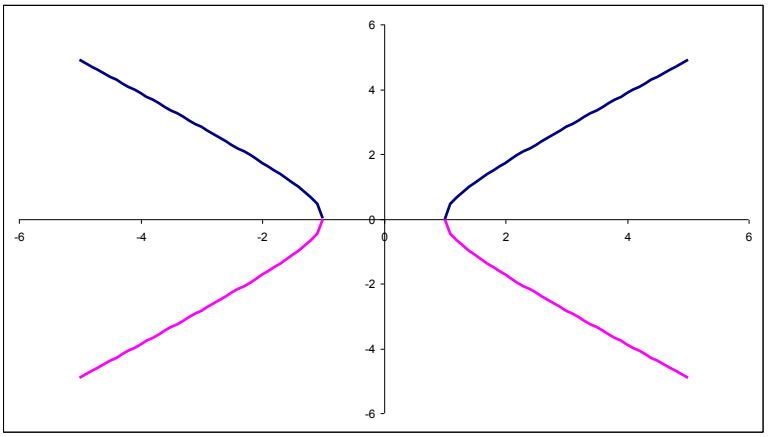
Definice funkce.

1. $f = \{[x,z]; x^2 - y^2 - 1 = 0\}$, $y^2 = x^2 - 1$, $|y| = \sqrt{x^2 - 1}$, $x \in (-\infty,-1> \cup <1,+\infty)$



Pro každé $x \in (-\infty,-1) \cup (1,+\infty)$ existují $y_1 = \sqrt{x^2 - 1}$, $y_2 = -\sqrt{x^2 - 1}$ a je při tom splněno $y_1^2 = x^2 - 1 = y_2^2$.

Na množině $x \in (-\infty,-1) \cup (1,+\infty)$ není f funkce.

2. $f(x) = x^2 - 5x + 6$.

Jedná se o funkci? Určete množiny, kde je funkce prostá a na nich definujte funkci inverzní.

Nechť f není funkce na svém definičním oboru, $y_1 = x_1^2 - 5x_1 + 6$, $y_2 = x_2^2 - 5x_2 + 6$, nechť $y_1 \neq y_2$ a $x_1 = x_2$. Pak $x_1^2 - 5x_1 + 6 = x_2^2 - 5x_2 + 6$ neboli $y_1 = y_2$, což je spor.

f je funkce v celém svém definičním oboru, tj. pro všechna reálná x.

f je prostá, jestliže pro $x_1 \neq x_2$ je $y_1 \neq y_2$.

Jedná se polynom 2. řádu s kořeny $x_1 = 2$, $x_2 = 3$. Funkce je prostá na intervalech $(-\infty,5/2)$ a $(5/2,+\infty)$.

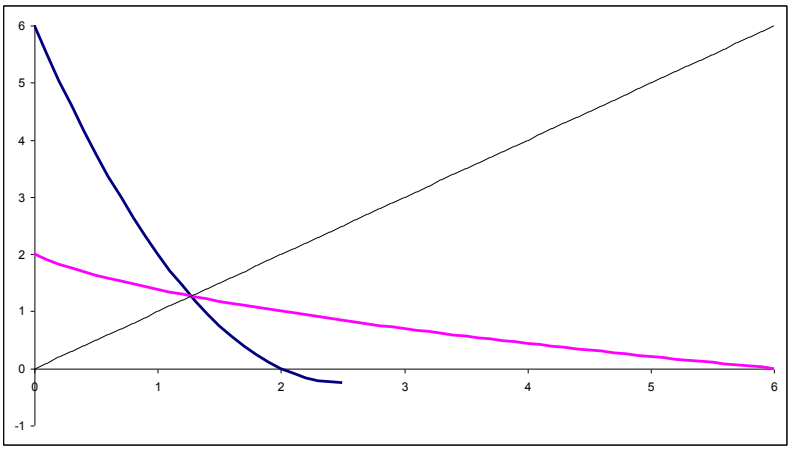
$y \geq -1/4$

$y = x^2 - 5x + 6$, $y - 6 = x^2 - 5x$, $y - 6 + 25/4 = x^2 - 5x + 25/4$, $y + 1/4 = (x - 5/2)^2$

$\sqrt{y + 1/4} = |x - 5/2|$.

