

# Derivation

## Meaning

$dy/dx$  . where  $y = f(x)$  &  $x$  is known as independent function and  $y$  is dependent to function  $x$ , so we differentiate  $y$  with respect to  $x$ .

Example :  $y = 2, d(2)/dx = 0$

Example :  $y = 3, d(3)/dx = 0$

## Types of functions

Constant function	The derivative of a constant function is always 0	This will be similar for all the values of $y$ where $y$ equals to a constant value.
Polynomial functions	<p>The derivative of polynomial function where, the variable has a power which is constant. So what does this mean?</p> <p>Let's assume the variable <math>y</math>, which is to be differentiated with respect to <math>x</math>, and the value of <math>y = x^4</math>, so</p> <p><math>d(x^n)/dx</math> , where <math>y = x^n = n \cdot x^{n-1}</math></p> <p><math>d(x^4)/dy = 4 \cdot x^{(4-1)}</math> <math>4x^3</math></p> <p>1. Here the power was brought at the beginning of the function and multiplied .</p> <p>2. The constant value 4 at the power, 1 was subtracted</p> <p>3. So, finally after both the steps : <math>4x^3</math>.</p>	Example : $y = 5x^4, d(5x^4)/dx = 5 \cdot (x^4-1) = 5x^3$ .
Trigonometrical functions	<p>Unlike Constant functions &amp; polynomial functions , trigonometric functions have some formulaes, which makes it handy for students</p> <p><math>d(\sin x)/dx = \cos x</math> <math>d(\cos x)/dx = -\sin x</math> <math>d(\tan x)/dx = \sec^2 x</math> <math>d(\csc x)/dx = -\operatorname{cosec}^2 x</math> <math>d(\sec x)/dx = \sec x \cdot \tan x</math> <math>d(\operatorname{cosec} x)/dx = -\operatorname{cosec} x \cdot \cot x</math></p>	<p>Example : <math>y = 2\cos x, d(2\cos x)/dx</math></p> <p><math>\Rightarrow 2 \cdot d(\cos x)/dx</math></p> <p><math>\Rightarrow 2 \cdot (-\sin(x))</math></p> <p><math>\Rightarrow -2\sin x</math></p>
Exponential functions	The exponential function $e^x$ is never affected with differentiation, $d(e^x)/dx = e^x$	
Logarithmic Functions	The logarithmic functions derivatives are $\ln x$ or $\log x$ with base $e$ is always equal to $1/x$	

## Product Rule

When two types of functions are multiplied, what do we do? We have rules for individual functions but what now?

For  $d(y)/dx$  , where  $y = u \cdot v$ , where  $u$  &  $v$  are two different functions , the formula will be:

$$d(uv)/dx = du/dx \cdot v + dv/dx \cdot u$$

1. We (differentiate the first functions) and multiply the value of second function then + add it with ( the differentiation of the second function multiplied by first function)

Example : Differentiate  $x \sin x$

$$d(x)/dz \cdot \sin x + d(\sin x)/dz \cdot x$$

$$\Rightarrow 1 \cdot \sin x + \cos(x) \cdot x$$

$$\Rightarrow \sin x + x \cos(x)$$

Don't get confused why "dz" has been used not "dx" because we already have the value  $x$  in the function, so I did not mean to use it this time, instead used dz.

## Quotient Rule

What do we do when two functions are divided and we need to differentiate them??

For  $d(y)/d(x)$  , where  $y = u/v$

1. What we can do is  $y = uv^{-1}$  and solve the same as above product rule, but if we dont want to use it, we can use the quotient rule :

$$dy/dx = [ d(u)/d(x) \cdot v - d(v)/d(x) \cdot u ] / v^2$$

Example : Differentiate  $\sin x / x$

$$\Rightarrow [ d(\sin x)/dz \cdot x - d(x)/dz \cdot \sin x ] / x^2$$

$$\Rightarrow [ \cos x \cdot x - \sin(x) \cdot 1 ] / x^2$$

## Rules

## Chain Rule

When two functions are embedded within the arguments/parameters of each other we need to apply the chain rule.

Example : Differentiate  $dy/dx$  , where  $y = \sin(\log x)$

$\Rightarrow$  At first we differentiate the value of  $\sin(\log x)$  which would be  $\cos(\log x)$  , remember not to differentiate  $\log(x)$  at once because here  $\log(x)$  is the value for  $\sin$ ,

$$\Rightarrow \text{Then, } \cos(\log x) \cdot d(\log x)/dx$$

$$\Rightarrow \cos(\log x) \cdot 1/x \text{ ( as differentiating } \log x \text{ will result into } 1/x \text{)}$$

$$\Rightarrow \cos \log x / x \text{ ( final answer)}$$