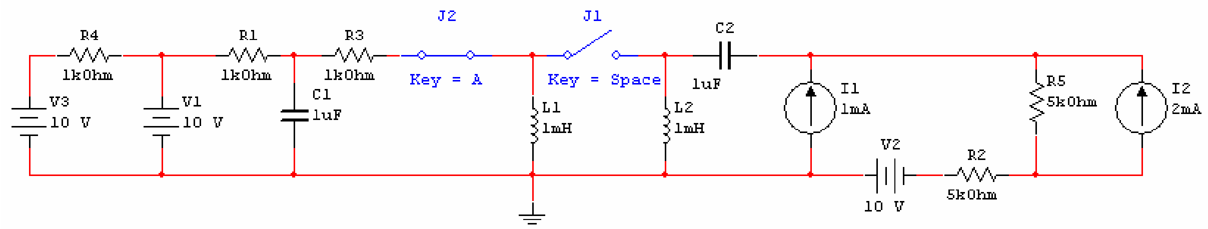
Universidad de Granada
Departamento de Electrónica y Tecnología de Computadores

ANÁLISIS DE CIRCUITOS

Ingeniería de Telecomunicación
Examen 11 de febrero de 2010

Duración: 2 horas 30 minutos
Responda a cada pregunta en hojas separadas
Lea detenidamente los enunciados antes de contestar

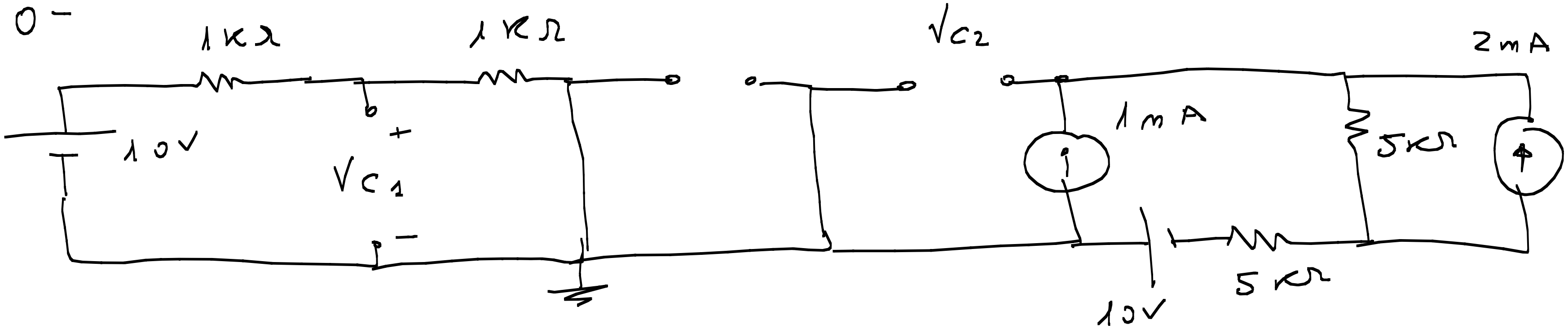
1. En el circuito de la figura los interruptores han permanecido conectados en la posición que se indica durante un tiempo largo. En t=0s cambian de posición (J2 se abre y J1 se cierra).
   a) Calcule la tensión en los condensadores (C1 y C2) en $t = 0^{-}$s, en $t = 0^{+}$s y en $t \to \infty$. **(1 punto)**
   b) Calcule la corriente en cada una de las bobinas (L1 y L2) en $t = 0^{-}$s, en $t = 0^{+}$s y en $t \to \infty$. **(1 punto)**
   c) Calcule y represente la evolución de la tensión entre los extremos del condensador (C1) en función del tiempo **(2 puntos)**



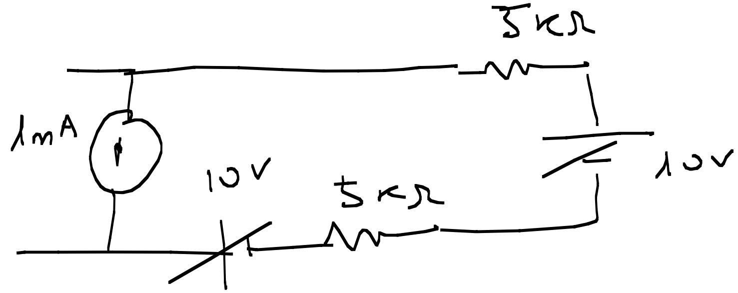
a) $V_{C1}(t=0^-)$ $V_{C1}(t=0^+)$ $V_{C1}(t\to\infty)$