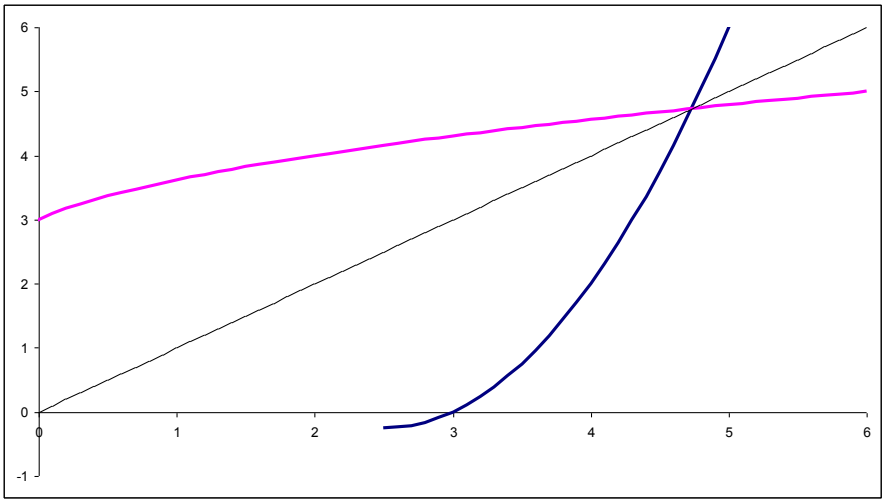
Pro $x \geq 5/2$ je $x = 5/2 + \sqrt{y + 1/4}$, pro $x \leq 5/2$ je $x = 5/2 - \sqrt{y + 1/4}$.

Inverzní funkce $f^{-1}(x) = 5/2 - \sqrt{x + 1/4}$, $D(f^{-1}) = <-1/4,+\infty)$, $H(f^{-1}) = (-\infty, 5/2>$,



nebo

$f^{-1}(x) = 5/2 + \sqrt{x + 1/4}$, $D(f^{-1}) = <-1/4, +\infty)$, $(f^{-1}) = < 5/2, +\infty)$.



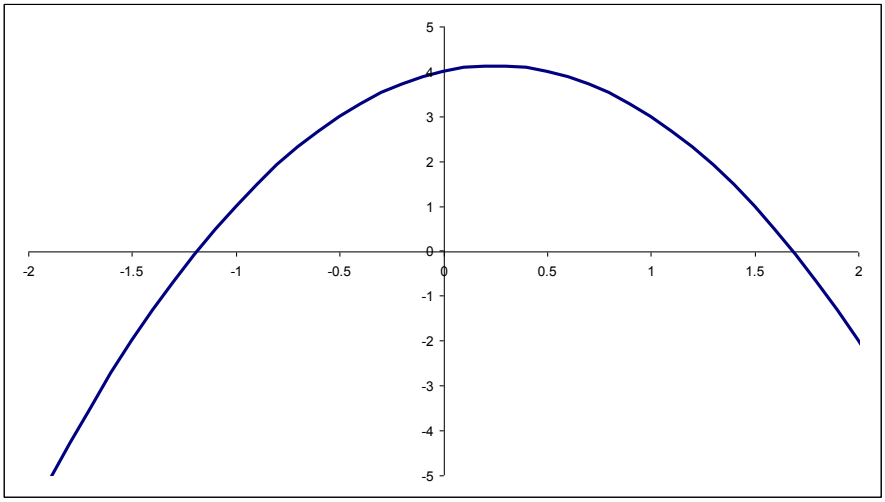
3. $f(x) = x^2 - 5x + 6$, $x \geq 5/2$

Pro každou dvojici x_1, x_2 platí: $x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$ Funkce je rostoucí na intervalu <5/2, +∞).

