CS121 Parallel Computing Lab 1

Breadth First Search using OpenMP

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Abstract

In this lab, I implement two parallel BFS algorithms using OpenMP, top-down BFS and hybrid BFS [1]. I also implement a relative fast serial algorithm and optimize the two parallel BFS algorithms using techniques like bitmap, prefix sum and so on.

1 Introduction

1.1 Compile & Build

All codes are tested under Ubuntu 20.04 LTS with g++ 9.3.0. You need a c++ compiler which support at least C++ 17 standard and GNU atomic compiler intrinsics and also need python3 to run bench and fetch scripts.
Run the benchmark by:

```
make bench
```

The scripts will first fetch the four graphs from the SNAP dataset website and clone the PaRMAT [2] to generate three RMAT graphs. Then compile my code and finally run benchmark on these graphs automatically.

1.2 Run

By directly running our program, you can see the helping information.
There are two modes in our program, benchmark mode and running mode.
In benchmark mode, you need to give 6 arguments in order: input file path, the format of the input file (can be chosen from SNAP, MatrixMarket and RMAT), benchmark file output path (save the benchmark result), -1 to inform our program to run in bench mode, bfs algorithm type (can be chosen from serial, topdown and hybrid) and finally thread nums. In running mode, you need to give 6 arguments in order: input file path, the format of the input file (can be chosen from SNAP, MatrixMarket and RMAT), output file path (save the running result), source node id, BFS algorithm type (can be chosen from serial, topdown and hybrid) and finally thread nums.

1.3 Benchmark Configures

I run my code on a cloud virtual machine provided by tencent cloud. The detailed information are:

Config	Value
CPU	AMD EPYC ROME^{TM} 7k62 (64 vCPU)
Memory	128 GB
OS	Ubuntu 20.04 LTS
Kernel	5.4.0-77-generic
Compiler	gcc version 9.3.0
Optimize	-O2

2 Implement Details

2.1 Serial Algorithm

Algorithm 1 Serial BFS

dist $\leftarrow \{-1, -1, \cdots, -1\}$
parent $\leftarrow \{-1, -1, \cdots, -1\}$
queue $\leftarrow$ {source}
dist[source] $\leftarrow 0$
parent[source] $\leftarrow$ source
while not queue.empty() **do**
  $u \leftarrow$ queue.pop()
  for $v \in$ neighbors(u) **do**
    if dist[v] $= -1$ **then**
      dist[v] $\leftarrow$ dist[u] $+ 1$
      parent[v] $\leftarrow u$
      queue.push(v)
    end if
  end for
end while

In the serial algorithm, we use a queue to implement BFS. For simplicity we use std::queue to implement.
One observation is that we do not need to maintain a visit array, we can just use dist array to check whether a vertex is already visited. By doing this we can avoid extra memory access and due to big cache in CPU this method is faster.
Another optimization is the way to store the graph. I store the graph with adjacent list but implement with raw arrays, a form like CSR format for storing sparse matrix.

2.2 Graph Storage

As I have introduced in the serial algorithm section, I store the graph in an adjacent list with array implementation.
That is I store all the neighbors in a whole huge array g. The neighbors of the same vertex are stored consecutively. And maintain a offset array o where vertex u's neighbors' storage indices lie between $[o[u], o[u+1])$. Actually, this is CSR format for storing sparse matrix.
Further more, from my observation, there are some unused vertices id in the graph. So, during the input process, I create a mapping to compress vertex id. This can reduce our memory cost. In the output, I just need to unmap to recover the original id.
Besides, also during the input process, I sort the same vertex's neighbors' id in increasing order, this can increase the probability that visiting sequence is adjacent.

2.3 Memory Management

I create a singleton class named MemoryManager to manage memory for big arrays. By doing this, we can simply avoid memory leak problems and reuse the memory during benchmark process, which will run bfs multiple times. The memory reuse can optimize time for memory allocation.

2.4 Top-down Parallel BFS

Algorithm 2 Top-down step

> **input:** vertices, frontier, next, parent, dist
> **for** $u \in$ frontier **do**
>   **for** $v \in$ neighbors(u) **do**
>   **if** dist$[v] = -1$ **then**
>   dist$[v] \leftarrow$ dist$[u] + 1$, parent$[v] \leftarrow u$
>   next $\leftarrow$ next $\cup \{v\}$
>   **end if**
>   **end for**
> **end for**

Algorithm 3 Top-down BFS

> **input:** vertices, source
> frontier $\leftarrow$ {source}
> next $\leftarrow \{\}$
> dist $\leftarrow \{-1, -1, \cdots, -1\}$
> parents $\leftarrow \{-1, -1, \cdots, -1\}$
> dist[source] $\leftarrow 0$
> **while** frontier $\neq \{\}$ **do**
>   top-down step
>   frontier $\leftarrow$ next
>   next $\leftarrow \{\}$
> **end while**

To implement this in OpenMP efficiently, we need to implement bitmap and prefix sum. We can use bitmap to reduce memory traffic when checking whether a vertex is already visited. And we can use prefix sum to put neighbors into the exact position in the next frontier. See bitmap and prefix sum section for more details.

```
while (!f->empty()) {
#pragma omp parallel for
  for (int i = 0; i < f->n; ++i)
    pi[i + 1] = deg(f->d[i]);
  prefixSumOmp(pi, f->n + 1);
  nf->n = pi[f->n];
#pragma omp parallel for schedule(guided)
  for (int i = 0; i < f->n; ++i) {
    // traverse and do topdown step
  }
  f->cullFrom(*nf, cb);
}
```

In the code above, f is the current frontier while nf is the next frontier. First we do a prefix sum on the degree array to get the next frontier's size. And by using this prefix sum array, we can put neighbors directly into the next frontier instead of using a critical section to push back one by one.
In the last step, we need to cull the visited vertices (duplicates) in the next frontier. Otherwise, the number of vertices in the frontier will explode. See the frontier culling section for more details.
Inside the top-down step, I use bit map to check whether a vertex has already been visited. This can be done through an atomic operation test-and-set. However, we can further optimize to avoid this. We first just do test operation in the bitmap, if not visited then check the dist array whether the value is -1 [3, 4]. By doing this, we can avoid the expensive atomic operation test-and-set. In experiments, this can gain $1\% \sim 4\%$ performance improvement. The improvement is not that huge because the cost of atomic operations are much lower in modern CPUs.