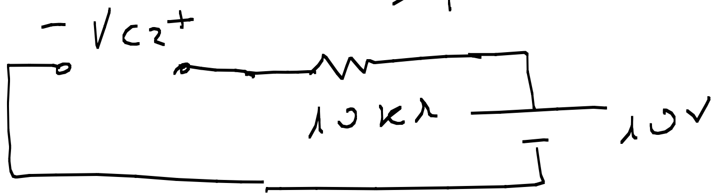
$V_{C2}(t=0^-)$ $V_{C2}(t=0^+)$ $V_{C2}(t\to\infty$

$t = 0^-$



$V_{C1}(t=0^-) = 10 \cdot \frac{1}{2} = 5V$



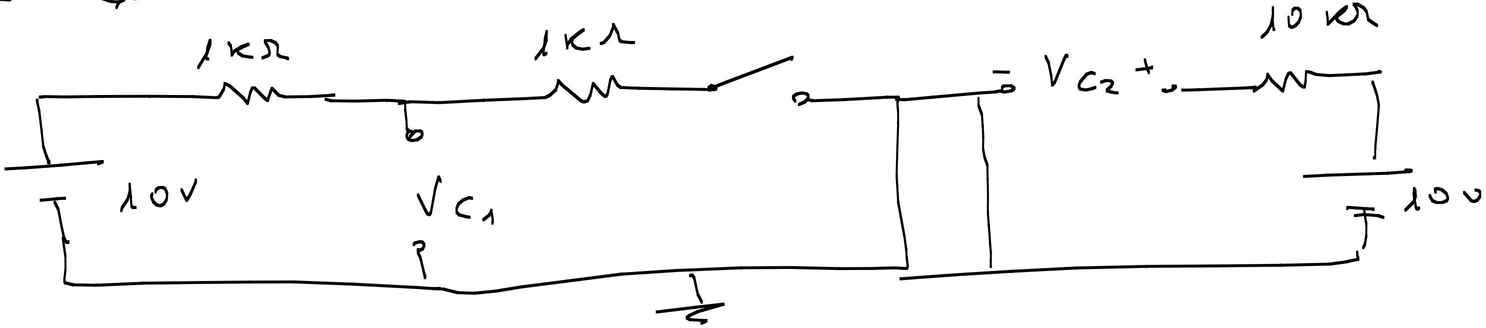


$V_{C2}(t=0^-) = 10\ V$

$t = 0^+$ $\quad V_{C1}(t=0^+) = V_{C1}(t=0^-) = 5\,V$

$V_{C2}(t=0^+) = V_{C2}(t=0^-) = 10\,V$

$t \to \infty$



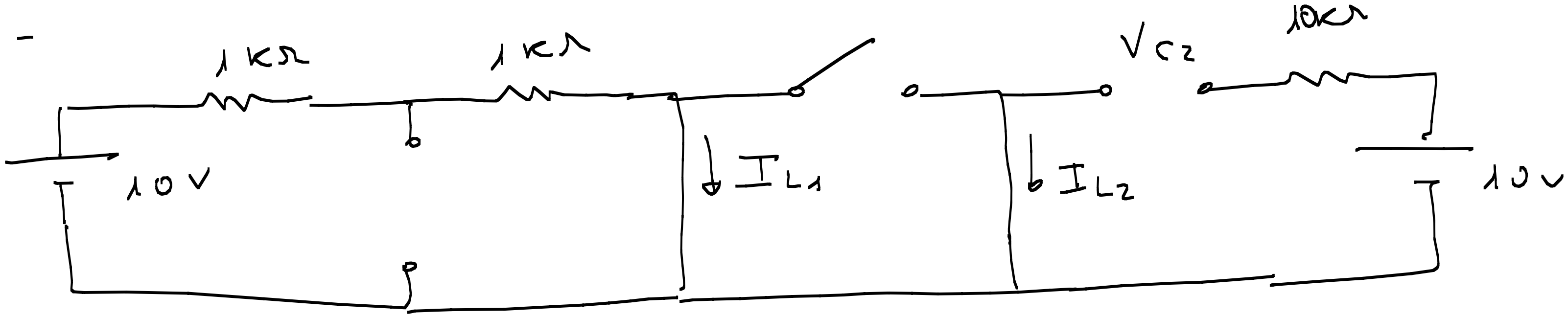
$V_{C2}(t \to \infty) = 10\,V$

$V_{C1}(t \to \infty) = 10\,V$

b) $I_{L1}(t=0^-)$ $\quad$ $I_{L1}(t=0^+)$ $\quad$ $I_{L1}(t\to\infty)$

$I_{L2}(t=0^-)$ $\quad$ $I_{L2}(t=0^+)$ $\quad$ $I_{L2}(t\to\infty)$

$t=0^-$



$-10 + 2 I_{L1} = 0$

$I_{L1}(t=0^-) = 5 \text{ mA}$

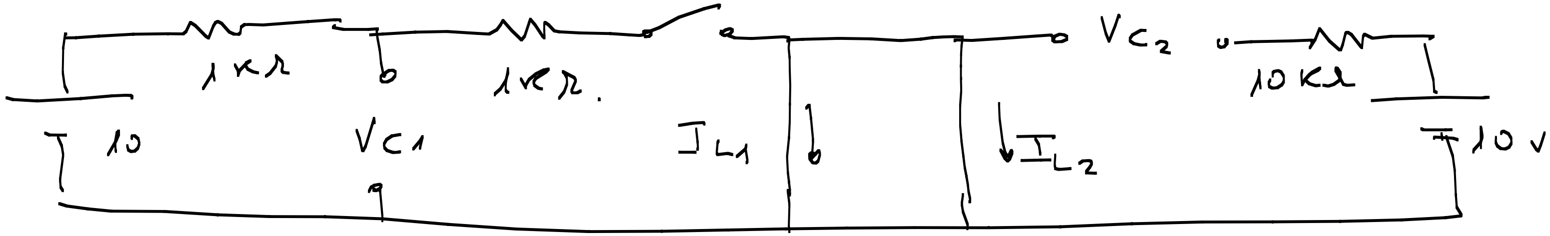
$I_{L2}(t=0^-) = 0 \text{ mA}$

$t=0^+$