4. Nakreslete graf funkce $f(x) = -2x^2 + x + 4$

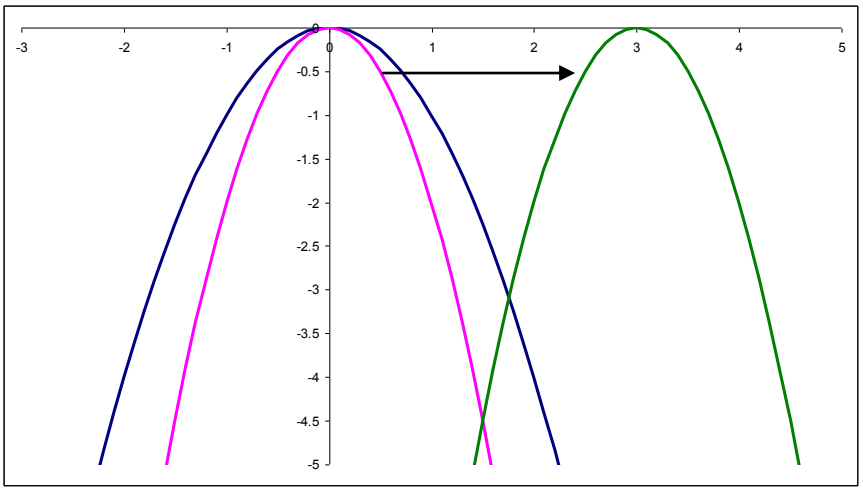
D = 33, $x_1 = \frac{-1+\sqrt{33}}{-4} = \frac{1}{4} - \frac{\sqrt{33}}{4}$, $x_2 = \frac{-1-\sqrt{33}}{-4} = \frac{1}{4} + \frac{\sqrt{33}}{4}$,

$f(0) = 4$



5. Nakreslete graf funkce $f(x) = -2x^2 + 12x - 18$

$f(x) = -2(x-3)^2$. Oproti funkci $f(x) = -x^2$ je parabola „užší“ a posunuta o 3 jednotky po ose x vpravo.

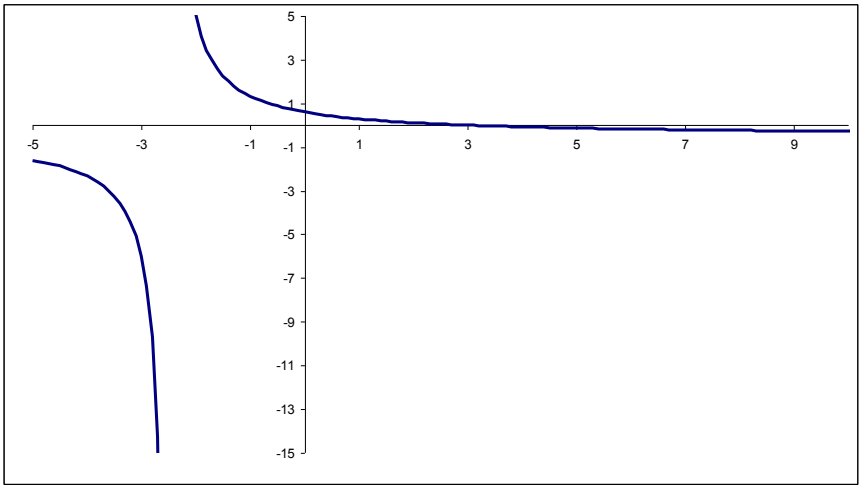


6. Nakreslete graf funkce $f(x) = \dfrac{3-x}{2x+5}$.

$x \neq -5/2$

Asymptoty: x = -5/2, y = -1/2.

$x = 0 \Rightarrow y = 3/5 > -1/2 \Rightarrow$ větve hyperboly umístěny v 1. a 3. kvadrantu.

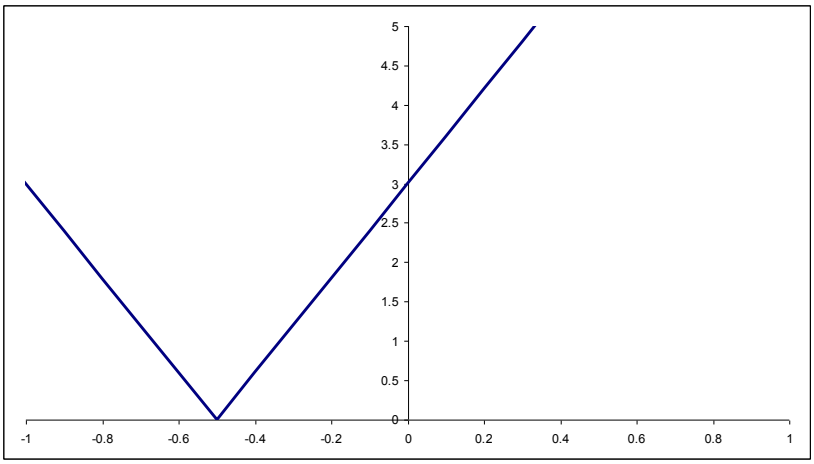


7. Nakreslete graf funkce $f(x) = 3|2x+1|$

Nulový bod x = - 1/2.