2.5 Hybrid Parallel BFS

Instead of just using conventional top-down BFS, we can gain further optimization in some graphs by combining top-down method and bottom-up method.

Algorithm 4 bottom-up step

input: vertices, frontier, next, parent, dist
for $v \in$ vertices **do**
  if $\text{dist}[v] = -1$ **then**
    for $u \in \text{neighbors}(v)$ **do**
      if $u \in$ frontier **then**
        $\text{parent}[v] \leftarrow u$
        $\text{dist}[v] \leftarrow \text{dist}[u] + 1$
        $\text{next} \leftarrow \text{next} \cap \{v\}$
        break
      end if
    end for
  end if
end for

During the parallel hybrid bfs, we maintain three variables: the number of edges to check from the frontier (m_f), the number of vertices in the frontier (n_f) and the number of edges to check from unexplored vertices (m_u). These metrics are easy and efficient to compute. In OpenMP, we can simply add reduction clause for these variables.
These variables are used to control whether we need to do top-down step or bottom-up step. We start bfs with top-down method, then when $m_f > \frac{m_u}{\alpha}$, we transfer to bottom-up method. When $n_f < \frac{n}{\beta}$, we transfer back to top-down method. We can find this hybrid parallel bfs performs better in some graphs.
Here, we choose $\alpha = 14$ and $\beta = 24$ follow the best parameter described in the paper [1].

2.6 Bit Map

Store size of int32 or int64 times 8 bool into one int. In C++, we can use a template class to easily switch between int32 and int64. The test and set operation of bitmap is easy to implement by just taking some bit operations.
An important one is test-and-set operation, here I use GNU compiler intrinsics:

```
__atomic_fetch_or(&d[index],
    bitOffset, __ATOMIC_SEQ_CST);
```

This is an atomic intrinsics operation provided by gcc, which is relatively fast. Actually, as I have discussed in top-down bfs section, we can avoid this to gain a better performance.

2.7 Prefix Sum in OpenMP

```
#pragma omp parallel
{
  int id = omp_get_thread_num();
  T s = 0;
#pragma omp for schedule(static)\
    nowait
  for (int i = 0; i < n; ++i) {
    s += a[i];
    a[i] = s;
  }
  buffer[id + 1] = s;

#pragma omp barrier
  T offset = 0;
  for (int i = 1; i < id + 1; ++i)
    offset += buffer[i];
#pragma omp for schedule(static)
  for (std::uint32_t i = 0; i < n; ++i)
    a[i] += offset;
}
```

First, we use different threads to prefix sum the numbers distributed to them locally. Then, store each thread's sum into a buffer, get the prefix sum in the buffer of the threads whose thread id is less than or equal to the current thread, then add the sum into each number it owns.

2.8 Frontier Culling

We need to cull the next frontier to avoid vertices explosion. This is a filter process, first we write into a buffer: 0 for this item should be filtered and 1 for this item should be preserved. Then we apply prefix sum on this buffer, after that, we can directly write preserved items into the correct position.

2.9 IO Optimization

Since we need to do a lot of experiments, we need to run bfs with 20 random sources and this should be run multiple times to reduce uncertainty and we need to run benchmark with number of threads from 1 to 64. In addition, the IO takes a very high percentage of the total running time.
In order to run experiments faster, we need to optimize the IO. As I introduced in the run code section, you need to pass the input file's format. This format is used in the IO optimization, my code can directly load graphs from SNAP and RMAT format. No need to convert it to Matrix-Market format.
While reading, I maintain a buffered reader, that is I create a large buffer, each time I use

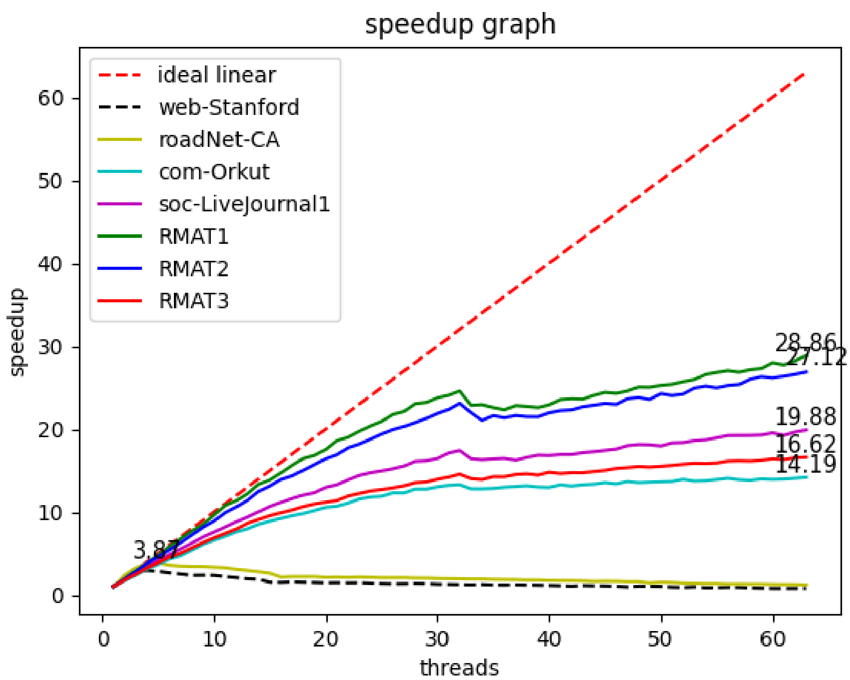
std::fread to read a large block of the file into the memory, then parse the the numbers manually, which is much faster than fscanf or fstream (more than 20 times faster).
By doing this, the first four graph all can be loaded (including pre-processings like id mapping construction) in 5 second and the RMAT can also be loaded in 1min.