$I_{L1}(t=0^+) = I_{L1}(t=0^-) = 5 \text{ mA}$

$I_{L2}(t=0^+) = I_{L1}(t=0^-) = 0 \text{ mA}$

$t \to \infty$



$I_{L1} + I_{L2} = 0$ $\qquad v_{L1}(t) = v_{L2}(t)$

$$L_1 \frac{d\, i_{L1}}{dt} = L_2 \cdot \frac{d\, i_{L2}}{dt}$$

$$\int L_1 \frac{d\, i_{L1}}{dt}\, dt = \int L_2 \frac{d\, i_{L2}}{dt}\, dt$$

$$\int_0^\infty L_1\, d i_{L1}(t) = L_1 \left[i_{L1}(\infty) - i_{L1}(0) \right] = L_1 i_{L1}(\infty) - L_1 \cdot 5\text{mA}$$

$$\int_0^\infty L_2\, d i_{L2}(t) = L_2 \left[i_{L2}(\infty) - i_{L2}(0) \right] = L_2 i_{L2}(\infty) - L_2 \cdot 0$$

$$\int_0^\infty L_1\, d i_{L1}(t) = \int_0^\infty L_2\, d i_{L2}(t)$$

$$\left.\begin{aligned} L_1 i_{L1}(\infty) - 5 L_1 &= L_2 i_{L2}(\infty) \\ i_{L1}(\infty) + i_{L2}(\infty) &= 0 \end{aligned}\right\}$$