$x \geq -1/2,\ f(x) = 6x+3,\ f(0) = 3,\ f(x) = 0 \Leftrightarrow x = -1/2$

$x \leq -1/2,\ f(x) = -6x-3,\ f(0) = -3,\ f(x) = 0 \Leftrightarrow x = -1/2$



8. Nakreslete graf funkce $f(x)=2|x+2|-3|2x+1|-5$

Nulové body: -2, -1/2

	(- ∞, -2 >	<- 2, -1/2 >	<- 1/2, +∞)
$x+2$	-	+	+
$2x+1$	-	-	+

$x\in(-\infty,-2>$

$f(x)=-2x-4+6x+3-5=4x-6$, $f(0)=-6$, $f(x)=0\Leftrightarrow x=3/2$, $f(-2)=-14$

$x\in<-2,-1/2>$

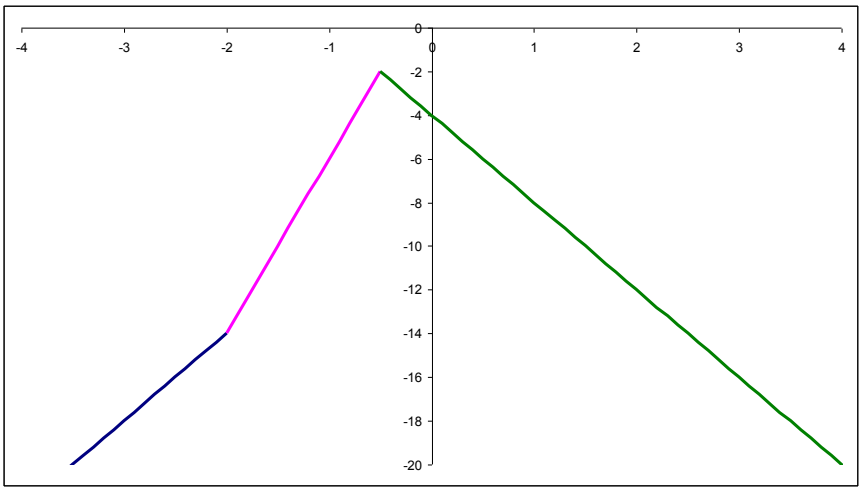
$f(x)=2x+4+6x+3-5=8x+2$,

$f(0)=2$, $f(x)=0\Leftrightarrow x=-1/4$, $f(-2)=-14\Rightarrow$navazuje, $f(-1/2)=-2$

$x\in<-1/2,+\infty)$

$f(x)=2x+4-6x-3-5=-4x-4$, $f(0)=-4$, $f(x)=0\Leftrightarrow x=-1$,

$f(-1/2)=-2\Rightarrow$navazuje



9. Nakreslete graf funkce $f(x)=2|(x+2)(x-1)|-3|(2x+1)x|-5x^2+1$

Nulové body: -2, -1/2, 0, 1

	(- ∞, -2 >	<- 2, -1/2 >	<- 1/2, 0 >	<0, 1 >	<1, +∞)
$(x+2)(x-1)$	+	-	-	-	+
$x(2x+1)$	+	+	-	+	+