3 Results

After running experiments, the benmark result is listed as follows:

Graph	Single Thread	Topdown, threads	Hybrid, threads	Speedup	Time (s)	StdDev
web-Stanford	227.185	675.243, 4	648.741, 5	2.972	0.00342469	1.38624
roadNet-CA	63.536	208.959, 4	204.252, 4	3.289	0.0274864	0.552123
com-Orkut	373.057	**5291.94, 64**	5280.75, 64	14.185	0.0225889	99.7608
soc-Livejournal1	72.3957	1439.49, 63	1439.56, 63	19.885	0.047927	15.9759
RMAT1	19.4047	559.64, 63	549.926, 64	**28.857**	1.78583	6.17689
RMAT2	33.6309	906.489, 64	912.044, 64	27.119	1.09644	11.5889
RMAT3	138.81	2293.06, 62	2307.07, 63	16.620	0.43345	1000.97

The performance are measured in MTEPS (millions of edges traversed per second), the baseline is the conventional top-down with single thread. We can find that the hybrid method only performs a little better in some graphs, it cannot significantly increase the peak performance. However, in the experiments, I find that the hybrid method can perform better in many thread nums.
In the first two graphs, our peak speedup is around 3, I think this is because the average degree of the graph is not much enough and the graph is also much smaller than the rest, we reach the peak performance using just 4 threads.
In the third graph, com-Orkut, our algorithm reach the peak performance of 5291.94 MTEPS, which is very efficient. And in the experiment, our performance is always increasing as the number of threads increase.
In the soc-Livejournal1 graph, we also obtain a relatively good speedup.
For the three big random graph, the absolute performance in MTEPS is not that high because of memory access, these graph need too much memory and the cache performance is not that good. However, our algorithm gains a very good speedup in these graphs. The performance almost always increases as the number of threads increase.
And in these graphs, we reach our peak speedup of 28.857, which is good enough and our algorithm is efficient.



References

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[2] Farzad Khorasani, Rajiv Gupta, and Laxmi N. Bhuyan. Scalable simd-efficient graph processing on gpus. In *Proceedings of the 24th International Conference on Parallel Architectures and Compilation Techniques*, PACT '15, pages 39–50, 2015.

[3] Sungpack Hong, Tayo Oguntebi, and Kunle Olukotun. Efficient parallel graph exploration on multi-core cpu and gpu. In *2011 International Conference on Parallel Architectures and Compilation Techniques*, pages 78–88, 2011.

[4] Charles E. Leiserson and Tao B. Schardl. A work-efficient parallel breadth-first search algorithm (or how to cope with the nondeterminism of reducers). In *Proceedings of the Twenty-Second Annual ACM Symposium on Parallelism in Algorithms and Architectures*, SPAA '10, page 303314, New York, NY, USA, 2010. Association for Computing Machinery.

4 Appendix

All the experiment data are listed on the next page.

Threads	MTEPS	StdDev	Time (s)
1	227.185	1.496	0.010179
2	433.386	1.517	0.005336
3	543.152	1.443	0.004258
4	675.243	1.386	0.003425
5	622.034	1.349	0.003718
6	593.315	1.319	0.003898
7	575.647	1.315	0.004017
8	539.476	1.412	0.004287
9	517.574	1.336	0.004468
10	536.526	1.307	0.004310
11	495.762	1.335	0.004665
12	427.408	1.769	0.005411
13	445.938	1.365	0.005186
14	410.805	1.385	0.005629
15	345.527	1.452	0.006693
16	338.506	1.467	0.006831
17	357.785	1.448	0.006463
18	347.207	1.465	0.006660
19	340.809	1.511	0.006785
20	323.520	1.554	0.007148
21	334.867	1.502	0.006906
22	319.326	1.545	0.007242
23	333.096	1.563	0.006942
24	319.401	1.542	0.007240
25	300.344	1.577	0.007699
26	291.765	1.583	0.007926
27	306.569	1.576	0.007543
28	303.731	1.606	0.007614
29	309.752	1.544	0.007466
30	293.676	1.546	0.007874
31	288.308	1.623	0.008021
32	280.844	1.577	0.008234
33	267.614	1.603	0.008641
34	251.999	1.622	0.009177
35	267.411	1.665	0.008648
36	269.803	1.592	0.008571
37	247.019	1.617	0.009362
38	270.403	1.579	0.008552
39	266.204	1.570	0.008687
40	259.412	1.587	0.008914
41	249.951	1.663	0.009252
42	234.790	1.652	0.009849
43	255.117	1.624	0.009064
44	247.736	1.585	0.009335
45	243.743	1.596	0.009487
46	231.102	1.621	0.010006
47	216.896	1.691	0.010662
48	236.481	1.596	0.009779
49	234.245	1.647	0.009872
50	230.345	1.610	0.010039
51	196.082	1.632	0.011794
52	202.631	1.676	0.011412
53	200.286	1.648	0.011546
54	193.876	1.688	0.011928
55	196.090	1.662	0.011793
56	208.272	1.659	0.011103
57	178.018	1.686	0.012990
58	189.542	1.679	0.012200
59	183.522	1.652	0.012601
60	175.918	1.721	0.013145
61	174.028	1.711	0.013288
62	177.212	1.658	0.013049
63	177.172	1.678	0.013052
64	166.524	1.705	0.013887

Table 1: web-Stanford top-down

Threads	MTEPS	StdDev	Time (s)
1	227.937	1.492	0.010145
2	490.973	1.428	0.004710
3	603.280	1.427	0.003833
4	605.104	1.412	0.003822
5	648.741	1.348	0.003565
6	607.349	1.355	0.003808
7	573.509	1.312	0.004032
8	539.257	1.293	0.004288
9	543.633	1.298	0.004254
10	539.715	1.322	0.004285
11	505.165	1.331	0.004578
12	476.320	1.352	0.004855
13	433.943	1.759	0.005329
14	430.489	1.545	0.005372
15	349.422	1.559	0.006618
16	345.103	1.461	0.006701
17	359.240	1.484	0.006437
18	350.233	1.486	0.006603
19	334.184	1.516	0.006920
20	331.572	1.518	0.006974
21	332.425	1.486	0.006956
22	328.930	1.564	0.007030
23	315.951	1.558	0.007319
24	320.247	1.589	0.007221
25	310.767	1.557	0.007441
26	303.474	1.583	0.007620
27	302.316	1.575	0.007649
28	313.654	1.596	0.007373
29	296.558	1.606	0.007798
30	289.216	1.607	0.007996
31	283.593	1.615	0.008154
32	283.091	1.598	0.008169
33	274.701	1.640	0.008418
34	285.365	1.630	0.008104
35	271.899	1.591	0.008505
36	249.157	1.596	0.009281
37	275.492	1.618	0.008394
38	264.243	1.610	0.008751
39	252.926	1.572	0.009143
40	255.612	1.635	0.009047
41	232.081	1.626	0.009964
42	237.053	1.619	0.009755
43	242.901	1.579	0.009520
44	235.143	1.613	0.009834
45	232.968	1.650	0.009926
46	234.023	1.667	0.009881
47	211.662	1.652	0.010925
48	226.766	1.678	0.010198
49	220.611	1.627	0.010482
50	213.731	1.684	0.010820
51	211.508	1.644	0.010933
52	194.382	1.662	0.011897
53	218.737	1.677	0.010572
54	190.824	1.733	0.012119
55	198.547	1.658	0.011647
56	191.752	1.662	0.012060
57	205.727	1.659	0.011241
58	177.468	1.644	0.013031
59	178.307	1.669	0.012969
60	173.671	1.610	0.013315
61	175.288	1.697	0.013193
62	177.268	1.674	0.013045
63	173.180	1.692	0.013353
64	168.822	1.683	0.013698

