Exercise 1. Context Free Languages:

Consider the language $L = \{0^n1^n \mid n \geq 0\}$.

1. Prove that L is context-free.

2. Construct a context-free grammar for L.

Solution.

1. To prove that L is context-free, we can construct a pushdown automaton (PDA) that recognizes it. The PDA's stack will keep track of the number of 0s encountered before encountering 1s. For each 0 encountered, we push a symbol onto the stack, and for each 1 encountered, we pop a symbol from the stack. We accept if the stack is empty when all symbols are read. Since we can construct a PDA for L, L is context-free.

2. A context-free grammar G for L can be defined as follows:

$$S \to 0S1$$
$$S \to \varepsilon$$

where S is the start symbol, and ε represents the empty string. These production rules generate strings of the form 0^n1^n, where $n \geq 0$. So, the context-free grammar G generates the language $L = \{0^n1^n \mid n \geq 0\}$.