$x \in (-\infty,-2> \cup <1,+\infty)$

$f(x) = 2x^2 + 2x - 4 - 6x^2 - 3x - 5x^2 + 1 = -9x^2 - x - 3$

D < 0 nemá kořeny

f(-2) = -37, f(1) = -13

$x \in <-2,-1/2> \cup <0,1>$

$f(x) = -2x^2 - 2x + 4 - 6x^2 - 3x - 5x^2 + 1 = -13x^2 - 5x + 5$

D = 285, x_1 = -1.68323, x_2 = 0.913996

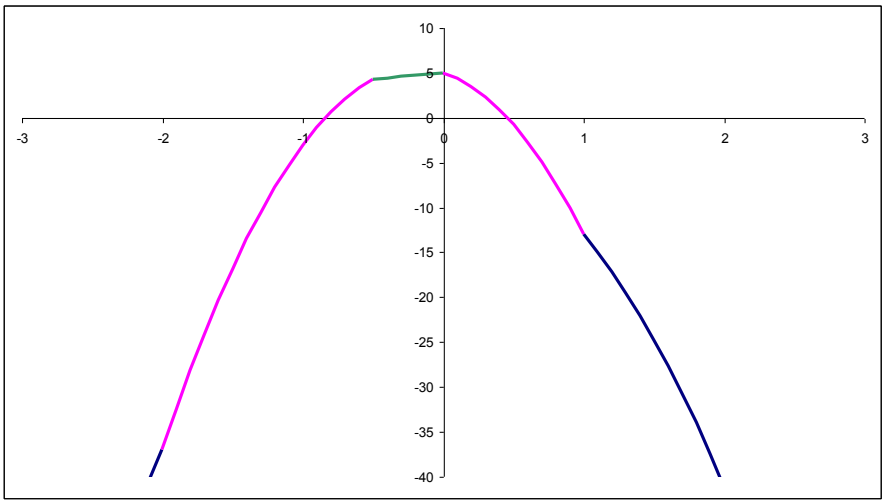
f(-2) = -37, f(1) = -13 $\Rightarrow$ navazuje, f(-1/2) = 17/4, f(0) = 5

$x \in <-1/2,0>$

$f(x) = -2x^2 - 2x + 4 + 6x^2 + 3x - 5x^2 + 1 = -x^2 + x + 5$

D = 21, x_1 = -1.79129, x_2 = 2.791288

f(-1/2) = 17/4, f(0) = 5$\Rightarrow$ navazuje



10. Nakreslete graf funkce $f(x) = 2|(x+2)(x-1)| + 1$

Nulové body: -2, 1

	(- ∞, -2 >	<- 2, 1 >	<1, +∞)
$(x+2)(x-1)$	+	-	+

$x \in (-\infty,-2> \cup <1,+\infty)$

$f(x) = 2x^2 + 2x - 4 + 1 = 2x^2 + 2x - 3$

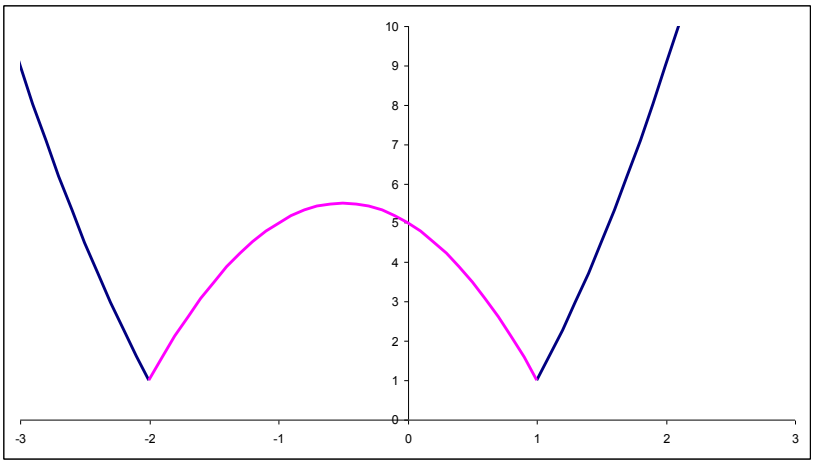
D = 28, x_1 = 0.822876, x_2 = -1.82288, f(-2) = 1, f(1) = 1

$x \in <-2,1>$

$f(x) = -2x^2 - 2x + 4 + 1 = -2x^2 - 2x + 5$

D = 44, x_1 = 1.158312, x_2 = -2.15831

f(-2) = 1, f(1) = 1$\Rightarrow$ navazuje, f(0) = 5



11. Nakreslete graf funkce $f(x) = \dfrac{|1-x|}{2|x|+3}$

Nulové body: 0, 1

	(- ∞, 0>	<0,1 >	<1, +∞)
1-x	+	+	-
x	-	+	+

$x \in (-\infty, 0>$

$f(x) = \dfrac{1-x}{3-2x}$, asymptoty $x = 3/2$, $y = 1/2$, $x = 0 \Rightarrow f(0) = 1/3 < 1/2 \Rightarrow$ 1. a 3. kvadrant

$x \in <0,1>$

$f(x) = \dfrac{1-x}{3+2x}$, asymptoty $x = -3/2$, $y = -1/2$, $x = 0 \Rightarrow f(0) = 1/3 > -1/2 \Rightarrow$ 1. a 3. kvadrant,

$f(1) = 0$

$x \in <1,+\infty)$

$f(x) = \dfrac{x-1}{3+2x}$, asymptoty $x = -3/2$, $y = -1/2$, $x = 1 \Rightarrow f(1) = 0 \Rightarrow$ navazuje,

$f(0) = -1/3 > -1//2 \Rightarrow$ 2. a 4. kvadrant