Table 2: web-Stanford hybrid

Threads	MTEPS	StdDev	Time (s)
1	63.536	0.046	0.087087
2	126.437	0.409	0.043763
3	170.751	0.390	0.032405
4	201.307	0.552	0.027486
5	208.959	0.384	0.026480
6	194.068	0.391	0.028512
7	187.587	0.476	0.029497
8	185.714	0.338	0.029794
9	184.917	0.368	0.029923
10	180.929	0.602	0.030582
11	176.582	0.425	0.031335
12	167.351	0.364	0.033064
13	159.158	0.667	0.034766
14	150.570	0.569	0.036749
15	132.198	1.517	0.041856
16	115.717	0.453	0.047817
17	119.304	0.464	0.046379
18	118.957	0.483	0.046514
19	120.476	0.558	0.045928
20	115.477	0.538	0.047916
21	116.835	0.488	0.047359
22	114.730	0.556	0.048228
23	117.491	0.518	0.047095
24	115.521	0.567	0.047898
25	111.179	0.546	0.049768
26	111.659	0.539	0.049554
27	113.130	0.548	0.048910
28	110.632	0.543	0.050015
29	110.502	0.555	0.050073
30	107.941	0.579	0.051261
31	106.466	0.547	0.051972
32	105.837	0.566	0.052281
33	104.126	0.551	0.053140
34	104.394	0.578	0.053003
35	101.750	0.572	0.054381
36	100.766	0.597	0.054911
37	99.318	0.623	0.055712
38	97.549	0.603	0.056723
39	97.685	0.609	0.056643
40	94.682	0.553	0.058440
41	93.789	0.573	0.058997
42	93.690	0.624	0.059059
43	90.454	0.605	0.061171
44	89.147	0.575	0.062068
45	86.353	0.592	0.064077
46	88.771	0.627	0.062331
47	85.654	0.551	0.064600
48	87.244	0.599	0.063423
49	76.117	0.557	0.072693
50	83.374	0.589	0.066366
51	76.310	0.556	0.072510
52	76.364	0.588	0.072458
53	75.438	0.585	0.073348
54	72.748	0.573	0.076061
55	70.582	0.583	0.078394
56	67.867	0.582	0.081530
57	71.462	0.591	0.077429
58	70.448	0.580	0.078543
59	70.590	0.651	0.078385
60	68.157	0.594	0.081183
61	67.993	0.630	0.081379
62	67.695	0.661	0.081737
63	65.103	0.731	0.084991
64	61.135	0.737	0.090509

Table 3: roadNet-CA top-down

Threads	MTEPS	StdDev	Time (s)
1	64.990	0.065	0.085139
2	129.330	0.248	0.042784
3	177.361	0.269	0.031197
4	204.252	0.387	0.027090
5	203.879	0.570	0.027140
6	193.994	0.385	0.028523
7	187.655	0.330	0.029486
8	185.081	0.317	0.029896
9	183.330	0.523	0.030182
10	178.658	0.343	0.030971
11	177.387	0.406	0.031193
12	166.612	0.387	0.033210
13	159.637	0.405	0.034661
14	151.499	0.751	0.036523
15	141.900	0.586	0.038994
16	117.185	0.460	0.047218
17	120.118	0.483	0.046065
18	120.118	0.490	0.046065
19	120.418	0.444	0.045950
20	114.444	0.537	0.048349
21	114.521	0.552	0.048316
22	114.788	0.496	0.048204
23	116.727	0.521	0.047403
24	114.623	0.509	0.048273
25	112.818	0.528	0.049045
26	112.871	0.526	0.049022
27	111.180	0.536	0.049768
28	109.971	0.512	0.050315
29	107.616	0.564	0.051416
30	104.108	0.638	0.053149
31	106.876	0.566	0.051772
32	104.293	0.588	0.053054
33	101.115	0.601	0.054722
34	103.115	0.568	0.053660
35	99.106	0.641	0.055831
36	100.189	0.618	0.055228
37	99.843	0.608	0.055419
38	94.562	0.588	0.058514
39	90.153	0.678	0.061376
40	93.856	0.583	0.058955
41	92.654	0.588	0.059719
42	86.854	0.539	0.063707
43	92.202	0.591	0.060012
44	89.118	0.686	0.062089
45	90.448	0.523	0.061176
46	89.653	0.558	0.061718
47	87.035	0.610	0.063575
48	77.941	0.709	0.070993
49	78.048	0.548	0.070895
50	74.819	0.575	0.073955
51	81.592	0.668	0.067816
52	76.831	0.720	0.072018
53	75.117	0.595	0.073661
54	73.212	0.595	0.075578
55	73.619	0.612	0.075161
56	70.787	0.636	0.078166
57	70.447	0.602	0.078545
58	69.615	0.605	0.079483
59	69.074	0.567	0.080105
60	68.695	0.744	0.080548
61	67.393	0.585	0.082103
62	67.668	0.619	0.081771
63	65.222	0.642	0.084837
64	63.638	0.714	0.086949

