 \log(\omega)$ $\qquad$ $\pm n \pi / 2$



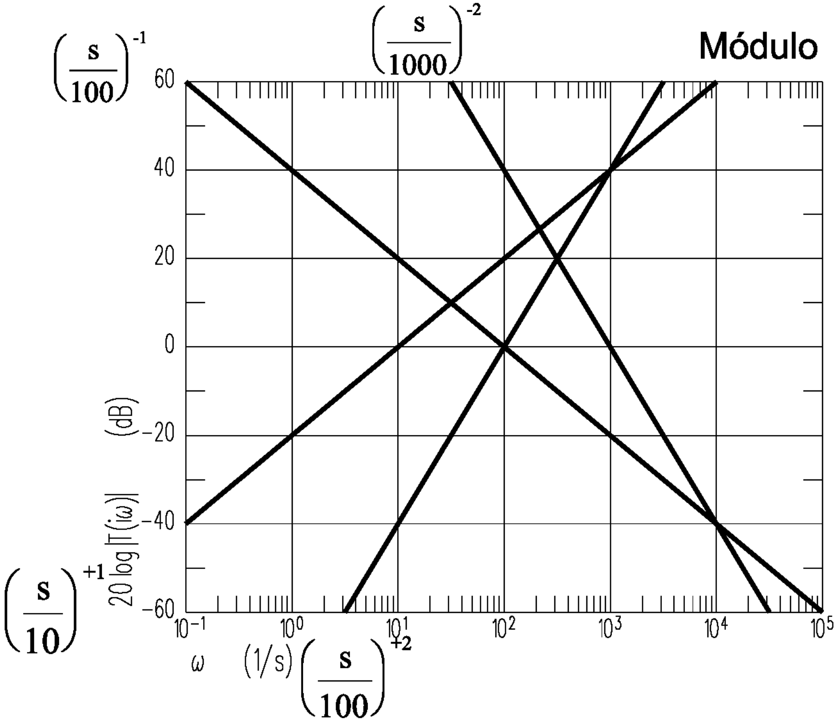


$T(s) = \left(\dfrac{s}{\omega_o}\right)^{\pm n}$ $\qquad$ $T(i\omega) = \left(i\dfrac{\omega}{\omega_o}\right)^{\pm n}$

Módulo $\qquad$ Fase

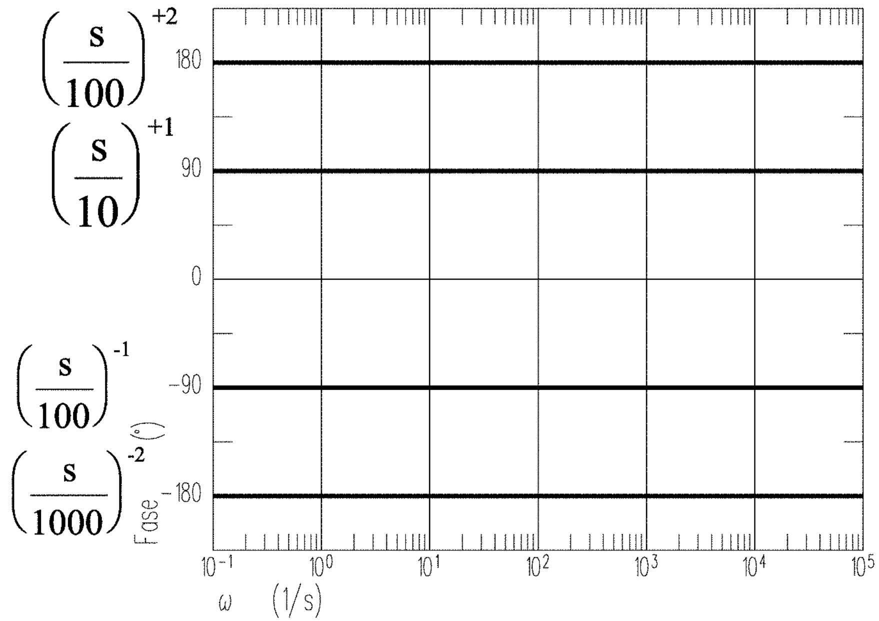
$\pm n\, 20 \log(\omega / \omega_o)$ $\qquad$ $\pm n\, \pi / 2$

