1 "Linear" gravity

Force is:

$$F = \frac{Gm_1m_2}{r} \tag{1}$$

Energy is:

$$E = \int F dr = Gm_1m_2 \ln(r) \tag{2}$$

Vis-viva equation becomes (divided through by m_2 so we substitute $m_1 = m$ for simplicity):

$$\epsilon = \frac{v_a^2}{2} + Gm\ln(r_a) = \frac{v_p^2}{2} + Gm\ln(r_p) \tag{3}$$

Rearranging:

$$\frac{v_a^2}{2} - \frac{v_p^2}{2} = Gm[\ln(r_p) - \ln(r_a)] \tag{4}$$

Because of reasons $h = r_pv_p = r_av_a =$ constant, thus $v_p = \frac{r_a}{r_p}v_a$:

$$\frac{1}{2}\Big(\frac{r_p^2 - r_a^2}{r_p^2}\Big)v_a^2 = Gm\ln\Big(\frac{r_p}{r_a}\Big) \tag{5}$$

Isolate kinetic energy at apoapsis because we can I guess:

$$\frac{1}{2}v_a^2 = Gm\ln\left(\frac{r_p}{r_a}\right)\left(\frac{r_p^2}{r_p^2 - r_a^2}\right) \tag{6}$$

Specific orbital energy thing was:

$$\epsilon = \frac{v^2}{2} + Gm\ln(r) = \frac{v_a^2}{2} + Gm\ln(r_a) \tag{7}$$

substitute the apoapsis energy guy in:

$$\frac{v^2}{2} + Gm\ln(r) = Gm\ln\left(\frac{r_p}{r_a}\right)\left(\frac{r_p^2}{r_p^2 - r_a^2}\right) + Gm\ln(r_a) \tag{8}$$

Isolate and simplify:

$$v^2 = 2Gm\left[\ln\left(\frac{r_p}{r_a}\right)\left(\frac{r_p^2}{r_p^2 - r_a^2}\right) + \ln\left(\frac{r_a}{r}\right)\right] \tag{9}$$

Singularity when $r_p = r_a$, limit is:

$$v^2 = 2Gm\left[\ln\left(\frac{r_a}{r}\right) + \frac{1}{2}\right] \tag{10}$$

No eccentricity means $r_a = r$:

$$v = \sqrt{Gm} \tag{11}$$