$$i_{L1}(\infty) = - i_{L2}(\infty)$$

$$L_1 \, i_{L1}(\infty) + L_2 \, i_{L1}(\infty) = 5 \, L_1$$

$$L_1 = L_2 = 1 \, mH$$

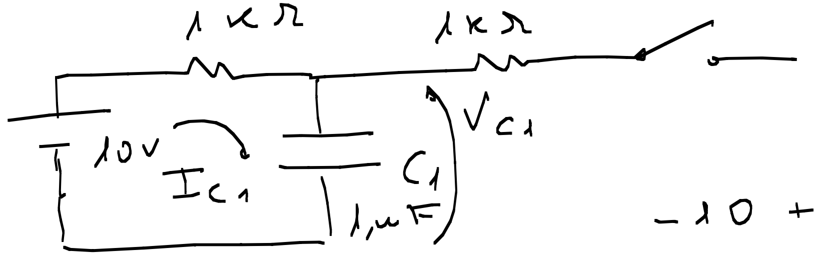
$$2 \, i_{L1}(\infty) = 5$$

$$i_{L1}(\infty) = 2,5 \, mA$$

$$i_{L2}(\infty) = -2,5 \, mA$$

c) $V_{C1}(t=0^-) = V_{C1}(t=0^+) = 5V$

$t > 0$



$$-10 + 1 \cdot I_{C1} + V_{C1} = 0$$

$$I_{C1} = C \frac{dV_{C1}}{dt}$$

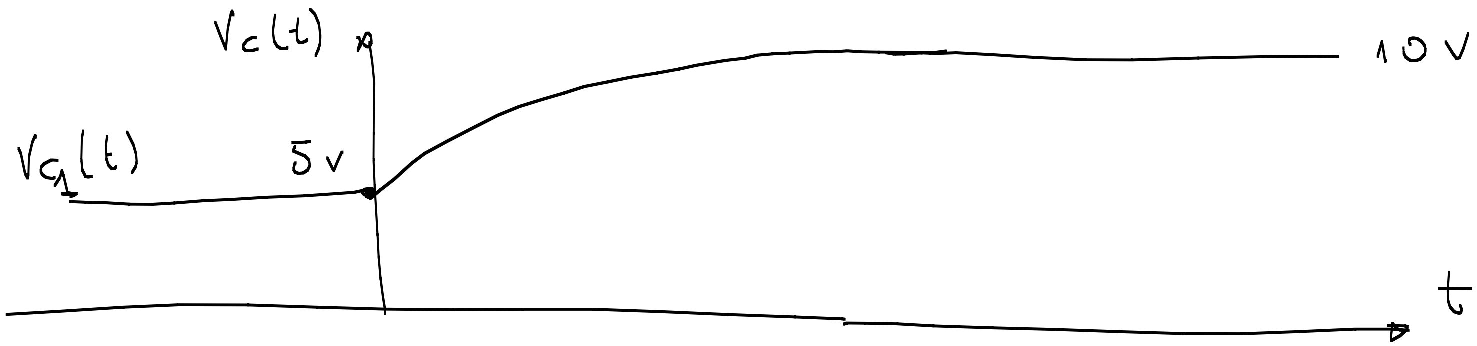
$$1 \cdot 10^3 \cdot 1 \cdot 10^{-6} \frac{dV_{C1}}{dt} + V_{C1} = 10$$

$$1 \cdot 10^{-3} \frac{dV_{C1}}{dt} + V_{C1} = 10$$

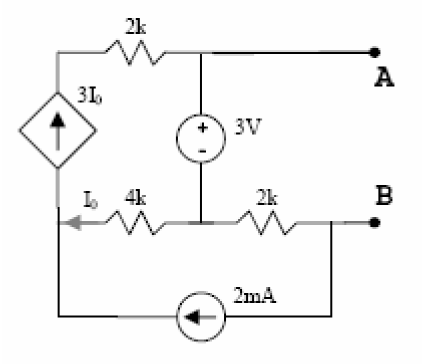
$$\tau \frac{dx}{dt} + x = K \qquad x(t) = K + (x_0 - K)\, e^{-\frac{t}{\tau}}$$

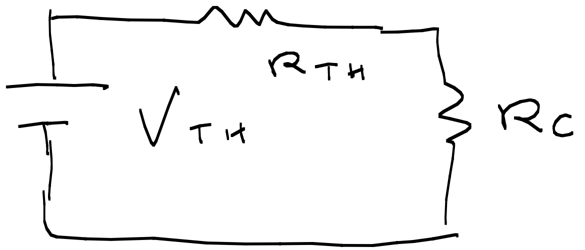
$$V_{C1}(t) = 10 + (5 - 10)\, e^{-\frac{t}{1 \cdot 10^{-3}}}$$