Módulo



13

Fase



14

Ejercicios

$$10\,s = \frac{s}{0,1}$$

$$\left(\frac{s}{10}\right)\left(\frac{s}{100}\right)$$

$$-100\left(\frac{s}{10}\right)^{-2}$$

$$\left(\frac{s}{100}\right)\left(\frac{s}{1000}\right)^{-2}$$

$$-10\left(\frac{s}{100}\right)^{3}$$

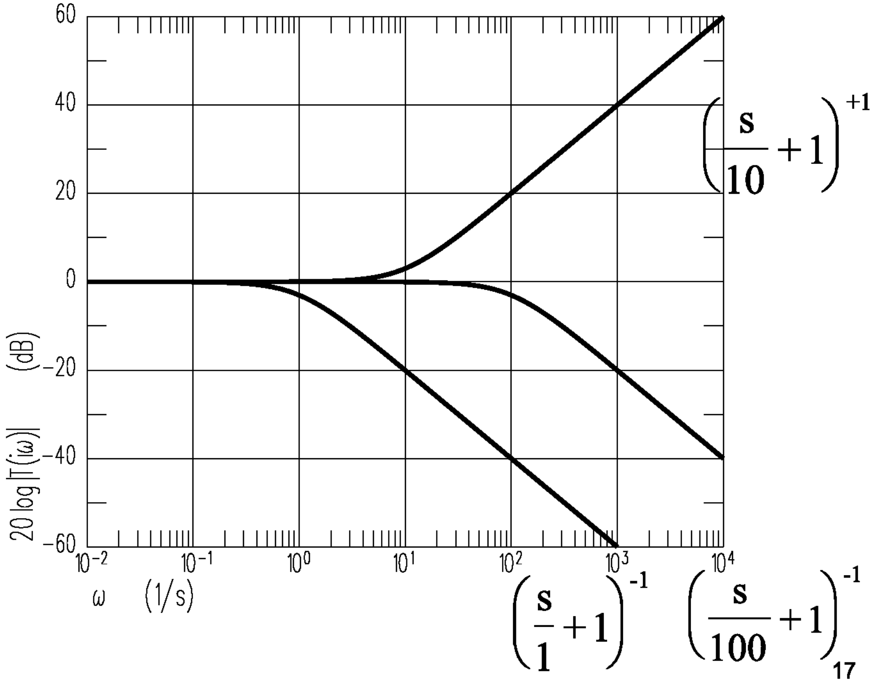
15

$$T(s) = \left(\frac{s}{\omega_o}+1\right)^{\pm 1} \qquad T(i\omega) = \left(1+i\frac{\omega}{\omega_o}\right)^{\pm 1}$$

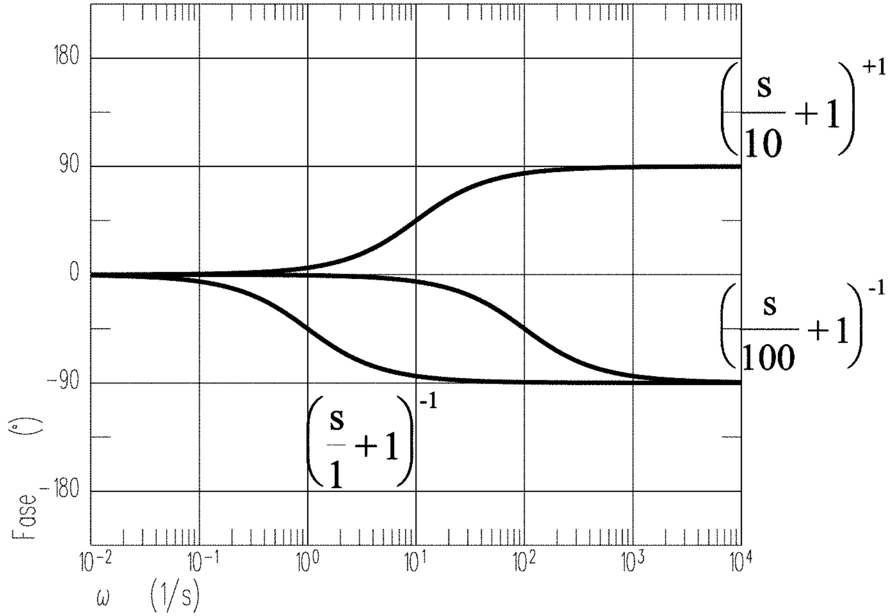
	Módulo	Fase
$\omega \ll \omega_o$	0	0
$\omega = \omega_o$	$\pm 20\log\sqrt{2}$	$\pm\pi/4$
$\omega \gg \omega_o$	$\pm 20\log(\omega/\omega_o)$	$\pm\pi/2$