Table 4: roadNet-CA hybrid

Threads	MTEPS	StdDev	Time (s)
1	374.800	117.463	0.312660
2	624.144	109.925	0.187753
3	860.299	108.191	0.136214
4	1165.000	108.844	0.100588
5	1379.340	108.573	0.084957
6	1558.860	105.412	0.075173
7	1762.510	106.749	0.066488
8	1988.230	106.858	0.058939
9	2271.510	108.503	0.051589
10	2424.630	108.177	0.048331
11	2623.320	109.033	0.044671
12	2835.580	108.332	0.041327
13	2958.730	108.960	0.039607
14	3178.190	108.092	0.036872
15	3297.570	108.338	0.035537
16	3415.700	108.912	0.034308
17	3563.230	109.133	0.032887
18	3654.710	110.269	0.032064
19	3760.860	109.977	0.031159
20	3937.910	110.243	0.029758
21	3986.240	110.746	0.029397
22	4130.730	109.232	0.028369
23	4314.060	109.433	0.027164
24	4392.380	108.789	0.026679
25	4425.170	109.782	0.026482
26	4502.310	109.851	0.026028
27	4585.530	108.050	0.025555
28	4759.110	109.216	0.024623
29	4757.550	109.650	0.024631
30	4816.790	109.150	0.024328
31	4872.720	108.502	0.024049
32	4853.710	108.059	0.024143
33	4756.910	107.608	0.024635
34	4769.110	108.101	0.024572
35	4788.540	107.560	0.024472
36	4841.110	106.958	0.024206
37	4872.620	107.060	0.024050
38	4905.430	107.290	0.023889
39	4822.300	106.460	0.024301
40	4820.570	106.199	0.024309
41	4951.120	105.491	0.023668
42	4893.700	104.647	0.023946
43	4935.560	105.384	0.023743
44	4978.230	105.226	0.023540
45	5046.930	105.418	0.023219
46	4989.740	103.508	0.023485
47	5026.400	103.785	0.023314
48	5047.380	103.994	0.023217
49	5000.770	104.425	0.023433
50	5090.570	103.988	0.023020
51	5103.990	104.108	0.022960
52	5174.240	104.239	0.022648
53	5121.430	102.051	0.022881
54	5101.020	102.773	0.022973
55	5136.910	102.116	0.022812
56	5235.040	101.863	0.022385
57	5148.830	100.991	0.022759
58	5154.040	102.371	0.022737
59	5220.430	101.220	0.022447
60	5138.460	100.481	0.022805
61	5213.890	100.581	0.022476
62	5250.480	99.694	0.022319
63	5291.940	98.893	0.022144
64	5258.370	98.453	0.022285

Table 5: com-Orkut top-down

Threads	MTEPS	StdDev	Time (s)
1	373.057	117.513	0.314121
2	643.040	110.912	0.182236
3	896.053	108.606	0.130779
4	1067.720	107.202	0.109752
5	1346.180	106.567	0.087050
6	1609.290	107.719	0.072818
7	1756.340	106.820	0.066721
8	2005.570	106.501	0.058430
9	2259.890	108.304	0.051854
10	2459.030	109.469	0.047655
11	2639.670	109.408	0.044394
12	2832.200	108.073	0.041376
13	2974.360	109.310	0.039398
14	3158.630	108.457	0.037100
15	3318.720	108.553	0.035310
16	3445.430	108.648	0.034012
17	3553.300	109.027	0.032979
18	3662.370	110.160	0.031997
19	3793.170	109.977	0.030894
20	3927.410	109.470	0.029838
21	3952.410	110.598	0.029649
22	4130.880	109.023	0.028368
23	4295.710	109.243	0.027280
24	4406.520	109.347	0.026594
25	4436.700	109.910	0.026413
26	4590.460	109.079	0.025528
27	4585.830	108.080	0.025554
28	4719.230	108.332	0.024831
29	4752.230	109.671	0.024659
30	4856.510	109.731	0.024130
31	4926.870	108.206	0.023785
32	4954.900	108.028	0.023650
33	4770.440	108.613	0.024565
34	4661.500	107.291	0.025139
35	4772.790	107.291	0.024553
36	4801.180	107.379	0.024408
37	4866.380	106.407	0.024081
38	4855.870	105.745	0.024133
39	4849.410	105.416	0.024165
40	4832.190	106.005	0.024251
41	4816.090	105.328	0.024332
42	4893.360	103.893	0.023948
43	4950.030	105.676	0.023674
44	4974.720	104.904	0.023556
45	4996.050	105.678	0.023456
46	4932.660	105.349	0.023757
47	5101.800	104.638	0.022969
48	4976.920	102.979	0.023546
49	5069.300	104.111	0.023117
50	5074.080	101.990	0.023095
51	5061.060	102.483	0.023154
52	5199.960	102.921	0.022536
53	5091.570	102.244	0.023016
54	5138.630	102.851	0.022805
55	5190.800	101.316	0.022575
56	5252.690	102.899	0.022309
57	5183.050	101.218	0.022609
58	5022.790	100.109	0.023331
59	5181.450	101.077	0.022616
60	5197.900	100.415	0.022545
61	5175.880	99.762	0.022641
62	5206.630	99.302	0.022507
63	5187.730	99.761	0.022589
64	5280.750	99.110	0.022191

Table 6: com-Orkut hybrid

Threads	MTEPS	StdDev	Time (s)
1	73.368	15.860	0.940379
2	129.099	15.884	0.534426
3	189.150	15.907	0.364756
4	237.009	15.928	0.291102
5	294.273	15.936	0.234455
6	350.570	15.927	0.196805
7	401.378	15.956	0.171892
8	450.747	15.950	0.153065
9	505.560	15.926	0.136470
10	547.939	15.944	0.125915
11	593.313	15.921	0.116286
12	638.654	15.927	0.108030
13	669.242	15.947	0.103092
14	725.609	15.924	0.095084
15	771.965	15.955	0.089374
16	806.785	15.954	0.085517
17	842.310	15.960	0.081910
18	871.806	15.955	0.079139
19	889.950	15.978	0.077525
20	929.610	15.948	0.074218
21	960.994	15.960	0.071794
22	1004.890	15.943	0.068658
23	1042.630	15.918	0.066173
24	1066.500	15.955	0.064692
25	1082.150	15.930	0.063756
26	1090.190	16.003	0.063286
27	1102.190	16.157	0.062597
28	1165.950	16.026	0.059174
29	1171.350	16.210	0.058901
30	1188.210	16.304	0.058065
31	1238.850	15.977	0.055692
32	1240.560	16.827	0.055615
33	1177.110	16.273	0.058613
34	1153.860	16.173	0.059794
35	1187.840	16.084	0.058084
36	1182.360	16.081	0.058353
37	1171.620	15.962	0.058888
38	1186.430	16.082	0.058153
39	1207.940	16.252	0.057117
40	1217.620	15.943	0.056663
41	1217.570	15.949	0.056665
42	1225.980	15.965	0.056276
43	1241.440	15.984	0.055575
44	1242.840	16.032	0.055513
45	1261.010	16.049	0.054713
46	1271.140	15.881	0.054277
47	1302.120	15.946	0.052986
48	1276.290	15.981	0.054058
49	1275.230	16.150	0.054103
50	1282.710	16.071	0.053787
51	1325.230	15.947	0.052062
52	1328.350	16.171	0.051940
53	1344.220	16.008	0.051326
54	1353.660	16.006	0.050968
55	1358.320	16.249	0.050793
56	1392.560	15.938	0.049544
57	1377.200	16.018	0.050097
58	1380.180	15.957	0.049989
59	1396.470	15.992	0.049406
60	1414.640	16.060	0.048771
61	1393.680	15.930	0.049505
62	1421.530	15.936	0.048535
63	1439.490	16.359	0.047929
64	1434.860	16.042	0.048084