$$V_{C1}(t) = 10 - 5\, e^{-1000t}$$


Vc(t)
Vc1(t)
5v
10 V
t

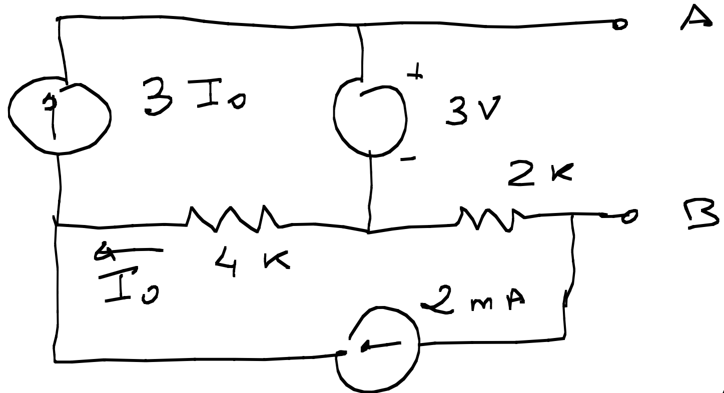
2. Determina el valor de la resistencia que colocada en los terminales A y B reciba la máxima potencia que pueda suministrarle el circuito **(1.5 puntos).**



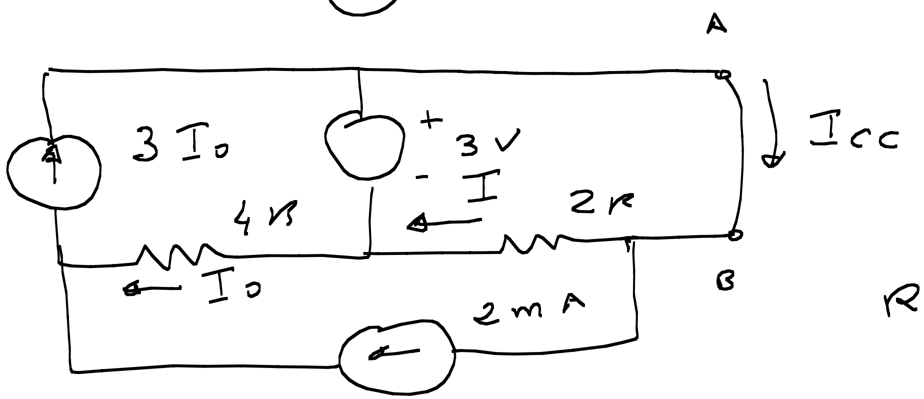


MÁXIMA POTENCIA

$$R_C = R_{TH}$$



$$V_{AB} = 3 + 2 \cdot 2 = 7\,V$$



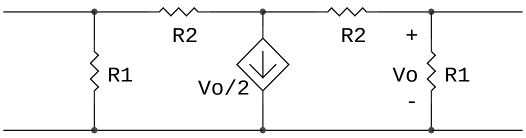
$$I = \frac{3}{2}$$

$$I_{cc} = 2 + I = 2 + \frac{3}{2} = \frac{7}{2}\,mA$$

$$R_{TH} = \frac{V_{AB}}{I_{cc}} = \frac{7}{\frac{7}{2}} = 2\,k\Omega$$

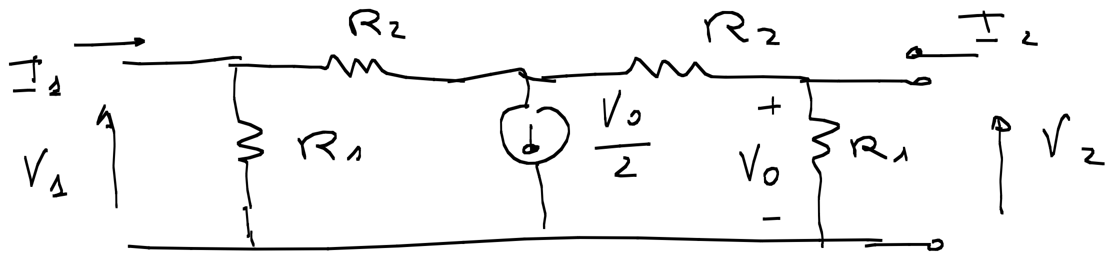
R_{C} máx potencia $= 2\,k\Omega$

3. Calcule el valor de las resistencias R1 y R2 del cuadripolo de la figura sabiendo el valor de los siguientes parámetros: y_{11}=0.2 Ω^{-1}, y_{21}=-0.1 Ω^{-1}. Complete la matriz de admitancias. **(1.5 puntos).**