16

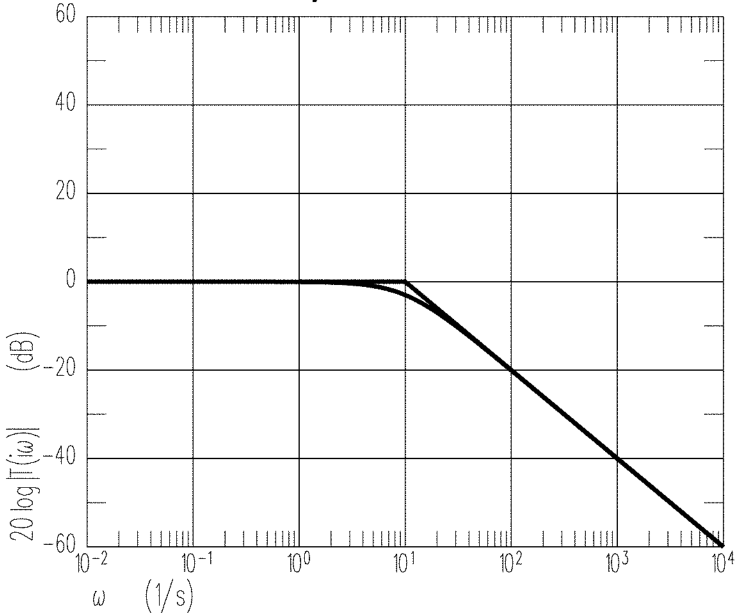
Módulo



Fase



Comportamiento asintótico



Ejercicios

$$(s+10)$$

$$-10\,\frac{\left(\frac{s}{100}\right)}{\left(\frac{s}{1000}+1\right)}$$

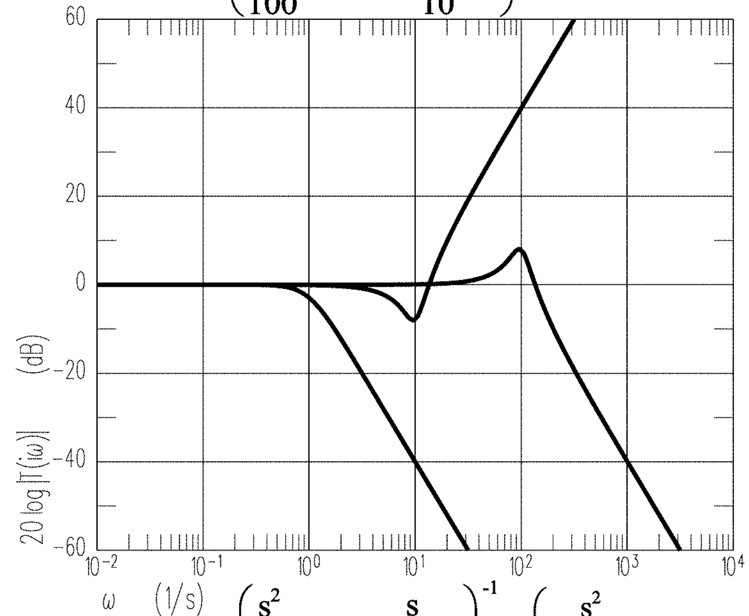
$$10\,\frac{s}{10}\,\frac{\left(\frac{s}{100}+1\right)}{\left(\frac{s}{10}+1\right)}$$

$$T(s) = \left(\frac{s^2}{\omega_o^2} + 2\delta\frac{s}{\omega_o} + 1\right)^{\pm 1} \qquad T(i\omega) = \left[\left(1 - \frac{\omega^2}{\omega_o^2}\right) + i2\delta\frac{\omega}{\omega_o}\right]^{\pm 1}$$

	Módulo	Fase
$\omega \ll \omega_o$	0	0
$\omega = \omega_o$	$\pm 20\log(2\delta)$	$\pm\pi/2$
$\omega \gg \omega_o$	$\pm 40\log(\omega/\omega_o)$	$\pm\pi$