Table 7: soc-LiveJournal1 top-down

Threads	MTEPS	StdDev	Time (s)
1	72.396	15.858	0.953010
2	132.750	15.904	0.519727
3	182.132	15.914	0.378813
4	243.076	15.911	0.283836
5	304.884	15.904	0.226295
6	345.457	15.984	0.199717
7	393.294	15.986	0.175425
8	449.555	15.959	0.153471
9	505.052	15.909	0.136607
10	546.486	15.921	0.126250
11	592.072	15.935	0.116529
12	635.937	15.913	0.108492
13	680.524	15.939	0.101383
14	724.712	15.937	0.095202
15	762.634	15.928	0.090468
16	807.839	15.931	0.085405
17	832.076	15.955	0.082918
18	858.285	15.956	0.080386
19	891.944	16.033	0.077352
20	938.725	15.931	0.073497
21	958.702	15.962	0.071966
22	1002.310	15.950	0.068835
23	1025.660	15.951	0.067268
24	1066.580	15.972	0.064687
25	1075.600	15.972	0.064144
26	1101.180	15.972	0.062655
27	1124.620	16.106	0.061349
28	1139.460	16.189	0.060550
29	1159.780	16.267	0.059489
30	1191.140	16.155	0.057923
31	1210.300	16.379	0.057005
32	1260.960	15.989	0.054715
33	1188.940	16.081	0.058030
34	1180.720	16.093	0.058433
35	1178.400	16.101	0.058549
36	1192.410	16.092	0.057861
37	1176.430	16.084	0.058647
38	1200.790	16.075	0.057457
39	1220.570	15.974	0.056526
40	1216.440	16.081	0.056718
41	1225.750	15.956	0.056287
42	1216.670	15.983	0.056707
43	1233.140	16.060	0.055950
44	1249.990	16.090	0.055195
45	1259.810	16.095	0.054765
46	1272.850	15.919	0.054204
47	1294.560	15.966	0.053295
48	1312.450	15.894	0.052569
49	1310.350	15.936	0.052653
50	1299.520	16.051	0.053092
51	1325.920	15.927	0.052035
52	1329.350	15.948	0.051900
53	1346.690	15.941	0.051232
54	1346.130	15.981	0.051254
55	1372.040	16.026	0.050285
56	1373.720	15.938	0.050224
57	1392.900	15.951	0.049533
58	1393.430	16.004	0.049514
59	1393.990	15.973	0.049494
60	1410.330	15.962	0.048920
61	1385.630	16.080	0.049792
62	1417.660	16.436	0.048667
63	1439.560	15.976	0.047927
64	1438.070	16.276	0.047977

Table 8: soc-LiveJournal1 hybrid

Threads	MTEPS	StdDev	Time (s)
1	19.521	4.101	51.228000
2	37.882	5.904	26.397700
3	55.030	5.622	18.171900
4	73.447	7.365	13.615300
5	92.997	5.802	10.753000
6	112.888	6.153	8.858320
7	131.358	6.668	7.612800
8	149.039	6.714	6.709640
9	163.408	7.524	6.119640
10	182.318	6.374	5.484910
11	202.021	6.795	4.949980
12	220.508	6.186	4.534980
13	236.631	6.320	4.225990
14	252.929	6.067	3.953670
15	268.971	6.986	3.717870
16	284.729	6.901	3.512110
17	303.350	5.941	3.296520
18	319.528	8.494	3.129620
19	325.427	6.405	3.072890
20	339.772	6.130	2.943150
21	360.853	6.414	2.771210
22	367.186	7.237	2.723420
23	380.964	6.901	2.624920
24	393.547	9.561	2.540990
25	405.692	6.864	2.464920
26	422.267	7.810	2.368170
27	422.012	8.737	2.369600
28	441.489	7.122	2.265060
29	447.322	7.414	2.235530
30	461.201	10.924	2.168250
31	466.518	13.858	2.143540
32	475.970	9.307	2.100970
33	439.709	50.910	2.274230
34	445.030	42.152	2.247040
35	429.559	40.470	2.327970
36	426.679	30.053	2.343680
37	442.835	35.315	2.258180
38	435.340	28.298	2.297050
39	438.183	22.657	2.282150
40	444.402	17.911	2.250210
41	445.331	15.765	2.245520
42	449.007	12.434	2.227140
43	453.613	14.126	2.204520
44	467.002	13.571	2.141320
45	466.047	10.651	2.145710
46	471.712	12.513	2.119940
47	473.165	10.823	2.113430
48	486.325	9.677	2.056240
49	485.337	9.849	2.060430
50	488.833	8.729	2.045690
51	491.995	9.150	2.032540
52	497.773	8.311	2.008950
53	501.799	6.882	1.992830
54	515.901	7.974	1.938360
55	513.224	6.029	1.948470
56	524.308	6.479	1.907280
57	521.201	6.527	1.918650
58	522.916	6.534	1.912350
59	529.551	7.863	1.888390
60	532.391	7.315	1.878320
61	537.004	6.713	1.862180
62	547.360	5.635	1.826950
63	559.964	6.177	1.785830
64	555.190	6.695	1.801190