$y_{11} = 0,2\ \Omega^{-1}$

$y_{21} = -0,1\ \Omega^{-1}$



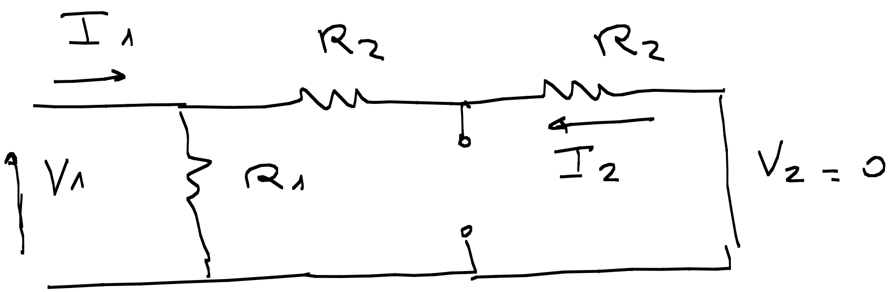
$I_1 = y_{11} V_1 + y_{12} V_2$

$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$

$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$

$I_2 = y_{21} V_1 + y_{22} V_2$

$V_2 = 0$



$V_1 = I_1 \cdot (R_1 // 2R_2) = \frac{2 \cdot R_1 R_2}{R_1 + 2R_2} I_1$

$\frac{I_1}{V_1} = \frac{R_1 + 2R_2}{2 R_1 R_2} = 0,2$

$V_1 = - I_2 \cdot 2 R_2$

$\frac{I_2}{V_1} = - \frac{1}{2 R_2} = -0,1 \rightarrow R_2 = \frac{1}{0,2} = 5\ \Omega$

$$R_1 + 2R_2 = 0,4\, R_1 R_2$$

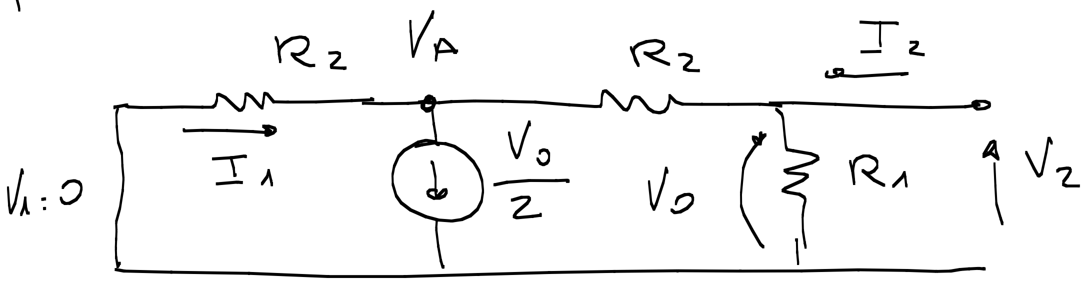
$$R_1 - 0,4 \cdot 5\, R_1 = -2R_2$$

$$R_1(1-2) = -10$$

$$R_1 = 10\,\Omega$$

$$y_{12} = \frac{I_1}{V_2}\bigg|_{V_1=0} \qquad y_{22} = \frac{I_2}{V_2}\bigg|_{V_1=0}$$

$V_1 = 0 \longrightarrow$



$$V_0 = V_2 \qquad I_1 = -\frac{V_A}{R_2} = -\frac{V_A}{5}$$

$$\frac{V_A - V_2}{5} = \frac{V_2}{10} - I_2 \qquad -\frac{V_A}{5} = \frac{V_2}{2} + \frac{V_A - V_2}{R_2}$$

$$\frac{V_2}{5} - \frac{V_2}{2} = \frac{V_A}{5} + \frac{V_A}{5}$$

$$\frac{2V_2 - 5V_2}{10} = \frac{2V_A}{5}$$

$$-3V_2 = 4V_A \qquad V_A = -\frac{3}{4}V_2$$

$$-\frac{3}{4}V_2 - V_2 = \frac{V_2}{2} - 5I_2$$

$$5I_2 = V_2\left(\frac{1}{2} + 1 + \frac{3}{4}\right) \qquad 5I_2 = V_2\,\frac{9}{4}$$

$$y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} \qquad y_{22} = \frac{I_2}{V_2} = \frac{9}{20}$$

$$I_1 = -\frac{V_A}{5} = -\frac{1}{5}\cdot\left(-\frac{3}{4}V_2\right)$$

$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} \qquad y_{12} = \frac{I_1}{V_2} = \frac{3}{20}$$