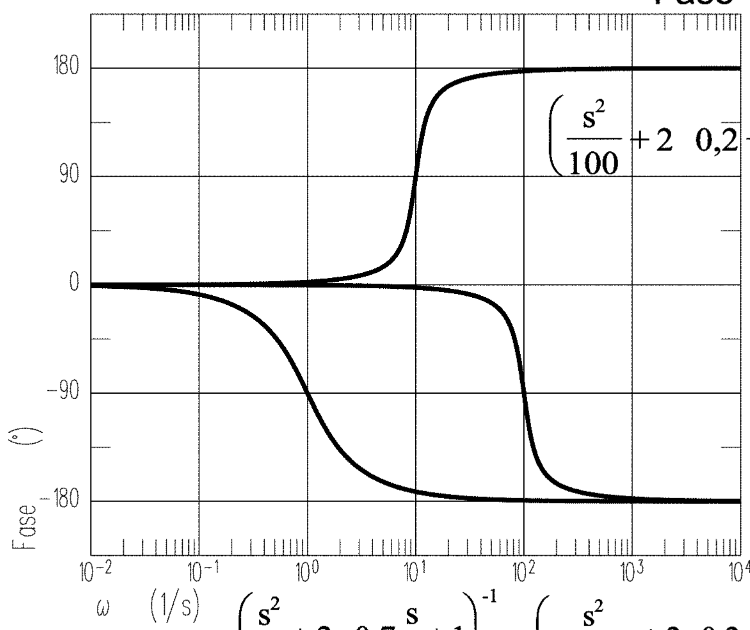
$$\left(\frac{s^2}{100} + 2\ \ 0{,}2\frac{s}{10} + 1\right)^{+1}$$

Módulo



$$\left(\frac{s^2}{1} + 2\ \ 0{,}7\frac{s}{1} + 1\right)^{-1} \qquad \left(\frac{s^2}{10000} + 2\ \ 0{,}2\frac{s}{100} + 1\right)^{-1}$$

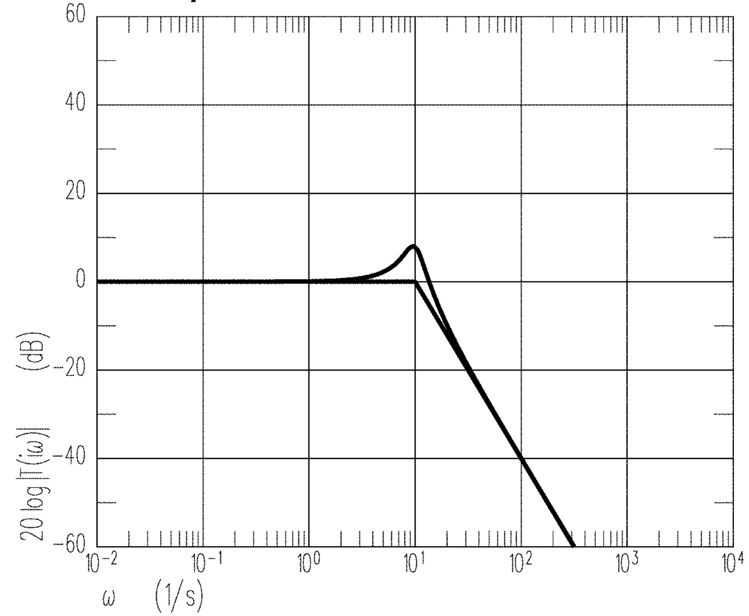
Fase



$$\left(\frac{s^2}{100} + 2\ \ 0{,}2\frac{s}{10} + 1\right)^{+1}$$

$$\left(\frac{s^2}{1} + 2\ \ 0{,}7\frac{s}{1} + 1\right)^{-1} \qquad \left(\frac{s^2}{10000} + 2\ \ 0{,}2\frac{s}{100} + 1\right)^{-1}$$