Table 9: RMAT1 top-down

Threads	MTEPS	StdDev	Time (s)
1	19.405	4.252	51.533900
2	37.173	4.397	26.900900
3	56.689	5.274	17.640100
4	72.911	5.845	13.715400
5	91.590	5.255	10.918200
6	110.255	5.647	9.069900
7	131.496	5.532	7.604780
8	143.844	7.743	6.951990
9	164.901	6.373	6.064230
10	186.793	6.296	5.353520
11	209.069	5.826	4.783120
12	220.390	6.341	4.537410
13	236.904	5.537	4.221120
14	258.905	8.833	3.862420
15	268.925	6.385	3.718510
16	285.682	5.551	3.500400
17	298.161	6.203	3.353890
18	314.012	7.011	3.184590
19	326.552	4.876	3.062300
20	340.236	5.159	2.939130
21	355.143	5.627	2.815770
22	369.233	5.086	2.708320
23	382.816	5.341	2.612220
24	395.252	6.696	2.530030
25	405.260	7.681	2.467550
26	416.232	5.321	2.402500
27	428.927	6.675	2.331400
28	447.088	5.442	2.236700
29	450.212	5.939	2.221180
30	460.474	6.404	2.171680
31	467.372	7.200	2.139620
32	477.293	8.373	2.095150
33	443.396	42.776	2.255320
34	440.649	46.564	2.269380
35	438.144	41.855	2.282350
36	432.953	35.810	2.309720
37	438.746	31.028	2.279220
38	440.853	29.715	2.268330
39	437.560	16.899	2.285400
40	444.561	17.321	2.249410
41	456.863	16.253	2.188840
42	458.977	14.721	2.178760
43	457.942	16.495	2.183680
44	462.043	12.828	2.164300
45	474.045	12.774	2.109500
46	471.393	10.527	2.121370
47	477.164	9.980	2.095720
48	481.110	9.889	2.078530
49	485.820	10.431	2.058380
50	490.257	8.383	2.039750
51	492.697	8.470	2.029650
52	498.862	6.801	2.004560
53	503.809	7.560	1.984880
54	507.818	8.116	1.969210
55	520.598	6.238	1.920870
56	524.687	5.863	1.905900
57	520.516	6.437	1.921170
58	526.881	7.792	1.897960
59	529.717	6.733	1.887800
60	543.165	6.287	1.841060
61	538.093	7.278	1.858410
62	543.672	6.763	1.839350
63	546.852	8.610	1.828650
64	549.926	9.015	1.818430

Table 10: RMAT1 hybrid

Threads	MTEPS	StdDev	Time (s)
1	33.631	5.728	29.734600
2	64.591	4.924	15.482000
3	93.858	5.799	10.654400
4	125.042	6.145	7.997300
5	155.562	8.427	6.428300
6	181.753	6.893	5.501970
7	210.118	6.738	4.759230
8	242.362	7.420	4.126050
9	274.229	7.576	3.646590
10	298.145	7.627	3.354080
11	332.420	7.867	3.008240
12	355.765	8.601	2.810850
13	383.835	8.600	2.605280
14	411.716	9.247	2.428860
15	441.416	9.248	2.265440
16	468.322	7.769	2.135290
17	483.065	6.301	2.070110
18	505.657	5.385	1.977630
19	522.136	8.047	1.915210
20	552.291	6.851	1.810640
21	564.488	7.204	1.771520
22	592.298	7.469	1.688340
23	610.758	7.283	1.637310
24	631.754	8.512	1.582890
25	641.516	6.797	1.558810
26	664.169	7.228	1.505640
27	682.696	8.228	1.464780
28	698.462	8.889	1.431720
29	715.788	13.046	1.397060
30	735.666	13.114	1.359310
31	751.906	13.634	1.329950
32	777.056	6.405	1.286910
33	687.974	48.334	1.453540
34	707.511	44.225	1.413410
35	710.919	40.403	1.406630
36	708.867	34.397	1.410700
37	727.803	30.832	1.374000
38	714.639	24.212	1.399310
39	722.558	23.270	1.383970
40	738.296	18.412	1.354470
41	747.254	17.272	1.338230
42	736.403	15.642	1.357950
43	757.945	14.966	1.319360
44	768.970	13.368	1.300440
45	763.677	13.478	1.309450
46	771.239	13.650	1.296610
47	793.869	14.521	1.259650
48	785.466	13.182	1.273130
49	791.969	13.570	1.262670
50	816.581	12.408	1.224620
51	805.759	13.210	1.241070
52	813.184	12.131	1.229730
53	822.550	11.978	1.215730
54	846.856	10.640	1.180840
55	839.318	10.988	1.191440
56	849.272	9.618	1.177480
57	852.981	11.281	1.172360
58	874.574	9.039	1.143410
59	862.164	12.621	1.159870
60	858.634	10.343	1.164640
61	880.983	11.538	1.135100
62	894.467	10.550	1.117980
63	902.116	8.968	1.108500
64	906.489	8.551	1.103160

Table 11: RMAT2 top-down

Threads	MTEPS	StdDev	Time (s)
1	33.869	6.809	29.525600
2	65.033	5.010	15.376700
3	94.976	8.900	10.529000
4	127.999	5.783	7.812570
5	152.636	8.116	6.551550
6	182.181	6.990	5.489030
7	211.679	10.552	4.724130
8	238.853	7.863	4.186670
9	268.520	7.463	3.724120
10	299.450	9.616	3.339450
11	333.508	6.943	2.998430
12	356.718	8.575	2.803340
13	383.905	9.623	2.604810
14	420.160	7.557	2.380050
15	441.039	6.586	2.267370
16	460.566	7.114	2.171240
17	479.232	6.768	2.086670
18	499.446	9.927	2.002220
19	528.590	6.636	1.891830
20	549.304	5.150	1.820480
21	568.343	8.030	1.759500
22	596.583	9.700	1.676210
23	611.469	7.889	1.635410
24	628.815	7.368	1.590290
25	652.038	7.374	1.533650
26	670.253	9.278	1.491970
27	677.253	9.180	1.476550
28	692.304	7.885	1.444450
29	713.981	10.841	1.400600
30	729.301	12.257	1.371180
31	745.130	13.383	1.342050
32	766.532	12.803	1.304580
33	738.898	32.456	1.353370
34	697.656	39.622	1.433370
35	727.388	35.850	1.374780
36	718.325	31.497	1.392130
37	720.785	31.785	1.387380
38	723.040	22.489	1.383050
39	721.951	20.845	1.385140
40	723.720	18.325	1.381750
41	725.700	17.619	1.377980
42	749.951	17.433	1.333420
43	762.633	16.243	1.311250
44	752.072	15.265	1.329660
45	776.353	12.062	1.288070
46	772.593	14.492	1.294340
47	795.267	12.049	1.257440
48	800.975	10.865	1.248480
49	792.194	10.457	1.262320
50	801.280	11.542	1.248000
51	808.875	11.703	1.236290
52	814.829	12.525	1.227250
53	838.536	9.556	1.192550
54	832.869	11.355	1.200670
55	839.701	11.893	1.190900
56	848.771	12.366	1.178170
57	853.224	8.522	1.172030
58	860.623	11.928	1.161950
59	884.984	10.320	1.129960
60	879.950	11.723	1.136430
61	887.108	8.797	1.127260
62	892.379	12.940	1.120600
63	904.162	9.922	1.106000
64	912.044	11.589	1.096440