4. Un circuito tiene una función de transferencia:

$$H(s)=\frac{V_o(s)}{V_i(s)}=\frac{2s}{s^2+3s+2}$$

a) Determine la salida estacionaria del circuito (Vo(t)) cuando se le aplica una señal $V_i(t)=\cos(5t)+\cos(10t)$ V. **(1 punto).**

b) En el circuito anterior determina la respuesta a una entrada escalón Vi(t)=10u(t) si los elementos del circuito no almacenaban energía (condiciones iniciales nulas). **(1 punto).**

c) Represente el diagrama de Bode en amplitud para dicha función de transferencia. ¿Qué tipo de filtro es el circuito? Determina el ancho de banda del filtro, frecuencia o frecuencias de corte y cuánto atenúa en cada década que aumente la frecuencia en la banda o bandas rechazadas. **(1 punto).**

Transformadas de posible utilidad:

$$\xrightarrow{L}$$

$u(t)$	$\dfrac{1}{s}$
$u(t)e^{at}$	$\dfrac{1}{s-a}$
$\sin\omega t$	$\dfrac{\omega}{s^2+\omega^2}$
$\cos\omega t$	$\dfrac{s}{s^2+\omega^2}$

$$\xleftarrow{L^{-1}}$$

a) $V_0(t)$? $\qquad V_i(t) = \cos 5t + \cos 10t$

$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{2s}{s^2 + 3s + 2} \qquad s = j\omega$$

$$H(j\omega) = \frac{\tilde{V}_o}{\tilde{V}_i}$$

$\tilde{V}_{o1} = \tilde{V}_{i1} \cdot H(j\omega)$ para $V_{i1}(t)$ de frecuencia $\omega = 5$

$$\left.\begin{aligned} |\tilde{V}_{o1}| &= |\tilde{V}_{i1} \cdot H(j\omega)| = |\tilde{V}_{i1}| \cdot |H(j\omega)| \\ \angle \tilde{V}_{o1} &= \angle \tilde{V}_{i1} \cdot H(j\omega) = \angle \tilde{V}_{i1} + \angle H(j\omega) \end{aligned}\right\} \omega = 5$$

$$\left.\begin{aligned} |\tilde{V}_{o2}| &= |\tilde{V}_{o2}| \cdot |H(j\omega)| \\ \angle \tilde{V}_{o2} &= \angle \tilde{V}_{o2} + \angle H(j\omega) \end{aligned}\right\} \omega = 10$$

$$H(s) = \frac{2s}{s^2 + 3s + 2}$$

$$H(5j) = \frac{10j}{-25 + 15j + 2} = \frac{10j}{-23 + 15j} \qquad |H(5j)| = \frac{10}{\sqrt{23^2 + 15^2}} = 0{,}364$$

$$\underline{|H(5j)} = 90^\circ - 146{,}88^\circ = -56{,}88^\circ$$

$$H(10j) = \frac{20j}{-100 + 30j + 2} = \frac{20j}{-98 + 30j} \qquad |H(10j)| = \frac{20}{\sqrt{98^2 + 30^2}} = 0{,}195$$

$$\underline{|H(10j)} = 90^\circ - 162{,}979^\circ = -72{,}979^\circ$$

$$|\tilde{V}_{o1}| = |\tilde{V}_{i1}| \cdot |H(5j)| = 1 \cdot 0,364 = 0,364$$

$$|\tilde{V}_{o2}| = |\tilde{V}_{i2}| \cdot |H(10j)| = 1 \cdot 0,195 = 0,195$$

$$\angle \tilde{V}_{o1} = \angle \tilde{V}_{i1} + \angle H(5j) = 0 - 56,88° = -56,88°$$

$$\angle \tilde{V}_{o2} = \angle \tilde{V}_{i2} + \angle H(10j) = 0 - 72,979° = -72,979°$$

$$\tilde{V}_{o1} = 0,364 \angle -56,88° \qquad \tilde{V}_{o2} = 0,195 \angle -72,979°$$

$$V_o(t) = 0,364 \cos(5t - 56,88°) + 0,195 \cos(10t - 72,979°)$$

b) $V_i(t) = 10\,u(t)$ $\qquad V_i(s) = \frac{10}{s}$

$$V_o(s) = V_i(s) \cdot \frac{2s}{s^2+3s+2} = \frac{10}{s} \cdot \frac{2s}{s^2+3s+2} = \frac{20}{(s+1)(s+2)}$$