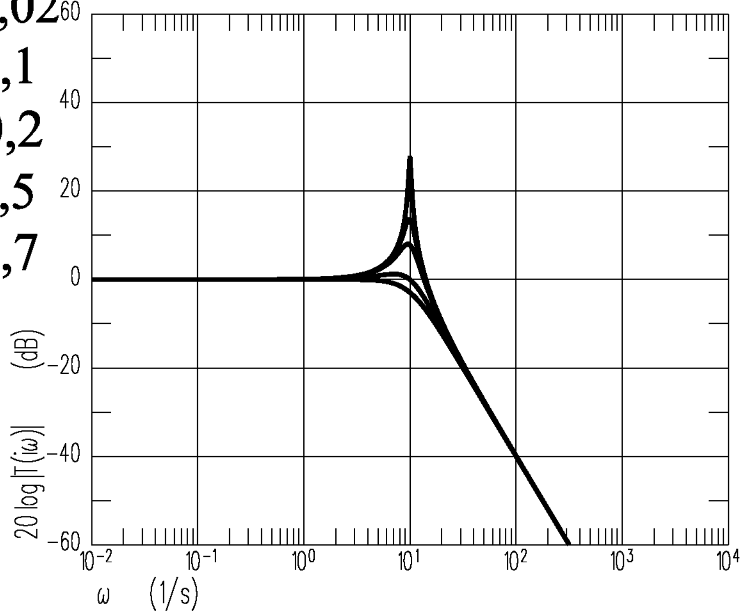
Comportamiento asintótico



Influencia de δ

Módulo

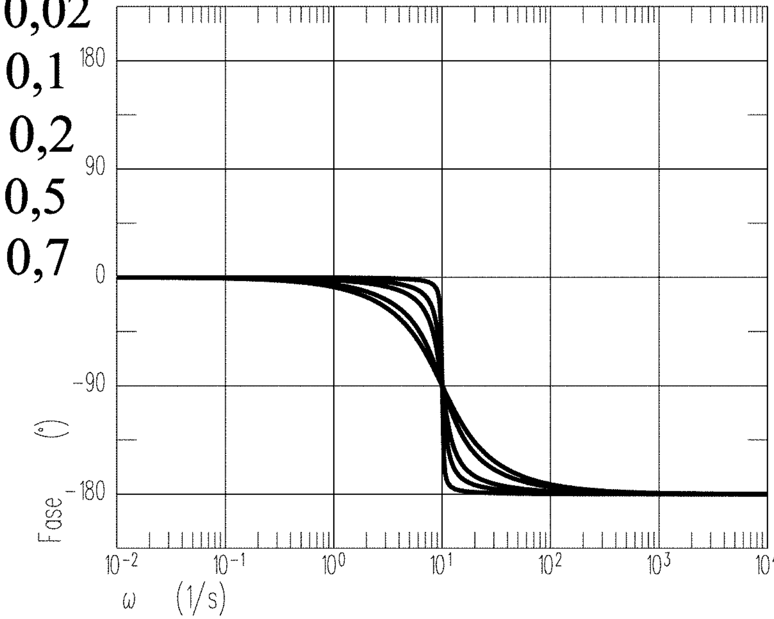
$\delta = 0{,}02$
$\delta = 0{,}1$
$\delta = 0{,}2$
$\delta = 0{,}5$
$\delta = 0{,}7$



Influencia de δ

Fase

$\delta = 0{,}02$
$\delta = 0{,}1$
$\delta = 0{,}2$
$\delta = 0{,}5$
$\delta = 0{,}7$



Ejercicios

$$(s^3 + 10\,s^2 + 100\,s)$$

$$\frac{(s+10)}{(s^2+s+1)}$$

$$-\frac{(s^2+2\,s+100)}{(s^2+s+1)}$$

$$\frac{(s+1)(s+100)}{(s^2+10\,s+100)}$$

$$\frac{(s^2+2\,s+4)}{\left(\frac{s^2}{100}+0{,}2\,s+4\right)}$$

Bode en una hoja de cálculo

Se hará un diagrama de Bode con datos similares a los de la práctica 2. Datos en el fichero bd2.prn

Frecuencia de corte :
LP1 = 1000 rad/s
LP2 = 10.000 rad/s

Ganancia :
LP1 = 2 (6 dB)
LP2 = 1 (0 dB)

Coef. amort. δ :
LP2 = 0,06

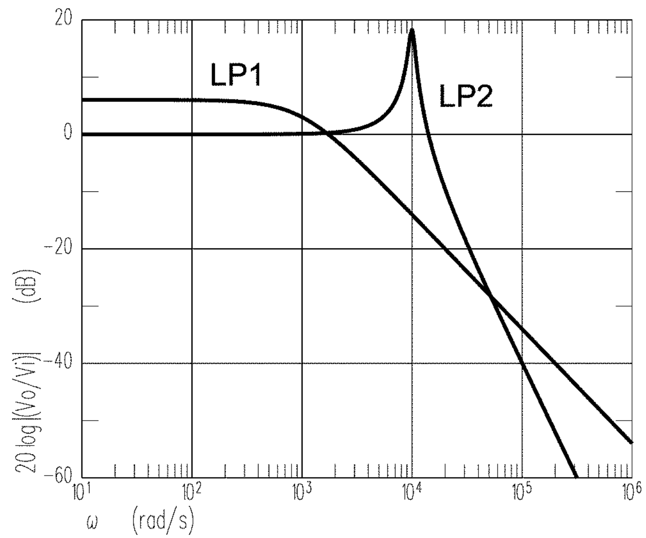


Diagrama de Bode

FFT Granada granada.net78.net

10-X-2011
S.O.: Win95
Res.: 800x600
Col.: 16bit

FIN