Table 12: RMAT2 hybrid

Threads	MTEPS	StdDev	Time (s)
1	139.513	1000.320	7.167770
2	252.527	1003.070	3.959970
3	337.229	1005.230	2.965350
4	451.668	1003.950	2.214020
5	541.001	1004.480	1.848430
6	610.628	1003.630	1.637660
7	694.811	1003.710	1.439240
8	781.503	1002.920	1.279580
9	876.805	1001.840	1.140500
10	952.196	1003.240	1.050200
11	1018.720	1003.000	0.981620
12	1089.440	1002.610	0.917900
13	1193.460	1002.250	0.837900
14	1259.430	1003.930	0.794011
15	1315.880	1001.670	0.759946
16	1365.860	1003.260	0.732137
17	1420.220	1003.500	0.704116
18	1478.600	1002.440	0.676316
19	1521.710	1002.900	0.657156
20	1548.770	1002.990	0.645675
21	1577.380	1003.010	0.633961
22	1649.840	1003.630	0.606118
23	1697.240	1003.040	0.589192
24	1732.230	1002.900	0.577289
25	1762.480	1002.970	0.567382
26	1791.110	1002.490	0.558312
27	1824.490	1002.930	0.548099
28	1876.980	1002.370	0.532769
29	1887.920	1003.850	0.529684
30	1936.170	1002.430	0.516484
31	1972.520	1003.480	0.506966
32	1990.610	1002.780	0.502359
33	1906.020	1002.060	0.524654
34	1930.320	1003.340	0.518050
35	1948.260	1002.440	0.513278
36	1964.180	1002.170	0.509119
37	2016.020	1003.000	0.496027
38	2027.320	1002.500	0.493262
39	2006.090	1002.290	0.498483
40	2046.920	1002.620	0.488540
41	2023.780	1002.050	0.494125
42	2028.330	1001.880	0.493016
43	2040.120	1001.850	0.490168
44	2059.840	1001.720	0.485474
45	2084.010	1000.980	0.479845
46	2098.670	1000.630	0.476492
47	2104.850	1001.660	0.475093
48	2113.140	1001.140	0.473229
49	2139.110	1001.750	0.467484
50	2137.420	1001.370	0.467855
51	2153.900	1001.380	0.464275
52	2179.400	1001.420	0.458843
53	2168.810	1001.780	0.461083
54	2198.700	1001.570	0.454815
55	2204.670	1001.330	0.453582
56	2209.730	1001.580	0.452544
57	2245.260	1000.970	0.445383
58	2236.450	1001.230	0.447137
59	2240.410	1000.910	0.446347
60	2261.870	1000.940	0.442112
61	2267.040	1000.780	0.441103
62	2293.060	1000.680	0.436099
63	2276.030	1001.450	0.439362
64	2249.800	1001.120	0.444484

Table 13: RMAT3 top-down

Threads	MTEPS	StdDev	Time (s)
1	138.810	1000.310	7.204090
2	249.458	1002.880	4.008690
3	348.716	1005.210	2.867660
4	435.654	1003.930	2.295400
5	533.869	1004.640	1.873120
6	631.060	1003.530	1.584630
7	698.035	1003.340	1.432590
8	784.288	1003.140	1.275040
9	876.542	1001.580	1.140850
10	955.088	1003.530	1.047020
11	1029.560	1003.310	0.971290
12	1102.040	1002.870	0.907406
13	1199.610	1002.690	0.833607
14	1265.200	1003.550	0.790387
15	1326.630	1001.850	0.753789
16	1374.710	1003.250	0.727428
17	1416.750	1003.640	0.705840
18	1476.330	1002.390	0.677356
19	1511.980	1003.040	0.661385
20	1551.910	1003.040	0.644367
21	1579.690	1003.590	0.633036
22	1656.080	1003.500	0.603837
23	1693.140	1003.120	0.590620
24	1738.920	1002.870	0.575070
25	1757.990	1002.540	0.568831
26	1779.710	1002.810	0.561889
27	1799.170	1003.140	0.555813
28	1874.260	1002.500	0.533544
29	1890.510	1003.490	0.528957
30	1917.300	1002.420	0.521568
31	1953.750	1002.990	0.511836
32	2021.600	1002.330	0.494657
33	1947.280	1003.110	0.513536
34	1931.590	1002.790	0.517709
35	1980.110	1002.320	0.505022
36	1978.090	1002.210	0.505538
37	1998.030	1001.590	0.500494
38	1994.150	1002.460	0.501466
39	2005.670	1001.620	0.498587
40	2053.710	1001.400	0.486923
41	2031.730	1001.750	0.492193
42	2045.390	1001.540	0.488905
43	2045.070	1002.150	0.488980
44	2064.210	1001.490	0.484447
45	2081.490	1001.290	0.480425
46	2116.370	1001.030	0.472508
47	2130.450	1000.720	0.469384
48	2148.450	1000.980	0.465453
49	2128.430	1001.540	0.469830
50	2151.990	1001.260	0.464687
51	2167.140	1000.810	0.461438
52	2187.330	1001.330	0.457179
53	2202.550	1001.100	0.454019
54	2186.060	1001.250	0.457445
55	2206.860	1001.410	0.453133
56	2240.150	1001.280	0.446399
57	2244.650	1000.960	0.445504
58	2240.740	1001.420	0.446282
59	2253.350	1000.660	0.443784
60	2277.890	1001.100	0.439003
61	2264.970	1000.260	0.441507
62	2295.820	1001.230	0.435574
63	2307.070	1000.970	0.433450
64	2279.100	1000.650	0.438770

Table 14: RMAT3 hybrid