$$s = \frac{-3 \pm \sqrt{9-8}}{2} = \begin{cases} \frac{-3+1}{2} = -1 \\ \frac{-3-1}{2} = -2 \end{cases}$$

$$\frac{20}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$$

$$A = (s+1) \cdot \frac{20}{(s+1)(s+2)}\Bigg|_{s=-1} = \frac{20}{1} = 20$$

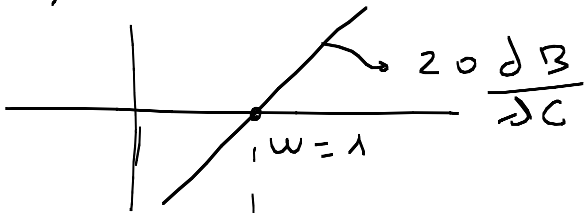
$$B = (s+2) \frac{20}{(s+1)(s+2)}\Bigg|_{s=-2} = \frac{20}{-1} = -20$$

$$V_o(s) = \frac{20}{(s+1)(s+2)} = \frac{20}{s+1} - \frac{20}{s+2}$$

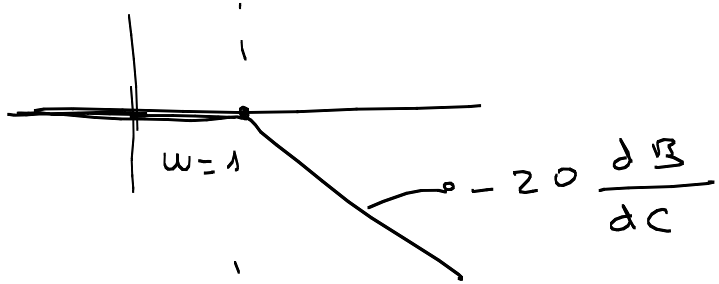
$$V_o(t) = 20 \cdot e^{-t} - 20\, e^{-2t}$$

c) $H(s) = \dfrac{V_o(s)}{V_i(s)} = \dfrac{s}{(s+1)\left(\frac{s}{2}+1\right)}$

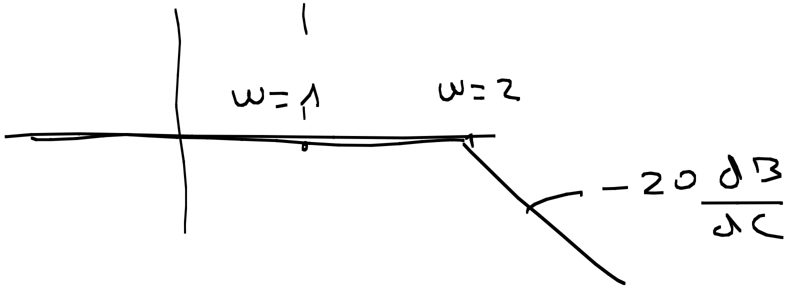
$s \longrightarrow$



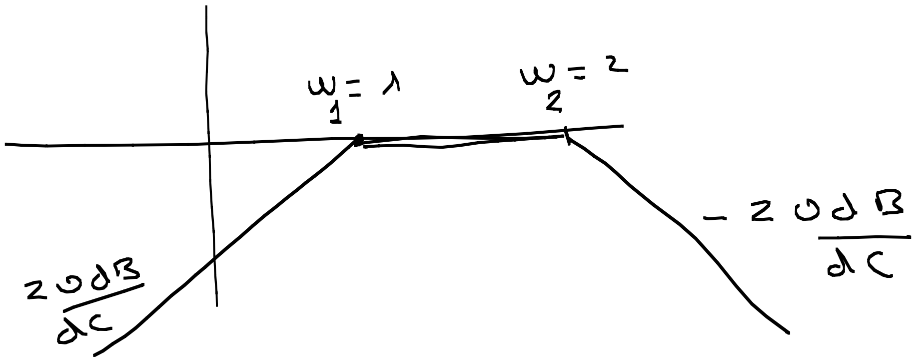
$\dfrac{1}{s+1} \longrightarrow$



$\dfrac{1}{\frac{s}{2}+1}$



$$\frac{s}{(s+1)\left(\frac{s}{2}+1\right)}$$



- FILTRO PASO BANDA
- $BW = \omega_2 - \omega_1$
- SE ATENÚA $-20 \frac{dB}{dc}$ A PARTIR DE ω_2